Documentation for the pyroll-thermal-2d Plugin

Max Weiner

January 12, 2023

1 Model Description

$$0 = \dot{q}_1 - \dot{q}_2 - \dot{q}_3 + \dot{q}_4 \tag{1}$$

$$\dot{m} = \varrho\pi \frac{\Delta x}{\Delta t} \left(\frac{\Delta r}{2}\right)^2 \tag{2}$$

$$\dot{q}_1 = c_\mathrm{p} \dot{m} T_i^n \tag{3}$$

$$\dot{q}_2 = c_\mathrm{p} \dot{m} T_i^{n+1} \tag{4}$$

$$\dot{q}_3 = -\lambda \frac{T_{i+1}^n - T_i^n}{\Delta r} \times 2\pi\Delta x \left(r_i + \frac{\Delta r}{2}\right) \tag{5}$$

$$\dot{q}_4 = -\lambda \frac{T_i^n - T_{i-1}^n}{\Delta r} \times 2\pi\Delta x \left(r_i - \frac{\Delta r}{2}\right) \tag{6}$$

$$\dot{q}_3 = 2\pi r \Delta x \left[-\alpha \left(T_\infty - T_\mathrm{S}\right) - \epsilon_0 \epsilon_\mathrm{r} \left({T_\infty}^4 - {T_\mathrm{S}}^4\right)\right] \tag{7}$$

$$2\lambda \frac{T_\mathrm{S} - T_I^n}{\Delta r} = \alpha \left(T_\infty - T_\mathrm{S}\right) + \epsilon_0 \epsilon_\mathrm{r} \left({T_\infty}^4 - {T_\mathrm{S}}^4\right) \tag{8}$$

for core layer $\dot{q}_4 = 0$

$$\Delta T_0 = \frac{\pi\lambda\Delta x \Delta t}{\varrho c_\mathrm{p} V} \left(T_1^n - T_0^n\right) \tag{9}$$

$$\Delta T_i = \frac{2\pi\lambda\Delta x \Delta t}{\varrho c_\mathrm{p} V \Delta r} \left[\left(T_{i+1}^n - T_i^n\right)\left(r_i + \frac{\Delta r}{2}\right) - \left(T_i^n - T_{i-1}^n\right)\left(r_i - \frac{\Delta r}{2}\right)\right] \tag{10}$$

$$\Delta T_I = \frac{2\pi\Delta x \Delta t}{\varrho c_\mathrm{p} V} \left[\left[\alpha \left(T_\infty - T_\mathrm{S}\right) + \epsilon_0 \epsilon_\mathrm{r} \left({T_\infty}^4 - {T_\mathrm{S}}^4\right)\right]\left(r_I + \frac{\Delta r}{2}\right) - \lambda \frac{T_I^n - T_{I-1}^n}{\Delta r}\left(r_I - \frac{\Delta r}{2}\right)\right] \tag{11}$$

2 Plugin Usage

Symbols

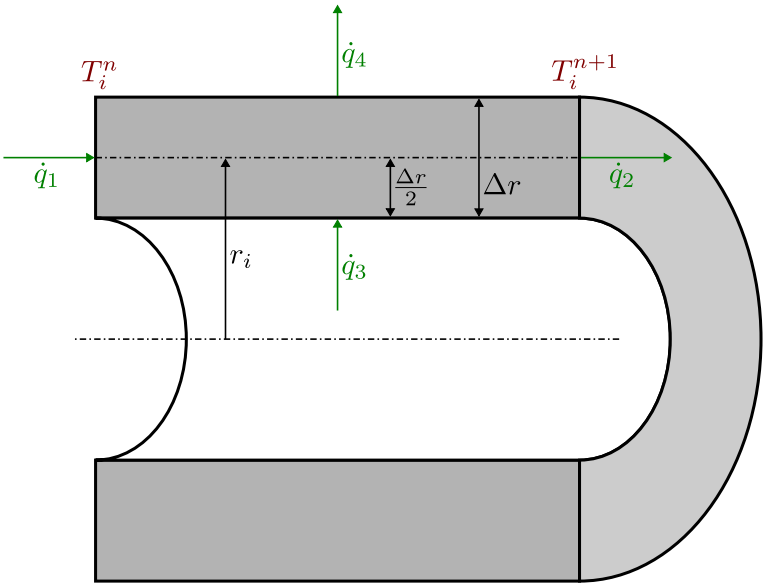

Figure 1: Heat Flows on a Disk Element Ring

Symbol	Description
t	Time
r	Radius coordinate in polar systems
$\dot{m}$	Mass flow in x-direction
ϱ	Density
c_p	Thermal Capacity
x	X Coordinate
Δx	Discretization width in x
Δt	Discretization width in time
T	Absolute temperature
T_S	Absolute surface temperature
T_∞	Environemnt temperature
Δr	Discretization width in radius
λ	Thermal conductivity
$\dot{q}$	Heat flow
α	Heat transfer coefficient
ϵ_0	Radiation coeffient of black radiator

Continued on next page

Table 0: (Continued)

ϵ_{r}	Relative radiation coeffient
i	Index of raster in radius
n	Index of raster in x
I	Maximum index of raster in radius
N	Maximum index of raster in x
ΔT	Increment of temperature
V	Volume of the disk element rep. layer