

Documentation for the pyroll-thermal-2d Plugin

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1 Model Description

1.1 Heat Flow Balance

see Figure 1, i index of layer in radial direction, n index of disk element in x-direction

$$0 = \dot{q}_1 - \dot{q}_2 - \dot{q}_3 + \dot{q}_4 + \dot{q}_S \quad (1)$$

heat flow contributions

$$\dot{q}_1 = \rho c_p \dot{V} T_i^n \quad (2)$$

$$\dot{q}_2 = \rho c_p \dot{V} T_i^{n+1} \quad (3)$$

$$\dot{q}_3 = -\lambda \frac{T_{i+1}^n - T_i^n}{\Delta r} \times 2\pi \left(r_i + \frac{\Delta r}{2} \right) \Delta x \quad (4)$$

$$\dot{q}_4 = -\lambda \frac{T_i^n - T_{i-1}^n}{\Delta r} \times 2\pi \left(r_i - \frac{\Delta r}{2} \right) \Delta x \quad (5)$$

$$\dot{q}_S = \eta_S \frac{k_f}{\eta_\varphi} \varphi \dot{V} \quad (6)$$

at surface different \dot{q}_3 , with surface temperature T_S

$$\dot{q}_3 = \left[-\alpha (T_\infty - T_S) - \epsilon_0 \epsilon_r (T_\infty^4 - T_S^4) \right] \times 2\pi \left(r_i + \frac{\Delta r}{2} \right) \Delta x \quad (7)$$

surface temperature estimation as stationary state between environment and outer layer, numerical solution

$$2\lambda \frac{T_S - T_i^n}{\Delta r} = \alpha (T_\infty - T_S) + \epsilon_0 \epsilon_r (T_\infty^4 - T_S^4) \quad (8)$$

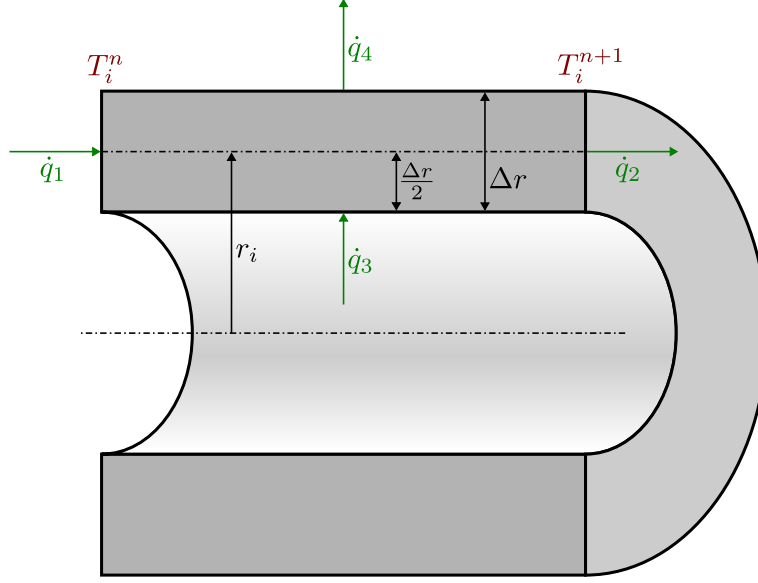


Figure 1: Heat Flows on a Disk Element Ring

1.2 Temperature Increment Functions

for core layer $\dot{q}_4 = 0$

$$\Delta T_0 = \frac{1}{\rho c_p \dot{V}_0} [\pi \lambda \Delta x (T_1^n - T_0^n) + \dot{q}_{s0}] \quad (9)$$

for intermediate layers with Equation 4

$$\Delta T_i = \frac{1}{\rho c_p \dot{V}_i} \left[\frac{2\pi \lambda \Delta x}{\Delta r} \left[(T_{i+1}^n - T_i^n) \left(r_i + \frac{\Delta r}{2} \right) - (T_i^n - T_{i-1}^n) \left(r_i - \frac{\Delta r}{2} \right) \right] + \dot{q}_{si} \right] \quad (10)$$

for surface layer with Equation 7

$$\Delta T_i = \frac{1}{\rho c_p \dot{V}_i} \left[2\pi \Delta x \left[\left[\alpha (T_\infty - T_S) + \epsilon_0 \epsilon_r (T_\infty^4 - T_S^4) \right] \left(r_i + \frac{\Delta r}{2} \right) - \lambda \frac{T_i^n - T_{i-1}^n}{\Delta r} \left(r_i - \frac{\Delta r}{2} \right) \right] + \dot{q}_{si} \right] \quad (11)$$

2 Plugin Usage

Symbols

Symbol	Description
α	Heat transfer coefficient
c_p	Thermal Capacity
ϵ_0	Radiation coefficient of black radiator
ϵ_r	Relative radiation coefficient
η_S	Efficiency of heat source by deformation
η_φ	Efficiency of deformation
i	Index of raster in radius
\hat{i}	Maximum index of raster in radius
k_f	Flow stress
λ	Thermal conductivity
\dot{m}	Mass flow in x-direction
n	Index of raster in x
\hat{n}	Maximum index of raster in x
φ	Equivalent strain
$\dot{\varphi}$	Equivalent strain rate
\dot{q}	Heat flow
\dot{q}_S	Heat source (generation)
r	Radius coordinate in polar systems
Δr	Discretization width in radius
ϱ	Density
t	Time
Δt	Discretization width in time
T	Absolute temperature
ΔT	Increment of temperature
T_∞	Environment temperature
T_S	Absolute surface temperature
V	Volume of the disk element rep. layer
\dot{V}	Volume flow
x	X Coordinate
Δx	Discretization width in x