Intersection of Two Lines

Given two lines:

Line 1: $a_1x + b_1y + c_1 = 0$

Line 2: $a_2x + b_2y + c_2 = 0$

Two lines are identical if:

$$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$$

Two lines are parallel if:

$$\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$$

If two lines intersect, then intersection coordinate is given by:

$$x = \frac{\begin{bmatrix} b1 & b2 \\ c1 & c2 \end{bmatrix}}{\begin{bmatrix} a1 & a2 \\ b1 & b2 \end{bmatrix}} \quad y = \frac{\begin{bmatrix} c1 & c2 \\ a1 & a2 \end{bmatrix}}{\begin{bmatrix} a1 & a2 \\ b1 & b2 \end{bmatrix}}$$

Intersection of Two Planes

Given two planes:

Plane 1: $A_1x + B_1y + C_1z + D_1 = 0$

Plane 2: $A_2x + B_2y + C_2z + D_2 = 0$

The normal vector of Plane 1 is n1 = (A_1, B_1, C_1) and Plane 2 is n2 = (A_2, B_2, C_2)

Two planes are identical if:

$$\frac{A1}{A2} = \frac{B1}{B2} = \frac{C1}{C2} = \frac{D1}{D2}$$

Two planes are parallel if:

$$\frac{A1}{A2} = \frac{B1}{B2} = \frac{C1}{C2} \neq \frac{D1}{D2}$$

If Two Planes intersect, then:

One of the coordinates could be set to zero, say C1 & C2 = 0 and then solve for the other two but this will only work if intersection line L intersects with the plane z = 0. It will be true when the z-coordinate of normal vector is non-zero. So, we have to select a nonzero coordinate of normal vector and then set the corresponding coordinate of P1 and P2 to 0.

If the intersection line L passes through z-axis then z coordinate can be set to zero.

So the equation is now $A_1x + B_1y + D_1 = 0$ and $A_2x + B_2y + D_2 = 0$

$$x = \frac{\begin{bmatrix} b1 & b2 \\ d1 & d2 \end{bmatrix}}{\begin{bmatrix} a1 & a2 \\ b1 & b2 \end{bmatrix}} \quad y = \frac{\begin{bmatrix} d1 & d2 \\ a1 & a2 \end{bmatrix}}{\begin{bmatrix} a1 & a2 \\ b1 & b2 \end{bmatrix}}$$

The vector notation of intersection line L is given by if intersection line passes through z axis, then a1*b2 - b1*a2 ≠ 0

$$L = \left(\frac{\begin{bmatrix} b1 & b2 \\ d1 & d2 \end{bmatrix}, \begin{bmatrix} d1 & d2 \\ a1 & a2 \end{bmatrix}, 0}{\begin{bmatrix} a1 & a2 \\ b1 & b2 \end{bmatrix}}\right) + s(n1 * n2)$$

The vector notation of intersection line L is given by if intersection line passes through y axis: then a1*c2 - c1*a2 ≠ 0

$$L = \left(\frac{\begin{bmatrix} c1 & c2 \\ d1 & d2 \end{bmatrix}, 0, \begin{bmatrix} d1 & d2 \\ a1 & a2 \end{bmatrix}}{\begin{bmatrix} a1 & a2 \\ c1 & c2 \end{bmatrix}}\right) + s(n1 * n2)$$

The vector notation of intersection line L is given by if intersection line passes through x axis: then b1*c2 - b2*c1 ≠ 0

$$L = \left(\frac{0, \begin{bmatrix} c1 & c2 \\ d1 & d2 \end{bmatrix}, \begin{bmatrix} d1 & d2 \\ b1 & b2 \end{bmatrix}}{\begin{bmatrix} b1 & b2 \\ c1 & c2 \end{bmatrix}}\right) + s(n1 * n2)$$

Result

Line Intersection Output

- line_1 = Line 0 1 1
  line_2 = Line 0 6 6

  Output:

```
*Main> line
Given two lines are identical. Hence infinite solutions exist
```

- line_1 = Line 2 5 6
  line_2 = Line 8 20 20

  Output:

```
*Main> line
Given Two Lines are Parallel
```

- line_1 = Line 2 0 6
  line_2 = Line 0 2 5

  Output:

```
*Main> line
Point2D {xCoord = -3.0, yCoord = -2.5}
```

Plane Intersection Output

- plane_1 = Plane 2 1 2 4

plane_2 = Plane 4 2 4 8

Output:

```
*Main> plane
Given Planes are identical
```

- plane_1 = Plane 2 1 2 4
  plane_2 = Plane 4 2 4 5

  Output:

```
*Main> plane
Given Planes are Parallel
```

- plane_1 = Plane 0 1 1 4
  plane_2 = Plane 4 0 5 2

  Output:

```
*Main> plane
ParamerticLine {point = Point3D {x = -0.5, y = -4.0, z = 0.0}, parameter = Point3D {x = -5.0, y = -4.0, z = 4.0}}
```

Here the output is in the form of Parametric Line equation i.e. **a + ƛ*b** where a = point and b = parameter. Thus equation of line is (-0.5, -4.0, 0.0) + ƛ* (-5.0, -4.0, 4.0) where ƛ is a real number.

References

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