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Data Availability
Data openly available in a public repository that does not issue DOIs. Source code and data are available on GitHub at the following URLs:

Project Home `https://github.com/pyroll-project`

Core Package `https://github.com/pyroll-project/pyroll-core`

Benchmark Input and Data `https://github.com/pyroll-project/pyroll-pub1-benchmark`

Conflicts of Interest
The authors declare no conflicts of interest.

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Keywords
Rolling Simulation; Open Source; Groove Rolling

Rolling Process Variation Estimation Using a Monte-Carlo Method

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1 Introduction

All technical processes are subject to certain variations. Knowledge and control of these variations is crucial for process stability and product quality. Sources of variations in a process are the variations already present in the input workpiece and newly generated variations due to imperfections of the process itself. In the case of wire and bar hot rolling industry regarded here, not only the product geometry, but also the final material properties sre highly sensitiv to the process conditions. Especially the temperature evolution of the workpiece within the rolling line has high impact on the microstructural transformation processes happening before, during and after forming. The product's microstructural state determines the mechanical properties and therefore its behavior in further processing and application. Current alloy concepts in the steel industry increase this problem, as the process window gets more narrow to achieve the property requirements for high-tech applications. The controlled combination of forming and thermally activated material transformation is commonly subsumed under the term thermo-mechanical treatment.

The classic way of analysing variational behavior with mathematical tooling is the error propagation using first order Taylor series expansions (see f.e. [1]). This concept is widely used in science and industry, especially to determine the error of indirect measurement processes. This procedure has two main problems. First, one needs to determine the first derivatives of the regarded function in each dimension, either analytically or numerically, which is rather problematic if the function is complicated. Second, the error is commonly represented in terms of the variance, so there is only information about the spread, but not about the shape of the distribution inherent.

The usage of a Monte-Carlo Method circumvents these problems, and shall be proposed with this work. The term Monte Carlo Method (MCM) generally refers to a class of methods, which are characterized by the use of random numbers. These methods are rather diverse and serve different purposes. Here, the term shall be used for the concept of drawing random numbers as input for a function and analysing the results of several evaluations of this function, with different random inputs, with statistical methods. A detailed overview on this type of Monte Carlo methods is given by Lemieux [2]. The nature of the function can be complicated, even of a black-box type, where nothing about the internals of the function is known but the input and

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output interfaces. In this case, Monte Carlo methods can provide valuable information about the behavior of the function while altering inputs. Also, the complete distribution of the input variables is included in the estimation and is reflected in the results.

Here, the function equals the simulation procedure, so it is generally known, but complicated. For example, it is generally not possible, to compute derivatives of the outputs in dependence on the inputs in an analytical way. Even numerical derivation is hard, due to the multi-dimensional nature of most natural or technical systems and the pronounced non-linear behavior of the function.

The use of Monte Carlo methods for the analysis of variations in technical processes was reported before in the field of assembly of complicated structures, like in mechanical engineering and building construction (f.e. [3, 4, 5, 6, 7, 8]). However, in the field of rolling processes, there was no such attempt yet to the knowledge of the authors. The authors have previously used a similar approach to model powder morphology influences in sintering processes [9, 10]. The current work shall show the possibility of the application of Monte Carlo methods for the analysis of process variations in rolling processes. The focus lies hereby on the estimation of the workpiece temperature evolution and its impact on the microstructure state of the final product. The influence of variations in the initial workpiece and within the regarded process route is analysed and evaluated. Due to the need of a large number of function evaluations (simulation runs), the evaluation speed of the process model is crucial to the applicability of this approach.

Rolling simulation is currently dominated by the use of finite element (FE) based models. These are offering high accuracy and high resolution results at the expense of high computational resource usage. So these methods are inconvenient for the current need. Therefore, one-dimensional approaches shall be used here. These offer less accuracy and limited resolution, but are computable within fractions of seconds on typical personal computer systems. The current work is based on the open-source rolling simulation framework PyRolL [11], developed by the authors, which is a fast, open and flexible software package mainly aimed at groove rolling in reduction passes. The models used for the different parts of the problem can be exchanged and extended with low effort to the users needs.

2 Methods

2.1 Experimental Procedure

The object of the current investigation is the operation of the experimental semi-continuous rolling plant located at the Institute of Metal Forming, TU Bergakademie Freiberg. It consists of a two-high reversing roughing stand and four continuous finishing stands. The pass schedule of the current work consists of 10 oval-round reversing passes followed by 4 oval-round continuous finishing passes. A 50 mm round workpiece made of C45 carbon steel is rolled down to 8 mm diameter, starting at 1150 °C, resp. 1423.15 K. Details of the schedule are provided in Table 1 and in the supplemental material [12]. The profile shapes appearing in the schedule are illustrated in Figure 1.

Online measurements of the process conditions and workpiece state are done regarding roll force and torque, as well as workpiece temperature. The latter is measured using pyrometers at several points in the plant: at entry and exit of the reversing stand and before, between and after the finishing stands. The sensor signals are collected as timelines, so that they can be automatically analysed afterwards as described in subsection 2.3.

To achieve statistical certainty, a number of 45 rolling trials were carried out. This enables an estimation of the variations appearing. A major source of variation in these trials is the manual

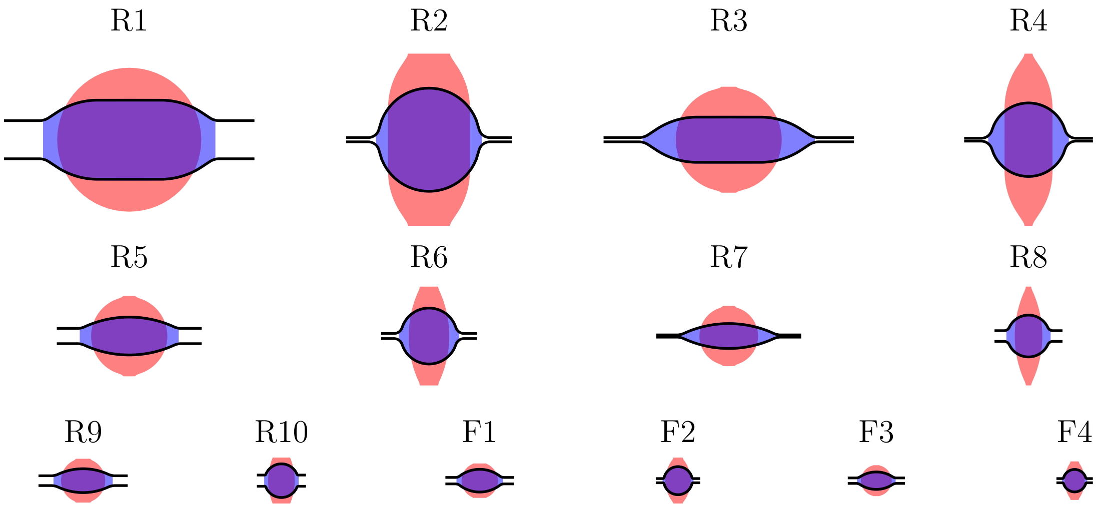


Figure 1: True Scaled Illustration of the Investigated Pass Sequence

Table 1: Principal Data of the Investigated Pass Sequence

#	Type	w mm	h mm	s mm	R_W mm	v $\mathrm{m\,s^{-1}}$
R1	swedish oval	60.0	28.0	13.5	160.5	1.0
R2	round	36.6	36.5	1.5	160.5	1.0
R3	swedish oval	60.0	16.0	1.5	160.5	2.0
R4	round	27.6	26.0	1.0	160.5	2.0
R5	circular oval	34.0	13.4	5.4	160.5	2.0
R6	round	20.4	19.8	1.8	160.5	2.0
R7	circular oval	34.0	8.8	0.8	160.5	2.0
R8	round	14.7	14.8	3.8	160.5	2.0
R9	circular oval	20.1	8.5	3.5	160.5	2.0
R10	round	11.3	12.0	4.0	160.5	2.0
F1	circular oval	15.6	8.1	2.3	107.5	7.8
F2	round	10.1	10.0	1.5	107.5	9.3
F3	circular oval	12.8	6.2	2.0	107.5	12.1
F4	round	8.1	8.0	1.5	85.0	15.8

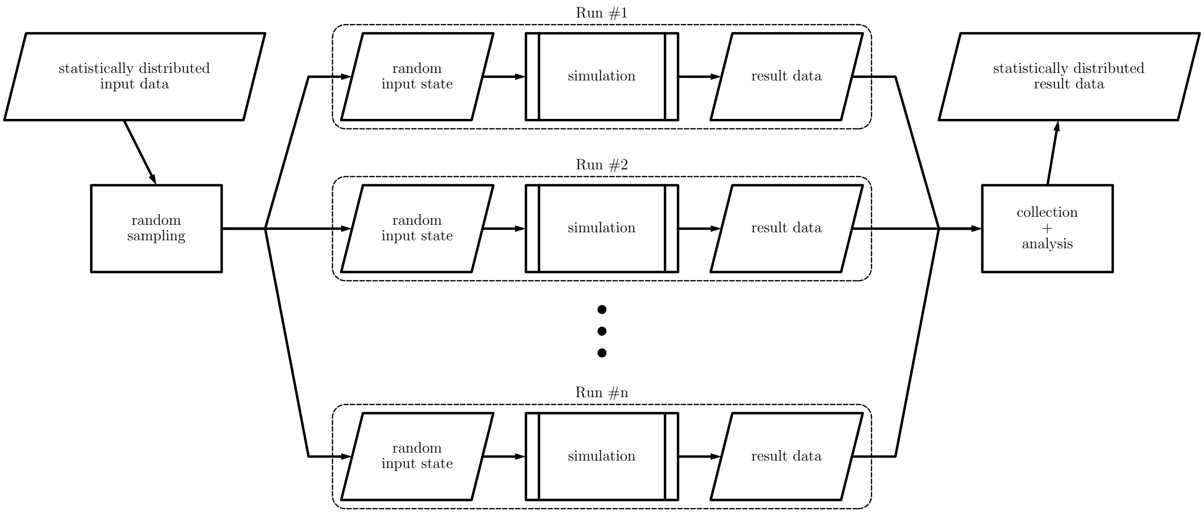


Figure 2: Chart of the Concept of Variation Estimation Using Monte Carlo Techniques

feeding of the workpiece into the reversing passes. The duration of those is scheduled with about 6 seconds. The actual time needed has to be investigated in this work, see subsection 2.3 for details.

2.2 Monte-Carlo Approach

The basic idea of the approach shown here is to simulate the rolling process several times with different input values, which are drawn by a random number generator according to predefined statistical distributions. Afterwards, the distribution of the results can be analysed by classic methods of descriptive statistics to obtain information about the process' variational behavior. The principle is shown in Figure 2.

This approach provides information about the overall variational behavior of the process. If a single source of variation is introduced in the input, the reaction of the process on this variable can be analysed. The count of variation sources introduced is generally unbounded. In contrast to classic Taylor series error propagation, the computational effort does not directly increase remarkably with increasing count of investigated parameters. However, an increase in sample size can be necessary to achieve sufficient certainty. The key problem is to obtain data describing the variations of the input variables. The tracing back of result variations to the input can be done using classic correlation methods of descriptive statistics, however, with the same typical caveats. The main benefit of the approach is, that no information about the internals of the simulation procedure is needed for variational analysis, especially there is no need for derivatives of result values in dependence on the input. The simulation procedure can generally be treated as black box with defined input and output interfaces.

2.3 Statistical Data Acquisition

As input for the Monte-Carlo approach statistical descriptions of the regarded input variables are needed. Regarding the geometric variations of the input workpiece, the diameter of the samples was determined at multiple spots using a calibre. The initial temperature of the samples was

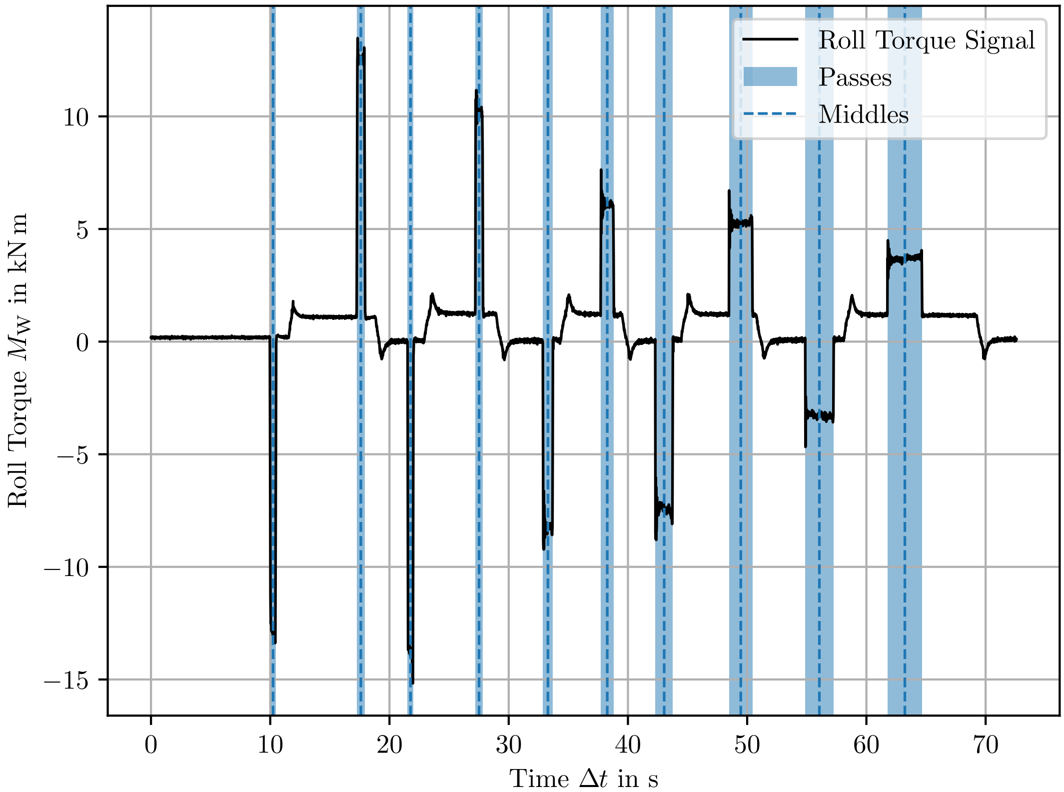


Figure 3: Example Roll Torque Signal With Automatically Determined Roll Pass Locations

determined using the pyrometer installed near the roll gap entry. Both were approximated using a normal distribution for sampling of random input values.

The question of varying inter-pass durations is crucial for scientific experiments on microstructure evolution, but currently often neglected. Mostly, only fixed durations between the reversing passes are included in the design calculations. Due to manual transport and feed of the workpiece to the following roll pass, the scheduled inter-pass durations are never realized in practice. Although, these deviations from the schedule influence the microstructure evolution of the sample, as well as the actual conditions in the roll passes. The current approach is aimed to help quantifying these deviations.

To obtain the inter-pass durations from the timeline data, the passes have to be identified automatically. This was done by analysing the roll torque signal as plotted in Figure 3. The original signal was first downsampled and smoothed. Then, a difference filter was applied and the peaks of the resulting signal were determined. These peaks denote start and end times of the roll passes, the middle time of those was used as time coordinate of the roll pass. The distances of those were used as the inter-pass durations.

For the approximative description of the durations' distribution, a gamma distribution was used, which is a generalized exponential distribution. The probability density function (PDF) of the gamma distribution is defined as in Equation 1, where Γ is the gamma function and $\alpha > 0$ and $\beta > 0$ are parameters.

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha - 1} \exp(-\beta x) \tag{1}$$

Since the gamma distribution is only defined for $x > 0$, but no inter-pass durations below a certain value occur due to technical restrictions, the distribution was modified by introducing a minimal inter-pass duration $\Delta t_{\mathrm{P_{min}}}$ with $x = \Delta t_{\mathrm{P}} - \Delta t_{\mathrm{P_{min}}}$. So there are three free parameters α, β and $\Delta t_{\mathrm{P_{min}}}$ for fitting of the distribution function. The fitting was done using least squares optimization of the PDF function on the density histogram of the data.

2.4 Core Simulation Procedure

In the current work, the open-source rolling simulation framework PyRolL [11] was used to simulate the rolling process. Generally, the shown approach can be used with every rolling simulation software available, since the procedure does not depend on any internals of the simulation. A fast simulation approach, however, is favourable, since the simulation has to be done several, up to hundreds of, times. The models used here are of one-dimensional type, thus, they lack of resolution in other directions as the rolling direction and provide only limited accuracy, but at the benefit of high solution speed. They typically combine empirical approaches with simplified analytical solutions. The simulation was done with the basic configuration of PyRolL, which includes the empirical roll force and torque model of Hensel and Spittel [13], an integral thermal model approach according to Hensel et al. [14], contact area estimation according to Zouhar [15] and roll flattening according to Hitchcock and Trinks [16]. Spreading was simulated using the equivalent flat pass according to Lendl [17, 18, 19] in conjunction with the spreading equation of Wusatowski [20]. Details of software construction and model equations are provided in the documentation of PyRolL [21].

The models most important for the following elaborations will be discussed in brief. The temperature change of the workpiece was calculated by an integral heat balance as proposed by Hensel et al. [14] and given in Equation 2.

$$\Delta T = \Delta T_{\mathrm{Convection}} + \Delta T_{\mathrm{Contact}} + \Delta T_{\varphi} + \Delta T_{\mathrm{Radiation}} \tag{2}$$

$\Delta T_{\mathrm{Convection}}$ is the temperature change by convective heat flows according to Equation 3 with $\alpha_{\mathrm{Convection}}$ as a heat transfer factor, T_∞ as the environment temperature, T as the current workpiece temperature and Δt as the duration of the process step.

$$\Delta T_{\mathrm{Convection}} = \alpha_{\mathrm{Convection}} \left(T_\infty - T\right) \Delta t \tag{3}$$

$\Delta T_{\mathrm{Contact}}$ is the temperature change by contact to the rolls according to Equation 4 with $\alpha_{\mathrm{Contact}}$ as a heat transfer factor and T_{R} as the temperature of the rolls.

$$\Delta T_{\mathrm{Contact}} = \alpha_{\mathrm{Contact}} \left(T_{\mathrm{R}} - T\right) \Delta t \tag{4}$$

$\Delta T_{\mathrm{Radiation}}$ is the temperature change by radiation according to Equation 5 with ϵ_0 as the Stefan's and Boltzmann's constant and ϵ_{r} as the relative radiation coefficient of the gray radiator.

$$\Delta T_{\mathrm{Radiation}} = \epsilon_0 \epsilon_{\mathrm{r}} \left(T_\infty^4 - T^4\right) \Delta t \tag{5}$$

ΔT_φ is the temperature change by deformation according to Equation 6 with k_{Wm} as the empirical deformation resistance and φ_{eq} as the equivalent plastic strain. Since the deformation resistance

Table 2: Material Data and Model COefficients Used in the Simulations

(a) Flow Stress Model of C45 acc. to [22]

$$k_\mathrm{f} = A \cdot \mathrm{e}^{m_1\vartheta} \cdot \varphi^{m_2} \cdot \dot{\varphi}^{m_3} \cdot \mathrm{e}^{m_4/\varphi} \cdot (1+\varphi)^{m_5\vartheta+m_6} \cdot \mathrm{e}^{m_7\varphi} \cdot (1+\dot{\varphi})^{m_8\vartheta} \cdot \vartheta^{m_9}$$

A	m_1	m_2	m_3	m_4	m_5	m_6
2731.39	−0.002 68	0.310 76	0	−0.000 56	0.000 46	0

m_7	m_8	m_9	ϑ	φ
−0.983 75	0.000 139	0	820 °C to 1200 °C	0.04 to 1.5

(b) Other Material Data and Model Coefficients

ϱ	c_p	α_Contact	$\alpha_\mathrm{Convection}$	ϵ_r
$\mathrm{kg\,m^{-3}}$	$\mathrm{J\,kg^{-1}\,K^{-1}}$	$\mathrm{W\,m^{-2}\,K^{-1}}$	$\mathrm{W\,m^{-2}\,K^{-1}}$	
7500.0	690.0	5000.0	10.0	0.8

is used here instead of the flow stress, this term also includes approximately the heat generation by inner and outer friction.

$$\Delta T_\varphi = 0.95 k_\mathrm{Wm} \varphi_\mathrm{eq} \tag{6}$$

The deformation resistance was taken as proposed by Hensel and Spittel [13] and given in Equation 7.

$$\frac{k_\mathrm{Wm}}{k_\mathrm{f\,eq}} = 0.9901 + 0.106\frac{A_\mathrm{c}}{A_\mathrm{eq}} + 0.0283\left(\frac{A_\mathrm{c}}{A_\mathrm{eq}}\right)^2 + 1.5718\exp\left[-2.4609\frac{A_\mathrm{c}}{A_\mathrm{eq}}\right] + 0.3117\exp\left[-15.625\left(\frac{A_\mathrm{c}}{A_\mathrm{eq}}\right)^2\right] \tag{7}$$

The material data and model coefficients used above were taken for the following simulations as in Table 2.

3 Results

3.1 Experimental Results

3.1.1 Input Workpiece Variation

3.1.2 Pause Duration Variation

3.2 Simulation Results

In the following, three questions shall be investigated and answered:

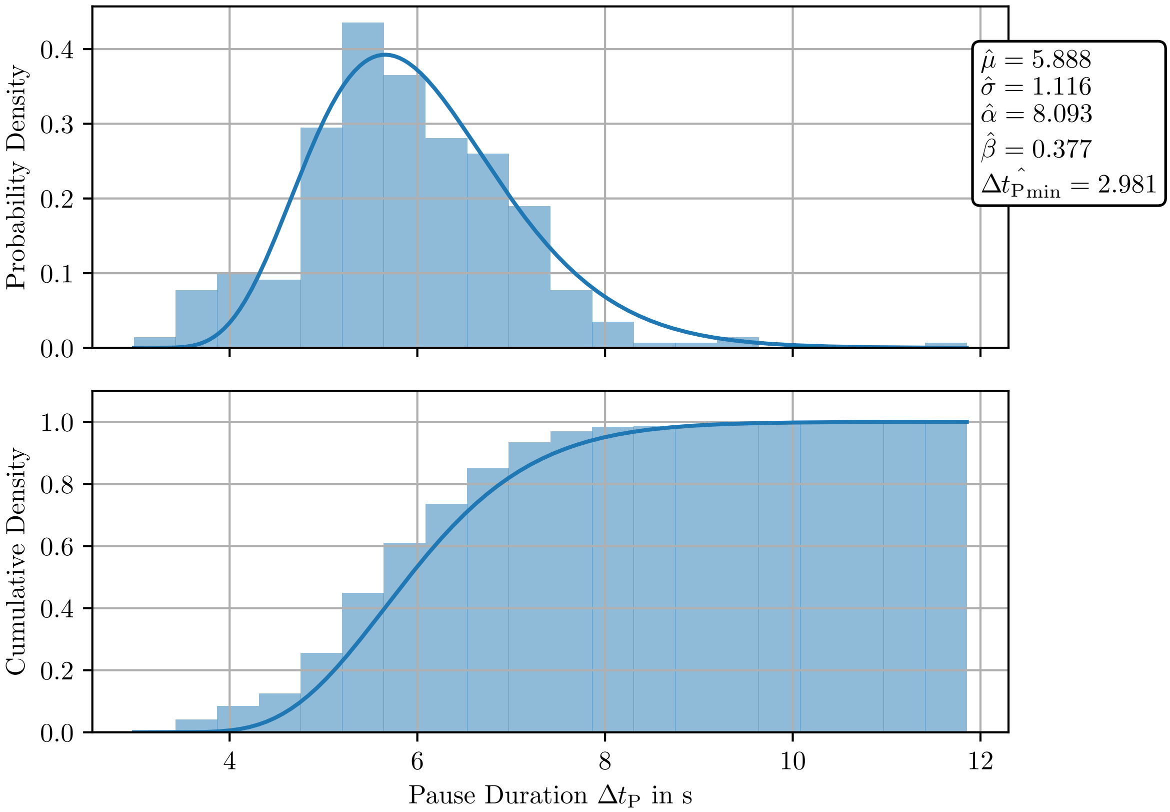


Figure 4: Density and Cumulative Histograms of Inter-Pass Durations With Fitted Gamma Distribution

1. What is the difference in behavior of variations sourced in the input workpiece and arising within the process?

2. What is the influence of elastic mill response on the variational behavior of the process?

3. Is there a minimum number of passes needed to eliminate variations of the input workpiece?

For this distinct simulations were carried out and compared with each other and the experimental data.

3.2.1 Different Sources of Variation

Two basic classes of variation sources in can be identified in rolling processes, or manufacturing processes in general: variations inherent to the input workpiece and variations arising in the regarded processes itself. These effect together the variation of the resulting product. To investigate the different behavior of them, two simulations shall be carried out and compared. The first one only regards variations of the input workpiece and how they evolve during the process. The second one introduces additional variations within the process in means of varying inter-stand pause durations between the reversing passes. These originate, as denoted before, in the manual handling of the workpiece for feeding into the next pass. The focus of the following analysis lies on the temperature evolution of the workpiece, since this is crucial for microstructure evolution and final material properties, and will, presumably, be heavily effected by varying pause durations.

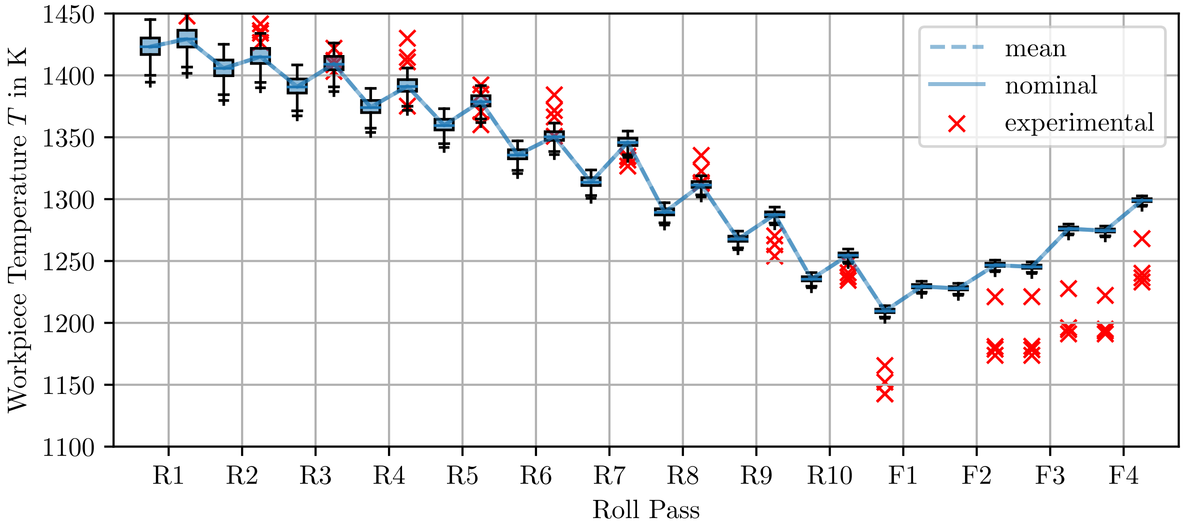
The temperature evolution of the first case is shown in Figure 5a. The variation of the input workpiece was taken as obtained in subsubsection 3.1.1. The box plots in the figure show the variation of the workpiece temperature, where the box marks a distance of $\hat{\sigma}$ to $\hat{\mu}$ and the whiskers a distance of $3\hat{\sigma}$. The variation of temperature decreases with each processing step and is remarkably small in the product. So there is something like a "natural" depression of variation in each process step.

The second case, however, is shown in Figure 5b. Here, the variation is not decreasing with each step, but increasing in the transport steps. In contrast, roll passes still decrease the variation. If the overall variation decreases in the process, depends, of course, on the ratio between decrease in passes and increase in transports. In this view, the goal of process design must be to prevent an overall increase of variation in the process. The main vantage point for this is to limit variation in pause durations.

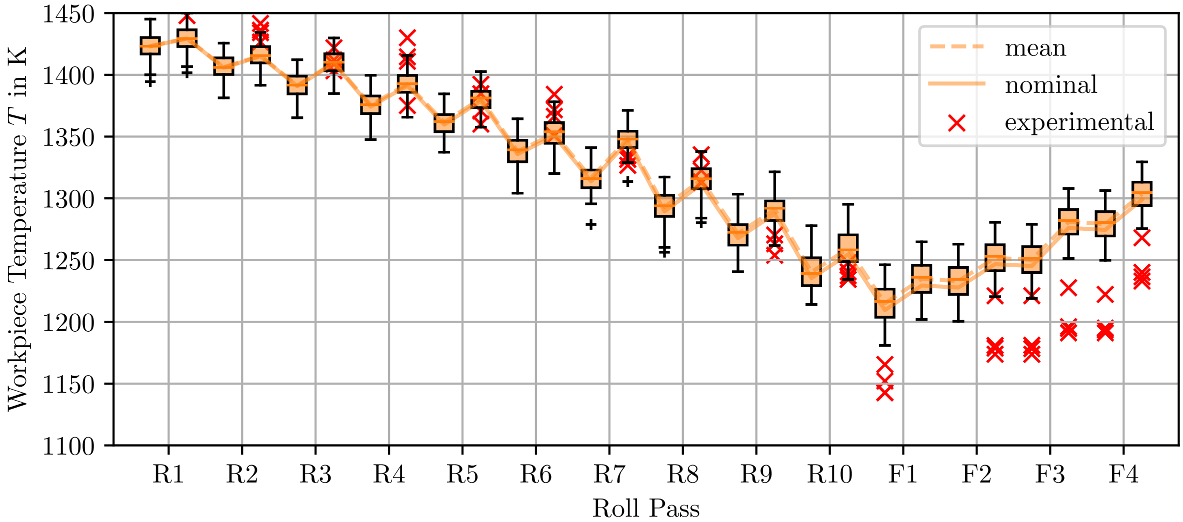
Figure 6 shows the evolution of variation of both cases in comparison. One can see, that the overall variation is increasing solely in the reversing transport for the second case. Note, that the influence of transports in oval cross-section shape is remarkably higher than those in round shape. This can be explained by the adverse surface area to volume ratio of oval cross-sections.

From this, the hypothesis that process steps with high influence on temperature have also high influence on the variation depression can be stated. This is proofed by plotting the relative depression in standard deviation per step as in Equation 8 over the change in temperature in the step. This correlation can especially be observed in varying only the input workpiece as shown in Figure 7a, where the variation depressions in roll passes and transports show an approximately linear correlation to the temperature changes. In the case of varying pause durations, the roll passes still show the same behavior, but the correlation in the transports is destroyed by the introduction of additional variation, as can be seen in Figure 7b.

$$\Delta\bar{\sigma} = \frac{|\Delta\sigma|}{|\sigma|} \tag{8}$$



(a) Under Influence of Input Workpiece Variation



(b) Under Influence of Input Workpiece Variation and Pause Duration Variation

Figure 5: Variation of Workpiece Temperature

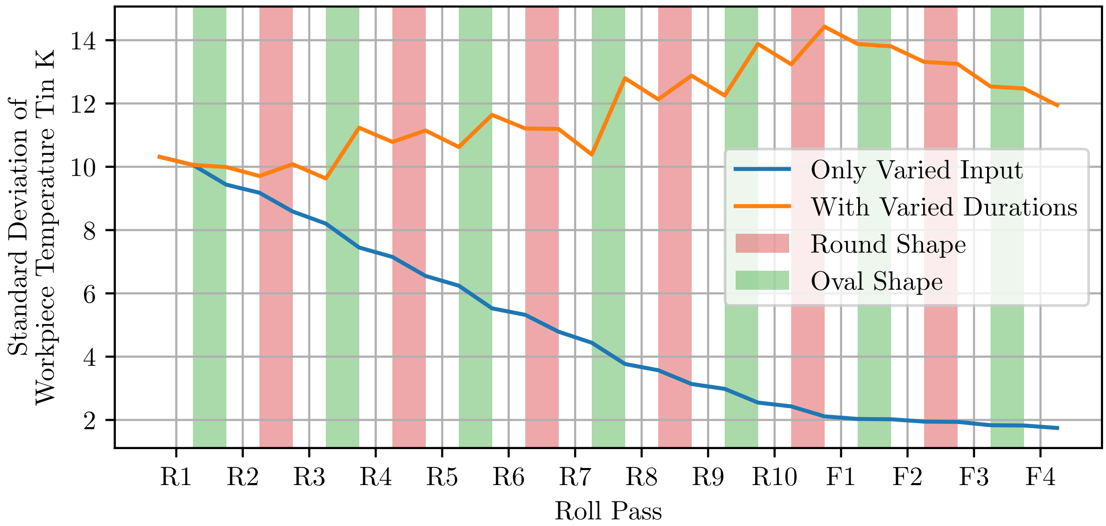


Figure 6: Comparison of Temperature Variation Evolution Between Input Variation and Process Variation

3.2.2 Elimination of Input Variation

A common statement found is that the variations in the input workpiece are eliminated after three to four passes. To validate this statement, simulations under different variations of the input workpiece have been carried out.

Figure 8a shows the depression of the filling ratio standard deviation along the pass sequence for different initial workpiece diameter variations. It is found that the variation decreases rapidly in the first three passes and is negligible small in the fourth, no matter what initial variation was applied. So regarding the geometry the validity of the former statement can be confirmed.

Figure 8b, however, shows the depression of temperature variation along the pass sequence for different initial workpiece temperature variations. Although, the variation is effectively depressed in the sequence, the variation of the output workpiece is still remarkably higher for higher input variations. So regarding the temperature evolutions, the statement can not be confirmed, especially in regard of microstructure evolution heavily effected by the temperature path taken.

4 Summary and Outlook

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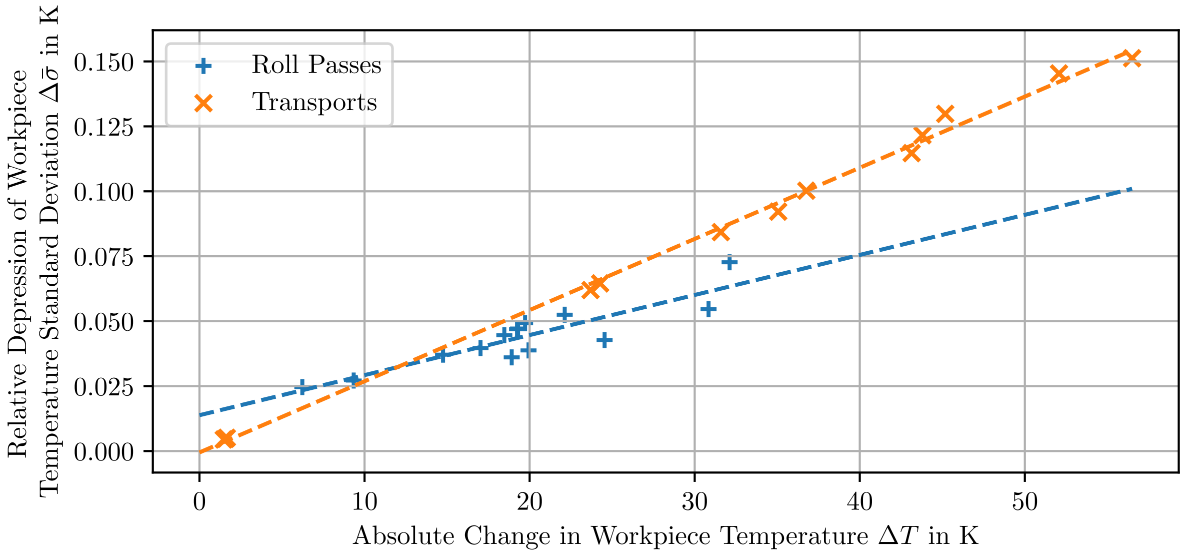
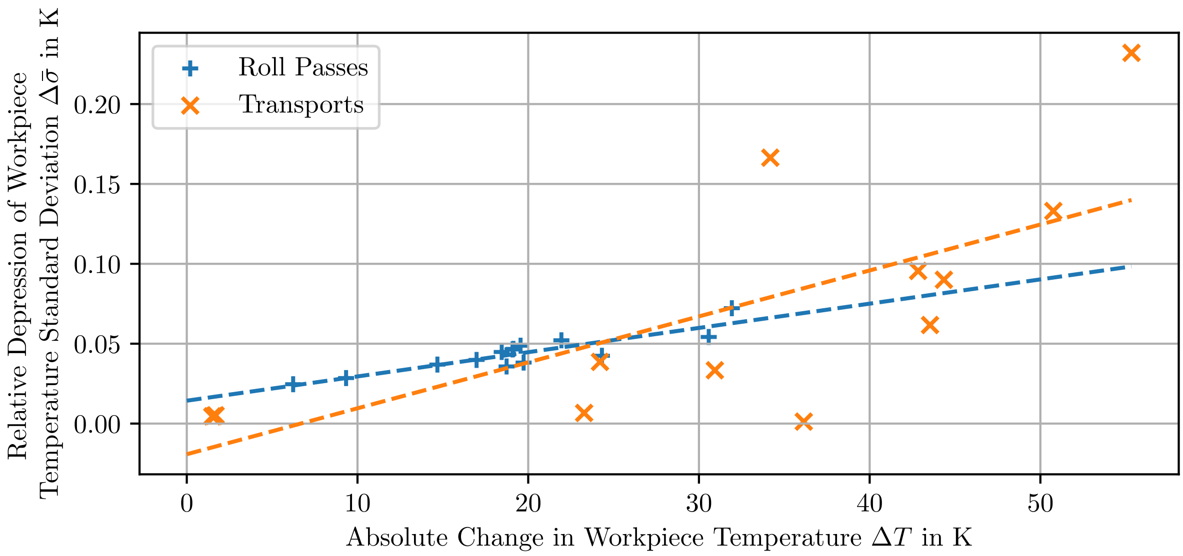
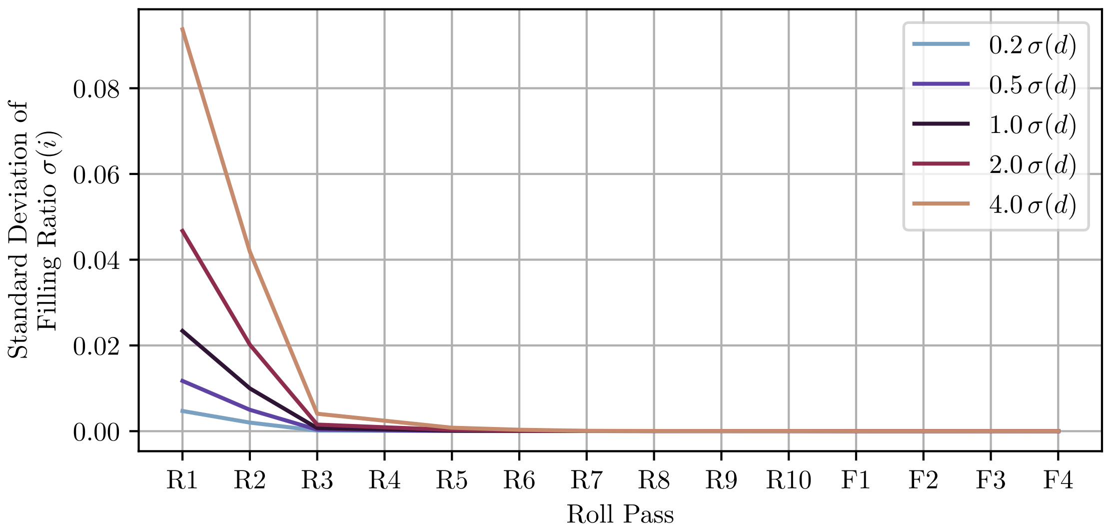


(a) Under Influence of Input Workpiece Variation

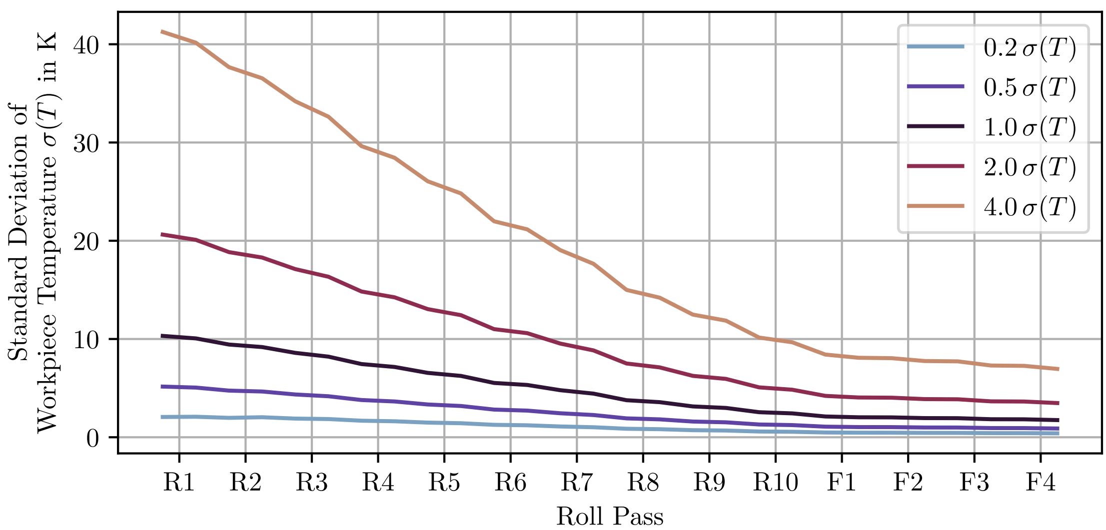


(b) Under Influence of Input Workpiece Variation and Pause Duration Variation

Figure 7: Correlation Between Change in Temperature Standard Deviation and Change in Temperature Per Unit



(a) Variation of Roll Pass Filling Ratios



(b) Variation of Workpiece Temperature

Figure 8: Depression of Workpiece Variation

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