Advanced Dynamics (wb2630-T1)

Q1 2014

Homework No. 5

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Date: 29-Sept-2014 **Deadline:** Before lecture on Oct 6.

Instructions:

- Show all work, clearly and in order, if you want to get full credit.
- Follow the homework rules as published on Blackboard.
- Justify your answers algebraically whenever possible to ensure full credit. Sketch all relevant graphs.
- Circle or otherwise indicate your final answers.
- Clearly distinguish between scalars and vectors, for example by underlining vectors ($\underline{x}$) and not underlining scalars (x). **If you use another notation, define it like this**: A vector will be written as ...., a scalar will be written as ....
- Success!

1. (*10 points*) ELLIPSOID OF INERTIA

Consider a rigid body having an inertia matrix

$$\mathbf{I} = \begin{pmatrix} 4 & 0 & \sqrt{3} \\ 0 & 5 & 0 \\ \sqrt{3} & 0 & 6 \end{pmatrix} \text{kgm}^2 \tag{1}$$

Find the principal moments of inertia and the directions of the corresponding principal axes (without using a computer or electronic calculator).

2. (*25 points*) ROLLING DISK

A thin disk of mass m, made of homogeneous material, is supported by a stiff arm of negligible mass via a ball bearing B, and it rotates around this arm with angular velocity of magnitude $\dot{\psi}$ (Fig. 1). The arm rotates around the Z-axis by means of ball bearing A, with angular velocity of magnitude $\dot{\phi}$. The directions X, Y, and Z are defined in an inertial frame, with Z pointing vertical. The directions of the unit vectors $\boldsymbol{e}_{x'}$, $\boldsymbol{e}_{y'}$, and $\boldsymbol{e}_{z'}$ are considered fixed to the disk. The disk rolls on the ground without slipping, which implies that $\dot{\psi} = k\dot{\phi}$, with $k = -(\frac{2R+r}{2r})$. Acceleration of gravity g points in negative Z-direction. No particular relationship is assumed between R and r.

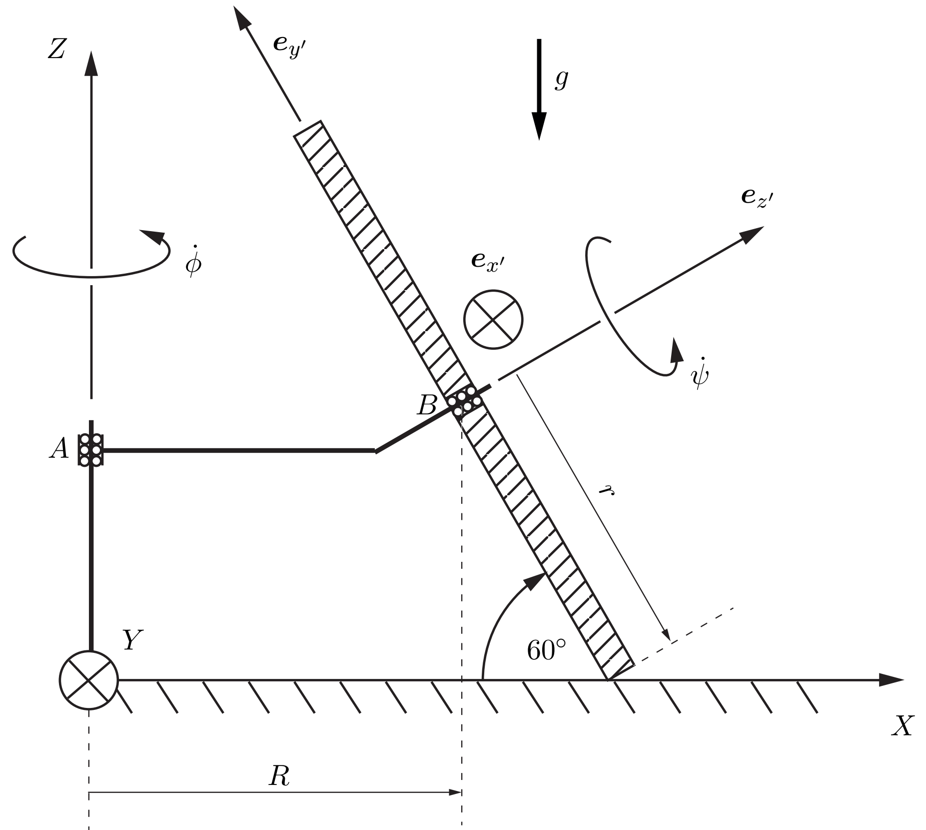

Figure 1: Rolling Disk

a. (*5 pts*) Find the angular momentum of the disk with respect to its center of mass, in the given configuration, expressed in the body-fixed coordinate system, as a function of the precession rate $\dot{\phi}$ and the given constants.

b. (*10 pts*) Draw two Free-Body Diagrams, one of the stiff arm and one of the disk, in the given configuration. Uniquely label all force and moment components and find all components (in both diagrams) that must be zero because of the bearing support and because of the fact that the arm is massless.

c. (*10 pts*) Using Newton's second law and the Euler equations (in the form of Greenwood equation 3.164), derive the equation(s) of motion for the disk, in terms of the precession acceleration $\ddot{\phi}$, and find all remaining force and moment components in your Free-Body Diagram of the disk, as functions of ϕ, $\dot{\phi}$, $\ddot{\phi}$ and the given constants.

3. (*25 points*) MODEL PLANE

A model plane of mass m is suspended from the ceiling of a room (Fig. 2) by a spring with linear characteristics. The spring has a resting length of l_0 and a spring constant k. The attachment point A at the ceiling is the origin of the fixed $\mathcal{N}$ (XYZ) coordinate system. The coordinates of the attachment point P on the plane are given as $(p_x,\, 0,\, p_z)$ in the local $\mathcal{B}$ (xyz) coordinate system that is fixed to the plane. The origin of the plane's local frame $\mathcal{B}$ is at the plane's center of mass location S, with coordinates s_X, s_Y, and s_Z in the global $\mathcal{N}$ frame. Gravity g points in positive Z-direction.

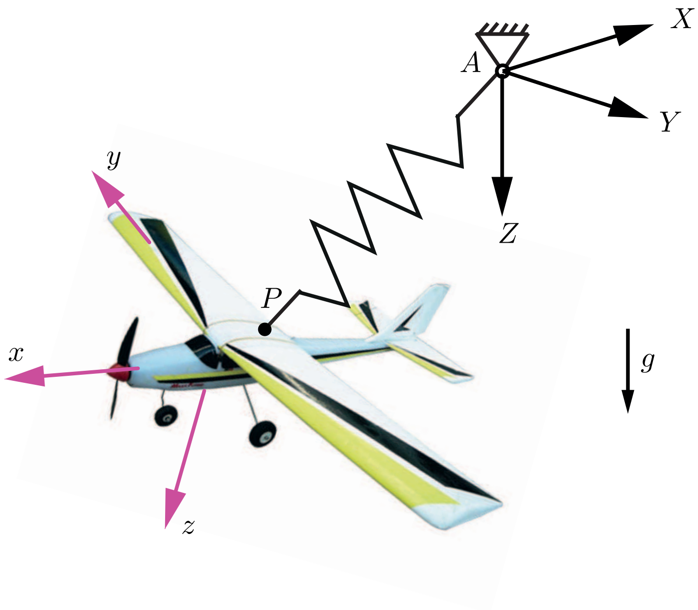

Figure 2: A model plane suspended from the ceiling.

At a time t_a, point P is at the location ${}^{\mathcal{N}}\boldsymbol{p} = \begin{pmatrix} 0 & l_0 & 2l_0 \end{pmatrix}^T$. The body-fixed coordinate directions coincide with the space-fixed coordinate directions at t_a, and the plane is momentarily only rotating about its local z-axis at a rate of ω_z. Assume the plane's mass distribution is symmetric about the body-fixed axes, and the mass moments of inertia about these axes are Ixx, Iyy, Izz. Find the angular acceleration vector ${}^{\mathcal{N}}\dot{\boldsymbol{\omega}}$ and the translational acceleration vector ${}^{\mathcal{N}}\ddot{\boldsymbol{s}}$ of the center of mass of the plane at the time t_a, as functions of the given parameters. Include a Free-Body Diagram of the plane in your reasoning.

4. (*40 points*) CONTROL MOMENT GYROSCOPE

Consider a flywheel (simplified as a thin disk of homogenous material, of mass m and radius r) rotating about its symmetry axis by a constant speed Ω and about a gimbal axis (defined by the direction vector $\boldsymbol{e}_g$) by a speed $\dot{\gamma}$ (Fig. 3). The gimbal structure is considered massless, and we do not consider the influence of gravity. The $\mathcal{F}$ frame, containing the stg axes, rotates with the gimbal, but not with the disk, such that $\boldsymbol{e}_g$ always stays parallel to $\boldsymbol{e}_z$ in the space-fixed $\mathcal{N}$-frame. The assembly is used as a control moment gyroscope (CMG), which means that it generates gyroscopic moments on its support structure when a motor rotates the flywheel about the gimbal. In the following, we will investigate this effect.

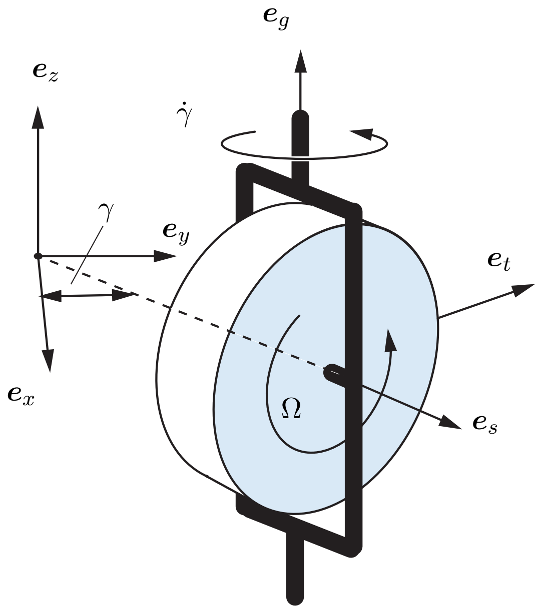

Figure 3: A control moment gyroscope (CMG) with its gimbal axis pointing in z direction. The gimbal angle γ is measured with respect to the x axis.

a. (*10 pts*) Calculate the angular momentum vector $\boldsymbol{H}$ of the disk with respect to its center of mass in the gimbal-fixed $\mathcal{F}$-frame (denoted ${}^{\mathcal{F}}\boldsymbol{H}$) and in the space-fixed $\mathcal{N}$-frame (denoted ${}^{\mathcal{N}}\boldsymbol{H}$), as a function of Ω, γ, and $\dot{\gamma}$.

b. (*10 pts*) Use the transport theorem (see slides week 1 or Greenwood equation 1.12) to calculate the derivative of angular momentum, ${}^{\mathcal{F}}\dot{\boldsymbol{H}}$, with components expressed in the gimbal-fixed $\mathcal{F}$-frame.

c. (*2 pts*) Find the external moment vector ${}^{\mathcal{F}}\boldsymbol{M}$ acting on the disk with respect to its center of mass, as function of Ω, γ, and $\dot{\gamma}$, with components expressed in the gimbal-fixed frame.

d. (*8 pts*) Apply the modified Euler equations (in the form of Greenwood equations 3.203 to 3.205) directly to check your result.

e. (*10 pts*) Assuming the gimbal axis is supported by a pair of ball bearings that are arranged symmetrical to the st-plane (so one above and one below the flywheel) and at a distance a from each other, calculate the magnitude of the resultant radial force F_r on each bearing, as a function of Ω and $\dot{\gamma}$.