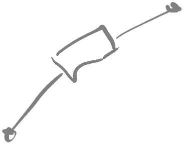
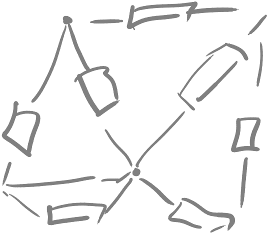
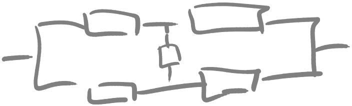
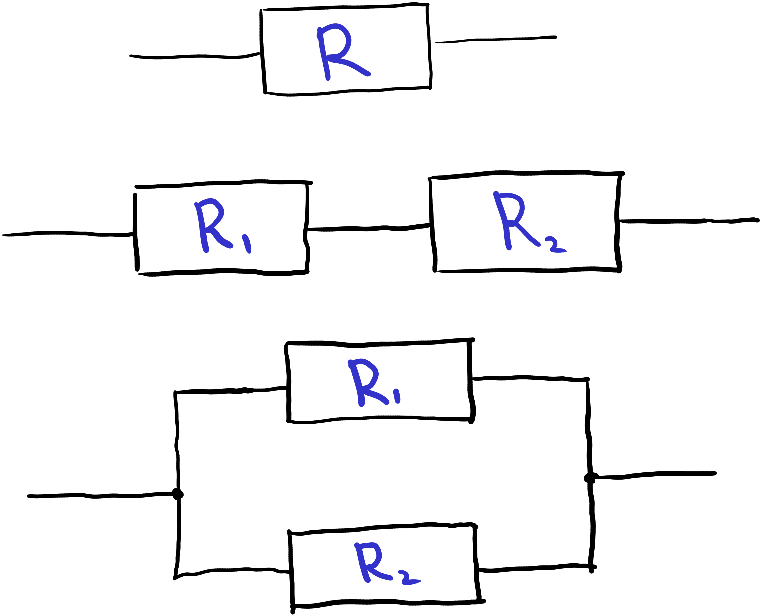
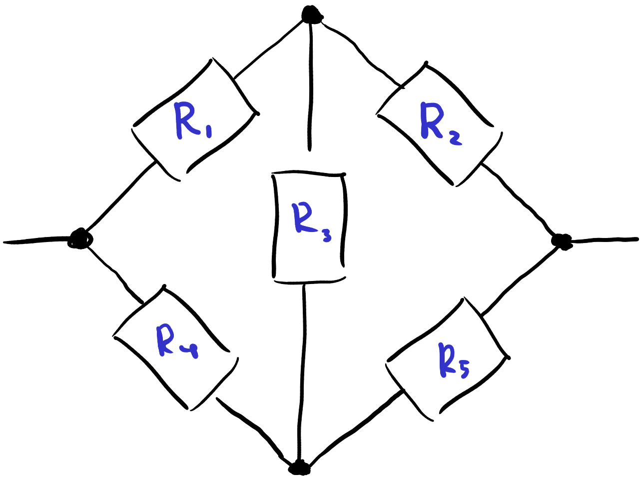


Сети сопротивлений (сюжет)

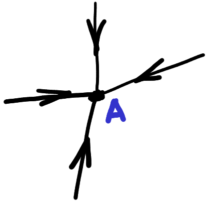





R
R1
R2
R1
R2

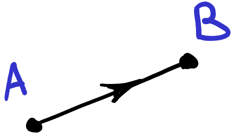

R1
R2
R3
R4
R5

I



$$\sum_{xA\in E} J_{xA}=0$$

II

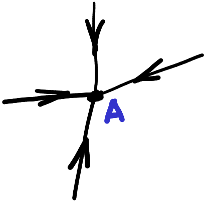


$$\varphi_B-\varphi_A+J_{AB}\cdot R_{AB}=0$$

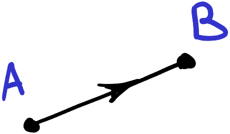
интегрирование эл-х цепей (системы ОДУ)

электростатика

I



II

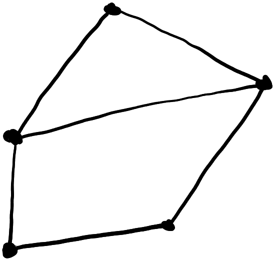


$$\sum_{XA\in E} J_{XA} = 0$$

$$\varphi_B - \varphi_A + J_{AB}\cdot R_{AB} = 0$$

$$J_{AB} = \frac{1}{R_{AB}}\cdot\varphi_A - \frac{1}{R_{AB}}\cdot\varphi_B$$

$$\sum_{XA\in E}\left(\frac{1}{R_{XA}}\varphi_A - \frac{1}{R_{XA}}\varphi_X\right) = 0$$



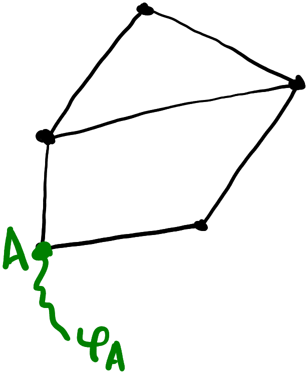
$$\forall y \in V:$$

$$\sum_{xy \in E} \left(\frac{1}{R_{xy}} \varphi_y - \frac{1}{R_{xy}} \varphi_x \right) = 0$$

$|V|$ ур-ий

$|V|$ неизв-х

система вырождена



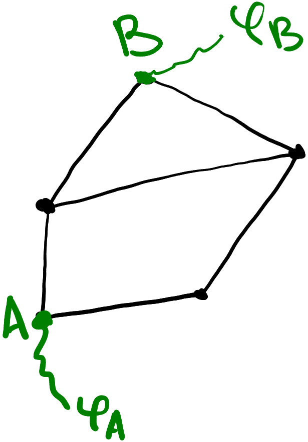
$$\forall y \in V/\{A\}:$$

$$\sum_{xy\in E}\left(\frac{1}{R_{xy}}\varphi_y - \frac{1}{R_{xy}}\varphi_x\right) = 0$$

$$+$$

$$\varphi_A = (\ldots)$$

не вырождена, но $J=0$



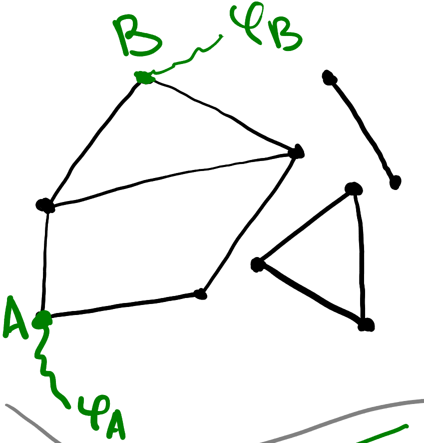
$$\forall Y \in V / \{A, B\}:$$

$$\sum_{XY \in E} \left(\frac{1}{R_{XY}} \varphi_Y - \frac{1}{R_{XY}} \varphi_X \right) = 0$$

$$+$$

$$\varphi_A = (\ldots), \quad \varphi_B = (\ldots)$$

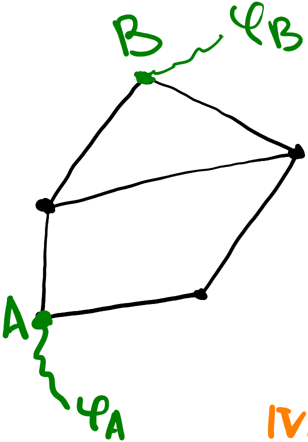
решение есть! но ...



снова вырождена.

разобраться с графами, циклами и совместностью ур-ий

потребовать связность и хотя бы одна $\varphi_A = (\ldots)$



$$\forall Y \in V / \{A, B\}:$$

$$\sum_{XY \in E} \left(\frac{1}{R_{XY}} \varphi_Y - \frac{1}{R_{XY}} \varphi_X \right) = 0$$

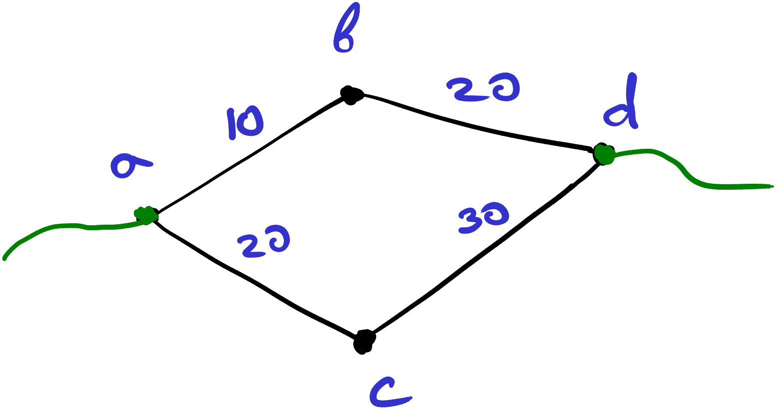
$$+$$

$$\varphi_A = (\ldots), \quad \varphi_B = (\ldots)$$

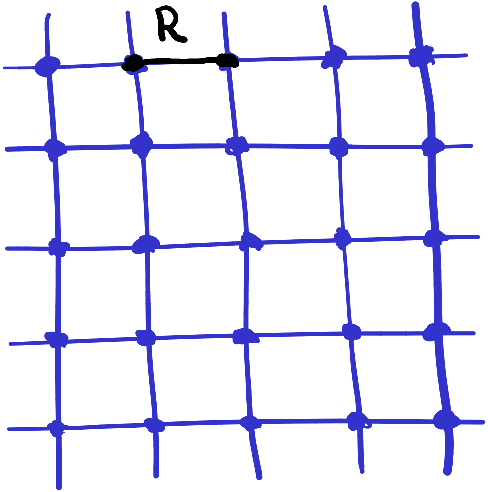
$|V|$ ур-ий

$|V|$ неизв-х, система замкнута.

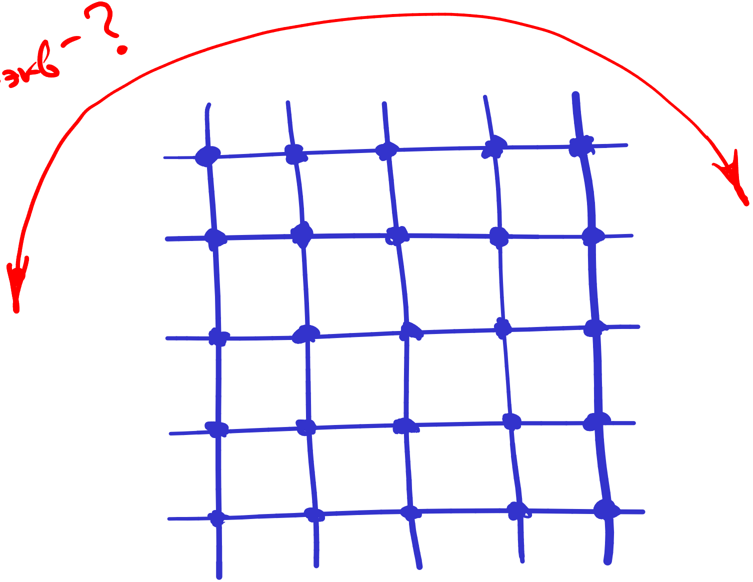
Далее — код.

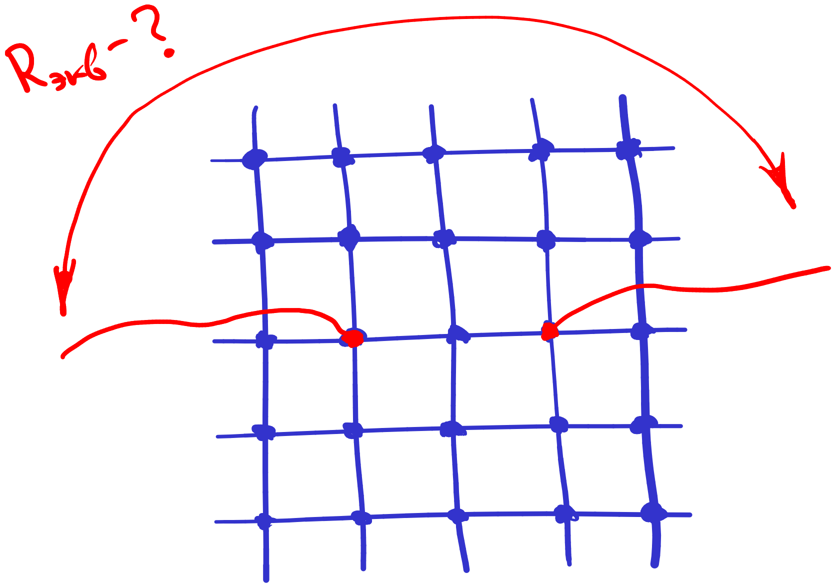


$$R_{eff} = \left(\frac{1}{30} + \frac{1}{50}\right)^{-1} = \frac{150}{8}$$


R

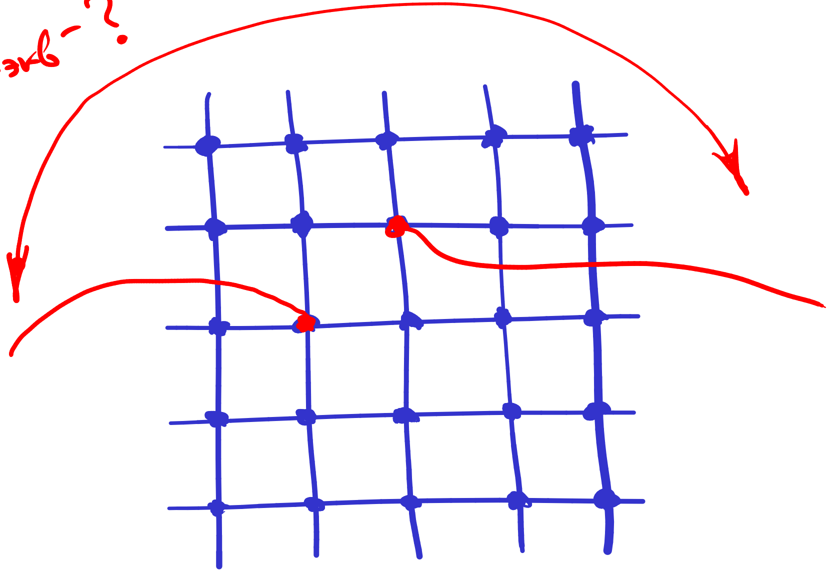
$R_{экв}$ – ?



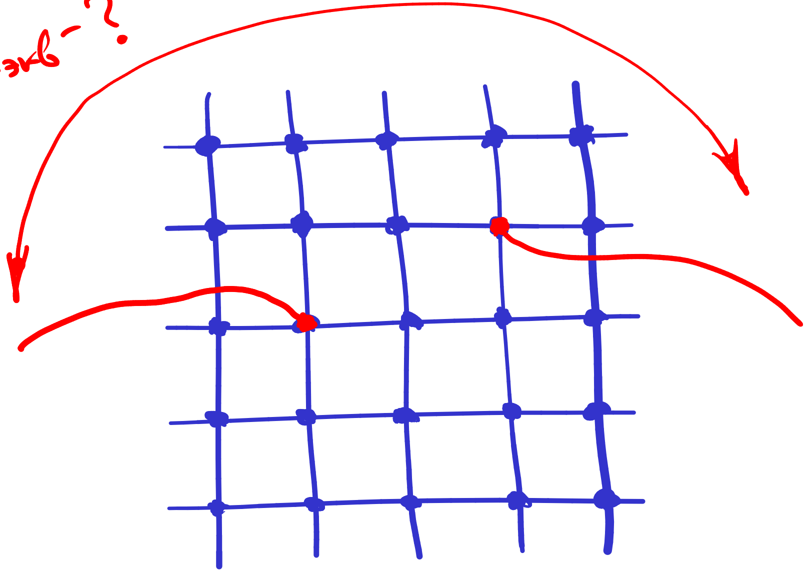


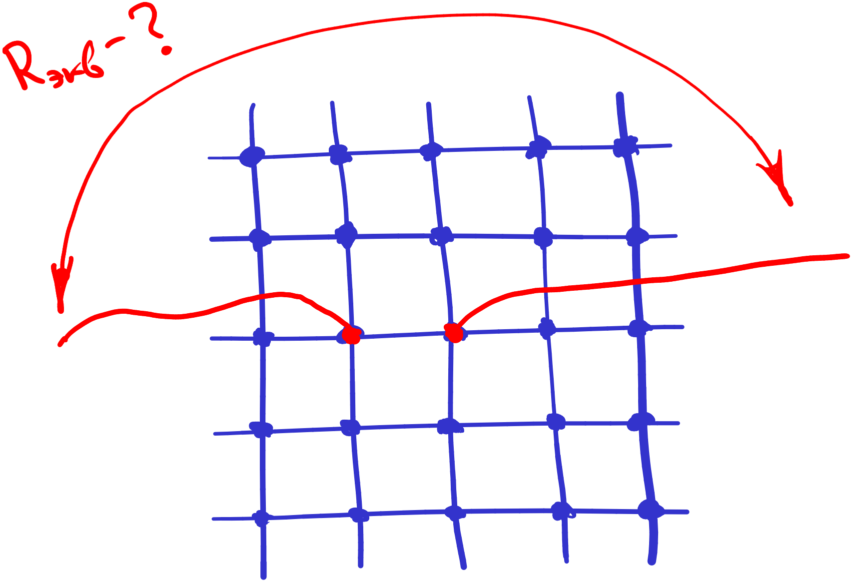
$R_{экв}$ - ?

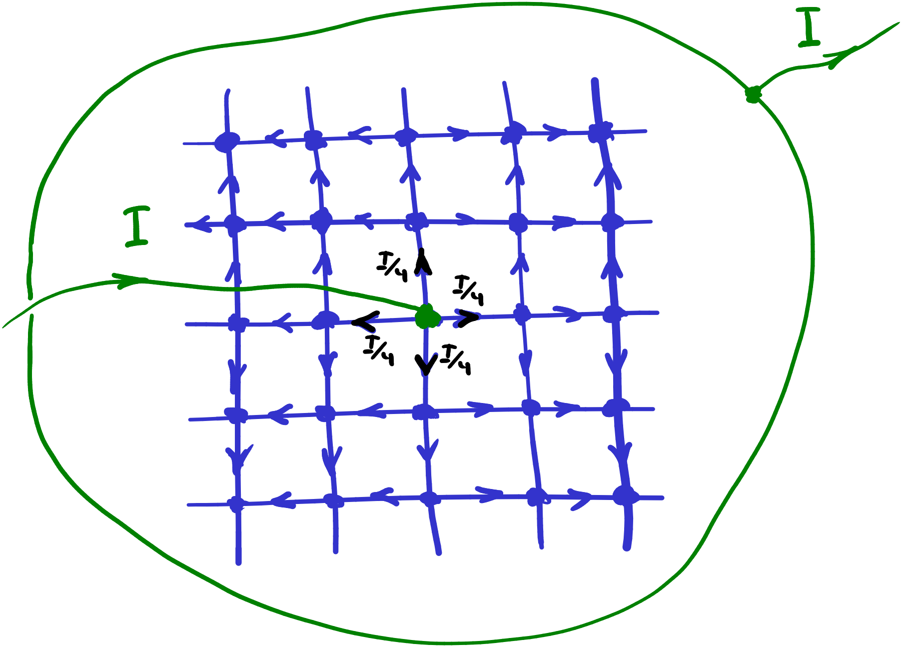
$R_{экв}$ – ?



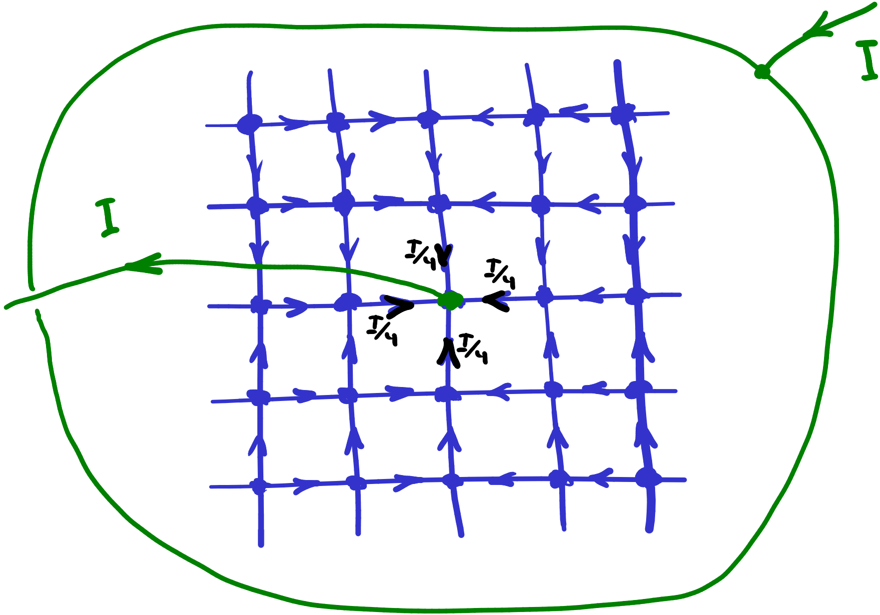
$R_{экв}$ – ?



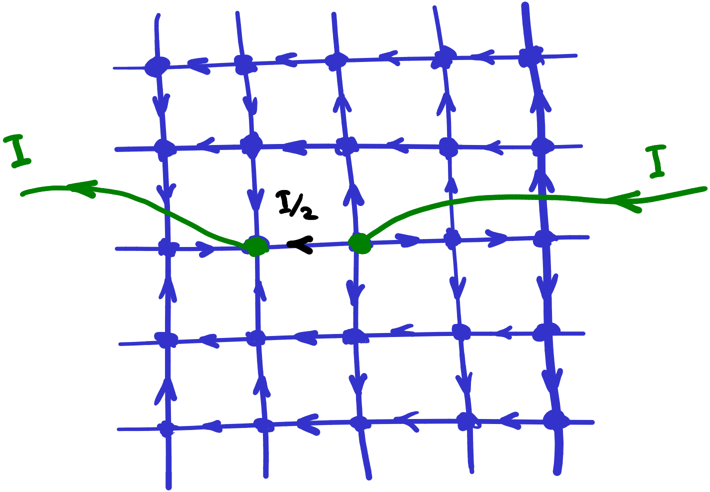

$R_{экв}$ – ?



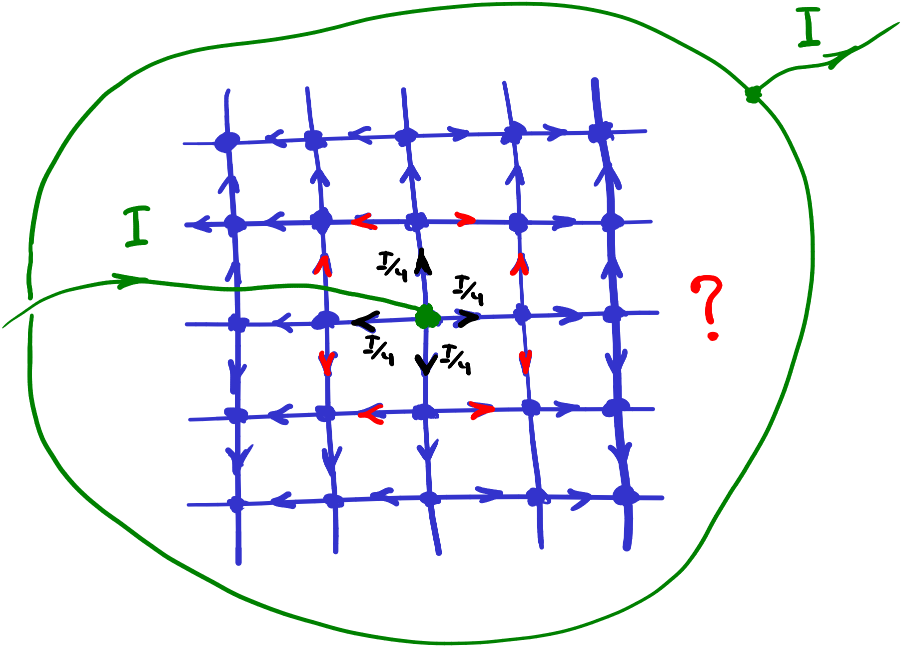
I
I
I/4
I/4
I/4
I/4



I
I
I/4
I/4
I/4
I/4

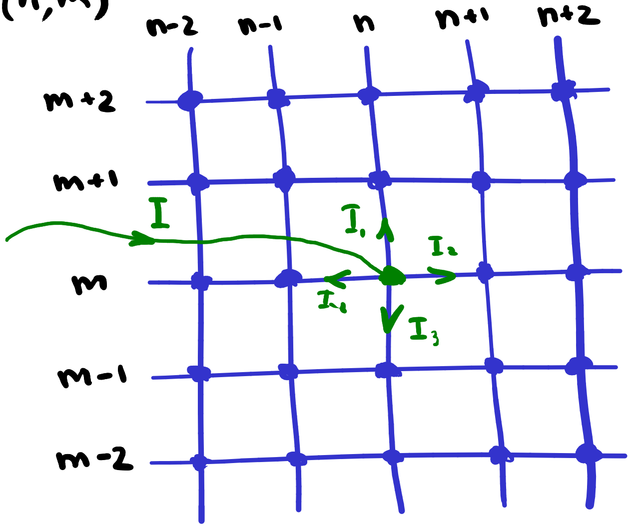


$$R_{экв} = \frac{I/2 \cdot R}{I} = R/2$$

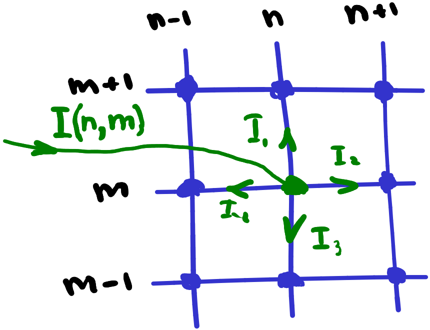


I
I
I/4
I/4
I/4
I/4
?

$\varphi(n,m)$

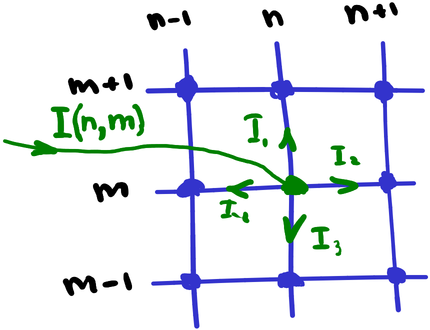


$\varphi(n,m)$



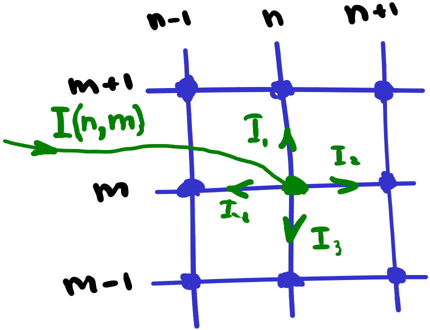
$\varphi(n,m)$

1. $I_1 + I_2 + I_3 + I_4 = I(n,m)$



$\varphi(n,m)$

1. $I_1+I_2+I_3+I_4=I(n,m)$

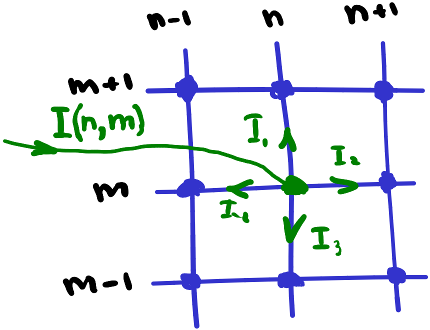


2. $I_1=\frac{1}{R}\left(\varphi(n,m+1)-\varphi(n,m)\right)$, $I_2=\frac{1}{R}\left(\varphi(n+1,m)-\varphi(n,m)\right)$,

$I_3=\frac{1}{R}\left(\varphi(n,m-1)-\varphi(n,m)\right)$, $I_4=\frac{1}{R}\left(\varphi(n-1,m)-\varphi(n,m)\right)$.

$\varphi(n,m)$

$$1.\ \underbrace{I_1+I_2+I_3+I_4=I(n,m)}$$

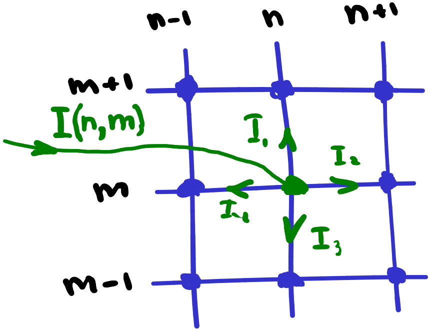


$$\overbrace{2.\ I_1=\frac{1}{R}\left(\varphi(n,m+1)-\varphi(n,m)\right),\ I_2=\frac{1}{R}\left(\varphi(n+1,m)-\varphi(n,m)\right),}$$

$$I_3=\frac{1}{R}\left(\varphi(n,m-1)-\varphi(n,m)\right),\ I_4=\frac{1}{R}\left(\varphi(n-1,m)-\varphi(n,m)\right).$$

$\varphi(n,m)$

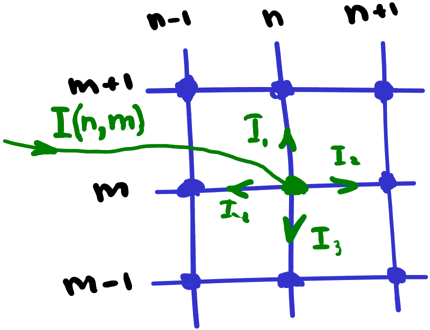
$$1.\ I_1 + I_2 + I_3 + I_4 = I(n,m)$$



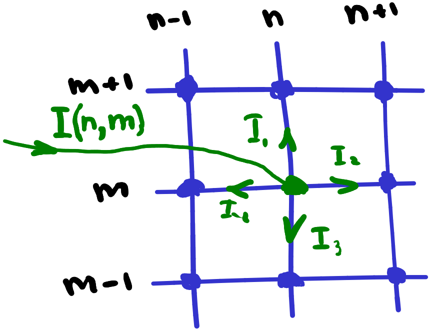
$$\varphi(n+1,m) + \varphi(n,m+1) + \varphi(n-1,m) + \varphi(n,m-1) - 4\varphi(n,m) = I(n,m)\cdot R$$

$$2.\ I_1 = \frac{1}{R}\big(\varphi(n,m+1) - \varphi(n,m)\big),\ I_2 = \frac{1}{R}\big(\varphi(n+1,m) - \varphi(n,m)\big),$$

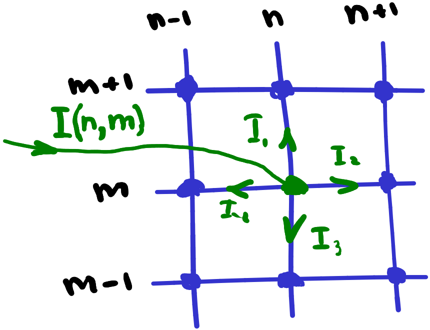
$$I_3 = \frac{1}{R}\big(\varphi(n,m-1) - \varphi(n,m)\big),\ I_4 = \frac{1}{R}\big(\varphi(n-1,m) - \varphi(n,m)\big).$$


n-1
n
n+1
m+1
I(n,m)
I_1
I_2
m
I_4
I_3
m-1

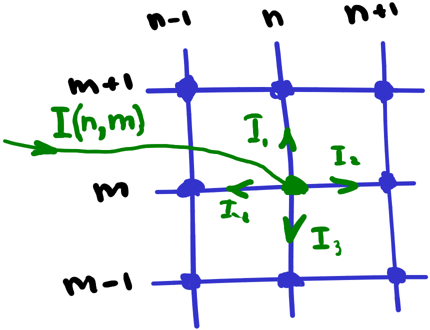
$$\varphi(n+1,m) + \varphi(n,m+1) + \varphi(n-1,m) + \varphi(n,m-1) - 4\varphi(n,m) = I(n,m) \cdot R$$



$$\underbrace{\varphi(n+1,m)+\varphi(n,m+1)+\varphi(n-1,m)+\varphi(n,m-1)-4\varphi(n,m)}_{\Delta\varphi(n,m)=I(n,m)\cdot R}=I(n,m)\cdot R$$



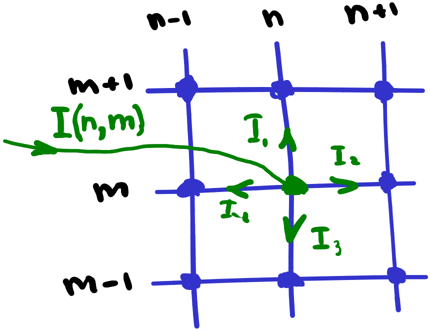
$$\underbrace{\varphi(n+1,m)+\varphi(n,m+1)+\varphi(n-1,m)+\varphi(n,m-1)-4\varphi(n,m)}_{\Delta\varphi(n,m)=I(n,m)\cdot R}=I(n,m)\cdot R$$



дискр. оператор лапласа

$$\underbrace{\varphi(n+1,m)+\varphi(n,m+1)+\varphi(n-1,m)+\varphi(n,m-1)-4\varphi(n,m)}_{\Delta\varphi(n,m)} = I(n,m)\cdot R$$

$$\Delta\varphi(n,m) = I(n,m)\cdot R$$



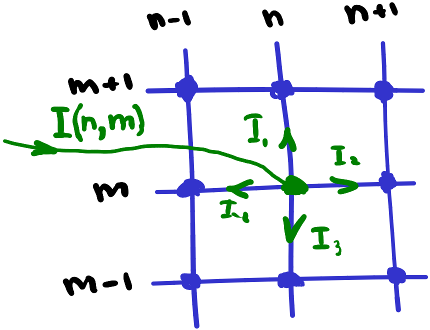
дискр. оператор Лапласа

Фунд. решение $u(n,m)$: $\Delta u(n,m) = \begin{cases} 1 & \text{при } n=m=0 \\ 0 & \text{иначе} \end{cases}$

тогда

$$\underbrace{\varphi(n+1,m)+\varphi(n,m+1)+\varphi(n-1,m)+\varphi(n,m-1)-4\varphi(n,m)}_{\Delta\varphi(n,m)} = I(n,m)\cdot R$$

$$\Delta\varphi(n,m) = I(n,m)\cdot R$$



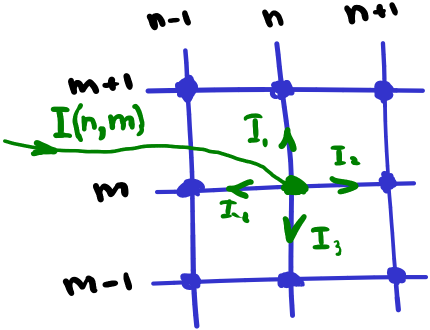
Дискр. оператор Лапласа

Фунд. решение $u(n,m)$: $\Delta u(n,m) = \begin{cases} 1 & \text{при } n=m=0 \\ 0 & \text{иначе} \end{cases}$

Тогда: $\varphi(n,m) = \sum\limits_{k,\ell} I(k,\ell)\cdot R\cdot u(n-k, m-\ell)$

$$\underbrace{\varphi(n+1,m)+\varphi(n,m+1)+\varphi(n-1,m)+\varphi(n,m-1)-4\varphi(n,m)}_{\Delta\varphi(n,m)} = I(n,m)\cdot R$$

$$\Delta\varphi(n,m) = I(n,m)\cdot R$$



дискр. оператор Лапласа

!

Фунд. решение $u(n,m)$: $\Delta u(n,m) = \begin{cases} 1 & \text{при } n=m=0 \\ 0 & \text{иначе} \end{cases}$

Тогда: $\varphi(n,m) = \sum\limits_{k,\ell} I(k,\ell)\cdot R\cdot u(n-k,m-\ell)$

$U(x,y)$ – ПФДВ для $u(n,m)$

$U(x,y)$ — ПФДВ для $u(n,m)$:

$$U(x,y)=\sum_{n,m} u(n,m)\cdot e^{-inx-imy}$$

$U(x,y)$ — ПФДВ для $u(n,m)$:

$$U(x,y) = \sum_{n,m} u(n,m) \cdot e^{-inx-imy}, \text{ тогда}$$

$U(x,y)$ — ПФДВ для $u(n,m)$:

$$U(x,y)=\sum_{n,m} u(n,m)\cdot e^{-inx-imy}\text{, тогда}$$

$$e^{ix}\cdot U(x,y)=\sum_{n,m} u(n+1,m)\cdot e^{-inx-imy}$$

$\mathcal{U}(x,y)$ — ПФДВ для $u(n,m)$:

$$\mathcal{U}(x,y)=\sum_{n,m} u(n,m)\cdot e^{-inx-imy},$$ тогда

$$e^{ix}\cdot\mathcal{U}(x,y)=\sum_{n,m} u(n+1,m)\cdot e^{-inx-imy}$$

$$e^{iy}\cdot\mathcal{U}(x,y)=\sum_{n,m} u(n,m+1)\cdot e^{-inx-imy}$$

$U(x,y)$ — ПФДВ для $u(n,m)$:

$$U(x,y)=\sum_{n,m} u(n,m)\cdot e^{-inx-imy}, \text{ тогда}$$

$$e^{ix}\cdot U(x,y)=\sum_{n,m} u(n+1,m)\cdot e^{-inx-imy}$$

$$e^{iy}\cdot U(x,y)=\sum_{n,m} u(n,m+1)\cdot e^{-inx-imy}$$

$$\left(e^{ix}+e^{iy}+e^{-ix}+e^{-iy}-4\right)\cdot U(x,y)=1$$

$U(x,y)$ — ПФДВ для $u(n,m)$:

$$U(x,y)=\sum_{n,m} u(n,m)\cdot e^{-inx-imy}, \text{ тогда}$$

$$e^{ix}\cdot U(x,y)=\sum_{n,m} u(n+1,m)\cdot e^{-inx-imy}$$

$$e^{iy}\cdot U(x,y)=\sum_{n,m} u(n,m+1)\cdot e^{-inx-imy}$$

$$\left(e^{ix}+e^{iy}+e^{-ix}+e^{-iy}-4\right)\cdot U(x,y)=1$$

$$U(x,y)=\frac{1}{2(\cos x+\cos y-2)}$$

$$u(n,m)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2\pi}\int_{-\pi}^{\pi} U(x,y)\cdot e^{i(nx+my)}\,dx\,dy$$

$$\mathcal{U}(x,y) = \frac{1}{2(\cos x + \cos y - 2)}$$

$$u(n,m) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathcal{U}(x,y) \cdot e^{i(nx+my)}\,dx\,dy$$

$$\mathcal{U}(x,y)=\frac{1}{2(\cos x+\cos y-2)}$$

$$u(n,m)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2\pi}\int_{-\pi}^{\pi}\mathcal{U}(x,y)\cdot e^{i(nx+my)}\,dx\,dy$$

$$u(n,m)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{e^{i(nx+my)}}{2(\cos x+\cos y-2)}\,dx\,dy$$

$$U(x,y) = \frac{1}{2(\cos x + \cos y - 2)}$$

$$u(n,m) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2\pi}\int_{-\pi}^{\pi} U(x,y)\cdot e^{i(nx+my)}\,dx\,dy$$

$$u(n,m) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{e^{i(nx+my)}}{2(\cos x + \cos y - 2)}\,dx\,dy$$

почти решение,
оно расходится, что
когда это останавливало?

$$u(n,m)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{e^{i(nx+my)}}{2(\cos x+\cos y-2)}\,dx\,dy$$

$$u(n,m) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{e^{i(nx+my)}}{2(\cos x + \cos y - 2)}\,dx\,dy$$

Рассмотрим $\tilde{u}(n,m) = u(n,m) - u(0,0)$:

$$u(n,m)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{e^{i(nx+my)}}{2(\cos x+\cos y-2)}\,dx\,dy$$

Рассмотрим $\tilde{u}(n,m)=u(n,m)-u(0,0)$:

$$\tilde{u}(n,m)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\cos(nx+my)-1}{2(\cos x+\cos y-2)}\,dx\,dy$$

$$u(n,m)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{e^{i(nx+my)}}{2(\cos x+\cos y-2)}\,dx\,dy$$

Рассмотрим $\tilde{u}(n,m)=u(n,m)-u(0,0)$:

$$\tilde{u}(n,m)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\cos(nx+my)-1}{2(\cos x+\cos y-2)}\,dx\,dy$$

Это тоже решение, т.к. $\Delta u(0,0)=0$.

$$\tilde{u}(n,m) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\cos(nx+my)-1}{2(\cos x+\cos y-2)}\,dx\,dy$$

$$\tilde{u}(n,m) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\cos(nx+my)-1}{2(\cos x+\cos y-2)}\,dx\,dy$$

$$\delta(n) = \tilde{u}(n+1,n+1) - \tilde{u}(n,n)$$

$$\tilde{u}(n,m)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\cos(nx+my)-1}{2(\cos x+\cos y-2)}\,dx\,dy$$

$$\delta(n)=\tilde{u}(n+1,n+1)-\tilde{u}(n,n)=$$

$$=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\cos((n+1)(x+y))-\cos(n(x+y))}{2(\cos x+\cos y-2)}\,dx\,dy$$

$$\tilde{u}(n,m) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\cos(nx+my)-1}{2(\cos x+\cos y-2)}\,dx\,dy$$

$$\delta(n) = \tilde{u}(n+1,n+1) - \tilde{u}(n,n) =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\cos((n+1)(x+y)) - \cos(n(x+y))}{2(\cos x+\cos y-2)}\,dx\,dy =$$

$$= \frac{1}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \frac{\sin\frac{x+y}{2}\,\sin\left((2n+1)\frac{x+y}{2}\right)}{2\left(1-\cos\frac{x+y}{2}\cos\frac{x-y}{2}\right)}\,dx\,dy$$

$$\tilde{u}(n,m)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\cos(nx+my)-1}{2(\cos x+\cos y-2)}\,dx\,dy$$

$$\delta(n)=\tilde{u}(n+1,n+1)-\tilde{u}(n,n)=$$

$$=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\cos((n+1)(x+y))-\cos(n(x+y))}{2(\cos x+\cos y-2)}\,dx\,dy=$$

$$\frac{x+y}{2}=\xi,\quad \frac{y-x}{2}=\eta$$

$$=\frac{1}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\frac{\sin\frac{x+y}{2}\,\sin\left((2n+1)\frac{x+y}{2}\right)}{2\left(1-\cos\frac{x+y}{2}\cos\frac{x-y}{2}\right)}\,dx\,dy$$

$$\tilde{u}(n,m) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\cos(nx+my)-1}{2(\cos x+\cos y-2)}\,dx\,dy$$

$$\delta(n) = \tilde{u}(n+1,n+1) - \tilde{u}(n,n) =$$

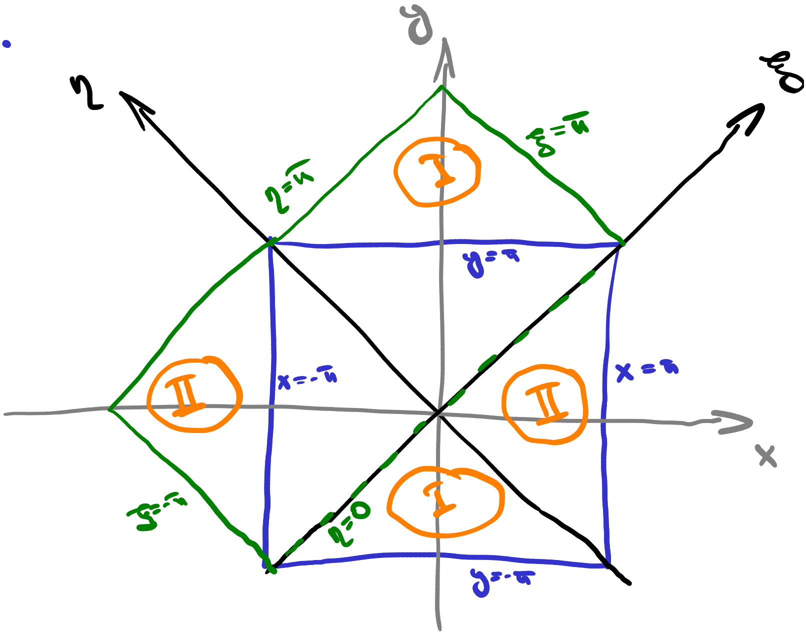
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\cos((n+1)(x+y)) - \cos(n(x+y))}{2(\cos x+\cos y-2)}\,dx\,dy =$$

$$\frac{x+y}{2} = \xi, \quad \frac{y-x}{2} = \eta$$

$$= \frac{1}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \frac{\sin\frac{x+y}{2}\,\sin\left((2n+1)\frac{x+y}{2}\right)}{2\left(1-\cos\frac{x+y}{2}\cos\frac{x-y}{2}\right)}\,dx\,dy =$$

$$= \frac{1}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{0}^{\pi} \frac{\sin\xi \cdot \sin(2n+1)\xi}{1-\cos\xi\,\cos\eta}\,d\xi\,d\eta$$

T.K.



$$\delta(n) = \tilde{u}(n+1,n+1) - \tilde{u}(n,n) =$$

$$= \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} \int_{0}^{\pi} \frac{\sin \xi \cdot \sin(2n+1)\xi}{1 - \cos\xi \cos\eta} \, d\xi \, d\eta$$

$$\delta(n) = \tilde{u}(n+1,n+1) - \tilde{u}(n,n) =$$

$$= \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi}\int_{0}^{\pi} \frac{\sin \xi \cdot \sin(2n+1)\xi}{1-\cos\xi \cos\eta}\, d\xi\, d\eta =$$

$$= \frac{1}{4\pi} \int_{-\pi}^{\pi} \frac{\sin \xi \cdot \sin(2n+1)\xi}{\sqrt{1-\cos^2\xi}}\, d\xi$$

$$\delta(n) = \tilde{u}(n+1,n+1) - \tilde{u}(n,n) =$$

$$= \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi}\int_{0}^{\pi} \frac{\sin\xi \cdot \sin(2n+1)\xi}{1-\cos\xi\cos\eta}\, d\xi\, d\eta =$$

$$= \frac{1}{4\pi} \int_{-\pi}^{\pi} \frac{\sin\xi \cdot \sin(2n+1)\xi}{\sqrt{1-\cos^2\xi}}\, d\xi =$$

$$= \frac{1}{2\pi} \int_{0}^{\pi} \sin(2n+1)\xi\, d\xi$$

$$\delta(n) = \tilde{u}(n+1,n+1) - \tilde{u}(n,n) =$$

$$= \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} \int_{0}^{\pi} \frac{\sin\xi \cdot \sin(2n+1)\xi}{1 - \cos\xi \cos\eta} \, d\xi \, d\eta =$$

$$= \frac{1}{4\pi} \int_{-\pi}^{\pi} \frac{\sin\xi \cdot \sin(2n+1)\xi}{\sqrt{1-\cos^2\xi}} \, d\xi =$$

$$= \frac{1}{2\pi} \int_{0}^{\pi} \sin(2n+1)\xi \, d\xi = \frac{1}{\pi(2n+1)}, \text{ т.е.}$$

$$\delta(n) = \tilde{u}(n+1,n+1) - \tilde{u}(n,n) =$$

$$= \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi}\int_{0}^{\pi} \frac{\sin\xi \cdot \sin(2n+1)\xi}{1-\cos\xi\,\cos\eta}\, d\xi\, d\eta =$$

$$= \frac{1}{4\pi} \int_{-\pi}^{\pi} \frac{\sin\xi \cdot \sin(2n+1)\xi}{\sqrt{1-\cos^2\xi}}\, d\xi =$$

$$= \frac{1}{2\pi} \int_{0}^{\pi} \sin(2n+1)\xi\, d\xi = \frac{1}{\pi(2n+1)}, \text{ т.е.}$$

$$\tilde{u}(0,0) = 0$$

$$\delta(n) = \tilde{u}(n+1,n+1) - \tilde{u}(n,n) =$$

$$= \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi}\int_{0}^{\pi} \frac{\sin\xi \cdot \sin(2n+1)\xi}{1-\cos\xi\cos\eta}\, d\xi\, d\eta =$$

$$= \frac{1}{4\pi} \int_{-\pi}^{\pi} \frac{\sin\xi \cdot \sin(2n+1)\xi}{\sqrt{1-\cos^2\xi}}\, d\xi =$$

$$= \frac{1}{2\pi} \int_{0}^{\pi} \sin(2n+1)\xi\, d\xi = \frac{1}{\pi(2n+1)}, \text{ т.е.}$$

$$\tilde{u}(0,0)=0, \quad \tilde{u}(1,1) = \frac{1}{\pi}$$

$$\delta(n)=\tilde{u}(n+1,n+1)-\tilde{u}(n,n)=$$

$$=\frac{1}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{0}^{\pi}\frac{\sin\xi\cdot\sin(2n+1)\xi}{1-\cos\xi\,\cos\eta}\,d\xi\,d\eta=$$

$$=\frac{1}{4\pi}\int_{-\pi}^{\pi}\frac{\sin\xi\cdot\sin(2n+1)\xi}{\sqrt{1-\cos^2\xi}}\,d\xi=$$

$$=\frac{1}{2\pi}\int_{0}^{\pi}\sin(2n+1)\xi\,d\xi=\frac{1}{\pi(2n+1)},\ \text{т.е.}$$

$$\tilde{u}(0,0)=0,\quad \tilde{u}(1,1)=\frac{1}{\pi},\quad \tilde{u}(2,2)=\frac{1}{\pi}\left(1+\frac{1}{3}\right),$$

$$\delta(n) = \tilde{u}(n+1,n+1) - \tilde{u}(n,n) =$$

$$= \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} \int_{0}^{\pi} \frac{\sin\xi \cdot \sin(2n+1)\xi}{1 - \cos\xi \cos\eta} \, d\xi \, d\eta =$$

$$= \frac{1}{4\pi} \int_{-\pi}^{\pi} \frac{\sin\xi \cdot \sin(2n+1)\xi}{\sqrt{1-\cos^2\xi}} \, d\xi =$$

$$= \frac{1}{2\pi} \int_{0}^{\pi} \sin(2n+1)\xi \, d\xi = \frac{1}{\pi(2n+1)}, \text{ т.е.}$$

$$\tilde{u}(0,0) = 0, \quad \tilde{u}(1,1) = \frac{1}{\pi}, \quad \tilde{u}(2,2) = \frac{1}{\pi}\left(1 + \frac{1}{3}\right),$$

$$\tilde{u}(n,n) = \frac{1}{\pi}\left(1 + \frac{1}{3} + \frac{1}{5} + \ldots + \frac{1}{2|n|-1}\right)$$

$u(n,m)$:

	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$
$m=2$	$4/3\pi$	b	a	b	$4/3\pi$
$m=1$	b	$1/\pi$	$1/4$	$1/\pi$	b
$m=0$	a	$1/4$	0	$1/4$	a
$m=-1$	b	$1/\pi$	$1/4$	$1/\pi$	b
$m=-2$	$4/3\pi$	b	a	b	$4/3\pi$

$u(n,m)$:

	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$
$m=2$	$4/3\pi$	b	a	b	$4/3\pi$
$m=1$	b	$1/\pi$	$1/4$	$1/\pi$	b
$m=0$	a	$1/4$	0	$1/4$	a
$m=-1$	b	$1/\pi$	$1/4$	$1/\pi$	b
$m=-2$	$4/3\pi$	b	a	b	$4/3\pi$

$$0+\frac{1}{\pi}+\frac{1}{\pi}+a-4\cdot\frac{1}{4}=0$$

$u(n,m)$:

	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$
$m=2$	$4/3\pi$	b	a	b	$4/3\pi$
$m=1$	b	$1/\pi$	$1/4$	$1/\pi$	b
$m=0$	a	$1/4$	0	$1/4$	a
$m=-1$	b	$1/\pi$	$1/4$	$1/\pi$	b
$m=-2$	$4/3\pi$	b	a	b	$4/3\pi$

$$0+\frac{1}{\pi}+\frac{1}{\pi}+a-4\cdot\frac{1}{4}=0$$

$$a=1-\frac{2}{\pi}$$

$u(n,m)$:

	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$
$m=2$	$4/3\pi$	b	a	b	$4/3\pi$
$m=1$	b	$1/\pi$	$1/4$	$1/\pi$	b
$m=0$	a	$1/4$	0	$1/4$	a
$m=-1$	b	$1/\pi$	$1/4$	$1/\pi$	b
$m=-2$	$4/3\pi$	b	a	b	$4/3\pi$

$$0+\frac{1}{\pi}+\frac{1}{\pi}+a-4\cdot\frac{1}{4}=0$$

$$a=1-\frac{2}{\pi}$$

$$\frac{1}{4}+\frac{1}{4}+b+b-4\cdot\frac{1}{\pi}=0$$

$u(n,m)$:

	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$
$m=2$	$4/3\pi$	b	a	b	$4/3\pi$
$m=1$	b	$1/\pi$	$1/4$	$1/\pi$	b
$m=0$	a	$1/4$	0	$1/4$	a
$m=-1$	b	$1/\pi$	$1/4$	$1/\pi$	b
$m=-2$	$4/3\pi$	b	a	b	$4/3\pi$

$$0+\frac{1}{\pi}+\frac{1}{\pi}+a-4\cdot\frac{1}{4}=0$$

$$a=1-\frac{2}{\pi}$$

$$\frac{1}{4}+\frac{1}{4}+b+b-4\cdot\frac{1}{\pi}=0$$

$$b=\frac{2}{\pi}-\frac{1}{4}$$

$u(n,m)$:

	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$
$m=2$	$4/3\pi$	b	a	b	$4/3\pi$
$m=1$	b	$1/\pi$	$1/4$	$1/\pi$	b
$m=0$	a	$1/4$	0	$1/4$	a
$m=-1$	b	$1/\pi$	$1/4$	$1/\pi$	b
$m=-2$	$4/3\pi$	b	a	b	$4/3\pi$

$$0+\frac{1}{\pi}+\frac{1}{\pi}+a-4\cdot\frac{1}{4}=0$$

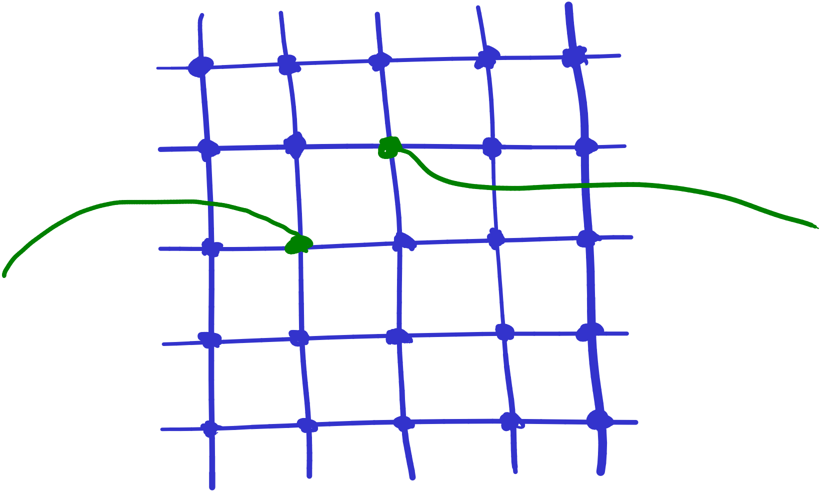
$$a=1-\frac{2}{\pi}$$

$$\frac{1}{4}+\frac{1}{4}+b+b-4\cdot\frac{1}{\pi}=0$$

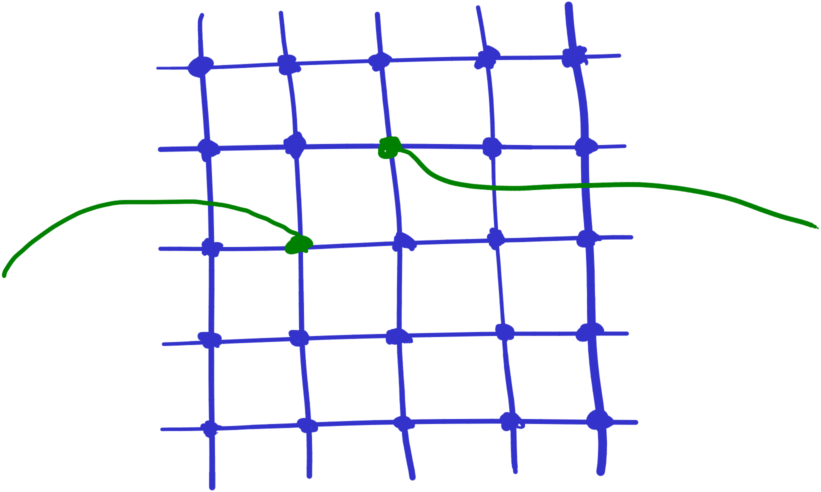
$$b=\frac{2}{\pi}-\frac{1}{4}$$

потенциал Соболева

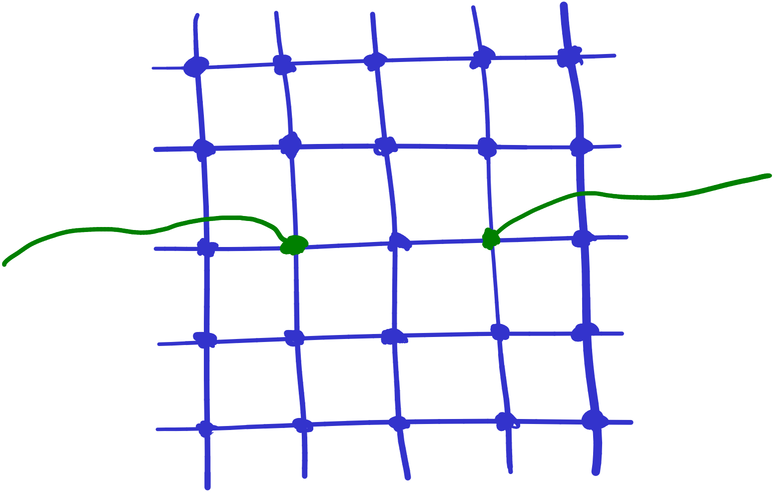
$R_{3\times 6} =$



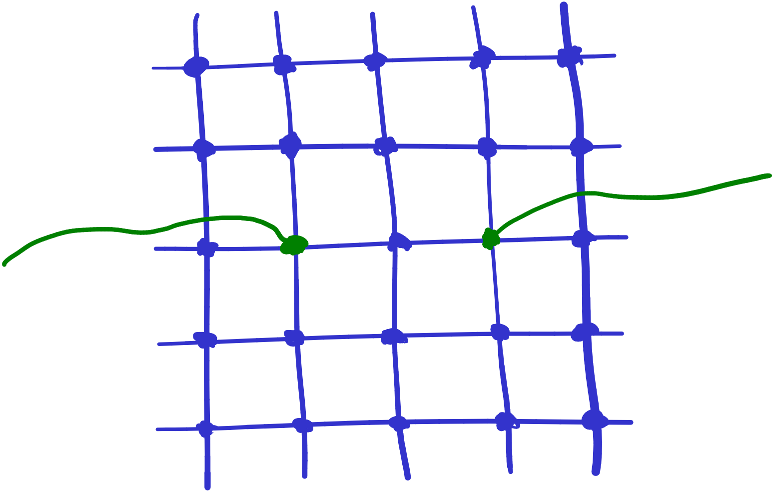
$$R_{эфф} = \frac{2}{\pi} R$$



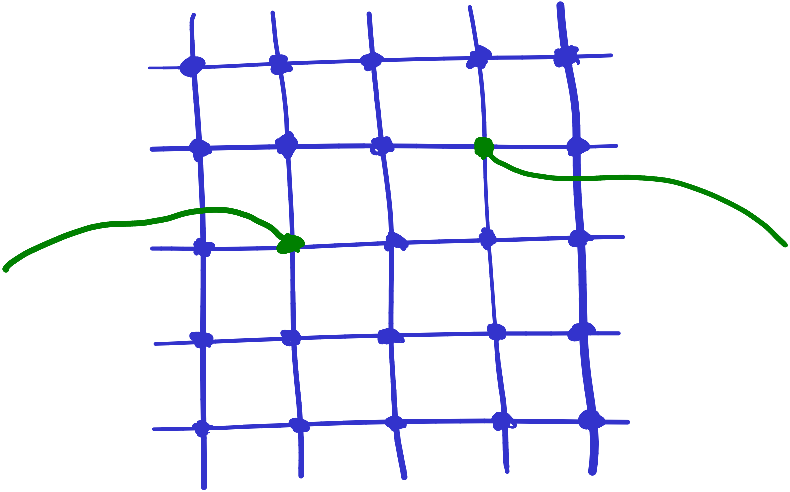
$R_{экв} =$



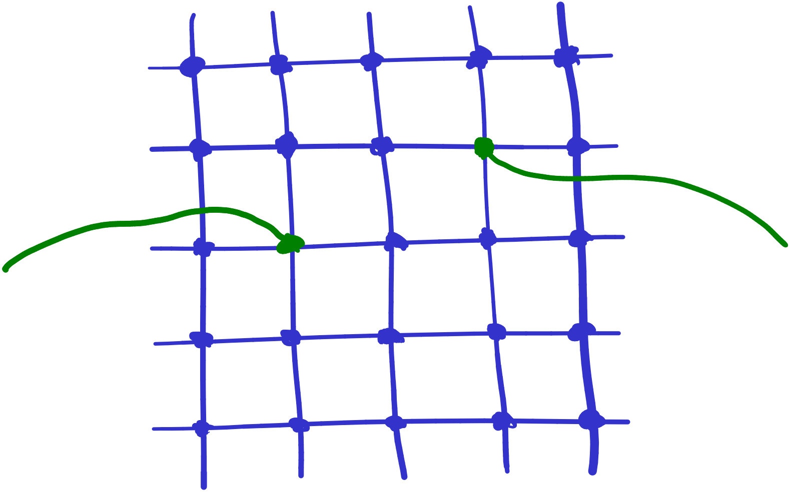
$$R_{экв} = 2aR = 2\left(1 - \frac{2}{n}\right)R$$



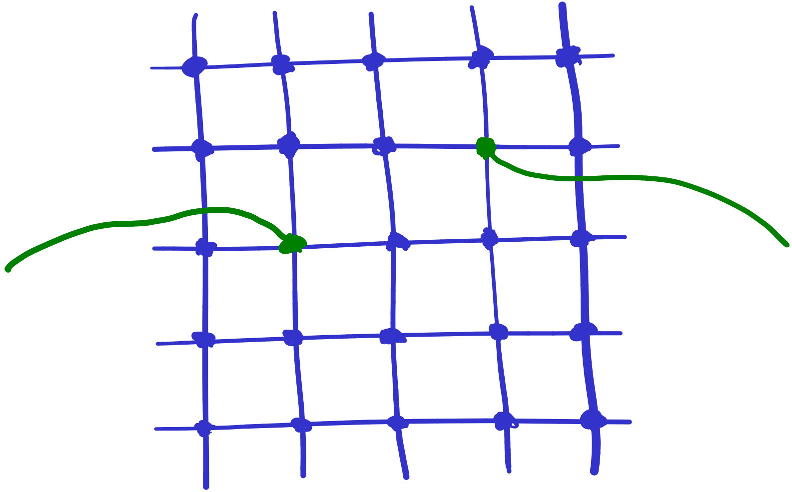
$R_{экв} =$



$$R_{экв} = 2b\,R = \left(\frac{4}{\pi} - \frac{1}{2}\right)R$$



$$R_{экв} = 2b\,R = \left(\frac{4}{\pi} - \frac{1}{2}\right)R$$



Смотри код...

Интересно 1:

$L_2[-\pi,\pi]$ ℓ_2

$$X(\omega)=\sum_n x_n e^{-i\omega n}$$

$$x_n=\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{i\omega n}\,d\omega$$

ПФДВ $\longleftrightarrow$ ряд Фурье

L_2 L_2

$$X(\omega)=\int_{\mathbb{R}} x(t)e^{-i\omega t}\,dt$$

$$x(t)=\frac{1}{2\pi}\int_{\mathbb{R}} X(\omega)e^{i\omega t}\,d\omega$$

ПФ $\longleftrightarrow$ ПФ

$\mathbb{Z}_N\to\mathbb{C}$ $\mathbb{Z}_N\to\mathbb{C}$

$$X_k=\sum_n x_n e^{-\frac{2\pi i}{N}kn}$$

$$x_n=\frac{1}{N}\sum_k X_k e^{\frac{2\pi i}{N}kn}$$

ДПФ $\longleftrightarrow$ ДПФ

Двойственность Понтрягина

Интересно 2:

$$\Delta\varphi(n,m) = f(n,m)$$

$$\Updownarrow$$

$$\min \sum_{n,m} \frac{1}{2}\big(\varphi(n+1,m) - \varphi(n,m)\big)^2 + \frac{1}{2}\big(\varphi(n,m+1) - \varphi(n,m)\big)^2 + \varphi(n,m)\cdot f(n,m)$$

мин. диссипации энергии

Интересно 3:

$\Delta \varphi(n,m) = 0$ — дискр. гарм. ф-ия

и аналитическая:

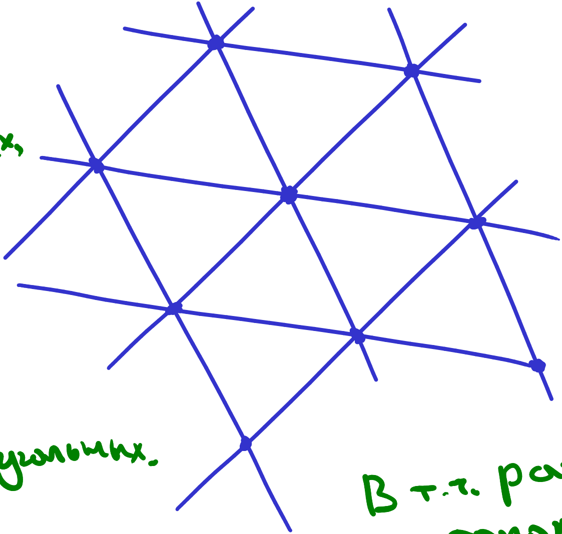
$$f(n+im) = D_n \varphi(n,m) + i D_m \varphi(n,m)$$

Условие Коши–Римана:

$$f(z+1) - f(z) = \frac{1}{i}\left(f(z+i) - f(z)\right)$$

Интересно 4 :

Гарм. ф-ии на решётках,



и не только прямоугольных.

В т.ч. разной топологии.

См. также:

1. Производящие ф-ии.
2. Случайные блуждания.
3. Алг. геометрия.

Лабораторная :