workshop 6

problem 5.2

See book.

> 💡 **Tip**
>
> $1\, dyne \cdot cm = 10^{-7} J$

Soln:

a) $W = \int p\,dv \fallingdotseq 314\,J$

b) Since there's only 1 mole, $pV = RT$.

$\Delta E = c_V \Delta T = \frac{3}{2} R \left(\frac{P_C V_C}{R} - \frac{P_A V_A}{R} \right)$

$= \frac{3}{2} (6-2) \times 10^9$ dynes $\cdot$ cm

$= 600\,J$.

c) $Q = \Delta E + W$ where ΔE is as calculated in (b), which is 600J.

W = area under curve

$= 400 + \frac{100}{2} \pi$

$\fallingdotseq 557\,J$

so $Q = 600 + 557 = 1157\,J$

problem 5.5

See book.

Soln:

a) Thermally insulated $\Rightarrow$ $Q = 0$

so $\Delta E = -W$

since $V \uparrow$, the gas is doing work to the piston, $\Delta E \downarrow$

since $\underbrace{E \propto T}$, when $E \downarrow$, T also $\downarrow$.

(proved in Eqn 3.12.11 in book).

Thus $T \downarrow$.

b). Entropy of gas $\uparrow$ since process is irreversible.

c). $\Delta E = -W = -mgh = -mg \dfrac{V_f - V_0}{A}$

$\Delta E = \nu C_v (T_f - T_0)$

$$\nu C_v (T_f - T_0) = -mg \frac{V_f - V_0}{A}.$$

In equilibrium, $p = \dfrac{mg}{A}$, $pV_f = \nu R T_f$ $\Rightarrow$ $V_f = \dfrac{\nu R T_f}{mg} A$

$$\nu C_v (T_f - T_0) = -mg \frac{\frac{\nu A R T_f}{mg} - V_0}{A}$$

$$= -\nu R T_f + \frac{mg V_0}{A}$$

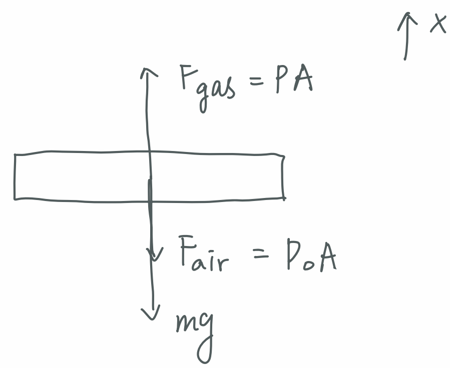
$$\nu (C_v + R) T_f = \nu C_v T_0 + \frac{mg V_0}{A}$$

$$\Rightarrow T_f = \frac{\nu C_v T_0 + \frac{mg V_0}{A}}{\nu (C_v + R)} = \frac{T_0 + \frac{mg V_0}{\nu C_v A}}{1 + \frac{R}{C_v}}$$

problem 5.6

> **Note**
>
> This is a variant of the Ruchardt experiment. I don't know if it's mentioned in class but see following video for more information.
>
> https://www.youtube.com/watch?v=e1VDAa4ttOc&ab_channel=Caltech%27sFeynmanLectureHall

Soln:



EOM: $\quad m\ddot{x} = PA - mg - P_0 A$

since the process is approximately adiabatic,

$$pV^\gamma = \text{constant} = \left(P_0 + \frac{mg}{A}\right) V_0^\gamma \quad \text{as it was originally.}$$

also $\quad m\ddot{x} = \left(P_0 + \frac{mg}{A}\right)\left(\frac{V_0}{V}\right)^\gamma A - mg - P_0 A$

$$= \left(P_0 + \frac{mg}{A}\right)\left(\frac{V_0}{Ax}\right)^\gamma A - mg - P_0 A \quad ①$$

$x = \frac{V_0}{A} + \eta$ where η is the small perturbation displacement

Expand about $\frac{V_0}{A}$:

$$\frac{1}{x^\gamma} = \frac{1}{\left(\frac{V_0}{A} + \eta\right)^\gamma}$$

Expansion of $(1-x)^{-n} = 1 + (-n)(-x)^1 + \frac{-n(-n-1)}{2!}(-x)^2 + \cdots$

$$= \frac{1}{\left(1 + \eta\frac{A}{V_0}\right)^\gamma}\left(\frac{A}{V_0}\right)^\gamma$$

$$= \left(\frac{A}{V_0}\right)^\gamma \left(1 - n\eta\frac{A}{V_0} + \cdots\right) \simeq \left(\frac{A}{V_0}\right)^\gamma \left(1 - n\eta\frac{A}{V_0}\right)$$

put it back to ① we have

$$m\ddot{x} = m\ddot{\eta} = (P_0 + \frac{mg}{A})(\frac{V_0}{Ax})^{\gamma} A - mg - P_0 A$$

$$= (P_0 + \frac{mg}{A}) \frac{AV_0^{\gamma}}{A^{\gamma}} \frac{A^{\gamma}}{V_0^{\gamma}} (1 - \gamma\eta\frac{A}{V_0}) - mg - P_0 A$$

$$= \cancel{P_0 A} + \cancel{mg} - \cancel{mg} - \cancel{P_0 A} - (P_0 + \frac{mg}{A}) \gamma\eta\frac{A}{V_0}$$

$$\Rightarrow m\ddot{\eta} = -(P_0 + \frac{mg}{A}) \gamma\eta\frac{A^2}{V_0}$$

Recall $\ddot{x} + \omega^2 x = 0$

so $\nu = \frac{\omega}{2\pi} = \frac{\sqrt{(P_0 + \frac{mg}{A})\frac{\gamma A^2}{V_0 m}}}{2\pi}$

$$\gamma = \frac{(2\pi\nu)^2 V_0 m}{(P_0 + \frac{mg}{A}) A^2} = \frac{4\pi^2\nu^2 V_0 m}{P_0 A^2 + mgA}$$