Advanced Dynamics (wb2630-T1)

Q1 2014

Homework No. 7

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Date: 13-Oct-2014 **Deadline:** Before lecture on Oct 20.

Instructions:

- Show all work, clearly and in order, if you want to get full credit.
- Follow the homework rules as published on Blackboard.
- Justify your answers algebraically whenever possible to ensure full credit. Sketch all relevant graphs.
- Circle or otherwise indicate your final answers.
- Clearly distinguish between scalars and vectors, for example by underlining vectors ($\underline{x}$) and not underlining scalars (x). **If you use another notation, define it like this**: A vector will be written as ...., a scalar will be written as ....
- Success!

1. (*15 points*) LADDER

A ladder (homogeneous mass m, length l) is leaning against a smooth wall (Fig. 1). At its foot, which stands on smooth ground, a force F is applied. Use the Lagrange equations to find the equations of motion for this system in terms of the angle α, the force F, and the given constants, in the form of $\ddot{\alpha} = f(\alpha, \dot{\alpha}, F)$.

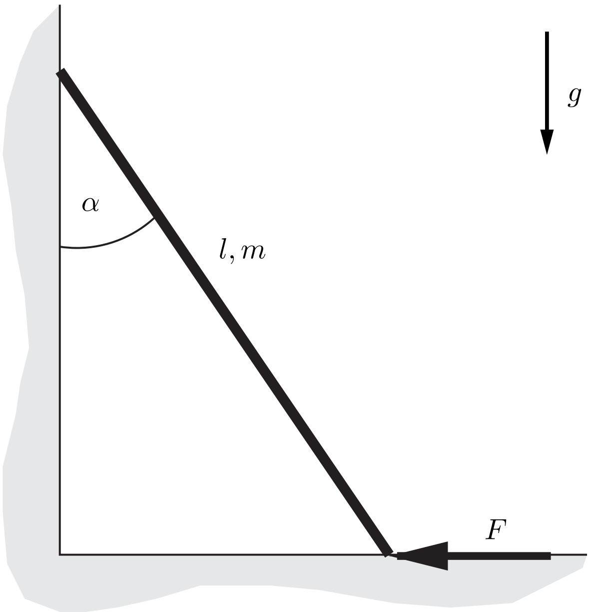

Figure 1: Ladder leaning against a wall.

2. (*15 points*) ROLLING DISK

A thin disk of mass m, made of homogeneous material, is supported by a stiff arm of negligible mass via a ball bearing B, and it rotates around this arm with angular velocity of magnitude $\dot{\psi}$ (Fig. 2). The arm rotates around the Z-axis by means of ball bearing A, with angular velocity of magnitude $\dot{\phi}$. The directions X, Y, and Z are defined in an inertial frame, with Z pointing vertical. The directions of the unit vectors $\boldsymbol{e}_{x'}$, $\boldsymbol{e}_{y'}$, and $\boldsymbol{e}_{z'}$ are considered fixed to the disk. The disk rolls on the ground without slipping, which implies that $\dot{\psi} = k\dot{\phi}$, with $k = -(\frac{2R+r}{2r})$. Acceleration of gravity g points in negative Z-direction. No particular relationship is assumed between R and r. An external force of magnitude F_e, pointing in direction of x', acts on the axle of the disk.

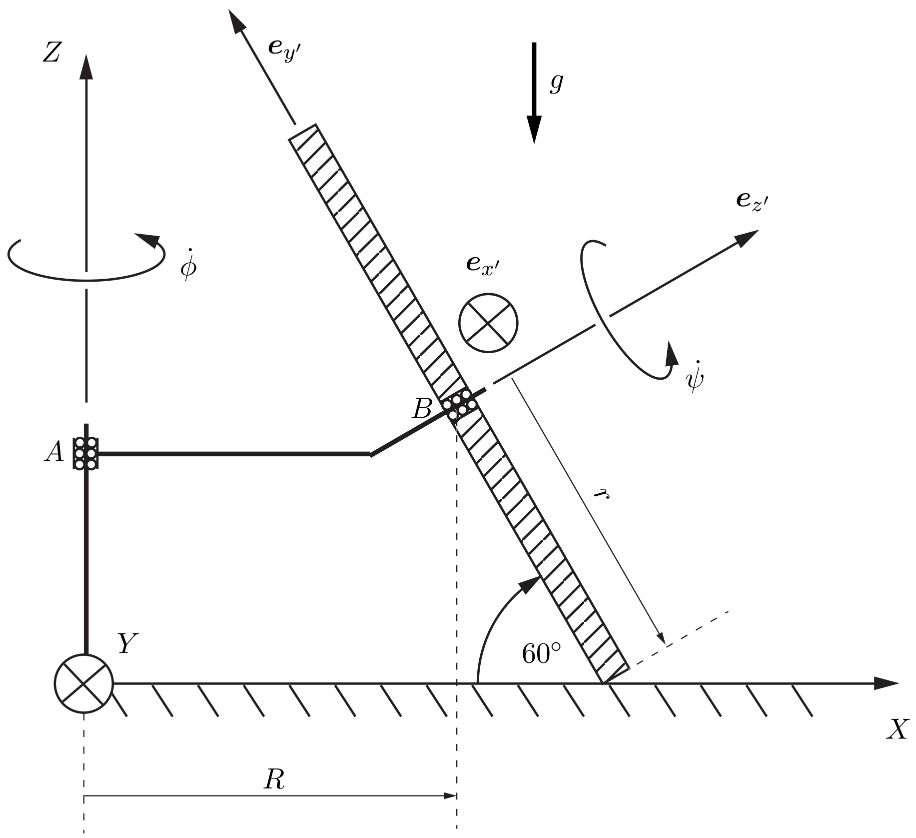

Figure 2: Rolling Disk

With ϕ as generalized coordinate, use the Lagrange equations to find $\ddot{\phi}$ as a function of ϕ, $\dot{\phi}$, the applied force F_e, and the given constants, so $\ddot{\phi} = f(\phi, \dot{\phi}, F_e)$. You may re-use your results from Homework 4, question 3a.

3. (*35 points*) TWO PARTICLES

Two particles (Fig. 3), of masses $2m$ and m, are connected by a rigid massless rod of length l. Particle 1 can slide without friction on a fixed horizontal rod. Gravity acts on the particles in downward direction.

a. (*15 pts*) Use the Lagrange equations to find the equations of motion for this system in terms of the generalized coordinates s and θ (with $\boldsymbol{q} = \begin{pmatrix} s & \theta \end{pmatrix}^T$), in the form $\ddot{\boldsymbol{q}} = \boldsymbol{f}(\boldsymbol{q}, \dot{\boldsymbol{q}})$.

b. (*20 pts*) Now change to an appropriate set of (dependent) generalized coordinates and use Lagrange multipliers to find the force that the horizontal rod exerts on particle 1. Indicate your definition of force direction clearly by means of an appropriate drawing.

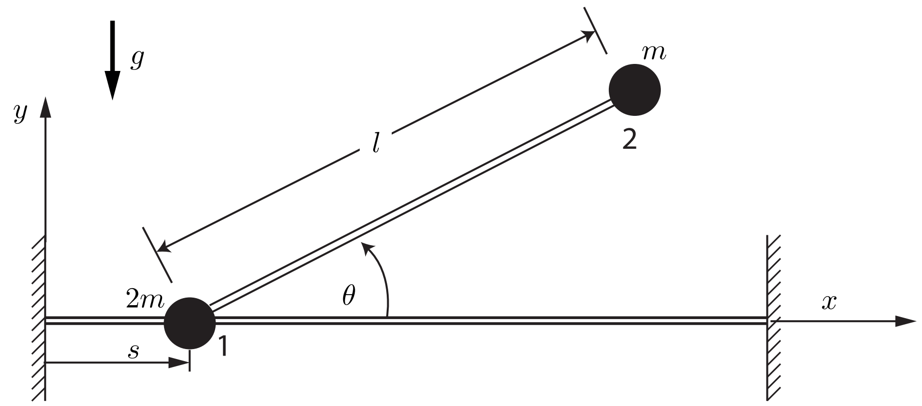

Figure 3: Two particles

4. (*35 points*) MODEL PLANE

A model plane of mass m is suspended from the ceiling of a room (Fig. 4) by a spring with linear characteristics. The spring has a resting length of $l_0 = 0\,\mathrm{m}$ (note the difference to previous homework!) and a spring constant k. The attachment point A at the ceiling is the origin of the space-fixed $\mathcal{N}$ (XYZ) coordinate system. The coordinates of the attachment point P on the plane are given as $(p_x,\, 0,\, 0)$ in the local $\mathcal{B}$ (xyz) coordinate system that is fixed to the plane. The origin of the plane's local frame $\mathcal{B}$ is at the plane's center of mass location S. Type I Euler angles ψ, θ, ϕ (convention following Greenwood) describe the plane's orientation with respect to the $\mathcal{N}$-frame. Gravity g points in positive Z-direction.

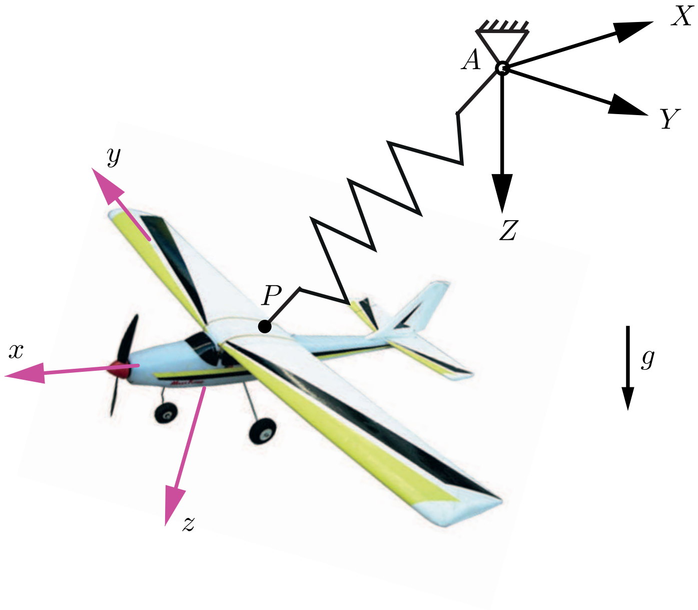

Figure 4: A model plane suspended from the ceiling.

Assume the plane's mass distribution is symmetric to the body-fixed xy and xz planes, and the mass moments of inertia about the body-fixed axes are $I_{xx} = I_{yy} = \frac{1}{2} I_p$, $I_{zz} = I_p$.

Consider a vector $\boldsymbol{q} = \begin{pmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 \end{pmatrix}^T$ of generalized coordinates that contains the Cartesian coordinates $q_1 = s_X$, $q_2 = s_Y$, and $q_3 = s_Z$ of the plane's center of mass, expressed in the space-fixed $\mathcal{N}$ frame, and the Euler angles $q_4 = \psi$, $q_5 = \theta$, and $q_6 = \phi$ describing the plane's orientation with respect to the $\mathcal{N}$-frame. Use the Lagrange equations to find the equations of motion in the (implicit) form $\boldsymbol{f}(\ddot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \boldsymbol{q}) = \boldsymbol{0}$.