Problem 1.

a. $L_{2,0} f(x_0) = \dfrac{x-0.3}{0-0.3} \cdot \dfrac{x-0.6}{0-0.6} e^{0} \cos 0$

$L_{2,1} f(x_1) = \dfrac{x-0}{0.3-0} \dfrac{x-0.6}{0.3-0.6} e^{0.6} \cos 0.9$

$L_{2,2} f(x_2) = \dfrac{x-0}{0.6-0} \dfrac{x-0.3}{0.6-0.3} e^{1.2} \cos 1.8$

$P_2(x) = L_{2,0} f(x_0) + L_{2,1} f(x_1) + L_{2,2} f(x_2) = -11.22x^2 + 3.868x + 1$

其中 $f'(x) = 2e^{2x}\cos 3x - 3e^{2x}\sin 3x = e^{2x}(2\cos 3x - 3\sin 3x)$

$f''(x) = e^{2x}[2(2\cos 3x - 3\sin 3x) - 6\sin 3x - 9\cos 3x]$

$= e^{2x}(-5\cos 3x - 12\sin 3x)$

$f'''(x) = e^{2x}(-10\cos 3x - 24\sin 3x + 15\sin 3x - 36\cos 3x)$

$= e^{2x}(-46\cos 3x - 9\sin 3x)$

$\therefore \varepsilon = \max\limits_{\xi\in[0,0.6]} \left| \dfrac{f'''(\xi)}{3!} \right| \max\limits_{x\in[0,0.6]} |(x-x_0)(x-x_1)(x-x_2)|$

$= \dfrac{65.65}{6} \times 0.01039$

$= 0.1137$

同理 b. $L_{2,k} = \prod\limits_{\substack{j=1 \\ j\neq k}}^{2} \dfrac{x-x_j}{(x_k-x_j)}$

$p(x) = \sum\limits_{k=0}^{2} L_{2,k} \cdot f(k)$

$= -0.1306x^2 + 0.8970x - 0.6365$

$f'''(x) = \dfrac{3\sin(\ln x) + \cos(\ln x)}{x^3}$

$\varepsilon = \max\limits_{\xi\in[2,2.6]} \left| \dfrac{f'''(x)}{3!} \right| \cdot \max\limits_{x\in[2,2.6]} |(x-x_0)(x-x_1)(x-x_2)|$

$= \dfrac{0.3358}{6} \times 0.24$

$= 0.01423$

Problem 2

$P(x) = a_0 + a_1x^1 + a_2x^2 + a_3x^3$

其中 $a_3 = 6$ 且給了4对点,分别代入

$$\begin{cases} a_0 = 0 \\ a_0 + 0.5a_1 + 0.25a_2 + 0.125a_3 = y \\ a_0 + a_1 + a_2 + a_3 = 3 \\ a_0 + 2a_1 + 4a_2 + 8a_3 = 2 \end{cases}$$

解线性方程 $\Rightarrow y = 4.25$

另,也可构建拉格朗日插值多项式

$P_3(x) = L_0 f(x_0) + L_1 f(x_1) + L_2 f(x_2) + L_3 f(x_3)$

$a_3 = \dfrac{y}{0.5(0.5-1)(0.5-2)} + \dfrac{3}{1\times(1-0.5)\times(1-2)} + \dfrac{2}{2\times(2-0.5)\times(2-1)}$

$= \dfrac{8}{3}y - 6 + \dfrac{2}{3}$

$= 6 \quad \Rightarrow y = \dfrac{34}{8}$

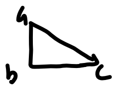
Problem 3

a.

$P_{1,3}(0.4) = \frac{1}{x_3-x_2}\left[(x-x_2)P_3 - (x-x_3)P_2\right]\Big|_{x=0.4}$

$= 4[-0.1\times 8 - (-0.35)P_2]$

$= 2.4$

$\Rightarrow P_2 = 4$ tips  知道2点可求第3点

同理. $P_{1,2,3}(0.4) = \frac{1}{x_3-x_1}\left[(x-x_1)P_{2,3} - (x-x_3)P_{1,2}\right]\Big|_{x=0.4}$

$= 2[0.15\times 2.4 - (-0.35)P_{1,2}]$

$= 2.96$

$\Rightarrow P_{1,2} = 3.2$

同理 $P_{0,1,2} = 3.08$

b.

$x_0=0$	P_0		
$x_1=1$	P_1	$P_{0,1}=5.1$	
$x_2=2$	P_2		$P_{0,1,2}=1.35$
$x_3=3$	P_3		$P_{1,2,3}=3$ $P_{0,1,2,3}$

其中 $P_{0,1,2} = \frac{1}{x_2-x_1}\left[(x-x_1)P_{0,2} - (x-x_2)P_{0,1}\right]$

$= 0.5[1.5\times 3.5 - 0.5\times 5.1]$

$= 1.35$

$P_{0,1,2,3} = \frac{1}{x_3-x_0}\left[(x-x_0)P_{1,2,3} - (x-x_3)P_{0,1,2}\right]$

$= \frac{1}{3}\times[2.5\times 3 - (-0.5)\times 1.35]$

$= 2.725$

Problem 4.

$x_0 = 0.0$	$f[x_0] = 1$		
		$f[x_0, x_1] = 5$	
$x_1 = 0.4$	$f[x_1] = 3$		$f[x_0, x_1, x_2] = \frac{50}{7}$
		$f[x_1, x_2] = 10$	
$x_2 = 0.7$	$f[x_2] = 6$		

Problem 5

2个区间三次样条插值共需8个方程

$$\begin{cases} S_0(x) = a_0 + b_0x + c_0x^2 + d_0x^3 & x\in[0,1] \\ S_1(x) = a_1 + b_1(x-1) + c_1(x-1)^2 + d_1(x-1)^3 & x\in[1,2] \end{cases}$$

自然样条:

$$\begin{cases} a_0 = 0 \quad a_1 = 1 \\ a_1 = a_0 + b_0 + c_0 + d_0 \\ 2 = a_1 + b_1 + c_1 + d_1 \\ b_1 = b_0 + 2c_0 + 3d_0 \\ c_1 = c_0 + 6d_0 \\ c_0 = 0 \quad c_1 = 0 \end{cases} \Rightarrow \begin{cases} a_0 = 0 \quad a_1 = 1 \\ b_0 = 1 \quad b_1 = 1 \\ c_0 = 0 \quad c_1 = 0 \\ d_0 = 0 \quad d_1 = 0 \end{cases}$$

$\therefore S = \begin{cases} x & x\in[0,1] \\ x & x\in[1,2] \end{cases}$

紧压.样条

$$\begin{cases} a_0 = 0 \quad a_1 = 1 \\ a_1 = a_0 + b_0 + c_0 + d_0 \\ 2 = a_1 + b_1 + c_1 + d_1 \\ b_1 = b_0 + 2c_0 + 3d_0 \\ c_1 = c_0 + 6d_0 \\ b_0 = 1 \\ b_1 + c_1 + d_1 = 1 \end{cases} \Rightarrow \begin{cases} a_0 = 0 \quad a_1 = 1 \\ b_0 = 1 \quad b_1 = 1 \\ c_0 = 0 \quad c_1 = 0 \\ d_0 = 0 \quad d_1 = 0 \end{cases}$$

$\therefore S = x \qquad x\in[0,2]$