浙江大學

Numerical Analysis Assignment #1

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1 problem1

引理:

介值定理:$f \in C[a,b]$,$\forall u$介于$f(a),f(b)$,则一定存在ξ ,使$f(\xi)=u$

证明：不妨设m,M为$f(x)$在$[a,b]$上的最小值与最大值，若$m=M$时，则$f(x)=const$，定理显然成立。当$m \le M$时，根据最值性定理，$\exists x_1,x_2$，使$f(x_1)=m,f(x_2)=M$。
不妨构造$g(x)=f(x)-u$，$g \in C[x_1,x_2]$，有:

$$\begin{cases} g(x_1)=f(x_1)-u=m-u<0 \\ g(x_2)=f(x_2)-u=M-u>0 \end{cases}$$

根据零点定理，$g \in C[x_1,x_2]$，$g(x_1)\cdot g(x_2)<0$,一定$\exists \xi \in [x_1,x_2]$，使$g(\xi)=0$,所以$f(\xi)=u$，即$\exists \xi$，使$f(\xi)=u$成立，原引理得证。

1.1 a.

不妨假设$f(x_1)\le f(x_2)$，设$u=\frac{f(x_1)+f(x_2)}{2}$，有$f(x_1)\le u \le f(x_2)$，根据引理，必然$\exists \xi \in [x_1,x_2]$，使$f(\xi)=u=\frac{f(x_1)+f(x_2)}{2}$.

1.2 b.

不妨假设$f(x_1)\le f(x_2)$，设$u=\frac{c_1f(x_1)+c_2f(x_2)}{c_1+c_2}$,

$$\begin{cases} f(x_1)-u=\frac{c_2}{c_1+c_2}(f(x_1)-f(x_2))\le 0 \\ f(x_2)-u=\frac{c_1}{c_1+c_2}(f(x_2)-f(x_1))\ge 0 \end{cases}$$

故，$f(x_1)\le u \le f(x_2)$，根据引理得，$\exists \xi \in [x_1,x_2]$，使$f(\xi)=u=\frac{c_1f(x_1)+c_2f(x_2)}{c_1+c_2}$.

1.3 c.

2 problem2

2.1 a.

$|f(x_0)-\tilde{f}(x_0)|=f'(\xi)\cdot(x_0+\epsilon-x_0)=f'(x_0+\theta\epsilon)\cdot\epsilon$，$\theta \in (0,1)$

当ϵ足够小时，我们可以将$f'(x_0+\theta\epsilon)$近似为$f'(x_0)$，$|f(x_0)-\tilde{f}(x_0)|=f'(x_0)\cdot\epsilon$

$|f(x_0)-\tilde{f}(x_0)|/|f(x_0)|=f'(\xi)\cdot(x_0+\epsilon-x_0)/f(x_0)=f'(x_0+\theta\epsilon)\cdot\epsilon/f(x_0),\theta \in (0,1)$

当ϵ足够小时，我们可以将$f'(x_0+\theta\epsilon)$近似为$f'(x_0)$，$|f(x_0)-\tilde{f}(x_0)|/f(x_0)=f'(x_0)\cdot\epsilon/f(x_0)$

2.2 b.

2.2.1 i.

absolute errors:
$f(x)=e^x$，$|f(x_0)-\tilde{f}(x_0)|=e^{x_0+\theta\epsilon}\cdot\epsilon$
又$f(x)=e^x\in C[-\infty,+\infty]$，
代入数据得，
$|f(x_0)-\tilde{f}(x_0)|\in(e^1\times 5\times 10^{-6}，e^{1+5\times 10^{-6}}\times 5\times 10^{-6})=(1.359140914226\times 10^{-5}，1.359147709951\times 10^{-5})$

relative errors: $|f(x_0)-\tilde{f}(x_0)|/|f(x_0)\in(5.000000000\times 10^{-6}，5.000025000\times 10^{-6})$

2.2.2 ii.

absolute errors: $|f(x_0)-\tilde{f}(x_0)|\in(4.99923846817\times 10^{-6}，4.99923847578\times 10^{-6})$
relative errors: $(2.8644980772\times 10^{-4}，2.8644980815\times 10^{-4})$

2.3 c.

2.3.1 i.

relative errors: $(0.11013232897403354982，0.11013287963705503669)$
absolute errors: $(0.04051541963787690798，0.04051562221548152959)$

2.3.2 ii.

relative errors: $(4.19534404480205\times 10^{-6}, 4.19535764538226\times 10^{-6})$
absolute errors: $(7.71173022668821\times 10^{-6}, 7.71175522678460\times 10^{-6})$

3 Problem 3

3.1 a.

3.1.1 i.

exactly: $\frac{4}{5}+\frac{1}{3}=\frac{17}{15}$

3.1.2 ii.

three-digit chopping: $0.800+0.333=0.113\times 10^1$
relative errors: $|\frac{17}{15}-0.113\times 10^1|/\frac{17}{15}=2.94\times 10^{-4}$

3.1.3 iii.

three-digit rounding: $0.800+0.333=0.113\times 10^1$
relative errors: $|\frac{17}{15}-0.113\times 10^1|/\frac{17}{15}=2.94\times 10^{-4}$

3.2 b.

3.2.1 i.

exactly: $(\frac{1}{3}+\frac{3}{11})-\frac{3}{20}=\frac{301}{660}$

3.2.2 ii.

three-digit chopping: $0.333 + 0.272 - 0.150 = 0.452$
relative errors: 8.90×10^{-3}

3.2.3 iii.

three-digit rounding: $0.333 + 0.273 - 0.150 = 0.453$
relative errors: 6.71×10^{-3}

4 Problem 4

4.1 a.

不妨设$g(x) = F(x) - c_1L_1 - c_2L_2 = c_1(L_1 + O(x^\alpha)) + c_2(L_2 + O(x^\beta)) - c_1L_1 - c_2L_2 = O(x^\alpha) + O(x^\beta)$
对于$\lim\limits_{x\to 0} \frac{g(x)}{x^\gamma} = \lim\limits_{x\to 0} O(x) + O(x^{|\alpha-\beta|}) = 0$
故$g(x) \sim x^\gamma$，证毕。

4.2 b.

不妨设$h(x) = G(x) - L_1 - L_2 = O((c_1x)^\alpha) + O((c_2x)^\beta)$
对于

$$\lim_{x\to 0} \frac{h(x)}{x^\gamma} = \begin{cases} \lim\limits_{x\to 0} c_1^\alpha O(x) + c_2^\beta O(x^{-\alpha+\beta}) = 0 & \gamma = \alpha \\ \lim\limits_{x\to 0} c_1^\alpha O(x^{\alpha-\beta}) + c_2^\beta O(x) = 0 & \gamma = \beta \end{cases}$$

故$h(x) \sim x^\gamma$，证毕。

5 Problem 5 and Problem 6

The code is include in the files

6 Problem 7

$|p1 - p| \geq |g(p_0) - p| \geq |g(p_0) - g(p)| = |(p_0 - p) \cdot g'(p + \epsilon(p_0 - p))|, \epsilon \in (0, 1)$
so,$|p_1 - p| = |p_0 - p| \cdot |g'(p + \epsilon(p_0 - p))|$,obviously,we can find a δ,than
$0 < |p_0 - p| < \delta$,and $|g'(p + \epsilon(p_0 - p))| > 1$,for $g \in C[a, b]$,than $|p_1 - p| > |p_0 - p|$ is proved.