

浙江大学



Numerical Analysis Assignment #2

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3.1 a.

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5.1 a.

5.2 b.

1 Problem 1

牛顿方法如下:

$$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)}$$

所以, 经过两次迭代, $p_2 = -0.8656841632$.

p_0 会导致 $f'(p_0) = 0$, 不能使用。

2 Problem 2

(I)

$$x_{k+1} = 2x_k - bx_k^2$$
$$\epsilon_{k+1} = \frac{\frac{1}{b} - x_{k+1}}{\frac{1}{b}} = 1 - bx_{k+1} = 1 - 2bx_k + b^2x_k^2 = (1 - bx_k)^2 = \epsilon_k^2$$

(II)

$$x_{k+1} = 2x_k - bx_k^2 \quad x \in (0, \frac{2}{b})$$

so we can find $x_1 \in (0, \frac{1}{b})$

对于

$$f(x) = 2x - bx^2$$

故存在 $k \in (0, 1)$, 我们有 $|g'(x)| = |1 - 2bx| < k$, 根据MVT, x 收敛至 $f(x)$ 的不动点 $\frac{1}{b}$.

3 Problem 3

the code is in the file, here only present the results. By the way, I try to make the procedure run more times, which can find the final root

3.1 a.

```

x0 = [0. 0. 0.]
x1 = [ 0.5          0.5         -0.52359878]
x2 = [ 0.50016669  0.25080364 -0.51738743]
x3 = [ 0.49994493  0.12620625 -0.52045512]
x4 = [ 0.49998624  0.06391304 -0.52200313]
x5 = [ 0.49999467  0.03277679 -0.52278013]
x6 = [ 0.49999742  0.01722919 -0.52316841]
x7 = [ 0.4999986   0.00949621 -0.52336156]
x8 = [ 0.49999916  0.00570987 -0.52345614]
x9 = [ 0.49999942  0.00396593 -0.52349971]
x10 = [ 0.49999951  0.00332332 -0.52351576]
x11 = [ 0.49999953  0.00320354 -0.52351875]
x12 = [ 0.49999953  0.00319907 -0.52351886]
x13 = [ 0.49999953  0.00319906 -0.52351886]
x14 = [ 0.49999953  0.00319906 -0.52351886]
x15 = [ 0.49999953  0.00319906 -0.52351886]

```

3.2 b.

```

[Running] python -u "d:\code\c\NA_A2\p3_b.py"
x0 = [0. 0. 0.]
x1 = [ 5.  37. -39.]
x2 = [ 4.35087719 18.49122807 -19.84210526]
x3 = [ 5.36382418  9.25545179 -11.61927596]
x4 = [ 5.6960556  4.66532836 -7.36138395]
x5 = [ 5.88282266  2.42727945 -5.31010212]
x6 = [ 5.96609477  1.41264746 -4.37874223]
x7 = [ 5.99518811  1.05856593 -4.05375405]
x8 = [ 5.99987186  1.00155958 -4.00143144]
x9 = [ 5.9999999  1.00000117 -4.00000107]
x10 = [ 6.  1. -4.]
x11 = [ 6.  1. -4.]

```

4 Problem 4

试了多个初值，结果处于没有收敛和迭代速度很慢两种情况，可能是由于步长设置不合理导致，代码已附在文件夹中

5 Problem 5

5.1 a.

$$J(x_1, x_2) = \begin{pmatrix} \frac{x_1}{5} & \frac{x_2}{5} \\ \frac{1+x_2^2}{10} & \frac{x_1 x_2}{5} \end{pmatrix} \text{取 } K = 0.95, \text{ 则当 } x_i \in D \text{ 时,}$$

$$\left| \frac{\partial g_i(x)}{\partial x_j} \right| \leq \left| \frac{\partial g_2(x)}{\partial x_2} \right| = \left| \frac{x_1 x_2}{5} \right| \leq \frac{9}{20} < \frac{0.95}{2}$$

由上述定理, 在 D 上不动点唯一.

5.2 b.

代入计算有:

$$\begin{aligned} x^{(0)} &= [0, 1]^t \\ x^{(1)} &= \left[\frac{9}{10}, \frac{8}{10} \right]^t \\ x^{(2)} &= [1.045, 0.9046]^t \end{aligned}$$