$$r_{XY} = \frac{\dfrac{\sum_i^n (X_i - \bar{X})(Y_i - \bar{Y})}{n}}{\sqrt{\dfrac{\sum_i^n (X_i - \bar{X})^2}{n}}\sqrt{\dfrac{\sum_i^n (Y_i - \bar{Y})^2}{n}}}$$

$$\frac{\sum_i^n (x_i - \bar{x})^2}{n} = \frac{\sum_i^n (x_i)^2 - 2\bar{x}\sum_i^n x_i + n\bar{x}^2}{n}$$

$$= \frac{\sum_i^n x_i^2}{n} - 2\bar{x}^2 + \bar{x}^2 = \frac{\sum_i^n x_i^2}{n} - \bar{x}^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$$

$$= \frac{n\sum x^2 - (\sum x)^2}{n^2}$$, 같은 방식으로 $$\frac{\sum_i^n (y_i - \bar{y})^2}{n} = \frac{n\sum y^2 - (\sum y)^2}{n^2}$$

$$\frac{\sum_i^n (x_i - \bar{x})(y_i - \bar{y})}{n} = \frac{\sum(xy)}{n} + \frac{n\bar{x}\bar{y}}{n} - \frac{\bar{x}\sum y + \bar{y}\sum x}{n}$$

$$= \frac{\sum xy}{n} + \cancel{\frac{\sum x}{n} \cdot \frac{\sum y}{n}} - \cancel{\frac{\sum x \sum y}{n^2}} - \frac{\sum y \sum x}{n^2}$$

$$= \frac{n(\sum xy) - (\sum x)(\sum y)}{n^2}$$

$$\frac{\dfrac{n(\sum xy) - (\sum x)(\sum y)}{\cancel{n^2}}}{\sqrt{\dfrac{n\sum x^2 - (\sum x)^2}{\cancel{n^2}}}\sqrt{\dfrac{n\sum y^2 - (\sum y)^2}{\cancel{n^2}}}} = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}}$$