Documentation for the pyroll-thermal-2d Plugin

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January 13, 2023

1 Model Description

$$0 = \dot{q}_1 - \dot{q}_2 - \dot{q}_3 + \dot{q}_4 \tag{1}$$

$$\dot{m} = \varrho \pi \frac{\Delta x}{\Delta t} \left(\frac{\Delta r}{2}\right)^2 \tag{2}$$

$$\dot{q}_1 = c_p \dot{m} T_i^n \tag{3}$$

$$\dot{q}_2 = c_p \dot{m} T_i^{n+1} \tag{4}$$

$$\dot{q}_3 = -\lambda \frac{T_{i+1}^n - T_i^n}{\Delta r} \times 2\pi \Delta x \left(r_i + \frac{\Delta r}{2} \right)$$
 (5)

$$\dot{q}_4 = -\lambda \frac{T_i^n - T_{i-1}^n}{\Delta r} \times 2\pi \Delta x \left(r_i - \frac{\Delta r}{2} \right) \tag{6}$$

$$\dot{q}_3 = 2\pi r \Delta x \left[-\alpha \left(T_{\infty} - T_{\rm S} \right) - \epsilon_0 \epsilon_{\rm r} \left(T_{\infty}^4 - T_{\rm S}^4 \right) \right] \tag{7}$$

$$2\lambda \frac{T_{\rm S} - T_I^n}{\Delta r} = \alpha \left(T_{\infty} - T_{\rm S} \right) + \epsilon_0 \epsilon_{\rm r} \left(T_{\infty}^4 - T_{\rm S}^4 \right) \tag{8}$$

for core layer $\dot{q}_4 = 0$

$$\Delta T_0 = \frac{\pi \lambda \Delta x \Delta t}{\varrho c_p V} \left(T_1^n - T_0^n \right) \tag{9}$$

$$\Delta T_i = \frac{2\pi\lambda\Delta x\Delta t}{\varrho c_p V\Delta r} \left[\left(T_{i+1}^n - T_i^n \right) \left(r_i + \frac{\Delta r}{2} \right) - \left(T_i^n - T_{i-1}^n \right) \left(r_i - \frac{\Delta r}{2} \right) \right]$$
(10)

$$\Delta T_{I} = \frac{2\pi\Delta x \Delta t}{\varrho c_{\rm p} V} \left[\left[\alpha \left(T_{\infty} - T_{\rm S} \right) + \epsilon_{0} \epsilon_{\rm r} \left(T_{\infty}^{4} - T_{\rm S}^{4} \right) \right] \left(r_{I} + \frac{\Delta r}{2} \right) - \lambda \frac{T_{I}^{n} - T_{I-1}^{n}}{\Delta r} \left(r_{I} - \frac{\Delta r}{2} \right) \right]$$

$$(11)$$

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2 Plugin Usage

Symbols

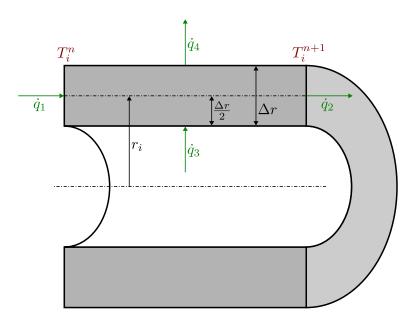


Figure 1: Heat Flows on a Disk Element Ring

Symbol	Description
α	Heat transfer coefficient
$c_{ m p}$	Thermal Capacity
\dot{m}	Mass flow in x-direction
\dot{q}	Heat flow
$\dot{q}_{ m S}$	Heat source (generation)
ϵ_0	Radiation coefficent of black radiator
$\epsilon_{ m r}$	Relative radiation coefficient
i	Index of raster in radius
I	Maximum index of raster in radius
λ	Thermal conductivity
n	Index of raster in x
N	Maximum index of raster in x
r	Radius coordinate in polar systems
Δr	Discretization width in radius
t	Time
Δt	Discretization width in time

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Table 0: (Continued)

\overline{T}	Absolute temperature
ΔT	Increment of temperature
T_{∞}	Environemnt temperature
$T_{ m S}$	Absolute surface temperature
V	Volume of the disk element rep. layer
ϱ	Density
x	X Coordinate
Δx	Discretization width in x