States of interest

- Side force Y
- Rolling moment l
- Yawing moment n

States to change

- Roll rate p
- Sideslip β
- Yaw rates r

Input parameters

C_{Y_p}

z_v : Distance from cg to vertical tail MAC in z direction

l_v : Distance from cg to vertical tail MAC in x direction

b_v : vertical tail span

r_1 : Fuselage depth at quarter chord of vertical

Roll rate derivatives

C_{Y_p}

distance from cg to vertical tail mean aerodynamic center

$$C_{Y_p} = 2\left(\frac{z_v \cos\alpha - l_v \sin\alpha}{b}\right) C_{Y_{\beta_v}} \quad (10.5) \text{ (Pg 42)}$$

From graph 10.12 k_v against $\frac{b_v}{2r_1}$ ← vertical tail span; fuselage depth at quarter chord of vertical tail

Area of single vertical tail

$$C_{Y_{\beta_v}} = -k_v \left(C_{L_{\alpha_v}}\right)\left(1 + \frac{d\sigma}{d\beta}\right)\eta_v \left(\frac{S_v}{S}\right) \quad (10.28) \text{ (Pg 11)}$$

estimated by 8.1.3.2 or found by controls team

wing quarter chord sweep angle; wing distance to fuselage centerline; wing aspect ratio

$$\left(1 + \frac{d\sigma}{d\beta}\right)\eta_v = 0.724 + 3.06\left[\left(\frac{S_v}{S}\right) \Big/ \left(1 + \cos\Lambda_{c/4}\right)\right] + 0.4\frac{z_w}{z_f} + 0.009A \quad (10.31) \text{ (Pg 14)}$$

(!!!) wing height of fuselage at wing root chord

C_{n_p}

$$C_{n_p} = C_{n_{p_w}} + C_{n_{p_v}} \quad (10.62) \text{ (Pg 46)}$$

wing total contribution from graph (Fig 10.37, Pg 47)

← flap deflection employed

$$C_{n_{p_w}} = -\left(\frac{C_{n_p}}{C_L}\right)_{C_L=0} C_L + \left(\frac{C_{n_p}}{\varepsilon_t}\right)\varepsilon_t + \left[\left(\frac{\Delta C_{n_p}}{a_{\delta f}}\right)(\delta_f)\right](a_{\delta_f})\,\delta_f$$

depends on the type of flap use (8.1.3.2) (Pg 1929)

$$a_{\delta_f} = \frac{\Delta C_l}{C_{l_\alpha}}\,\delta_f$$

← lift curve slope

$\left[\left(\frac{\Delta C_{n_p}}{a_{\delta f}}\right)\delta_f\right]$ ← Contribution due to symmetry, (Need λ taper ratio) can be found in (fig 10.38, Pg 48)

$$\left(\frac{C_{n_p}}{C_L}\right)_{\substack{C_L=0\\M}} = \left[\frac{A + 4\cos\Lambda_{c/4}}{AB + 4\cos\Lambda_{c/4}}\right]\left[\frac{AB + \frac{1}{2}(AB + \cos\Lambda_{c/4})\tan^2\Lambda_{c/4}}{A + \frac{1}{2}(A + \cos\Lambda_{c/4})\tan^2\Lambda_{c/4}}\right]\left(\frac{C_{n_p}}{C_L}\right)_{\substack{C_L=0\\M=0}} \quad (10.63) \text{ (Pg 47)}$$

$$B = \left[1 - M^2\cos^2\Lambda_{c/4}\right]^{\frac{1}{2}} \quad (10.64) \text{ (Pg 47)}$$

Distance from the c.g. to aerodynamic center for aft of c.g.

$$\left(\frac{C_{n_p}}{C_L}\right)_{\substack{C_L=0\\M=0}} = \frac{-1}{6}\left[\frac{A + 6(A + \cos\Lambda_{c/4})\left(\frac{\bar{x}}{\bar{c}}\frac{\tan\Lambda_{c/4}}{A} + \frac{\tan^2\Lambda_{c/4}}{12}\right)}{A + 4\cos\Lambda_{c/4}}\right]$$

vertical tail:

$$C_{n_{p_v}} = -\left(\frac{2}{b^2}\right)(l_v\cos\alpha + z_v\sin\alpha)(z_v\cos\alpha - l_v\sin\alpha - z_v)\,C_{y_{\beta_v}}$$

written before

C_{l_p}

$$C_{l_p} = \frac{4C_{L\alpha w}}{S_w b_w^2}\int_0^{b_w/2} c y^2\,dy$$

Yaw rate derivatives

C_{n_r}

$$C_{n_r} = \underbrace{C_{n_{r_w}} + C_{n_{r_v}}}_{\text{roll rate}}$$

(from aero team)

zero-lift drag coefficient of the wing

$$C_{n_{r_w}} = \left(\frac{C_{n_r}}{C_L^2}\right)(C_{L_w})^2 + \left(\frac{C_{n_r}}{C_{D_0}}\right)C_{D_{0_w}}$$

From graph figure 10.44 (pg 58)

Assume equal to entire lift coefficient for preliminary design

Found from graph figure 10.45 (pg 59)

vertical tail:

$$C_{n_{r_v}} = \left(\frac{2}{b^2}\right)(l_v\cos\alpha + z_v\sin\alpha)^2 C_{y_{\beta_v}}$$

written before

C_{Y_r}

written before

$$C_{Y_r} = -2\, C_{Y_{\beta_v}} (l_v \cos\alpha + z_v \sin\alpha)\frac{1}{b}$$

C_{l_r}

$$C_{l_r} = C_{l_{r_w}} + C_{l_{r_v}}$$

Slope of

dihedral angle

From graph (figure 10.42) (Pg 55)

wing twist

Effects of symmetric flap deflection on the rolling moment due to roll rate (Fig 10.43) (Pg 56)

(10.82) (Pg 53)

$$C_{l_{r_w}} = C_{L_w}\left(\frac{C_{l_r}}{C_L}\right)_{\substack{C_L=0\\M}} + \left(\frac{\Delta C_{l_r}}{\Gamma}\right)\Gamma + \left(\frac{\Delta C_{l_r}}{\varepsilon_t}\right)\varepsilon_t + \left(\frac{\Delta C_{l_r}}{a_{\delta_f}}\right) a_{\delta_f}\delta_f$$

Slope of rolling moment due to roll rate

Slope of rolling moment due to yaw rate at Mach = 0 (low speed)

$$\left(\frac{C_{l_r}}{C_L}\right)_{\substack{C_L=0\\M}} = \frac{1 + \dfrac{A(1-B^2)}{2B(AB+2\cos\Lambda_{c/4})} + \dfrac{AB+2\cos\Lambda_{c/4}}{AB+4\cos\Lambda_{c/4}}\,\dfrac{\tan^2\Lambda_{c/4}}{8}}{1 + \dfrac{A+2\cos\Lambda_{c/4}}{A+4\cos\Lambda_{c/4}}\,\dfrac{\tan^2\Lambda_{c/4}}{8}} \left(\frac{C_{l_r}}{C_L}\right)_{\substack{C_L=0\\M=0}}$$

From graph (Figure 10.41) (Pg 55)

$$\frac{\Delta C_{l_r}}{\Gamma} = 0.083(\pi A \sin\Lambda_{c/4})/(A + 4\cos\Lambda_{c/4}) \quad (10.84)\ (\text{Pg } 54)$$

vertical tail

$$C_{l_{r_v}} = -\left(\frac{2}{b^2}\right)(l_v\cos\alpha + z_v\sin\alpha)(z_v\cos\alpha - l_v\sin\alpha)\, C_{Y_{\beta_v}}$$

Rate of sideslip angle derivatives

$C_{Y_{\dot\beta}}$

wing ac to v-tail ac (mean) in x-direction

wing ac to v-tail ac (mean) in -direction

(10.46) (pg 26)

$$C_{Y_{\dot\beta}} = 2(C_{L_{\alpha_v}})\left(\frac{d\sigma}{d\beta}\right)\left(\frac{S_v}{S}\right)(l_p\cos\alpha_f + z_p\sin\alpha_f)/b$$

angle of attack of the fuselage

Wing dihedral angle

From Fig 10.33 (pg 40)

$$\frac{d\sigma}{d\beta} = (\sigma_{\beta_\alpha})\, a_f + (\sigma_{\beta_\Gamma})\left(\frac{\Gamma}{57.3}\right) - (\sigma_{\beta_{\varepsilon_t}})\,\varepsilon_t + (\sigma_{\beta_{wf}})$$

sidewash contribution due to α From graph (Fig 10.3) (Pg 30)

sidewash contribution due to σ_{β_Γ} From graph (Fig 10.31) (Pg 31)

sidewash contribution due to ε_t From graph (Fig 10.31) (Pg 31)

wing twist angle

C_{l_β}

(!) Close to zero?

$$C_{l_\beta} = C_{y_\beta}(z_p \cos\alpha_f - l_p \sin\alpha_f)/b \qquad (10.49)\ (\text{pg } 10.46)$$

Written before

C_{n_β}

(!) Close to zero?

$$C_{n_\beta} = C_{y_\beta}(l_p \cos\alpha_f - z_p \sin\alpha_f)/b \qquad (10.49)\ (\text{pg } 10.46)$$

Written before

Propeller correction (Pg 12-13)

- Axis of rotation makes an angle with freestream
  - ↳ produce side force Y_p and yawing moment n
    - ↳ Altering C_{n_α}, C_{n_β}, and C_{m_α}

Definition of stability derivative coefficient (Warren Phillip)

$$C_{n_p} \equiv \frac{n_p}{\rho\left(\frac{\omega}{2\pi}\right)^2 d_p^5} \qquad C_{Y_p} \equiv \frac{Y_p}{\rho\left(\frac{\omega}{2\pi}\right)^2 d_p^4}$$

free stream velocity

$$\text{Advance ratio } J = \frac{2\pi V_\infty}{\omega d_p}$$

Propeller diameter

propeller angular

$$(\Delta C_n)_p = \frac{2d_p^3}{S_w b_w}\frac{(C_{n_\alpha})_p}{J^2}\alpha_p - \frac{y_{bp}}{b_w} f_T C_D + \frac{2d_p^2 l_p}{S_w b_w}\frac{(C_{N_\alpha})_p}{J^2}\beta_p$$

$$\alpha_p = \alpha - \varepsilon_{dp} + \alpha_{0p}$$

$$\beta_p = \beta - \varepsilon_{sp}$$

(Just for report) (Not required for calculation)

downwash gradient at the position of the propeller

Change in C_{n_α}

$$(\Delta C_{n_\alpha})_p = \frac{2d_p^3}{S_w b_w}(1-\varepsilon_{d,\alpha})_p \frac{(C_{n_\alpha})_p}{J^2}$$

sidewash gradient of the position of the propeller

Change in C_{n_β}

$$(\Delta C_{n_\beta})_p = \frac{2d_p^2 l_p}{S_w b_w}(1-\varepsilon_{s,\beta})_p \frac{(C_{N_\alpha})_p}{J^2}$$

Yawing moment varies linearly with α_p

$$C_{n_p} = (C_{n_\alpha})_p \alpha_p$$

Sideforce coefficient varies linearly with β_p

$$C_{Y_p} = (C_{Y_\beta})_p \beta_p$$

Fuselage Correction (Pg 9–10)

- Impacts more on yaw stability

Maximum cross-sectional area of the fuselage

(!!!) Estimate 25%–30% of fuselage length

diameter of circle $d_f = 2\sqrt{\frac{S_f}{\pi}}$

$$\Delta (C_{n_\beta})_f = 2 \frac{S_f \, l_f}{S_w \, b_w} \left[1 - 1.76 \left(\frac{d_f}{c_f} \right)^{\frac{3}{2}} \right]$$

Defined as the fuselage length

State-space model

$$\dot{u} = \frac{\bar{q}S}{m}C_x - qw - g\sin\theta + \frac{T}{m}$$

$$\dot{w} = \frac{\bar{q}S}{m}C_z + qu + g\cos\theta$$

$$\dot{q} = \frac{\bar{q}S\bar{c}}{I_y}C_m$$

$$\dot{\theta} = q$$

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & 0 & 0 & -g \\ 0 & \frac{1}{m}\frac{\partial Z}{\partial w} & \left(u_0 + \frac{1}{m}\frac{\partial Z}{\partial q}\right) & 0 \\ 0 & \frac{1}{I_{yy}}\frac{\partial M}{\partial w} & \frac{1}{I_y}\frac{\partial M}{\partial q} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{1}{m}\frac{\partial X}{\partial \delta_e} \\ \frac{1}{m}\frac{\partial Z}{\partial \delta_e} \\ \frac{1}{I_{yy}}\frac{\partial M}{\partial \delta_e} \\ 0 \end{bmatrix} \delta_e$$