Advanced Dynamics (wb2630-T1)

Q1 2014

Homework No. 3

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Date: 15-Sept-2014 **Deadline:** Before lecture on Sept 22.

<u>Instructions</u>:

- Show all work, clearly and in order, if you want to get full credit.
- Follow the homework rules as published on Blackboard.
- Justify your answers algebraically whenever possible to ensure full credit. Sketch all relevant graphs.
- Circle or otherwise indicate your final answers.
- Clearly distinguish between scalars and vectors, for example by underlining vectors ($\underline{x}$) and not underlining scalars (x). **If you use another notation, define it like this**: A vector will be written as ...., a scalar will be written as ....
- Success!

1. (*25 points*) AXIS, ANGLE OF ROTATION

Here we follow the notation from Greenwood Ch3.1. Consider a rigid body with a body-fixed frame x', y', z' initially aligned with a space-fixed frame X, Y, Z. After a rotation, the x'-axis is aligned with the Y-axis, the y'-axis is aligned with the Z-axis and the z'-axis is aligned with the X-axis.

a. (*5 pts*) Determine the rotation matrix $\mathbf{C}$ (with $\boldsymbol{r}' = \mathbf{C}\boldsymbol{r}$).

b. (*10 pts*) Determine the axis and angle of rotation. Draw this axis in a picture where you see the space-fixed frame, and the body-fixed frame after the rotation. Discuss your result.

c. (*5 pts*) Determine the Euler type I angles, ψ, θ, and ϕ, for this rotation. Draw three pictures with the two frames, for the three successive rotations (like a cartoon).

d. (*5 pts*) Determine the Euler type II angles, ϕ, θ, and ψ, for this rotation. Draw three pictures with the two frames, for the three successive rotations (like a cartoon).

2. (*15 points*) ANGULAR VELOCITY AND ACCELERATION

A rigid body rotates in space, and this rotation with respect to the fixed frame $\mathcal{N}$ is described by the principal axis with unit direction vector $\boldsymbol{a}$ and the principal angle ϕ. Both are functions of time t, with:

$$\boldsymbol{a}(t) = \begin{pmatrix} \sin(kt) \\ \cos(kt) \\ 0 \end{pmatrix} \quad \text{and} \quad \phi(t) = t^2, \tag{1}$$

where k is a given real constant.

a. (*5 pts*) Find the angular velocity vector ${}^{\mathcal{N}}\boldsymbol{\omega}$ of the rigid body, expressed in the fixed frame $\mathcal{N}$.

b. (*5 pts*) Find the angular acceleration ${}^{\mathcal{N}}\dot{\boldsymbol{\omega}}$ of the rigid body in the fixed frame $\mathcal{N}$.

c. (*5 pts*) Find the *relative* angular acceleration ${}^{\mathcal{N}}(\dot{\boldsymbol{\omega}})_r$ of the rigid body with respect to the body-fixed frame $\mathcal{B}$, with components expressed in the ${}^{\mathcal{N}}$-frame.

3. (*40 points*) ROLLING DISK

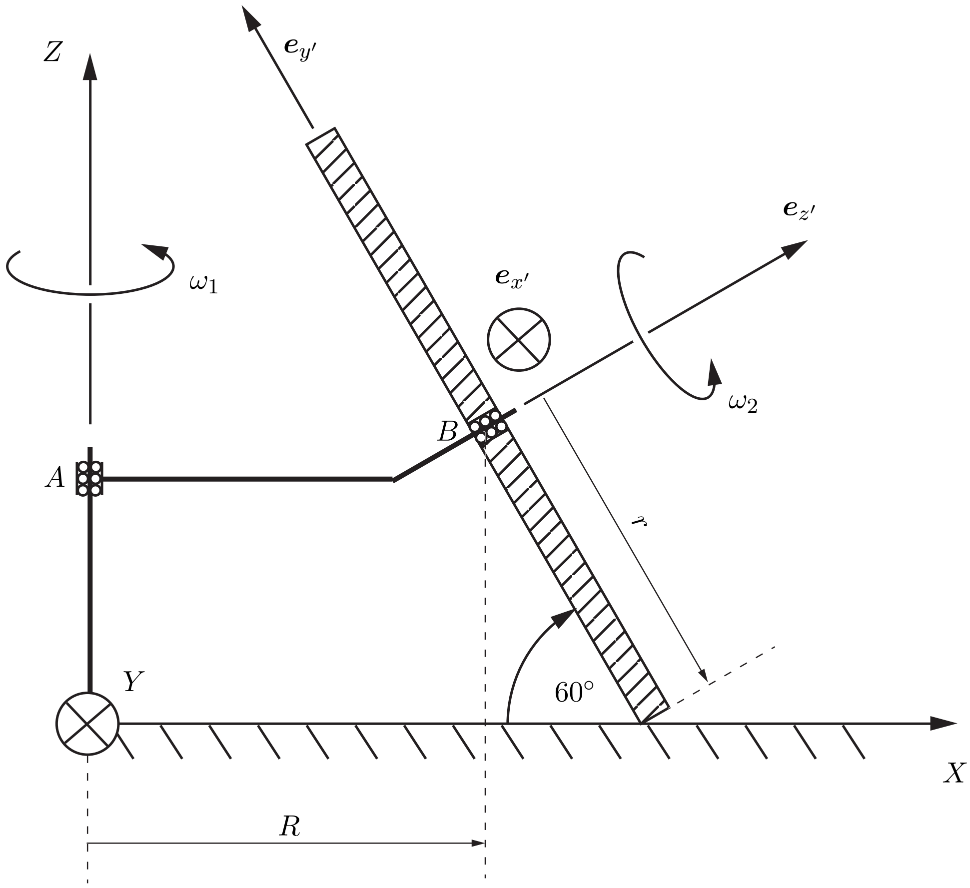

Figure 1: Rolling Disk

A thin disk is supported by a stiff arm via a ball bearing B, and it rotates around this arm with constant angular velocity of magnitude ω_2. The arm rotates around the Z-axis by means of ball bearing A, with constant angular velocity of magnitude ω_1. The directions X, Y, and Z are defined in an inertial frame, with Z pointing vertical. The directions of the unit vectors $\boldsymbol{e}_{x'}$, $\boldsymbol{e}_{y'}$, and $\boldsymbol{e}_{z'}$ are considered fixed to the disk. The disk rolls on the ground without slipping.

a. (*15 pts*) Find the rotation matrix and the type II Euler angles to map vectors expressed in the XYZ frame to their expressions in the $x'y'z'$ frame, for the configuration shown in the figure.

b. (*10 pts*) Find ω_2 in function of ω_1 and the given parameters.

c. (*10 pts*) Find the distance R from the Z-axis to point B, such that the *instantaneous* axis of rotation of the disk is horizontal.

d. (*5 pts*) Find the (principal) axis of rotation, as described by the unit vector $\boldsymbol{a}$, and the (principal) angle of rotation ϕ, for the configuration shown in the figure.

4. (*20 points*) FRAME MAPPING

Consider an inertial reference coordinate system XYZ (denoted by $\mathcal{N}$). Two further, local Cartesian coordinate systems are given: uvw (this frame is denoted by $\mathcal{B}$) and rst (this frame is denoted by $\mathcal{F}$). Expressed in the $\mathcal{N}$ frame, the unit vectors of the $\mathcal{F}$-frame are:

$$ {}^{\mathcal{N}}\boldsymbol{r} = \frac{1}{4}\begin{pmatrix} 3 \\ -2 \\ \sqrt{3} \end{pmatrix}, {}^{\mathcal{N}}\boldsymbol{s} = \frac{1}{2}\begin{pmatrix} -1 \\ 0 \\ \sqrt{3} \end{pmatrix}, {}^{\mathcal{N}}\boldsymbol{t} = \frac{-1}{4}\begin{pmatrix} \sqrt{3} \\ 2\sqrt{3} \\ 1 \end{pmatrix} \tag{2} $$

Expressed in the $\mathcal{B}$-frame, the unit vectors of the $\mathcal{F}$-frame are:

$$ {}^{\mathcal{B}}\boldsymbol{r} = \frac{-1}{12}\begin{pmatrix} 2\sqrt{3}+1 \\ 4\sqrt{3}+2 \\ 5\sqrt{3}-2 \end{pmatrix}, {}^{\mathcal{B}}\boldsymbol{s} = \frac{1}{12\sqrt{2}}\begin{pmatrix} 3\sqrt{3}-6 \\ 6\sqrt{3} \\ -6\sqrt{3}-3 \end{pmatrix}, {}^{\mathcal{B}}\boldsymbol{t} = \frac{1}{12\sqrt{2}}\begin{pmatrix} \sqrt{3}+14 \\ 2\sqrt{3}-8 \\ -2\sqrt{3}-1 \end{pmatrix} \tag{3} $$

a. (*5 pts*) Find the Euler parameter (quaternion) representations of the rotation from the $\mathcal{N}$-frame to the $\mathcal{F}$-frame, and for the rotation from the $\mathcal{F}$-frame to the $\mathcal{B}$-frame.

b. (*10 pts*) Use quaternion algebra to find the combined rotation that maps vectors ${}^{\mathcal{N}}\boldsymbol{p}$ expressed in $\mathcal{N}$ to their representation ${}^{\mathcal{B}}\boldsymbol{p}$ in $\mathcal{B}$.

c. (*5 pts*) Transform your result to find the combined rotation matrix ${}^{\mathcal{B}}\mathbf{C}_{\mathcal{N}}$ from $\mathcal{N}$ to $\mathcal{B}$ with

$$ {}^{\mathcal{B}}\boldsymbol{p} = {}^{\mathcal{B}}\mathbf{C}_{\mathcal{N}} \cdot {}^{\mathcal{N}}\boldsymbol{p}. \tag{4} $$