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Q.1

$$\frac{d}{dx}\left(a\frac{du}{dx}\right) + \frac{d^2}{dx^2}\left(b\frac{d^2u}{dx^2}\right) + u - x^2 = 0$$

$0 < x < 1$

$$0 = \int_0^1 w\left[-\frac{d}{dx}\left(a\frac{du}{dx}\right) + \frac{d^2}{dx^2}\left(b\frac{d^2u}{dx^2}\right) + u - x^2\right]dx$$

$$0 = \int_0^1 \left[\frac{dw}{dx}\left(a\frac{du}{dx}\right) - \frac{dw}{dx}\frac{d}{dx}\left(b\frac{d^2u}{dx^2}\right) + cwu - wx^2\right]dx$$

$$+ \left[-w\left(a\frac{du}{dx}\right)\right]_0^1 + \left[w\cdot\frac{d}{dx}\left(b\frac{d^2u}{dx^2}\right)\right]\Big|_0^1$$

$$0 = \int_0^1 \frac{dw}{dx}\left(a\frac{du}{dx}\right) + \frac{d^2w}{dx^2}\left(\frac{bd^2u}{dx^2}\right) + cwu$$

$$- wx^2\Big]dx + \left\{w\left[-a\frac{du}{dx} + \frac{d}{dx}\left(b\frac{d^2u}{dx^2}\right)\right]\right\}_0^1$$

$$+ \left[\frac{dw}{dx}\cdot b\frac{d^2u}{dx^2}\right]_0^1$$

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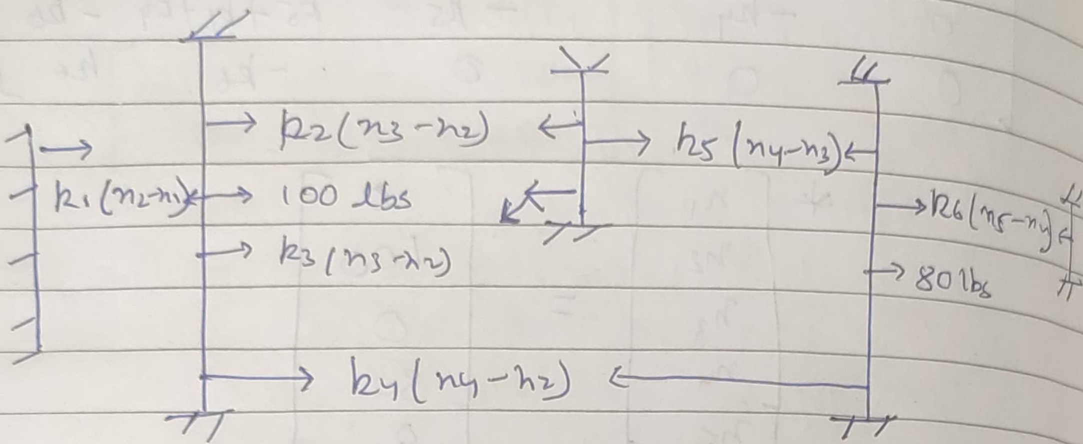
$$K_{ij} = \int_0^1 \left[\frac{d\psi_i}{dx} \left(a \frac{d\psi_j}{dx} \right) + \frac{d^2\psi_i}{dx^2} \left(b \cdot \frac{d^2\psi}{dx^2} \right) + c\,\psi_i \psi_j \right] dx$$

$$F_i = \int_0^1 \psi_i x^2 \, dx + w \left[-a \frac{du}{dx} + \frac{d}{dx} \cdot \frac{d^2u}{dx^2} \cdot b \right]_0^1 + \frac{dw}{dx} \left[b \cdot \frac{d^2u}{dx^2} \right]_0^1$$

The interpolation requires a 4-term polynomial i.e. a cubic one

Q.2

Free Body Diagram :



Let x_1 to x_5 be displacements
The Equations of motions from its eq^{bm} are

$$k_1x_1 - k_1x_2 = 0$$

and

$$(k_1+k_2+k_3)x_2 - k_1x_1 - k_2x_3 - k_3x_3$$
$$- k_4x_2 - k_4x_4 = 0$$

and

$$(k_2+k_3+k_5)x_3 - (k_2+k_3)x_2 - k_5x_4 = 0$$

and

$$(k_5+k_6+k_4)x_4 - k_5x_3 - k_6x_5 - k_4x_2 = 0$$

and

$$k_6x_5 - k_6x_4 = 80$$

Therefore in Matrix form :

$$\begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1+k_2+k_3+k_4 & -(k_2+k_3) & -k_4 & 0 \\ 0 & -(k_2+k_3) & (k_2+k_3+k_5) & -k_5 & 0 \\ 0 & -k_4 & -k_5 & k_5+k_6+k_4 & -k_6 \\ 0 & 0 & 0 & -k_6 & k_6 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 0 \\ 80 \\ 0 \end{bmatrix}$$

Keeping Values given

$$\begin{bmatrix} -60 & -60 & 0 & 0 & 0 \\ -60 & 340 & -120 & -150 & 0 \\ 0 & -130 & 250 & -120 & 0 \\ 0 & -150 & -120 & 450 & -180 \\ 0 & 0 & 0 & -180 & 180 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 0 \\ 80 \\ 0 \end{bmatrix}$$

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Cancelling out Ist, 5^{th} rows and column as they are fixed

$$\begin{bmatrix} 340 & -130 & -150 \\ -130 & 250 & -120 \\ -150 & -130 & 450 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 80 \end{bmatrix}$$

$$\left.\begin{aligned} x_2 &= 0.91 \text{ in} \\ x_3 &= 0.8077 \text{ in} \\ x_4 &= 0.6966 \text{ in} \end{aligned}\right\} \quad \begin{aligned} x_1 &= 0 \\ x_5 &= 0 \end{aligned}$$

displacements

Reaction forces

$$\begin{bmatrix} -60 & -60 & 0 & 0 & 0 \\ 0 & 0 & 0 & -180 & 180 \end{bmatrix} \begin{bmatrix} 0 \\ 0.91 \\ 0.8077 \\ 0.6966 \end{bmatrix} = \begin{bmatrix} R_1 \\ R_5 \end{bmatrix}$$

$\therefore R_1 \approx -54.6$ lbs

$R_5 = -125.388$ lbs

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Q.3

Shape functions are the polynomials meant to describe the variation of primary variable along the domain of element.

Characteristics of Shape f^n:

1) Value of shape f^n of particular node is one and is zero to all other nodes.
2) Sum of all shape f^n is one
3) Sum of the derivative of all the shape f^{ns} for a particular primary variable is zero.

The polynomial type of interpolation functions are mostly used due to the following reasons:

1) It is easy to formulate and computerize the finite element Eq^{ns}
2) It is easy to perform differentiation or integration.
3) The accuracy of the results can be improved by increasing the order of the polynomial.

Q.4

For a quadratic approximation

$u(x) = a + bx + cx^2$ —— (1)

Choosing two nodes at midpoints and one at the midpoint.

$u_1 = u(x_1) = a + bx_1 + cx_1^2$
$u_2 = u(x_2) = a + bx_2 + cx_2^2$
$u_3 = u(x_3) = a + bx_3 + cx_3^2$

Solving the above system ; and substituting the values of a, b & c in (1) we get

$$u(x) = \psi_1(x) u_1 + \psi_2 u_2 + \psi_3(x) u_3$$

$$= \sum_{j=1}^{3} \psi_j(x) u_j$$

$\psi_j \rightarrow$ Lagrange Interpolation functions

To express in terms of local coordinate $\bar{x}$,

$$\psi_1(\bar{x}) = \left(1 - \frac{\bar{x}}{h_e}\right)\left(1 - \frac{2\bar{x}}{h_e}\right)$$

$$\psi_2 = (\bar{x}) = 4\frac{\bar{x}}{h_e}\left(1 - \frac{\bar{x}}{h_e}\right)$$

$$\psi_3 = (\bar{x}) = \frac{-\bar{x}}{h_e}\left(1 - \frac{2\bar{x}}{h_e}\right)$$

Since $u_i = U(x_i)$, was a requirement, this means that it is zero at all other points and hence satisfies the kronecker-delta property

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