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Research article

Weight-restricted approach on constant returns to scale DEA models: Efficiency of internet banking in Turkey

Abdullah A. Kraidi [a,*], Sahand Daneshvar [a], Kehinde Adewale Adesina [b,c]

[a] *Department of Industrial Engineering- Engineering Faculty- Eastern Mediterranean University, TRNC, Via Mersin 10, Gazimagusa, 99628, Turkey*
[b] *Department of Industrial Engineering – Near East University, Nicosia/KKTC, Via Mersin 10, Turkey*
[c] *Food Engineering – Near East University, Nicosia/KKTC, Via Mersin 10, Turkey*

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ABSTRACT

The performance measurement of decision-making units (DMUs) can be optimised by strategically choosing the optimal input and output weights. This approach circumvents the potential complexities associated with establishing a common basis for ranking homogeneous units and managing the results from multiple efficient frontiers. Efficient frontier ranking is simplified by limiting the weights based on the intrinsic or technical connections between relevant inputs and outputs. It also creates a coherent framework that coordinates various considerations across all under-evaluated DMUs and variables. This method effectively examines the complexities related to the inputs and outputs of homogeneous units using conventional data envelopment analysis (DEA) models to evaluate the efficiency of these units.

In the banking sector, the relationship between inputs and outputs is complex. This complexity requires careful attention, particularly in cases where the existence of one input or output is conditional on the existence of another. This dynamic is accentuated by the integration of advanced fintech solutions, which inherently provide significant benefits to banking operations. The use of appropriate analytical models for banking data is important. The synergy between advanced financial technologies and analytical methods has reached its peak in reducing losses and strengthening profits. This, in turn, facilitates the optimal allocation of financial resources to growth-oriented projects.

The empirical application of weight-restricted DEA models to Turkey's mobile-internet-banking sector underscores their efficacy. These modified models contrast with the standard DEA versions, which often encounter challenges stemming from the allocation of weights to pivotal variables. This study demonstrates the tangible benefits of our approach and its potential to revolutionise the efficiency assessment landscape in the banking sector.

The proposed unconventional model which is applied to Turkish Internet banking for the period extending from 2006 to 2018 and which takes a comprehensive weight restriction approach to DEA and encompasses all relevant input and output variables, resulted in strictly positive weights and a reduced count of inflated efficient units along with their scores. This is practically beneficent to banking sector in so far as it facilitates decision-making and related economic and financial planning, and as it proves to be a reliable benchmark by local or international banks intending to provide or develop Internet banking services.

* Corresponding author. Department of Industrial Engineering- Engineering Faculty- Eastern Mediterranean University, TRNC, Gazimagusa, Turkey.

E-mail addresses: 15600119@emu.edu.tr (A.A. Kraidi), sahand.danrshvar@emu.edu.tr (S. Daneshvar), kehinde.adesina@neu.edu.tr (K.A. Adesina).

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1. Introduction

Data envelopment analysis (DEA) quantitatively measures entities' performance through decision-making units (DMUs), and evaluates their aptitude for converting multiple inputs to outputs. This technique facilitates benchmarking and forecasting. Farrell (1957) proposed the basic DEA-efficient frontier model as in equation (1) [1].

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \tag{1}$$

It was further improved by Charnes, Cooper, and Rhodes in 1978 [2], who considered multiple inputs to obtain one or more outputs using CCR models. They proposed that an efficient frontier has constant returns to scale (CRS) characteristics, implying that an increase in input would yield a proportionate increase in output. A modification was made by Banker, Charnes, and Cooper in 1984 [3] in the form of a BCC model with a variable return to scale (VRS) frontier. The motivation was that an increase in a unit's input may not always trigger a proportional change in its output.

DEA, as a linear programming, non-stochastic, non-parametric, and relative efficiency measuring technique, can identify the highest possible efficiency ratio of multi response experiments. The referral envelops the peer units and benchmarks (creating a frontier for evaluating peer units). These peer units are allotted relative values between zero and one, denoting strict inefficiency and efficiency points for the DMUs, respectively. The range between 0 and 1 can be partitioned into weak efficient points (WEP) and strong efficient points (SEP) to determine the inset frontier dynamics [4,5]. The DEA specifies the amounts and areas to improve inefficient units by analysing each DMU's performance by converting inputs to outputs [6]. In such scenarios, an inefficient unit should produce its current outputs with fewer inputs (input minimisation) or generate higher outputs with the same inputs (output maximisation). In other words, input or output orientations are matters of focus when using the minimum input to generate a given output, or in the possible increase in outputs from fixed inputs.

DEA identifies a decrease or increase in the percentage requirements for a unit to be on the frontier and is efficient. Owing to weight selection, such nonlinear programming results in multiple efficient DMUs. This problem has been targeted by DMUs ranking methods, such as super-efficiency [7], cross-efficiency [8,9], and common weights [10,11]. Nevertheless, the shortcomings of these methods have stimulated further efforts to enhance their technical, allocative, scale, and overall efficiencies. It is worth mentioning that the input-oriented CRS model assumptions are valid only when all the units under investigation operate at the optimal scale. In contrast, the VRS was predicated on the presence of operational constraints which forced the units to operate at a non-optimal scale. Standard DEA models were marred by generating zero weights and multiple efficient units resulting from the flexible choice of variables.

A flexible choice of the input and output weight sets creates multiple efficient units. Ranking methods can be used to address this issue. The weight constraint notion was first proposed by Thompson et al. [12] as a means of reducing multiple efficient units in a system. This multiplicity is commonly encountered when measuring the efficiency of banking systems. The problem of multiple efficient units (originally generated from the standard DEA CRS application to bank data) was solved using a restriction procedure [2,13,14]. Other solutions were later developed [15], targeting the problems of weight flexibility and biased assessment of DMUs based on selected inputs and outputs.

In standard DEA applications to banking data, according to the number of attempts by scholars, multiple efficient and non-efficient units have resulted. The multiplicity of efficient units is sometimes problematic, as it denotes biased results of the model and may cause confusion. Considering the weights obtained from such applications, some units appear with zero weight, although they have high efficiency scores. This zero weight phenomenon, when considered by laymen, denotes that such variables have no value. This is a clear indication that the standard DEA requires modifications to solve this problem. This is because DEA basically functions by identifying the easiest and shortest ways to rank variables. This problem has led to previous attempts to apply restrictions to the weights in different methods to show the real weights and effects of the variables. This ultimately led to results which were different from those obtained by applying standard models. This is quite evident when the present work selected the variables which have more direct impacts on Internet-banking functioning, which are used in Internet-banking evaluation using different statistical programs and applications, such as E-Views. The present study selected these variables and incorporated them to evaluate the efficiency of Internet banking using DEA. This is the first attempt to apply such variables to the DEA of Internet banking in Turkey. This application is of great benefit and value from an economic and banking perspective, on the one hand, and is valuable in terms of model improvement and hypothesis verification, on the other hand.

The Fintech Futures Report of 2018 ranked Turkey as the 17th largest and fastest growing economy during and after the ravaging economic crises which occurred between 2006 and 2017. According to the Irving Fischer Committee 2020, Turkey witnessed 305 innovative fintech collaborations with banks since 2017. This figure surpasses that of the US's and most European countries [16,17]. A core success factor was the outstanding increase of mobile-banking active users from 6.7 m in 2014 to 49.3 m in 2019 within a population of 83.16 m [18,19] corresponding to 95 % mobile phone users as at the end of 2014. 45 % Of that population were mobile phone users for online banking, 62 % of that number specifically paying online. This percentage was the highest in the Middle East [20]. In 2015, it was estimated that Turkey had the world's highest number of Internet-banking users (65 % of the world's) [20], thereby marking digital banking the main distribution hub [21]. Since then, most banks have been key players in the initiative, rapidly propelling Turkey towards the transformation into a fully cashless economy.

There is continuing interest in adopting various DEA approaches to investigate banking efficiency in Turkey, and more specifically, Internet banking. The research applied DEA with varying aims, scopes (institutes data type, size, number, ownership, and place-country-specific or cross-country and time), influencing factors (reforms, crises, technological advances), and methodologies for ranking banking efficiency. However, DEA applications to mobile Internet banking in Turkey remain scarce despite its innovative banking sector; refer to Babuscu et al. [22], Arslan & Ergec [23], Boyacioglu et al. [24], Mirshab et al. [25], Tatlidil & Unvan [26], Eken & Kale [27], Yilmaz [28], Gokgoz [29], Karaca & Yayar [30], Babacan [22], Eken et al. [31], Gokdemir et al. [32], Guneş & Yi [33], Yildirim [34], Ata & Bogan [35], Batir & Gungor [36], Buyukakin & Pehlivanoglu [37], Bahar et al. [38], and Kahveci & Wolfs [39].

This study, accordingly applies DEA to mobile Internet banking in Turkey employing non-negative variable weights restriction of sets to enhance the evaluation and ranking of efficient frontiers. The purpose of this study is to show that a modified DEA model is a reliable tool for measuring the efficiency of internet banking, especially in fast-growing banking systems such as that of Turkey's.

This study aims to answer the question which is relevant to a more effective measurement of banking efficiency using weights. This stems from the fact that the DEA provides a straightforward measure of efficiency. In other words, DEA gives full efficiency scores to some variables and less or no efficiency scores to others, regardless of the significance of these variables. This indicates that lower efficiency scores do not necessarily reflect the real significance of variables. Thus, the model was modified by assigning weights to the variables to see resultant score efficiency; it is reliability and applicability to other banks. The present study first applies weight restriction conjointly and then individually to identify one set of generated positive weights (of highly dependent inputs and outputs) which units have in common to evaluate the DMUs' efficiency and ranking. The remainder of the paper is structured as follows. Section 2 (Theory and Calculation) comprises the standard DEA model, considerations for a weight-restriction approach, DMUs' efficiency evaluation, DMUs' ranking procedure, and a numerical illustration of the proposed model. Section 3 presents the Results, Discussion, and Conclusions.

2. Theory, Calculation

2.1. DEA model specifications

Multiplier side of CCR model is a CRS model which is used in this study is suggested by Charnes et al. (1978) [2].

$$\text{Min} \quad \sum_{r=1}^{s} U_r \mathscr{Y}_{ro} \tag{2}$$

$$\text{S.t} \sum_{i=1}^{m} V_i \mathscr{X}_{io} = 1$$

$$\sum_{r=1}^{s} U_r \mathscr{Y}_{rj} - \sum_{i=1}^{m} V_i \mathscr{X}_{ij} + d_j = 0, j = 1, \dots, n$$

$$U_{r,} \ V_i \ , d_j \geq 0 \ \forall i, \forall r, \forall j$$

Where:

- $\mathscr{Y}_{rj}$ is output r produced by the jth DMU,
- $\mathscr{X}_{ij}$ is input i used by the jth DMU,
- $\sum_{r=1}^{s} U_r^* \mathscr{Y}_{ro} = \Theta^*_o$ is the efficiency score of DMU_O under evaluated,

Model 2 generated dissimilar weights, resulting in weight choice flexibility and efficient unit multiplicity. These issues create uncommon grounds for efficiency comparisons, inadequate rankings, and difficulties in DMU discrimination.

2.2. Weight-restriction approach

The data were subjected to correlation analysis using E-Views. The results are obtained using two weight restriction approaches: conjoined weight restriction (CWR) and individual weight restriction. The CWR approach joins the input and output weights, and identifies a common set of weights which must transcend the input and output weight dissimilarities using a common bound. This is indicated in Model 3 3 provided by Charnes and Cooper [37] as:

$$\text{Min} = \frac{\sum_j^n = 1dj}{\alpha} \tag{3}$$

$$\text{S.t} \sum_{r=1}^{s} U_r \mathscr{Y}_{rj} - \sum_{i=1}^{m} V_i \mathscr{X}_{ij} + d_j = 0, j = 1, \dots, n$$

$$\alpha \leq V_i \ \leq 1, i = 1, \dots, m$$

$$\alpha \leq U_r \ \leq 1, r = 1, \dots, s$$

$U_{r,}\ V_i\ , \alpha \geq 0\ \forall i, \forall r$

d_j is a slack variable for each unit, and α indicates the minimum bound for input and output weights. Interestingly, the second and third constraints enforce multipliers variation between the minimal bound α and the maximal 1. Moreover, the summation of the variables $\sum_j^n = 1dj$ in the objective function is reduced by d_j while simultaneously maximising. This function simultaneously lessens the sum of all units' derivation by lessening the variable while utilising α maximisation to identify a positive lower bound for common weights amongst all feasible multipliers. Accordingly, (1) strictly positive weights are generated, (2) weight differences are disabled, (3) input and output weights prior to information are no longer required for the bound specification, and (4) the number of efficient units decreases (as opposed to the number obtained by classical CCR).

Model (2) can be linearized using the method of Charnes and Cooper's (1962) method [37]. In other words, weight restriction generates strictly positive weights, and is therefore feasible within the context of the following conjecture or hypothesis:

H2. Model (2) optimally generates strictly positive weights, and is therefore feasible.

Proof. Using Model (2), If DMU_d is a reference for evaluating DMU_0 in

$U_r^d > 0$ and $V_i^d > 0$ (i = 1, …m, r = 1, …s),

Then, U_r^d, V_i^d, $\alpha = Min_{i,r}\ \{U_r^d, V_i^d\}$ proves to be feasible in the model where $\alpha > 0$

In the individual weight-restriction approach, the weight-restriction model is expanded through a linearized model to identify the optimum positive weight for a group of common weights among the feasible multipliers. This individual restriction of input and output weights, coupled with the identified optimum positive weight, simultaneously forbids weight discrepancies (simultaneously). This is encapsulated in Model 4 which was provided by Yekta et al. [40] as follows;

$$\text{Min} = \frac{\sum_j^n = 1dj}{\alpha} \tag{4}$$

$$\text{S.t.} \sum_{r=1}^{s} U_r\ \mathscr{Y}_{rj} - \sum_{i=1}^{m} V_i \mathscr{X}_{ij} + d_j = 0, j = 1, \dots, n$$

$$W_1\ \leq V_i\ \leq 1, i = 1, \dots, m$$

$$W_0\ \leq U_r\ \leq 1, r = 1, \dots, s$$

$$W_1\ \geq \alpha$$

$$W_0\ \geq \alpha$$

$$W_{1,} W_{0,} U_{r,}\ V_i\ , \alpha \geq 0\ \forall i, \forall r$$

d_j in the first constraint represents each DMU's slack. The second constraint forces the input multipliers to vary between bounds $W_{1,}$ and 1: The third constraint forces the output multiplier to vary between $W_{0,}$ and 1 The fourth and fifth constraints specify the lower bound of output weights for the positive variable α. Model (3) acts in a similar way to Model (2), enabling the search into one group of common weights through maximisation of α, minimisation of $\sum_{j=1}^n dj\ = 1$, and additionally, a reduction of the distance between W_0 and W_i . The following alterations of variables would enable an adjustment of Model 4, resulting in a linear version, namely Model 5, provided by Yekta et al. [40] which would contribute to optimal and feasible results:

$$\text{r} = \frac{1}{\alpha}, \widetilde{u_r} = \text{r}\ u_r\ , \check{V}i = rv_i\ , \check{d}j = rd_j\ , \check{w}i = rw_i\ , \check{w}_0\ = rw_0 \tag{5}$$

$$\text{Min} \sum_{j=1}^{n} \check{d}j$$

$$\text{S.t.} \sum_{r=1}^{s} \widetilde{u_r}\ \mathscr{Y}_{rj} - \sum_{i=1}^{m} \widetilde{u_r}\ \mathscr{X}_{ij} + \widetilde{d_j} = 0, j = 1, \dots, n$$

$$w_{\tilde{1}}\ \leq v_{\tilde{1}}\ \leq r, i = 1, \dots, m$$

$$w_{\tilde{0}}\ \leq v_{\tilde{1}}\ \leq r, r = 1, \dots, s$$

$$w_{\tilde{1}}\ \geq 1$$

$$w_{\tilde{0}}\ \geq 1$$

$$w_{\tilde{1}}\ \geq 1$$

$$w_{\tilde{1}}, w_{\tilde{0}}, u_{\tilde{r}}, v_{\tilde{i}}, r \geq 0, \forall i, \forall r$$

Similarly, the feasibility of Model 5 was checked for conformity with the following conjecture:

H3. . Model 4 is feasible and generates the positive weights.

In all the previously mentioned models, the standard models were modified by considering lower and/or upper bounds on the weights. However, in some cases, inputs and outputs can be highly interdependent, and the existence of one depends on the existence of the other. For example, in banking, interest rates cannot include empty accounts. This type of relationship can be reflected by defining an appropriate weight restriction to prevent an abnormally positive weight from a highly dependent input or output in the applied model. In the next section, we modify the conventional CRS DEA model to fit the bank data under validation.

2.3. Data collection

When considering each bank as a decision-making unit within the context of an internet-banking system, various factors can be identified which can be classified as inputs and outputs. Certainly, these factors can be considered as inputs in the context of mobile-banking systems because of their influential roles in shaping and determining the operational environment and banking performance. From a banking perspective, specifically that of Turkish banks, such inputs and outputs were used by the banks and the Central Bank of Turkey to evaluate Internet-banking efficiency as part of evaluation tools or methods other than DEA. From an economic standpoint, these inputs and outputs are the main tools or units used by banks to decide whether to incorporate Internet-banking services. The variables in question, which are listed below, are those which were used by the banks to make a financial decision relevant to incorporating Internet-banking services. The following is a breakdown of why each of these factors can be categorised as an input.

2.3.1. Number of customers

The number of customers is a crucial input because it directly affects the demand for mobile-banking services. The higher the number of customers, the more transactions, engagements, and interactions are likely to occur through mobile-banking platforms. The customer base can influence the overall transaction volume, mobile-banking usage patterns, and the need for scalability of the system to accommodate varying levels of demand.

2.3.2. Number of branches

The number of branches indicates the physical presence of a bank network. This input can influence the accessibility and reach of mobile-banking services. More branches can potentially lead to wider coverage and customer convenience, thus affecting the adoption and utilisation of mobile-banking services.

2.3.3. Number of employees

This workforce contributes directly to the management and maintenance of mobile-banking systems. This is an input, because the efficiency of the system depends on the resources available to manage it. Adequate staffing can ensure timely customer support, system maintenance, and the introduction of new features, all of which can impact user experience.

2.3.4. Average number of banks providing mobile- and internet-banking services

This factor represents the competitive landscape and industry trends in the adoption of digital banking. This is an input because it provides context for the bank's mobile-banking strategy. The average number of banks offering digital services can influence a bank's decision to adopt or enhance its mobile-banking services, leading to differentiation and competitiveness.

2.3.5. Inflation

Inflation affects the costs of the resources, technology, and infrastructure required for mobile-banking operations. This is an input because it affects the operational costs of the system. Inflation can influence pricing strategies, budget allocations, and technology investments, all of which can affect the efficiency and affordability of mobile-banking services.

These factors are inputs because they contribute to the setup, operation, and performance of mobile-banking systems. They have direct and indirect effects on customer engagement, service availability, operational costs, strategic decisions, and competitiveness within the digital banking landscape. By considering these inputs, a mobile-banking system can be optimised to meet customer demands, provide seamless experiences, and adapt to changing market conditions.

However, these factors can be considered outputs in the context of mobile- and Internet-banking systems because of the reflective nature of the system's performance, impact on the economy, and the outcomes they represent. Here, we explain why each of these factors can be categorised as an output.

2.3.6. Non-financial mobile- and internet-banking transactions

Non-financial transactions conducted through mobile and internet banking indicate the extent of customer engagement and usage of the system beyond monetary transactions. Nonfinancial transactions, such as checking balances, account enquiries, information updates, showcase user activity, and interaction with the platform indicate the value and convenience that the system provides to customers.

2.3.7. Number of mobile- and internet-banking transactions

The total number of transactions conducted through mobile and internet banking reflects the level of user adoption and usage. The higher the number of transactions, the more the platform is used for various financial activities, highlighting its effectiveness and popularity among customers.

2.3.8. Volume of mobile- and internet-banking transactions

Transaction volume signifies the magnitude of the financial activities conducted through mobile- and internet-banking system. A larger transaction volume indicates a higher level of digital financial operations, which contribute to efficiency and cost savings for both customers and banking institutions.

2.3.9. Exchange rate of national currency (TL) to USD

The exchange rate represents the value of the national currency compared to the USD and reflects economic conditions. The exchange rate indirectly signifies a country's economic stability and its impact on international trade and investments, which can influence financial behaviour, including the use of digital banking services.

2.3.10. GDP by expenditure in constant prices

The GDP by expenditure indicates a country's economic output at constant prices, reflecting overall economic health. The GDP reflects economic activity, which in turn can influence consumer spending patterns, investment decisions, and, consequently, the use of digital banking services.

In summary, they represent the outcomes and impacts of mobile and Internet systems. They reflect user behaviour, adoption rates, financial activities, and the system's contribution to a broader economic landscape. By monitoring these outputs, banks can gain insights into the success of their digital banking initiatives, level of customer satisfaction, and the system's role in shaping economic indicators.

These inputs also interact with each other. The number of branches and employees might affect customer acquisition, and the presence of multiple banks offering digital services could influence the banks' decisions to adopt these services.

The outputs were interconnected. Transaction volumes contribute to nonfinancial transactions and overall transaction counts. Transaction values and economic factors influence exchange rates and GDP, thereby creating a feedback loop.

Clearly, the inputs directly affect the outputs. For instance, a higher number of customers, branches, and employees may lead to increased transaction volumes and engagement. Market trends (average number of banks offering digital services) and economic conditions (inflation) influence outputs such as transaction values, exchange rates, and GDP. Inputs not only influence outputs, but can also interact with them. A bank's expansion (more branches and employees) can affect its transaction volumes. Additionally, economic conditions (inflation) may affect both transaction behaviour and exchange rates.

In summary, the inputs drive the outputs and interact both individually and collectively. Similarly, outputs are interconnected and influenced by inputs and external factors. The relationships among inputs, outputs, and their interactions create a complex interplay within the banking ecosystem, where various factors shape operational outcomes and performance metrics.

3. Numerical illustrations

The quarterly reports of banks in Turkey for a period of 13 years extending from 2006 to 2018, were published by The Banks Association of Turkey (TBB) and were selected to illustrate the models proposed in this study. The data under investigation were

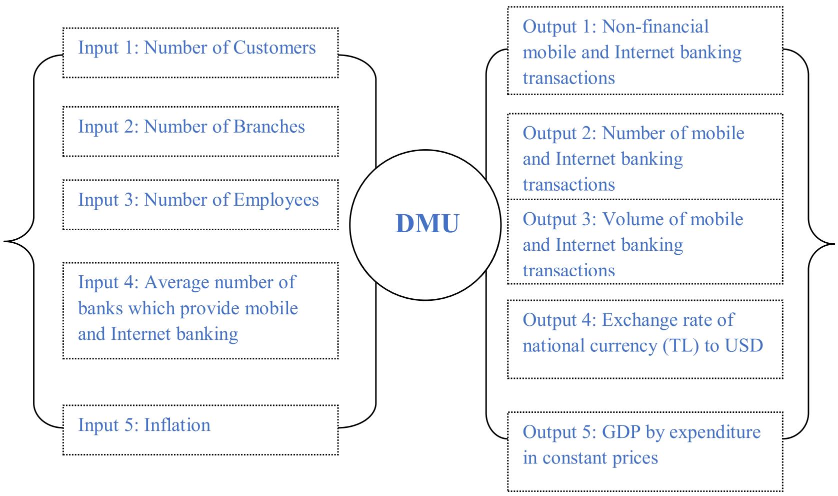

Fig. 1. Inputs and Outputs selected from the TBB Dataset.

obtained from balance sheets and income statements and were subsequently analysed using the PIM-DEA software. A set of five inputs and five outputs were considered (Fig. 1).

52 DMUs are formed. Each DMU represents quarterly data compiled over a period of 13 years (2006–2018; Table 1). The DEA models used in this study were input-oriented.

The customer base of any bank should naturally exceed the number of branches. Bank branches are established with the primary objective of attracting and serving customers. Thus, the presence of bank branches is linked to the presence of customers. As a result, the weight assigned to output 2 should not surpass that of output 1 *($U_2 \leq U_1$)*. This assertion is rooted in the observation that the total volume of funds transferred across all branches of each bank far exceeds the volume of currency transactions conducted through Internet or mobile-banking channels. Consequently, the combined weight of output 4 and input 4 must not exceed the combined weight of output 2 and input 2 *($U_4 + V_4 \leq U_2 + V_2$)*. Fluctuations in GDP significantly impact the scale of a bank's assets, subsequently influencing factors such as the number of potential customers, workforce size, branch network, and even the strategic decision to

Table 1
Input and outputs for 52 Turkish banks.

DMU	INPUTS					OUTPUTS				
	1 (10^6)	2 (10^3)	3 (10^5)	4 (%)	5	1 (10^5)	2 (10^4)	3 (10^5)	4	5 (10^{11})
1	2.68	6.28	1.30	57.45	8.08	2.72	4.40	1.10	1.326	2.43
2	2.83	6.45	1.34	57.45	9.61	2.84	4.82	1.49	1.452	2.50
3	3.08	6.55	1.36	57.45	10.83	2.43	4.81	1.50	1.494	2.45
4	3.37	6.80	1.38	56.52	9.83	2.93	4.87	1.41	1.449	2.53
5	3.46	6.93	1.40	56.52	10.32	1.72	5.47	1.54	1.403	2.62
6	3.58	7.13	1.43	56.52	9.51	1.66	5.72	1.72	1.333	2.58
7	4.02	7.32	1.49	56.52	7.14	1.55	5.65	1.76	1.279	2.54
8	4.27	7.57	1.53	56.52	8.16	1.50	5.82	1.68	1.184	2.67
9	4.59	7.80	1.58	56.52	8.81	1.95	6.28	1.86	1.199	2.78
10	4.80	8.11	1.61	56.52	10.34	1.88	6.08	1.93	1.256	2.64
11	4.98	8.48	1.65	56.52	11.65	1.98	6.66	2.03	1.204	2.56
12	5.17	8.73	1.66	57.78	10.93	2.30	6.72	2.01	1.536	2.53
13	5.42	8.77	1.66	57.78	8.37	2.63	7.15	2.04	1.651	2.43
14	5.59	8.79	1.66	57.78	5.70	2.46	7.81	2.20	1.564	2.48
15	5.75	8.84	1.65	57.78	5.33	2.47	7.89	2.22	1.492	2.52
16	5.97	8.97	1.67	57.78	5.71	2.53	8.22	2.38	1.481	2.57
17	6.01	8.97	1.69	57.78	9.29	2.40	8.09	2.33	1.501	2.61
18	6.15	9.09	1.71	57.78	9.22	2.51	7.82	2.54	1.532	2.68
19	6.35	9.28	1.72	57.78	8.38	2.57	7.75	2.54	1.508	2.74
20	6.69	9.40	1.73	57.78	7.43	2.55	8.56	2.89	1.455	2.84
21	7.23	9.52	1.75	56.82	4.34	3.36	8.95	3.62	1.570	2.91
22	7.47	9.65	1.76	56.82	5.89	3.57	9.12	3.71	1.559	2.98
23	7.83	9.78	1.76	56.82	6.37	4.61	9.42	4.07	1.728	3.06
24	8.61	9.77	1.76	56.82	9.20	5.37	10.19	4.12	1.829	3.09
25	9.33	9.82	1.76	56.82	10.49	5.93	10.58	4.05	1.788	3.08
26	9.47	9.92	1.78	56.82	9.42	5.89	10.89	4.37	1.797	3.13
27	9.86	1.00	1.79	56.82	9.10	6.12	10.45	4.45	1.795	3.18
28	1.06	1.02	1.81	57.78	6.77	6.47	11.45	4.83	1.786	3.22
29	1.12	1.03	1.84	57.78	7.21	7.34	11.70	4.66	1.781	3.34
30	1.14	1.05	1.87	57.78	6.98	7.94	12.13	5.33	1.840	3.43
31	1.20	1.08	1.91	57.78	8.31	8.01	12.21	5.74	1.971	3.47
32	1.24	1.10	1.92	60.00	7.48	8.03	12.70	5.80	2.026	3.50
33	1.31	1.10	1.93	60.00	8.01	8.26	12.99	5.97	2.217	3.59
34	1.32	1.11	1.94	58.70	9.40	8.42	12.99	6.21	2.113	3.53
35	1.37	1.11	0.94	60.87	9.24	8.48	12.78	6.52	2.163	3.62
36	1.43	1.12	1.96	59.57	8.76	7.72	13.74	6.91	2.263	3.67
37	1.54	1.12	1.97	59.57	7.47	8.55	14.04	6.94	2.462	3.74
38	1.57	1.12	1.95	59.57	7.74	7.64	14.08	7.54	2.668	3.80
39	1.62	1.12	1.97	59.57	7.30	7.28	13.17	7.70	2.855	3.84
40	1.74	1.12	1.96	59.57	8.16	7.84	14.13	8.46	2.906	3.89
41	1.85	1.11	1.96	59.57	8.60	7.98	14.33	8.16	2.946	3.91
42	1.83	1.10	1.95	59.57	6.93	8.23	14.50	9.31	2.897	3.95
43	1.87	1.09	1.93	59.57	8.04	7.45	13.52	9.03	2.961	3.84
44	2.04	1.07	1.92	57.45	7.57	8.63	14.90	10.78	3.286	4.05
45	1.38	1.07	1.92	57.45	10.21	9.22	14.05	10.72	3.696	4.11
46	1.33	1.07	1.91	58.70	11.50	7.89	1.20	11.2	3.580	4.19
47	1.28	1.06	1.90	58.70	10.56	8.31	12.64	11.16	3.515	4.26
48	1.31	1.05	1.89	57.45	12.27	8.56	13.68	13.19	3.803	4.36
49	1.35	1.05	1.88	57.45	10.28	9.03	14.00	13.06	3.817	4.40
50	1.27	1.05	1.89	57.45	12.79	7.78	13.00	13.79	4.373	4.41
51	1.26	1.05	1.88	57.45	19.44	8.50	12.74	14.62	5.665	4.34
52	1.25	1.04	1.87	57.45	22.37	7.99	13.30	14.22	5.504	4.23

Source: Bank Association of Turkey (https://www.tbb.org.tr/tr/bankacilik/banka-ve-sektor-bilgileri/banka-bilgileri/bankalar/64).

embrace or eschew Internet banking. Therefore, it follows that the weight attributed to input 4 should not surpass that of output 5 $(V_4 \leq U_5)$. The ability of a bank to effectively manage the quantity and magnitude of Internet-banking transactions often contributes to fluctuations in inflation rates, whether upward or downward. Consequently, prominence of the third output should be greater than that of the fifth input $(V_5 \leq U_3)$. Lastly, owing to the direct impact of currency exchange rates on the variability of GDP, the significance of output 4 should inherently exceed that of output 5 $(U_5 \leq U_4)$.

The proposed model which is combined with the abovementioned five new restrictions for TBB data based on defined inputs and outputs, is given below.

$$Max \sum_{r=1}^{5} U_r \mathscr{Y}_{ro} \tag{6}$$

$$s.t \sum_{i=1}^{5} V_i \mathscr{X}_{io} = 1$$

$$\sum_{r=1}^{5} U_r \mathscr{Y}_{rj} - \sum_{i=1}^{5} V_i \mathscr{X}_{ij} \leq 0 \quad j = 1, \ldots., 52$$

$$U_2 \leq U_1$$

$$U_4 + V_4 \leq U_2 + V_2$$

$$V_4 \leq U_5$$

$$V_5 \leq U_3$$

$$U_5 \leq U_4$$

$$U_r \geq 0 \quad r = 1, \ldots., 5$$

$$V_i \geq 0 \quad i = 1, \ldots.., 5$$

The addition of a new constraint to a linear programming problem may render it unfeasible. The newly added weight restrictions in Model 6 are considered in such a manner that this model is feasible for all 52 under-evaluated DMUS. The new model is expected to modify the biased efficiency values achieved without the above restrictions. Table 1 lists the input and output values of the under-evaluated DMUs. Both the conventional and modified CRS DEA models were applied to these data.

4. Limitations, results, and discussions

Proposing a new weight-restricted CCR model for evaluating the internet-banking efficiency of Turkish banks (Model 6) may have some possible limitations which could be addressed as follows.

In addition to the choice of inputs and outputs, data availability and quality are the most important limitations. Selecting the right set of input and output variables is crucial for accurately measuring the efficiency. If important or irrelevant variables are overlooked, the validity of the results would be affected. The accuracy and availability of the data used for the input and output variables can significantly affect the results of Model 6. If the data are incomplete, inconsistent, or inaccurately measured, biased efficiency scores can be obtained.

Although the weighted approach allows for a more nuanced analysis, determining the appropriate weights can be challenging. The weights assigned to the inputs and outputs can influence the final efficiency scores, and different weighting schemes may yield different results. Moreover, assigning weights may involve subjective judgment, which can introduce bias into the model.

The linearity and constant returns to scale assumptions of the proposed model may not perfectly align with the real-world behaviour of Turkish banks' internet-banking operations. The model assumes that all banks are similar and that their activities can be measured and compared using the same set of inputs and outputs. However, the diversity of banking operations could lead to variations in results.

Obviously, Internet banking is evolving rapidly. The model may not capture the dynamic changes and advancements that occur constantly in technology and customer preferences. External factors such as regulatory changes, economic conditions, and market competition can impact a bank's efficiency but may not be accounted for in the model.

A model efficiency profile generated with the outcome of the conventional DEA and proposed DEA weight-restriction models is presented in Fig. 2. The sensitivity of the proposed model was observed at intersections in the profile. The points of intersection indicate that similar frontiers are obtained by both the conventional DEA and proposed restricted model. The propensity of the proposed model can be explicitly deciphered through two distinct features/distributions: intersections and discrepancies in the profile. There is a wide distribution between the two models for DMUs 1–20 shows a large discrepancy or deficiency orchestrated by the shortcomings associated with the convectional DEA CCR model. Among these DMUs (i.e. 1-20), conventional CCR marks DMUs 1, 2, 5, 6, 9, 14, 15, 16 as 100 % efficient. By contrast, the proposed weight-restriction model revealed the inefficiency of these DMUs.

A similar wide discrepancy was observed within DMUs 23–29, where 24, 45, and 26 were promoted as efficient but were otherwise inefficient when applying the proposed model. Conversely, close proximity was observed within DMUs 31–100. This reveals a region within the given inputs where the discrepancy is minimal. Overall, the conventional CCR model showed that 23 of the 52 are efficient. However, the proposed model reveals only 11 efficient DMUs. This result indicates that the null hypothesis cannot be rejected, and that there is a need to support the standard CCR model with the adjusted version.

The proposed restricted weight reduces the bloated values of efficiencies produced by the standard CCR model; therefore, a feasible DMU must be efficient for both models. Table 2 lists the feasible DMUs obtained by using the proposed model. This feasibility was based on 100 % efficiency. Nonetheless, other DMUs can also be considered on the frontier. Thus, an efficiency range of 90–100 % was considered; with 90–94 % as weakly efficient points (WEP), 95–98 as efficient points (EP) and 98–100 % as strongly efficient points (SEP). The reduction effect of the proposed model on these DMUs could be viewed as insignificant, although the efficiency values were reduced. By this partitioning, 23 DMUs are on the frontier, with 12 SEP, 7 WEP, and 4 EP. This classification is necessary to provide a panoramic view of the propensity of efficient DMUs. These results agree with those of previous studies [4,5].

The proposed model enables the CCR model to push inefficient DMUs by changing their inputs or outputs. This view aims to achieve higher productivity through the technical efficiency of DMUs. In Appendix Table A1, Targets of CCR, the target gives the changes obtained in inputs and/or outputs due to imposing weight-restrictions in the model. Increase or decrease in input-output values extracted from TBB and target values. Positive impacts were observed on outputs, whereas negative impacts were observed on inputs. This is one of the benefits of the proposed model, and is in line with the aims of this study. Therefore, it is convenient to state that the negative effects do not translate into poor effects. This indicates that the proposed model has a diminishing effect on the inputs.

The proposed model envisages or anticipates the negative effects on inputs and positive effects on outputs. The extent of the impact was measured by the degree of positive and negative gains (Fig. 3). High positive and negative values indicate high and low impacts, respectively. Conversely, low positive and negative values indicate low and high impacts, respectively. Therefore, as depicted in Fig. 3, output 3 is the most impacted one with a gain of 10.92 % while output 1 is the least impacted with a gain of −1.01 %. This implies that 1.01 %, 5.80 %, 10.92 %, 2.44 %, and 3.89 % contributed to outputs 1, 2, 3, 4, and 5, respectively. Similarly, input 1 is the most impacted one with a gain of −3.22 % while input 2 is the least impacted with a gain of −1.88 %. This means the proposed model diminished inputs 1, 2, 3, 4, and 5 by −3.89 %, −1.88 %, −3.02 %, −3.22, and −1.92 respectively.

The gain was translated to provide the anticipated improvement, as shown in Fig. 4. Anticipated improvement of inputs 1, 2, 3, 4 and 5, is by −0.82, −0.93, −1.39, −1.43, and −0.48 respectively, with input 4 the most to be improved. Outputs 1, 2, 3, 4, and 5 show anticipated improvements of 0.83, 1.18, 2.16, 0.85, and 1.19, respectively, with Output 3 expected to be the most improved. The anticipated improvement shows the extent to which outputs are expected to be improved by the positive impact of the restricted weight application. Similarly, the anticipated improvement in the inputs indicates the extent to which they can be improved by the negative impact of the proposed model.

Applying the standard DEA to mobile-internet-banking data in this study, a number of zero values of input-output weights for DMUs have resulted in defiance of their positions on the frontier. However, by applying cross-efficiency, peer appraisal (as presented in Table 1) reveals a discrimination impact. Although DMUs overrated by CCR model are connected without weight restrictions, over 97 % of them have been impacted (except DMU 49 which shows huge superiority). It can also be deduced from the cross-efficiency matrix (Table A2, Cross Efficiencies of CCR) that 90 % the DMU's have high efficiency scores compared to the 10 %, with efficiency values

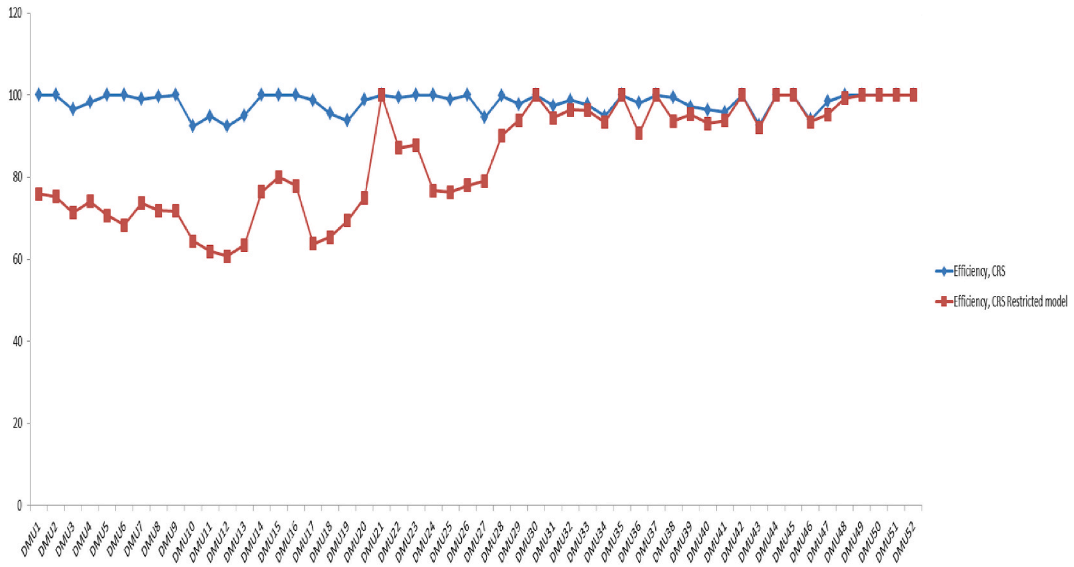

Fig. 2. Models Profile showing the Efficiency of the DMUs for both Conventional CRS DEA and CRS weight-restricted DEA models.

Table 2
Feasible DMUs through reduced discrepancy, partitioning and cross-efficiency.

Name	Efficiency, CRS (%)	Efficiency, CRS Restricted model (%)	Discrepancy	Deduction	Partitioning	Peer Appraisal e_k (%)	Order of Importance (Ranking)
DMU1	100	75.9	Reduced	Infeasible	–	87.06	
DMU2	100	75.22	Reduced	Infeasible	–	87.96	
DMU3	96.45	71.35	Reduced	Infeasible	–	82.84	
DMU4	98.29	74.05	Reduced	Infeasible	–	84.36	
DMU5	100	70.66	Reduced	Infeasible	–	87.35	
DMU6	100	68.35	Reduced	Infeasible	–	88.32	
DMU7	99.06	73.72	Reduced	Infeasible	–	86.45	
DMU8	99.63	71.85	Reduced	Infeasible	–	86.46	
DMU9	100	71.76	Reduced	Infeasible	–	87.91	
DMU10	92.4	64.41	Reduced	Infeasible	–	80.80	
DMU11	94.83	61.87	Reduced	Infeasible	–	80.44	
DMU12	92.45	60.75	Reduced	Infeasible	–	79.79	
DMU13	95.07	63.49	Reduced	Infeasible	–	82.67	
DMU14	100	76.47	Reduced	Infeasible	–	89.69	
DMU15	100	80.06	Reduced	Infeasible	–	90.27	
DMU16	100	77.91	Reduced	Infeasible	–	91.05	
DMU17	98.78	63.79	Reduced	Infeasible	–	85.79	
DMU18	95.52	65.4	Reduced	Infeasible	–	84.46	
DMU19	93.75	69.41	Reduced	Infeasible	–	84.65	
DMU20	98.84	74.97	Reduced	Infeasible	–	89.66	
DMU21	**100**	**100**	**Intersected**	**Feasible**	**SEP**	**95.31**	**4**
DMU22	99.4	87.15	Reduced	Infeasible	–	93.34	
DMU23	100	87.84	Reduced	Infeasible	–	94.19	
DMU24	100	76.73	Reduced	Infeasible	–	91.38	
DMU25	98.91	76.29	Reduced	Infeasible	–	88.53	
DMU26	100	78.02	Reduced	Infeasible	–	90.92	
DMU27	94.59	79.09	Reduced	Infeasible	–	88.11	
DMU28	99.84	90.15	Reduced	Infeasible	–	92.39	
DMU29	97.77	93.9	Reduced	Infeasible	WEP	91.72	
DMU30	**100**	**100**	**Intersected**	**Feasible**	**SEP**	**93.87**	**5**
DMU31	97.39	94.45	Reduced	Infeasible	WEP	90.84	
DMU32	98.78	96.36	Reduced	Infeasible	EP	91.23	
DMU33	97.66	96.33	Reduced	Infeasible	EP	90.31	
DMU34	94.83	93.41	Reduced	Infeasible	WEP	88.39	
DMU35	**100**	**100**	**Intersected**	**Feasible**	**SEP**	**90.84**	**7**
DMU36	98.01	90.68	Reduced	Infeasible	–	88.61	
DMU37	**100**	**100**	**Intersected**	**Feasible**	**SEP**	**88.42**	**9**
DMU38	99.45	93.68	Reduced	Infeasible	WEP	87.65	
DMU39	97.22	95.28	Reduced	Infeasible	EP	83.60	
DMU40	96.35	93.1	Reduced	Infeasible	WEP	83.33	
DMU41	95.86	93.76	Reduced	Infeasible	WEP	81.06	
DMU42	**100**	**100**	**Intersected**	**Feasible**	**SEP**	**84.13**	**10**
DMU43	92.73	91.98	Reduced	Infeasible	–	77.95	
DMU44	**100**	**100**	**Intersected**	**Feasible**	**SEP**	**81.36**	**11**
DMU45	**100**	**100**	**Intersected**	**Feasible**	**SEP**	**96.58**	**3**
DMU46	93.96	93.43	Reduced	Infeasible	WEP	49.43	
DMU47	98.44	95.26	Reduced	Infeasible	EP	95.54	
DMU48	100	99.25	Reduced	Infeasible	SEP	98.30	
DMU49	**100**	**100**	**Intersected**	**Feasible**	**SEP**	**100**	**1**
DMU50	**100**	**100**	**Intersected**	**Feasible**	**SEP**	**97.10**	**2**
DMU51	**100**	**100**	**Intersected**	**Feasible**	**SEP**	**91.00**	**6**
DMU52	**100**	**100**	**Intersected**	**Feasible**	**SEP**	**89.95**	**8**

below 50 of the standard CCR model. Interestingly, integrating the proposed restriction model with peer appraisal cross-efficiency provides a glimpse into the order of importance (ranking) of feasible and efficient DMUs. The rankings of the 11 feasible and efficient DMUs are as follows: DMU49, DMU50, DMU45, DMU21, DMU30, DMU51, DMU35, DMU52, DMU37, DMU42, and DMU44. Therefore, it is asserted that DMU 49 is promoted as the most important DMU, having been ranked first and best among the 11 feasible, intersected, and strongly efficient points (SEP) by the proposed models.

Another point which is worth mentioning, quarterly efficiency score as per Table 2 are volatile, namely those obtained from standard model application in comparison to modified model application whose readings showed less volatility. For example, DMUs 9 to 12, which represent the quarters of 2008 (the global economic crisis year), gave volatile results in the standard model application, showing inconsistent readings, which is somewhat illogical. This is due to the expectation that the model should convey varying results. From an economic perspective, some quarterly results should be predictable in advance, as they are related to economic,

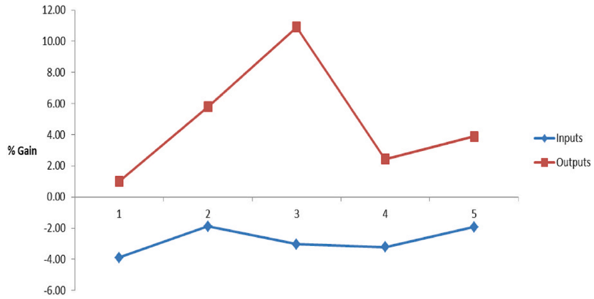

Fig. 3. Percentage Gain on Input-Output Showing the Proposed model's Impact.

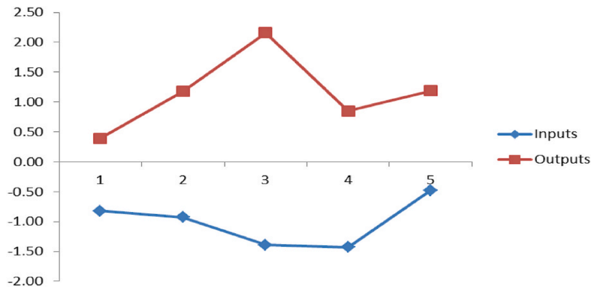

Fig. 4. Anticipated improvement due to the proposed Model's weights restriction.

political, environmental, and geographical conditions both locally, regionally, and globally. The modified model overcomes this shortcoming, which is evident by comparing the reading of the same quarterly data which started inefficiently and showed a consistent and gradual efficiency score decrease.

5. Conclusions

The proposed restricted weighted model offers a nuanced and comprehensive approach for assessing efficiency. By incorporating weight restrictions, the model accurately accounts for the relative importance of the various input and output factors. This precision ensures that the evaluation reflects the intricate dynamics of the internet-banking sector. This aids in identifying the most influential factors contributing to the efficiency of internet-banking operations. By understanding the significance of each input and output, banks can allocate resources more effectively, thereby enhancing their overall performance. The insights derived from the evaluation guide strategic decision-making. Banks can strategically focus on areas where improvements are most impactful, such as enhancing customer engagement, optimising branch networks, and improving technology infrastructure. This targeted approach could lead to improved outcomes.

Maintaining a competitive edge is crucial in a rapidly evolving digital landscape. A comprehensive evaluation allows banks to benchmark their performance against industry leaders, identify areas for improvement, and implement measures to enhance market standing. Efficiency in internet banking translates into a better customer experience. By evaluating and enhancing their efficiency, banks can provide seamless online services, quicker response times, and user-friendly interfaces, thereby contributing to customer satisfaction and retention. A robust internet-banking sector can have a positive impact on the broader economy. It encourages cashless transactions, fosters financial inclusion, and promotes economic growth by facilitating efficient fund transfers and financial activities.

The history of the Turkish banking sector has been marked by continuous growth and resilience, driven by an ongoing commitment to efficiency improvements, especially through the adoption of advanced fintech solutions such as mobile internet banking. This study investigates the efficiency of Turkey's mobile-internet-banking landscape using applied to two decades of data from pioneering service-oriented banks, which encompass approximately half of the country's banking institutions. Notably, these banks, among the largest and most active banks in the country, have demonstrated a noticeable increase in efficiency, subsequently exerting a positive influence on the overall performance of the entire sector. A particular highlight is the last 12-month period under scrutiny, showing the highest efficiency scores, which is attributed to strategic enhancements and the vigorous promotion of mobile internet banking. Despite the significance of both analytical tools and the impact of internet banking, the application of DEA to Turkey's mobile-internet-banking realm remains relatively unexplored. This is in contrast to the prevailing research which often focuses on digital banking from ownership or comparative country perspectives, employing diverse analytical methodologies.

This study encompasses three interrelated categories of inputs and outputs, covering elements related to banks, those specifically pertinent to mobile internet banking, and those generally linked to the economy. These interconnections elucidate the efficiency of services, which, in turn, reverberate through the banking sector and provider banks, ultimately contributing to sustainable economic growth. Examination of the proposed model highlights Input 4 (average number of banks offering mobile internet banking) as the most significantly improved, followed by Input 3 (number of employees), Input 2 (number of branches), Input 1 (number of customers), and Input 5 (inflation). Correspondingly, output 3 (volume of mobile internet banking) exhibits substantial improvement, followed by output 5 (GDP by expenditure at constant prices), output 2 (number of mobile- and internet-banking transactions), output 4 (exchange rate of national currency to USD), and output 1 (non-financial mobile internet banking). The independence of these variables underscores the importance of members of the Turkish Banking Association (TBB) prioritising inputs and outputs in this sequence to optimise mobile-internet-banking operations. However, a holistic understanding of prioritisation orders necessitates the inclusion of additional variables.

Moreover, the study presents several indications for the escalating efficiency of mobile internet banking: (1) the successful adoption by banks and acceptance by customers has notably diminished bank branch expenses; (2) the solution attests to the full potential of Turkey's digital transformation, demonstrated by the government's collection of fees through this channel, contributing to a predominantly cashless society; (3) the government's reduced expenditure on replacing damaged currency bills, reallocating these funds towards growth initiatives; and (4) the banking sector's robustness in times of crises, underscoring its role as the cornerstone of the economy. A significant aspect of this study is the introduction of a weight-restriction approach as an alternative to conventional DEA models, which facilitates the derivation of shared weights which enable efficient DMU ranking. Future research could examine the impact of self-assessment cross-efficiency on the outcomes of this study.

In summary, the proposed model adopts a comprehensive approach which encompasses all relevant input and output variables, resulting in strictly positive weights and a reduced count of inflated efficient units along with their scores. Nonetheless, ongoing research is poised to unveil the optimal prioritisation of input-output variables. To fortify the resilience of the model, it is imperative to integrate the synergies and interdependencies among the inputs and outputs. The inclusion of other DEA models, such as benevolent DEA, DEA games, and DEA-integrated models, such as DAE-Taguchi and DEA bootstrap, would undoubtedly contribute to the robustness of the proposed framework.

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Data availability statement

Data will be made available on request.

The use of generative AI and AI-assisted technologies

The authors also declare that there are no artificial intelligence (AI)- or AI-assisted technologies for writing this research or producing images or figures.

CRediT authorship contribution statement

Abdullah A. Kraidi: Writing – review & editing, Writing – original draft. **Sahand Daneshvar:** Supervision, Formal analysis. **Kehinde Adewale Adesina:** Supervision, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.heliyon.2024.e31008.

Appendix

Table A1
CCR model Target inputs and outputs of the CCR model

Name	In1	In1 T	Index2 Value	Index2 Target	Index3 Value	Index3 Target	Index4 Value	Index4 Target	Index5 Value	Index5 Target	Index6 Value	Index6 Target	Index7 Value	Index7 Target	Index8 Value	Index8 Target	Index9 Value	Index9 Target	Index10 Value	Index10 Target
DMU1	0.55	0.55	0.23	0.23	0.07	0.07	0.3	0.3	0.3	0.3	0.36	0.36	0.94	0.94	0.66	0.66	0.56	0.56	0.13	0.13
DMU2	0.57	0.57	0.26	0.26	0.1	0.1	0.32	0.32	0.31	0.31	0.43	0.43	0.94	0.94	0.68	0.68	0.57	0.57	0.14	0.14
DMU3	0.56	0.56	0.26	0.26	0.1	0.12	0.32	0.32	0.26	0.31	0.48	0.42	0.94	0.91	0.69	0.66	0.58	0.56	0.15	0.15
DMU4	0.57	0.57	0.26	0.27	0.1	0.14	0.33	0.34	0.32	0.32	0.44	0.43	0.93	0.91	0.7	0.67	0.61	0.58	0.17	0.16
DMU5	0.59	0.59	0.25	0.25	0.11	0.11	0.37	0.37	0.19	0.19	0.46	0.46	0.93	0.93	0.71	0.71	0.62	0.62	0.17	0.17
DMU6	0.58	0.58	0.24	0.24	0.12	0.12	0.38	0.38	0.18	0.18	0.43	0.43	0.93	0.93	0.72	0.72	0.63	0.63	0.18	0.18
DMU7	0.58	0.58	0.23	0.24	0.12	0.13	0.38	0.38	0.17	0.31	0.32	0.32	0.93	0.92	0.76	0.71	0.65	0.63	0.2	0.2
DMU8	0.61	0.61	0.21	0.25	0.11	0.15	0.39	0.39	0.16	0.3	0.36	0.36	0.93	0.93	0.78	0.74	0.67	0.64	0.21	0.21
DMU9	0.63	0.63	0.21	0.21	0.13	0.13	0.42	0.42	0.21	0.21	0.39	0.39	0.93	0.93	0.8	0.8	0.69	0.69	0.22	0.22
DMU10	0.6	0.6	0.22	0.28	0.13	0.2	0.41	0.41	0.2	0.26	0.46	0.43	0.93	0.86	0.82	0.69	0.72	0.61	0.24	0.22
DMU11	0.58	0.58	0.21	0.29	0.14	0.19	0.45	0.45	0.21	0.25	0.52	0.4	0.93	0.88	0.84	0.73	0.75	0.66	0.24	0.23
DMU12	0.57	0.57	0.27	0.28	0.14	0.19	0.45	0.45	0.25	0.25	0.49	0.38	0.95	0.88	0.84	0.73	0.78	0.67	0.25	0.23
DMU13	0.55	0.56	0.29	0.29	0.14	0.17	0.48	0.48	0.28	0.28	0.37	0.31	0.95	0.9	0.84	0.78	0.78	0.71	0.27	0.25
DMU14	0.56	0.56	0.28	0.28	0.15	0.15	0.52	0.52	0.27	0.27	0.25	0.25	0.95	0.95	0.84	0.84	0.78	0.78	0.27	0.27
DMU15	0.57	0.57	0.26	0.26	0.15	0.15	0.53	0.53	0.27	0.27	0.24	0.24	0.95	0.95	0.84	0.84	0.79	0.79	0.28	0.28
DMU16	0.58	0.58	0.26	0.26	0.16	0.16	0.55	0.55	0.27	0.27	0.26	0.26	0.95	0.95	0.85	0.85	0.8	0.8	0.29	0.29
DMU17	0.59	0.59	0.26	0.29	0.16	0.19	0.54	0.54	0.26	0.29	0.42	0.3	0.95	0.94	0.86	0.83	0.8	0.78	0.29	0.29
DMU18	0.61	0.61	0.27	0.33	0.17	0.24	0.52	0.52	0.27	0.31	0.41	0.39	0.95	0.91	0.87	0.79	0.81	0.74	0.3	0.29
DMU19	0.62	0.62	0.27	0.31	0.17	0.26	0.52	0.52	0.28	0.34	0.37	0.35	0.95	0.89	0.87	0.77	0.83	0.71	0.31	0.29
DMU20	0.64	0.64	0.26	0.32	0.2	0.27	0.57	0.57	0.28	0.36	0.33	0.33	0.95	0.94	0.88	0.84	0.84	0.78	0.33	0.32
DMU21	0.66	0.66	0.28	0.28	0.25	0.25	0.6	0.6	0.36	0.36	0.19	0.19	0.93	0.93	0.89	0.89	0.85	0.85	0.35	0.35
DMU22	0.68	0.68	0.28	0.32	0.25	0.3	0.61	0.61	0.39	0.41	0.26	0.26	0.93	0.93	0.9	0.86	0.86	0.82	0.37	0.36
DMU23	0.69	0.69	0.3	0.3	0.28	0.28	0.63	0.63	0.5	0.5	0.28	0.28	0.93	0.93	0.89	0.89	0.87	0.87	0.38	0.38
DMU24	0.7	0.7	0.32	0.32	0.28	0.28	0.68	0.68	0.58	0.58	0.41	0.41	0.93	0.93	0.9	0.9	0.87	0.87	0.42	0.42
DMU25	0.7	0.75	0.32	0.39	0.28	0.39	0.71	0.71	0.64	0.64	0.47	0.42	0.93	0.92	0.9	0.89	0.87	0.86	0.46	0.45
DMU26	0.71	0.71	0.32	0.32	0.3	0.3	0.73	0.73	0.64	0.64	0.42	0.42	0.93	0.93	0.91	0.91	0.88	0.88	0.46	0.46
DMU27	0.72	0.73	0.32	0.38	0.3	0.41	0.7	0.7	0.66	0.66	0.41	0.38	0.93	0.88	0.91	0.85	0.89	0.83	0.48	0.46
DMU28	0.73	0.78	0.32	0.37	0.33	0.42	0.77	0.77	0.7	0.73	0.3	0.3	0.95	0.94	0.92	0.92	0.9	0.9	0.52	0.52
DMU29	0.76	0.77	0.31	0.36	0.32	0.42	0.79	0.79	0.8	0.8	0.32	0.32	0.95	0.92	0.93	0.91	0.92	0.9	0.55	0.54
DMU30	0.78	0.78	0.32	0.32	0.36	0.36	0.81	0.81	0.86	0.86	0.31	0.31	0.95	0.95	0.95	0.95	0.93	0.93	0.56	0.56
DMU31	0.79	0.79	0.35	0.38	0.39	0.43	0.82	0.82	0.87	0.87	0.37	0.33	0.95	0.92	0.97	0.93	0.96	0.91	0.59	0.57
DMU32	0.79	0.81	0.36	0.36	0.4	0.41	0.85	0.85	0.87	0.89	0.33	0.33	0.99	0.97	0.98	0.96	0.98	0.96	0.61	0.6
DMU33	0.81	0.84	0.39	0.42	0.41	0.49	0.87	0.87	0.9	0.91	0.36	0.35	0.99	0.96	0.98	0.96	0.98	0.95	0.64	0.63
DMU34	0.8	0.89	0.37	0.54	0.42	0.68	0.87	0.87	0.91	0.91	0.42	0.4	0.96	0.91	0.98	0.93	0.99	0.9	0.65	0.61
DMU35	0.82	0.82	0.38	0.38	0.45	0.45	0.86	0.86	0.92	0.92	0.41	0.41	1	1	0.48	0.48	0.99	0.99	0.67	0.67
DMU36	0.83	0.9	0.4	0.52	0.47	0.64	0.92	0.92	0.84	0.94	0.39	0.38	0.98	0.96	0.99	0.97	0.99	0.96	0.7	0.69
DMU37	0.85	0.85	0.43	0.43	0.47	0.47	0.94	0.94	0.93	0.93	0.33	0.33	0.98	0.98	1	1	1	1	0.76	0.76
DMU38	0.86	0.87	0.47	0.47	0.52	0.54	0.94	0.94	0.83	0.93	0.35	0.34	0.98	0.97	1	0.99	1	0.98	0.77	0.76
DMU39	0.87	0.87	0.5	0.5	0.53	0.62	0.88	0.9	0.79	0.82	0.33	0.32	0.98	0.95	1	0.95	1	0.94	0.79	0.77
DMU40	0.88	0.89	0.51	0.52	0.58	0.64	0.95	0.95	0.85	0.92	0.36	0.35	0.98	0.94	1	0.96	0.99	0.95	0.85	0.82
DMU41	0.89	0.92	0.52	0.58	0.56	0.74	0.97	0.97	0.86	0.94	0.38	0.37	0.98	0.94	0.99	0.95	0.99	0.94	0.91	0.87
DMU42	0.9	0.9	0.51	0.51	0.64	0.64	0.97	0.97	0.89	0.89	0.31	0.31	0.98	0.98	0.99	0.99	0.98	0.98	0.9	0.9
DMU43	0.87	0.87	0.52	0.54	0.62	0.69	0.91	0.92	0.81	0.88	0.36	0.33	0.98	0.9	0.98	0.91	0.97	0.9	0.92	0.85
DMU44	0.92	0.92	0.58	0.58	0.74	0.74	1	1	0.94	0.94	0.34	0.34	0.94	0.94	0.97	0.97	0.96	0.96	1	1
DMU45	0.93	0.93	0.65	0.65	0.73	0.73	0.94	0.94	1	1	0.46	0.46	0.94	0.94	0.97	0.97	0.95	0.95	0.68	0.68
DMU46	0.95	0.95	0.63	0.68	0.77	0.86	0.08	0.86	0.86	0.88	0.51	0.48	0.96	0.91	0.97	0.91	0.95	0.89	0.65	0.61
DMU47	0.97	0.97	0.62	0.64	0.76	0.82	0.85	0.88	0.9	0.9	0.47	0.46	0.96	0.95	0.97	0.94	0.94	0.91	0.63	0.62
DMU48	0.99	0.99	0.67	0.67	0.9	0.9	0.92	0.92	0.93	0.93	0.55	0.55	0.94	0.94	0.96	0.96	0.93	0.93	0.64	0.64

(*continued on next page*)

Table A1 (*continued*)

Name	In1	In1 T	Index2 Value	Index2 Target	Index3 Value	Index3 Target	Index4 Value	Index4 Target	Index5 Value	Index5 Target	Index6 Value	Index6 Target	Index7 Value	Index7 Target	Index8 Value	Index8 Target	Index9 Value	Index9 Target	Index10 Value	Index10 Target
DMU49	1	1	0.67	0.67	0.89	0.89	0.94	0.94	0.98	0.98	0.46	0.46	0.94	0.94	0.96	0.96	0.93	0.93	0.66	0.66
DMU50	1	1	0.77	0.77	0.94	0.94	0.87	0.87	0.84	0.84	0.57	0.57	0.94	0.94	0.96	0.96	0.93	0.93	0.62	0.62
DMU51	0.98	0.98	1	1	1	1	0.86	0.86	0.92	0.92	0.87	0.87	0.94	0.94	0.96	0.96	0.93	0.93	0.62	0.62
DMU52	0.96	0.96	0.97	0.97	0.97	0.97	0.89	0.89	0.87	0.87	1	1	0.94	0.94	0.95	0.95	0.93	0.93	0.61	0.61

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