TU Delft Delft University of Technology

Advanced Dynamics (wb2630-T1)

Q1 2014

Homework No. 4

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Homework Coordinator: Dr. ir. Arend L. Schwab

Date: 22-Sept-2014 **Deadline:** Before lecture on Sept 29.

Instructions:

- Show all work, clearly and in order, if you want to get full credit.
- Follow the homework rules as published on Blackboard.
- Justify your answers algebraically whenever possible to ensure full credit. Sketch all relevant graphs.
- Circle or otherwise indicate your final answers.
- Clearly distinguish between scalars and vectors, for example by underlining vectors ($\underline{x}$) and not underlining scalars (x). **If you use another notation, define it like this**: A vector will be written as ...., a scalar will be written as ....
- Success!

1. (*15 points*) KINETIC ENERGY 3D

Do problem 3.16(a) from Greenwood (without finding the mass matrix).

2. (*25 points*) INERTIA DYAD

A brick with mass m and length $2a$, width $2b$, and height $2c$, has its centre of mass located at the origin and is initially orientated with its length along the X-axis and its height along the Z-axis. The body-fixed frame $x'y'z'$ is initially aligned with the space-fixed frame XYZ.

a. (*10 pts*) Determine the inertia dyad (mass moments of inertia) of the brick in the body-fixed frame.

b. (*10 pts*) The brick is rotated by the Euler type I angles $\psi = \pi/2$, $\theta = 0$, and $\phi = \pi/2$. Determine the inertia dyad of the brick after rotation expressed in the space-fixed frame.

c. (*5 pts*) Draw the brick in the initial configuration and next to it the brick in the rotated configuration and check your result.

3. (*30 points*) ROLLING DISK

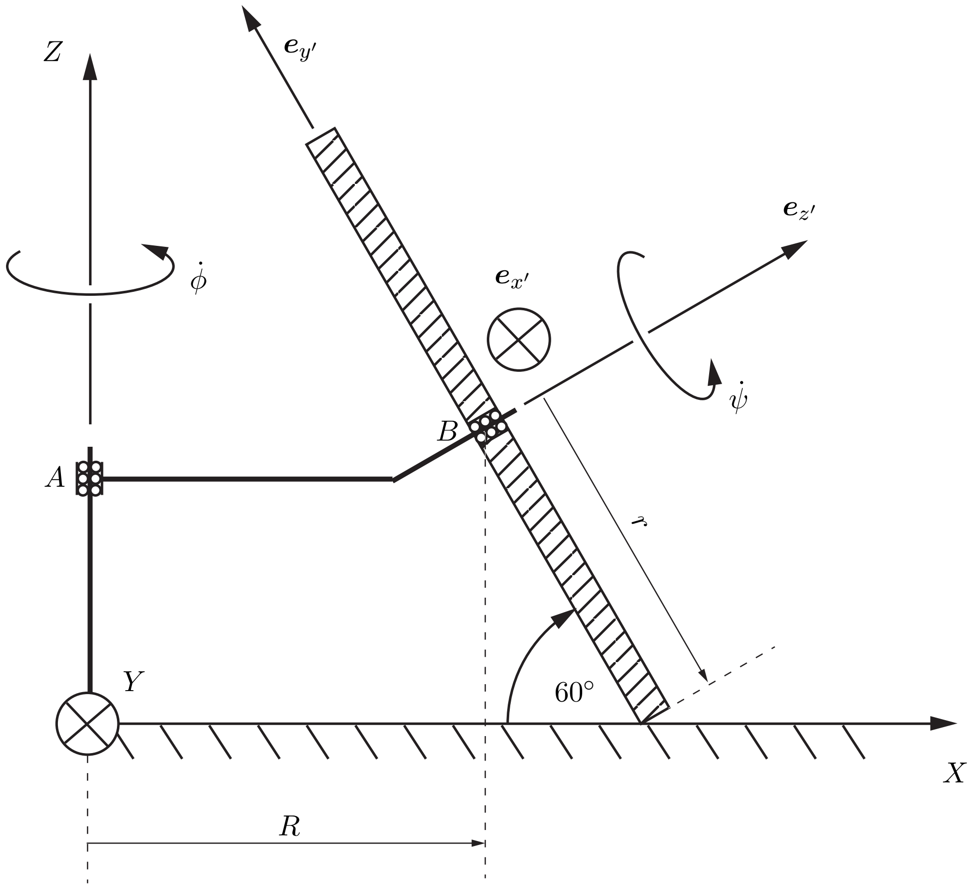

Figure 1: Rolling Disk

A thin disk of mas m, made of homogeneous material, is supported by a stiff arm of negligible mass via a ball bearing B, and it rotates around this arm with angular velocity of magnitude $\dot{\psi}$. The arm rotates around the Z-axis by means of ball bearing A, with angular velocity of magnitude $\dot{\phi}$. The directions X, Y, and Z are defined in an inertial frame, with Z pointing vertical. The directions of the unit vectors $\boldsymbol{e}_{x'}$, $\boldsymbol{e}_{y'}$, and $\boldsymbol{e}_{z'}$ are considered fixed to the disk. The disk rolls on the ground without slipping, which implies that $\dot{\psi} = -(\frac{2R+r}{2r})\dot{\phi}$. No particular relationship is assumed between R and r.

a. (*15 pts*) Find the kinetic energy contained in the disk, as a function of the precession rate $\dot{\phi}$ and the given constants.

b. (*2 pts*) What is the number of independent degrees of freedom of the disk in this system (Explain your answer)?

c. (*13 pts*) Derive the equation(s) of motion for the disk, in terms of the precession acceleration $\ddot{\phi}$.

4. (*30 points*) MODEL PLANE

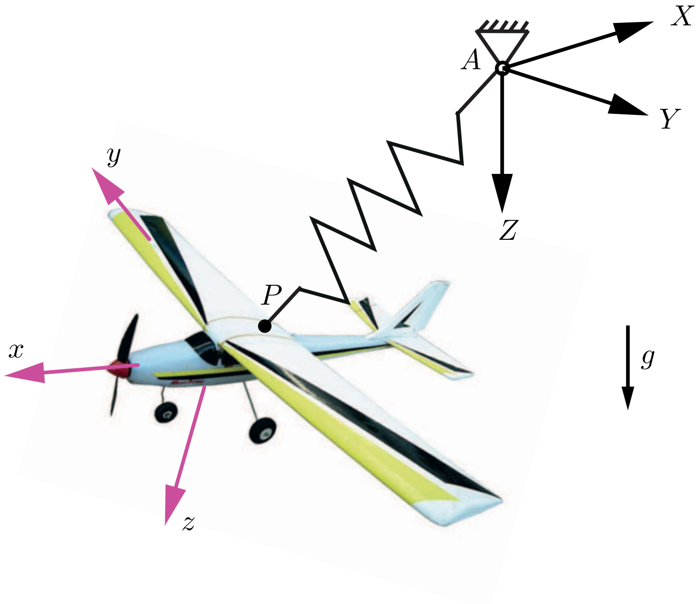

Figure 2: A model plane suspended from the ceiling.

A model plane of mass m is suspended from the ceiling of a room by a spring with linear characteristics. The spring has a resting length of l_0 and a spring constant k. The attachment point A at the ceiling is the origin of the fixed $\mathcal{N}$ (XYZ) coordinate system. The coordinates of the attachment point P on the plane are given as $(p_x,\ 0,\ p_z)$ in the local $\mathcal{B}$ (xyz) coordinate system that is fixed to the plane. The origin of the plane's local frame $\mathcal{B}$ is at the plane's center of mass location S, with coordinates s_X, s_Y, and s_Z in the global $\mathcal{N}$ frame. Gravity g points in positive Z-direction.

The plane is released at a time t_0, with point P originally at the location ${}^{\mathcal{N}}\boldsymbol{p} = \begin{pmatrix} 0 & 0 & -2l_0 \end{pmatrix}^T$ (components expressed in the space-fixed frame $\mathcal{N}$), with the plane having an initial translational speed v_0 and no angular velocity.

At a time t_e, point P is at the location ${}^{\mathcal{N}}\boldsymbol{p} = \begin{pmatrix} l_0 & 0 & -l_0 \end{pmatrix}^T$, the center of mass of the plane is momentarily at rest, and the plane is rotating only about its local y-axis. The body-fixed coordinate directions coincide with the space-fixed coordinate directions at the moment of release, t_0, and again at the considered instant t_e. Assume the plane's mass distribution is symmetric about the body-fixed axes, and the mass moments of inertia about these axes are Ixx, Iyy, Izz. Find the magnitude ω of the angular velocity vector of the plane, as a function of the given parameters.