

# 浙江大学



## Numerical Analysis Assignment #3

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**1 Problem 1**

1.1 a.

1.2 b.

**2 Problem 2**

**3 Problem 3**

**4 Problem 4**

4.1 a.

4.2 b.

**5 Problem 5**

5.1 a.

5.2 b.

**6 Problem 6**

**7 Problem 7**

## 1 Problem 1

### 1.1 a.

$$\begin{aligned}\|x - \tilde{x}\|_{\infty} &= \|(0.2, 0.5, -0.4)^T\|_{\infty} = 0.5 \\ A\tilde{x} &= (1, -1.3, 1.8)^T \\ \|A\tilde{x} - b\|_{\infty} &= 0.3\end{aligned}$$

### 1.2 b.

$$\begin{aligned}\|x - \tilde{x}\|_{\infty} &= \|(0.33, 0.9, -0.8)^T\|_{\infty} = 0.9 \\ A\tilde{x} &= (1.27, -1.16, 2.21)^T \\ \|A\tilde{x} - b\|_{\infty} &= 0.27\end{aligned}$$

## 2 Problem 2

$$\|A\|_2 = \rho(A \cdot A^T)^{\frac{1}{2}}$$

if A is symmetric, then

$$\begin{aligned}A \cdot A^T &= A^2 = f(A) \\ \rho(A \cdot A^T) &= \lambda'_{\max} = f(\lambda_{\max}) = \lambda_{\max}^2\end{aligned}$$

so

$$\|A\|_2 = \rho(A)$$

## 3 Problem 3

the code is in the file.

编写代码遇到问题:

- 将numpy数组切片后, 寻找非零最小项的下标

```
list = a[i:, i]
# 找到非零最小值的下标
[[k]] = np.where(list ==
np.min(np.abs(list[np.nonzero(list)])))
p = k + i # 避免了前i行的影响
```

- 依次放入列表X获得解

```

xn = a[n-1][n] / a[n-1][n-1]
X = [xn]

# 求根并放入列表x
for k in range(n-2, -1, -1):
    b=0
    cnt = 0
    for j in range(k+1,n):
        b = b + a[k][j] * X[-1-cnt]
        cnt += 1
    x = (a[k][n] - b) / a[k][k]
    X.append(x)

```

## 4 Problem 4

the code is in the file.

### 4.1 a.

```

[Running] python -u "d:\code\c\NA_A3\tempCodeRunnerFile.py"
x 0 = [ 1.25      -1.33333333  0.2       ]
x 1 = [ 1.63333333 -0.98333333  0.23333333]
x 2 = [ 1.55416667 -0.86666667 -0.06      ]
x 3 = [ 1.45166667 -0.79527778 -0.075     ]

```

### 4.2 b.

```

[Running] python -u "d:\code\c\NA_A3\tempCodeRunnerFile.py"
x 0 = [-2.  2.  0.]
x 1 = [-1.  1. -1.]
x 2 = [-1.75  1.75 -0.5 ]
x 3 = [-1.25  1.25 -0.875]

```

## 5 Problem 5

the code is in the file.

### 5.1 a.

GS:

```
[Running] python -u "d:\code\c\NA_A3\p5_a_GS.py"
x 0 = [ 0.33333333 -0.16666667 0.5 ]
x 1 = [ 0.11111111 -0.22222222 0.61904762]
x 2 = [ 0.05291005 -0.23280423 0.64852608]
x 3 = [ 0.03955656 -0.23595364 0.65559875]
x 4 = [ 0.0361492 -0.23660752 0.65733928]
x 5 = [ 0.03535107 -0.23678863 0.65775895]
The procedure is successful!
```

Jacobi:

```
[Running] python -u "d:\code\c\NA_A3\p5_a_Jacobi.py"
x 0 = [0.33333333 0. 0.57142857]
x 1 = [ 0.14285714 -0.35714286 0.42857143]
x 2 = [ 0.07142857 -0.21428571 0.66326531]
x 3 = [ 0.04081633 -0.25680272 0.63265306]
x 4 = [ 0.03684807 -0.23129252 0.66399417]
x 5 = [ 0.03490444 -0.23975543 0.6547619 ]
x 6 = [ 0.03516089 -0.23570619 0.65922185]
x 7 = [ 0.03502399 -0.23732106 0.65737656]
x 8 = [ 0.03510079 -0.23663751 0.65812732]
The procedure is successful!
```

## 5.2 b.

GS:

```
[Running] python -u "d:\code\c\NA_A3\p5_b_GS.py"
x 0 = [0.9 0.79 0.758]
x 1 = [0.979 0.9495 0.7899]
x 2 = [0.99495 0.957475 0.791495]
x 3 = [0.9957475 0.95787375 0.79157475]
x 4 = [0.99578738 0.95789369 0.79157874]
The procedure is successful!
```

Jacobi:

```
[Running] python -u "d:\code\c\NA_A3\p5_b_Jacobi.py"
x 0 = [0.9 0.7 0.6]
x 1 = [0.97 0.91 0.74]
x 2 = [0.991 0.945 0.782]
x 3 = [0.9945 0.9555 0.789 ]
x 4 = [0.99555 0.95725 0.7911 ]
x 5 = [0.995725 0.957775 0.79145 ]
x 6 = [0.995775 0.9578625 0.791555 ]
x 7 = [0.99578625 0.95788875 0.7915725 ]
The procedure is successful!
```

## 6 Problem 6

反证法：假设不为线性无关，那么一定存在

$$k_1x_1 + k_2x_2 = 0 \quad (1)$$

$$Ak_1x_1 + Ak_2x_2 = 0$$

$$\rho_1k_1x_1 + \rho_2k_2x_2 = 0 \quad (2)$$

将 (1) 式代入

$$k_1x_1(\rho_1 - \rho_2) = 0$$

与题目矛盾，故假设不成立。

## 7 Problem 7

不妨设  $A$  不可逆，则  $\det[A] = 0$ ，对于  $A \cdot x = 0$ ，存在非零解  $X = (x_1, x_2, \dots, x_n)$ ，令  $x_k = \max\{x_1, x_2, \dots, x_n\}$

$$\sum_{j \neq k}^n a_{kj}x_j = -a_{kk}x_k$$

又  $A$  为对角严格占优矩阵，所以

$$\left| \sum_{j \neq k}^n a_{kj}x_j \right| \leq |x_k| \sum_{j \neq k}^n |a_{kj}| < |a_{kk}| |x_k|$$

故两者矛盾， $A$  一定可逆。