



# Natural Language Processing

## 第八周 XFormer

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# Overview



## CONTENTS

01

More Attention Mechanisms

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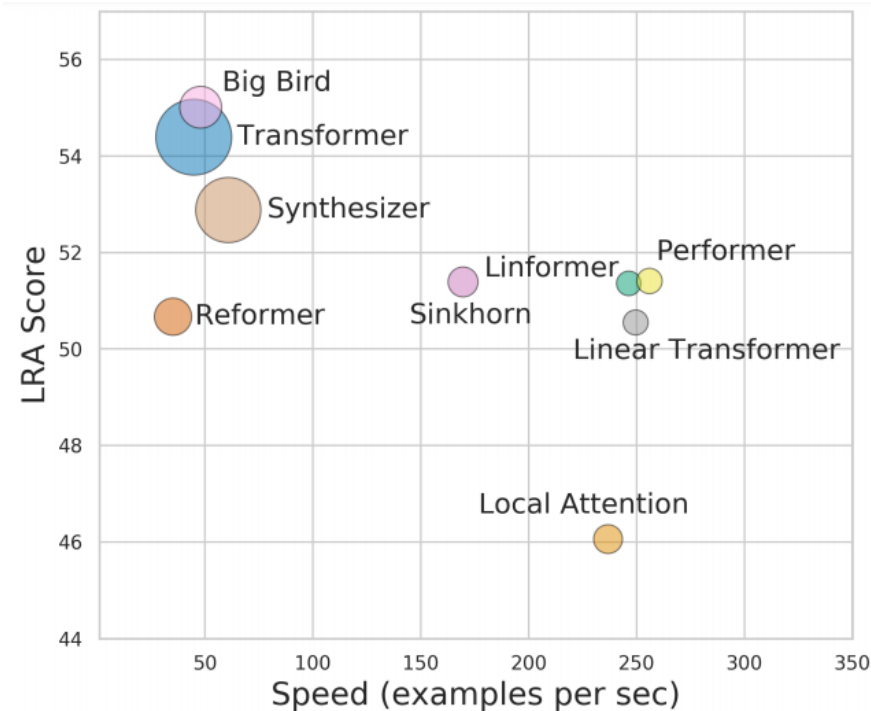
01

More Attention Mechanisms

更多的自注意力机制

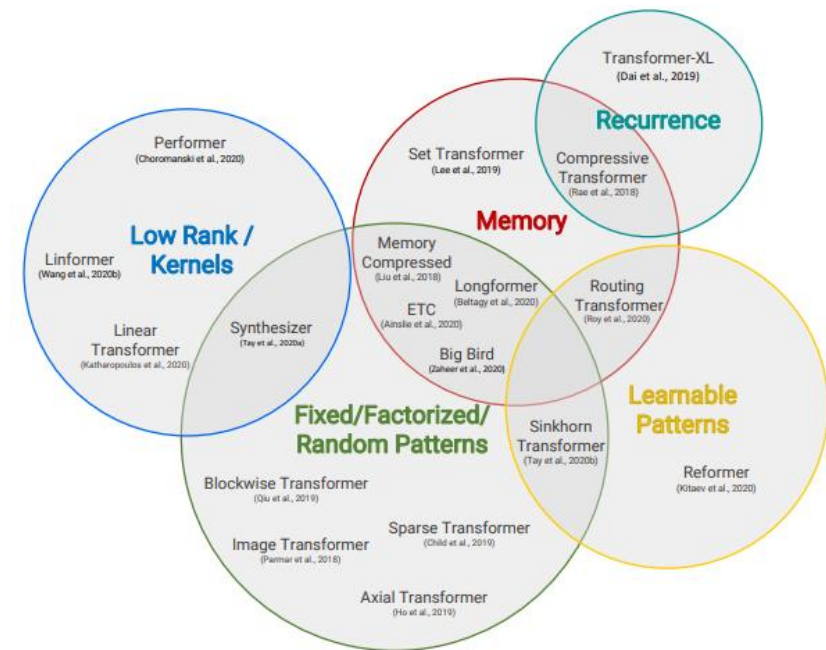
Spring 2023

# To Learn More ...



Long Range Arena: A Benchmark for Efficient Transformers

<https://arxiv.org/abs/2011.04006>



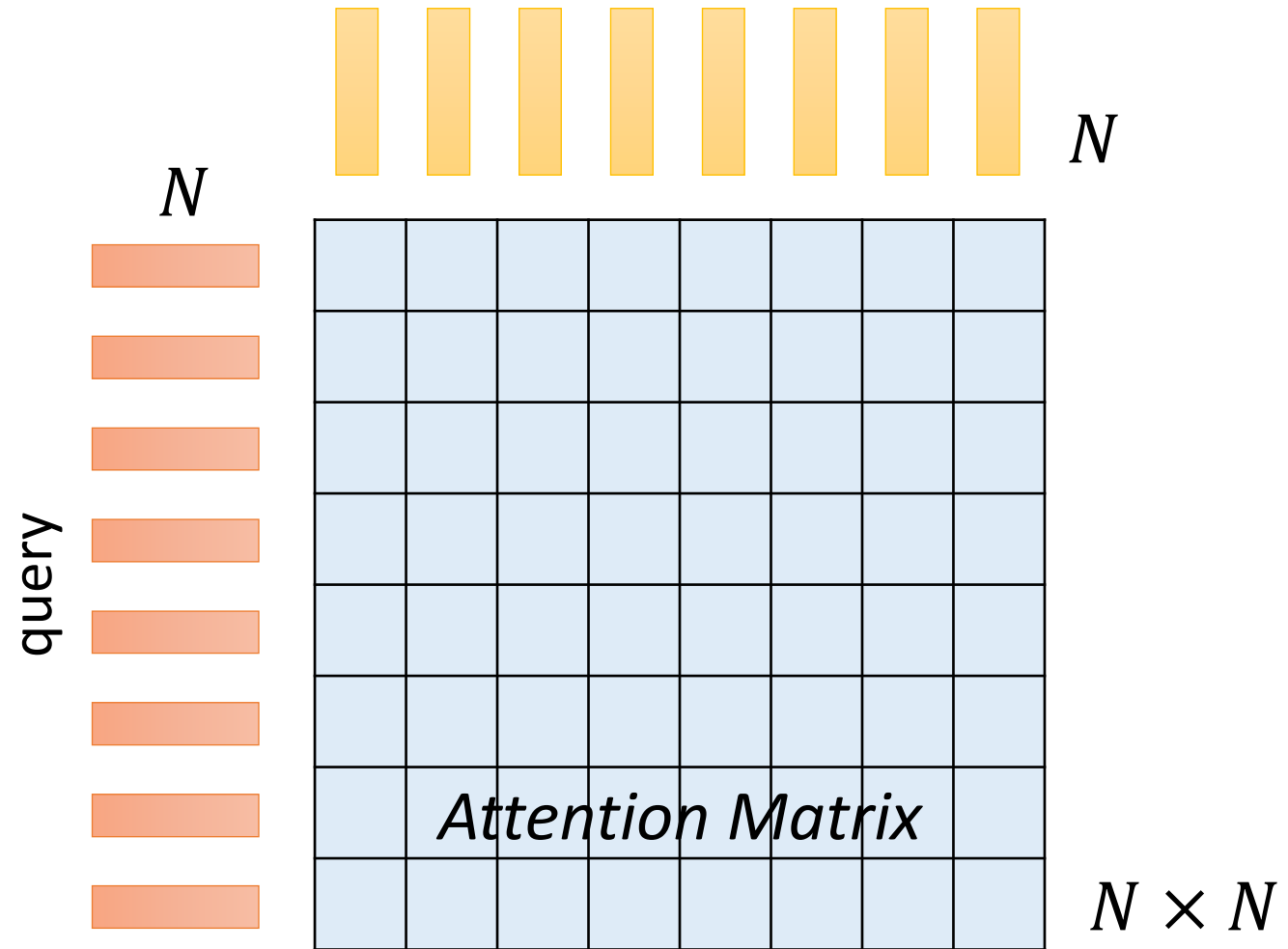
Efficient Transformers: A Survey

<https://arxiv.org/abs/2009.06732>

# How to make self-attention efficient?



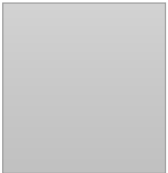
Sequence length =  $N$



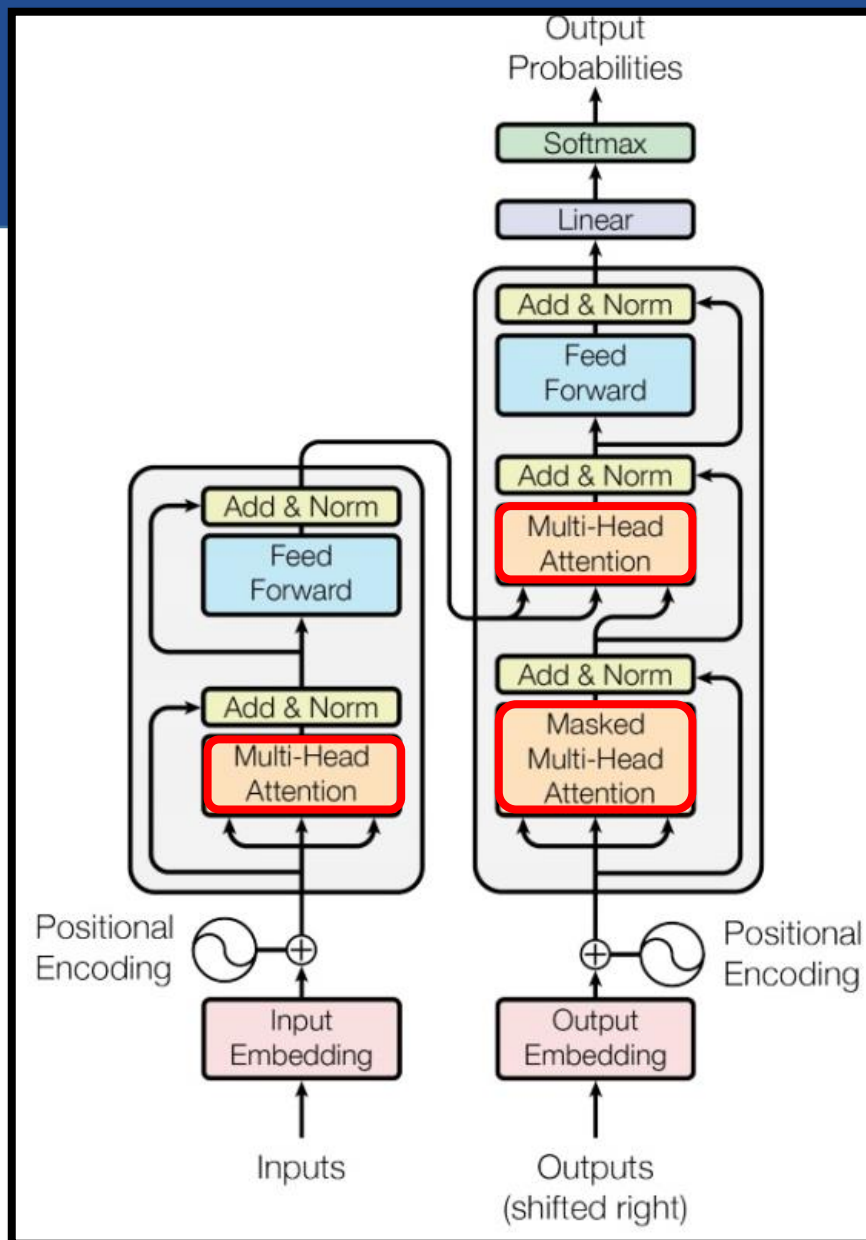
# Notice



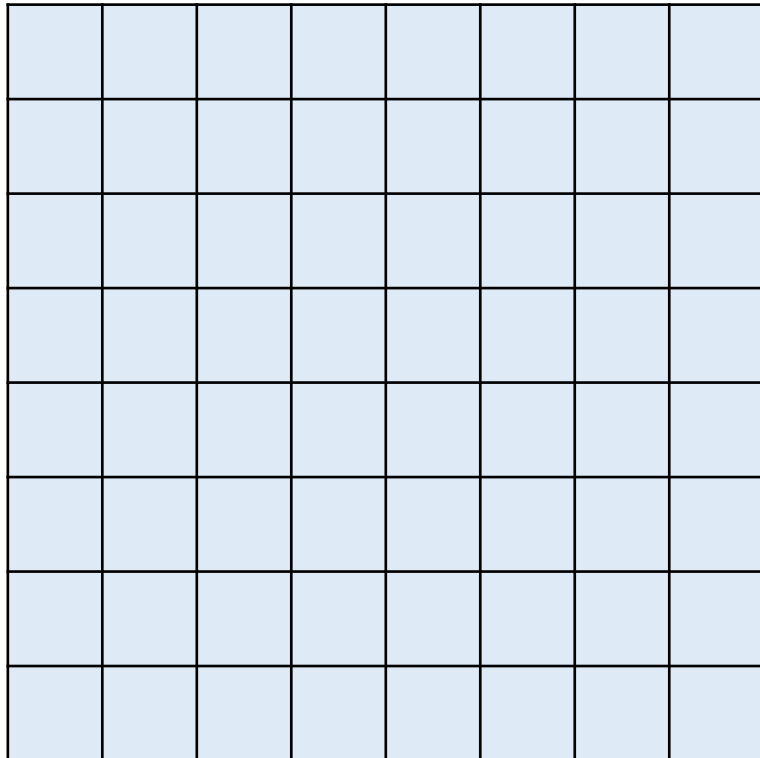
- Self-attention is only a module in a larger network. 自注意力机制仅仅知识大网络的一个模块。
- Self-attention dominates computation when  $N$  is large. 当 $N$ 非常大的时候, 自注意力机制的计算量显著上升。
- Usually developed for image processing

256  256

$N = 256 * 256$

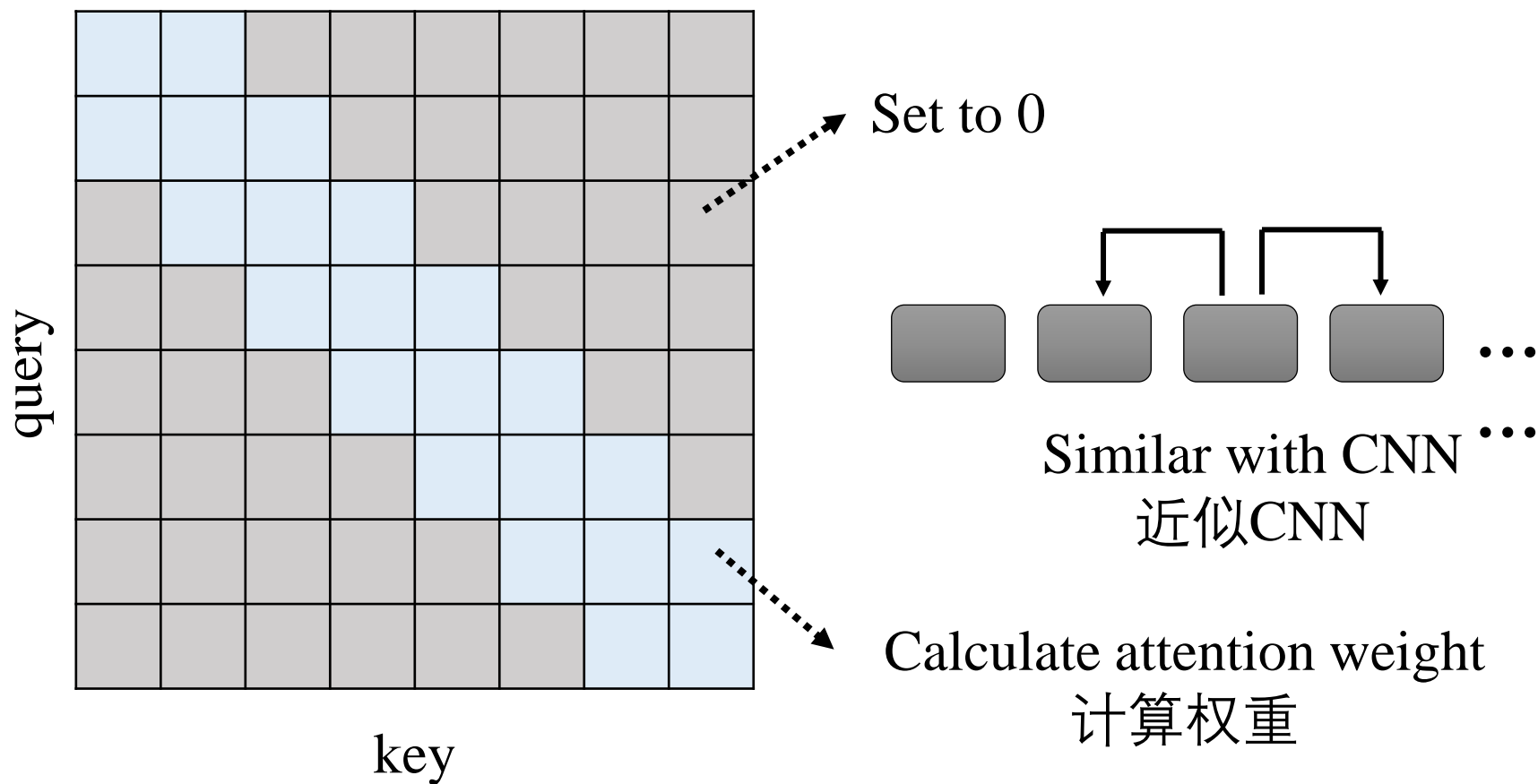


# Skip Some Calculations with Human Knowledge



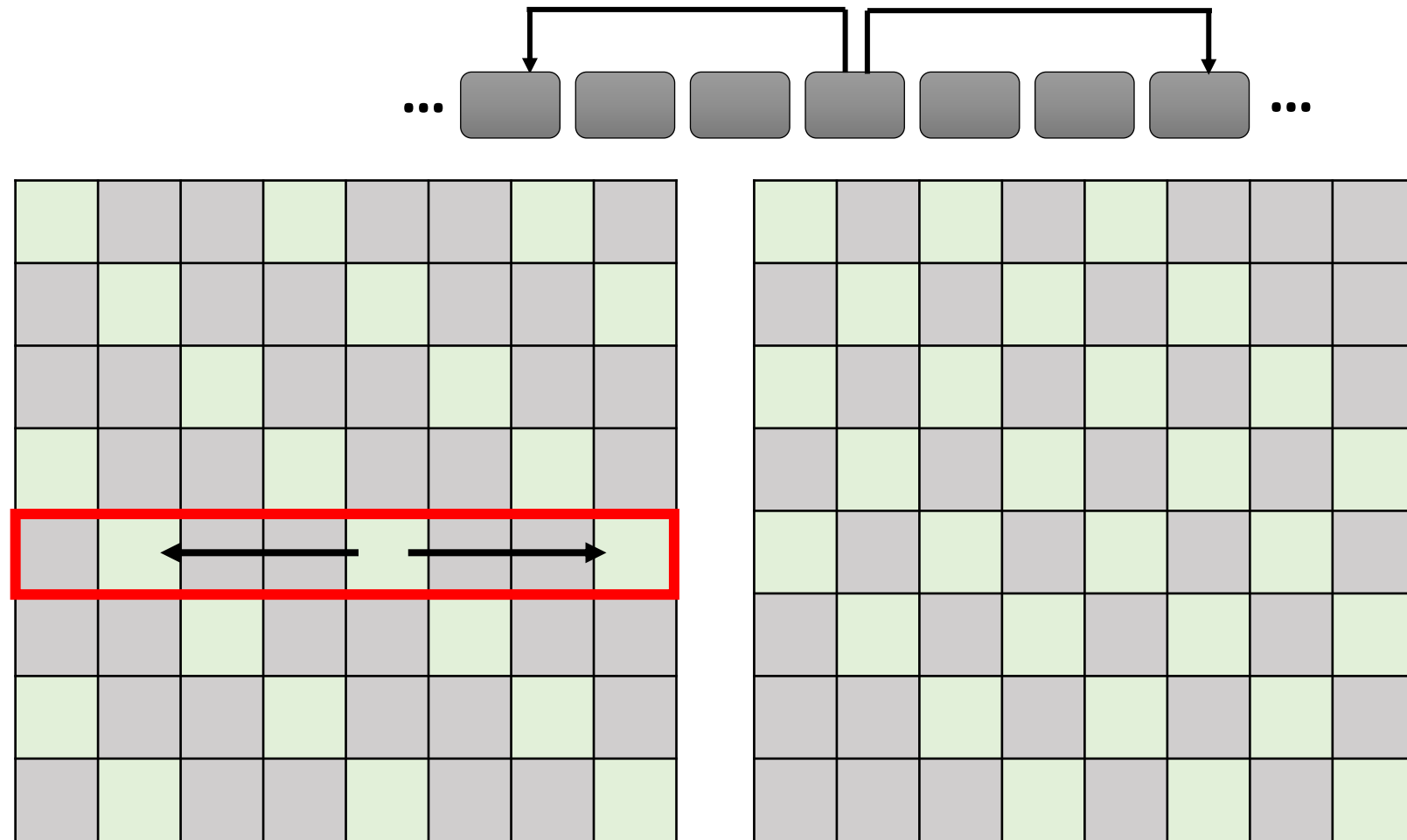
Can we fill in some values  
with human knowledge?

# Local Attention / Truncated Attention





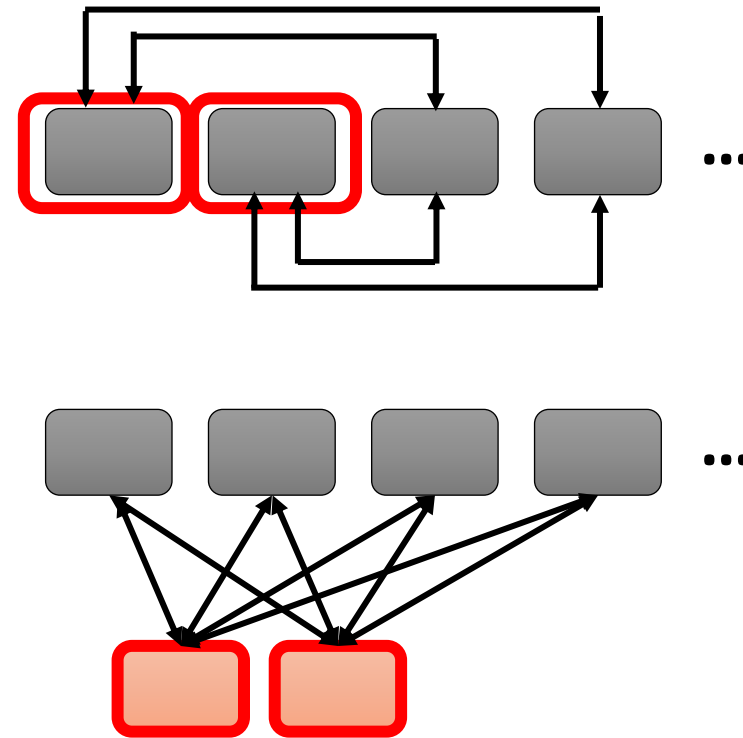
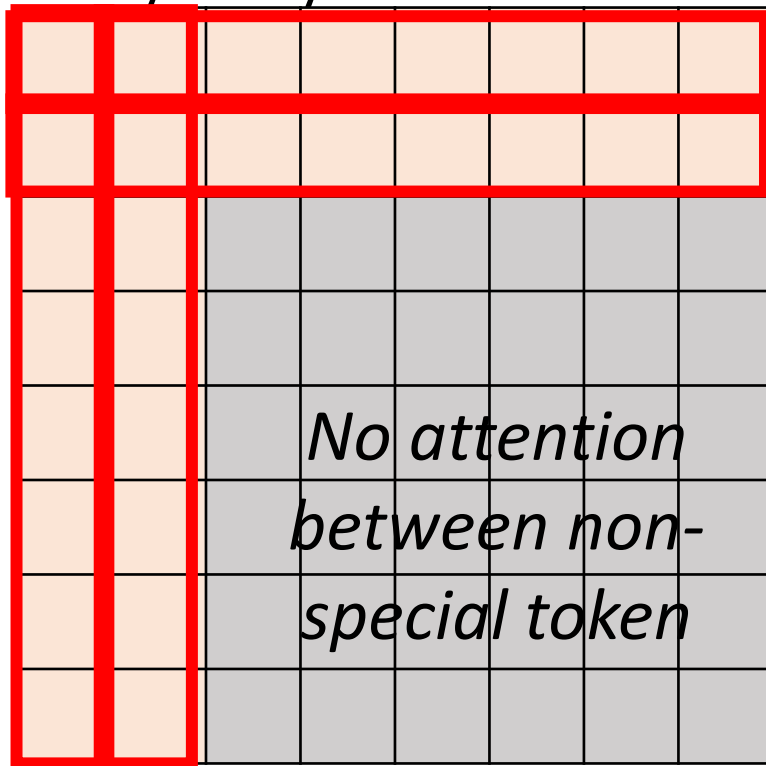
# Stride Attention



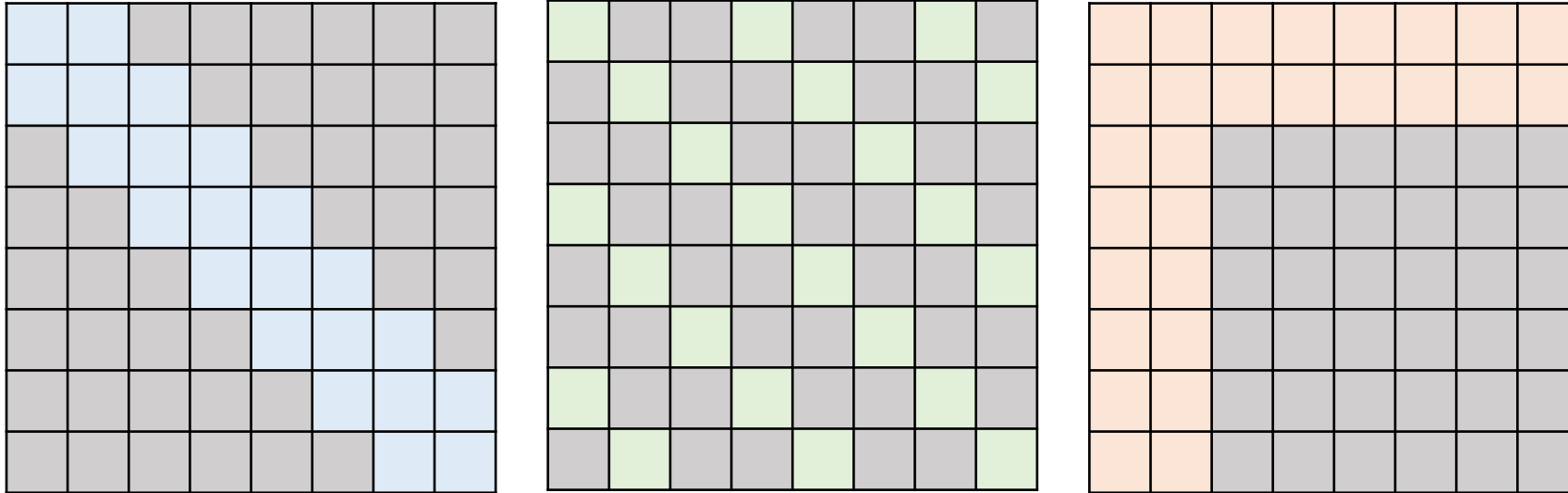
# Global Attention

Add special token into original sequence 在原句中新增特殊的token

- Attend to every token → collect global information 搜集全局信息
- Attended by every token → it knows global information



# Many Different Choices ...



**Different heads use different patterns.**

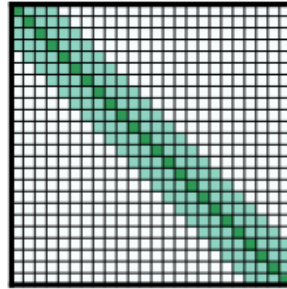
不同的头利用不同的特征。

# Many Different Choices ...

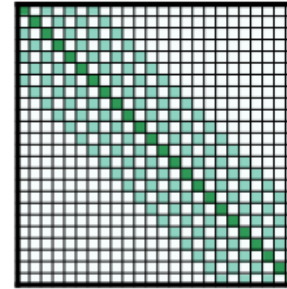


- LongFormer

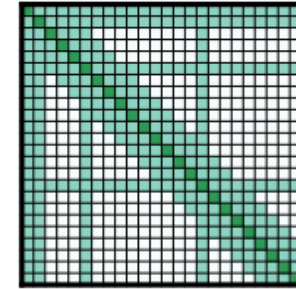
<https://arxiv.org/abs/2004.05150>



(b) Sliding window attention



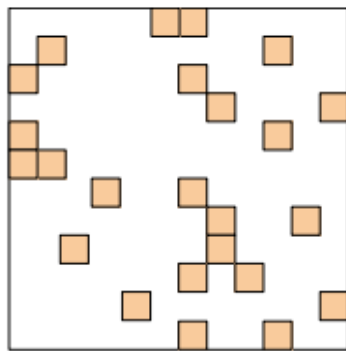
(c) Dilated sliding window



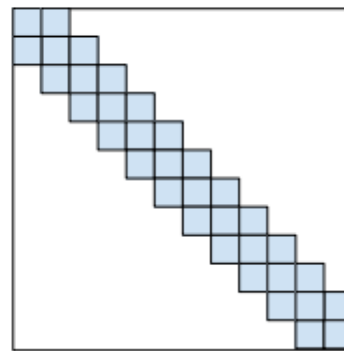
(d) Global+sliding window

- Big Bird

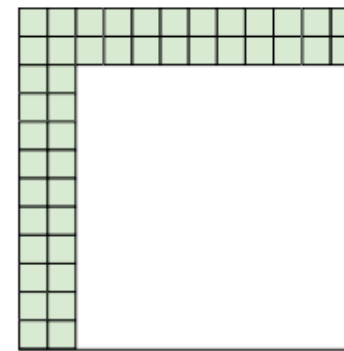
<https://arxiv.org/abs/2007.14062>



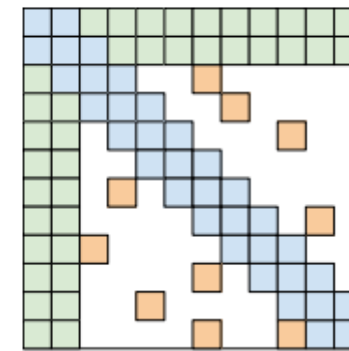
(a) Random attention



(b) Window attention

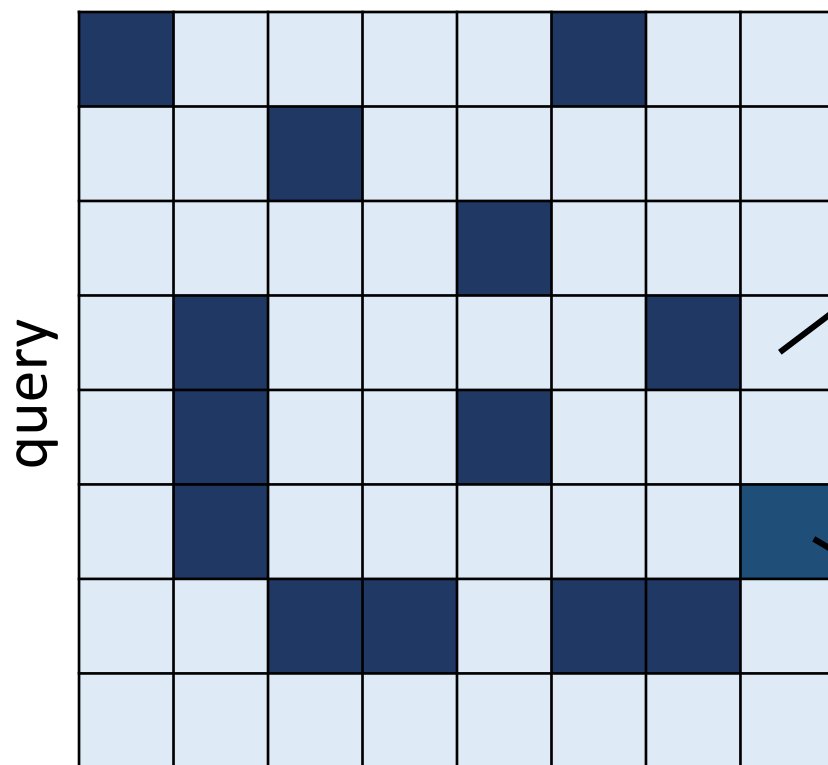


(c) Global Attention



(d) BIGBIRD

# Can we only focus on Critical Parts?



small value 小值

- Directly set to 0 直接归零
- Smaller influence on results  
对结果影响较小

large value 大值

How to quickly estimate the portion with small attention weights?

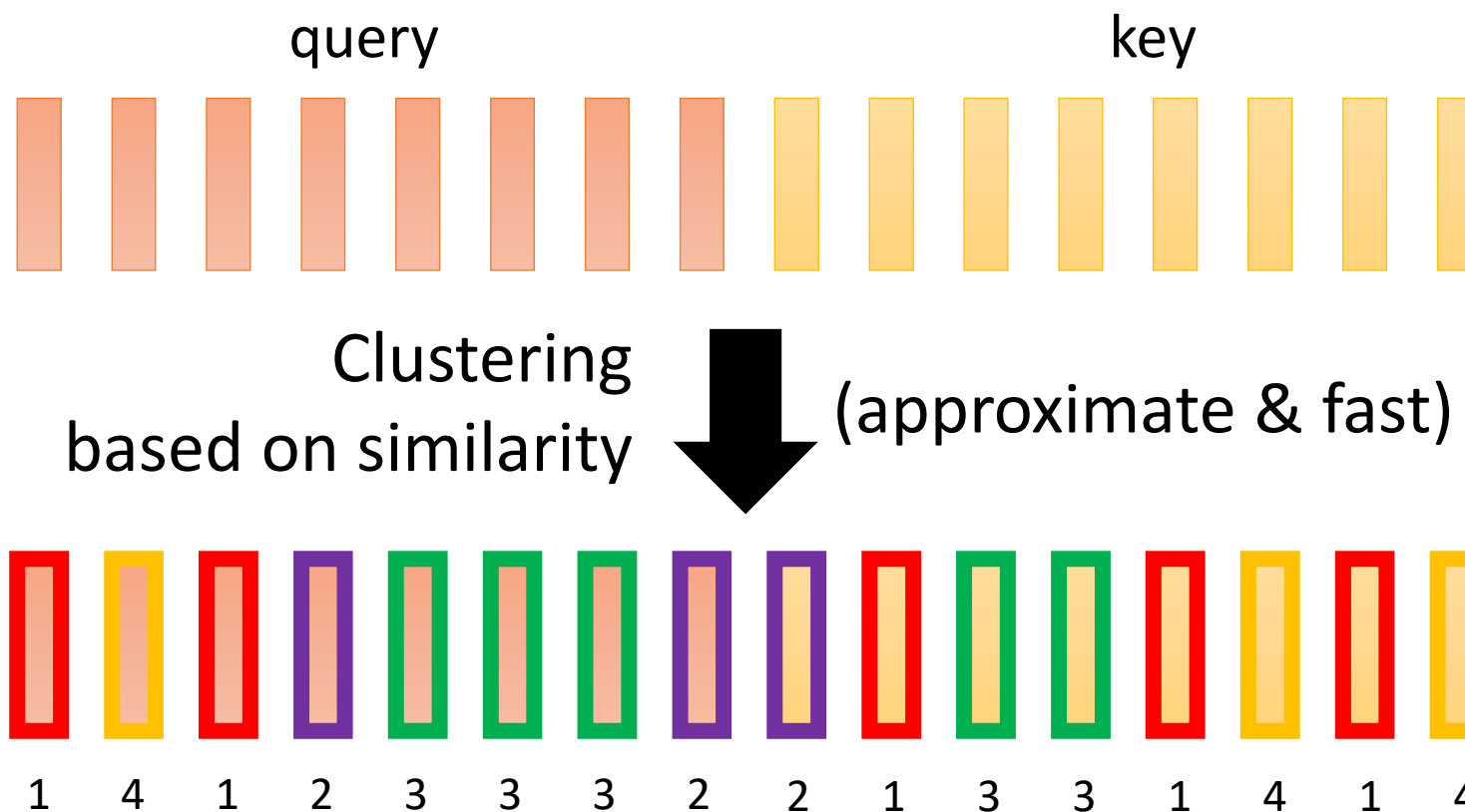
# Clustering

Reformer

<https://openreview.net/forum?id=rkgNKkHtvB>



## Step 1



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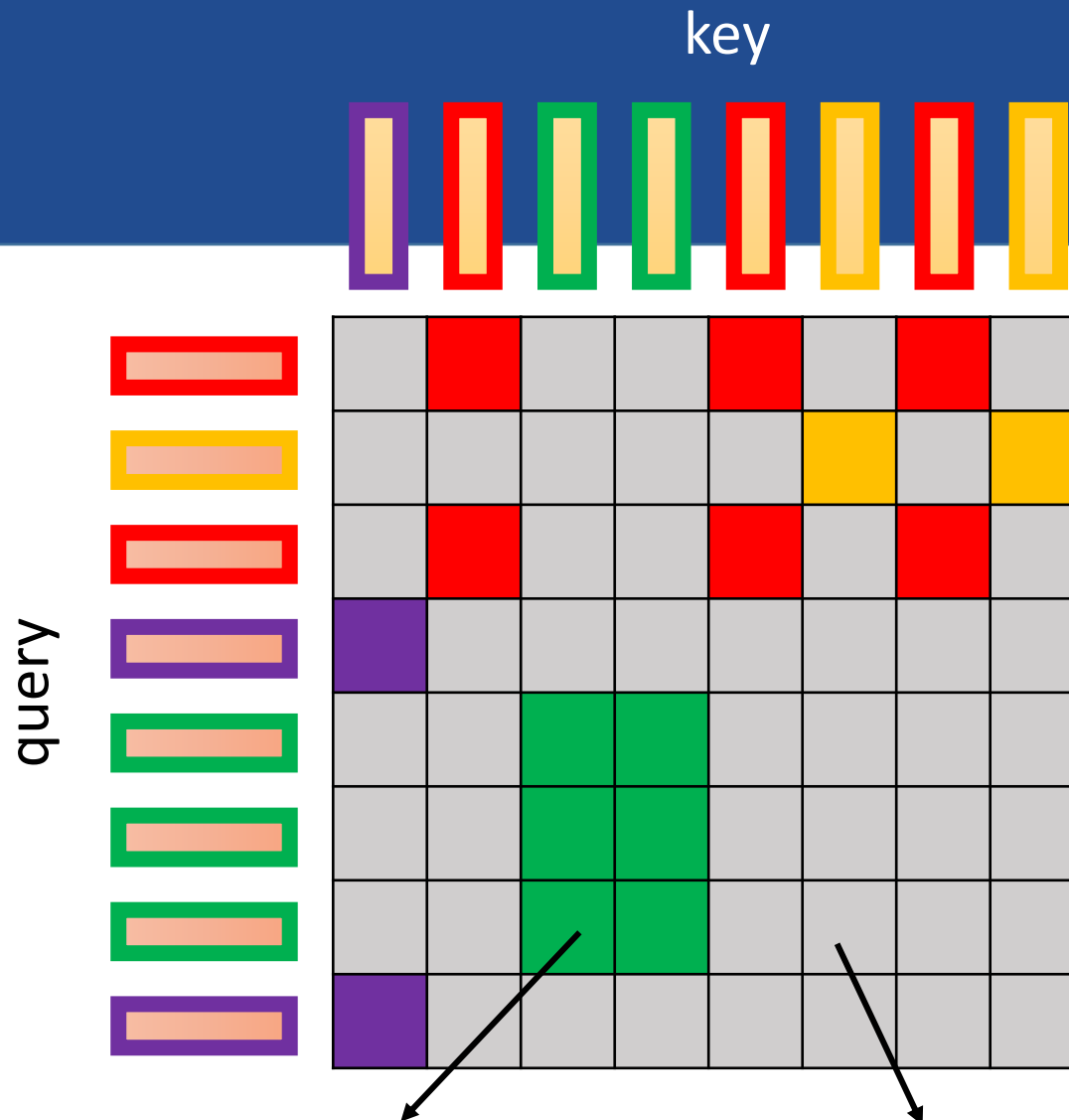
Routing Transformer

<https://arxiv.org/abs/2003.05997>

# Clustering



Step 2

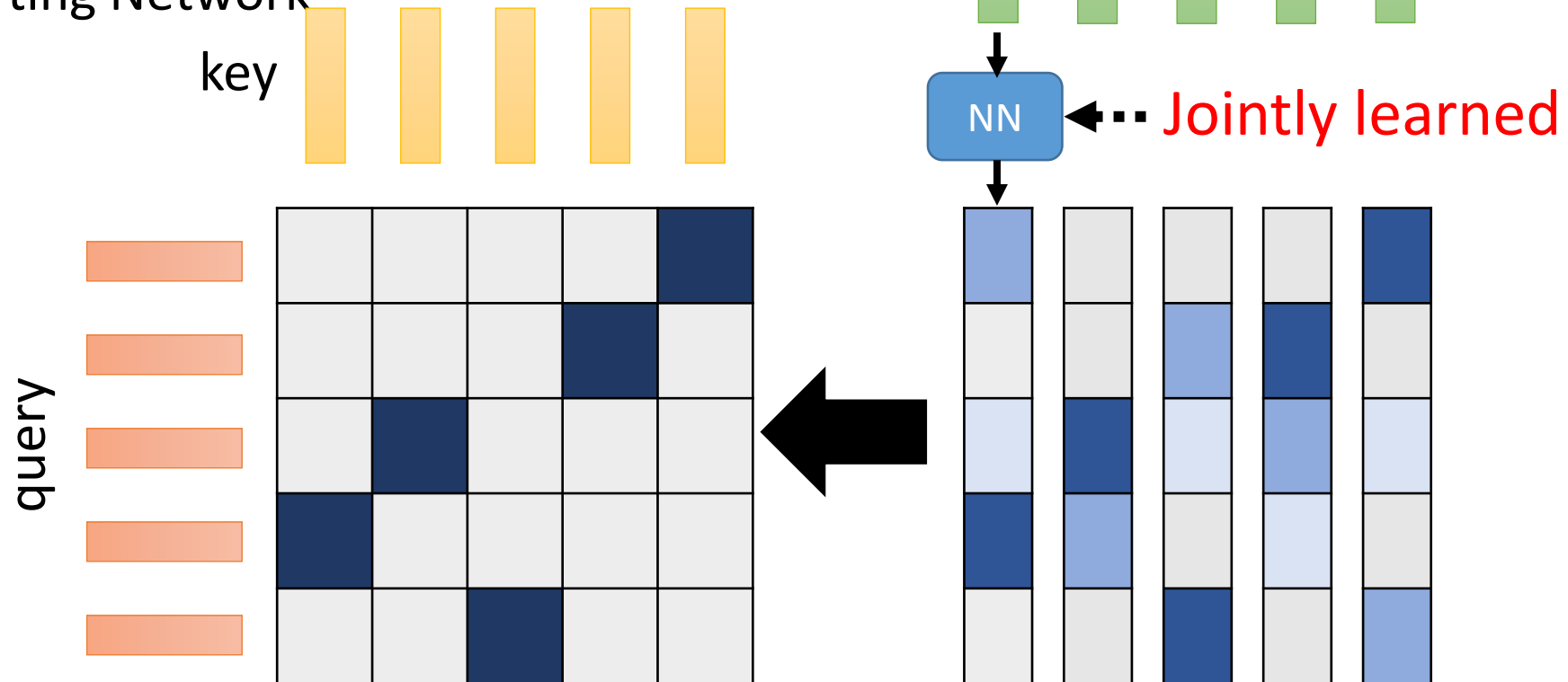


Belong to the same cluster, then  
calculate attention weight

Not the same cluster, set to 0

# Learnable Patterns

Sinkhorn Sorting Network

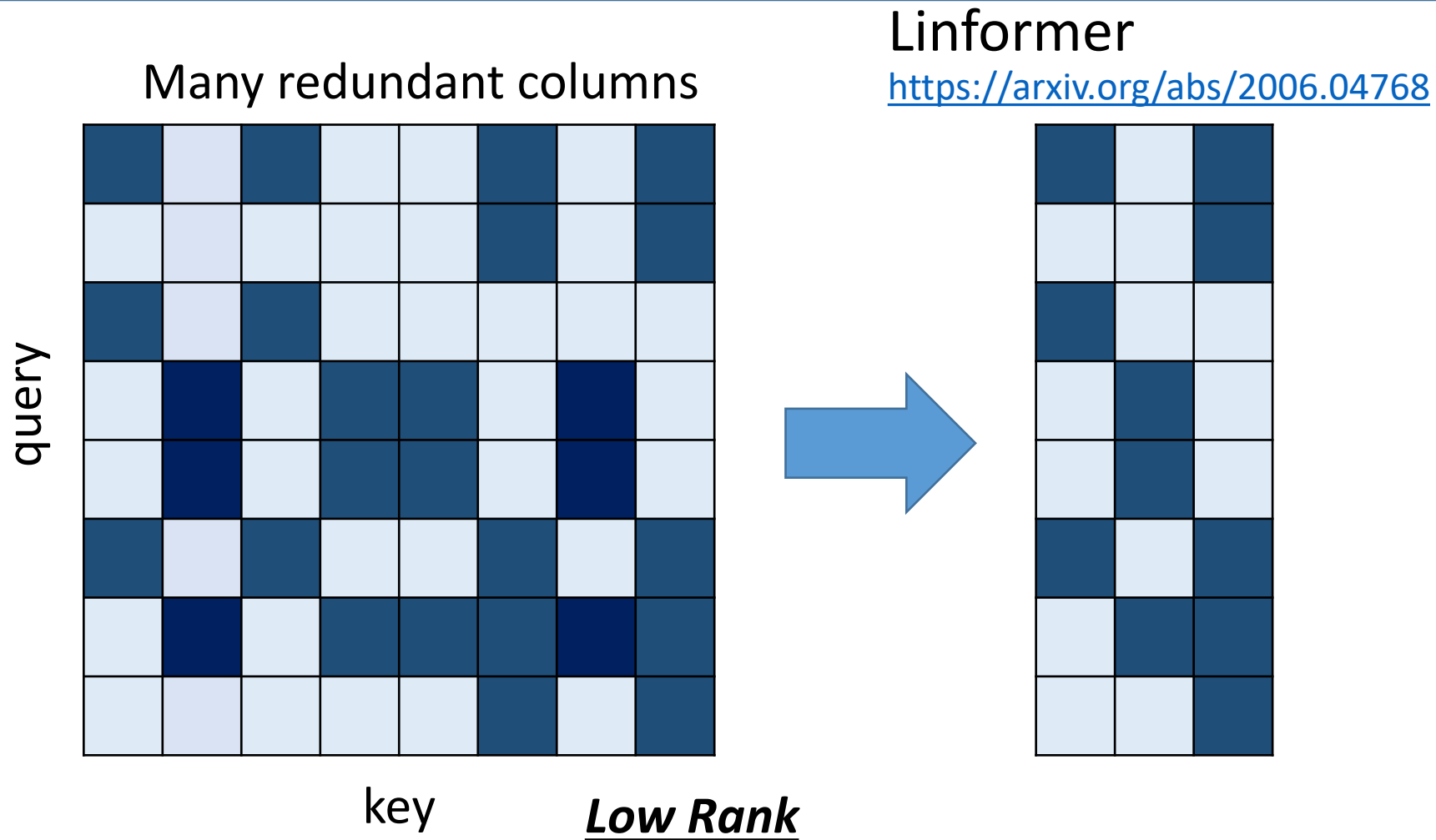


A grid should be skipped or not is decided by another learned module

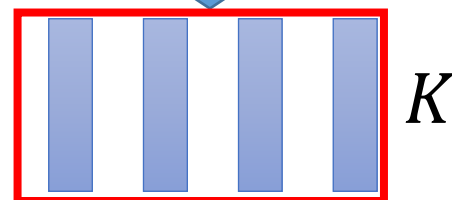
<https://arxiv.org/abs/2002.11296>  
(simplified version)



# Do we need full attention matrix?



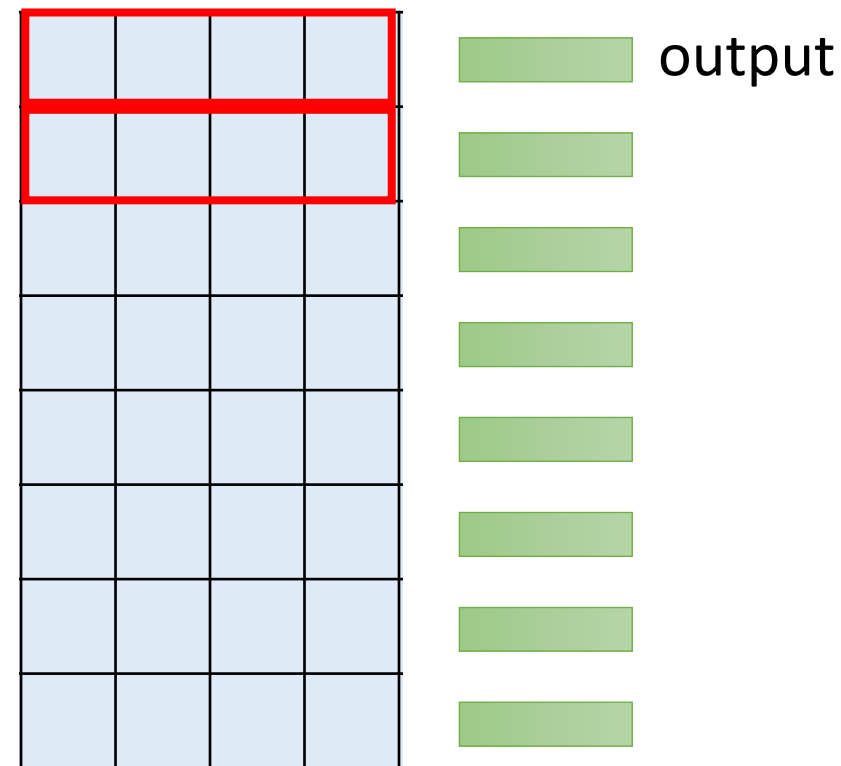
# Clustering



Can we reduce the number of queries?  
可以减少Q的数量吗?

query

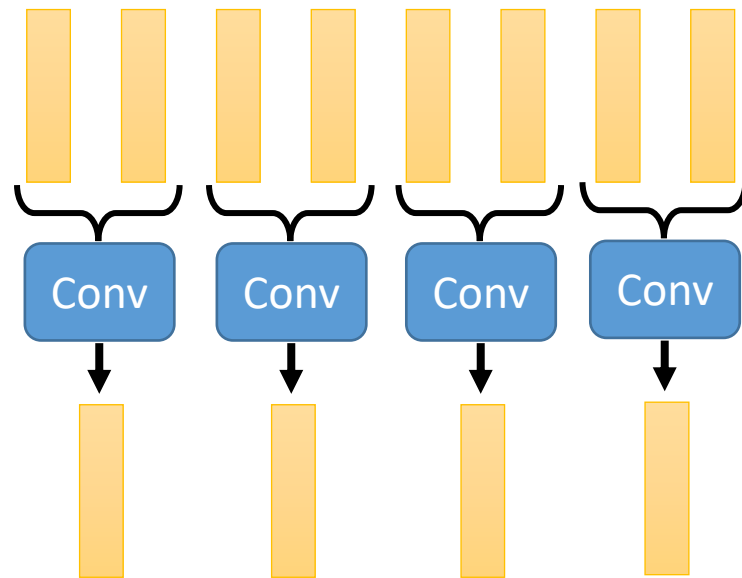
change output sequence length  
改变序列长度



# Reduce Number of Keys

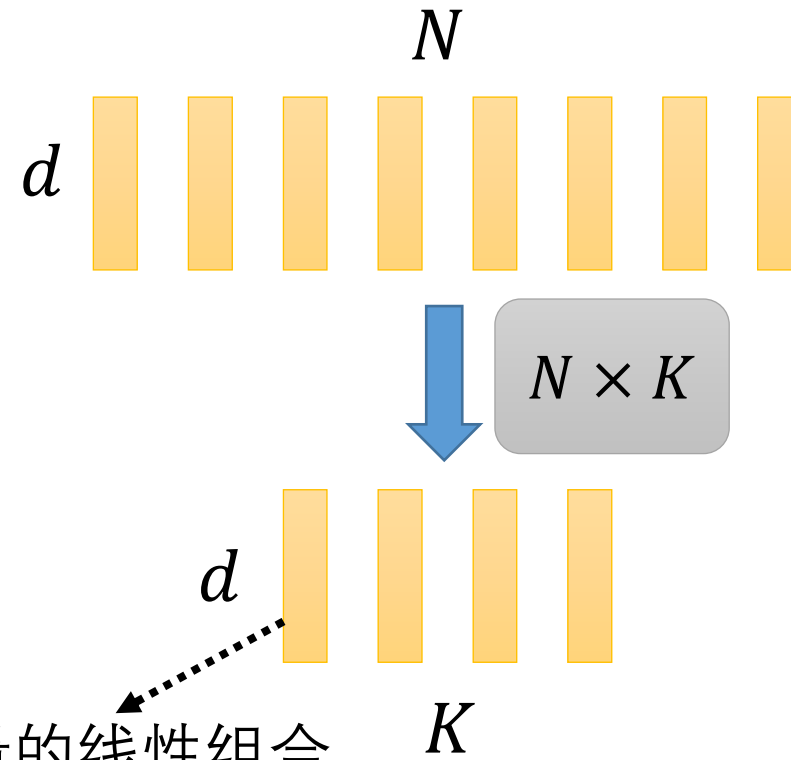
## Compressed Attention

<https://arxiv.org/abs/1801.10198>



## Linformer

<https://arxiv.org/abs/2006.04768>



Linear combination of  $N$  vectors  $N$ 个矢量的线性组合

# Attention Mechanism is three-matrix Multiplication



Review

$d \times N$

Q

=

$W^q$

I

$d \times N$

K

=

$W^k$

I

$d' \times N$

V

=

$W^v$

I

$A'$



softmax  
ignore

A

=

$K^T$

Q

$A'$

O

=

V

# Attention Mechanism is three-matrix Multiplication



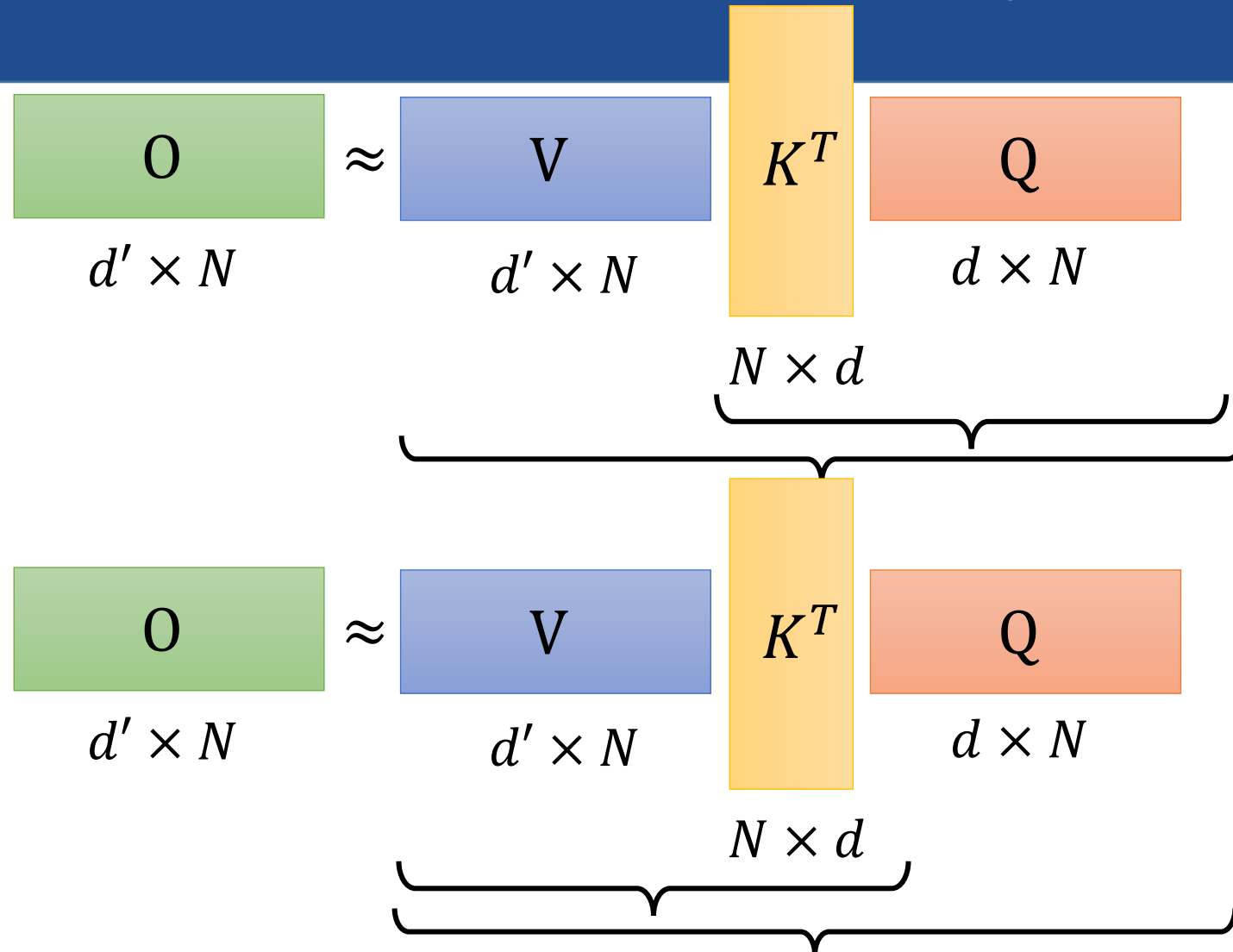
Review

$$\begin{array}{lcl} d \times N & \boxed{Q} & = \boxed{W^q} \boxed{I} \\ d \times N & \boxed{K} & = \boxed{W^k} \boxed{I} \\ d' \times N & \boxed{V} & = \boxed{W^v} \boxed{I} \end{array}$$

$$\begin{array}{ccccc} \boxed{O} & \approx & \boxed{V} & \boxed{K^T} & \boxed{Q} \\ d' \times N & & d' \times N & N \times d & d \times N \end{array}$$

A large curly brace spans the bottom of the matrices  $V$ ,  $K^T$ , and  $Q$ .

# Attention Mechanism is three-matrix Multiplication



What is the  
difference?  
区别?

# Review Linear Algebra

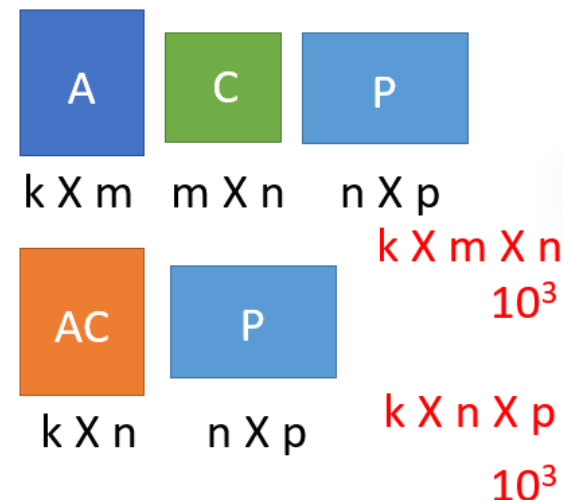
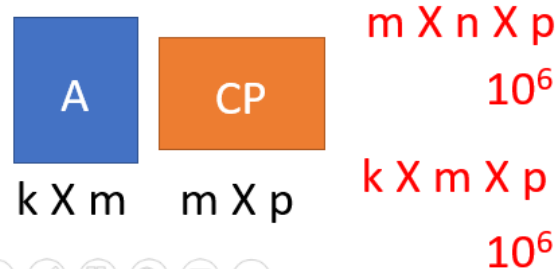
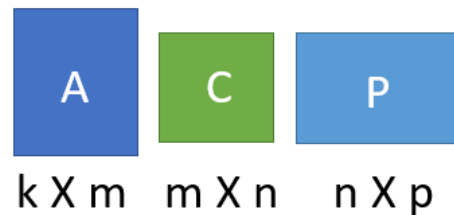


## Practical Issue

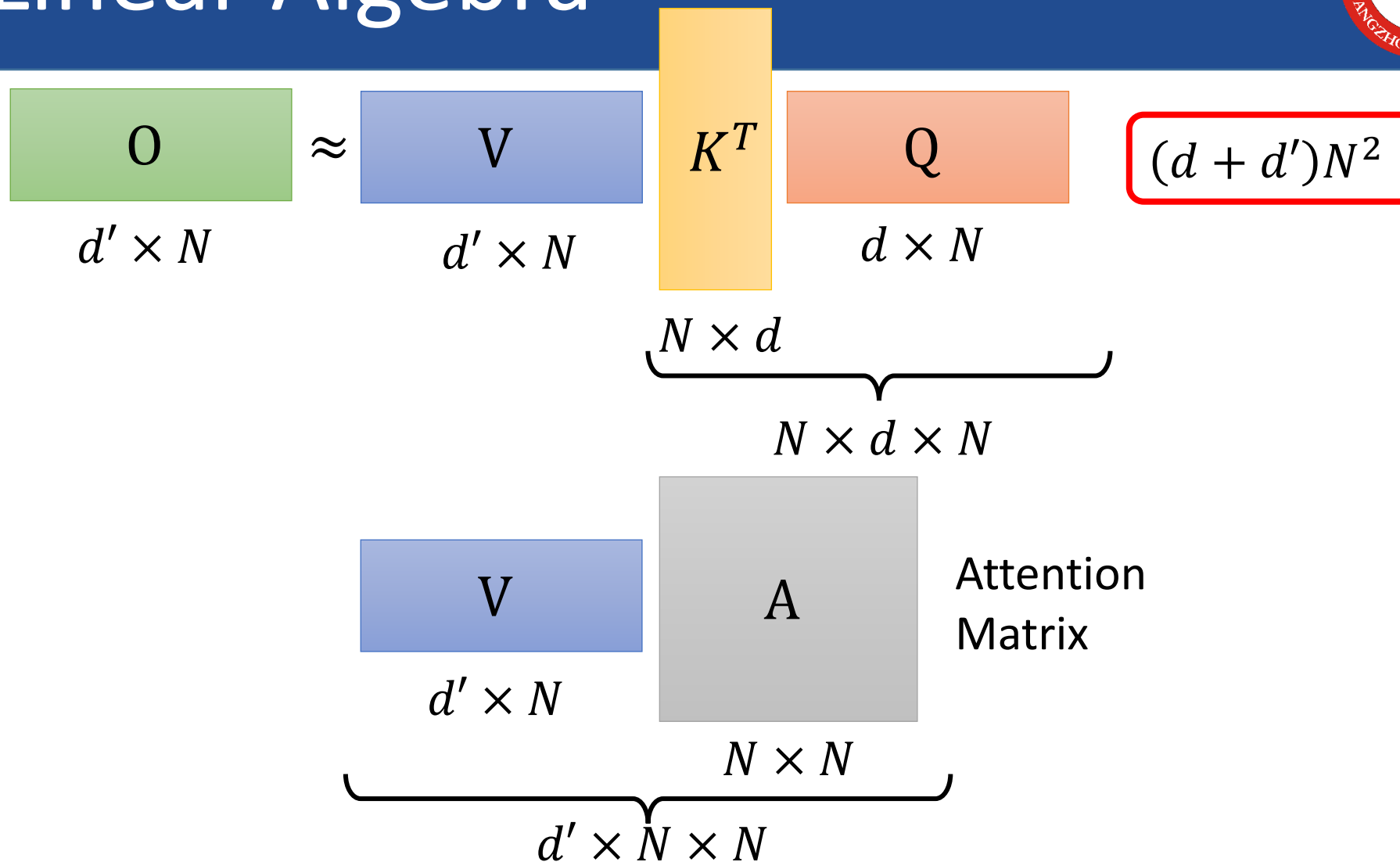
$k=1$     $m=1000$

$n=1$     $p=1000$

- Let  $A$  and  $B$  be  $k \times m$  matrices,  $C$  be an  $m \times n$  matrix, and  $P$  and  $Q$  be  $n \times p$  matrices
  - $A(CP) = (AC)P$

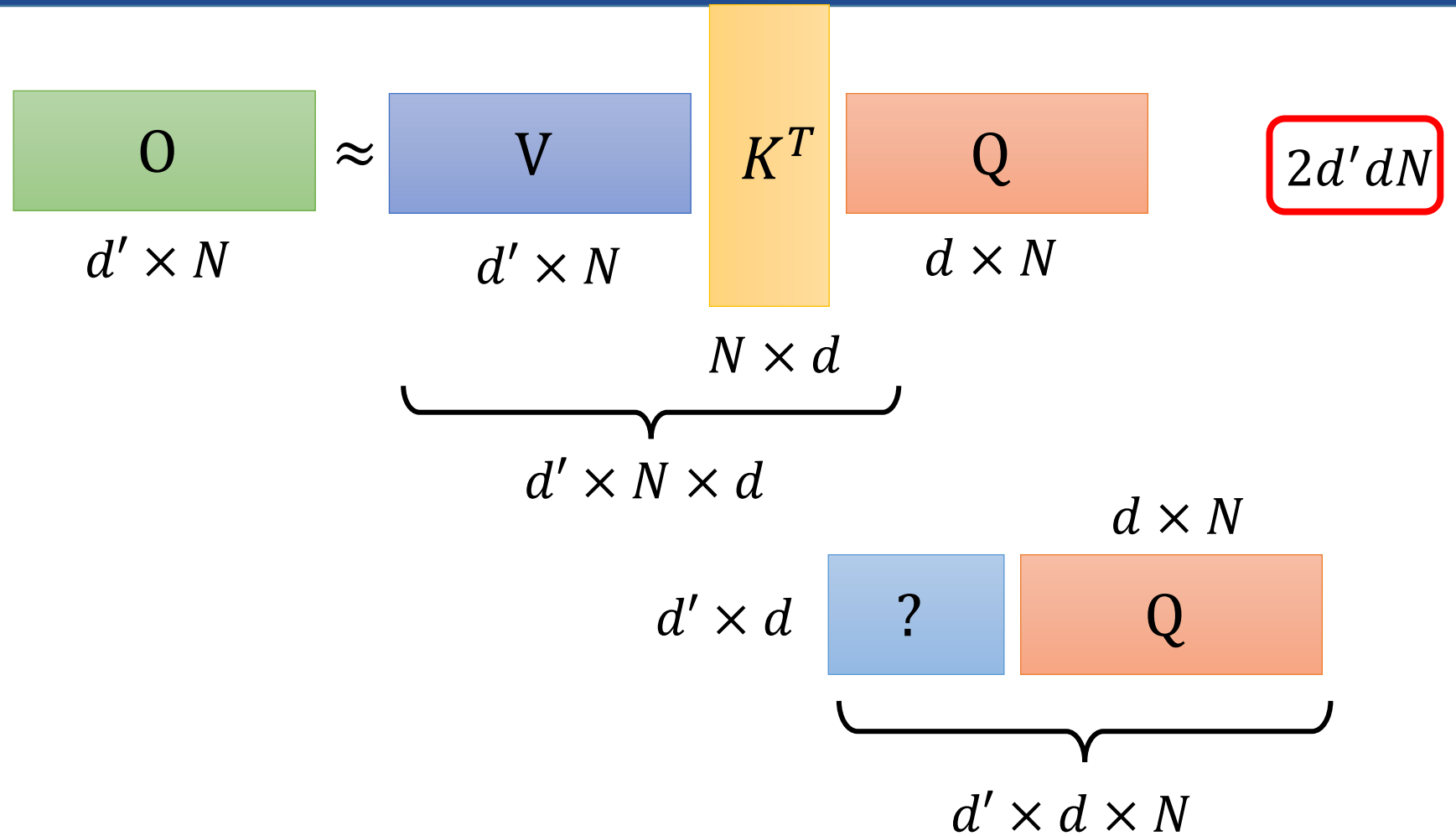


# Review Linear Algebra





# Review Linear Algebra



# Review Linear Algebra



$$\begin{array}{c} \boxed{0} \approx \boxed{V} \boxed{K^T} \boxed{Q} \\ d \times N \quad d \times N \quad N \times d \quad d \times N \end{array} \quad (d + d')N^2$$

$\underbrace{\hspace{10em}}_{N \times d} \quad \vee$

$$\begin{array}{c} \boxed{0} \approx \boxed{V} \boxed{K^T} \boxed{Q} \\ d \times N \quad d \times N \quad N \times d \quad d \times N \end{array} \quad 2d'dN$$

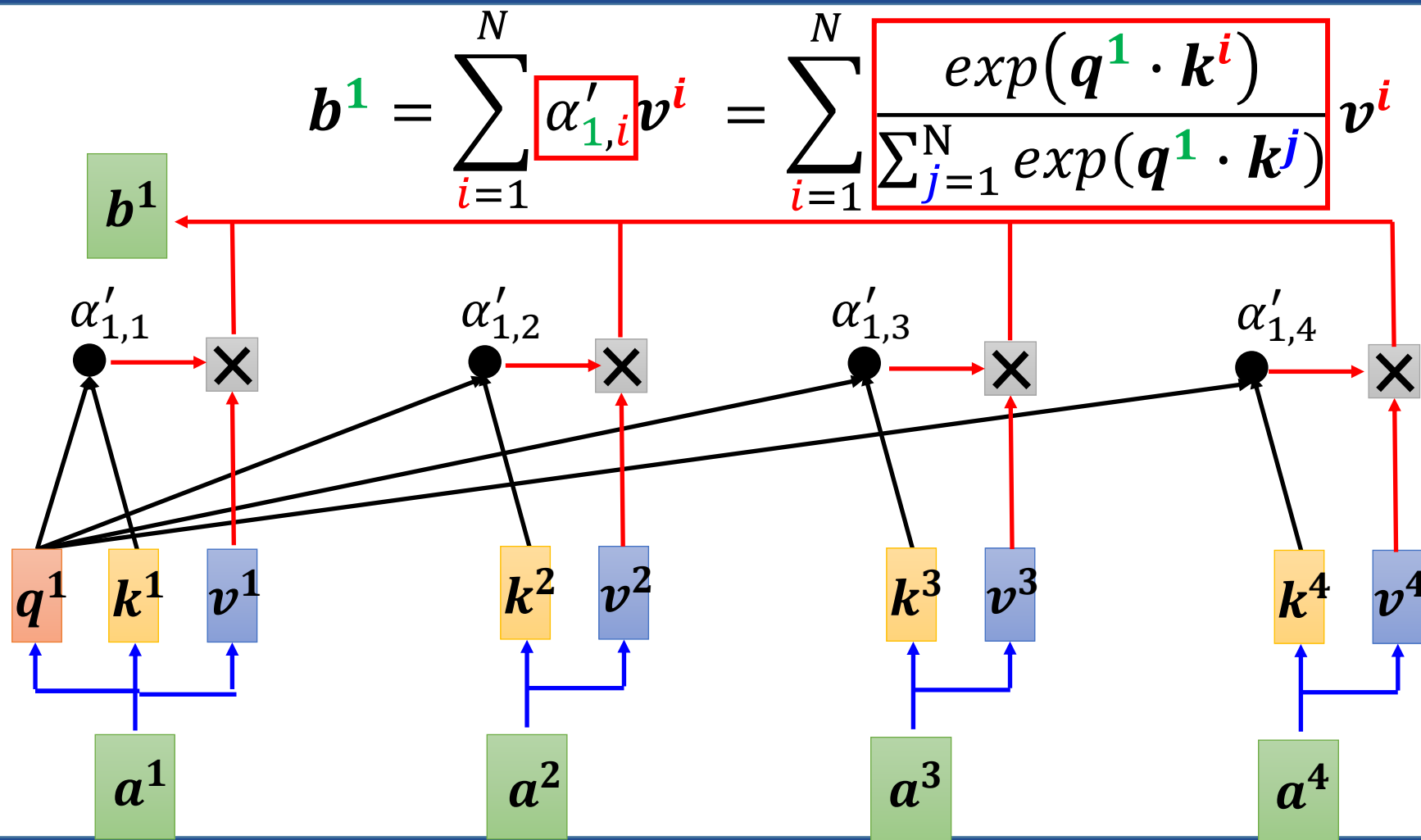
$\underbrace{\hspace{10em}}_{N \times d}$

# Review Linear Algebra



Let's put softmax back ...  
把Softmax激活函数放回来

# Softmax



# Softmax



$$\mathbf{b}^1 = \sum_{i=1}^N \alpha'_{1,i} \mathbf{v}^i = \sum_{i=1}^N \frac{\exp(\mathbf{q}^1 \cdot \mathbf{k}^i)}{\sum_{j=1}^N \exp(\mathbf{q}^1 \cdot \mathbf{k}^j)} \mathbf{v}^i$$

$$\begin{aligned} \exp(\mathbf{q} \cdot \mathbf{k}) \\ \approx \phi(\mathbf{q}) \cdot \phi(\mathbf{k}) \end{aligned}$$

$\mathbf{q} \rightarrow \phi \rightarrow \phi(\mathbf{q})$

$$= \sum_{i=1}^N \frac{\phi(\mathbf{q}^1) \cdot \phi(\mathbf{k}^i)}{\sum_{j=1}^N \phi(\mathbf{q}^1) \cdot \phi(\mathbf{k}^j)} \mathbf{v}^i$$

$$= \frac{\sum_{i=1}^N [\phi(\mathbf{q}^1) \cdot \phi(\mathbf{k}^i)] \mathbf{v}^i}{\sum_{j=1}^N \phi(\mathbf{q}^1) \cdot \phi(\mathbf{k}^j)}$$

$$\phi(\mathbf{q}^1) \cdot \sum_{j=1}^N \phi(\mathbf{k}^j) \rightarrow \phi(\mathbf{q}^1)$$

# Softmax



$$b^{\mathbf{1}} = \sum_{i=1}^N \alpha'_{1,i} v^i = \frac{\sum_{i=1}^N [\phi(q^{\mathbf{1}}) \cdot \phi(k^i)] v^i}{\phi(q^{\mathbf{1}}) \cdot \sum_{j=1}^N \phi(k^j)}$$

$$\sum_{i=1}^N [\phi(q^{\mathbf{1}}) \cdot \phi(k^i)] v^i$$

$$\phi(q^{\mathbf{1}}) = \begin{bmatrix} q_1^{\mathbf{1}} \\ q_2^{\mathbf{1}} \\ \vdots \end{bmatrix} \quad \phi(k^{\mathbf{1}}) = \begin{bmatrix} k_1^{\mathbf{1}} \\ k_2^{\mathbf{1}} \\ \vdots \end{bmatrix}$$

$$= [\phi(q^{\mathbf{1}}) \cdot \phi(k^{\mathbf{1}})] v^{\mathbf{1}} + [\phi(q^{\mathbf{1}}) \cdot \phi(k^{\mathbf{2}})] v^{\mathbf{2}} + \dots$$

$$= (q_1^{\mathbf{1}} k_1^{\mathbf{1}} + q_2^{\mathbf{1}} k_2^{\mathbf{1}} + \dots) v^{\mathbf{1}} + (q_1^{\mathbf{1}} k_1^{\mathbf{2}} + q_2^{\mathbf{1}} k_2^{\mathbf{2}} + \dots) v^{\mathbf{2}} + \dots$$

$$= \underline{q_1^{\mathbf{1}} k_1^{\mathbf{1}} v^{\mathbf{1}}} + \underline{q_2^{\mathbf{1}} k_2^{\mathbf{1}} v^{\mathbf{1}}} + \dots + \underline{q_1^{\mathbf{1}} k_1^{\mathbf{2}} v^{\mathbf{2}}} + \underline{q_2^{\mathbf{1}} k_2^{\mathbf{2}} v^{\mathbf{2}}} + \dots + \dots$$

$$= q_1^{\mathbf{1}} (k_1^{\mathbf{1}} v^{\mathbf{1}} + k_1^{\mathbf{2}} v^{\mathbf{2}} + \dots) + q_2^{\mathbf{1}} (k_2^{\mathbf{1}} v^{\mathbf{1}} + k_2^{\mathbf{2}} v^{\mathbf{2}} + \dots)$$

# Softmax



$$\mathbf{b}^1 = \sum_{i=1}^N \alpha'_{1,i} \mathbf{v}^i = \frac{\sum_{i=1}^N [\phi(\mathbf{q}^1) \cdot \phi(\mathbf{k}^i)] \mathbf{v}^i}{\phi(\mathbf{q}^1) \cdot \sum_{j=1}^N \phi(\mathbf{k}^j)}$$

$$\sum_{i=1}^N [\phi(\mathbf{q}^1) \cdot \phi(\mathbf{k}^i)] \mathbf{v}^i$$

$$\phi(\mathbf{q}^1) = \begin{bmatrix} q_1^1 \\ q_2^1 \\ \vdots \end{bmatrix}$$

M dim

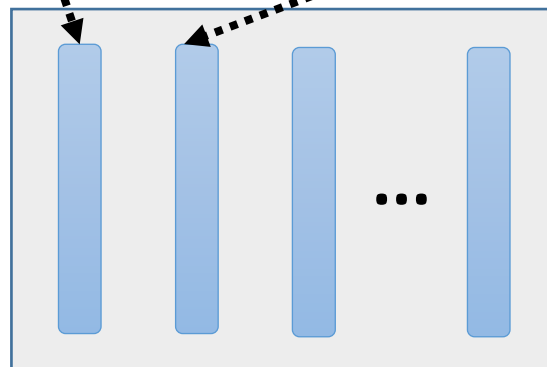
$$\phi(\mathbf{k}^1) = \begin{bmatrix} k_1^1 \\ k_2^1 \\ \vdots \end{bmatrix}$$

$$= q_1^1 (k_1^1 \mathbf{v}^1 + k_2^1 \mathbf{v}^2 + \dots) + q_2^1 (k_2^1 \mathbf{v}^1 + k_2^2 \mathbf{v}^2 + \dots)$$

$$\sum_{j=1}^N k_1^j \mathbf{v}^j$$

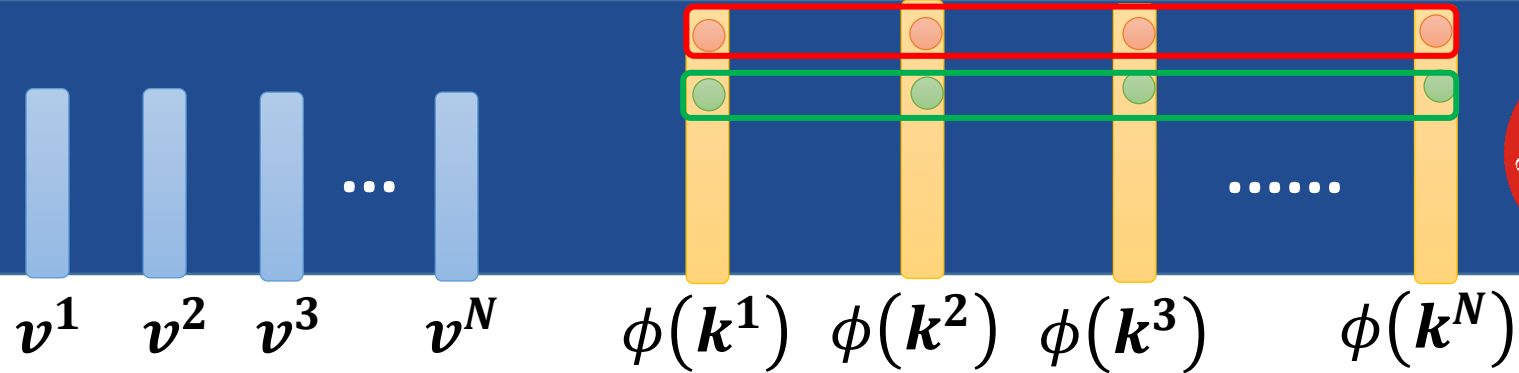
$$\sum_{j=1}^N k_2^j \mathbf{v}^j$$

M vectors



$\phi(\mathbf{q}^1)$

# Softmax



$$\sum_{j=1}^N k_1^j v^j \quad \sum_{j=1}^N k_2^j v^j$$

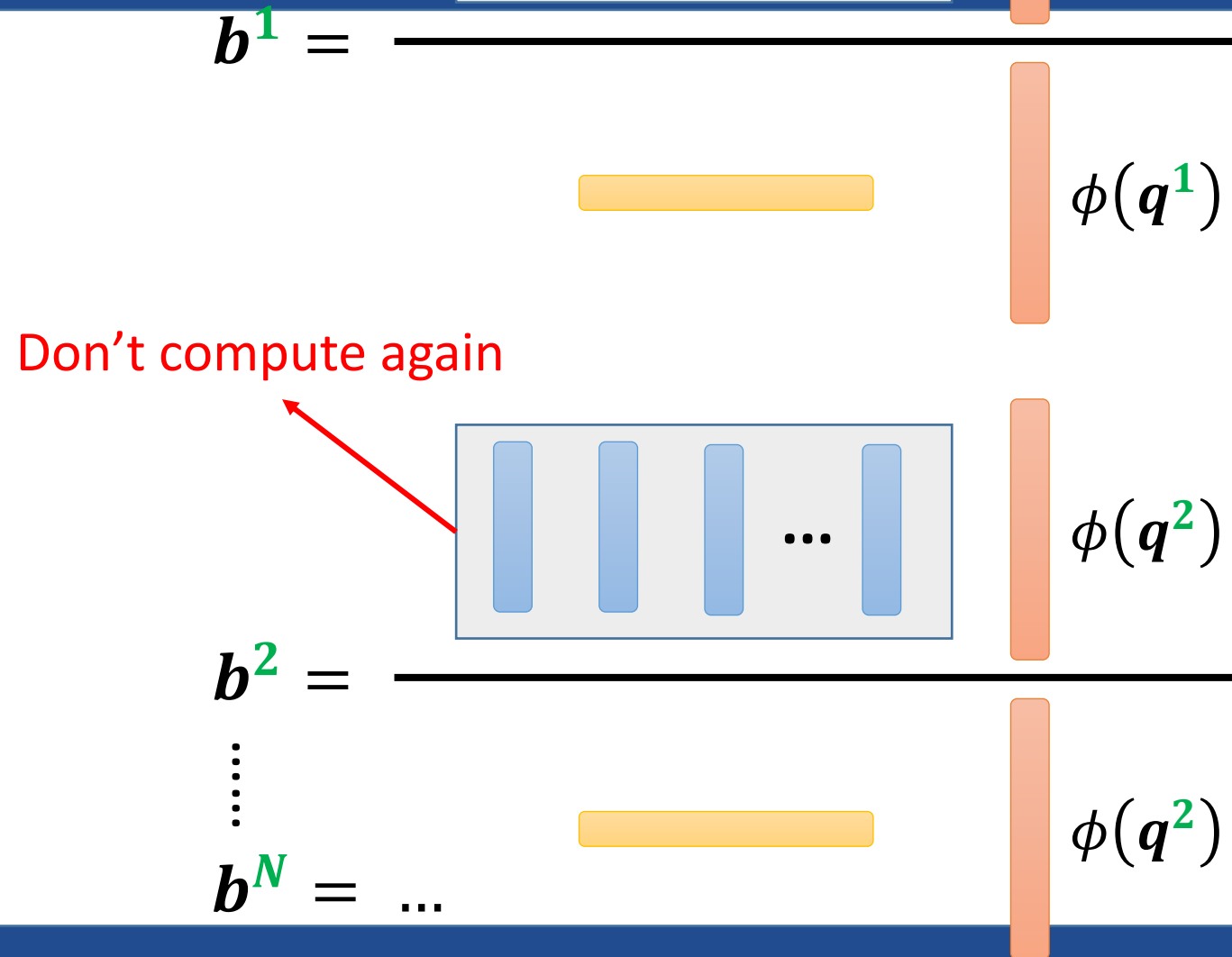
$M$  vectors

$$b^1 = \frac{\sum_{j=1}^N \phi(k^j)}{\phi(q^1)}$$

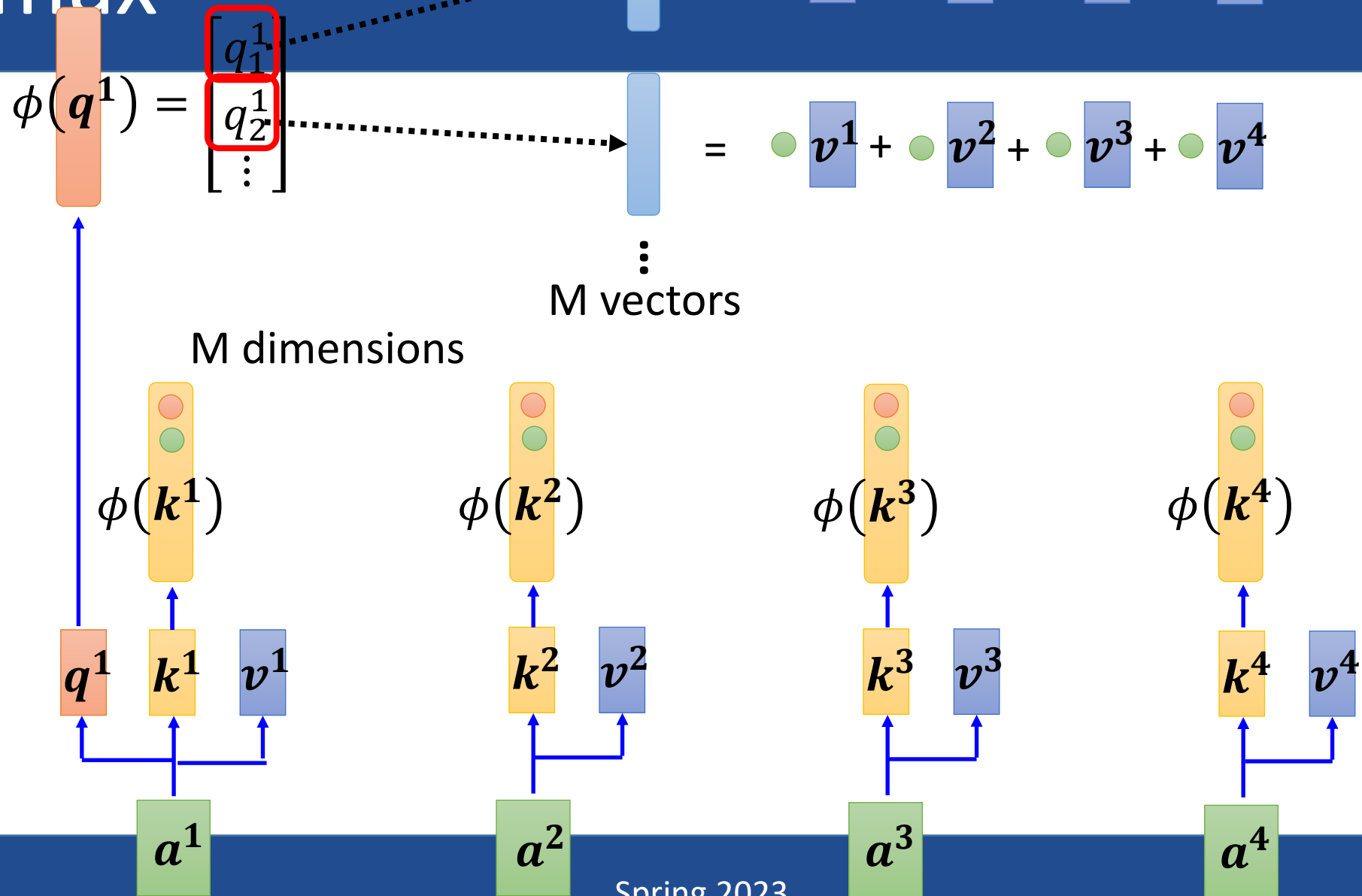
Diagram illustrating the calculation of the bias term  $b^1$ . The numerator is the sum of feature vectors  $\phi(k^j)$  for  $j=1$  to  $N$ . The denominator is the feature vector  $\phi(q^1)$ . Both the numerator and denominator are  $M$  dimensional.



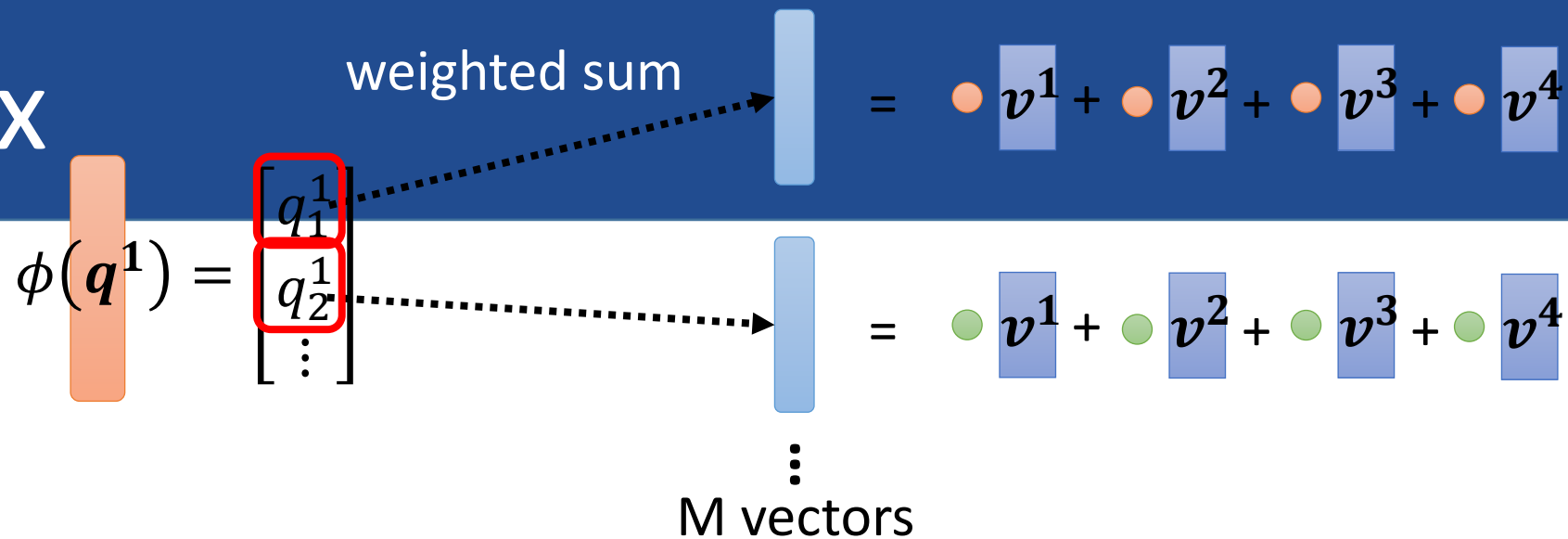
# Softmax



# Softmax

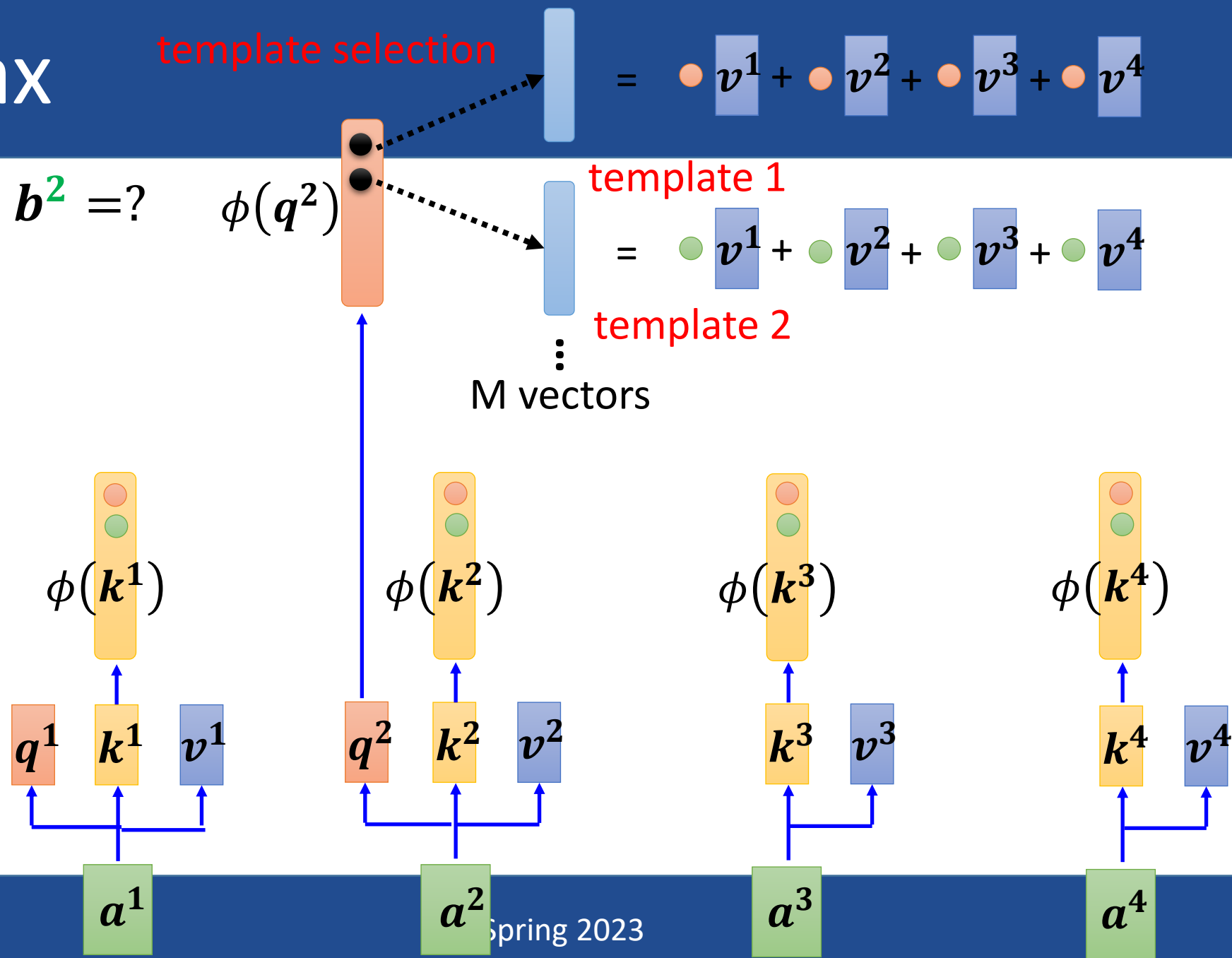


# Softmax

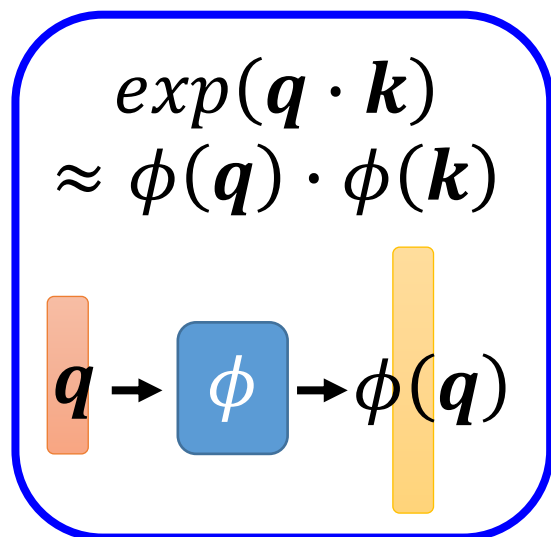


$$b^1 = \frac{\sum_{j=1}^N \phi(k^j)}{\phi(q^1)}$$

# Softmax



# Realization



- Efficient attention

<https://arxiv.org/pdf/1812.01243.pdf>

- Linear Transformer

<https://linear-transformers.com/>

- Random Feature Attention

<https://arxiv.org/pdf/2103.02143.pdf>

- Performer

<https://arxiv.org/pdf/2009.14794.pdf>

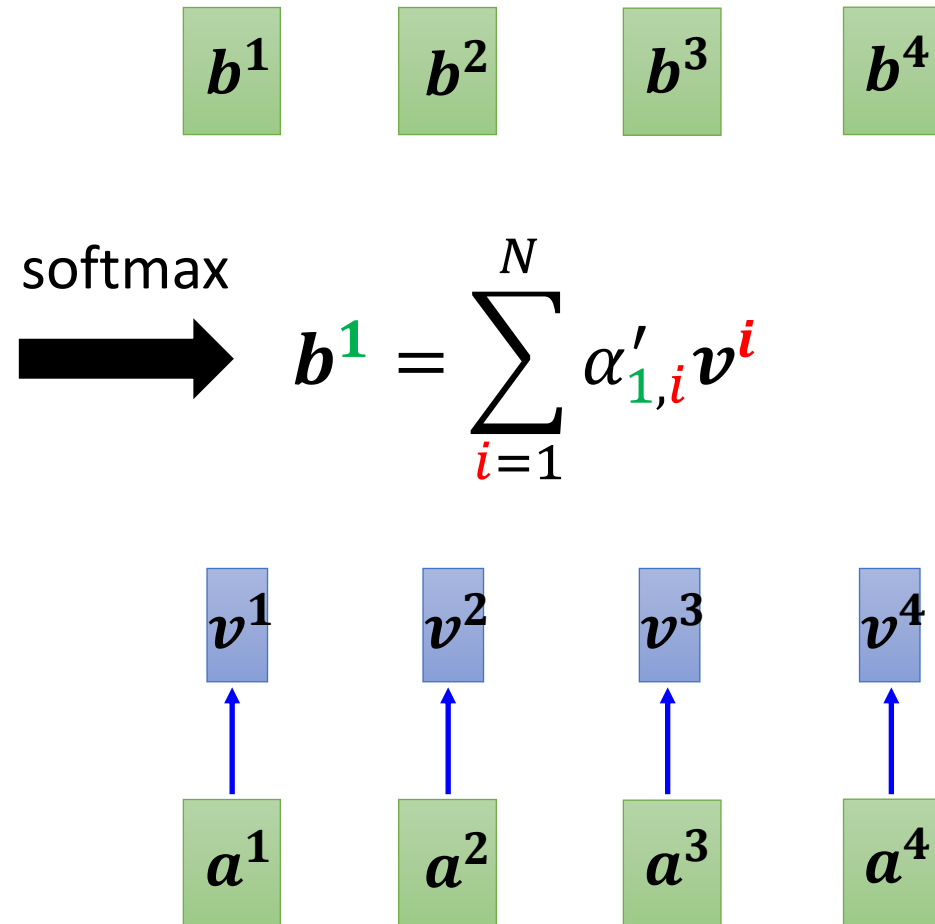
# Do we need q and k to compute attention? Synthesizer!



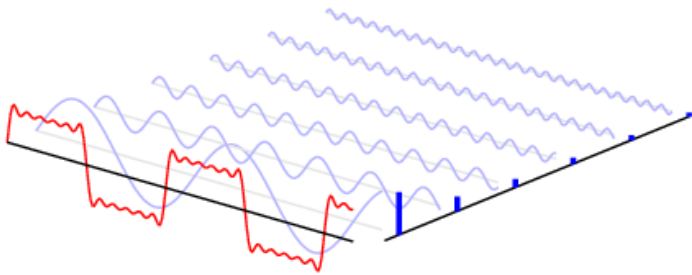
$\alpha_{1,1}$	$\alpha_{1,2}$	$\alpha_{1,3}$	$\alpha_{1,4}$
$\alpha_{1,2}$	$\alpha_{2,2}$	$\alpha_{2,3}$	$\alpha_{2,4}$
$\alpha_{1,3}$	$\alpha_{2,3}$	$\alpha_{3,3}$	$\alpha_{3,4}$
$\alpha_{1,4}$	$\alpha_{2,4}$	$\alpha_{3,4}$	$\alpha_{4,4}$

~~From q and k?~~

They are network  
parameters!



# Attention-Free



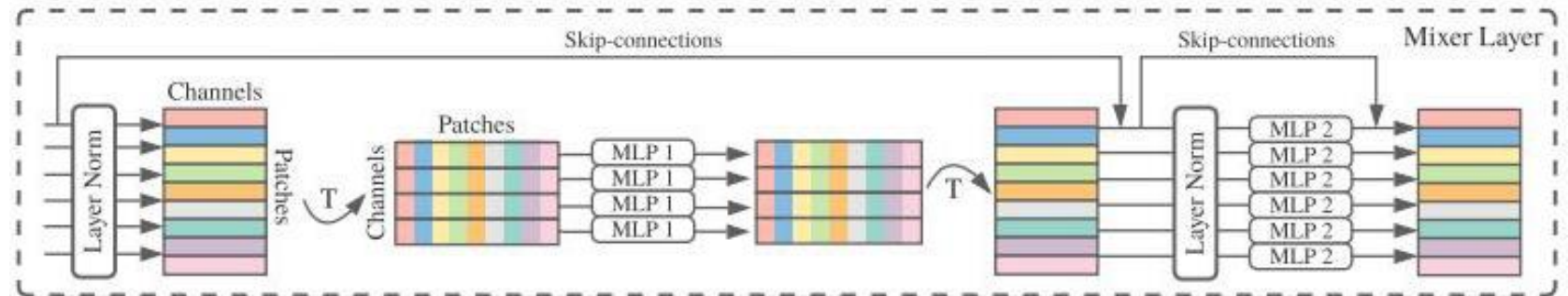
- Fnet: Mixing tokens with fourier transforms

<https://arxiv.org/abs/2105.03824>

- Pay Attention to MLPs <https://arxiv.org/abs/2105.08050>

- MLP-Mixer: An all-MLP Architecture for Vision

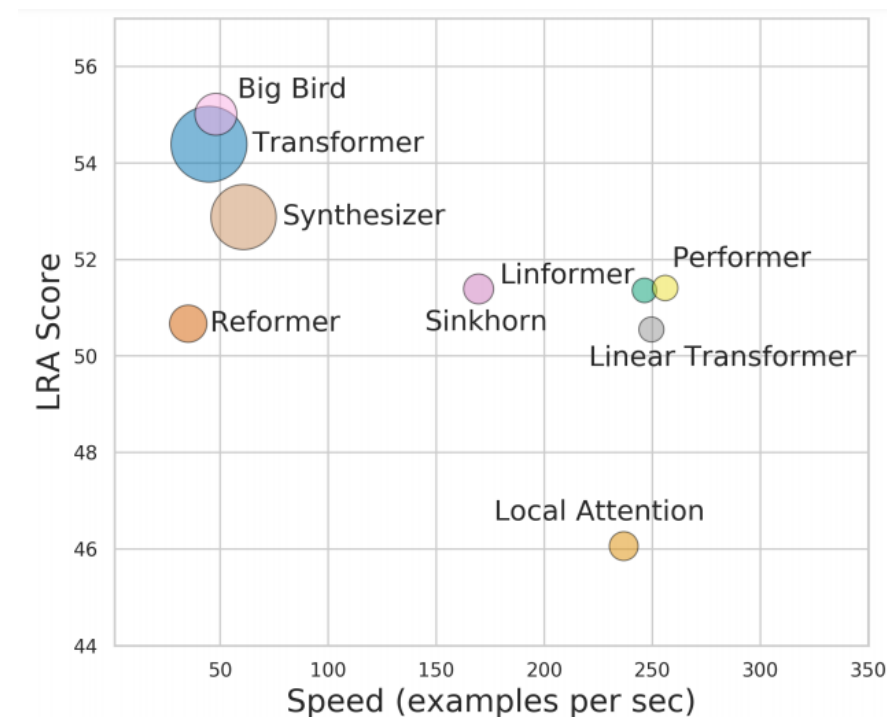
<https://arxiv.org/abs/2105.01601>



# Summary



- Human knowledge
  - Local Attention, Big Bird
- Clustering
  - Reformer
- Learnable Pattern
  - SinFform
- Representative key
  - LinFormer
- k,q first  $\rightarrow$  v,k first
  - Linear Transformer, Performer
- New framework
  - Synthesizer





# Q&A



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