# Recoloring Images for Vision Deficient Viewers

Phillip Yommer (pyommer@g.clemson.edu)

December 7, 2017

## Abstract

### Project

CPSC 4040: Computer Graphics Images
Final Programming Project
A Problem in Color Mapping to a Limited Pallet

### Background

This program attempts to correct the colors of an image for color vision deficient viewers based on the confusion lines of the CIE 1931 XYZ color space. The idea for this project was inspired from Daniel Flück's Colorblindor color blindness test described at http://www.color-blindness.com/2007/01/18/. Flück's goal is "...to introduce a color blindness test based on the confusion lines of the CIE 1931 color space..." [Fluck].

The channel values of an RGB pixel correspond to red, green, and blue intensities in the color of a pixel. The channel values of an LMS pixel correspond to the wavelengths of light perceived by the human eye through its long, medium, and short rod and cone cells. Color vision deficiencies occur when a person's long, medium, or short rod or cone cells either respond poorly or not at all to light stimuli. Color matching functions can be used to determine the CIE 1931 color space coordinates that correspond to color values of wavelengths of light defined in the LMS space. Given the visible range of an average person's LMS receptors and the average range of color vision deficient viewers' LMS receptors, the CIE XYZ color space can be used to map colors from the LMS color space of visible light. [LMS Color Space]

Pixels can be transformed from the RGB to the CIE XYZ color space via transformation matrices. Different color vision deficiencies have defined "lines of confusion" that correspond to white points in the CIE XYZ color space. The confusion lines intersect at three different copunctual points for the three types of color vision deficiencies. Given these copunctual points, transformation matrices can be determined to transform all pixels in an image to a guaranteed pixel in the visible portion of the CIE 1931 color space for a color deficient viewer.

# Color Spaces

## Color Vision and the LMS Color Space

The LMS color space corresponds pixel channel values to wavelengths of visible light perceived by the human eye through its long, medium, and short rod and cone cells. The rod cells are receptors for distinguishing light at normal intensities and the cone cells are receptors for distinguishing light at softer intensities. The long, medium, and short types of receptors indicate the wavelengths of light to which they can respond.

Color vision deficiencies occur when a person's long, medium, or short rod or cone cells either respond poorly or not at all to light stimuli. Color matching functions can be used to determine the CIE 1931 color space coordinates that correspond to color values of wavelengths of light defined in the LMS space. Given the visible range of an average person's LMS receptors and the average range of color vision deficient viewers' LMS receptors, the CIE XYZ color space can be used to map colors from the LMS color space of visible light. [LMS Color Space]

## The RGB Color Space

The RGB color space defines pixels as channels of color intensities for red, green, and blue. Pixel channel values are assumed to be in the range $[0, 1]$, which correspond to an integer in the range $[0, 255]$.

When pixels are read in RGB space, the pixels are vectors of length 3. When pixels are read in RGBA space, the pixels are vectors of length 4 and hold the alpha channel value of the pixel. The RGBA color space is depicted in Figure 1 [RGB Color Space]. The RGB color space corresponds to the top-most line of the RGBA space, where the alpha value is 1.

When color space transformations are applied to RGBA pixels, the RGB values of the pixel are stored in a temporary RGB pixel, transformed as RGB pixels, worked on as RGB pixels, and stored as output pixels of the new RGB pixel and the original alpha (A) value, maintaining the alpha value of all pixels from the input image to the output image.

## The CIE 1931 Color Space

Created by the International Commission on Illumination as a result from experiments done by William David Wright and John Guild, the CIE 1931 color spaces were the first defined quantitative links between distributions of wavelengths in the electromagnetic visible spectrum, and physiological perceived colors in human color vision [CIE 1931 Color Space].

The CIE 1931 color space represents colors as wavelengths of light (measured in nanometers) at defined $x, y$ coordinates on the CIE 1931 color space plane. The CIE color-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ describe the chromatic response of the CIE standard observer. These color-matching functions are used to eliminate the variable due to distribution of cones and other differences in the eye. The CIE Standard Observer is defined to represent an average human's eye. The CIE 1931 color space is shown in Figure 2 and the corresponding CIE color-matching functions are depicted in Figure 3, from Wikipedia [CIE 1931 Color Space].

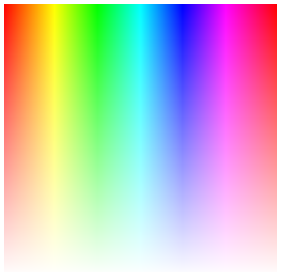


**Figure 1**: The RGBA Color Space

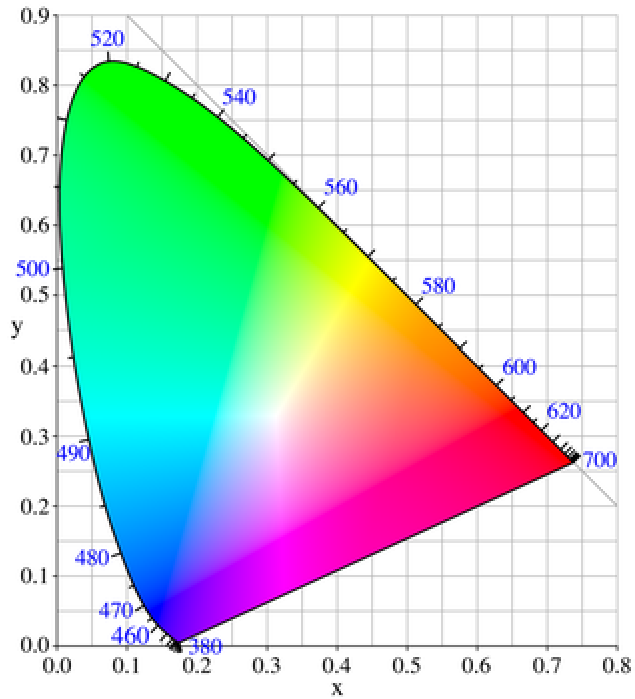



**Figure 2**: The CIE XYZ Color Space

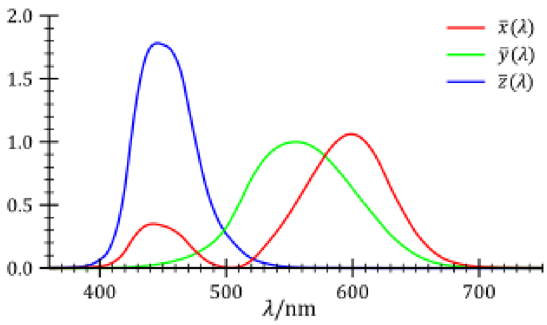



**Figure 3**: Color Mapping Functions

### Pixel Representations

A pixel in the RGB color space represents pixel colors as $R$, $G$, and $B$ channel valuess corresponding to intensities of red, green, and blue, respectively. RGB pixels are described by the vector:

$$\vec{P}_{RGB} = \begin{pmatrix} R \\ G \\ B \end{pmatrix}, \forall 0 \leq R, G, B \leq 1$$

where $R, G, B$ are the channel values.

A pixel in the CIE XYZ color space represents . XYZ pixels are described by the vector:

$$\vec{P}_{XYZ} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix},$$

where $X$, $Y$, and $Z$ represent coordinate locations in CIE XYZ space.

### Converting Pixels between RGB and CIE XYZ Color Spaces

Transformation matrices can be applied to RGB pixels to convert them to CIE XYZ coordinates and vice-versa.

The RGB to XYZ transformation matrix is defined as:

$$T_{RGB} = \begin{pmatrix} 3.2406 & -1.5372 & -0.4986 \\ -0.9689 & 1.8758 & 0.0415 \\ 0.0557 & -0.0204 & 1.057 \end{pmatrix}$$

from [CIE 1931 Color Space]. The XYZ pixel corresponding to the RGB pixel $P_{RGB}$ is determined via the transformation:

$$P_{XYZ} = T_{RGB} \times P_{RGB}$$

The XYZ to RGB transformation matrix is defined as:

$$T_{XYZ} = \begin{pmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{pmatrix}$$

from [CIE 1931 Color Space]. The XYZ pixel corresponding to the RGB pixel $P_{XYZ}$ is determined via the transformation:

$$P_{RGB} = T_{XYZ} \times P_{XYZ}$$

## The Protanopia, Deuteranopia, and Tritanopia Color Spaces

### Color Vision Deficiencies

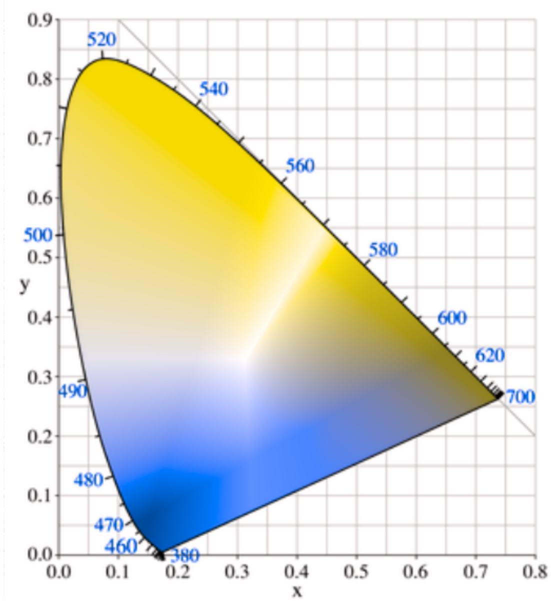
Protanopic color vision deficiency is associated with a person's red receptors of their eye. Protanopia is lacking the red receptors entirely. Protanomaly is the presence of deficient red receptors.

Deuteranopic color vision deficiency is associated with a person's green receptors of their eye. Deuteranopia is lacking the green receptors entirely. Deuteranomaly is the presence of deficient green receptors.

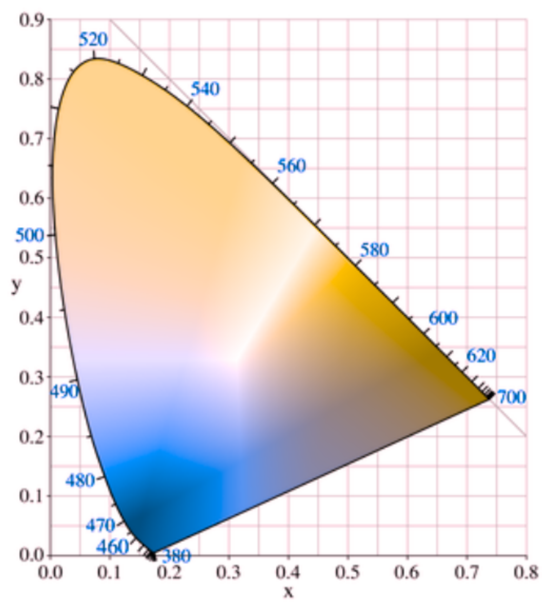
Tritanopic color vision deficiency is associated with a person's blue receptors of their eye. Tritanopia is lacking the blue receptors entirely. Tritanomaly is the presence of deficient blue receptors.

**Color Spaces**

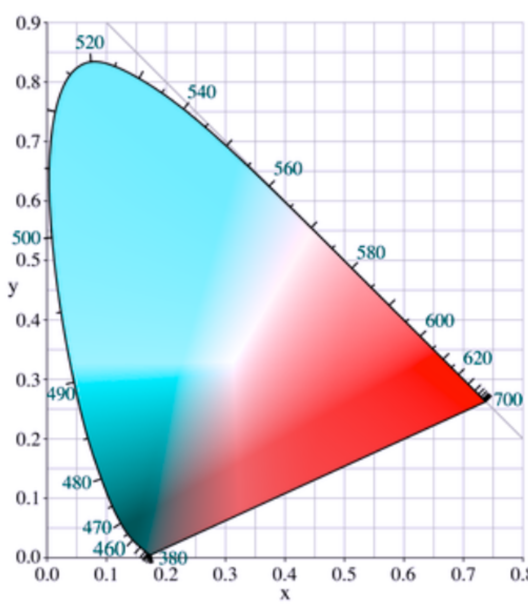
The Protanopia, Deuteranopia, and Tritanopia color spaces are depicted in the CIE 1931 color space in Figure 4.




(a) Protanopia CIE XYZ Color Space




(b) Deuteranopia CIE XYZ Color Space

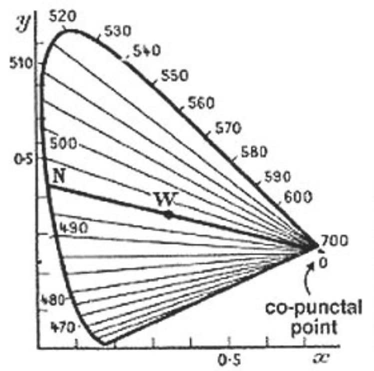



(c) Tritanopia CIE XYZ Color Space

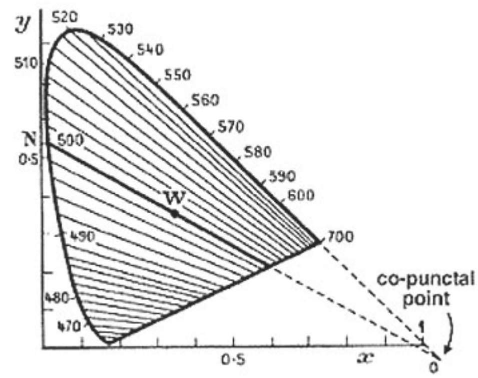
**Figure 4**: The CIE XYZ color spaces for color deficiencies.

### Confusion Lines

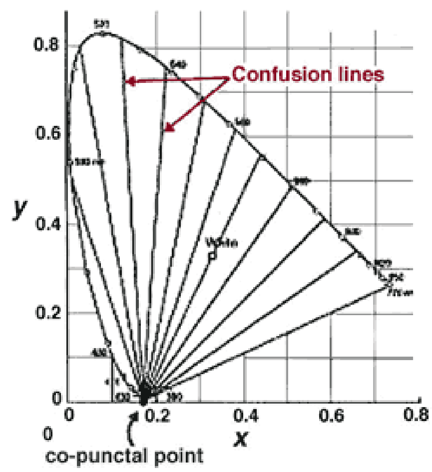
The confusion lines for a color vision deficiency are lines in the CIE 1931 color space, along which viewers with the color deficiency will see white, while to anyone without a color vision deficiency will see the standard CIE XYZ colors. The Protanopia, Deuteranopia, and Tritanopia confusion lines are depicted in the CIE 1931 color space in Figure 5.




(a) Protanopia Confusion Lines




(b) Deuteranopia Confusion Lines




(c) Tritanopia Confusion Lines

**Figure 5**: The confusion lines for color deficiencies [Fluck].

### Coponctual Points

The copunctual point for a color vision deficiency is the point in the CIE 1931 color space where the confusion lines for that color deficiency intersect, depicted in the CIE 1931 color space in Figure 5.

The copunctal points are defined for the three types of color vision deficiencies as:

$$p_0 = \begin{cases} (0.747, 0.253, 0.0) & \text{for Protanopia} \\ (1.08, -0.8, 0.0) & \text{for Deuteranopia} \\ (0.171, 0.0, 0.0) & \text{for Tritanopia} \end{cases}$$

# Implementation

The CIE 1931 color space was selected to be the color space in which to recolor pixels. A few reasons for this selection include:

1. The mathematical relationships that define these color spaces are essential tools for color management, important when dealing with color inks, illuminated displays, and recording devices such as digital cameras [CIE 1931 Color Space]. The CIE 1931 color space is an acceptable color space to represent three main formats a color vision deficient viewer would be interested in viewing, camera, monitor, and paper.
2. The defined average subspaces of XYZ that contain colors interpretable by color vision deficient viewers. This gave a basis of the expected palette to see in the output image and allows pixels to be guaranteed viewable by color vision deficient viewers.
3. The defined transformation matrices for converting XYZ pixels to color vision deficient viewable XYZ pixels. This gave a clear method of implementation to follow. Convert pixels from RGB to XYZ to color vision deficient viewable color spaces via transformation matrices.

## Mathematical Descriptions

### Defining Viewable Pixels

For a color vision deficiency $d$, a "viewable" pixel color is one in which the corresponding XYZ pixel coordinate describes a non-white pixel color in the XYZ subspace for the color vision deficiency.

### Transformation Matrices

The XYZ to color deficient-corrected XYZ transformation matrices can be derrived from the copunctal points for the different color vision deficiencies via some function $\delta(p_0) \rightarrow T_d$. The following transformation matrices are from [Kalloniatis and Luu].

1. The XYZ to Protanopia XYZ transformation matrix:

$$T_{Protanopia} = T_1 = \begin{pmatrix} 0.567 & 0.433 & 0.0 \\ 0.558 & 0.442 & 0.0 \\ 0.0 & 0.242 & 0.478 \end{pmatrix}$$

2. The XYZ to Deuteranopia XYZ transformation matrix:

$$T_{Deuteranopia} = T_2 = \begin{pmatrix} 0.625 & 0.375 & 0.0 \\ 0.7 & 0.3 & 0.0 \\ 0.0 & 0.3 & 0.7 \end{pmatrix}$$

3. The XYZ to Tritanopia XYZ transformation matrix:

$$T_{Tritanopia} = T_3 = \begin{pmatrix} 0.95 & 0.05 & 0.0 \\ 0.0 & 0.433 & 0.567 \\ 0.0 & 0.475 & 0.525 \end{pmatrix}$$

**Transforming Pixel Colors**

Transforming pixels from some XYZ coordinate location to a viewable XYZ pixel for some color vision deficiency is done applying the transformation:

$$P_d = T_d \times P_{XYZ},$$

for the color vision deficiency correcting transformation matrix $T_d$, where

$$T_d = \begin{cases} T_1 = T_{Protanopia}, & \text{for Protanopia} \\ T_2 = T_{Deuteranopia}, & \text{for Deuteranopia} \\ T_3 = T_{Tritanopia}, & \text{for Tritanopia} \end{cases}$$

The recolored pixels of the output image are determined from the three functions $g(\vec{p})$, $h(\vec{p})$, and $f(\vec{p})$:

1. $g(\vec{p})$ is the XYZ-to-RGB transformation function, defined as:

$$g(\vec{p}) = r\vec{g}b = T_{XYZ} \times \vec{p},\ \forall \vec{p} \in P_{XYZ},$$

   where $r\vec{g}b$ represents a pixel in RGB space.

2. $h(\vec{p})$ are the RGB-to-XYZ transformation function, defined as:

$$h(\vec{p}) = x\vec{y}z = T_{RGB} \times \vec{p},\ \forall \vec{p} \in P_{RGB},$$

   where $x\vec{y}z$ represents a pixel in XYZ space.

3. $f(T, \vec{p})$ is the XYZ to color vision deficiency subspace transformation function, defined as:

$$f_d(\vec{x}) = f(T_d, \vec{x}) = T_d \times \vec{x},\ \forall T_d, \vec{x}\ :\ \{T_d \in \{T_1, T_2, T_3\},\ \vec{x} = x\vec{y}z \in P_{XYZ}\}.$$

## Code Implementation

### Defined Vectors and Matrices

Vectors are stored in `vector_t` type variables, where `vector_t` is defined as a `vector` of `double`s, `vector` < `double` >.

Matrices are stored in `matrix_t` type variables, where `matrix_t` is defined as a `vector` of `vector`s, `vector` < `vector_t` >.

### Representing Pixels and Pixmaps

Pixels are stored in the defined `vector_t` type variables.

Pixmaps are stored in `pixmap_t` type variables, where `pixmap_t` is defined as a `vector` of `matrix`s, `vector` < `matrix_t` >.

### The Three Main Functions

▼ `readImage`()

- Reads image RGBA pixel data into a `pixmap_t` matrix.
- Handles pixels in the RGBA color space.

▼ `writeImage`() and `displayImage`()

- Write and display image pixel data from a `pixmap_t` matrix.
- Handles pixels in the RGB or RGBA color space.

▼ `recolorImage`()

- Takes an input image `pixmap_t` to be recolored and a mode `int` that specifies the color vision deficiency to use.
- Converts each pixel to a pixel in the XYZ subspace corresponding to the specified mode.
  1. Applies the $T_{RGB}$ transformation matrix to the RGB pixel channels to convert the pixels to their corresponding XYZ pixel coordinates via $\vec{P}_{XYZ} = T_{RGB} \times \vec{P}_{RGB}$.
  2. Applies the $T_i$ transformation matrix to the XYZ pixel coordinates to transform the pixels to some XYZ pixel coordinate via $\vec{P}_{XYZ} = T_d \times \vec{P}_{XYZ}$, that is guaranteed viewable by a viewer with color vision deficiency $d$, where $i \in \{Protanopia, Deuteranopia, Tritanopia\}$.
  3. Converts each pixel from the CIE XYZ to the appropriate subset of CIE XYZ color space by applying the corresponding transformation matrix $T_1$, $T_2$, or $T_3$.

4. Converts each pixel from the CIE XYZ to the RGB color space by applying the $T_{XYZ}$ transformation matrix.

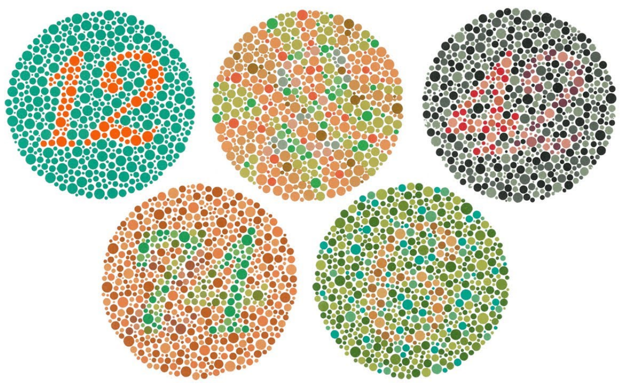
– Applies the formula:

$$P_{out} = T_{XYZ} \times (T_d \times (T_{RGB} \times P_{in}))$$

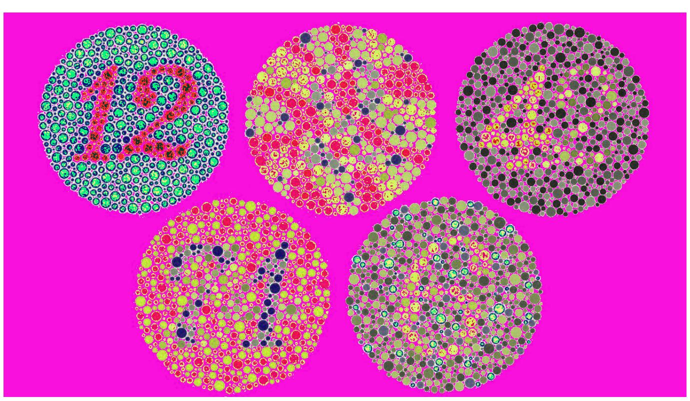
# Results

## Color Vision Deficiency Test Image

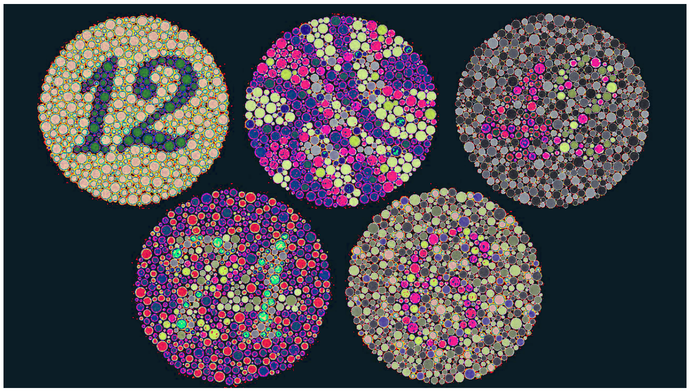
The '12' is a control, everyone should see it. The other numbers, '2', '42', '74', and '6' should be hard or impossible for color vision deficient viewers to interpret.



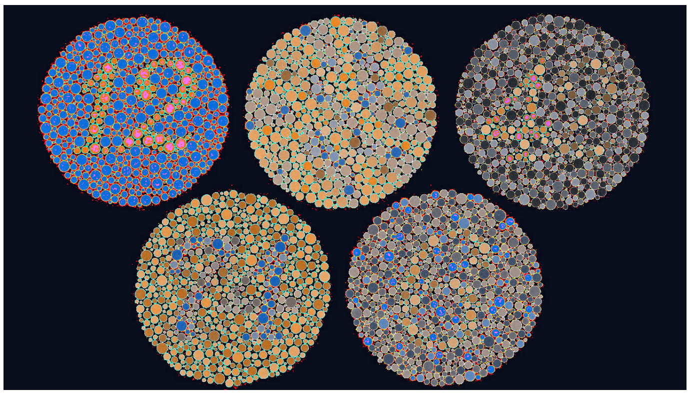
(a) Original Color Vision Test Image



(b) Protanopia Corrected



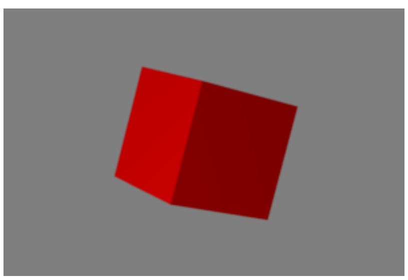
(c) Deuteranopia Corrected



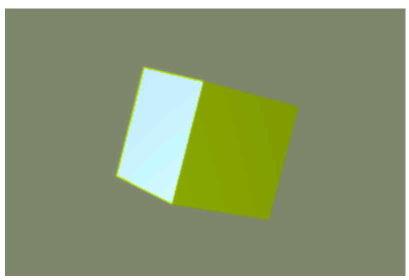
(d) Tritanopia Corrected

**Figure 6**: A test image designed to test for color vision deficiencies.

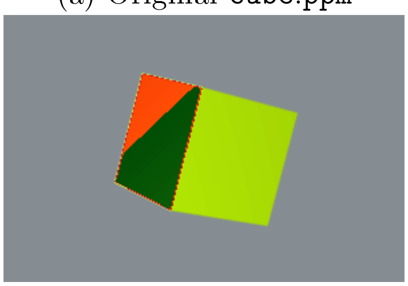
### Cube



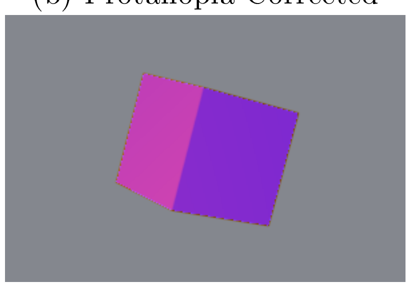
(a) Original `cube.ppm`



(b) Protanopia Corrected



(c) Deuteranopia Corrected

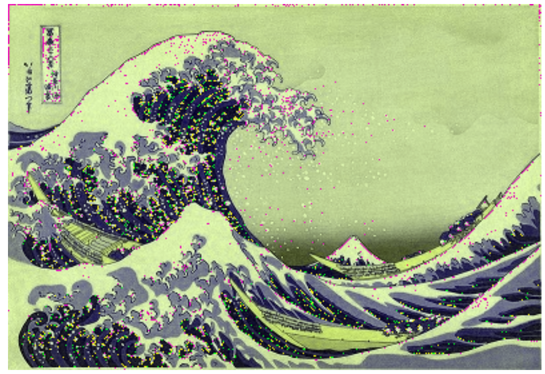


(d) Tritanopia Corrected

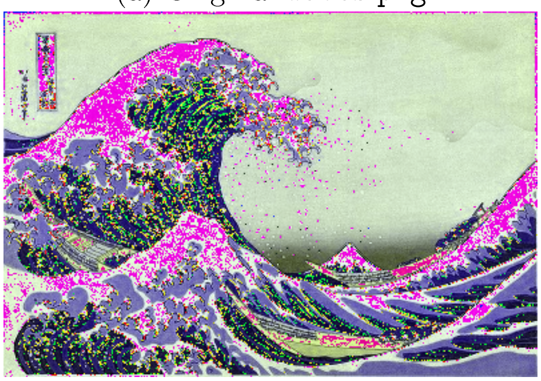
**Figure 7**: An example image of a cube.

### Waves

(a) Original `waves.png`



(b) Protanopia Corrected



(c) Deuteranopia Corrected

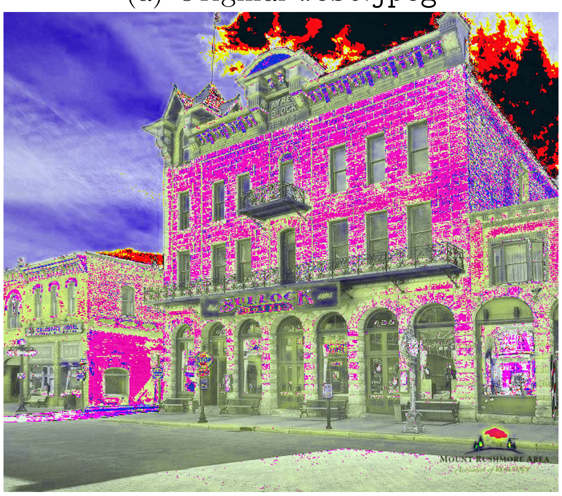
(d) Tritanopia Corrected

**Figure 7**: An example image, `waves.png`.

**West**

(a) Original `west.jpeg`

(b) Protanopia Corrected

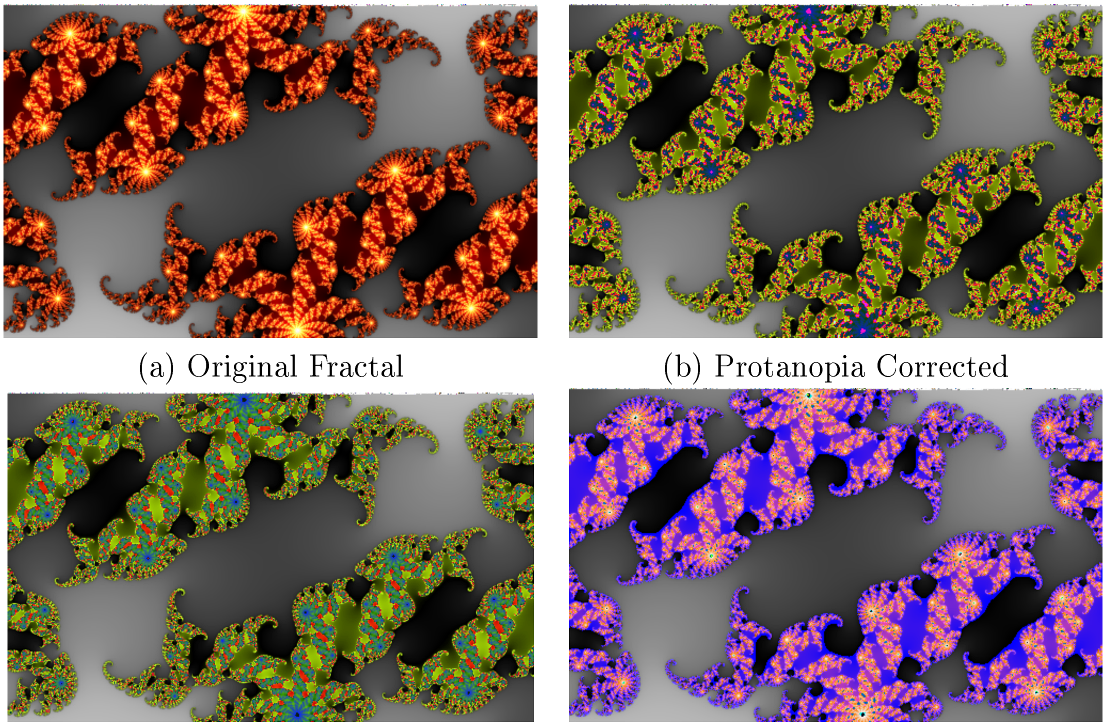


(c) Deuteranopia Corrected

(d) Tritanopia Corrected

**Figure 8**: An example image, `west.jpeg`.

**Fractal**



(a) Original Fractal

(b) Protanopia Corrected

(c) Deuteranopia Corrected

(d) Tritanopia Corrected

**Figure 9**: An example image of a fractal.

## Problems

- Noise
  - This is clearly noticeable in the `west.jpeg` and `waves.png` image color corrections.
- Testing
  - Lack of color vision deficient viewers to test the output of the recolored images lead to difficulty in determining the accuracy of the recoloring engine.
  - Different color spaces were implemented in testing, but created more problems than they solved.
    - The sRGB space was used with no visibly noticeable change to require implementing the space further.
    - The LMS space was considered in the early stages to be used to guarantee pixels in the output recolored image were inside the range of wavelengths for any viewers specified LMS cone and rod receptors. The CIE XYZ space was chosen over this upon finding the color vision deficient subspaces of XYZ and their corresponding transformation matrices.

## Solutions

- ▼ Implementation using the CIE Lab color space instead of, or in addition to, the implemented version with the CIE XYZ color space.

# References


[House] Donald H. House, The Digital Image, Computer Graphics Images

[Fluck] Daniel Flück, Colorblindor Test, http://www.color-blindness.com/2007/01/18/

[Kalloniatis and Luu] Michael Kalloniatis and Charles Luu, Color Perception, http://webvision.med.utah.edu/book/part-viii-gabac-receptors/color-perception/

[Judd] Deane B. Judd, Fundamental Studies of Color Vision from 1860 to 1960, http://www.pnas.org/content/55/6/1313.full.pdf

[RGB Color Space] Wikipedia, RGB Color Space, https://en.wikipedia.org/wiki/RGB_color_space

[CIE 1931 Color Space] Wikipedia, CIE 1931 Color Space, https://en.wikipedia.org/wiki/CIE_1931_color_space

[LMS Color Space] Wikipedia, LMS Color Space,

[Color Blindness] Wikipedia, Color Blindness, https://en.wikipedia.org/wiki/Color_blindness

[1]