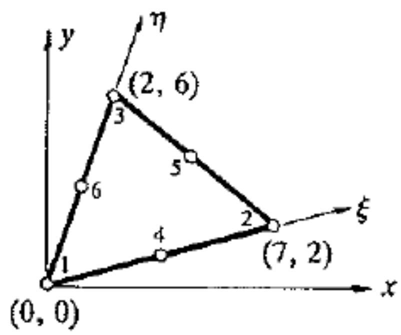
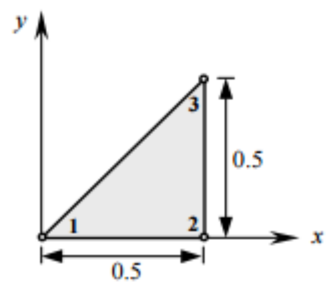
TEST-2

Paper VIII.1 "Fluidity in nature: Computational Interpretations"

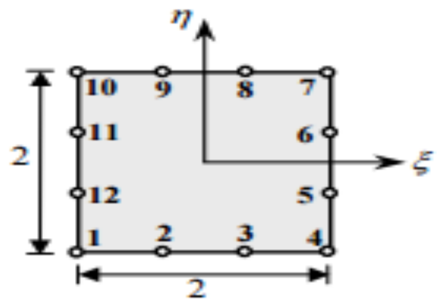
1. Consider the quadratic triangular element shown in the figure. Evaluate the integral of the product $\left(\frac{\partial \psi_1}{\partial x}\right)\left(\frac{\partial \psi_1}{\partial y}\right)$ numerically. Which formulation you will use in this case, sub-parametric or iso-parametric, give justifications in support of your answer? **(5 marks)**



2. If the nodal values of the triangular element shown in figure given below are $u_1=0.2645$, $u_2=0.2172$, $u_3=0.1800$, then compute 'u' at the point $(x, y) = (0.375, 0.375)$. **(5 marks)**



3. Calculate shape functions at the nodes 12 and 6 of the following quadratic rectangular element in terms of the natural co-ordinate system $-1 \leq \xi,\ \eta \leq 1$. **(5 marks)**



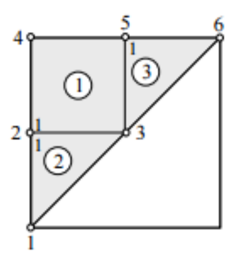
4. Consider the steady state heat transfer in a square region as shown in the figure. The governing equation is given by; **(10 marks)**

$$-\frac{\partial}{\partial x}\left(k\frac{\partial u}{\partial x}\right)-\frac{\partial}{\partial y}\left(k\frac{\partial u}{\partial y}\right)=f_0$$

The boundary conditions for the problem are;

$$u(0,y)=y^2,\ \ u(x,0)=x^2,\quad u(1,y)=1-y,\quad u(x,1)=1-x$$

Assuming k=1 and f_0=2, determine the unknown nodal values of 'u' using the mesh of a rectangle and two triangles, use the concept of symmetry about the line y=x while solving the problem.



You can directly use the stiffness matrix for this question. Element matrices for linear rectangular and triangular elements for above governing equation are as follows;

$$K^e_{ij}=\frac{k}{6}\begin{bmatrix}4 & -1 & -2 & -1\\ -1 & 4 & -1 & -2\\ -2 & -1 & 4 & -1\\ -1 & -2 & -1 & 4\end{bmatrix},\quad f_i^e=\frac{1}{4}f_0ab\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}^T,$$

$$K^e_{ij}=\frac{1}{2}\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix},\quad f_i^e=\frac{f_0ab}{6}\begin{bmatrix}1\\1\\1\end{bmatrix}\quad (a, b \textit{ are the element length in } x, y \textit{ direction respectively.})$$