Documentation for the pyroll-ring-model-thermal Plugin

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1 Model Description

1.1 Ring Model

The following derivations are based on the ring model approach. For details on this approach read also the respective documentation[1].

1.2 Heat Flow Balance

As illustrated in Figure 1, the heat flow balance of each ring on a disk element can be built as in Equation 1.

$$0 = \dot{q}_{1i} - \dot{q}_{2i} - \dot{q}_{3i} + \dot{q}_{4i} + \dot{q}_{\mathrm{S}i} \tag{1}$$

The distinct heat contributions can be expressed as in the following. $\dot{q}_{1i}$ and $\dot{q}_{2i}$ are convective flows caused by workpiece material entry and exit in and from the disk element, where ϱ is the mass density, c_p is the thermal capacity, $\dot{V}_i$ the volume flow through the ring of the disk element, and T_i the absolute temperature of the respective ring. i is the index of the ring in the interval $[0, \hat{\imath}]$, where $\hat{\imath} = n - 1$ with the count of rings n. k, however, is the index of the disk, whose definition region is of no matter here. As all quantities except the temperatures are considered constant within one disk element, resp. ring, the index k is neglected. But, all may vary from disk to disk.

$$\dot{q}_{1i} = \varrho c_\mathrm{p} \dot{V}_i T_i^k \tag{2}$$

$$\dot{q}_{2i} = \varrho c_\mathrm{p} \dot{V}_i T_i^{k+1} \tag{3}$$

$\dot{q}_{3i}$ and $\dot{q}_{4i}$ are the conductive flows between the rings, with λ as the thermal capacity, r_i as the radius coordinate of the rings center line, R_j as the radius coordinates of the

[1] `https://github.com/pyroll-project/pyroll-ring-model`

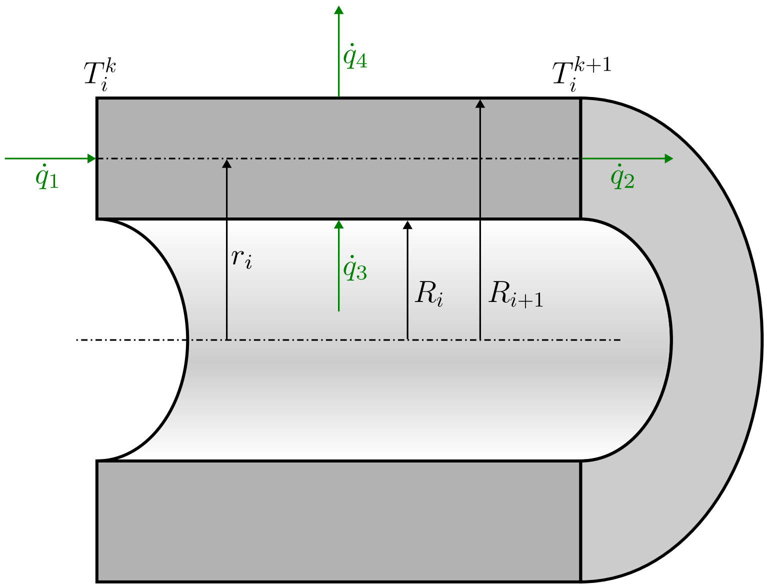

Figure 1: Heat Flows on a Disk Element Ring

ring's boundaries and the disk width in rolling direction Δx. j is the index of the ring boundary in the interval $[0, \hat{\jmath}]$, where $\hat{\jmath} = \hat{\imath} + 1$.

$$\dot{q}_{3i} = -\lambda \frac{T_{i+1}^k - T_i^k}{r_{i+1} - r_i} \times 2\pi R_{i+1} \Delta x \tag{4}$$

$$\dot{q}_{4i} = -\lambda \frac{T_i^k - T_{i-1}^k}{r_i - r_{i-1}} \times 2\pi R_i \Delta x \tag{5}$$

Heat generation by deformation is respected by the source term $\dot{q}_{\mathrm{S}}$, with the efficiency of heat generation η_{S}, the efficiency of deformation η_φ, the flow stress k_{f} the deformed volume V and the equivalent strain rate $\dot{\varphi}$.

$$\dot{q}_{\mathrm{S}i} = \eta_{\mathrm{S}} \frac{k_{\mathrm{f}}}{\eta_\varphi} \dot{\varphi} V_i \tag{6}$$

The volume flow $\dot{V}$ through the ring is calculated from the cross section A_i and the material flow velocity v. The velocity is approximated by the quotient $\frac{\Delta x}{\Delta t}$ with the time step Δt. This enables elimination of Δx from the resulting equations, so the model becomes independent of the actual spacial axis in rolling direction. The step functions below are therefore formulated in terms of the time increment Δt. Especially in transport units the spacial axis may not be defined, but the time axis will.

$$\dot{V}_i = A_i v = A_i \frac{\Delta x}{\Delta t} \tag{7}$$

The cross-section of each ring is calculated as in Equation 8. Note, that $R_0 = 0$.

$$A_i = \pi \left(R_{i+1}^2 - R_i^2\right) \tag{8}$$

The outermost ring (surface ring) exchanges heat with the environment, so $\dot{q}_{3,\hat{\imath}}$ is defined differently. The definition used here includes heat transfer according to a heat transfer coefficient concept and gray body radiation, with the heat transfer coefficient α, the Stefan-Boltzmann radiation constant ϵ_0 and the relative radiation coefficient of the gray body ϵ_r. T_S is the absolute temperature of the surface.

$$\dot{q}_{3\hat{\imath}} = \left[-\alpha\left(T_\infty - T_\mathrm{S}\right) - \epsilon_0\epsilon_\mathrm{r}\left({T_\infty}^4 - {T_\mathrm{S}}^4\right)\right] \times 2\pi R_{\hat{\imath}+1}\Delta x \tag{9}$$

Since the surface is infinitesimally narrow, it has no heat capacity. Therefore, the heat flows on both sides must be equal. The T_S is approximated by equalizing the conductive flow from the outer ring and the heat transfer to the environment as in Equation 10. This is a scalar non-linear equation in T_S and can be solved f.e. by Newton's method.

$$\lambda\frac{T_\mathrm{S} - T_{\hat{\imath}}^k}{R_{\hat{\imath}+1} - r_{\hat{\imath}}} = \alpha\left(T_\infty - T_\mathrm{S}\right) + \epsilon_0\epsilon_\mathrm{r}\left({T_\infty}^4 - {T_\mathrm{S}}^4\right) \tag{10}$$

1.3 Temperature Increment Functions

Taking the equations shown above, the increment functions for the temperatures in each ring are formulated. $\Delta T_i = T_i^{k+1} - T_i^k$ is the respective temperature increment. The following equations are mostly equal for roll passes and transports, with the following differences:

- The heat transfer coefficients α take different values, as in transports convective heat flow is regarded and in roll passes solid body contact.
- In roll passes, radiation only occurs at the free surface at the sides of the profile and is negligible in comparison to the solid body contact, so $\epsilon_\mathrm{r} = 0$.
- In transport no deformation occurs, so $\dot{\varphi} = 0$ leading to a vanishing source term.

The core ring has no inner boundary, so $\dot{q}_{40} = 0$.

$$\Delta T_0 = \frac{\Delta t}{\varrho c_\mathrm{p} A_0}\left[\pi\lambda\left(T_1^k - T_0^k\right) + \eta_\mathrm{S}\frac{k_\mathrm{f}}{\eta_\varphi}\dot{\varphi}A_0\right] \tag{11}$$

The intermediate rings take $\dot{q}_{3i}$ as in Equation 4.

$$\Delta T_i = \frac{\Delta t}{\varrho c_\mathrm{p} A_i}\left[2\pi\lambda\left[\frac{T_{i+1}^k - T_i^k}{r_{i+1} - r_i}R_{i+1} - \frac{T_i^k - T_{i-1}^k}{r_i - r_{i-1}}R_i\right] + \eta_\mathrm{S}\frac{k_\mathrm{f}}{\eta_\varphi}\dot{\varphi}A_i\right] \tag{12}$$

The surface ring takes $\dot{q}_{3\hat{\imath}}$ as in Equation 9 with respect to Equation 10.

$$\Delta T_{\hat{\imath}} = \frac{\Delta t}{\varrho c_{\mathrm{p}} A_{\hat{\imath}}} \left[2\pi \left[\left[\alpha \left(T_\infty - T_{\mathrm{S}} \right) + \epsilon_0 \epsilon_{\mathrm{r}} \left({T_\infty}^4 - {T_{\mathrm{S}}}^4 \right) \right] R_{\hat{\imath}+1} - \lambda \frac{T_{\hat{\imath}}^k - T_{\hat{\imath}-1}^k}{r_{\hat{\imath}} - r_{\hat{\imath}-1}} R_{\hat{\imath}} \right] \right.$$
$$\left. + \eta_{\mathrm{S}} \frac{k_{\mathrm{f}}}{\eta_\varphi} \dot{\varphi} A_{\hat{\imath}} \right] \quad (13)$$

2 Plugin Usage

`Unit.OutProfile.temperature` and `Unit.OutProfile.ring_temperatures` are added to `root_hooks`.

2.1 Roll Passes

2.1.1 Additional Hooks

`RollPass.heat_transfer_coefficient` represents the heat transfer coefficient α for the contact of workpiece and rolls, implemented with default value $6000\,\mathrm{W}^2\,\mathrm{m}^{-1}\,\mathrm{K}^{-1}$

`RollPass.deformation_heat_efficiency` represents the efficiency of heat generation by deformation η_{S}, implemented with default value 0.95

2.1.2 Provided Implementations

`RollPass.OutProfile.ring_temperatures`

`RollPass.DiskElement.OutProfile.ring_temperatures` calculates the temperature evolution according to the equations Equation 11, Equation 12 and Equation 13 as described above

`RollPass.Profile.surface_temperature`

`RollPass.DiskElement.Profile.surface_temperature` calculates the surface temperature by solving Equation 10 as described above

2.2 Transports

2.2.1 Additional Hooks

`Transport.heat_transfer_coefficient` represents the heat transfer coefficient α for convection transfer to the atmosphere, implemented with default value $15\,\mathrm{W}^2\,\mathrm{m}^{-1}\,\mathrm{K}^{-1}$

`Transport.relative_radiation_coefficient` the relative radiation coefficient ϵ_{r}, implemented with default value 0.8

2.2.2 Provided Implementations

`Transport.OutProfile.ring_temperatures`

`Transport.DiskElement.OutProfile.ring_temperatures` calculates the temperature evolution according to the equations Equation 11, Equation 12 and Equation 13 as described above

`Transport.Profile.surface_temperature`

`Transport.DiskElement.Profile.surface_temperature` calculates the surface temperature by solving Equation 10 as described above

Symbols

Symbol	Description
A	Cross section
α	Heat transfer coefficient
c_p	Thermal Capacity
ϵ_0	Radiation coeffient of black radiator
ϵ_r	Relative radiation coeffient
η_S	Efficiency of heat source by deformation
η_φ	Efficiency of deformation
i	Index of the ring
$\hat{\imath}$	Maximum index of the ring
j	Index of the ring boundary
$\hat{\jmath}$	Maximum index of the ring boundary
k	Index of the disk element
k_f	Flow stress
λ	Thermal conductivity
$\dot{m}$	Mass flow in x-direction
n	Count of rings
φ	Equivalent strain
$\dot{\varphi}$	Equivalent strain rate
$\dot{q}$	Heat flow
$\dot{q}_\mathrm{S}$	Heat source (generation)

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Table 0: (Continued)

r	Radius coordinate of a ring's center line
R	Radius coordinate of a ring's boundary line
Δr	Discretization width in radius
ϱ	Density
t	Time
Δt	Discretization width in time
T	Absolute temperature
ΔT	Increment of temperature
T_∞	Environemnt temperature
T_S	Absolute surface temperature
V	Volume of the disk element rep. layer
v	Velocity of material flow
$\dot{V}$	Volume flow
x	X Coordinate
Δx	Discretization width in x