Equations

June 9, 2020

0.1 NCX mice model

0.1.1 Equations

Voltage activated Na current This is the sodium curent,calculated from a single (file ?) channel with constant field formulation of current.

$k_1 = 0.025 \times e^{\frac{em+90}{12}}$
$f_o = f_o + (f_r k_1 - f_o(\frac{1}{2} + \frac{1}{4k_1}))dt$
$f_r = f_r + ((1 - f_r)\frac{0.15}{k_1} - f_r k_1)dt$
$k_{em1} = e^{\frac{em}{26}} \quad (K_{em} = e^{\frac{EmF}{2RT}}, \frac{RT}{F} = \frac{(8.31 JmolK^{-1})*(273+21)K}{9.64*10^4 Cmol^{-1}} = 25.43)$
$i_{na} = -600\ f_o^2\ em\ \frac{(n_o - n_i K_{em1})}{(100+n_o+n_i)(K_{em1}-1.0)}$

```
k1=0.025*exp((em+90)/12);
fo=fo+ (fr*k1-fo*(0.5+0.25/k1))*dt;
fr=fr+((1-fr)*0.15/k1 - fr*k1)*dt;
kem1 = exp( em / 26.0);
ina =(-600* fo^2 )* em * (no  - ni * kem1) /( (100 + no + ni)*(kem1 - 1.0 ));
```

Voltage activated Ca current This is voltage activated Ca current

$k_{ca} = 0.025 e^{\frac{(em+60)}{12}}$
$f_{oc} = f_{oc} + (f_{rc} k_{ca} - f_{oc}(\frac{1}{2} + \frac{1}{4k_{ca}}))dt$
$f_{rc} = f_{rc} + ((1 - f_{rc})\frac{0.15}{kca} - f_r k_{ca})dt$
$i_{ca} = -900\ f_o^2\ em\ \frac{(c_o - c_i k_{em1}^2)}{(100+c_o+c_i)(k_{em1}^2-1.0)}$

```
kca=0.025*exp((em+60)/12);
foc=foc+ (frc*kca-foc*(0.5+0.25/kca))*dt;
frc=frc+((1-frc)*0.15/kca - fr*kca)*dt;
ica= -900* fo^2 * em * (co  - ci * kem1^2) /( (100 + co + ci)*(kem1^2 - 1.0 ));
```

0.1.2 Open probability of inward rectifier K channel

This is an inward rectifier K channel probability of being open

$f_{irk} = \frac{1}{1+exp(\frac{em+60}{15})}$

```
firk=1/(1+exp((em+60)/15));
```

0.1.3 Open probability of a delayed rectifier K channel

This is calculation of the opening probability of a delayed K channel

$k_{edk} = exp(\frac{em+42}{10})$

$f_{dk1} = f_{dk1} + ((1 - f_{dk1} - f_{dk2} - f_{dk3})k_{edk} + \frac{f_{dk2}}{k_{edk}} - f_{dk1}(k_{edk} + \frac{15}{k_{edk}}))0.0003dt$

$f_{dk2} = f_{dk2} + (f_{dk1}k_{edk} - 15\frac{fdk2}{kedk})0.0003dt$

$f_{dk3} = f_{dk3} + (f_{dk2}k_{edk} - 15\frac{fdk3}{kedk})0.0003dt$

```
kedk=exp((em+42)/10);
fdk1=fdk1+((1-fdk1-fdk2-fdk3)*kedk + fdk2/kedk  -fdk1*(kedk+15/kedk))*0.0003*dt;
fdk2=  fdk2 + (fdk1*kedk-15*fdk2/kedk)*0.0003*dt;
fdk3=  fdk3 + (fdk2*kedk-15*fdk3/kedk)*0.0003*dt;
```

0.1.4 Total K current

Which is the sum of inward rectifier and delayed K current. Like Na channel, the current is caculated from a constant field equation for a single file channel

$i_k = -(250f_{irk} + 220f_{dk3})em\frac{(k_o - k_i k_{em1})}{((50+k_o+k_i)(k_{em1}-1.0))}$

```
ik =- (250*firk+fdk3*220) * em * (ko  - ki * kem1) /( (50 + ko + ki)*(kem1 - 1.0
));
```

0.1.5 Na/K pump function

This is a primitive Na/K pump function which is assumed to be just proportional to a hill equation with 3 Na binding.

$i_{pump} = \frac{n_i^3}{(ni^3+20^3)} \times 6 \times 200$

```
ipump=ni^3/(ni^3+20^3)*6*200;
```

0.2 Na/Ca exchange current

This is the primitive Na/Ca exchange system employe.Two Na compete with 1 Ca and 1 Na binding independently from Ca, similar to what John Reeves found 40 years ago.

$d_{out} = 1 + \frac{c_o}{0.01} + \frac{no}{20(1+\frac{n_o}{20})}$

$d_{in} = 1 + \frac{c_i}{0.01} + \frac{ni}{20(1+\frac{n_i}{20})}$

$f_{co} = \frac{c_o}{0.1 \times d_{out}}$

$f_{2no} = \frac{n_o \times n_o}{20 \times 20 \times d_{out}}$

$f_{ci} = \frac{ci}{0.01 \times d_{in}}$

$f_{2ni} = \frac{n_i \times n_i}{20 \times 20 \times d_{out}}$

$f_{3ni} = \frac{f_{2ni} \times n_i}{(n_i+30)}$

$f_{3no} = \frac{f_{2no} \times n_o}{(n_o+30)}$

$k_{em} = e^{\frac{em}{55}}$

$i_{ncx} = \frac{80(f_{co} \times f_{3ni} \times k_{em} - f_{ci}\frac{f_{3no}}{k_{em}})}{f_{co} + (f_{3ni} \times k_{em}) + f_{ci} + \frac{f_{3no}}{K_{em}}}$

```
dout=1+co/0.01+no/20*(1+no/20);
din=1+ci/0.01+ni/20*(1+ni/20);
fco=co/0.01/dout;
f2no=no*no/20/20/dout;
fci=ci/0.01/din;
f2ni=ni*ni/20/20/din;
f3ni=f2ni*ni/(ni+30);
f3no=f2no*no/(no+30);
kem=exp(em/55);
incx=80*(fco*f3ni*kem-fci*f3no/kem)/(fco+f3ni*kem+fci+f3no/kem);
```

0.2.1 SR Ca pump function

This is the SR Ca pump function as it depends on binding Ca on the cytoplasmic side (fcain) and in the SR lumen (fcasr)

$f_{cain} = \frac{c_i}{c_i+0.002}$
$f_{casr} = \frac{casr}{casr+2}$
$f_{rel} = -\frac{i_{ca}}{(-i_{ca}+100)}$
$f_{srinact} = f_{srinact} + ((1 - f_{srinact}) \times f_{rel} \times 2 - f_{srinact} \times 0.005)dt$
$d_{casr} = (0.015 \times f_{cain} - 0.002 \times f_{casr}) - f_{rel} \times (1 - f_{srinact}) \times casr \times 0.13$
$casr = (casr + d_{casr}10)dt$

```
fcain=ci/(ci+0.002);
fcasr=casr/(casr+2);
frel=-ica/(-ica+100);
fsrinact=fsrinact+((1-fsrinact)*frel*2-fsrinact*0.005)*dt;
dcasr=(0.015*fcain-0.002*fcasr)-frel*(1-fsrinact)*casr*0.13;
casr=casr+dcasr*10*dt;
```

0.2.2 This is the cytoplasmic Na concentration

$n_i = n_i - (i_{na} + i_{pump} \times 3 + i_{ncx} \times 3) \times 10^{-6} \times dt$

```
ni=ni-(ina+ipump*3+incx*3)*10^-6*dt ;
```

0.2.3 This is the cytoplasmic K concentration

$k_i = k_i - (i_k - 2i_{pump}) \times 10^{-6} \times dt$

```
ki=ki-(ik-ipump*2)*10^-6*dt;
```

0.2.4 This is the total cytoplasmic Ca concentration

And the free Ca concentration, assumed to be 40 times less

$c_{itot} = c_{itot} - ((\frac{i_{ica}}{2} - i_{ncx}) \times 10^{-6} + d_{casr}) \times dt$
$c_i = \frac{c_{itot}}{40}$

```
citot=citot-((ica/2-incx)*10^-6 +dcasr)*dt;
ci=citot/40;
```

0.2.5 Calculation of the membrane potentail

This is the membrane potential calculated from total charges in the cytoplasm. The number, 120000 converts charge excess to Em and reflects the capacitance of the cell

$em = (n_i + k_i + 2c_{itot} + 2\frac{casr}{10} - anion) \times 12000$

```
em=(ni+ki+2*citot+2*casr/10-anion)*12000;
```