South Dakota School of Mines and Technology

Advanced Topics in AI, Fall 2022

CSC 549 - M01

Exam 2



1. Consider the following pseudocode for a faulty SARSA algorithm. Find all the mistakes in the algorithm. Explain why they are mistakes and correct them.

The image below shows the faults I have identified in the SARSA Algorithm.

```
procedure SARSA( number of episodes N \in \mathbb{N}
                          discount factor \lambda \in (0,1]
                          learning rate \alpha_n = \frac{1}{\log(n+1)})
    Initialize matrices Q(s,a) and n(s,a) to 0, \forall s,a
    for episode k \in 1, 2, 3, \dots, n do
         t \leftarrow 1
         Initialize s_1
         Choose a_1 from a uniform distribution over the actions (2)
         while Episode k is not finished do
              Take action a_t: observe reward r_t and next state s_{t+1}
             Choose a_{t+1} from s_{t+1} using \mu_t: an \varepsilon-greedy policy with respect to Q
              if The current state is terminal then

    Compute target value

                                                  y_t = 0 Terminal reward should be
              else
                                                                       This is off-policy, but Sarsa should be on-policy
                                       y_t = r_t + \max_a Q(s_{t+1}, a)
              end if
             n(s_t, a_t) \leftarrow n(s_t, a_t) + 1
              Update Q function:
                      Q(s_{t+1}, a_{t+1}) \leftarrow Q(s_t, a_t) - \alpha_{n(s_t, a_t)} (y_t - Q(s_t, a_t))
         end while
    end for
end procedure
```

- (i) The learning rate ($\alpha_n=\frac{1}{log(n+1)}$) would be greater than 1 if the base of the log term is not **2**.
- (ii) Choosing a_1 from a uniform distribution over the actions would result in an off -policy action which is not what we want. A more appropriate choice would be to **choose** a_1 **from** s_1 **using the policy** μ_t : an ϵ -greedy policy w.r.t. Q.
- (iii) This algorithm contains all the characteristics of a SARSA (λ) algorithm; so I feel it is safe to assume that this algorithm was designed to replicate a SARSA (λ) algorithm. What is missing here is the backtracking update (**Eligibility Trace Update**). The corrections for (3) are shown below:

For all
$$s \in S$$
, $a \in A(s)$:
$$Q(s,a) \leftarrow Q(s,a) + \alpha \ y_t \ n(s,a)$$

$$n(s,a) \leftarrow \lambda n(s,a)$$

^{*} here we assumed that $\lambda = 1$

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2. Your friend found a variant of SARSA which is defined through a sequence of policies π_t (where t >= 1), and consist of just changing (in the previous algorithm after corrections) the way the target is computed. The target becomes:

$$y_t = r_t + \lambda \sum_{a} \pi_t(a | s_{t+1}) Q(S_{t+1}, a)$$

Where $\pi_t(a \mid s)$ is the probability that a is selected in state s under policy π_t .

- a) What sequence of policies (π_i) should you choose so that the corresponding variant of SARSA is on-policy? The variant is called Expected SARSA.
- A. Expected SARSA would still be on-policy if the chosen action (a) by π_t resulted in the largest value of $\pi_t(a \mid s)$ $Q(S_{t+1}, a)$ when compared to other actions in the state. Or in other words, a chosen should reflect the dominant $\pi_t(a \mid s)$ $Q(S_{t+1}, a)$ in the term $\sum_a \pi_t(a \mid s_{t+1})$ $Q(S_{t+1}, a)$. This would only happen if the sequence of

policies followed are determined by an ϵ -greedy policy w.r.t. Q where ϵ = 1.

- b) Consider an off policy variant of SARSA corresponding to a stationary policy $\pi = \pi_t \forall t$. Under this algorithm, do the Q values converge? If so, what are the limiting Q values? Justify your answer.
- A. In the trivial case, if the stationary policy being followed doesn't reach the terminal state, the Q values will not converge as states which are looped over would have a Q value of $-\infty$.

The Q values would converge and would be dictated by the Q value of the terminal state. The Q values for each state would converge to the sum of the Q values of the next state plus the reward for taking the action that leads to that next state. For example, the Q value of the state just before the Terminal State in the policy would

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have a limiting Q value of the Terminal State (0) plus the the reward for taking the action that led to the Terminal State (-1) which equals to -1.

My thought process behind this was based of value iteration from Chapter 4. When the episodes reach infinity, the update $(\alpha(y_t - Q(s_t, a_t)))$ should tend to 0. And from observing the grid world problem, the stable value of each state in the policy was somewhat of the form:

$$Q(s_t, a_t) = Q(s_{t+1}, a_t) + r_t$$

And since the terminal states Value function is fixed at 0, it dictates the values of all preceding states by this equation.