Documentation for the pyroll-ring-model-thermal Plugin

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1 Model Description

1.1 Ring Model

The following derivations are based on the ring model approach. For details on this approach read also the respective documentation[1]. Refer especially to the statements about the symbols r_i, R_j, I_j and A_i used in the same meaning here.

1.2 Heat Flow Balance

As illustrated in Figure 1, the heat flow balance of each ring on a disk element can be built as in Equation 1.

$$0 = \dot{q}_{1i} - \dot{q}_{2i} - \dot{q}_{3i} + \dot{q}_{4i} + \dot{q}_{\mathrm{S}i} \tag{1}$$

The distinct heat contributions can be expressed as in the following. $\dot{q}_{1i}$ and $\dot{q}_{2i}$ are convective flows caused by workpiece material entry and exit in and from the disk element, where ϱ is the mass density, c_p is the thermal capacity, $\dot{V}_i$ the volume flow through the ring of the disk element, and T_i the absolute temperature of the respective ring. i is the index of the ring in the interval $[0, \hat{\imath}]$, where $\hat{\imath} = n - 1$ with the count of rings n. k, however, is the index of the disk, whose definition region is of no matter here. As all quantities except the temperatures are considered constant within one disk element, resp. ring, the index k is neglected. But, all may vary from disk to disk.

$$\dot{q}_{1i} = \varrho c_\mathrm{p} \dot{V}_i T_i^k \tag{2}$$

$$\dot{q}_{2i} = \varrho c_\mathrm{p} \dot{V}_i T_i^{k+1} \tag{3}$$

$\dot{q}_{3i}$ and $\dot{q}_{4i}$ are the conductive flows between the rings, with λ as the thermal capacity, r_i as the radius coordinate of the rings center line, R_j as the radius coordinates of the

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[1] `https://github.com/pyroll-project/pyroll-ring-model`

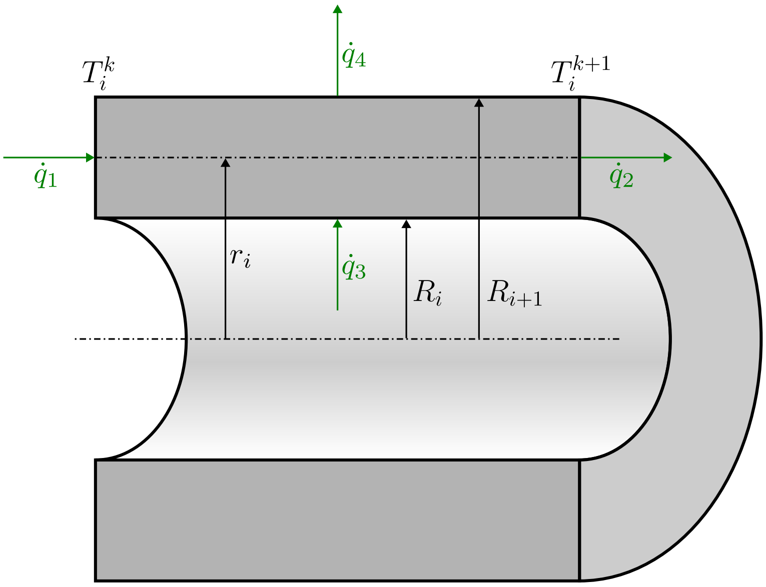


Figure 1: Heat Flows on a Disk Element Ring

ring's boundaries and the disk width in rolling direction Δx. j is the index of the ring boundary in the interval $[0, \hat{\jmath}]$, where $\hat{\jmath} = \hat{\imath} + 1$.

$$\dot{q}_{3i} = -\lambda \frac{T_{i+1}^k - T_i^k}{r_{i+1} - r_i} \times I_{i+1} \Delta x \tag{4}$$

$$\dot{q}_{4i} = -\lambda \frac{T_i^k - T_{i-1}^k}{r_i - r_{i-1}} \times I_i \Delta x \tag{5}$$

The interface length I_j is the length of the projected ring boundaries. There is no explicit equation for calculation of their values, but they are obtained numerically by integrating the path along their coordinate points, see section 3.

Heat generation by deformation is respected by the source term $\dot{q}_\mathrm{S}$, with the efficiency of heat generation η_S, the efficiency of deformation η_φ, the flow stress k_f the deformed volume V and the equivalent strain rate $\dot{\varphi}$.

$$\dot{q}_{\mathrm{S}i} = \eta_\mathrm{S} \frac{k_\mathrm{f}}{\eta_\varphi} \dot{\varphi} V_i \tag{6}$$

The volume flow $\dot{V}$ through the ring is calculated from the cross section A_i and the material flow velocity v. The velocity is approximated by the quotient $\frac{\Delta x}{\Delta t}$ with the time step Δt. This enables elimination of Δx from the resulting equations, so the model becomes independent of the actual spacial axis in rolling direction. The step functions below are therefore formulated in terms of the time increment Δt. Especially in transport units the spacial axis may not be defined, but the time axis will. For the cross-sections

A_i there is no explicit equation, but they are determined numerically. See section 3 for details.

$$\dot{V}_i = A_i v = A_i \frac{\Delta x}{\Delta t} \tag{7}$$

The outermost ring (surface ring) exchanges heat with the environment, so $\dot{q}_{3,\hat{\imath}}$ is defined differently. The definition used here includes heat transfer according to a heat transfer coefficient concept and gray body radiation, with the heat transfer coefficient α, the Stefan-Boltzmann radiation constant ϵ_0 and the relative radiation coefficient of the gray body ϵ_r. T_S is the absolute temperature of the surface.

$$\dot{q}_{3\hat{\imath}} = \left[-\alpha \left(T_\infty - T_\mathrm{S}\right) - \epsilon_0 \epsilon_\mathrm{r} \left({T_\infty}^4 - {T_\mathrm{S}}^4\right)\right] \times I_{\hat{\imath}+1} \Delta x \tag{8}$$

Since the surface is infinitesimally narrow, it has no heat capacity. Therefore, the heat flows on both sides must be equal. The T_S is approximated by equalizing the conductive flow from the outer ring and the heat transfer to the environment as in Equation 9. This is a scalar non-linear equation in T_S and can be solved f.e. by Newton's method.

$$\lambda \frac{T_\mathrm{S} - T_{\hat{\imath}}^k}{R_{\hat{\imath}+1} - r_{\hat{\imath}}} = \alpha \left(T_\infty - T_\mathrm{S}\right) + \epsilon_0 \epsilon_\mathrm{r} \left({T_\infty}^4 - {T_\mathrm{S}}^4\right) \tag{9}$$

1.3 Temperature Increment Functions

Taking the equations shown above, the increment functions for the temperatures in each ring are formulated. $\Delta T_i = T_i^{k+1} - T_i^k$ is the respective temperature increment. The following equations are mostly equal for roll passes and transports, with the following differences:

- The heat transfer coefficients α take different values, as in transports convective heat flow is regarded and in roll passes solid body contact.
- In roll passes, radiation only occurs at the free surface at the sides of the profile and is negligible in comparison to the solid body contact, so $\epsilon_\mathrm{r} = 0$.
- In transport no deformation occurs, so $\dot{\varphi} = 0$ leading to a vanishing source term.

The core ring has no inner boundary, so $\dot{q}_{40} = 0$.

$$\Delta T_0 = \frac{\Delta t}{\varrho c_\mathrm{p} A_0} \left[\lambda \frac{T_1^k - T_0^k}{r_1 - r_0} I_1 + \eta_\mathrm{S} \frac{k_\mathrm{f}}{\eta_\varphi} \dot{\varphi} A_0\right] \tag{10}$$

The intermediate rings take $\dot{q}_{3i}$ as in Equation 4.

$$\Delta T_i = \frac{\Delta t}{\varrho c_\mathrm{p} A_i} \left[\lambda \left[\frac{T_{i+1}^k - T_i^k}{r_{i+1} - r_i} I_{i+1} - \frac{T_i^k - T_{i-1}^k}{r_i - r_{i-1}} I_i\right] + \eta_\mathrm{S} \frac{k_\mathrm{f}}{\eta_\varphi} \dot{\varphi} A_i\right] \tag{11}$$

The surface ring takes $\dot{q}_{3\hat{\imath}}$ as in Equation 8 with respect to Equation 9.

$$\Delta T_{\hat{\imath}} = \frac{\Delta t}{\varrho c_{\mathrm{p}} A_{\hat{\imath}}} \left[\left[\alpha \left(T_{\infty} - T_{\mathrm{S}} \right) + \epsilon_0 \epsilon_{\mathrm{r}} \left({T_{\infty}}^4 - {T_{\mathrm{S}}}^4 \right) \right] I_{\hat{\imath}+1} - \lambda \frac{T_{\hat{\imath}}^k - T_{\hat{\imath}-1}^k}{r_{\hat{\imath}} - r_{\hat{\imath}-1}} I_{\hat{\imath}} + \eta_{\mathrm{S}} \frac{k_{\mathrm{f}}}{\eta_{\varphi}} \dot{\varphi} A_{\hat{\imath}} \right] \tag{12}$$

2 Plugin Usage

Load this plugin by

```
import pyroll.ring_model_thermal
```

To use this plugin, remember to define `Profile.density`, `Profile.thermal_capacity`, `Profile.thermal_conductivity` on your input profile or as hook functions.

If you are using disk elements, this plugin will calculate the temperature evolution stepwise on each disk element. Otherwise, the units will be treated as one step using the same equations as for a disk element. Therefore, the hook `Profile.ring_temperatures` is implemented on the disk elements as well as on the respective unit's outgoing profiles, as listed below.

If pyroll.report is installed, two types of plots are added to the report. For pass sequences, a plot displaying the evolution of surface, core and mean temperatures in dependence on the cumulative process time is generated similar to Figure 2a. For disked units (currently roll passes and transports) a plot displaying the evolution of the temperature distribution within the unit is generated similar to Figure 2b. The latter shows the temperature curve of each profile in the unit (including the disk elements') indicated by color. The red curve indicates the in profile, the blue one the out profile. The disk elements' profiles are colored gradual between these two. This offers a pseudo-3D illustration of the temperature evolution.

3 Implementaion Details

The interface lengths I_j and ring cross-sections A_i are obtained by the usage of shapely's capabilities and the `Profile.ring_contours` and `Profile.ring_sections` hooks.

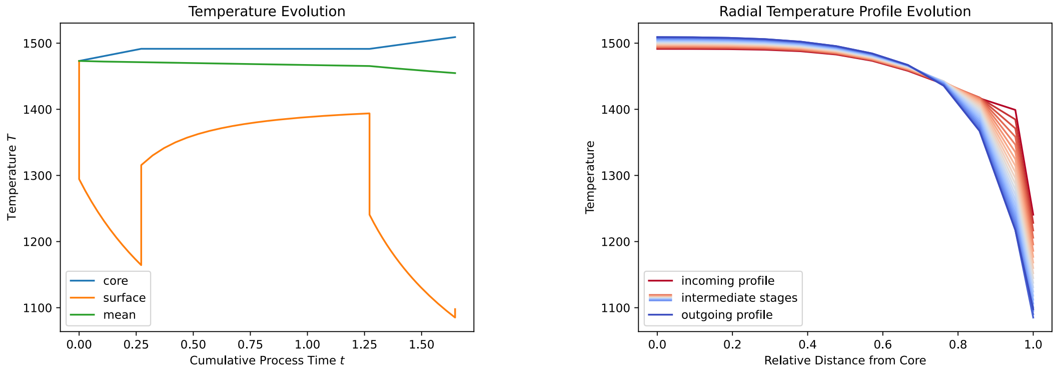
The plugin provides several new hooks, as well as, additional hook functions for existing ones as listed below. Additionally to the listed ones, some technical hook functions are provided to copy data from incoming profiles and disk elements and other subunits.

`Unit.OutProfile.temperature` and `Unit.OutProfile.ring_temperatures` are added to `root_hooks`.

3.1 Profiles

3.1.1 Additional Hooks

`Profile.ring_temperatures` a numpy array of the rings' local temperatures according to the ring model standard



(a) Evolution of Key Temperatures Along a Pass Sequence

(b) Evolution of the Temperature Distribution Within a Profile

Figure 2: Examples of Plots Added to the Report

3.1.2 Provided Implementations

`Profile.ring_temperatures` creates the `Profile.ring_temperatures` array from a single `Profile.temperature` value as a homogeneous state. Meant mainly for creation of an initial state, if only a single temperature is given on the input profile.

`Profile.temperature` calculates the nominal temperature of the profile as arithmetic mean of the ring temperatures

`Profile.surface_temperature` returns the outer ring's temperature as approximation for the surface temperature, if no further information is given. This implementation is overridden by the implementations of the respective unit profiles.

`core_temperature` returns the core ring's temperature as core temperature

3.2 Roll Passes

3.2.1 Additional Hooks

`RollPass.heat_transfer_coefficient` represents the heat transfer coefficient α for the contact of workpiece and rolls, implemented with default value $6000\,\mathrm{W}^{2}\,\mathrm{m}^{-1}\,\mathrm{K}^{-1}$

`RollPass.deformation_heat_efficiency` represents the efficiency of heat generation by deformation η_{S}, implemented with default value 0.95

3.2.2 Provided Implementations

`RollPass.OutProfile.ring_temperatures`

`RollPass.DiskElement.OutProfile.ring_temperatures` calculates the temperature evolution according to the equations Equation 10, Equation 11 and Equation 12 as described above

`RollPass.Profile.surface_temperature`

`RollPass.DiskElement.Profile.surface_temperature` calculates the surface temperature by solving Equation 9 as described above

3.3 Transports

3.3.1 Additional Hooks

`Transport.heat_transfer_coefficient` represents the heat transfer coefficient α for convection transfer to the atmosphere, implemented with default value $15\,\mathrm{W}^2\,\mathrm{m}^{-1}\,\mathrm{K}^{-1}$

`Transport.relative_radiation_coefficient` the relative radiation coefficient ϵ_r, implemented with default value 0.8

3.3.2 Provided Implementations

`Transport.OutProfile.ring_temperatures`

`Transport.DiskElement.OutProfile.ring_temperatures` calculates the temperature evolution according to the equations Equation 10, Equation 11 and Equation 12 as described above

`Transport.Profile.surface_temperature`

`Transport.DiskElement.Profile.surface_temperature` calculates the surface temperature by solving Equation 9 as described above