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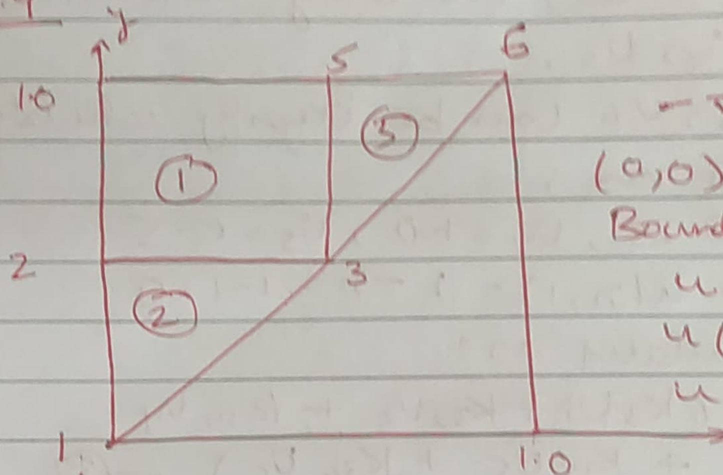
Q.4




$-\nabla^2 u = 2$

$(0,0), (1,0), (1,1), (0,1)$

Boundary condition =

$u(0,y) = y^2$

$u(x,0) = x^2$

$u(1,y) = 1-y$

$u(x,1) = 1-x$

Given eqⁿ can be written as

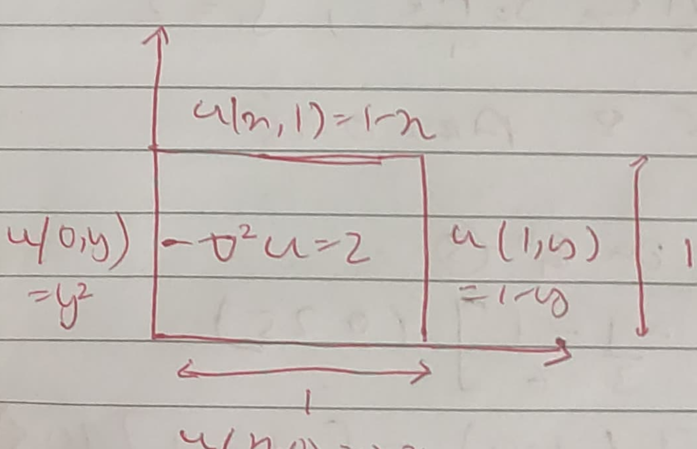
$-\nabla^2 u = f_0$ & $f_0 = 2$ or

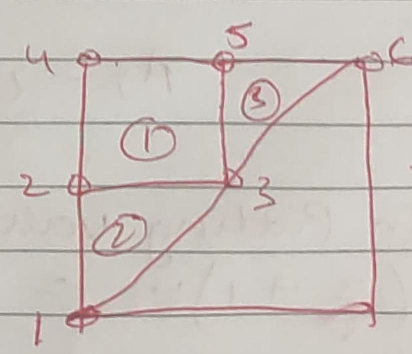
$$-\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial y}\right) = f_0 = 2 \quad (1)$$

Boundary Conditions are

$u(0,y) = y^2 \quad u(x,0) = x^2$

$u(1,y) = 1-y \quad u(x,1) = 1-x$

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For the mesh in ②, the only unknown modal value is $U_3$.
$U_1, U_2, U_4, U_5, U_6$ can be easily calculated & calculations are:
$U_1 = 0, \quad U_2 = 0.25, \quad U_4 = 1.0, \quad U_5 = 0.5$
$U_6 = u(1,1) = u(x,1) = 1 - x = 1 - 1 = 0$
$U_3$ can be calculated as:
$$K_{33} U_3 = F_3 - (K_{31} U_1 + K_{32} U_2 + K_{34} U_4 + K_{35} U_5 + K_{36} U_6) \quad — ③$$

$K_{31} = K^2_{32} = 0, \quad K_{32} = K^2_{31} + K^2_{21} = \frac{-1}{3} - \frac{1}{6}$

$K_{33} = K^2_{22} + K^2_{33} + k^2_{22} = \frac{4}{6} + \frac{1}{2} + \frac{1}{2}$

$K_{34} = K^2_{24} = \frac{-2}{6}$

$K_{35} = K^1_{23} + K^3_{21} = \frac{-1}{6} - \frac{1}{2}$

$K_{36} = K^3_{22} = 0$
$f_3 = 0 + 2 \frac{S_0 A_T}{3} + \frac{S_0 A_R}{4}; \quad S_0 = 2$

$A_T = 0.125 \quad \& \quad A_R = 0.25$

On Putting values in Eqⁿ ③
$$\left(\frac{2}{3} + 1\right) U_3 = -\left[\frac{-1}{3} - \frac{1}{6}\right](0.25) - \left[\frac{-2}{6}\right](1.0) - \left[\frac{-1}{6} - \frac{1}{2}\right](0.5) + \frac{7}{12}(0.5)$$

$$\Rightarrow \frac{5U_3}{3} = \frac{1}{6} + \frac{2}{6} + \frac{2}{6} + \frac{7}{24}$$

$$\Rightarrow \frac{5U_3}{\cancel{3}} = \frac{27}{\cancel{24}\,8}$$

$$\Rightarrow U_3 = \frac{27}{40} = 0.675 \text{ Ans}$$

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Q.2 The functions $u(x,y)$ & its derivatives in the finite element method are given by (for any element)

$$u(x,y) = \sum_{j=1}^{n} u_j \psi_j(x,y)$$

$$\frac{\partial u}{\partial x} = \sum_{j=1}^{n} u_j \frac{\partial \psi_j}{\partial y}$$

The derivatives for the linear triangular element are element-wise constant.

We have

$\alpha_1 = 0.25$ $\quad \alpha_2 = 0.0$ $\quad \alpha_3 = 0.0$

~~$\alpha_1 = 0.5$~~ $\quad \beta_1 = -0.5$ $\quad \beta_3 = 0.0$

$\gamma_1 = 0.0$ $\quad \gamma_2 = -0.5$ $\quad \gamma_3 = 0.5$

Interpolation functions become

$(2A = \alpha_1 + \alpha_2 + \alpha_3 = 0.25)$

$\psi_1 = (1-2x)$, $\quad \psi_2 = 2(x-y)$ $\quad \psi_3 = 2y$

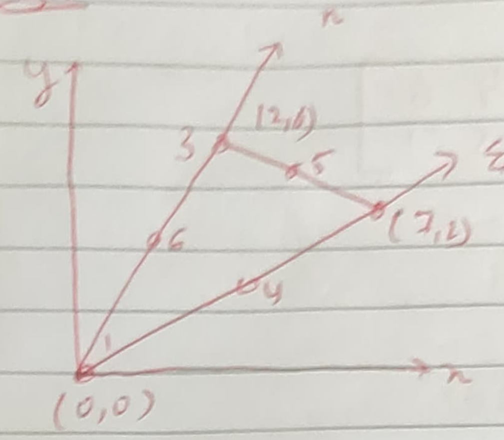
& the req. value of $u$ and its derivates are

$u(0.375, 0.375) = 0.2645 \times 0.25 + 0.1800 \times 0.75 = 0.2011$

$$\frac{\partial u}{\partial x} = u_1(-2.0) + u_2(2.0) + 0 = -0.0946$$

$$\frac{\partial u}{\partial y} = 0 + u_2(-2.0) + u_3(2.0) = -0.0744$$

Q.1




The element has straight edges, its geometry is defined by the interpolate functions of the corner nodes (i.e. a subparametric formulation can be used)

We cannot use only three corner nodes to describe the geometry exactly (hence an isometric formulation must be used)

$$x = \sum_{i=1}^{3} x_i L_i = 7L_2 + 2L_3$$
$$= 2 - 2L_1 + 5L_2$$

$$y = \sum_{i=1}^{3} y_i L_i = 2L_2 + 6L_3$$
$$= 6 - 6L_1 - 4L_2$$

$$[g] = \begin{bmatrix} -2 & -6 \\ 5 & -4 \end{bmatrix}, \quad [g]^{-1} = \frac{1}{38}\begin{bmatrix} -4 & 6 \\ -5 & -2 \end{bmatrix}$$

The global derivatives -

$$\begin{Bmatrix} \dfrac{\partial \psi_1}{\partial x} \\ \dfrac{\partial \psi_2}{\partial y} \end{Bmatrix} = \frac{1}{38}\begin{bmatrix} -4 & 6 \\ -5 & -3 \end{bmatrix}$$

$$\begin{Bmatrix} \dfrac{\partial \psi_1}{\partial L_1} \\ \dfrac{\partial \psi_2}{\partial L_2} \end{Bmatrix} = \begin{Bmatrix} \dfrac{-4(4L_1-1)}{38} \\ \dfrac{-5(4L_1-1)}{38} \end{Bmatrix}$$

where $\psi_1 = L_1(2L_1-1)$

$$\frac{\psi_1}{\partial L_1} = 4L_1 - 1$$

$$\frac{\partial \psi_1}{\partial L_2} = 0$$

For the point (2,4) the area coordinates i.e. $L_i$ can be calculated from (1)

$$2 = 7L_2 + 2L_3 \quad , \quad 4 = 2L_2 + 6L_3$$

Once $L_2$ and $L_3$ have been computed,

$$L_1 = 1 - L_2 - L_3$$

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Hence, $L_1 = \frac{5}{19}$ $L_2 = \frac{2}{19}$ $L_3 = \frac{12}{19}$

Finally $\frac{\partial \varphi_1}{\partial x} = \frac{-2}{19}\left(\frac{20}{19} - 1\right)$

$= \frac{-2}{361}$

$\frac{\partial \varphi_1}{\partial y} = \frac{-5}{38}\left(4 \times \frac{5}{19} - 1\right)$

$= \frac{-5}{722}$