Documentation for the pyroll-thermal-2d Plugin

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1 Model Description

1.1 Ring Model

The following derivations are based on the ring model approach. For details on this approach read also the respective documentation[1].

1.2 Heat Flow Balance

see Figure 1, i index of layer in radial direction, k index of disk element in x-direction

$$0 = \dot{q}_1 - \dot{q}_2 - \dot{q}_3 + \dot{q}_4 + \dot{q}_\mathrm{S} \tag{1}$$

heat flow contributions

$$\dot{q}_1 = \varrho c_\mathrm{p} \dot{V}_i T_i^k \tag{2}$$

$$\dot{q}_2 = \varrho c_\mathrm{p} \dot{V}_i T_i^{k+1} \tag{3}$$

$$\dot{q}_3 = -\lambda \frac{T_{i+1}^k - T_i^k}{r_{i+1} - r_i} \times 2\pi R_{i+1} \Delta x \tag{4}$$

$$\dot{q}_4 = -\lambda \frac{T_i^k - T_{i-1}^k}{r_i - r_{i-1}} \times 2\pi R_i \Delta x \tag{5}$$

$$\dot{q}_\mathrm{S} = \eta_\mathrm{S} \frac{k_\mathrm{f}}{\eta_\varphi} \dot{\varphi} V \tag{6}$$

where the volume flow $\dot{V}$

[1] https://github.com/pyroll-project/pyroll-ring-model

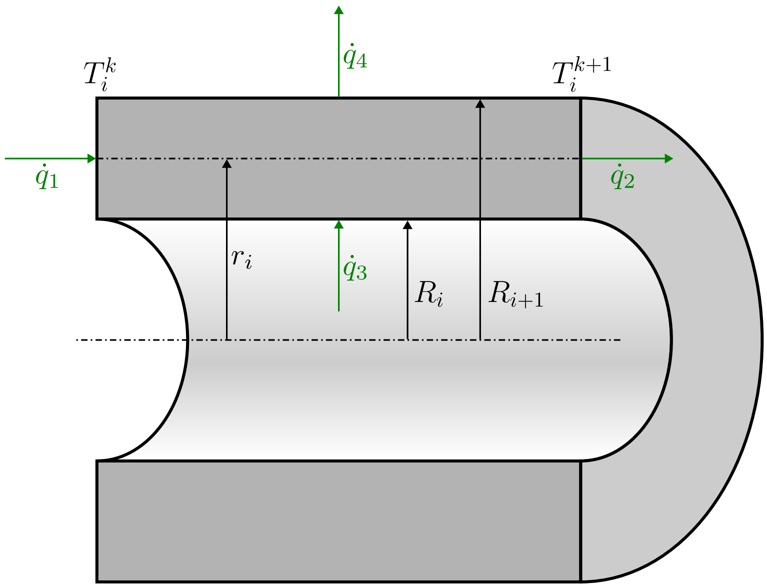

Figure 1: Heat Flows on a Disk Element Ring

$$\dot{V}_i = A_i \frac{\Delta x}{\Delta t} \tag{7}$$

$$A_i = \pi \left(R_{i+1}^2 - R_i^2 \right) \tag{8}$$

at surface different $\dot{q}_3$, with surface temperature T_S

$$\dot{q}_3 = \left[-\alpha \left(T_\infty - T_\mathrm{S} \right) - \epsilon_0 \epsilon_\mathrm{r} \left({T_\infty}^4 - {T_\mathrm{S}}^4 \right) \right] \times 2\pi R_{\hat{\imath}+1} \Delta x \tag{9}$$

surface temperature estimation as stationary state between environment and outer layer, numerical solution

$$\lambda \frac{T_\mathrm{S} - T_{\hat{\imath}}^k}{R_{\hat{\imath}+1} - r_{\hat{\imath}}} = \alpha \left(T_\infty - T_\mathrm{S} \right) + \epsilon_0 \epsilon_\mathrm{r} \left({T_\infty}^4 - {T_\mathrm{S}}^4 \right) \tag{10}$$

1.3 Temperature Increment Functions

for core layer $\dot{q}_4 = 0$

$$\Delta T_0 = \frac{\Delta t}{\varrho c_\mathrm{p} A_0} \left[\pi \lambda \left(T_1^k - T_0^k \right) + \eta_\mathrm{S} \frac{k_\mathrm{f}}{\eta_\varphi} \dot{\varphi} A_0 \right] \tag{11}$$

for intermediate layers with Equation 4

$$\Delta T_i = \frac{\Delta t}{\varrho c_\mathrm{p} A_i} \left[2\pi \lambda \left[\frac{T_{i+1}^k - T_i^k}{r_{i+1} - r_i} R_{i+1} - \frac{T_i^k - T_{i-1}^k}{r_i - r_{i-1}} R_i \right] + \eta_\mathrm{S} \frac{k_\mathrm{f}}{\eta_\varphi} \dot{\varphi} A_i \right] \tag{12}$$

for surface layer with Equation 9

$$\Delta T_{\hat{\imath}} = \frac{\Delta t}{\varrho c_{\mathrm{p}} A_{\hat{\imath}}} \left[2\pi \left[\left[\alpha \left(T_\infty - T_{\mathrm{S}} \right) + \epsilon_0 \epsilon_{\mathrm{r}} \left({T_\infty}^4 - {T_{\mathrm{S}}}^4 \right) \right] R_{\hat{\imath}+1} - \lambda \frac{T^k_{\hat{\imath}} - T^k_{\hat{\imath}-1}}{r_{\hat{\imath}} - r_{\hat{\imath}-1}} R_{\hat{\imath}} \right] + \eta_{\mathrm{S}} \frac{k_{\mathrm{f}}}{\eta_\varphi} \dot{\varphi} A_{\hat{\imath}} \right] \tag{13}$$

2 Plugin Usage

Symbols

Symbol	Description
A	Cross section
α	Heat transfer coefficient
c_{p}	Thermal Capacity
ϵ_0	Radiation coeffient of black radiator
ϵ_{r}	Relative radiation coeffient
η_{S}	Efficiency of heat source by deformation
η_φ	Efficiency of deformation
i	Index of the ring
$\hat{\imath}$	Maximum index of the ring
j	Index of the ring boundary
$\hat{\jmath}$	Maximum index of the ring boundary
k	Index of the disk element
k_{f}	Flow stress
λ	Thermal conductivity
$\dot{m}$	Mass flow in x-direction
φ	Equivalent strain
$\dot{\varphi}$	Equivalent strain rate
$\dot{q}$	Heat flow
$\dot{q}_{\mathrm{S}}$	Heat source (generation)
r	Radius coordinate of a ring's center line
R	Radius coordinate of a ring's boundary line

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Table 0: (Continued)

Δr	Discretization width in radius
ϱ	Density
t	Time
Δt	Discretization width in time
T	Absolute temperature
ΔT	Increment of temperature
T_∞	Environemnt temperature
T_S	Absolute surface temperature
V	Volume of the disk element rep. layer
$\dot{V}$	Volume flow
x	X Coordinate
Δx	Discretization width in x