Advanced Dynamics (wb2630-T1)

Q1 2014

Homework No. 2

Main Instructor: Dr.-Ing. Heike Vallery
Homework Coordinator: Dr. ir. Arend L. Schwab

Date: 08-Sept-2014 **Deadline:** Before lecture on Sept 15.

Instructions:

- Show all work, clearly and in order, if you want to get full credit.
- Follow the homework rules as published on Blackboard.
- Justify your answers algebraically whenever possible to ensure full credit. Sketch all relevant graphs.
- Circle or otherwise indicate your final answers.
- Clearly distinguish between scalars and vectors, for example by underlining vectors ($\underline{x}$) and not underlining scalars (x). **If you use another notation, define it like this**: A vector will be written as ...., a scalar will be written as ....
- Success!

1. (*15 points*) FRAME MAPPING

Consider an inertial reference coordinate system XYZ. Two further, local Cartesian coordinate systems are given: uvy and rst. Expressed in the XYZ system, the unit vectors of the two local coordinate systems are:

$$\boldsymbol{u} = \frac{1}{3}\begin{pmatrix}1\\2\\-2\end{pmatrix}, \boldsymbol{v} = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}, \boldsymbol{w} = \frac{1}{3\sqrt{2}}\begin{pmatrix}4\\-1\\1\end{pmatrix} \tag{1}$$

and

$$\boldsymbol{r} = \frac{1}{4}\begin{pmatrix}3\\-2\\\sqrt{3}\end{pmatrix}, \boldsymbol{s} = \frac{1}{2}\begin{pmatrix}-1\\0\\\sqrt{3}\end{pmatrix}, \boldsymbol{t} = \frac{-1}{4}\begin{pmatrix}\sqrt{3}\\2\sqrt{3}\\1\end{pmatrix} \tag{2}$$

Find the rotation matrix that describes the orientation of the uvw frame relative to the rst frame, and the rotation matrices that map vectors in the XYZ frame into respective uvw or rst frame vectors.

2. (*30 points*) MODEL PLANE

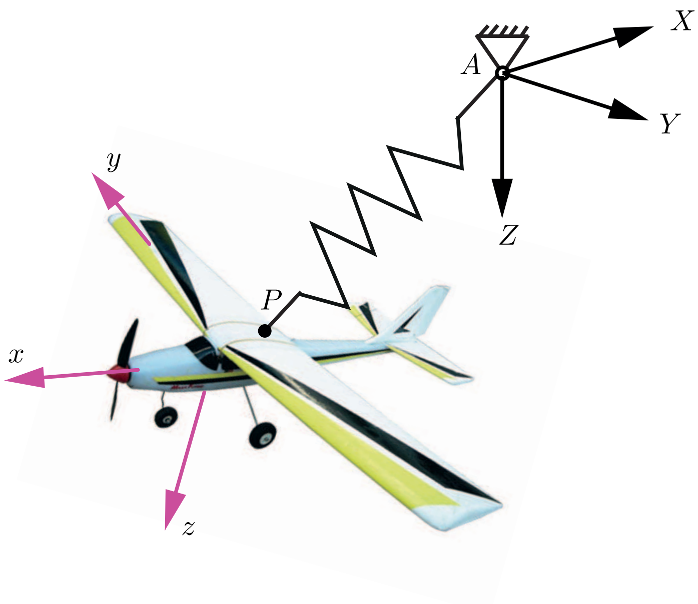

Figure 1: A model plane suspended from the ceiling.

A model plane is suspended from the ceiling of a room by a spring. The spring has a resting length of l_0. The attachment point A at the ceiling is the origin of the fixed XYZ coordinate system. The coordinates of the attachment point P on the plane are given as $(p_x, 0, p_z)$ in the local xyz coordinate system that is fixed to the plane. Assuming that the plane's orientation with respect to the XYZ coordinate system is described by type I Euler angles, with $\psi = \pi$, $\theta = \pi/3$, and $\phi = -\pi/4$, and that the origin of the plane's local coordinate system is at the position S, with coordinates s_X, s_Y, and s_Z in the global XYZ coordinate system, find the potential energy momentarily stored in the spring in function of the given parameters.

3. (*30 points*) GYRO BACKPACK

Falls are among the most frequent causes of hospitalization and death among the elderly. A key factor leading to falls is degraded balance control capability. At TU Delft, we are investigating a wearable robotic solution for patients with balance disorders, which is based on an assembly of control moment gyroscopes that is mounted onto the upper body via a backpack. This assembly of gyroscopes can generate a controllable moment vector on the person.
In this problem, we will consider the movement of the human (Fig. 2), pivoting about the ankle joint while standing on a flat surface.

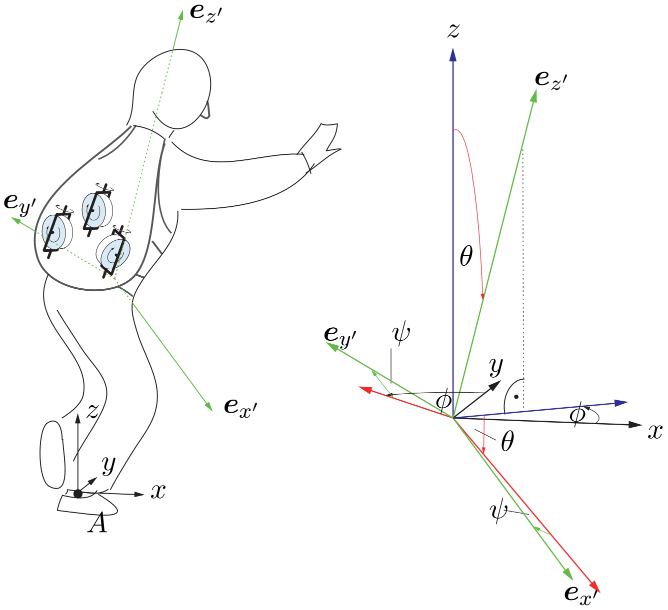

Figure 2: Gyroscopic support for human balance. The directions x, y, and z are defined with respect to an inertial frame, with z pointing vertical. The directions x', y', and z' are defined with respect to the human trunk, with x' pointing in anterior direction, which is aligned with the x axis if all angles are zero, and z' pointing along the human's longitudinal axis. Successive rotations by the Euler angles ϕ, θ, and ψ map the x, y, z coordinate system to the x', y', z' coordinate system.

The origin of the body-fixed coordinate system is in the center of mass of the body.
At a certain instant in time which we call $t = 0$, the orientation of the body-fixed coordinate system expressed in the global xyz reference system is given by $\mathbf{e}'_x = (0.7615, -0.6304, -0.1504)$, $\mathbf{e}'_y = (0.6454, 0.7589, 0.0868)$, and $\mathbf{e}'_z = (0.0594, -0.1632, 0.9848)$.
The initial angular velocity vector at $t = 0$ expressed in the global reference coordinate system is given by $\boldsymbol{\omega} = (-0.0297, 0.0816, -0.4924)$ rad/s.

a. (*5 pts*) Draw a cans-in-series representation as equivalent alternative for Fig. 2, right.

b. (*5 pts*) Determine for $t = 0$ the rotation matrix $\mathbf{R}$ which transforms the body-fixed coordinates $\boldsymbol{x}'$ into the reference system coordinates $\boldsymbol{x}$ as in $\boldsymbol{x} = \mathbf{R}\boldsymbol{x}'$.

c. (*10 pts*) Determine for $t = 0$ from this $\mathbf{R}$ the associated Euler angles (zxz) ϕ, θ, and ψ.

d. (*5 pts*) Determine for $t = 0$ the angular velocity vector $\boldsymbol{\omega}'$ expressed in the body-fixed coordinate system.

e. (*5 pts*) Determine for $t = 0$ the rate of change of the Euler angles: $(\dot{\phi}, \dot{\theta}, \dot{\psi})$.

4. (*25 points*) BRYANT ANGLES

Yet another way of describing orientation by Euler angles are the so-called Bryant angles. Here we follow the notation used by Greenwood Ch3.1. The order of rotation for Bryant angles is as follows: (1) α about the X-axis; (2) β about the y'-axis; (3) γ about the z''-axis.

a. (*10 pts*) Derive the overall rotation matrix $\mathbf{C}$ for these Bryant angles.

b. (*10 pts*) Express the time derivates of these Bryant angles in terms of the Bryant angles α, β, γ, and the components of the absolute angular velocity ω_x, ω_y and ω_z.

c. (*5 pts*) At which set of Bryant angles does the singular configuration occur? Draw a figure to check this result.