Workshop 2

Problem 1.20

Consider N similar antennas emitting linearly polarized electromagnetic radiation of wavelength λ and velocity c. The antennas are located along the x axis at a speparation λ from each other. An observer is located on the x axis at a great distance fromt eh antennas. When a *single* antenna radiates, the observer measures an *intensity* (i.e., mean-square electric-field amplitude) equal to I.

(a) If all the antennas are driven in phase by the same generator of frequency $\nu = \frac{c}{\lambda}$, what's the total intensity measured by the observer?

Hint: Superposition of waves

The antennas add in phase so that the total amplitude is $E_t = NE$, and since the intensity is proportional to E_t^2, $I_t = N^2 I$.

(b) If the antennas all radiate at the same frequency $\nu = \frac{c}{\lambda}$ but with completely random phases, what's the mean intensity measured by the observer?

Hint: use vector to represent different direction of amplitudes

We are looking for $\bar{E_t^2}$.

Each amplitude from each antenna has either a positive amplitude or a negative amplitude. We use $\hat{s_i}$ to stand for the direction of the i^{th} signal's amplitude. Thus, E_t can be written as $E_t = \sum_{i=1}^{N} E\hat{s}_i$.

Thus

$$\bar{E_t^2} = \overline{\sum_{i=1}^{N} (E\hat{s}_i)^2} = E^2 \sum_{i}^{N} \bar{\hat{s_i}}^2 + E^2 \sum_{i} \sum_{j \neq i} \overline{\hat{s}_i \cdot \hat{s}_j} \tag{1}$$

Since the phases are random so $\hat{s}_i \cdot \hat{s}_j$ should have equal probability of being 1 or -1. Thus, summing them up, we should get 0.

$\hat{s_i}^2$ should just be 1. After being summed up N times, it's N. Thus, in the end, The mean value of E_t^2 should be NE^2. In another word, the total intensity $I_t = NI$.

Problem 1.21

***Simplified*: Radar signals are emitted and reflected. The returning signals are picked up by a machine as a faint signal with definite amplitude a_s. However, a random fluctuating signal, for some complicated reasons, is also picked up by the machine with amplitude a_n. The machine thus registered a total amplitude of $a = a_s + a_n$. $\bar{a_n} = 0$ since a_n is equally likely to be positive as negative. However, there is considerable probability that a_n attains values considerably in excess of a_s; i.e., the root-mean-square amplitude $\overline{(a_n^2)}^{\frac{1}{2}}$ can be considerably greater than the signal a_s of interest. Suppose that the rms amplitude is $\overline{(a_n^2)}^{\frac{1}{2}} = 1000a_s$. Then the fluctuating signal a_n constitues a background of "noise" makes observation of desired echo signal essentially impossible.**

Suppose that N such signals are sent out in succession and that the total amplitudes a picked up at the machine after the signals are all added together before being displayed on the machine. The resulting amplitude must then have the form $A = A_s + A_n$, where A_n is the noice amplitude (with $\bar{A_n} = 0$) and $\bar{A} = A_s$ represents the resultant echo-signal amplitude.

How many signals must be sent out before rms amplitude reach A_s, i.e. $\overline{(A_n^2)}^{\frac{1}{2}} = A_s$ so that the signal becomes detectable?

$$\overline{A_n^2} = \sum_i^N \overline{a_{n_i}^2} + \sum_i^N \sum_{j \neq i}^N \bar{a_{ni}} \bar{a_{nj}} \tag{2}$$

Since $\bar{a_n} = 0$, the second term on RHS vanish, so

$$\overline{A_n^2} = N\overline{a_{n_i}^2} \tag{3}$$

We also know $\overline{(a_n^2)}^{\frac{1}{2}} = 1000a_s$, so

$$\overline{A_n^2}^{\frac{1}{2}} = N^{\frac{1}{2}}(\overline{a_{n_i}^2})^{\frac{1}{2}} = N^{\frac{1}{2}}1000a_s \tag{4}$$

However,

$$\overline{A_n^2}^{\frac{1}{2}} = A_s = Na_s \tag{5}$$

Thus, we find that $Na_s = N^{\frac{1}{2}}1000a_s \longrightarrow N = 10^6$.

Problem 1.24

(a) A particle is equally likely to lie anywhere on the circumference of a circle. Consider any straight line in the plane of the circle and passing through its center. Denote by the θ the angle between the z axis and the straight line connecting the center of the circle to the particle. What's the probability that this angle lies between θ and $\theta + d\theta$?

Hint: In another word, what's the probability of lying between θ and $\theta + d\theta$ out of 2π?

$$W(\theta)d\theta = \frac{d\theta}{2\pi} \tag{6}$$

(b) A particle is equally likely to lie anywhere on the surface of a sphere. Consider any line through the center of this sphere as the z axis. Denote by θ the angle between this z axis and the straight line connecting the center of sphere to the particle. What's the probability that this angle lies between θ and $\theta + d\theta$?

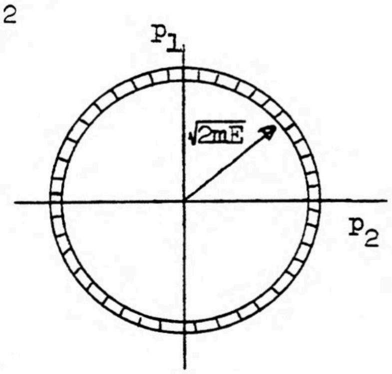
Hint: draw the picture of places where they satisfy to have angle θ between z axis and the line go through these places and the origin. Then draw a region that's $d\theta$.

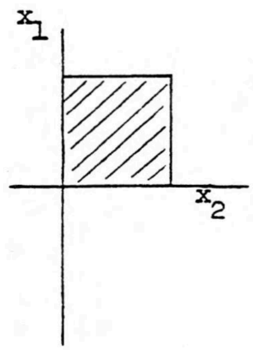
$$W(\theta)d\theta = \frac{2\pi a sin(\theta) a d\theta}{4\pi a^2} = \frac{\sin(\theta)}{2} \tag{7}$$

Problem 2.2

Consider a system consisting of two weakly interacting particles, each of mass m and free to move in one direction. Denote the respective postition corrdinates of the two particles by x_1 and x_2, their respective momenta by p_1 and p_2. The particles are confined in a box with end walls located at x = 0 and x = L. The total energy of the system is known to lie between E and $E + \delta E$. Since it's difficult to draw a 4D phase space, draw seperately the part of the phase space invovling x_1 and x_2, p_1 and p_2. Indicate on these diagrams the regions of phase space accessible to the system.

2.2





Since $p_1^2 + p_2^2 = 2mE$, the radius in p space is $\sqrt{2mE}$.