Documentation for the pyroll-thermal-2d Plugin

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1 Model Description

1.1 Heat Flow Balance

see Figure 1, i index of layer in radial direction, n index of disk element in x-direction

$$0 = \dot{q}_1 - \dot{q}_2 - \dot{q}_3 + \dot{q}_4 + \dot{q}_\mathrm{S} \tag{1}$$

heat flow contributions

$$\dot{q}_1 = \varrho c_\mathrm{p} \dot{V} T_i^n \tag{2}$$

$$\dot{q}_2 = \varrho c_\mathrm{p} \dot{V} T_i^{n+1} \tag{3}$$

$$\dot{q}_3 = -\lambda \frac{T_{i+1}^n - T_i^n}{\Delta r} \times 2\pi \left(r_i + \frac{\Delta r}{2} \right) \Delta x \tag{4}$$

$$\dot{q}_4 = -\lambda \frac{T_i^n - T_{i-1}^n}{\Delta r} \times 2\pi \left(r_i - \frac{\Delta r}{2} \right) \Delta x \tag{5}$$

$$\dot{q}_\mathrm{S} = \eta_\mathrm{S} \frac{k_\mathrm{f}}{\eta_\varphi} \dot{\varphi} \tag{6}$$

at surface different $\dot{q}_3$, with surface temperature T_S

$$\dot{q}_3 = \left[-\alpha \left(T_\infty - T_\mathrm{S} \right) - \epsilon_0 \epsilon_\mathrm{r} \left({T_\infty}^4 - {T_\mathrm{S}}^4 \right) \right] \times 2\pi \left(r_i + \frac{\Delta r}{2} \right) \Delta x \tag{7}$$

surface temperature estimation as stationary state between environment and outer layer, numerical solution

$$2\lambda \frac{T_\mathrm{S} - T_{\hat{i}}^n}{\Delta r} = \alpha \left(T_\infty - T_\mathrm{S} \right) + \epsilon_0 \epsilon_\mathrm{r} \left({T_\infty}^4 - {T_\mathrm{S}}^4 \right) \tag{8}$$

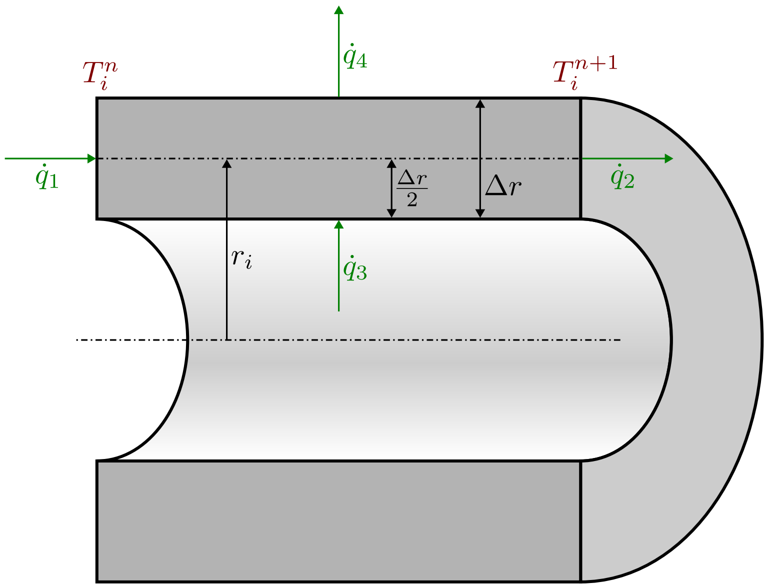

Figure 1: Heat Flows on a Disk Element Ring

1.2 Temperature Increment Functions

for core layer $\dot{q}_4 = 0$

$$\Delta T_0 = \frac{1}{\varrho c_\mathrm{p} \dot{V}} \left[\pi \lambda \Delta x \left(T_1^n - T_0^n \right) + \dot{q}_\mathrm{S} \right] \tag{9}$$

for intermediate layers with Equation 4

$$\Delta T_i = \frac{1}{\varrho c_\mathrm{p} \dot{V}} \left[\frac{2\pi\lambda\Delta x}{\Delta r} \left[\left(T_{i+1}^n - T_i^n \right) \left(r_i + \frac{\Delta r}{2} \right) - \left(T_i^n - T_{i-1}^n \right) \left(r_i - \frac{\Delta r}{2} \right) \right] + \dot{q}_\mathrm{S} \right] \tag{10}$$

for surface layer with Equation 7

$$\Delta T_i = \frac{1}{\varrho c_\mathrm{p} \dot{V}} \left[2\pi\Delta x \left[\left[-\alpha \left(T_\infty - T_\mathrm{S} \right) - \epsilon_0 \epsilon_\mathrm{r} \left({T_\infty}^4 - {T_\mathrm{S}}^4 \right) \right] \left(r_i + \frac{\Delta r}{2} \right) - \lambda \frac{T_i^n - T_{i-1}^n}{\Delta r} \left(r_i - \frac{\Delta r}{2} \right) \right] + \dot{q}_\mathrm{S} \right] \tag{11}$$

2 Plugin Usage

Symbols

Symbol	Description
α	Heat transfer coefficient
c_{p}	Thermal Capacity
ϵ_0	Radiation coeffient of black radiator
ϵ_{r}	Relative radiation coeffient
η_{S}	Efficiency of heat source by deformation
η_{φ}	Efficiency of deformation
i	Index of raster in radius
$\hat{\imath}$	Maximum index of raster in radius
k_{f}	Flow stress
λ	Thermal conductivity
$\dot{m}$	Mass flow in x-direction
n	Index of raster in x
$\hat{n}$	Maximum index of raster in x
φ	Equivalent strain
$\dot{\varphi}$	Equivalent strain rate
$\dot{q}$	Heat flow
$\dot{q}_{\mathrm{S}}$	Heat source (generation)
r	Radius coordinate in polar systems
Δr	Discretization width in radius
ϱ	Density
t	Time
Δt	Discretization width in time
T	Absolute temperature
ΔT	Increment of temperature
T_{∞}	Environemnt temperature
T_{S}	Absolute surface temperature
V	Volume of the disk element rep. layer
$\dot{V}$	Volume flow
x	X Coordinate
Δx	Discretization width in x