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博 士 論 文

一般化運動鏈之數目合成

On the Number Synthesis of Generalized Kinematic Chains

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本論文業經審查及口試合格特此證明

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中 華 民 國 103 年 6 月 6 日

摘要 (Abstract in Chinese)

本研究之目的是針對單接頭的一般化運動鏈、運動鏈、及呆鏈進行數目合成。基於圖學理論和運動鏈矩陣，提出四種確認演算法來判斷運動鏈的類型，以及三種列舉演算法來合成各種類型的運動鏈。並根據所提出的演算法發展電腦軟體，合成與繪製各種一般化運動鏈、運動鏈、及呆鏈圖譜，作為機構概念設計或創意性機構設計之用。

首先介紹機構運動鏈與圖學理論相關術語與定義，舉例說明其涵義，據此針對單接頭運動鏈之數目合成方法，做詳細的文獻回顧與探討。接著，對本研究所用之基本理論與概念如連桿類配、縮桿類配、及多接頭連桿鄰接矩陣合成、連桿鄰接矩陣合成等舉例說明並探究。以機構運動鏈文獻與基本理論為基礎，提出確認分離連桿演算法來刪除分離連桿運動鏈，以及確認庫拉托夫斯基(Kuratowski)圖形演算法是來刪除非平面圖形，並提出一種繪圖演算法以進行繪製機構運動鏈與圖。基於以上述的演算法，本研究提出一種列舉演算法可合成與繪製所有一般化運動鏈圖譜。再者，利用三桿基本呆鏈與退化運動鏈演算法，檢驗運動鏈是否為退化運動鏈，並提出運動鏈與呆鏈列舉演算法，合成出各種運動鏈與呆鏈圖譜。最後，根據以上所提出的演算法，發展出一套電腦軟體供機構設計者使用，可產生各種連桿與接頭數目之非同構單接頭機構運動鏈圖譜。

藉由本研究所提出的演算法，不只可精確地獲得所有機構運動鏈圖譜，且可簡化機構概念設計程序。

關鍵字：機構運動鏈、數目合成、構造合成、圖學理論、機構概念設計

Abstract

This work provides four checking algorithms and three enumeration algorithms based on graph theory and kinematic matrices for the number synthesis and the construction of the necessary atlases of generalized kinematic chains, kinematic chains, and rigid chains for the conceptual design of mechanisms. Furthermore, a computer program is developed to synthesize and sketch the various atlases of generalized kinematic chains, kinematic chains, and rigid chains with simple joints based on the proposed algorithms.

Firstly, the fundamental definitions and terminology regarding kinematic chains of mechanisms and graph theory are introduced. The literature survey on the number synthesis of various kinematic chains with simple joints is reviewed and studied. Then, the basic theories and concepts such as link assortments, contracted link assortments, and the synthesis of multiple link adjacency matrices and link adjacency matrices are presented and illustrated with examples. The algorithm for checking cut-links is provided for the elimination of chains with cut-links. The algorithm for checking Kuratowski graphs is proposed to delete non-planar graphs. And, a sketching algorithm is developed to sketch kinematic graphs or chains based on the basic contracted graphs. According to these algorithms, an enumeration algorithm is proposed for the construction of generalized kinematic chains. Furthermore, two algorithms for checking three-bar basic rigid chains and degenerate kinematic chains are put forward to discard the degenerate kinematic chains. Then, two enumeration algorithms are proposed for the synthesis of kinematic chains and rigid chains. Finally, a computer program based on the proposed algorithms is developed to synthesize and sketch various non-isomorphic generalized kinematic chains, kinematic chains, and rigid chains with simple joints.

The proposed algorithms and computer programs not only can precisely obtain various atlases of generalized kinematic chains, kinematic chains, and rigid chains, but also can simplify the methodology for the conceptual design of mechanisms.

Keywords: kinematic chains of mechanisms, number synthesis, structural synthesis, graph theory, conceptual design of mechanisms.

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Nomenclature

Symbols	**Definitions**
A_L	Link assortment
A_{ML}	Multiple link assortment
A_{CL}	Contracted link assortment
CLAM	Contracted link adjacency matrix
DA	Dyad amplification operation
DOF	Degrees of freedom
Dim	Dimension of multiple link adjacency matrix
$deg(V)$ or dV	Degrees of vertex
E	Edge
e_{ij}	Element of matrix
G	Graph
G	Graphization operation
G_L	Line graph
H	Hypergraph
J_1	Number of 1-DOF joints or pairs
J_2	Number of 2-DOF joints or pairs
J_m	Half the total number of joints of the multiple links
J'_m	Number of joints between two multiple links
JS	Joint simplification operation
K_L	Kinematic link
$K_{3,3}$, K_5	Kuratowski graphs
L	Loop

LAM	Link adjacency matrix
m_{max}	Maximum number of joints incident to a link
MLAM	Multiple link adjacency matrix
MTM	Mechanism topology matrix
N_{Ci}	Number of contracted links with *i* series joints
NewDim	Dimension of matrix
N_J	Number of joints
N_L	Number of links
N_m	Number of multiple links
P_G	Permutation group
P_{GL}	Link permutation group
S	Set
S_{Ji}	Contracted link with *i* series joints
TLAM	Transformation link adjacency matrix
TSM	Topological structure of a mechanism
V	Vertex
W	Walk
Y	Hyperedge

Chapter 1 Introduction

Various mechanisms have been designated to fulfill the requirements of human being in our daily life. The creation of all possible or new mechanisms is a challenge work. In the process of systematic creative design, mechanisms are often generalized into the corresponding generalized kinematic chains. This drives the need for providing various atlases of generalized kinematic chains, including kinematic chains of mechanisms and rigid chains of structures. This chapter introduces the motivation and objectives of this work, and the organization of the dissertation.

1.1 Motivation and Objectives

Traditionally, the creation of mechanisms to satisfy given requirements and constraints relies on the knowledge, experience, inspiration, imagination, intuition, and/or ingenuity of the designers. Many design concepts are available in references and handbooks for adaption as new designs or the modification of existing mechanisms. Two major systematic approaches, for the creative design, conceptual design, or structural synthesis of mechanisms, have been developed in the past decades. One approach is based on the concept of generalization and specialization [1-4], while the other involves the separation of the structure and the function [5-6]. In both methodologies, the generation of atlas of kinematic chains and/or graphs is a major step of the design process.

In order to describe the relationship between links and joints in a mechanism, the topological structure of a mechanism (TSM) should be identified. The TSM is characterized by its types and numbers of links and joints, and the incidences between them, and it can be represented by its mechanism topology matrix (MTM) [4]. An MTM can be transformed into its corresponding kinematic chain or graph. Furthermore, the kinematic chains of mechanisms can be classified into different types, e. g. with simple joints or multiple joints, fractionated or non-fractionated, degenerate, rigid chains, and so on.

The purpose of this work is to study the structural and number synthesis of generalized kinematic chains, kinematic chains, and rigid chains with simple joints based on graph theory. Some methods of the number synthesis of kinematic chains are designed for manual implementation, and are therefore applicable only to the synthesis of kinematic chains with a limited number of links and joints. Accordingly, it is needed to implement the enumeration algorithms by means of computer programs to automatically synthesize and sketch various atlases of generalized kinematic chains, kinematic chains, and rigid chains.

1.2 Dissertation Organization

The organization of this dissertation is shown in Figure 1.1 with the following eight chapters:

- Chapter 1 is an introduction to this dissertation including the motivation, objectives, and dissertation organization.
- Chapter 2 interprets the terminology and definitions regarding links and joints, graph theory, kinematic chains, link assortments, kinematic matrices, permutation groups, and so forth.
- Chapter 3 reviews the related literatures of the number synthesis of kinematic chains with simple joints. Various methods, such as intuition or visual inspection, Franke's condensed notation, graph theory, contracted graphs, Baranov trusses, transformation of binary chains, matrices, group theory, stratified method, and so on are introduced in this chapter.
- Chapter 4 studies some basic theories (e.g. link assortments, contracted link assortments, multiple link adjacency matrix synthesis, link adjacency matrix synthesis, and so on) for

the construction of the generalized kinematic chains, kinematic chains, and rigid chains.

- Chapter 5 proposes an enumeration algorithm for the number synthesis of generalized kinematic chains with simple joints. The algorithms for checking cut-links and Kuratowski graphs, and a sketching algorithm are presented. Based on these algorithms, an enumeration algorithm is proposed for the construction of generalized kinematic chains.
- Chapter 6 presents two algorithms for checking three-bar basic rigid chains and degenerate kinematic chains to identify the degenerate kinematic chains. Two enumeration algorithms for the number synthesis of kinematic and rigid chains with simple joints are proposed, respectively.
- Chapter 7 develops a computer program to automatically sketch the generalized kinematic chains, kinematic chains, and rigid chains, with examples. Furthermore, the numbers of generalized kinematic chains, kinematic chains, and rigid chains are listed and tabulated.
- Chapter 8 is the conclusions and suggestion to this work.
- Appendix A is the basic contracted graphs with 1, 2, 3, and 4 loops.
- Appendix B is the user interface of the computer program.
- Appendix C is the atlases of generalized kinematic chains.
- Appendix D is the atlases of kinematic chains.
- Appendix E is the atlases of rigid chains.

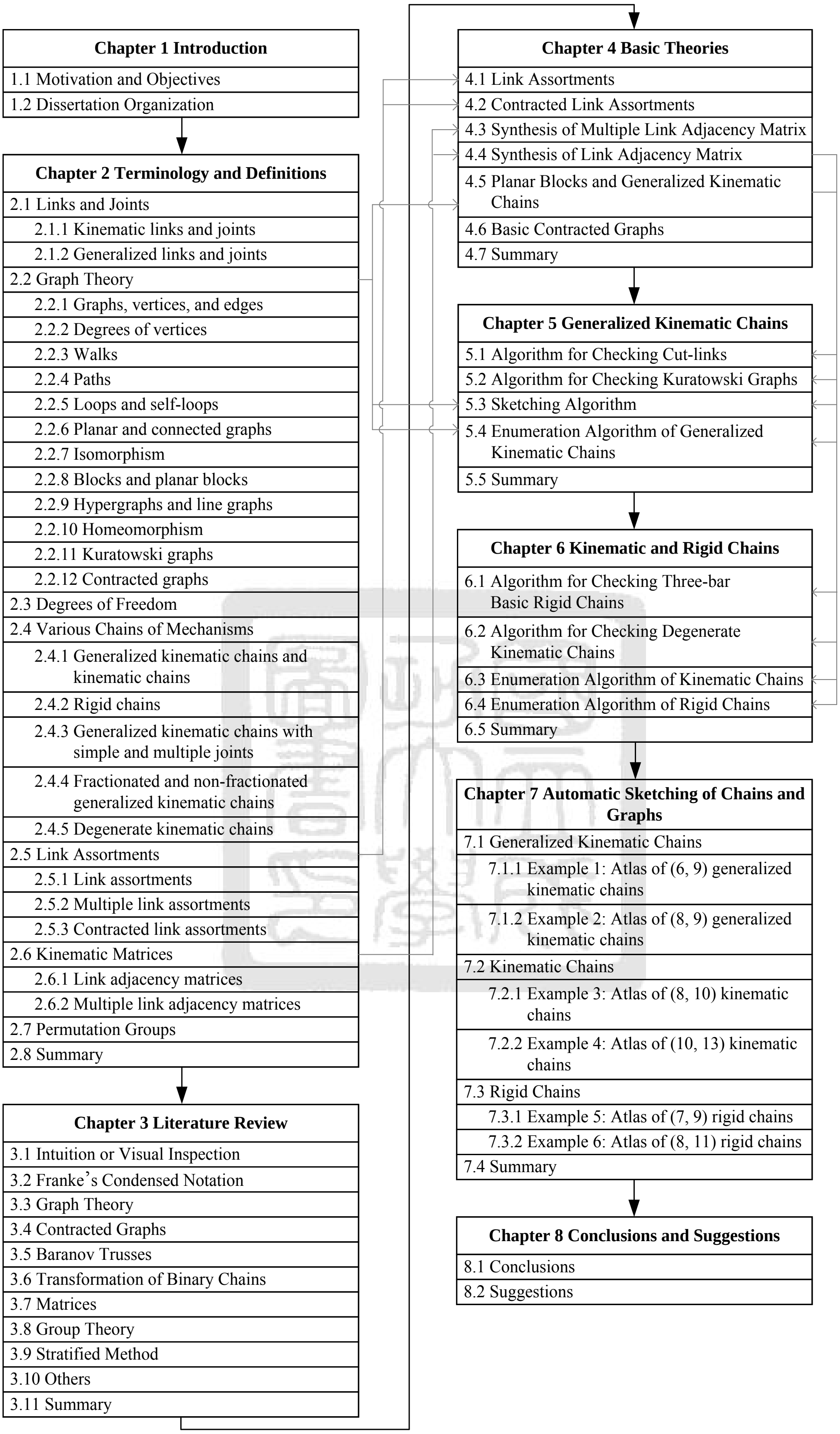


Figure 1.1 Dissertation organization

Chapter 2 Terminology and Definitions

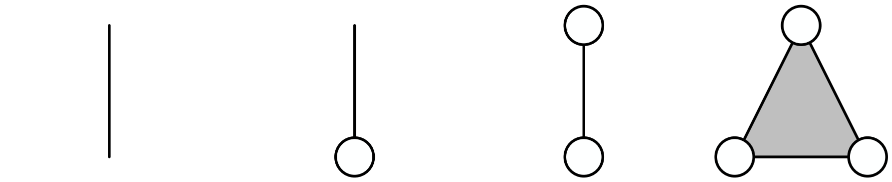
This chapter introduces some fundamental definitions and terminology from References [4, 6-9] that are essential for the structure and number synthesis of generalized kinematic chains, kinematic chains, rigid chains, and graphs.

2.1 Links and Joints

This section presents the definitions of kinematic links and joints for mechanisms, and generalized links and joints for generalized kinematic chains, kinematic chains, and rigid chains.

2.1.1 Kinematic links and joints

A material body can be defined as a rigid body if the distance between any two points of the body remains constant. The individual rigid bodies making up a machine or mechanism are called members or links [6]. A kinematic link, K_L, is a rigid member for holding its joints apart and transmitting motions and forces. In general, any rigid mechanical member is a kinematic link, and it can be identified based on the number of incident joints. A separated link is one with zero incident joints. A singular link is one with one incident joint. A binary link is one with two incident joints. A ternary link is one with three incident joints. An i-link is one with i incident joints, and so on. Graphically, a link with i incident joints is symbolized by an i-side shaded polygon with small circles on the vertices indicating incident joints. The schematic representations of a separated link, singular link, binary link, and ternary link are shown in Figure 2.1. Kinematic links can be identified as sliders, gears, rollers, cams, belts, ropes, keys, rivets, flexible members (as springs), or compression members, and so on.



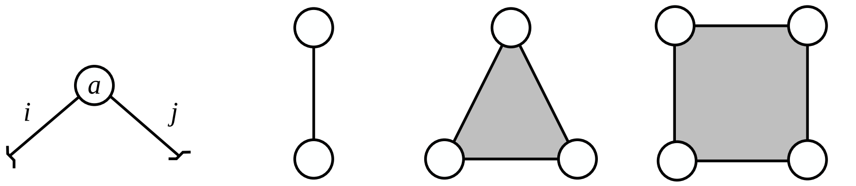
(a) Separated link (b) Singular link (c) Binary link (d) Ternary link

Figure 2.1 Type of kinematic links

In order to make mechanical members functional, they must be connected by certain means. A mechanical member is connected to another member which is called an element. Two elements that belong to two different members and are connected together form a kinematic joint or pair, such as revolute pair, prismatic pair, rolling pair, gear pair, cam pair, screw pair, and so forth.

2.1.2 Generalized links and joints

The term generalized joint is used to describe any joint in general, and it may be a revolute joint, a prismatic joint, a gear joint, or any other form of joint. Figure 2.2(a) shows a simple generalized joint (*a*) with two incident links (*i* and *j*). Similarly, the term generalized link is used to describe any link with incident generalized joints, and it may be a binary link, a ternary link, a quaternary link, etc. Graphically, a generalized link with N_J incident joints is symbolized by a shaded, N_J-sided polygon with small circles on the vertices indicating incident joints. Figures 2.2(b), (c), and (d) show graphical representations of a binary, ternary, and quaternary generalized link, respectively.



(a) Generalized joint (b) Binary link (c) Ternary link (d) Quaternary link

Figure 2.2 Representations of generalized joints and links

2.2 Graph Theory

Graph theory has been widely applied to different fields, such as chemistry, network, electrical engineering, civil engineering, and so on. Similarly, the topological structures or structure synthesis of mechanisms can be represented utilizing graph theory. In this section, some basic concepts and terminology of graph theory [7-9] are introduced and defined.

2.2.1 Graphs, vertices, and edges

A graph G of order p consists of a finite nonempty set $V=V(G)$ of p vertices together with a specified set E of q unordered pairs of distinct vertices. A pair $X=\{V_1, V_2\}$ of vertices in E is called an edge of G, and X is said to join V_1 and V_2. The vertices V_1 and V_2 are adjacent and vertex V_1 and edge X are incident with each other. A graph with p vertices and q edges is called a (p, q) graph. Graphically, a vertex is symbolized by a small dot and an edge is by a line. For a (6, 7) graph shown in Figure 2.3(a), vertices 2 and 6 are adjacent, and vertex 6 is incident to edges a and g.

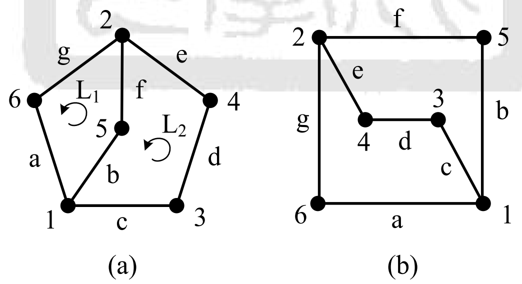


Figure 2.3 Two (6, 7) graphs

2.2.2 Degrees of vertices

The degree of a vertex V in a graph G is defined as the number of edges incident with vertex V, denoted as $deg(V)$ or dV. A vertex of zero degrees is called an isolated vertex. A vertex of two degrees is a binary vertex, a vertex of three degrees is a ternary vertex, and so on. For the (6, 7) graph shown in Figure 2.3(a), the degree of vertex 2 is three, and it is a ternary vertex; the degree of vertex 6 is two, and it is a binary vertex; and, the total degrees of the (6, 7) graph is fourteen.

2.2.3 Walks

A walk W of a graph G is an alternating sequence of vertices and edges, $W=(V_1, E_1, V_2, E_2, \ldots, V_i, E_i)$, beginning and ending with vertices, in which each edge is incident with the two vertices immediately preceding and following it. A walk W may also be simply denoted by a sequence of vertices $W=(V_1, V_2, \ldots, V_i)$, whose consecutive vertices are adjacent. For the (6, 7) graph shown in Figure 2.3(a), (V_1, V_6, V_2, V_5), $(V_1, V_5, V_2, V_4, V_3)$, and $(V_1, V_6, V_2, V_4, V_3)$ are walks.

2.2.4 Paths

A walk of a graph is defined as a path if the vertices in the graph are distinct, i.e., each vertex of the path is unique. The length of a path is defined as the number of edges in the path. Furthermore, a vertex-path is defined as a path that begins and ends with a vertex, and an edge-path is defined as a path that begins and ends with an edge. For example, the sequence 1, a, 6, g, 2, e, 4 shown in Figure 2.3(a) is a path with a length of 3.

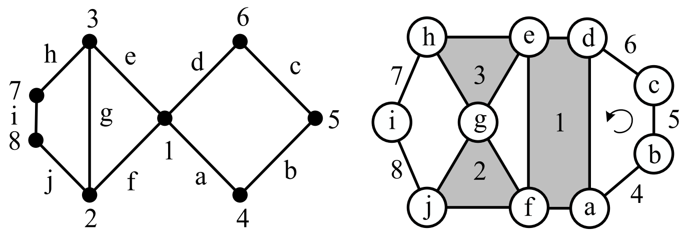
2.2.5 Loops and self-loops

A closed walk is defined as a loop if the n vertices are distinct for $n \geq 3$. Besides, the number of loops (L) of a graph is expressed as:

$$L = E - V + 1 \tag{2.1}$$

For example, the closed walk V_1, V_6, V_2, V_5, V_1 shown in Figure 2.3(a) is a loop (L_1). Based on the Equation (2.1), the number of loop is 2 ($7-6+1$).

If a string of binary vertices in a graph is incident to the same vertices or edges and forms a cycle, it is called a self-loop. For an (8, 10) graph shown in Figure 2.4(a), multiple vertex 1 and a string of binary vertices 4, 5, 6 form a self-loop.



(a) (b)

Figure 2.4 An (8, 10) graph and its chain

2.2.6 Planar and connected graphs

In graph theory, a planar graph is said to be embedded in a plane, i.e., the graph can be drawn on a plane surface so that no edge crossings. Such a graph is called a planar graph. Otherwise, it is called a non-planar graph. Moreover, a connected graph is a graph in which any two vertices are joined by a path, i.e., every vertex in connected graph is connected to every other vertex by at least one path. For example, the (6, 7) graph shown in Figure 2.3(a) is a planar and connected graph, and a (6, 9) graph shown in Figure 2.5 is a non-planar graph.

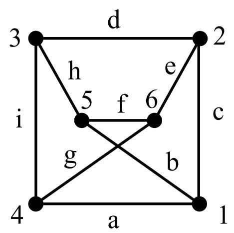


Figure 2.5 A (6, 9) graph

2.2.7 Isomorphism

Two graphs, G_1 and G_2, are the same and recognized as isomorphic if there is a one to one correspondence between their vertex sets which preserves adjacency. For every vertex in graph G_1, there can be found a corresponding vertex in graph G_2. For the two (6, 7) graphs shown in Figures 2.3(a) and (b), since they have the same adjacency, they are isomorphic graphs.

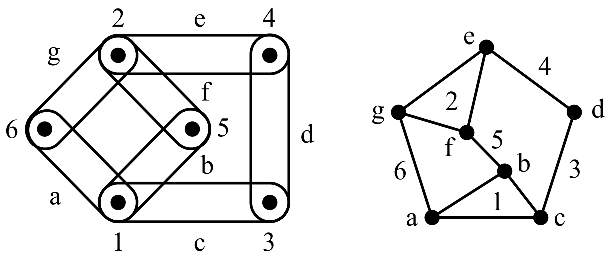
2.2.8 Blocks and planar blocks

A block is a graph and a maximal non-separable subgraph which is connected, nontrivial, and without any cut-vertex. It is also called a 2-connected graph or non-separable graph. In addition, a planar block is a block which can be drawn in the plane with no edge crossings. A planar block with p vertices and q edges is called a (p, q) planar block. For example, the graph shown in Figure 2.3(a) is a (6, 7) planar block.

2.2.9 Hypergraphs and line graphs

A hypergraph H is a pair of sets (V, Y) in which V is a set of elements called vertices and Y is a nonempty set of elements called hyperedges. Furthermore, it is a graph in which hyperedges may connect more than two vertices and no two hyperedges consist of the same set of vertices. Hypergraphs are drawn by representing each hyperedge as a closed curve containing its vertices.

For a graph G, its line graph G_L has the edges of graph G as its vertex set. The two vertices V_1 and V_2 of G_L are adjacent when each edge E_i of graph G is the set consisting of the two vertices that it joins; the intersection $V_1 \cap V_2$ is a singleton. In other words, vertices V_1 and V_2 of G_L are joined by an edge in G_L if edges E_1 and E_2 of G are incident with just one common vertex of G. For the (6, 7) graph shown in Figure 2.3(a), its corresponding hypergraph and line graph are shown in Figures 2.6(a) and (b), respectively.

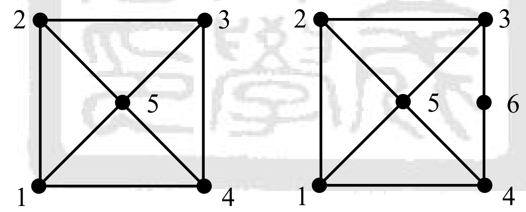


(a) Hypergraph (b) Line graph

Figure 2.6 A (6, 7) hypergraph and line graph

2.2.10 Homeomorphism

Two graphs, G_1 and G_2, are homeomorphic if both can be obtained from the same graph by inserting the binary vertices into its edges. For example, the (5, 8) graph and the (6, 9) graph shown in Figure 2.7 are homeomorphic. Note that the homeomorphic graphs is an equivalence relation.

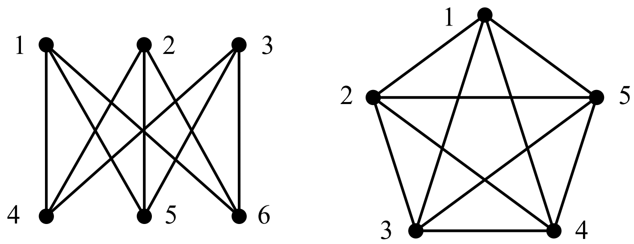


(a) (5, 8) graph (b) (6, 9) graph

Figure 2.7 A (6, 9) graph homeomorphic to a (5, 8) graph

2.2.11 Kuratowski graphs

A graph, G, is a planar graph if and only if it does not contain a subgraph homeomorphic to $K_{3,3}$ or K_5, as shown in Figures 2.8(a) and (b), respectively. They are also called Kuratowski graphs. If a graph contains a subgraph homeomorphic to either of these, it is also non-planar. The (6, 9) graph shown in Figure 2.5 is a $K_{3,3}$ graph.

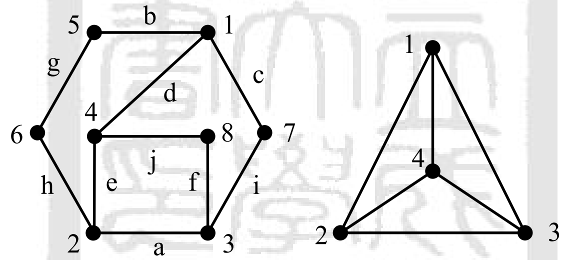


(a) $K_{3,3}$ (b) K_5

Figure 2.8 Kuratowski graphs

2.2.12 Contracted graphs

A contracted graph is a graph comprising only vertices with more than two degrees (binary vertices) and is obtained by contracting all binary vertices until no binary vertices exist in the graph. For example, the (8, 10) graph shown in Figure 2.9(a), its corresponding contracted graph is shown in Figure 2.9(b), respectively.



(a) (b)

Figure 2.9 An (8, 10) graph and its corresponding contracted graph

2.3 Degrees of Freedom

The number of degrees of freedom (DOF) of a mechanism determines how many independent inputs the mechanism must have. In another words, it is the number of independent coordinates needed to specify the relative positions of members in the mechanism. The number of DOF, of a planar mechanism with N_L links and N_J joints is given as:

$$\text{DOF} = 3(N_L - 1) - 2J_1 - J_2 \tag{2.2}$$

where N_L is the number of links in the mechanism or chain, J_1 is the number of 1-DOF joints or pairs, and J_2 is the number of 2-DOF joints or pairs.

2.4 Various Chains of Mechanisms

This section introduces various types of chains of mechanisms such as generalized kinematic chains, kinematic chains, rigid chains, fractionated or non-fractionated chains, simple-jointed and multiple-jointed chains, and degenerate kinematic chains.

2.4.1 Generalized kinematic chains and kinematic chains

A generalized kinematic chain consists of generalized links connected by generalized joints. Moreover, the chain is connected, closed, has no cut-links (or cut-joints), and contains only simple joints. A generalized kinematic chain with N_L generalized links and N_J generalized joints is known as a (N_L, N_J) generalized kinematic chain. Figure 2.10 shows the atlas for (6, 7) generalized kinematic chains.

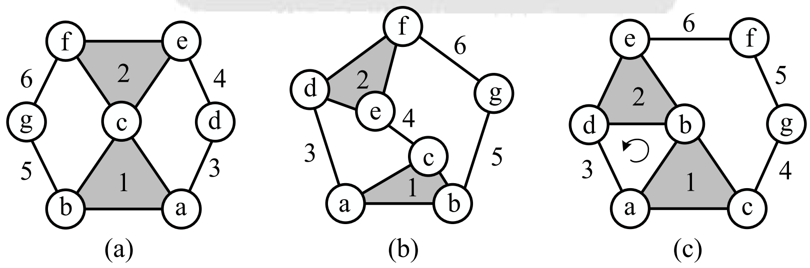
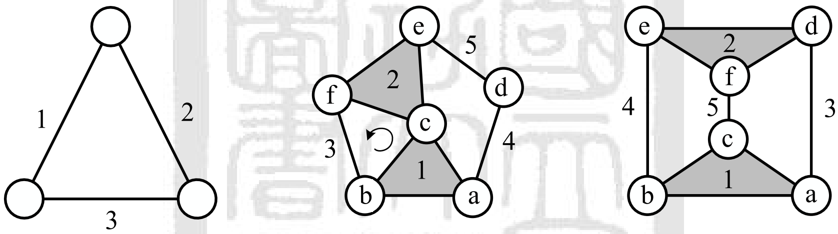


Figure 2.10 Atlas of (6, 7) generalized kinematic chains

A kinematic chain is a generalized kinematic chain in which the type of joints is specified, the number of DOF is positive, and the motion of the chain with one member grounded is constrained. For example, the two chains shown in Figures 2(a) and (b) are kinematic chains if all joints are specified as revolute joints or joints with 1 DOF.

2.4.2 Rigid chains

If a generalized kinematic chain contains revolute joints, has a non-positive DOF, and has an over-constrained motion with one member grounded, it is referred to as a rigid chain. In addition, a basic rigid chain is a rigid chain without any rigid sub-chain. The simplest basic rigid chain is the (3, 3) chain as shown in Figure 2.11(a). For example, the (5, 6) chain shown in Figure 2.11(b), with zero DOFs, is a five-bar rigid chain. Since it contains a three-bar basic rigid chain (link 1-2-3), it is not a five-bar basic rigid chain; nevertheless, the (5, 6) chain shown in Figure 2.11(c) is a basic rigid chain.



(a) The (3, 3) basic rigid chain (b) A (5, 6) rigid chain (c) A (5, 6) basic rigid chain

Figure 2.11 Three rigid chains

2.4.3 Generalized kinematic chains with simple and multiple joints

All generalized joints with two incident generalized links in a generalized kinematic chain is designated as a generalized kinematic chains with simple joints, whereas all generalized joints with more than two incident generalized links in a generalized kinematic chain is designated as a generalized kinematic chain with multiple joints. Graphically, a multiple joint in a kinematic chain is symbolized by concentric circles. For instance, two (6, 7) generalized kinematic chains with simple joints are shown in Figures 2.10(a) and (b), and a 6-link generalized kinematic chain with a multiple joint a is shown in Figure 2.12.

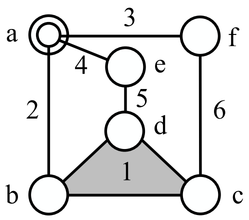


Figure 2.12 A 6-link generalized kinematic chain with a multiple joint

2.4.4 Fractionated and non-fractionated generalized kinematic chains

A cut-link (or cut-joint) is a link (or joint) in a chain whose removal results in a disconnected chain. The chain is designated as a fractionated generalized kinematic chain, if it has cut-links; otherwise, the chain is a non-fractionated generalized kinematic chain. For example, Figures 2.10(a) and (b) show two non-fractionated generalized kinematic chains; and for the (8, 10) fractionate chain shown in Figure 2.4(b), link 1 is a cut-link.

2.4.5 Degenerate kinematic chains

A kinematic chain with positive DOFs and a basic rigid chain is a degenerate kinematic chain. A degenerate kinematic chain can be transformed into a chain with fewer links by replacing the basic rigid chain with a single link. For example, the (10, 13) kinematic chain shown in Figure 2.13(a) contains a 7-link basic rigid chain shown in Figure 2.13(b), and this chain is degenerated into the (4, 4) kinematic chain as shown in Figure 2.13(c).

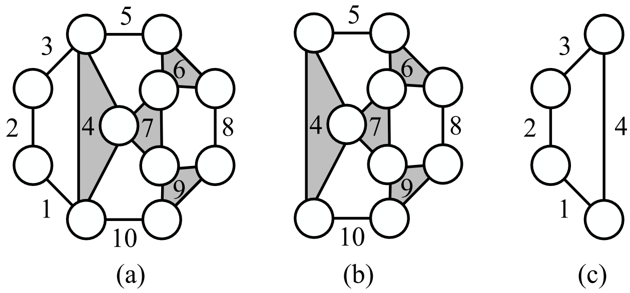


Figure 2.13 Degeneration of a (10, 13) kinematic chain

2.5 Link Assortments

In this section, the link assortments and contracted link assortments of generalized kinematic chains are introduced.

2.5.1 Link assortments

The link assortment, A_L, of a generalized kinematic chain is the type and the number of links in the chain. It is a set of numbers consisting of the numbers of binary links (N_{L2}), ternary links (N_{L3}), quaternary links (N_{L4}), etc., and is expressed as: A_L=[N_{L2}/N_{L3}/N_{L4}/…/N_{Li}/…]. The atlas of various generalized kinematic chains can be synthesized by assembling the corresponding link assortments.

2.5.2 Multiple link assortments

The multiple link assortment, A_{ML}, of a generalized kinematic chain is the type and the number of multiple links in the chain. It is a set of numbers consisting of the numbers of ternary links (N_{L3}), quaternary links (N_{L4}), and so on, and is expressed as: A_{ML}=[N_{L3}/N_{L4}/…/N_{Li}/…]. The atlas of various generalized kinematic chains can be sketched by assembling the corresponding multiple link assortments.

2.5.3 Contracted link assortments

A contracted link is a string of binary links in a generalized kinematic chain. The contracted link assortment, A_{CL}, of a generalized kinematic chain is the number of contracted links. It is a set of numbers consisting of the numbers of contracted links with i series joints (N_{Ci}) and it can be expressed as: A_{CL}=[N_{C2}/N_{C3}/N_{C4}/…/N_{Ci}/…].

2.6 Kinematic Matrices

The concept of matrices is a powerful tool for the representation of the topological structures of various chains. The definitions of link adjacency matrix and multiple link adjacency matrix are introduced.

2.6.1 Link adjacency matrices

The link adjacency matrix, LAM, of a generalized kinematic chain with N_L links and N_J joints is an $N_L \times N_L$ matrix with its elements $e_{ij}=1$ if link i is adjacent to link j, and $e_{ij}=0$ otherwise. For the (6, 7) generalized kinematic chain shown in Figure 2.10(a), its link adjacency matrix, LAM, is:

$$\text{LAM} = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

2.6.2 Multiple link adjacency matrices

The multiple link adjacency matrix, MLAM, of a generalized kinematic chain with N_m multiple links and N_J joints is an $N_m \times N_m$ matrix with its elements $e_{ij}=n_1n_2n_3\ldots n_x$ if multiple link i is adjacent to multiple link j with n series joints and x is the number of different types of contracted links, and $e_{ij}=0$ otherwise. Therefore, the dimension of the MLAM depends on the number of multiple links. For the (6, 7) generalized kinematic chain shown in Figure 2.10(a), its multiple link adjacency matrix, MLAM, is:

$$\text{MLAM} = \begin{bmatrix} 0 & 331 \\ 331 & 0 \end{bmatrix}$$

in which elements $e_{12}= e_{21}=331$ mean that two contracted links with three series joints (S_{J3}) are adjacent to two ternary links and one contracted link with one series joint (S_{J1}) is adjacent to two ternary links.

2.7 Permutation Groups

A permutation group, P_G, is a finite group whose elements are permutations of a given set and whose group operation consists the permutation in P_G. Two permutations form a group only if one is the identity element and the other is a permutation involution, i.e., a permutation which is its own inverse. The usual composition of mappings provides a binary operation for permutations on the same set. A set S and its binary operation must satisfy the following axioms: (1) closed, (2) associativity, (3) existence of an identity element, and (4) existence of inverses. When a collection of permutations satisfy the above conditions, it is called a permutation group. The permutation groups are used to demonstrate the symmetry of the topological structure of mechanisms. A bisection (one-to-one and onto) mapping from a finite set S onto itself is called a permutation P_G. For example, the sequence (a, c, d, b, e) is a permutation of the set S = (a, b, c, d, e) in which a is transformed into a, b is transformed into c, c is transformed into d, d is transformed into b, and e is transformed into e. In this permutation, $a \to a$ forms a cycle, denoted by [a] with a length of one, $b \to c \to d \to b$ forms another cycle [b c d] with a length of three, and $e \to e$ forms a cycle [e] with a length of one. The cyclic representation of this permutation is denoted by P_G=[a][b c d][e].

For the (6, 7) Watt chain shown in Figure 2.10(a), links 1 and 2 have three incident joints and all of their adjacency links have two incident joints. Therefore, links 1 and 2 have the same attribute, and so do links 3, 4, 5, and 6. The link permutation group P_{GL} of the chain is:

$$P_{GL} = \{P_{L1}, P_{L2}, P_{L3}, P_{L4}\} \tag{2.3}$$

where

$$P_{L1} = [1][2][3][4][5][6] \tag{2.3a}$$

$$P_{L2} = [12][34][56] \tag{2.3b}$$

$$P_{L3} = [1][2][35][46] \tag{2.3c}$$

$$P_{L4} = [12][36][45] \tag{2.3d}$$

2.8 Summary

In this chapter, some basic definitions and terminology regarding generalized kinematic chains, kinematic chains, rigid chains, and graphs are introduced. The fundamental definitions from graph theory are used to recognize planar or non-planar graph and to sketch various chains. Furthermore, the definitions of different types of chains in mechanisms and structures are presented in order to synthesize various chains. The link assortments, contracted link assortments, and kinematic matrices are introduced for solving such problems which will be presented in details in the subsequent chapters.

In particular, the generalized kinematic chains, kinematic chains, and rigid chains cannot have any cut-links in the chains. Furthermore, the kinematic chains should be identified without the degenerate kinematic chains. The basic difference between kinematic chains and rigid chains is the numbers of degrees of freedom. If the degrees of freedom is more than zero, the chain is a kinematic chain; otherwise, it is a rigid chain. Therefore, the generalized kinematic chains, kinematic chains, and rigid chains have different constraints. The subsequent chapters will propose some algorithms or methods to eliminate invalid chains.

Chapter 3 Literature Review

In the past long years, many scholars focused on the study of number synthesis, structural synthesis, structural analysis, isomorphism detection problem of generalized kinematic chains and kinematic chains, and some on the literature review of kinematic chains [10-14]. Available approaches, such as, intuition or visual inspection, Franke's condensed notation, graph theory, contracted graphs, Baranov trusses, transformation of binary chains, matrices, group theory, stratified method, and others for the structural synthesis and number synthesis of generalized kinematic chains and kinematic chains with simple joints are reviewed and discussed in this chapter.

3.1 Intuition or Visual Inspection

In the early study for the number synthesis of kinematic chains, some pioneering scholars used their intuition or inspiration to obtain or enumerate the kinematic chains. In 1885, Gruebler [15] was the first to present 12 kinematic chains with 8 links and 1 DOF. In 1921, Alt [16] revealed extra 4 kinematic chains with 8 links and 1 DOF. In 1917, Klein [10] constructed 228 kinematic chains with 10 links and 1 DOF. In 1955, Hain [17] presented the entire 16 kinematic chains with 8 links and 1 DOF. In 1964, Crossley [18] proposed an algorithm for listing the combinations of types of links based on solving Grüebler's equation, along with the kinematic chains with up to 9 links. In which, a few kinematic chains exist cut-links, i.e., they are fractionated kinematic chains. In 1953, Alt [19] synthesized 226 kinematic chains with 10 links and 1 DOF. In 1966, Crossley [19] indicated that Alt's collection of 226 kinematic chains with 10 links and 1 DOF have 16 isomorphic chains, and he presented the total number as 230. In 1976, Kiper and Schian [20] provided 6856 kinematic chains with 12 links and 1 DOF.

The major disadvantage for the number synthesis of kinematic chains based on intuition or visual inspection is that such a method cannot guarantee to generate all kinematic chains and eliminate the isomorphic chains.

3.2 Franke's Condensed Notation

In 1958, Franke [21] described an alternative symbolic notation to represent and enumerate non-isomorphic kinematic chains. In Franke's condensed notation, a circle with an appropriate number represents a polygonal member (multiple link), and one or more binary members (binary links) in series are indicated by a line with a corresponding number. Note that the number zero was used to indicate direct adjacency of two polygonal members (multiple links). For example, Figures 3.1(a) and (b) show a kinematic chain with 12 links and 16 joints and its corresponding Franke's symbolic notation, respectively.

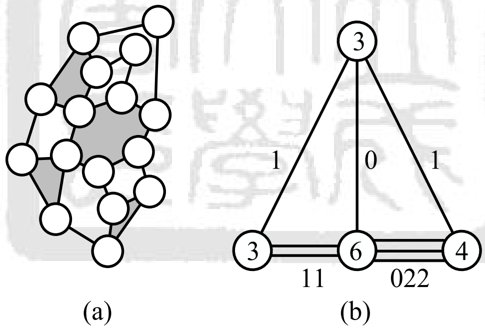


Figure 3.1 A 12-link and 16-joint kinematic chain and its corresponding Franke's notation [22]

There are four steps in Franke's condensed notation method for the structural synthesis of the kinematic chains. First, all possible combinations of number of the members (link assortments) are listed based on Chebyshev-Gruebler theory. Then, using Franke's symbolic notation, the available circles into "molecules" are arranged and sketched. These circles represent polygonal members (multiple links), and the circles are connected by straight lines in all possible combinations. Placing the number on the circles is equal to the number of connecting lines. Next, each line is assigned a number to connect between polygonal members (multiple links). The sum of all attached numbers is equal to the total number of binary links. Finally, all possible combinations and permutations are tabulated. In this method, the degenerate and isomorphic chains need to be discarded. In 1958, Franke [21] synthesized 2 kinematic chains with 6 links and 1 DOF and 16 kinematic chains with 8 links and 1 DOF.

In 1966, Davies and Crossley [22] applied Franke's condensed notation to construct the atlases of 7-, 9-, 10-, 11-link kinematic chains, respectively. They presented 35 kinematic chains with 9 links and 1 DOF and 230 kinematic chains with 10 links and 1 DOF; however, the atlas of 11-link kinematic chains was not available. In 1969, Haas and Crossley [23] proposed a structural permutation method based on Franke's condensed notation to construct kinematic chains, by constructing parts of kinematic chains with 13, 15 links and 4 DOFs. In 1971, Soni [24] presented the structural analysis of two general constraint kinematic chains with up to 3 loops utilizing Franke's condensed notation.

Franke's condensed notation is not easy to represent the topological structure of mechanisms. Furthermore, such a method applied visual inspection to construct kinematic chains, including eliminating isomorphic and degenerate kinematic chains.

3.3 Graph Theory

In the 1960s, many researchers [25-37] demonstrated the feasibility of synthesizing the kinematic structure of mechanisms with closed chains by means of graph theory.

In 1964, Crossley [25] presented an approach and proved the complete sets of possible permutations of links with 4, 6, 8, and 10 links based on graph theory, and he applied the concept of edge-vertex incidence matrix to discard isomorphic chains. However, the author only synthesized 222 kinematic chains with 10 links and 1 DOF. In 1966, Freudenstein and Dobrjanskyj [26] proposed a method based on graph theory for the type (number) synthesis of mechanisms and used the vertex-vertex matrix to identify isomorphic chains. The 16 kinematic chains with 8 links and 1 DOF and their corresponding graphs are listed. In 1967, Dobrjanskyj and Freudenstein [27] sketched kinematic chains based on their vertex-edge incidence matrices and graph theory, in which some kinematic chains have crossing links. A 10-link kinematic chain and its vertex-edge matrix are shown in Figure 3.2.

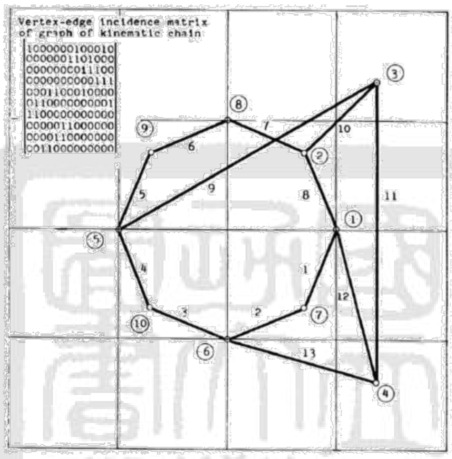


Figure 3.2 A 10-link kinematic chain and its vertex-edge incidence matrix [27]

In 1973, Huang and Soni [28] performed structural synthesis and analysis of planar kinematic chains by applying graph theory and Polya’s theory. The proposed method enumerated 2-loop kinematic chains, and the vertex-edge incidence matrix was used to test isomorphism. In 1984, Mayourian and Freudenstein [29] presented the atlas of graphs of the kinematic chains (planar blocks) with up to 6 links based on the basic restrictions on the graphs. The restriction equations are applied to obtain the numbers of vertices and edges, and acquire the admissible kinematic chains (planar blocks). In 1985, Olson et al. [30] developed a computer-aided type synthesis process based on graph theory and vertex-edge incidence matrices for the automatic sketching of closed-loop kinematic chains with simple joints and without any crossing link. Such a method cannot guarantee to obtain all kinematic chains with no crossing links. In 1986, Sohn and Freudenstein [31] applied the dual graph concept to perform the computer-generated enumeration of the kinematic structures of mechanisms with up to 3 DOFs and 5 loops. They also presented a method based on the vertex-vertex incidence matrix to eliminate isomorphic chains. In 1987, Sugimoto et al. [32] proposed a computer algorithm for the generation of all possible kinematic chains with a given number of DOF.

In 1991, Tempea [33] presented a numerical and structural synthesis method to determine basic kinematic chains with 4 loops and 2 DOFs based on graph theory and Assur group concept. An 11-link and 2-DOF basic kinematic chain shown in Figure 3.3(c) is obtained by comprising the addition of an open kinematic chain, Figure 3.3(a) and the Assur group, Figure 3.3(b). In 1994, Belfiore and Pennestri [34] developed a sketching procedure based on graph embedding methods for obtaining kinematic chains with up to 10 links, 1 DOF, and no crossing links. Such a method was limited to synthesize kinematic chains with 10 links and 1 DOF. In 2012, Nie et al. [35] derived an additional method with 2 links and 3 pairs for the type synthesis of planar closed kinematic chains with up to 12 links, and some kinematic chains with 13 links or 14 links based on graph theory. They also presented a sequence-similarity method for isomorphic identification and a loop method for the identification of basic rigid chains, in which few of kinematic chains have cut-links. In 2013, Pucheta et al. [36-37] developed a computerized method incorporating a loop-based algorithm and a force-directed algorithm for sketching non-fractionated kinematic chains and their associated graphs with minimal edge crossings.

Graph theory is an excellent tool for representing the topological structure of mechanisms and kinematic chains, and it provides a mathematic model to synthesize the kinematic chains. In addition, the number synthesis of kinematic chains based on graph theory is particularly applicable for developing computer programs.

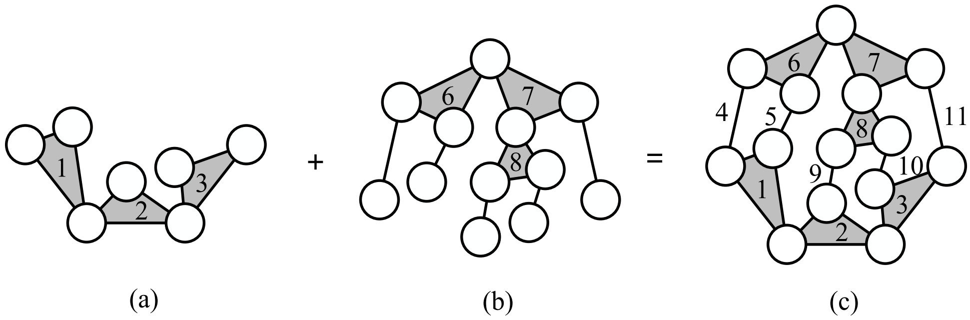


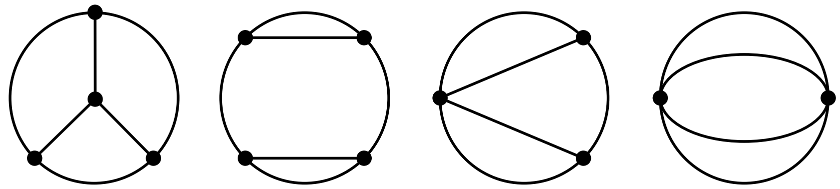
Figure 3.3 An 11-link and 2-DOF basic kinematic chain is constructed by an opened kinematic chain and an Assur group

3.4 Contracted Graphs

The contracted graph method for the number synthesis of kinematic chains consists of three main steps [38]. First, all possible combinations of the types of links (link assortments) are listed, i.e., the number of binary links, ternary links, quaternary links, and so on. Then, all different contracted graphs are constructed for each combination. Finally, all non-isomorphic kinematic chains are obtained by adding the binary links.

In 1967, Woo [39] presented a mathematical formulation approach to enumerate kinematic chains with 10 links and 13 joints based on the concepts of contraction map (contracted graph). The possible number of adding vertices to contracted graphs is determined based on Burnside theorem and permutation groups. The vertex-to-vertex incidence matrix was applied to check isomorphic graphs. In 1967, Freudenstein [40] proposed a method for the structural classification and enumeration of graphs (or mechanisms) based on the concepts of Polya’s theory and permutation groups. In 1970, Chen and Chen [41] applied graph theory and the concept of elementary degree-invariant transformation (EDI-transformation) to enumerate isomorphic kinematic chains with a given DOF and up to 10 links. In 1971, Tempea [42] presented the atlases of 141 contracted graphs with up to five loops by means of manually sketching for the structural synthesis of kinematic chains with lower or higher pairs. Figure

3.4 shows the atlas of contracted graphs with three loops. In 1974, Sanger [43] derived the characteristics of planar kinematic chains with up to two links of maximum degree and having total freedom. In 1989, Yan and Hwang [44] proposed an algorithm to automatic sketching kinematic chains for the minimization of the number of crossings based on the concepts of basic contracted kinematic chains with the associated basic contracted kinematic chains up to five loops without any isomorphic kinematic chains.



(a) (b) (c) (d)

Figure 3.4 Atlas of contracted graphs with three loops

In 1988, Yan and Hsu [45] proposed a method for the synthesis of contracted graphs for kinematic chains with multiple joints based on the idea of transforming a multiple joint in a kinematic chain into a solid polygon in the corresponding kinematic graph. The method can be extended to synthesize the kinematic chains with simple joints based on the concept of contracted graphs. The numbers of non-isomorphic basic kinematic chains with one and two multiple joints up to ten links are listed. In 1994, Lee and Yoon [38] developed a computerized program based on a recursive partitioning procedure for the enumeration of basic non-fractionated kinematic chains with a given number of links and DOFs. In addition, an algorithm was proposed for the detection of the embedded rigid structure.

The number and complexity of contracted graphs are much lesser than the kinematic chains. Through the process of number synthesis of kinematic chains, the atlases of contracted graphs should be constructed first, then the kinematic chains can be derived from each contracted graph. However, such a method cannot guarantee to obtain all possible contracted graphs as a databank for the number synthesis of kinematic chains.

3.5 Baranov Trusses

In 1952, the Russian kinematician Baranov who first announced the concept of Baranov trusses [46]. Since 1971, Manolescu and Tempea [47-48] presented a method to synthesize the planar kinematic chains with simple and multiple joints based on the transformation of Baranov trusses and approach "graphization" (*G*). There are two alternative steps. One is called joint simplification (*JS*). A link incident to a multiple joint is expanded, the (n)-link becomes ($n+1$)-link, and then a joint is reduced to become a simple joint. Another step is called dyad amplification (*DA*), and a dyad binary link is added between two joints. The two steps are executed until the planar kinematic chains with only simple joints. In such an approach, 6-, 8-, 9-, and 10-link kinematic chains were synthesized. Nevertheless, 40 different 9-link kinematic chains have 5 fractionated kinematic chains, i.e., kinematic chains have cut-links. Manolescu [49-51] further proposed a method for the structural synthesis of planar kinematic chains with simple and multiple joints based on the concept of Baranov trusses. Two planar kinematic chains with 6 links and 1 DOF shown in Figure 3.5 were constructed from the Baranov truss by using the operation of *G*, *DA*, and *JS*.

Baranov trusses method cannot precisely obtain all possible kinematic chains since the operations of *JS* and *DA* are not straightforward to perform. In addition, the method is unable to avoid generating the isomorphic chains.

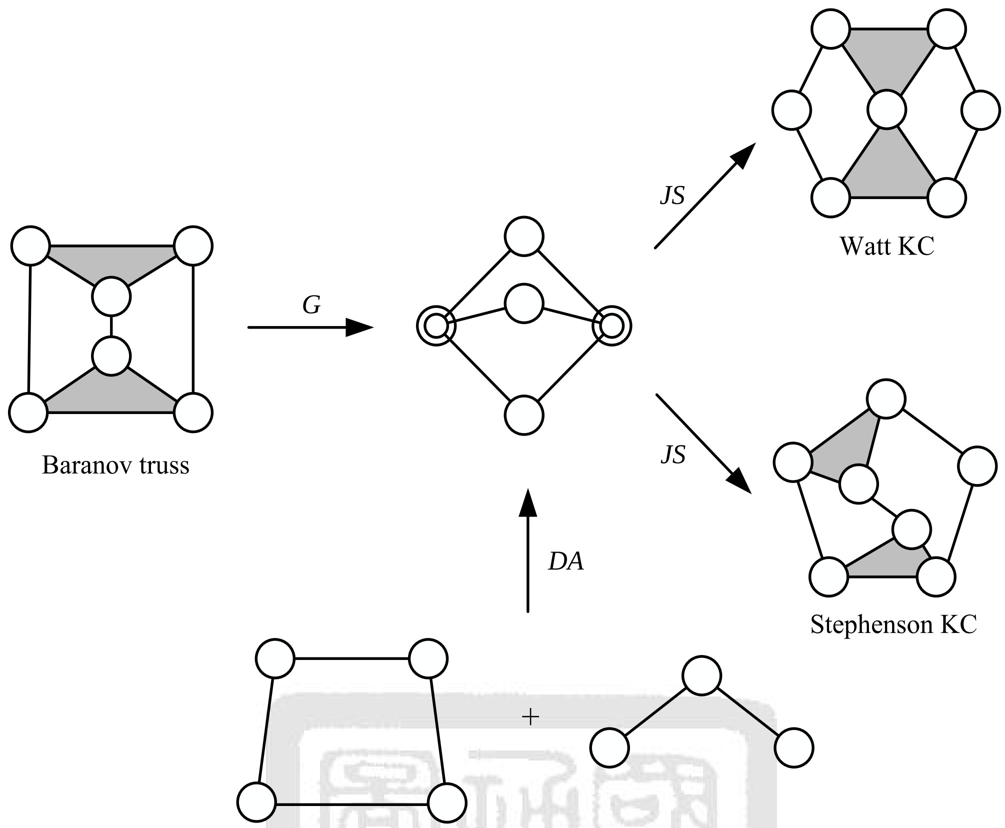


Figure 3.5 Construction of atlas of (6, 7) kinematic chains by using the operations *G*, *DA*, and *JS*

3.6 Transformation of Binary Chains

In 1979, Mruthyunjaya [52] proposed a method based on transformation of binary chains for the structural synthesis and derivation of possible simple- and multiple-jointed chains with positive or negative DOFs. A binary chain is one which consists of only binary links with simple as well as multiple joints. This method involves firstly, the possible combinations of types of binary chains should be determined, i.e., the number of binary joints, ternary joints quaternary joints, and so on, in the chain. Then, each binary chain needs to be transformed into all possible ways by reducing the multiplicity of the multiple joints in the chain. This procedure is almost the same as the operations "*JS*" in Manolescu's method [47]. For instance, a transformation of a ternary joint is formed by links 1, 2, and 3 shown in Figure 3.6(a). It can be transformed into two simple joints between links 1, 2 and links 1, 3 shown in Figure 3.6(b). The transformation can be denoted as "*1T2-3*". Finally, different chains are listed based on the transformation of binary chains. For the binary chain shown in Figure 3.7(a), it can be transformed into the Watt chain shown in Figure 3.7(b) and the Stephenson chain shown in Figure 3.7(c) as a result of the transformations *3T2-6*, *1T4-5* and *3T2-6*, *4T1-5*, respectively.

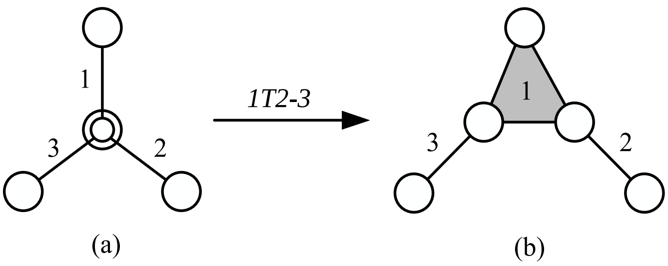


Figure 3.6 Transformation *1T2-3* of multiple joints

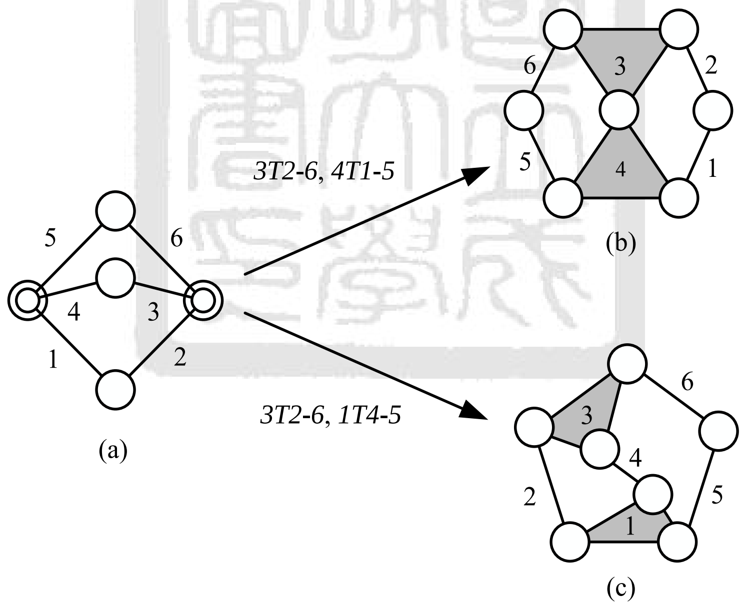


Figure 3.7 Atlas of (6, 7) kinematic chains by the transformation of the binary chain

In 1984, Mruthyunjaya [53-55] used the binary chain transformation method and matrices to develop a computerized approach to synthesize 97 simple-jointed kinematic chains with 10 links and 3 DOFs. In 1992, Mruthyunjaya et al. [56] applied the method in

Reference [53] and a method for the identification of isomorphic chains in Reference [57] to synthesize the kinematic chains with 11 links and 2 DOFs.

Such a method is similar to the reverse of the contraction of the multiple links into the multiple joints. The method for the transformation of binary chains began from a simple graph of binary links, then a number notation represented the transformation of multiple joints. However, the isomorphism detection problem in the method was not considered. The isomorphic chains may be generated while the binary chains are constructed and transformed.

3.7 Matrices

In 1955, Denavit and Hartenberg [58] firstly introduced a matrix representation of the loop equations for lower-pair mechanisms. Although the Denavit and Hartenberg notation is one of the most well-known matrix representations of kinematic structures, various other matrix representations are also proposed later. In 1964, Crossley [25] proposed an edge-vertex incidence matrix of a kinematic chain to identify the isomorphism. In 1966, Freudenstein and Dobrjanskyj [26] applied the vertex-vertex matrix to examine the isomorphic chains. In 1974, Raicu [59] presented the structural analysis of the simple-jointed kinematic chains with 3 to 5 links based on the concepts of an associated matrix (link adjacency matrix). For example, a chain with 6 links and 7 joints is shown in Figure 3.7(b), and its associated matrix can be expressed as follows:

$$M = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

In 1979, Mruthyunjaya and Raghavan [60] proposed a matrix representation of kinematic chains for determining the structural characteristics of kinematic chains and mechanisms based on Bocher's formulate and algebraic tests for the identification of the partial DOFs. Besides, the characteristic polynomial of the associated matrix with the kinematic chain was presented for detecting isomorphic chains. The matrix representations were also applied to represent multiple-jointed kinematic chains. In 1990, Yan and Hwang [61] developed an algorithm along with a computer program for the structural synthesis of kinematic chains with up to 12 links and 7 DOFs without isomorphic chains based on permutation groups and contracted link adjacency matrices (CLAMs). For a CLAM, its diagonal elements e_{ii}, called link element, represents the type of link i. The value of e_{ii} is equal to t if a multiple link with t joints and e_{ii} is equal to $-u$ if link i is a contracted link with u binary links. The off-diagonal elements e_{ij}, called joint element, represents the number of joints incident between link i and link j. Note that the value of e_{ij} can only be 0, 1, or 2. For the kinematic chain with 10 links and 13 joints shown in Figure 3.8, and its CLAM is expressed as follows:

$$\text{CLAM} = \begin{bmatrix} 4 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 4 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 3 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 3 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & -2 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & -2 \end{bmatrix}$$

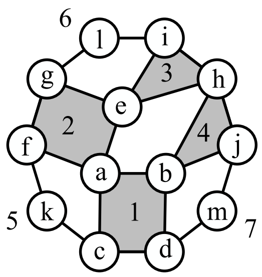


Figure 3.8 A (10, 13) inematic chain

In 1991, Hwang et al. [62-63] presented a method and developed a computer-aided algorithm based on the generation of the CLAMs for the number synthesis of planar kinematic chains with simple joints and up to 12 links without degenerate and isomorphic chains. In 1996, Mauskar and Krishnamurty [64] developed a computer program to automatic sketch kinematic chains with 6 to 11 links and single or multiple DOFs based on their inter-loop relationship and link-link adjacency matrix. The link-link adjacency of the mechanism is as an input data to create a link-joint incidence matrix. The loop matrix is determined by using the orthogonality property. Then, the proper loop matrix is detected and constructed for the generation of a kinematic chain without crossing links. In 2008, Hsieh et al. [65] developed a computer program for the structural synthesis of generalized kinematic chains with simple joints and without any cut-links (or cut-joints) by utilizing the CLAM and the multiple link adjacency matrix (MLAM). For the (10, 13) kinematic chain shown in Figure 3.8, its MLAM is expressed as follows:

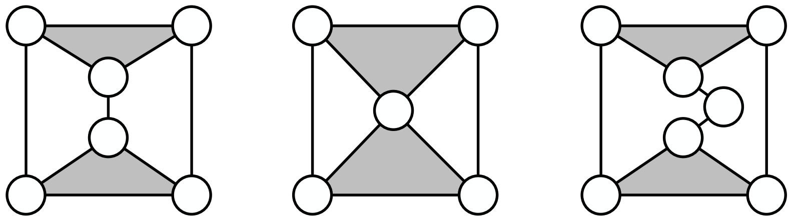
$$\text{MLAM} = \begin{bmatrix} 0 & 31 & 0 & 31 \\ 31 & 0 & 31 & 0 \\ 0 & 31 & 0 & 1 \\ 31 & 0 & 1 & 0 \end{bmatrix}$$

For the MLAM, elements e_{12}=e_{21}=31 mean that a contracted link with three series joints (two binary links) are adjacent to two quaternary links (links 1 and 2) and one contracted link with one series joint (one simple joint) is adjacent to two quaternary links (links 1 and 2). Elements e_{34}=e_{43}=1 mean that links 3 and 4 are adjacent to each other. Furthermore, the method of repeatable assignment is applied to recognize the isomorphic chains. The dimension of MLAM is smaller than CLAM and LAM. Therefore, it can reduce the memory space of computation and eliminate the isomorphic chains through the process.

For programming convenience, the topological structure of a chain or mechanism can be represented and expressed in a matrix form. Since LAM, CLAM, and MLAM are symmetric matrices, only the upper (or lower) triangular portion of the matrix need to be synthesized. Furthermore, the isomorphic and degenerate chains can be identified and discarded through the matrices method. As a result, the matrix method should be the most powerful approach for the number synthesis of kinematic chains.

3.8 Group Theory

Tuttle et al. [66-68] proposed a method for the enumeration of non-fractionated, pin-jointed kinematic chains with specified numbers of links, loops and DOFs based on the concept of symmetry group theory. First, all possible sets of higher links (types of multiple links) are determined. Then, all needed non-isomorphic bases (the completely connected sets of multiple links) of each partition need is found. Third, the symmetry group of each base is determined. Fourth, all possible sets of operations, contraction and expansions of the simple bonds (binary links) of each base, is constructed; and the number of links, and DOFs are the proper numbers. Next, for each set of operations label each bond (binary link) and then delete all isomorphic sets of labelled operations. Finally, all chains including DOF=0 sub-chains (rigid chains) are eliminated. For instance, the kinematic chain with 6 links and 1 DOF can be constructed by applying the base structure shown in Figure 3.9(a). Note that the base structure is not necessarily a kinematic chain. A (4, 5) chain, shown in Figure 3.9(b), can be obtained based on the base structure shown in Figure 3.9(a) by contracting the middle simple bond (binary link) into a joint. This is designated as the operation of contraction. A (6, 7) kinematic chain, shown in Figure 3.9(c), can be obtained based on the base structure shown in Figure 3.9(a) by expanding the middle simple bond into a series of binary links. This is designated as the operation of expansion.



(a) A base structure (b) A (4, 5) chain (c) A (6, 7) kinematic chain

Figure 3.9 A base structure generates different kinematic chains

Such a method may produce the problems of isomorphic bases and the symmetry group of a given base. In addition, the rigid chains or degenerate kinematic chains are hard to identified and eliminated. In 1996, Tuttle [69] presented a completely automatic procedure for the structural synthesis of the kinematic chains with all planar, non-fractionated, without any rigid chains and the isomorphism problem. Tuttle and coauthors introduced and applied the finite symmetry group theory for the number synthesis of kinematic chains to enumerate non-isomorphic kinematic chains exhaustively.

3.9 Stratified Method

In 1990, Fang and Freudenstein [70] proposed the stratified method for the enumeration of planar kinematic chains with 1 DOF and 2 to 4 independent loops. The method was developed for determining the stratified adjacency matrix and establishing the stratified code of the graph based on the concept of permutation groups. The stratified code is reconverted to the stratified adjacency matrix, then all mechanisms are reconstructed. Figure 3.10 shows levels of abstraction and their corresponding stratified adjacency matrices. The elements of the stratified adjacency matrix of the colored graph show the types of joints (1: revolute, 2: prismatic, 0: none). Condensing the element e_{12}=((2 1 1) (1)) in Figure 3.10(a) from the colored level to the monochrome level, element (2 1 1) is replaced by the number 3 and element (1) is replaced by the number 1. Therefore, element e_{12} of the stratified adjacency matrix of the monochrome graph is (3 1). The elements of the stratified adjacency matrix of the contracted graph show the number of strings between vertices *i* and *j*. In 2002 and 2005, Butcher and Hartman [71-72] extended the method in Reference [70] by means of the hierarchical representation and searching the simplified graph. The work is accomplished by enumerating the simplified and contracted graphs from which the kinematic chains with simple joints and up to 14 links were built up. Moreover, a rule-based method for deleting degenerate kinematic chains was proposed.

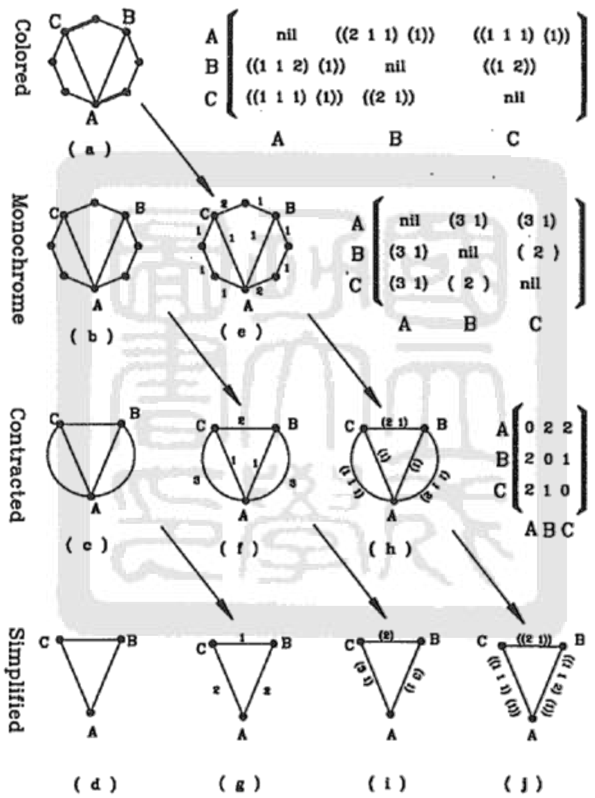


Figure 3.10 Levels of abstraction and corresponding stratified adjacency matrices [70]

The stratified representation is not efficient by reason of enormous numbers of links and joints. The method is good for the graphs or chains with a “medium” number of links and joints.

3.10 Others

In 1983, Earl and Rooney [73] presented a method based on sub-graph replacement operations for generating the interchange and direct graphs of kinematic chains with binary revolute joints and binary links. However, the method cannot guarantee to construct all possible kinematic chains without isomorphic chains. In 1990, Chieng and Hoeltzel [74] applied a computational model for solving the numerically continuous mechanism sketching problem as a discrete domain problem and for the avoidance of crossings based on a combinatorial approach. The proposed method is capable of generating mechanisms with non-crossing links. However, the loop information should be determined and an optimization method is presented to identify the proper independent loops for sketching the non-crossing links mechanisms. Besides, the large independent loop information and the optimization method results in decreasing the efficiency of computation and sketching.

In 1993, Sethi and Agrawal [75] proposed a 6-step classification method based on the multi-graph (contracted graph) theory and the concept of structural properties to classify the kinematic chains into different families for the selection of a basic structure of a mechanism. Such a method is complicated to construct the kinematic chains. In 2001, Rao and Deshmukh [76] derived a method based on the concept of loop formation for the structural synthesis of non-isomorphic kinematic chains. For example, a 10-link kinematic chain shown in Figure 3.11(a) was constructed by the combination of a basic loop, Figure 3.11(b), and a mode, Figure 3.11(c). However, the authors claimed that it is easy to obtain all possible modes. In 2006, Sunkari and Schmidt [77] proposed a synthesis algorithm using group theory for the isomorphism-free generation of planar non-fractionated kinematic chains. The proposed algorithm is performed by applying a McKay-type algorithm and comprising a degeneracy testing algorithm. In 2010, Martins et al. [78] used a method based on the concept of Assur group for the generation of planar fractionated kinematic chains with up to 4 loops and 6

DOFs. Since 2005, Ding and coauthors [79-93] proposed a computer-aided approach for the structural synthesis of planar non-fractionated and fractionated kinematic chains with different DOFs based on contracted graphs [87, 88, 89, 90, 93], perimeter topological graphs [80, 82, 85], characteristic representation code [79, 80, 85], isomorphism identification [79, 81, 82, 83, 85, 87, 91], and rigid sub-chain detection [81, 83, 86, 91, 93]. The proposed method can eliminate isomorphic, degenerate, and rigid chains exhaustively.

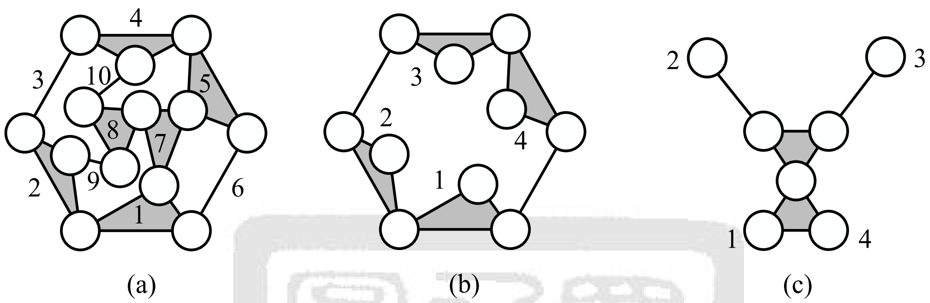


Figure 3.11 A 10-link kinematic chain constructed by the combination of a basic loop and a mode

3.11 Summary

The known results of the enumeration of non-fractionated planar and non-planar kinematic chains with simple joints, with 6 to 15 links and 1 to 7 DOFs by scholars in various publications are summarized as listed in Table 3.1. It is worth mentioning that the numbers of kinematic chains presented by Tuttle [66-68] are less than the other authors. However, the results of Tuttle are much similar to Lee and Yoon [38]. Tuttle eliminated the kinematic chains with crossing links. Therefore, the reason is due to the different definition of kinematic chains. In addition, references [10, 19, 25] based on the intuition or visual inspection method did not meet the right number of the (10, 13) kinematic chains. Reference [15] synthesized only 12 kinematic chains with 8 links and 1 DOF based on their visual method. In addition, Kiper and Schian [20] presented 6856 kinematic chains with 12 links and 1 DOF, but Hwang et al. [61-

63] synthesized 6862 non-degenerate and non-isomorphic kinematic chains with 12 links and 1 DOF. The main reason for the different results is due to the different methods or the different definitions of kinematic chains by the authors. Different methods may synthesize some invalid kinematic chains or eliminate some valid kinematic chains.

This chapter presents an exhaust search regarding literature review on the number synthesis of non-fractionated kinematic chains with simple joints. Various available methods, such as intuition or visual inspection, Franke's notation, graph theory, Baranov trusses, transformation of binary chains, matrices, group theory, stratified method, and other methods are studied, presented and discussed. In addition, computer programs for the automatic synthesis (and sketching) of kinematic chains are developed based on the proposed methods. Some differences in the total numbers of kinematic chains are enumerated by different authors mainly due to different definitions of kinematic chains; such as defining a kinematic chain as a kinematic chain with simple-jointed or multiple-jointed, fractionated or non-fractionated, planar or non-planar, and so forth. Furthermore, since the isomorphic and degenerate chains must be identified and eliminated, the atlas of kinematic chains are reckoned and synthesized.

Table 3.1 Numbers of kinematic chains

Links	Joints	DOFs	Planar Kinematic Chains				Kinematic Chains			
			Common Results	Refs.	Other Results	Refs.	Common Results	Refs.	Other Results	Refs.
6	7	1	2	[36, 37, 38, 68, 69, 77, 78]			2	[18, 19, 21, 22, 25, 26, 35, 38, 40, 41, 50, 51, 52, 61, 62, 63, 66, 71, 72, 76, 78, 89]		
7	8	2	3	[36, 37, 38, 68, 69, 77, 78]			4	[23, 35, 61, 62, 63, 76, 78]	3	[18, 38, 41, 50, 51, 66, 90]
8	9	3	5	[37, 38, 68, 69, 77, 78]			7	[35, 61, 62, 63, 78]	5	[38, 66]
	10	1	16	[36, 37, 38, 67, 68, 69, 77, 78]			16	[17, 21, 22, 25, 26, 27, 35, 38, 40, 44, 56, 61, 62, 63, 66, 70, 71, 72, 76, 78, 89]	12	[15]
9	10	4	6	[38, 77]			10	[35, 61, 62, 63]	6	[38]
	11	2	35	[36, 37, 38, 67, 68, 69, 77, 78]			40	[35, 48, 61, 62, 63, 76, 78]	35	[22, 23, 38, 66, 90]
10	11	5	8	[78]			14	[35, 61, 62, 63, 78]		
	12	3	74	[38, 67, 68, 69, 77, 78]			98	[36, 63, 64, 65, 73, 80]	74	[38, 66, 71]
									97	[54, 72]
	13	1	219	[38, 66, 68]	230	[69, 77, 78]	230	[19, 20, 22, 23, 35, 38, 39, 40, 41, 61, 62, 63, 66, 68, 70, 71, 72, 76, 78, 89]	222	[25]
									226	[19]
									228	[10]
11	12	6	10	[78]			19	[35, 61, 62, 63, 78]		
	13	4	126	[38, 77, 78]			189	[35, 61, 62, 63, 78]	126	[40]
	14	2	727	[38, 66, 68]	753	[69, 77, 78]	839	[35, 61, 62, 63, 78]	753	[38, 66, 68, 90]
12	13	7	12				24	[61, 62, 63]		
	14	5	212	[78]			354	[61, 62, 63, 78]		
	15	3	1,904	[66, 68]	1,902	[38]	2,442	[61, 62, 63]	1,962	[38]
									1,964	[66, 68]
					1,962	[69, 77, 78]			2,422	[78]
	16	1	5,918	[38, 68]	6,856	[77]	6,856	[20, 21, 38, 68, 69, 71, 72]	6,862	[61, 62, 63]

Table 3.1 Numbers of kinematic chains (cont.)

Links	Joints	DOFs	Planar Kinematic Chains				Kinematic Chains			
			Common Results	**Refs.**	**Other Results**	**Refs.**	**Common Results**	**Refs.**	**Other Results**	**Refs.**
13	**16**	**4**	**4,356**	[77, 78]	**4,239**	[38]	**5,915**	[78]	**4,356**	[38]
									5,951	[63]
	17	**2**	**24,271**	[68]	**24,270**	[38]	**27,496**	[38, 69, 90]	**27,497**	[68]
									29,704	[63]
14	**18**	**3**	**74,593**	[38]	**74,611**	[68]	**83,547**	[38, 69]	**83,565**	[68]
					83,547	[77]				
	19	**1**	**183,230**	[38]			**318,162**	[72, 89]	**275,255**	[38]
15	**18**	**4**	**194,413**	[38]	**216,291**	[77]	**216,291**	[28]		
	19	**2**	**1,432,730**	[77]			**1,432,732**	[90]		

Chapter 4 Basic Theories

The related basic theories for constructing and sketching generalized kinematic chains, kinematic chains, and rigid chains are presented in this chapter.

4.1 Link Assortments

The link assortments of a generalized kinematic chain or kinematic chains with N_L links and N_J joints can be obtained by solving the following two equations:

$$N_{L2} + N_{L3} + ... + N_{Li} + ... + N_{Lm} = N_L \tag{4.1}$$

$$2N_{L2} + 3N_{L3} + ... + iN_{Li} + ... + mN_{Lm} = 2N_J \tag{4.2}$$

where m is the maximum number of joints incident to a link and is constrained by the following [94]:

$$m_{\max} = \begin{cases} N_J - N_L + 2 & \text{when } N_L \le N_J \le 2N_L - 3 \\ N_L - 1 & \text{when } 2N_L - 3 \le N_J \le N_L(N_L - 1)/2 \end{cases} \tag{4.3}$$

and the number of joints N_J is constrained by the following expression:

$$N_L \le N_J \le N_L(N_L - 1)/2 \tag{4.4}$$

For the (5, 6) generalized kinematic chains shown in Figures 2.11(b) and (c), based on Equation (4.3), m_{max} is:

$$m_{\max} = N_J - N_L + 2 = 3$$

Therefore, Equations (4.1) and (4.2) become:

$$N_{L2} + N_{L3} = 5$$

$$2N_{L2} + 3N_{L3} = 12$$

By solving these two equations, the link assortment A_L is:

$$A_L = [3/2]$$

It consists of three binary links and two ternary links as shown in Figure 4.1.

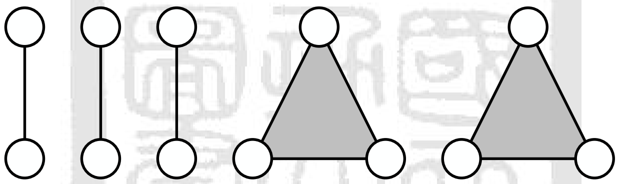


Figure 4.1 Type of links in (5, 6) generalized kinematic chains

4.2 Contracted Link Assortments

The contracted link assortment of a generalized kinematic chain or kinematic chains with N_L links and N_J joints must satisfy the following two equations [4]:

$$N_{C2} + N_{C3} + ... + N_{Ci} = N_C \tag{4.5}$$

$$N_{C2} + 2N_{C3} + ... + (i-1)N_{Ci} = N_{L2} \tag{4.6}$$

where i can be determined by:

$$1 < i \le N_{L2} - N_C + 2 \tag{4.7}$$

and the range of N_C can be defined as:

$$\max.\ \{1,\ J_m - J'_m\} \le N_C \le \min.\ \{N_{L2},\ J_m\} \tag{4.8}$$

where J_m is half the total number of joints of the multiple links and can be determined as follows:

$$J_m = 1.5N_{L3} + 2N_{L4} + \ldots + 0.5iN_{Li} + \ldots + 0.5mN_{Lm} \tag{4.9}$$

and J'_m is the number of joints between two multiple links and can be determined as follows:

$$J'_m = \begin{cases} 0 & \text{if } N_m = 1 \\ \frac{1}{2}\left[3(N_m - 1) - 1\right] & \text{if } N_m = 2,4,6,\ldots \\ \frac{1}{2}\left[3(N_m - 1) - 2\right] & \text{if } N_m = 3,5,7,\ldots \end{cases} \tag{4.10}$$

N_{C1} can be obtained by the following expression:

$$N_{C1} = N_J - 2N_{C2} - 3N_{C3} - \ldots - iN_{Ci} \tag{4.11}$$

Based on Equations (4.5)-(4.10), all possible contracted link assortments can be obtained for a given link assortment. For the (5, 6) generalized kinematic chains based on Equations (4.7)-(4.9), the range of N_C is:

$$2 \le N_C \le 3$$

Therefore, Equations (4.5) and (4.6) become:

$$N_{C2} + N_{C3} = N_C$$

$$N_{C2} + 2N_{C3} = 3$$

By solving these two equations, the contracted link assortments A_{CL} are:

$$A_{CL} = [1/1],[3/0]$$

The first contracted link assortment consists of one contracted link with two series joints and one contracted link with three series joints shown in Figure 4.2(a). The second contracted link assortment consists of three contracted links with two series joints as shown in Figure 4.2(b).

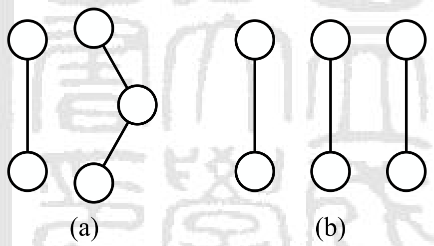


Figure 4.2 Type of contracted links in (5, 6) generalized kinematic chains

4.3 Synthesis of Multiple Link Adjacency Matrices

After the link assortments and the contracted link assortments are obtained, the MLAMs can be synthesized. For each generalized kinematic chain or kinematic chain, it corresponds to one link assortment and one contracted link assortment. Furthermore, each link assortment and contracted link assortment may construct different generalized kinematic chains or kinematic chains. In order to assign perfectly and avoid making an isomorphism, the method of repeatable assignment with non-isomorphism [67] is used for synthesizing the MLAM.

Therefore, some constraints for the synthesis of MLAMs are described as follows:

1. A multiple link with *m* joints (with *n* self-loops) must be adjacent to other multiple links by *m*-2*n* contracted links. If not, the generalized kinematic chain or kinematic chains would be a disconnected chain.
2. A multiple link with *m* joints (without self-loops) must be adjacent to other multiple links by *m* contracted links. If a generalized kinematic chain has three or more multiple links with *m* joints (without self-loops), any two of the multiple links cannot be adjacent to *m* contracted links. If not, the generalized kinematic chain or kinematic chains would be a disconnected chain.
3. One contracted link with one series joint (S_{J1}) and two contracted links with two series joints (S_{J2}) cannot form a self-loop with a multiple link. The contracted links cannot be assigned to the diagonal elements of the MLAM. Therefore, the diagonal elements of the MLAM should be zero to avoid self-loops or cut-links in the generalized kinematic chains or kinematic chains.
4. There can only exist one contracted link with one series joint (S_{J1}) between two multiple links and, the generalized kinematic chains or kinematic chains cannot be assigned to more than two contracted links with one series joint (S_{J1}). Therefore, the elements cannot be assigned two "1"s in the off-diagonal of the MLAM.

If the MLAM does not satisfy the above constraints, the matrix should be deleted. Therefore, the MLAM can be synthesized based on the link assortments, the contracted link assortments, and the above constraints.

Figure 4.3 illustrates the procedure for the synthesis of MLAMs. The details of each step are presented as follows:

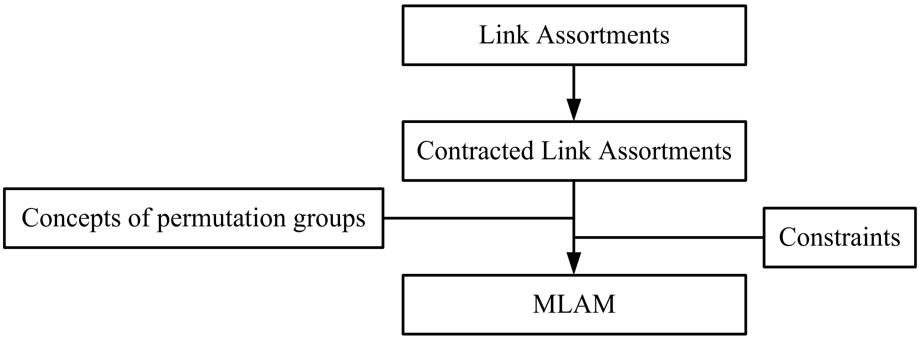


Figure 4.3 Synthesis procedure of MLAMs

Step 1. Link assortments

Based on Equations (4.1)-(4.4), the link assortments can be obtained.

For the (5, 6) generalized kinematic chains, the link assortment is A_L = [3/2].

Step 2. Contracted link assortments

Based on Equations (4.5)-(4.10), the contracted link assortments can be obtained. Moreover, the number of contracted links with one series joint (N_{C1}) can be obtained from Equation (4.11).

For the (5, 6) generalized kinematic chain, the contracted link assortments are A_{CL}=[1/1], [3/0]. Thus, N_{C1} is found from Equation (4.11) and N_{C1}=[1], [0], respectively.

Step 3. MLAM

The MLAM can be obtained based on the contacted link assortments obtained in Step 2, the concepts of permutation groups, and the above constraints.

In such a case, there are two ternary links and three binary links based on its link assortment. Therefore, the dimension of its MLAM is two. Based on the first contracted link assortment A_{CL}=[1/1] and N_{C1}=1, the elements of MLAM consist of one contracted link with three series joints (S_{J3}), one contracted link with two series joints (S_{J2}), and one contracted link with one series joint (S_{J1}).

Let the MLAM have the form as follows:

$$\text{MLAM} = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$$

The elements e={3, 2, 1} can be the set of contracted link types and can be assigned to the elements of the MLAM based on the permutation groups and repeatable assignment with non-isomorphism [63, 67]. The possible MLAM is as follows:

$$\text{MLAM} = \begin{bmatrix} 0 & 321 \\ 321 & 0 \end{bmatrix}$$

For MLAM=$\begin{bmatrix} 0 & 321 \\ 321 & 0 \end{bmatrix}$, e_{12}=e_{21}=321 mean that a contracted link with three series joints (S_{J3}) is adjacent to two ternary links, a contracted link with two series joints (S_{J2}) is adjacent to two ternary links, and a contracted link with one series joint (S_{J1}) is adjacent to two ternary links. The (5, 6) generalized kinematic chain can be obtained as shown in Figure 2.11(b).

Similarly, based on the second contracted link assortment A_{CL}=[3/0] and N_{C1}=0, the elements of the MLAM consist of three contracted links with two series joints (S_{J2}). The elements e={2, 2, 2} can be assigned to the elements of the MLAM, and the possible MLAM is as follows:

$$\text{MLAM} = \begin{bmatrix} 0 & 222 \\ 222 & 0 \end{bmatrix}$$

For MLAM=$\begin{bmatrix} 0 & 222 \\ 222 & 0 \end{bmatrix}$, e_{12}=e_{21}=222 mean that three contracted links with two series joints (S_{J2}) are adjacent to two ternary links. Therefore, the chain can be obtained as shown in Figure 2.11(c).

4.4 Synthesis of Link Adjacency Matrices

This section introduces the algorithm for the synthesis of link adjacency matrix (LAM). As shown in Figure 4.4, the LAM is synthesized by transforming the multiple link adjacency matrix (MLAM). The main steps in the synthesis algorithm are described as follows:

Step 1. Input an MLAM and set initial values *Dim* and *NewDim*

Since the MLAM and LAM are both symmetric matrices, the LAM can be synthesized by calculating the elements of either the upper triangular matrix or the lower triangular matrix. Each element of the upper triangular MLAM should be inputted. Besides, the value *Dim* is the dimension of the MLAM and is a constant. The value *NewDim* is a variable. The meaning of the value *NewDim* is to expand the dimension of the matrix. The element e_{ij} of MLAM is imported to perform the next step.

Step 2. Calculate LAM[*i, j*] and LAM[*j, i*]

The values x and e_{ij} can be obtained based on the following two equations:

$$x = e_{ij} - e_{ij}/10 \times 10 \tag{4.12}$$

$$e_{ij} = e_{ij}/10 \tag{4.13}$$

where e_{ij} is the element of the MLAM[i, j] and the value $e_{ij}/10$ must be an integer. The purpose of Equation (4.12) is to save the element of the last digit, i.e., the one before the decimal point. The meaning of the value x is to find out the adjacency relationship of the contracted links. The purpose of Equation (4.13) is to save the remaining digits. It means to save the other types of contracted links in the chain. If the value x=0, i.e., there are no relationship between link i and link j, go back to Step 1 and perform another element. If the value $x=1$, i.e., link i is adjacent to link j. Then, LAM[i, j]=LAM[j, i]=1. Otherwise, go to Step 3.

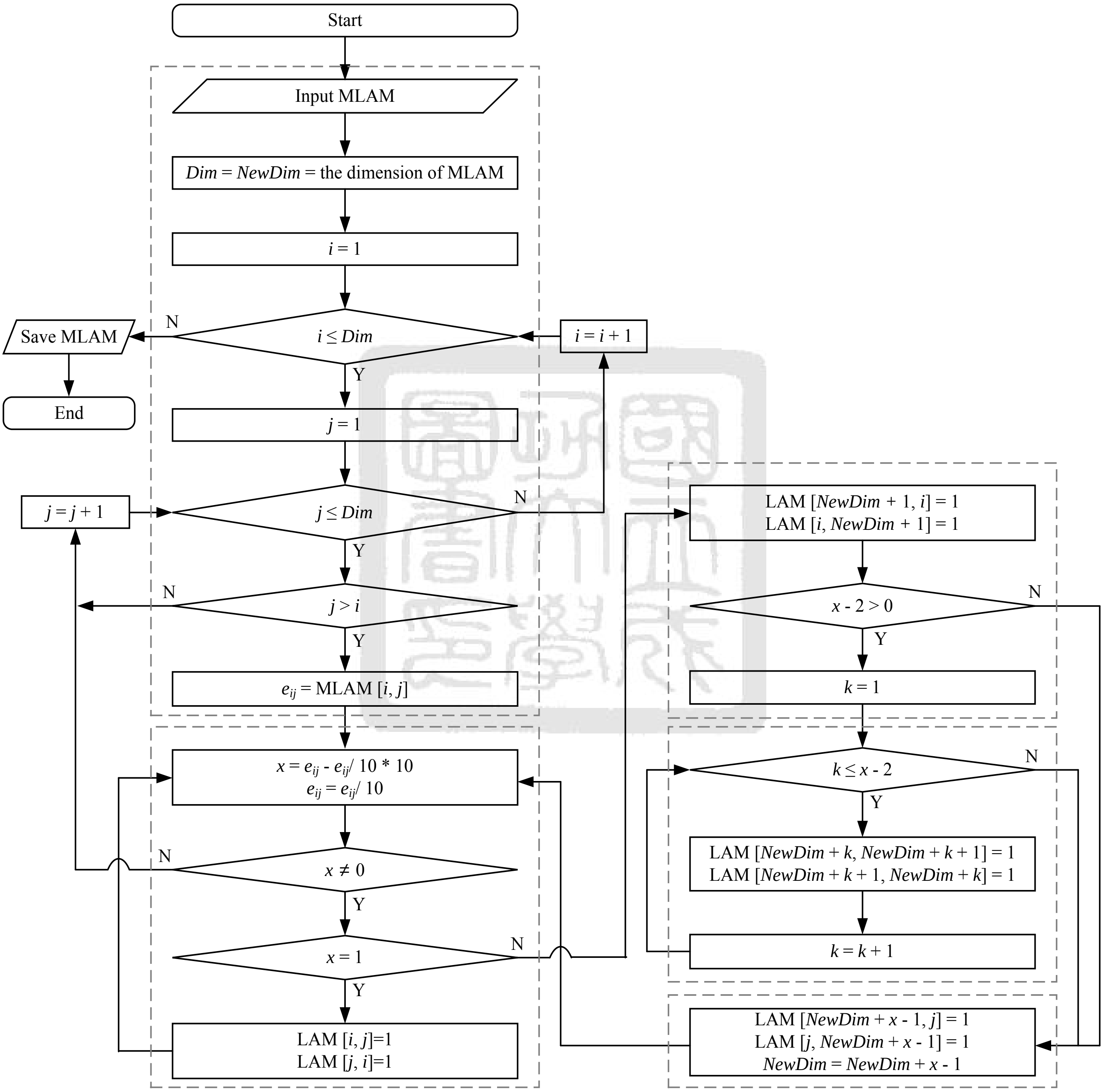


Figure 4.4 Synthesis algorithm of LAMs

Step 3. Calculate LAM[*NewDim*+1, *i*] and LAM[*i*, *NewDim*+1]

Since the value $x \neq 1$, LAM[*NewDim*+1, *i*]=LAM[*i*, *NewDim*+1]=1. If the value $x-2>0$, value $k=1$ and go to Step 4. Otherwise, go to Step 5.

Step 4. Calculate LAM[*NewDim*+*k*, *NewDim*+*k*+1] and LAM[*NewDim*+*k*+1, *NewDim*+*k*]

If the value $k \leq x-2$, LAM[$NewDim+k$, $NewDim+k+1$]=LAM[$NewDim+k+1$, *NewDim*+*k*]=1. Then, the new value $k=k+1$, and repeat this step again. If no, go to Step 5.

Step 5. Calculate LAM[*NewDim*+*x*-1, *j*] and LAM[*j*, *NewDim*+*x*-1]

Since the value $x-2 \leq 0$ or $k > x-2$, LAM[$NewDim + x - 1$, *j*]=LAM[*j*, *NewDim*+*x*−1]=1. Then, the new value $NewDim = NewDim + x - 1$ and go back to Step 2. Once all of the elements have been determined, the LAM can be obtained and saved.

The purpose of Steps 3 and 5 are to obtain the relationship between the binary links and multiple links. The purpose of Step 4 is to obtain the relationship between two binary links. Through LAM synthesis algorithm, the LAM can be synthesized and obtained. The (5, 6) generalized kinematic chain shown in Figure 2.11(b) is applied to illustrate the algorithm. The MLAM of the (5, 6) generalized kinematic chain is expressed as: $\text{MLAM} = \begin{bmatrix} 0 & 321 \\ 321 & 0 \end{bmatrix}$.

1. Input the MLAM. Since the dimension of the MLAM is two, the value $Dim = 2$. Furthermore, the $e_{12} = 321$ is as an input data and go to Step 2.

2. Based on Equations (4.12) and (4.13), the values x and e_{ij} can be obtained as follows: $x = 321 - 321/10 \times 10 = 1$, $e_{12} = 321/10 = 32$. The value $x = 1$, i.e., link 1 is adjacent to link 2. Then, LAM[*i*, *j*]=LAM[*j*, *i*]=LAM[1, 2]=LAM[2, 1]=1 and repeat Step 2 again.

3. The new $e_{12} = 32$, the new values x and e_{12} are obtained as $x = 32 - 32/10 \times 10 = 2$, $e_{12} = 32/10 = 3$. The value $x = 3$, go to Step 3.
4. Since the value $x = 2$ and $NewDim = 2$, LAM[$NewDim+1$, i] and LAM[i, $NewDim+1$]= LAM[3, 1]=LAM[1, 3]=1. Furthermore, the value $x - 2 = 0$ $(2 - 2 = 0)$, go to Step 5.
5. Since the value $x - 2 = 0$, LAM[$NewDim+x-1$, j]=LAM[j, $NewDim+x-1$]=LAM[3, 2]=LAM[2, 3]=1. Then, the new value $NewDim = 3$, go back to Step 2.
6. The new values x and e_{ij} are as $x = 3 - 3/10 \times 10 = 3$, $e_{12} = 3/10 = 0$ .The value $x = 3$, go to Step 3.
7. Since the value $x = 3$ and $NewDim = 3$, LAM[$NewDim+1$, i]=LAM[i, $NewDim+1$]= LAM[4, 1]=LAM[1, 4]=1. Furthermore, the value $x - 2 > 0$ $(3 - 2 > 0)$, so the value $k = 1$ and go to Step 4.
8. Since the value $k = x - 2$ $(1 = 3 - 2)$, LAM[$NewDim+k$, $NewDim+k+1$]=LAM [$NewDim+k+1$, $NewDim+k$]=LAM[4, 5]=LAM[5, 4]=1. Then, the new value $k = 2$, and go to Step 5.
9. Since the value $k > x - 2$ $(2 > 3 - 2)$, LAM[$NewDim+x-1$, j]=LAM[j, $NewDim+x-1$]=LAM[5, 2]=LAM[2, 5]=1. Then, the new value $NewDim = 5$ and go back to Step 2.
10. Since the new value x=0, all elements of the upper triangular MLAM have been determined. Furthermore, other elements of the upper triangular MLAM are all zero, go to save the LAM. The LAM of the (5, 6) generalized kinematic chain shown in Figure 2.11(b) can be expressed as:

$$\mathrm{LAM}_{(5,\ 6)} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Through LAM synthesis algorithm, another (5, 6) generalized kinematic chains as shown Figure 2. 11(c) can be synthesized and expressed as:

$$\mathrm{LAM}_{(5,\ 6)} = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Once the LAM are determined, the relationship among the links can be known. The various kinematic chains can be sketched based on the corresponding LAM.

4.5 Planar Blocks and Generalized Kinematic Chains

Since the early 1960s, graph theory has been applied to the structural synthesis and analysis of various types of chains and mechanisms. Based on the terminology and definitions in Section 2.2, the generalized kinematic chains can be synthesized by means of transforming the planar blocks. A planar block can be transformed into its corresponding generalized kinematic chains by representing the vertices and edges of the planar blocks with links and joints, respectively, in which two links in the chain are adjacent whenever the corresponding vertices in the graph are adjacent. For a given planar block, the following process describes how to construct the corresponding generalized kinematic chains from planar blocks based on some of the concepts from the hypergraphs and line graphs [95]:

Step 1. For each vertex, list those edges incident with the vertex.

Step 2. Construct the corresponding line graphs G_L described in Section 2.2.9.

Step 3. Replace each vertex of G_L with a small circle and replace each complete subgraph of G_L which is determined by a vertex of a planar block of degree at least three by a shaded polygon. This is done by removing the interior edges to obtain a perimeter polygon and then shading the interior of this polygon.

The resulting configuration is the corresponding generalized kinematic chain for the planar block. Every generalized kinematic chain has a unique associated planar block and each planar block generates a unique generalized kinematic chain. In addition, every theorem and concept about planar blocks transforms into a statement is valid for every generalized kinematic chain. For the atlas of (5, 6) planar blocks shown in Figure 4.5, the corresponding atlas of (5, 6) generalized kinematic chains can be constructed as follows:

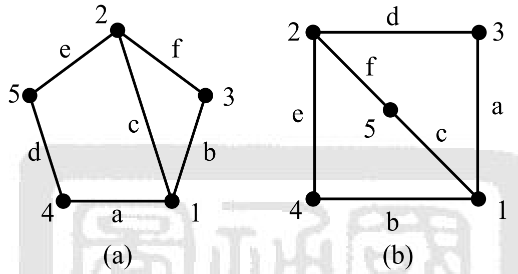


Figure 4.5 Atlas of (5, 6) planar blocks

Step 1. List those edges incident with the vertices. For the (5, 6) planar block shown in Figure 4.5(a), V_1=(a, b, c), V_2=(c, e, f), V_3=(b, f), V_4=(a, d), V_5=(d, e). For the (5, 6) planar block shown in Figure 4.5(b), V_1=(a, b, c), V_2=(d, e, f), V_3=(a, d), V_4=(b, e), V_5=(c, f).

Step 2. Construct the atlas of (5, 6) line graphs as shown in Figure 4.6.

Step 3. Replace each vertices of G_L with a small circle, remove the interior edges to obtain the perimeter polygon, and shade the interior of this polygon.

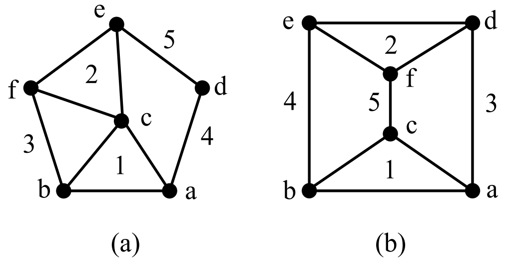


Figure 4.6 Atlas of (6, 7) line graphs

The result is the atlas of (5, 6) generalized kinematic chains as shown in Figures 2.11(b) and (c). Through these three steps, the planar blocks can be transformed into its corresponding generalized kinematic chains.

4.6 Basic Contracted Graphs

A basic contracted graph is a graph which consists of multiple vertices (links), i.e., ternary vertices (links), quaternary vertices (links), and so forth. The basic contracted graphs can be obtained by contracting binary vertices (links) until no binary vertices (links). The atlas of basic contracted graphs with up to five loops were presented by Tempea [42], then Yan and Hwang [44] obtained 19 basic contracted graphs with up to five loops. Based on the numbers of loops, Equation (2.1), and its multiple link assortments in Section 2.5.2, the basic contracted graphs are constructed and classified. Therefore, the basic contracted graphs with two to four loops are presented in Appendix A and those with five loops are in Reference [42]. This is the data bank for the sketching of the generalized kinematic chains, kinematic chains, and rigid chains. For the (10, 13) kinematic chains as shown in Figure 3.11, the corresponding graph is shown in Figure 4.7(a). The basic contracted graph as shown in Figure 4.7(b) is synthesized by contracting binary vertices 3, 6, 9, and 10.

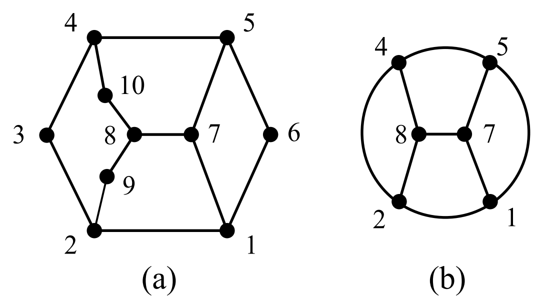


Figure 4.7 A basic contracted graph of (5, 6) generalized kinematic chains

4.7 Summary

Some basic theories regarding the generalized kinematic chains and kinematic chains of mechanisms and rigid chains of structures are put forward in this chapter. The link assortment is to solve the numbers and types of links in the chains. Besides, the contracted link assortment is to solve the number of contracted links with different series joints. Based on the link assortments, its corresponding contracted link assortments are obtained. In other words, each link assortment is correspond to one or many contracted link assortments.

For each link assortment and contracted link assortment, the MLAMs are synthesized and obtained. Similarly, each link assortment and contracted link assortment are correspond to one or many MLAMs. Different MLAMs are synthesized based on the concept of permutation groups and the constraints which are described in Section 4.3. The repeatable assignment of the elements in MLAMs are avoided by utilizing the concept of permutation groups. For this reason, the isomorphic kinematic chains of mechanisms are eliminated. Furthermore, each MLAM is transformed into one LAM based on the LAM synthesis algorithm. The LAM can help us to sketch the generalized kinematic chains, kinematic chains, and rigid chains.

Every generalized kinematic chain has a unique associated planar block. Each planar block generates a unique generalized kinematic chain. Therefore, a planar block corresponds to one generalized kinematic chain. The theorem helps us to construct and sketch the generalized kinematic chains. These basic theories and equations will be applied in the following Chapters 5 and 6.

Chapter 5 Generalized Kinematic Chains

This chapter presents an enumeration algorithm to construct and sketch the atlas of generalized kinematic chains by utilizing the concepts of MLAMs, LAMs and graph theory. Based on the definitions in Chapter 2, a generalized kinematic chain is a chain with no cut-links and crossing links. Therefore, the algorithms for checking cut-links and Kuratowski graphs are put forwarded. The results of this work can generate all non-isomorphic and non-fractionated generalized kinematic chains. Besides, examples are provided to illustrate the algorithms for checking cut-links and Kuratowski graphs, and the enumeration algorithm of generalized kinematic chains.

5.1 Algorithm for Checking Cut-links

An algorithm for checking cut-links is presented to identify the chains with cut-links. Based on the terminology and definitions in Chapter 2, the chain with cut-links is known as a fractionated kinematic chain (see more details in Section 2.4.4). If the chain exists cut-links, the chain should be deleted; otherwise, the chain would be a disconnected chain.

The algorithm for checking cut-links is mainly to remove the link in order and then examine the off-diagonal elements in the MLAM. Accordingly, a chain without cut-links means that its off-diagonal elements of the matrix must be zero. The chains with or without cut-links can be determined by means of the algorithm for checking cut-links, as shown in Figure 5.1. Each step of the algorithm is described as follows:

Step 1. Input an MLAM to execute the algorithm and go to Step 2.

Step 2. Replace all elements of link i in the MLAM with 0 and assume $i = 1, 2, \ldots, n$. Then, go to Step 3.

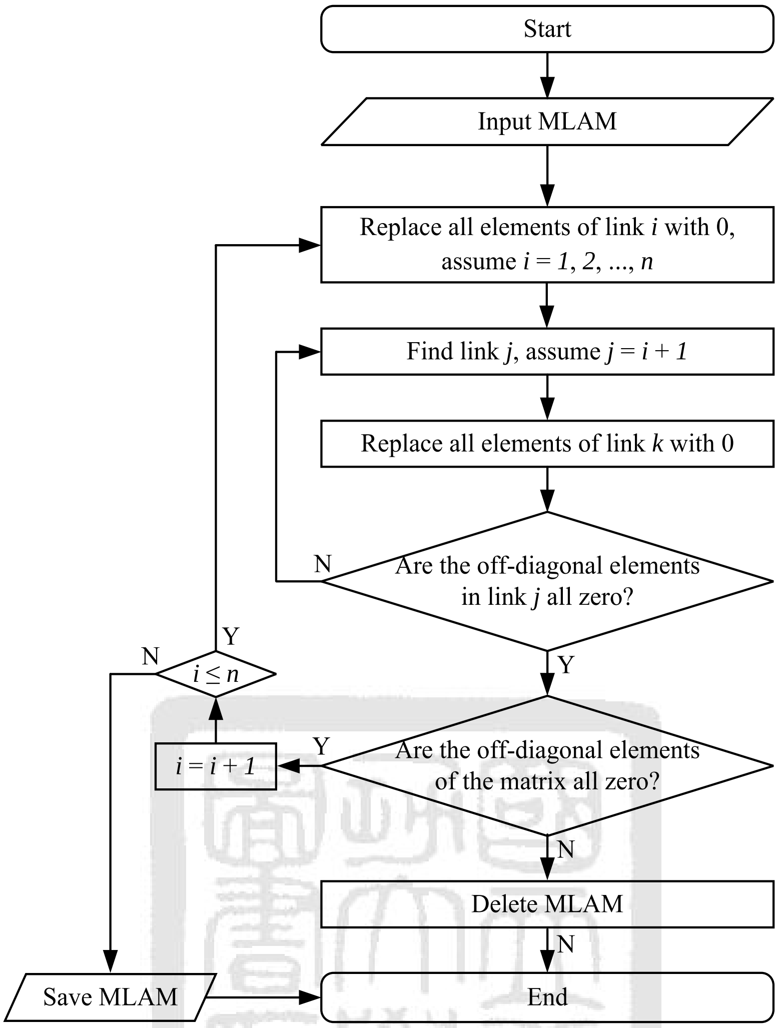


Figure 5.1 Algorithm for checking cut-links

Step 3. Check the elements of link j ($j = i + 1$) in the matrix that is a link adjacent to link k. The row k in the matrix should be added to the row j (row k ＋ row j) and go to Step 4.

Step 4. Replace all elements of link k in the matrix with 0, then go to Step 5.

Step 5. If the off-diagonal elements in link k are all zero, go to Step 6; otherwise, go to Step 3.

Step 6. If the off-diagonal elements of the matrix are all zero, $i = i + 1$ and go to Step 7; otherwise, the MLAM must be deleted and stop the algorithm.

Step 7. If $i \le n$, go to Step 2; otherwise, save the MLAM and stop the algorithm.

Here is an example to illustrate the algorithm for checking cut-links. The MLAM of a (7, 11) chain shown in Figure 5.2 is expressed as below:

$$\text{MLAM} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 21 & 0 & 0 & 0 \\ 1 & 21 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

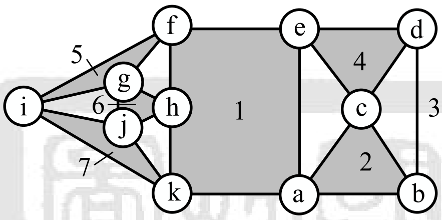


Figure 5.2 A (7, 11) chain

1. Input an MLAM to execute the algorithm and go to Step 2.
2. Start from link 1, all of elements in link 1 (row and column 1) have to be replaced with 0. The matrix is as below:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 21 & 0 & 0 & 0 \\ 0 & 21 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

and go to Step 3.

3. The element of link 2 ($j = i + 1$) in the matrix is adjacent to link 3. Row 3 in the matrix should be added to row 2 (row 3 + row 2). Then, the matrix is as below:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 21 & 21 & 0 & 0 & 0 \\ 0 & 21 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

and go to Step 4.

4. All elements in link 3 (row and column 3) in the matrix have to replace with 0. The matrix is as below:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 21 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

and go to Step 5.

5. Since the off-diagonal elements in link 2 (row and column 2) are all zero, go to Step 6.
6. Since the off-diagonal elements of the matrix are not all zero, this chain must have cut-links, and link *1* is a cut-link. Consequently, the chain with a cut-link is not a generalized kinematic chain or kinematic chain, it should be eliminated.

In a brief summary, the MLAM is as an input data to execute the algorithm for checking cut-links. There are not too many equations or functions to execute the algorithm. Therefore, the algorithm for checking cut-links is straightforward to identify the chains with cut-links.

5.2 Algorithm for Checking Kuratowski Graphs

Since there is a one-to-one correspondence between generalized kinematic chains and planar blocks, graph theory is applied to construct the generalized kinematic chains. The algorithm for checking Kuratowski graphs utilizing the MLAMs as an input data is proposed to examine the planar or non-planar blocks or graphs. If a graph has subgraph homeomorphic to $K_{3,3}$ or K_5 as shown in Figures 2.8(a) and (b), respectively, the graph is a non-planar graph; otherwise, it is a planar graph.

If the dimension of the MLAM is more than six, the dimension of the MLAM should be reduced to 5×5 or 6×6 matrices. Then, the matrices need to be identified whether the $K_{3,3}$ or K_5 matric is included in the matrix or not. If the dimension of the MLAM is less than five, the graphs are all planar. The MLAMs of $K_{3,3}$ and K_5 are expressed as follows:

$$\mathrm{MLAM}_{K3,3} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{MLAM}_{K5} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

The algorithm for checking Kuratowski graphs is shown in Figure 5.3 and each step is described as follows:

Step 1. Input an MLAM to execute the algorithm and go to Step 2.

Step 2. Replace all elements that are more than 1 with 1 in the MLAM. The new matrix is called the Transformation Link Adjacency Matrix (TLAM) and go to Step 3.

Step 3. If the sum of each row is more than 2, go to Step 4; otherwise, go to Step 8.

Step 4. If the dimension of the matrix is more than 6, go to Step 7; otherwise, go to Step 5.

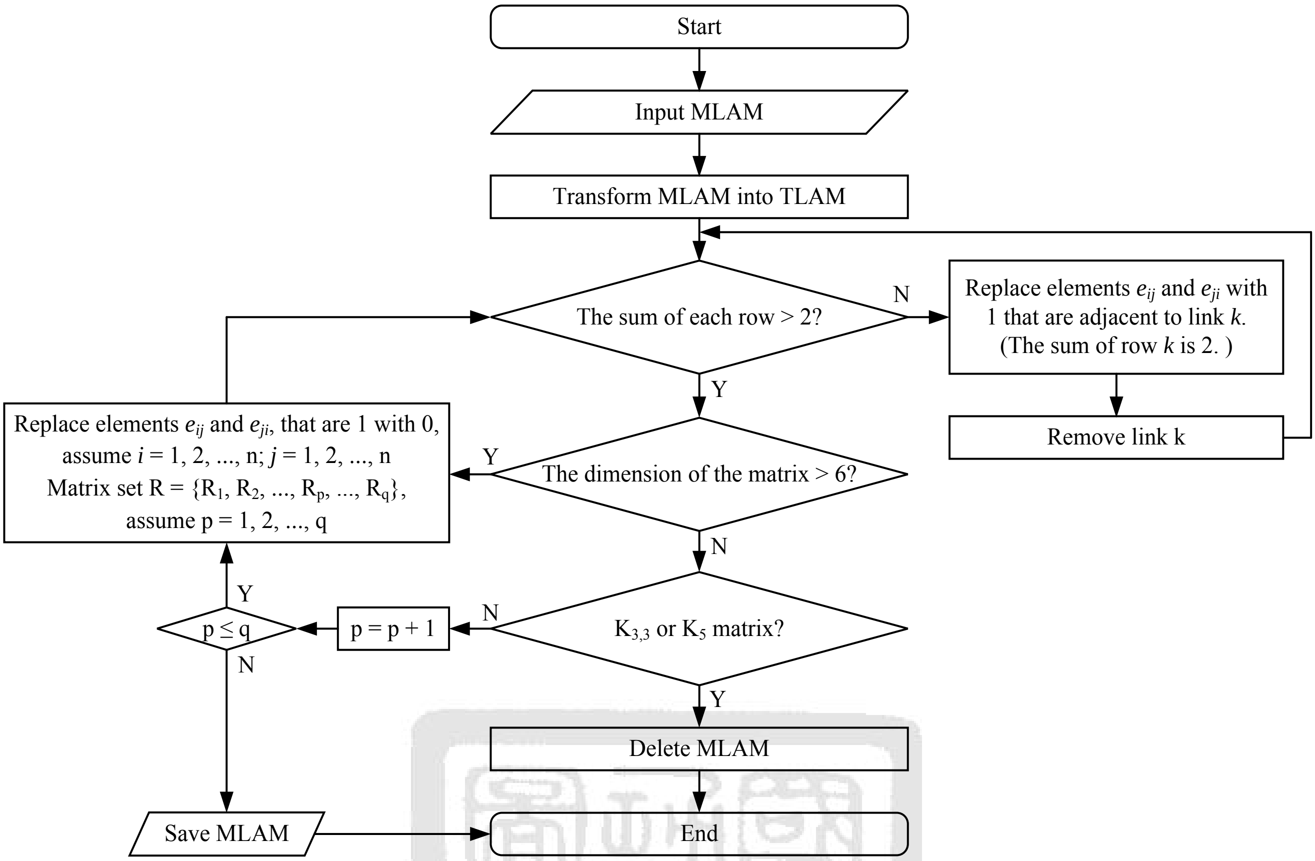


Figure 5.3 Algorithm for checking Kuratowski graphs

Step 5. If matrices $K_{3,3}$ or K_5 is included in the matrix, the MLAM should be deleted. It means that the graph is a non-planar one. If no, $p = p + 1$ and go to Step 6.

Step 6. If $p \leq q$, go to Step 7. If all elements have been determined and the matrices $K_{3,3}$ or K_5 are not exist, then the MLAM can be applied to construct the generalized kinematic chain.

Step 7. Replace elements e_{ij} and e_{ji} that are 1 (and assume i = 1, 2, …, n; j = 1, 2, …, n) with 0 and then go to Step 3. Note that number n is the dimension of the matrix. Furthermore, it will produce p matrices (R = {R_1, R_2, …, R_p, …, R_q}) and the value q depends on the sum of the elements in the matrix.

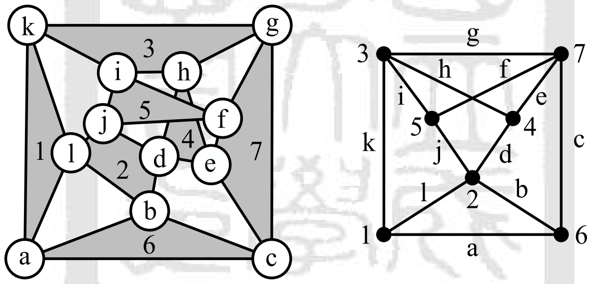
Step 8. Replace elements e_{ij} and e_{ji} that are adjacent to link k with 1, and the sum of row k is equal to two. Then, go to Step 9.

Step 9. Delete the row and column k and then go to Step 3.

The MLAM is used as an input data to perform the algorithm for checking Kuratowski graphs. Based on the algorithm, non-planar graphs can be eliminated.

Two examples are provided to illustrate the algorithm for checking Kuratowski graphs. For the first example, the MLAM of the (7, 12) chain shown in Figure 5.4(a) is expressed as below:

$$\mathrm{MLAM}_{(7,12)} = \mathrm{TLAM} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$



(a) (7, 12) chain (b) (7, 12) graph

Figure 5.4 A (7, 12) chain and its graph

1. Input an MLAM of the (7, 12) chain and go to Step 2.
2. Since all of elements in the MLAM are 1 or 0, the MLAM is also a TLAM and go to Step 3.
3. Since the sum of each row is more than 2, go to Step 4.
4. Since the dimension of the matrix is more than 6, go to Step 7.
5. Replace elements e_{12} and e_{21} that are 1 with 0 and the matrix R_1 is as below:

$$R_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Then, go to Step 3.

6. Since the sum of row 1 is two, go to Step 8.

7. Replace elements e_{36} and e_{63} that are adjacent to link 1 with 1, the matrix is as below:

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Then, go to Step 9.

8. Since the sum of row 1 is two, row and column 1 can be deleted. The matrix is as below:

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Then, go to Step 3.

9. Since the sum of each row is more than 2, go to Step 4.

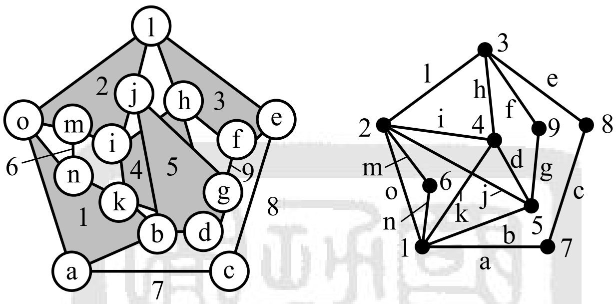
10. Since the dimension of the matrix is 6, go to Step 5.

11. Since $K_{3,3}$ matrix is included in the matrix, the graph is a non-planar one. The chain and

its graph are shown in Figures 5.4(a) and (b), respectively.

For the second example, the MLAM of the (9, 15) chain shown in Figure 5.5(a) is expressed as:

$$\mathrm{MLAM}_{(9,15)} = \begin{bmatrix} 0 & 21 & 3 & 1 & 1 \\ 21 & 0 & 1 & 1 & 1 \\ 3 & 1 & 0 & 2 & 1 \\ 1 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$



(a) (9, 15) chain (b) (9, 15) graph

Figure 5.5 A (9, 15) chain and its graph

1. Input the MLAM of the (9, 15) chain and go to Step 2.
2. The elements that are more than 1 should be replaced with 1 in the MLAM. The transformation link adjacency matrix (TLAM) is as below:

$$\mathrm{TLAM} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Then, go to Step 3.

3. Since the sum of each row is more than 2, go to Step 4.
4. Since the dimension of the matrix is 5, go to Step 5.

5. Since the TLAM is the K_5 matrix, the graph is a non-planar one. The chain and its graph are shown in Figures 5.5(a) and (b), respectively

All non-planar graphs can be deleted. The atlases of planar blocks and generalized kinematic chains can be constructed by means of the algorithm for checking Kuratowski graphs.

5.3 Sketching Algorithm

An algorithm for sketching various types of generalized kinematic chains is proposed based on its corresponding LAM and graph theory. Figure 5.6 shows the algorithm and each step is illustrated with an example.

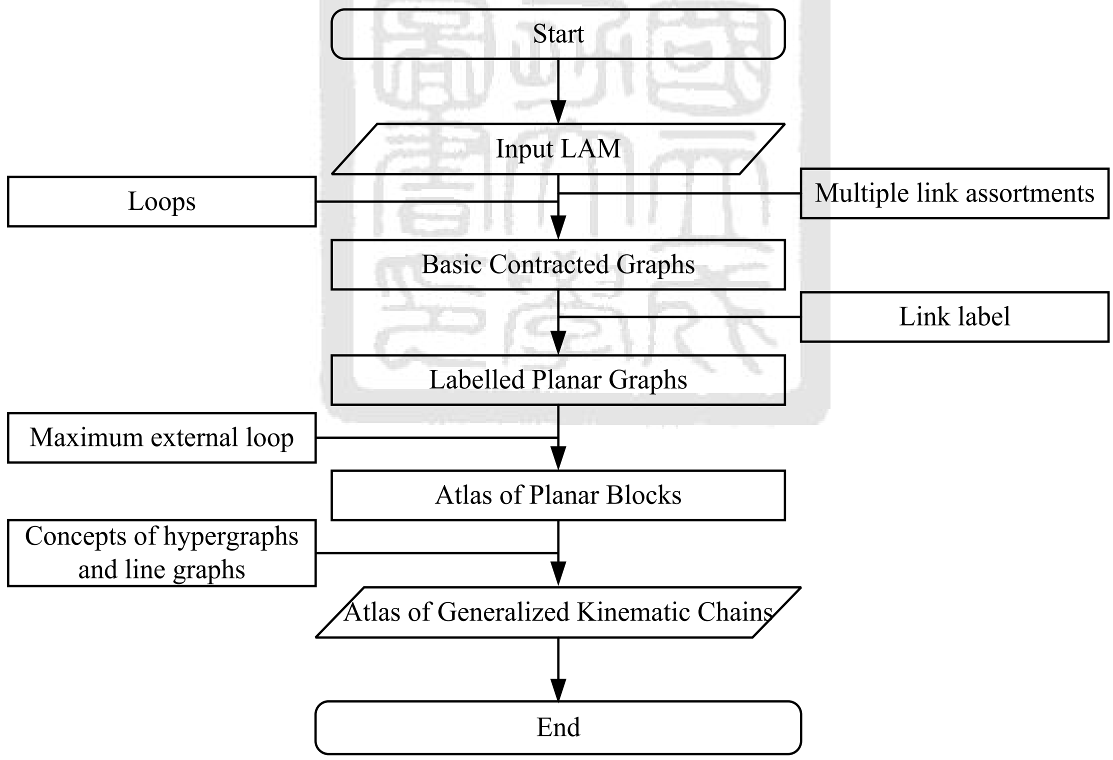


Figure 5.6 Sketching algorithm

Step 1. Link adjacency matrix

The LAM of a generalized kinematic chain is as an input data. Based on the LAM synthesis algorithm in Section 4.4, the LAM can be obtained.

The two LAMs of the (5, 6) generalized kinematic chains are synthesized as follows:

$$\mathrm{LAM}_{(5,\ 6)\_1} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}, \quad \mathrm{LAM}_{(5,\ 6)\_2} = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Step 2. Basic contracted graphs

Based on the multiple link assortments and the number of loops, the basic contracted graphs can be identified from Appendix A. Based on Equation (2.1), the number of loops is two. Since the multiple link assortment A_{ML} = [2], there are only two ternary vertices in the graph. Therefore, the basic contracted graph with two loops and two ternary vertices can be obtained from Appendix A, as shown in Figure 5.7(a).

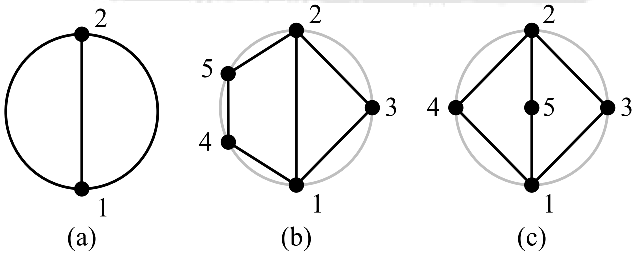


Figure 5.7 Labelled planar graphs with five vertices and six edges

Step 3. Labelled planar graphs

Based on the LAMs, binary vertices need to be added to the edges of its basic contracted graph. Then, vertices x and y are connected by an edge which are adjacent to each other based on it LAM. Thus, the labelled planar graphs can be obtained.

From Step 2, the ternary vertices can be labelled "1" and "2" in the basic contracted graphs shown in Figure 5.7(a). Then, binary vertices must be added in the graph as shown in Figures 5.7(b) and (c). For Figure 5.7(b), binary vertex 3 should be added between vertices 1, 2, and connected the vertices 1, 3, 2 by edges based on the $LAM_{(5,6)\_1}$. Binary vertices 4, 5 should be added between vertices 1, 2, and connected the vertices 1, 4, 5, 2 by edges based on the $LAM_{(5,6)\_1}$. For Figure 5.7(c), binary vertices 3, 4, and 5 should be added between vertices 1 and 2, and connected the vertices by edges based on the $LAM_{(5,6)\_2}$. Therefore, the labelled planar graphs with five vertices and six edges can be obtained as shown in Figures 5.7(b) and (c).

Step 4. Atlas of planar blocks

Based on the labelled planar graphs obtained in Step 3, the maximum external loop can be obtained. The outer circle can be deleted. Then, the planar blocks should be redrawn and the atlas of planar blocks can be sketched.

For the (5, 6) labelled planar graphs shown in Figure 5.7(b), the maximum external loop consists of vertices 1, 3, 2, 5, 4. Since there are five vertices in the external loop, this is a pentagon. Therefore, the graph is redrawn as shown in Figure 4.5(a). For the (5, 6) labelled planar graphs shown in Figure 5.7(c), the maximum external loop consists of vertices 1, 3, 2, 4. Since there are four vertices in the external loop, this is a quadrangle. The graph is redrawn as shown in Figure 4.5(b). Therefore, the atlas of (5, 6) planar blocks can be sketched as shown in Figure 4.5.

Step 5. Atlas of generalized kinematic chains

Each planar block can be transformed into a single generalized kinematic chain based on the atlas of planar blocks obtained in Step 4 and some of the concepts from the hypergraphs and line graphs [95]. Besides, a generalized kinematic chain can be transformed from its corresponding planar block based on the proposed method in Section 4.5.

For the atlas of (5, 6) planar blocks shown in Figure 4.5, the atlas of generalized kinematic chains can be obtained as shown in Figures 2.11(b) and (c), respectively.

Based on the proposed sketching algorithm, the aesthetic characteristics of generalized kinematic chains and planar blocks with no link or edge crossings can be sketched and obtained.

5.4 Enumeration Algorithm of Generalized Kinematic Chains

Based on the algorithms for checking cut-links and Kuratowski graph, and the sketching algorithm, the enumeration algorithm of generalized kinematic chains is proposed for the construction of atlases of generalized kinematic chains as shown in Figure 5.8. In what follows, each step is illustrated with an example.

Step 1. Input numbers of links and joints

The atlas of (6, 7) generalized kinematic chains is selected as an illustrative example.

Step 2. Calculate link assortments

The link assortments, $A_L=\{A_{L1}, A_{L2}, \ldots, A_{Lp}\}$, can be obtained and assume $i = 1, 2, \ldots, p$, based on Equations (4.1)-(4.4). Then, go to Step 3.

The link assortment of (6, 7) generalized kinematic chains is A_L=[4/2]. It consists of four binary links and two ternary links as shown in Figure 5.9.

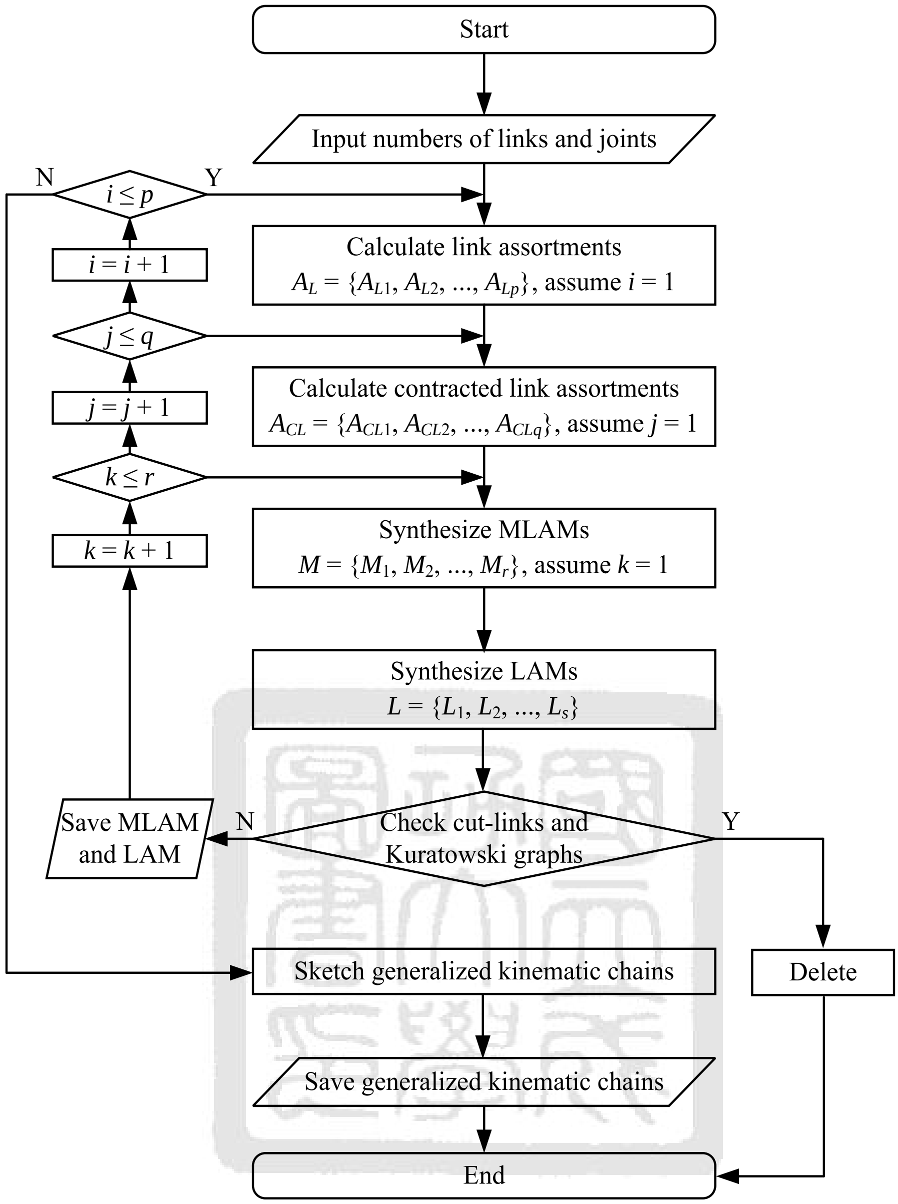


Figure 5.8 Enumeration algorithm of generalized kinematic chains

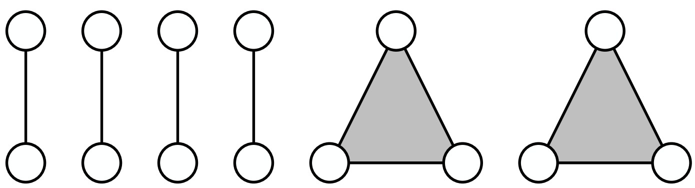


Figure 5.9 Type of links in (6, 7) generalized kinematic chains

Step 3. Calculate contracted link assortments

Based on Equations (4.5)-(4.10), the contracted link assortments, A_{CL}={A_{CL1}, A_{CL2}, …, A_{CLq}}, can be obtained and assume j = 1, 2, …, q. Then, go to Step 4.

Based on link assortment A_L=[4/2], the corresponding contracted link assortments are A_{CL1}=[0/2/0], A_{CL2}=[2/1/0], and A_{CL3}=[1/0/1]. The first contracted link assortment consists of two contracted links with three series joints as shown in Figure 5.10(a). The second contracted link assortment consists of two contracted links with two series joints and one contracted link with three series joints as shown in Figure 5.10(b). The third contracted link assortment consists of one contracted link with two series joints and one contracted link with four series joints as shown in Figure 5.10(c).

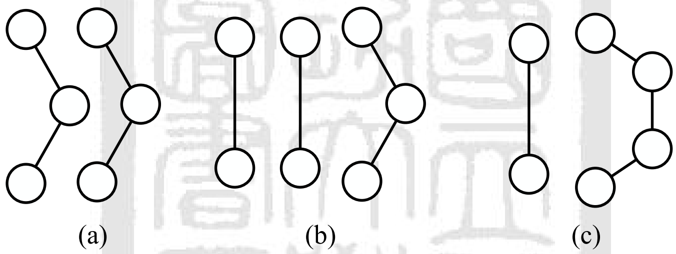


Figure 5.10 Type of contracted links in (6, 7) generalized kinematic chains

Step 4. Synthesize MLAMs

Based on Section 4.3, the MLAMs, M={M_1, M_2, …, M_k, …, M_r}, can be synthesized and assume k = 1, 2, …, r. Then, go to Step 5.

From Step 2, there are two ternary links and four binary links in the (6, 7) generalized kinematic chains. Since there are two multiple links, the dimension of the MLAM is two.

For the (6, 7) generalized kinematic chains, three MLAMs can be synthesized based on its corresponding link assortment and contracted link assortment. The MLAMs of the (6, 7) generalized kinematic chains are expressed as:

$$M_1 = \begin{bmatrix} 0 & 311 \\ 311 & 0 \end{bmatrix},\ M_2 = \begin{bmatrix} 0 & 322 \\ 322 & 0 \end{bmatrix},\ M_3 = \begin{bmatrix} 0 & 421 \\ 421 & 0 \end{bmatrix}$$

Step 5. Synthesize LAMs

Based on Section 4.4, the LAMs, L={L_1, L_2, …, L_s}, can be synthesized. Then, go to Step 6.

For the (6, 7) generalized kinematic chains, three LAMs can be synthesized based on the corresponding MLAMs. The LAMs of the (6, 7) generalized kinematic chains are expressed as:

$$L_1 = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix},\ L_2 = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix},\ L_3 = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Step 6. Check cut-links and Kuratowski graphs

The chains with cut-links and Kuratowski graphs should be deleted based on the algorithms for checking cut-links and Kuratowski graphs. If the chain has cut-links and is a Kuratowski graph (non-planar graph), delete the chain; otherwise, save the MLAM, LAM and $k = k+1$. If $k \le r$, go back to Step 4; otherwise, $j = j+1$. If $j \le q$, go back to Step 3; otherwise, $i = i+1$. If $i \le p$, go back to Step 2; otherwise, go to Step 7.

For the (6, 7) generalized kinematic chains, the dimension of the MLAM is two. Accordingly, the three graphs are all planar. Through the algorithm for checking cut-links, the off-diagonal elements of the three MLAMs are all zero. Therefore, the chains have no cut-links. These three MLAMs and LAMs should be saved and applied to construct and sketch the (6, 7) generalized kinematic chains.

Step 7. Sketch generalized kinematic chains

All generalized kinematic chains can be sketched based on their corresponding LAMs and the sketching algorithm in Section 5.3.

Based on Equation (2.1), the number of loops is two. Since the multiple link assortment is A_{ML} = [2], the basic contracted graph can be obtained from Appendix A, as shown in Figure 5.7(a). Based on each LAM and the basic contracted graph, the labelled planar graphs shown in Figures 5.11(a)-(c) can be synthesized. Then, the outer circle can be deleted and the planar blocks are redrawn. The atlas of (6, 7) planar blocks can be obtained as shown in Figures 5.12(a)-(c). Each planar block can be transformed into a single generalized kinematic chain based on Section 4.5. Therefore, the atlas of (6, 7) generalized kinematic chains can be sketched as shown in Figures 2.10(a)-(c), respectively.

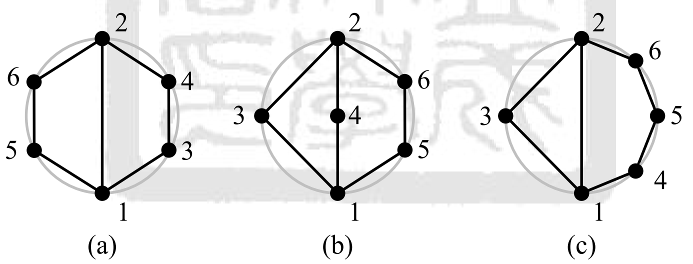


Figure 5.11 Labelled planar graphs with six links and seven joints

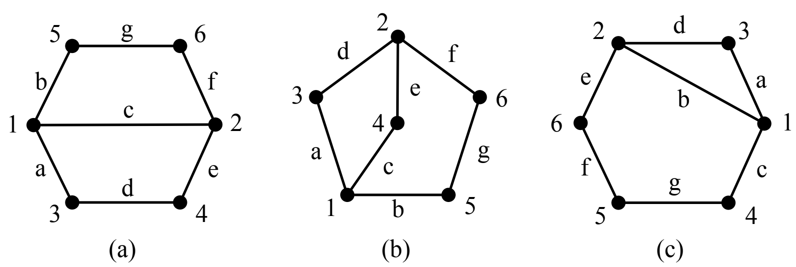


Figure 5.12 Atlas of (6, 7) planar blocks

5.5 Summary

The algorithms for checking cut-links and Kuratowski graphs, the sketching algorithm, and the enumeration algorithm of generalized kinematic chains are proposed for the synthesis of generalized kinematic chains. Since a generalized kinematic chain is a connected chain with no cut-links, the algorithm for checking cut-links is presented for deleting the chains with cut-links. Besides, the algorithm for checking Kuratowski graphs is proposed to eliminate non-planar blocks or graphs. The problem regarding the enumeration of planar blocks is solved in this work. Accordingly, all planar blocks are synthesized based on MLAMs and the algorithm for checking Kuratowski graphs. Since there has a one-to-one correspondence between generalized kinematic chains and planar blocks, a planar block can be converted to a generalized kinematic chain. Therefore, the generalized kinematic chains are synthesized by utilizing the enumeration algorithm of generalized kinematic chains.

The sketching algorithm is proposed for drawing different types of generalized kinematic chains and planar blocks based on its corresponding LAM and basic contracted graphs. However, the basic contracted graphs should be obtained first and is as the data bank to sketch different types of generalized kinematic chains and planar blocks.

The MLAMs or LAMs are applied as an input data to execute both the algorithms for checking cut-links and Kuratowski graphs, and the enumeration algorithm of generalized kinematic chains. As a result, it will be easy to develop the computer program.

Chapter 6 Kinematic Chains and Rigid Chains

This chapter proposes enumeration algorithms to construct and sketch the atlases of kinematic chains and rigid chains by utilizing the concepts of MLAMs and LAMs. Based on the definitions in chapter 2, a kinematic chain is a non-degenerate kinematic chain with positive DOFs, and the rigid chain is a chain with non-positive DOFs. Therefore, the algorithms for checking three-bar basic rigid chains and degenerate kinematic chains are proposed for the identification of degenerate kinematic chains. The results of this work can obtain all non-isomorphic and non-fractionated kinematic chains and rigid chains. Examples are provided to illustrate the algorithms.

6.1 Algorithm for Checking Three-bar Basic Rigid Chains

The purpose of this section is to propose an algorithm for checking three-bar basic rigid chains to examine whether the chains have three-bar basic rigid chains or not. If any three links are adjacent to each other, the chain may have a three-bar loop and is regarded as a degenerate kinematic chain. A degenerate kinematic chain with three-bar basic rigid chains can be detected by its configuration as shown in Figure 2.11(b). Accordingly, the algorithm shown in Figure 6.1 is proposed and each step is described as follows:

Step 1. Input an LAM to execute the algorithm and go to Step 2.

$$\mathrm{LAM}_{(8,10)} = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

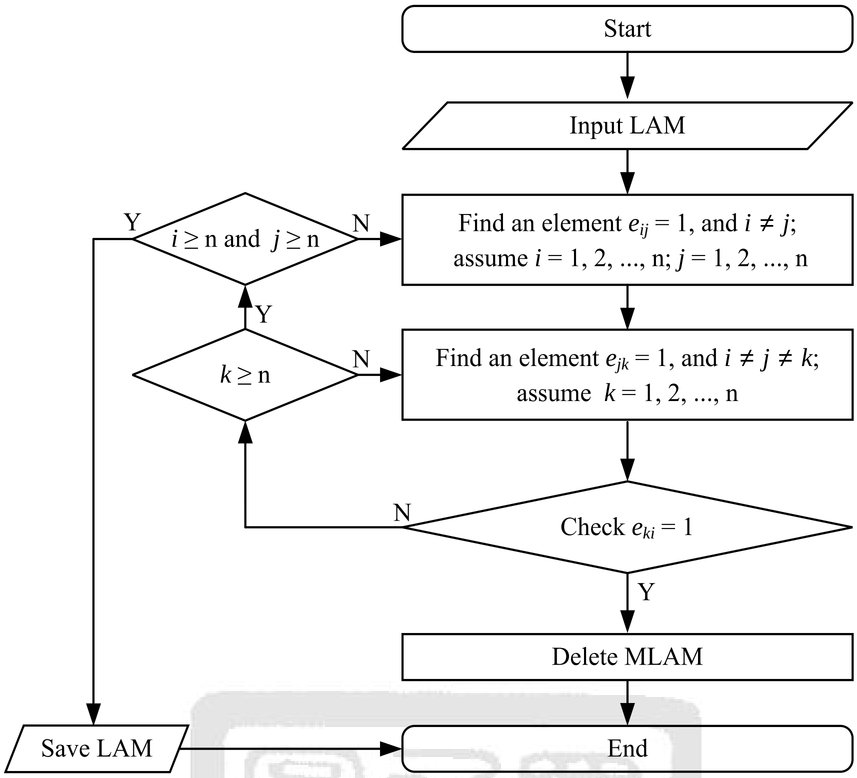


Figure 6.1 Algorithm for checking three-bar basic rigid chains

Step 2. Find an element $e_{ij} = 1$, $i \neq j$, and assume i = 1, 2, …, n; j = 1, 2, …, n. The value n is the dimension of the LAM. Then, go to Step 3.

Step 3. Find an element e_{jk} which link j is adjacent to link k. If the element $e_{jk} = 1$, $i \neq j \neq k$, and assume k = 1, 2, …, n, then go to Step 4.

Step 4. Check the element e_{ki}. If element $e_{ki} = 1$ (it means link k is adjacent to link i), the LAM should be deleted. Accordingly, the chain has three-bar basic rigid chains and it is a degenerate kinematic chain. If the element $e_{ki} = 0$, go to Step 5.

Step 5. If $k \geq n$, go to Step 6; otherwise, go to Step 3.

Step 6. If $i \geq n$ and $j \geq n$, go to save the LAM; otherwise, go to Step 2.

Here is an example to illustrate the algorithm for checking three-bar basic rigid chains. The LAM of an (8, 10) chain shown in Figure 6.2 is expressed as below:

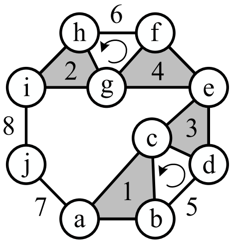


Figure 6.2 An (8, 10) chain

1. Input the LAM of the (8, 10) chain to execute the algorithm and go to Step 2.
2. Find an element $e_{ij} = e_{13} = 1$ and $i \geq j$, go to Step 3.
3. Find an element $e_{jk} = e_{34} = 1$ and $i \neq j \neq k$, go to Step 4.
4. Since element $e_{ki} = e_{41} = 0$, go to Step 5.
5. Since $k = 4 < 8$ (the dimension of the LAM), go to Step 6.
6. Since $i = 1$ and $j = 3 < 8$ (the dimension of the LAM), go to Step 3.
7. Find other element $e_{15} = 1$ and $i \neq j$, go to Step 3.
8. Find an element $e_{53} = 1$ and $i \neq j \neq k$, go to Step 4.
9. Since element $e_{31} = 1$, the LAM should be deleted. It means the chain has a three-bar loop (links 1-3-5) and it is a degenerate kinematic chain. Then, the algorithm goes to the end.

The LAM is used as an input data to execute the algorithm for checking three-bar basic rigid chains which can be successfully performed to check the degenerate kinematic chains.

6.2 Algorithm for Checking Degenerate Kinematic Chains

In the past years, some methods [10, 22, 25, 39, 48, 52-55, 61-63, 71-72, 81, 83, 86, 91, 93, 97, 98] have been proposed for the identification of degenerate kinematic chains or basic rigid chains. In 1991, Hwang and Hwang [98] developed a computer program based on the proposed method for the automatic detection of degenerate kinematic chains. According to the theorem in reference [98], all m contracted links $(m \geq 2)$ should be removed in order to simplify the kinematic chains. After all m contracted links are removed, only one contracted link remained; removing a binary link can destroy one loop of the new kinematic chain. Therefore, an algorithm for checking degenerate kinematic chains is proposed to extend their work. In the proposed algorithm, all m $(m \geq 1)$ contracted links should be removed. After m $(m \geq 1)$ contracted links are removed, then identify the DOFs for the new kinematic chain. If the $\text{DOFs} \leq 0$, the kinematic chain must contain a basic rigid sub-chain with n links. It is definitely a degenerate kinematic chain. The algorithm for checking degenerate kinematic chains is shown in Figure 6.3 and each step is described as follows:

Step 1. Input an MLAM as an input data and go to Step 2.

Step 2. If element $e_{ij} \geq 2$, go to Step 3. If no, go to Step 10.

Step 3. If the dimension of the matrix is 2, go to Step 9; otherwise, go to Step 4.

Step 4. Replace one digit of the elements e_{ij} and e_{ji}, that are no less than 2 with 0 (assume i = 1, 2, …, n; j = 1, 2, …, n), then go to Step 5. Note that the value n is the dimension of the MLAM. Furthermore, it will produce p matrices (R = {R_1, R_2, …, R_p}) and the value p depends on all of digits in the MLAM which are more than 1. For example, if $\text{MLAM} = \begin{bmatrix} 0 & 431 \\ 431 & 0 \end{bmatrix}$, it needs to replace two times and obtain two matrices R as follows: $R_1 = \begin{bmatrix} 0 & 31 \\ 31 & 0 \end{bmatrix}$, $R_2 = \begin{bmatrix} 0 & 41 \\ 41 & 0 \end{bmatrix}$. Since the digit 1 is less than 2, there is no matrix $R_2 = \begin{bmatrix} 0 & 43 \\ 43 & 0 \end{bmatrix}$.

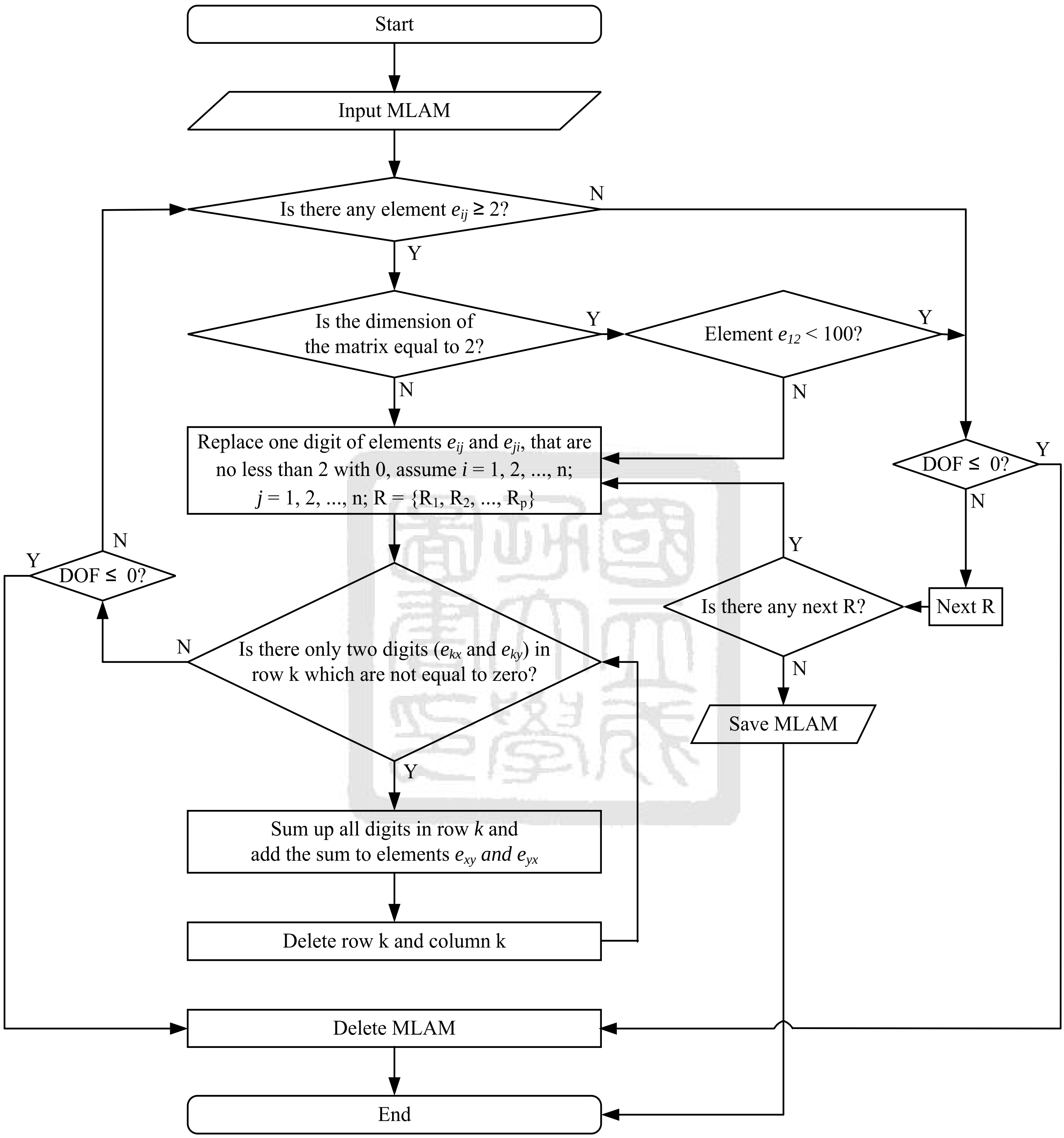


Figure 6.3 Algorithm for checking degenerate kinematic chains

Step 5. If there are only two digits (e_{kx} and e_{ky}) in row k which are not equal to zero, go to Step 6; otherwise go to Step 8.

Step 6. Sum up all digits in row k and add the sum after the elements e_{xy} and e_{yx}. Then, go to Step 7.

Step 7. Delete row k and column k, go to Step 5.

Step 8. If $\text{DOFs} \le 0$, the MLAM should be deleted and then go to an end; otherwise, go to Step 2.

Step 9. If element $e_{12} \le 100$, go to Step 10; otherwise, go to Step 4.

Step 10. If $\text{DOFs} \le 0$, the MLAM should be deleted and then go to an end; otherwise, go to Step 11.

Step 11. Execute next matrix R and go to Step 12.

Step 12. If there is another matrix R need to execute, go to Step 4. If no, go to save the MLAM and then go to an end.

The MLAM is used as an input data to perform the algorithm for checking degenerate kinematic chains. Based on the algorithm, all degenerate kinematic chains can be eliminated.

Two examples are provided to illustrate the algorithm. For the first example, the MLAM of an (8, 10) chain shown in Figure 6.4 is expressed as below:

$$\text{MLAM} = \begin{bmatrix} 0 & 4 & 1 & 1 \\ 4 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix}$$

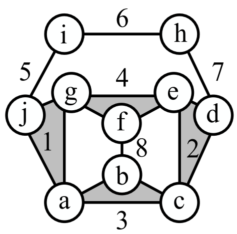


Figure 6.4 An (8, 10) degenerate kinematic chain

1. Input the MLAM of the (8, 10) chain and go to Step 2.
2. Since the element $e_{12} = 4 > 2$, go to Step 3.
3. Since the dimension of the matrix is more than 2, go to Step 4.
4. Replace the elements e_{12} and e_{21} with 0. The matrix R_1 can be obtained and expressed as follows:

$$R_1 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix}$$

Then, go to Step 5.

5. There are only two digits in the row 1, i.e., elements e_{13} and e_{14}. Then, go to Step 6.
6. The sum of all digits in row 1 are two $(e_{13} + e_{14} = 1 + 1)$ and add the sum two after the elements e_{34} and e_{43}. Therefore, elements e_{34} and e_{43} are transformed into 22 and go to Step 7.
7. Delete row and column 1, the matrix is expressed as:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 22 \\ 1 & 22 & 0 \end{bmatrix}$$

Then, go to Step 5.

8. There are only two digits in row 1, i.e., elements e_{12} and e_{13}. Then, go to Step 6.
9. The sum of all digits in row 1 are two $(e_{12} + e_{13} = 1 + 1)$ and add the sum two after the elements e_{23} and e_{32}. Therefore, elements e_{23} and e_{32} are transformed into 222 and go to Step 7.
10. Delete row and column *1*, the matrix is expressed as:

$$\begin{bmatrix} 0 & 222 \\ 222 & 0 \end{bmatrix}$$

Then, go to Step 5.

11. Since there are three digits in rows 1 and 2, go to Step 8.
12. Based on the matrix, the dimension is 2, this means two multiple links. Element e_{12} is 222, this means three contracted links with two series joints (three binary links). Therefore, there are five links (two ternary links and three binary links). Furthermore, the sum of upper triangular matrix is 6. There are six joints in the chain. Accordingly, the chain can be transformed into the (5, 6) basic rigid chain as shown in Figure 6.5(a). Based on Equation (2.2), the number of DOFs is zero. Since the (8, 10) chain is a degenerate kinematic chain, the chain must be deleted.

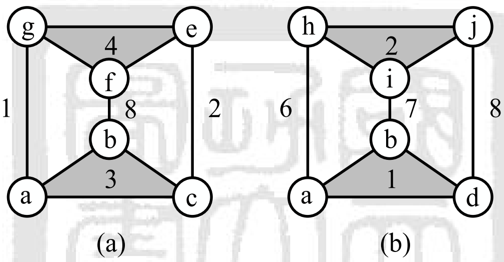


Figure 6.5 Two (5, 6) basic rigid chains

For the second example, the MLAM of an (8, 10) chain shown in Figure 6.6 is expressed as below:

$$\text{MLAM} = \begin{bmatrix} 0 & 4222 \\ 4222 & 0 \end{bmatrix}$$

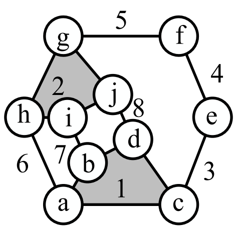


Figure 6.6 An (8, 10) degenerate kinematic chain

1. Input the MLAM of the (8, 10) chain and go to Step 2.
2. Since the element $e_{12} > 2$, go to Step 3.
3. Since the dimension of the matrix is 2, go to Step 9.
4. Since the element e_{12} is $4222 \geq 100$, go to Step 4.
5. Replace one digit of the elements e_{12} and e_{21} with 0. The matrix R_1 can be obtained and expressed as follows:

$$R_1 = \begin{bmatrix} 0 & 222 \\ 222 & 0 \end{bmatrix}$$

   Then, go to Step 5.
6. Since there are three digits in elements e_{12} and e_{21}, go to Step 8.
7. Based on the matrix R_1, the dimension is 2, this means two multiple links. Element e_{12} is 222, this means three contracted links with two series joints. Therefore, there are five links (two ternary links and three binary links). Furthermore, the sum of upper triangular matrix R_1 is 6. There are six joints in the chain. Accordingly, the chain can be transformed into a (5, 6) basic rigid chain as shown in Figure 6.5(b). Based on Equation (2.2), the number of DOFs is zero. Since the (8, 10) chain is a degenerate kinematic chain, it must be deleted.

The MLAM is used as an input data to execute the algorithm. Based on the algorithm for checking degenerate kinematic chains, all of degenerate kinematic chains are eliminated.

6.3 Enumeration Algorithm of Kinematic Chains

Based on the algorithm for checking three-bar basic rigid chains and degenerate kinematic chains, an enumeration algorithm shown in Figure 6.7 is proposed for the construction of kinematic chains. Each step is illustrated with an example as follows.

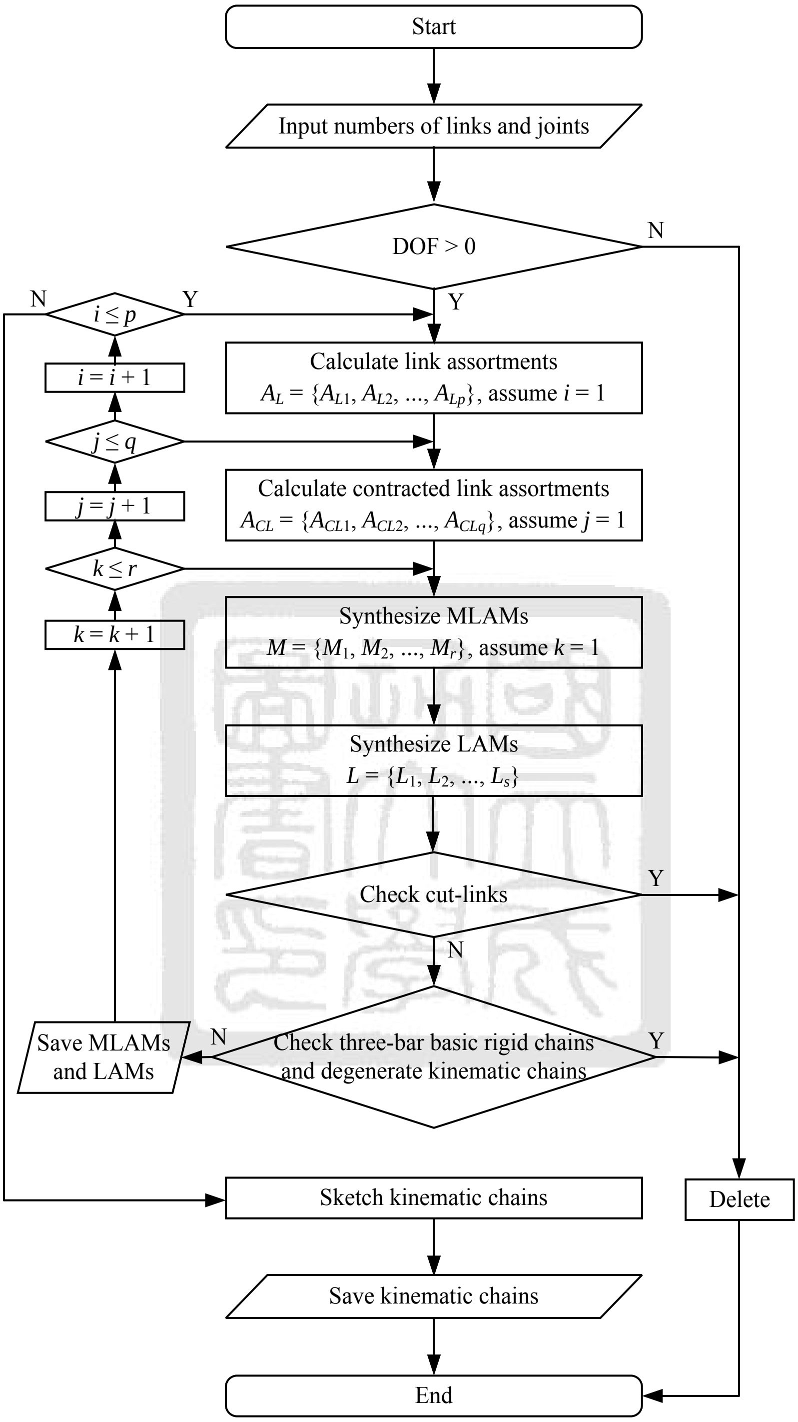


Figure 6.7　Enumeration algorithm of kinematic chains

Step 1. Input numbers of links and joints

The atlas of (7, 8) kinematic chains is selected as an illustrative example.

Step 2. Calculate DOFs

Based on Equation (2.2), the number of DOF can be obtained. If $\mathrm{DOF} > 0$, go to Step 3; otherwise, go to the end.

For the chains with 7 links and 8 joints, the DOF is 2. Therefore, the chains with 7 link and 8 joints can be constructed and go to Step 3.

Step 3. Calculate link assortments

The link assortments can be obtained, A_L={A_{L1}, A_{L2}, …, A_{Lp}}, and assume i = 1, 2, …, p, based on Equations (4.1)-(4.4). Then, go to Step 4.

The link assortment of (7, 8) kinematic chains is A_L=[5/2]. It consists of five binary links and two ternary links as shown in Figure 6.8.

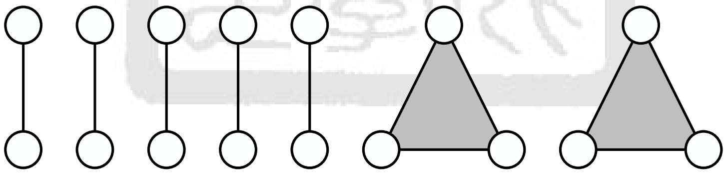


Figure 6.8 Type of links in (7, 8) kinematic chains

Step 4. Calculate contracted link assortments

Based on Equations (4.5)-(4.10), the contracted link assortments are obtained, A_{CL}={A_{CL1}, A_{CL2}, …, A_{CLq}}, and assume j = 1, 2, …, q. Then, go to Step 5.

For the (7, 8) kinematic chains, the contracted link assortments are A_{CL1}=[0/1/1/0], A_{CL2}=[1/0/0/1], A_{CL3}=[1/2/0/0], and A_{CL4}=[2/0/1/0]. The first contracted link assortment consists of one contracted link with three series joints and one contracted link with four series joints as shown in Figure 6.9(a). The second contracted link assortment consists of one contracted link with two series joints and one contracted link with five series joints as shown in Figure 6. 9(b). The third contracted link assortment consists of one contracted link with two series joints and two contracted links with three series joints as shown in Figure 6.9(c). The forth contracted link assortment consists of two contracted links with two series joints and one contracted link with four series joints as shown in Figure 6.9(d).

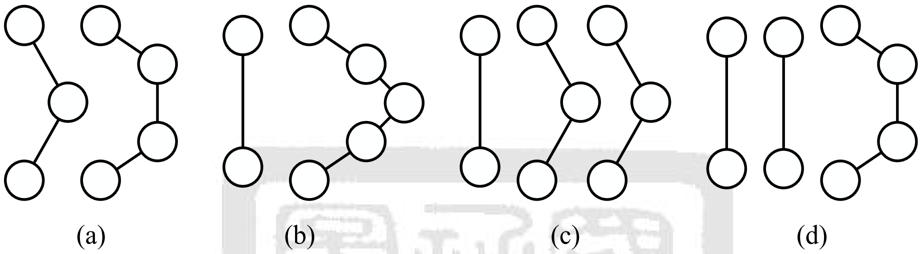


Figure 6.9 Type of contracted links in (7, 8) kinematic chains

Step 5. Synthesize MLAMs

Based on Section 4.3, the MLAMs, $M=\{M_1, M_2, \ldots, M_k, \ldots, M_r\}$, can be synthesized and assume $k = 1, 2, \ldots, r$. Then, go to Step 6.

From Step 3, there are five binary links and two ternary links in the (7, 8) kinematic chains. Since there are two multiple links, the dimension of MLAMs is two.

For the (7, 8) kinematic chains, four MLAMs are synthesized based on its corresponding link assortment and contracted link assortments. The MLAMs of the (7, 8) kinematic chains are expressed as:

$$M_1=\begin{bmatrix} 0 & 431 \\ 431 & 0 \end{bmatrix},\ M_2=\begin{bmatrix} 0 & 521 \\ 521 & 0 \end{bmatrix},\ M_3=\begin{bmatrix} 0 & 332 \\ 332 & 0 \end{bmatrix},\ M_4=\begin{bmatrix} 0 & 422 \\ 422 & 0 \end{bmatrix}$$

Step 6. Synthesize LAMs

Based on the LAM synthesis algorithm in Section 4.4, the LAMs, $L=\{L_1, L_2, \ldots, L_S\}$, can be synthesized. Then, go to Step 7.

For the (7, 8) kinematic chains, four LAMs can be synthesized based on its corresponding MLAMs. The LAMs of the (7, 8) kinematic chains are expressed as:

$$L_1=\begin{bmatrix} 0&1&1&0&0&1&0\\ 1&0&0&0&1&0&1\\ 1&0&0&1&0&0&0\\ 0&0&1&0&1&0&0\\ 0&1&0&1&0&0&0\\ 1&0&0&0&0&0&1\\ 0&1&0&0&0&1&0 \end{bmatrix},\quad L_2=\begin{bmatrix} 0&1&1&0&0&0&1\\ 1&0&0&0&0&1&1\\ 1&0&0&1&0&0&0\\ 0&0&1&0&1&0&0\\ 0&0&0&1&0&1&0\\ 0&1&0&0&1&0&0\\ 1&1&0&0&0&0&0 \end{bmatrix}$$

$$L_3=\begin{bmatrix} 0&0&1&0&1&0&1\\ 0&0&0&1&0&1&1\\ 1&0&0&1&0&0&0\\ 0&1&1&0&0&0&0\\ 1&0&0&0&0&1&0\\ 0&1&0&0&1&0&0\\ 1&1&0&0&0&0&0 \end{bmatrix},\quad L_4=\begin{bmatrix} 0&0&1&0&0&1&1\\ 0&0&0&0&1&1&1\\ 1&0&0&1&0&0&0\\ 0&0&1&0&1&0&0\\ 0&1&0&1&0&0&0\\ 1&1&0&0&0&0&0\\ 1&1&0&0&0&0&0 \end{bmatrix}$$

Step 7. Check cut-links

The chains with cut-links should be deleted based on the algorithm for checking cut-links. If the chain has no cut-links, go to Step 8; otherwise, delete the MLAM and LAM.

For the (7, 8) kinematic chains, the off-diagonal elements of these four MLAMs are all zero through the algorithm for checking cut-links in Section 5.1. Therefore, the kinematic chains have no cut-links.

Step 8. Check three-bar basic rigid chains and degenerate kinematic chains

The degenerate kinematic chains should be eliminated based on the algorithms for checking three-bar basic rigid chains and degenerate kinematic chains. If the chain is a degenerate kinematic chain, the chains must be deleted and go to an end; otherwise, go to save MLAMs, LAMs, and $k = k + 1$. If $k \le r$, go to Step 5; otherwise, $j = j + 1$. If $j \le q$, go to Step 4; otherwise, $i = i + 1$. If $i \le p$, go to Step 3; otherwise, go to Step 9.

For the second LAM of the (7, 8) chain, the elements e_{12}, e_{27}, and e_{71} are 1. Therefore, it means that links 1, 2, and 7 are adjacent to each other. There must has a 3-bar loops in the chain shown in Figure 6.10 and this is a degenerate kinematic chain. For other (7, 8) chains, they do not have three-bar loops and are not degenerate kinematic chains. Accordingly, these three MLAMs and LAMs are applied to sketch the kinematic chains.

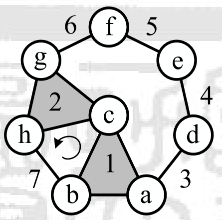


Figure 6.10 A (7, 8) degenerate kinematic chain

Step 9. Sketch kinematic chains

All kinematic chains can be obtained and sketched based on their corresponding LAMs and the sketching algorithm proposed in Section 5.3

Based on Equation (2.1), the number of loops is two. Since the multiple link assortment is A_{ML}=[2], the basic contracted graph can be obtained from Appendix A, as shown in Figure 5.7(a). Based on each LAM and the basic contracted graph, the labelled graphs can be synthesized as shown in Figures 6.11(a)-(c). Then, the outer circle can be deleted and the labelled graphs are redrawn. Based on the concepts of hypergraphs and line graphs, the atlas of (7, 8) kinematic chains can be obtained as shown in Figures 6.12(a)-(c), respectively.

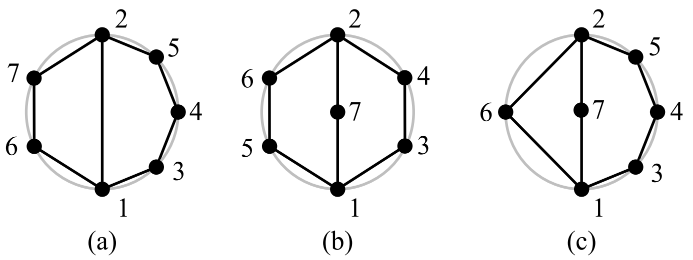


Figure 6.11 Labelled (7, 8) graphs

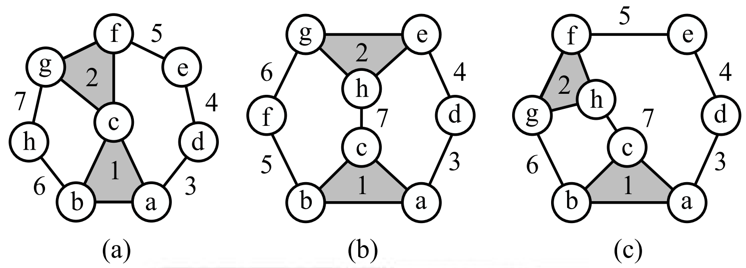


Figure 6.12 Atlas of (7, 8) kinematic chains

Through the enumeration algorithm of kinematic chains, the kinematic chains with different links and joints can be obtained based on the MLAMs and LAMs.

6.4 Enumeration Algorithm of Rigid Chains

An enumeration algorithm of rigid chains is proposed for the construction of atlases of rigid chains as shown in Figure 6.13. In what follows, each step is illustrated with an example.

Step 1. Input numbers of links and joints

The atlas of (6, 8) rigid chains is selected as an illustrative example.

Step 2. Calculate DOFs

Based on Equation (2.2), the number of DOF can be obtained. If $\text{DOF} \le 0$, go to Step 3; otherwise, go to the end.

For the chains with 6 links and 8 joints, the DOF is equal to −1. Therefore, the chains with 6 link and 8 joints are rigid chains.

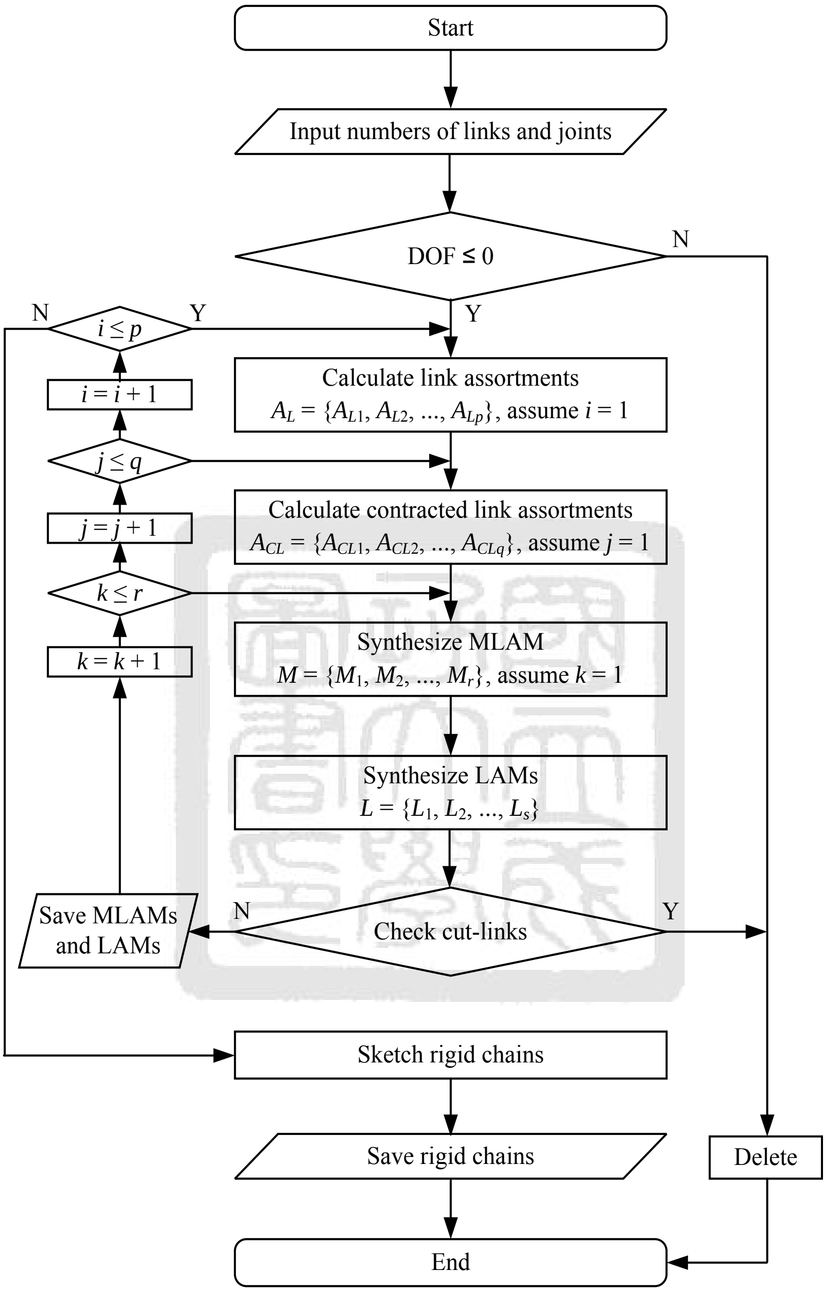


Figure 6.13 Enumeration algorithm of rigid chains

Step 3. Calculate link assortments

The link assortments, $A_L=\{A_{L1}, A_{L2}, \ldots, A_{Lp}\}$, can be obtained, and assume $i = 1, 2, \ldots, p$, based on Equations (4.1)-(4.4). Then, go to Step 4.

The link assortments of (6, 8) rigid chains are A_L=[2/4/0], [3/2/1], and [4/0/2].

Step 4. Calculate the contracted link assortments

Based on Equations (4.5)-(4.10), the contracted link assortments, $A_{CL}=\{A_{CL1}, A_{CL2}, \ldots, A_{CLq}\}$, can be obtained, and assume $j = 1, 2, \ldots, q$. Then, go to Step 5.

For the (6, 8) rigid chains, based on the first link assortment A_L=[2/4/0], the corresponding contracted link assortments are A_{CL1}=[0/1], and A_{CL2}=[2/0]. Based on the second link assortment A_L=[3/2/1], the corresponding contracted link assortments are A_{CL1}=[1/1], and A_{CL2}=[3/0]. Based on the third link assortment A_L=[4/0/2], the corresponding contracted link assortments are A_{CL1}=[2/1], and A_{CL2}=[4/0].

Step 5. Synthesize MLAMs

Based on the link assortments, the contracted link assortments, and the MLAM synthesis algorithm in Section 4.3, the MLAMs, $M=\{M_1, M_2, \ldots, M_k, \ldots, M_r\}$, can be synthesized and assume $k = 1, 2, \ldots, r$. Then, go to Step 6.

For the (6, 8) rigid chains, nine feasible MLAMs can be synthesized based on the corresponding link assortments and contracted link assortments.

Step 6. Synthesize LAMs

Based on the LAM synthesis algorithm in Section 4.4, the LAMs, $L=\{L_1, L_2, \ldots, L_S\}$, are synthesized. Then, go to Step 7.

For the (6, 8) rigid chains, nine LAMs can be synthesized based on its corresponding MLAMs.

Step 7. Check cut-links

The chains with cut-links should be eliminated based on the algorithm for checking cut-links in Section 5.1. If the chain has cut-links, it should be deleted and go to the end; otherwise, save the MLAM, LAM, and $k = k+1$. If $k \le r$, go to Step 5; otherwise, $j = j+1$. If $j \le q$, go to Step 4; otherwise, $i = i+1$. If $i \le p$, go to Step 3. If all feasible MLAMs and LAMs are synthesized, go to Step 8.

Through the algorithm for checking cut-links, the off-diagonal elements of the MLAMs are all zero. Therefore, the chains do not have cut-links. These nine LAMs can be applied to construct and sketch the atlas of (6, 8) rigid chains.

Step 8. Sketch rigid chains

All of rigid chains can be obtained and sketched based on their corresponding LAMs and the sketching algorithm proposed in Section 5.3

Based on Equation (2.1), the number of loops is three. Since the multiple link assortments are A_{ML}=[4/0], [2/1], and [0/2], the basic contracted graphs with three loops can be obtained from Appendix A, as shown in Figure 3.4. Based on each LAM and the basic contracted graphs, the labelled graphs can be synthesized as shown in Figures 6.14(a)-(i). Then, the outer circle can be deleted and the labelled graphs are redrawn. Based on the concepts of hypergraphs and line graphs, the atlas of (6, 8) rigid chains can be obtained as shown in Figures 6.15(a)-(i), respectively.

Through the enumeration algorithm of rigid chains, the rigid chains with different links and joints can be obtained based on the MLAMs and LAMs.

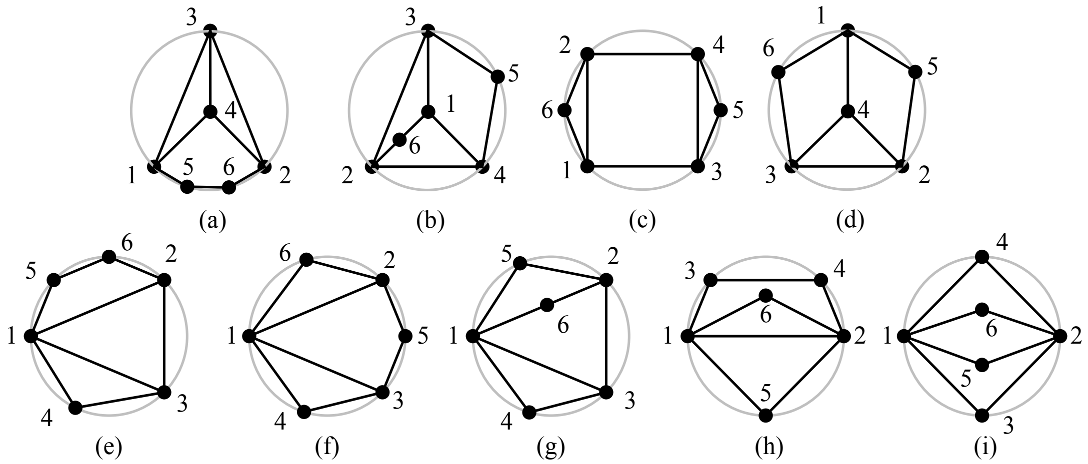


Figure 6.14 Labelled (6, 8) graphs

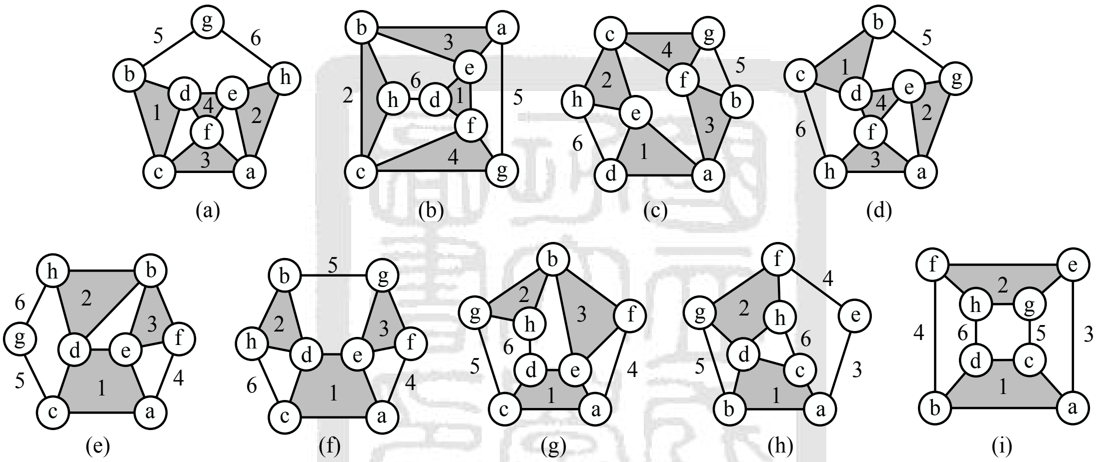


Figure 6.15 Atlas of (6, 8) rigid chains

6.5 Summary

The algorithms for checking three-bar basic rigid chains and degenerate kinematic chains are proposed to eliminate the degenerate kinematic chains. The algorithm for checking three-bar basic rigid chains is to discard the chains with the (3, 3) basic rigid chain. The algorithm for checking degenerate kinematic chains is put forward by removing binary links step by step. All of degenerate kinematic chains can be successfully eliminated by utilizing these two algorithms.

Since a kinematic chain is a chain without cut-links and basic rigid sub-chains, its enumeration algorithm is proposed based on MLAMs, LAMs, the algorithms for checking cut-links, three-bar basic rigid chains, and degenerate kinematic chains. Furthermore, the enumeration algorithm of rigid chains is presented for the synthesis of rigid chains with non-positive DOFs based on MLAMs, LAMs, and the algorithm for checking cut-links.

The LAMs are used as an input data to execute the algorithm for checking three-bar basic rigid chains and the MLAMs are used as an input data to perform the algorithm for checking degenerate kinematic chains. Each algorithm proposed in this chapter is illustrated with an example. The proposed algorithms can be applied to develop the computer program in the following chapter 7.

Chapter 7 Automatic Sketching of Chains and Graphs

The algorithms proposed in Chapters 5 and 6 are implemented into a computer program in order to facilitate the automatic sketching of generalized kinematic chains, kinematic chains, rigid chains, and planar blocks. The computer program is developed using Microsoft Visual Studio 2012 with programming language C# running on the Windows Platform. The program is executed on a PC with an Intel ® Core™ i7-3770 CPU 3.40GHz processor and 8GB RAM. Given only the numbers of links and joints as input data, the program automatically computes the link assortments, contracted link assortments, MLAMs and LAMs, and the user interface provides 27 different functions, including link assortments, contracted link assortments, sketching generalized kinematic chains, kinematic chains and rigid chains, labeling links and joints, changing the color of links, and so on, as shown in Figure 7.1. A detailed description of each of these functions is provided in Appendix B. The following sections present six illustrative examples of the computer program for generalized kinematic chains, kinematic chains, and rigid chains with different numbers of links and joints.

7.1 Generalized Kinematic Chains

Two examples are provided to illustrate the computer program for the generation of generalized kinematic chains.

7.1.1 Example 1: Atlas of (6, 9) generalized kinematic chains

For the input data with Links=6 and Joints=9, the link assortments are A_{L1}=[0/6/0/0], A_{L2}=[1/4/1/0], A_{L3}=[2/2/2/0], A_{L4}=[3/0/3/0], A_{L5}=[2/3/0/1], A_{L6}=[3/1/1/1], and A_{L7}=[4/0/0/2]. For the first link assortment, A_{L1}=[0/6/0/0], there are no contracted link assortments. For the second link assortment, A_{L2}=[1/4/1/0], the contracted link assortment is [1/0]. For the third link assortment, A_{L3}=[2/2/2/0], the contracted link

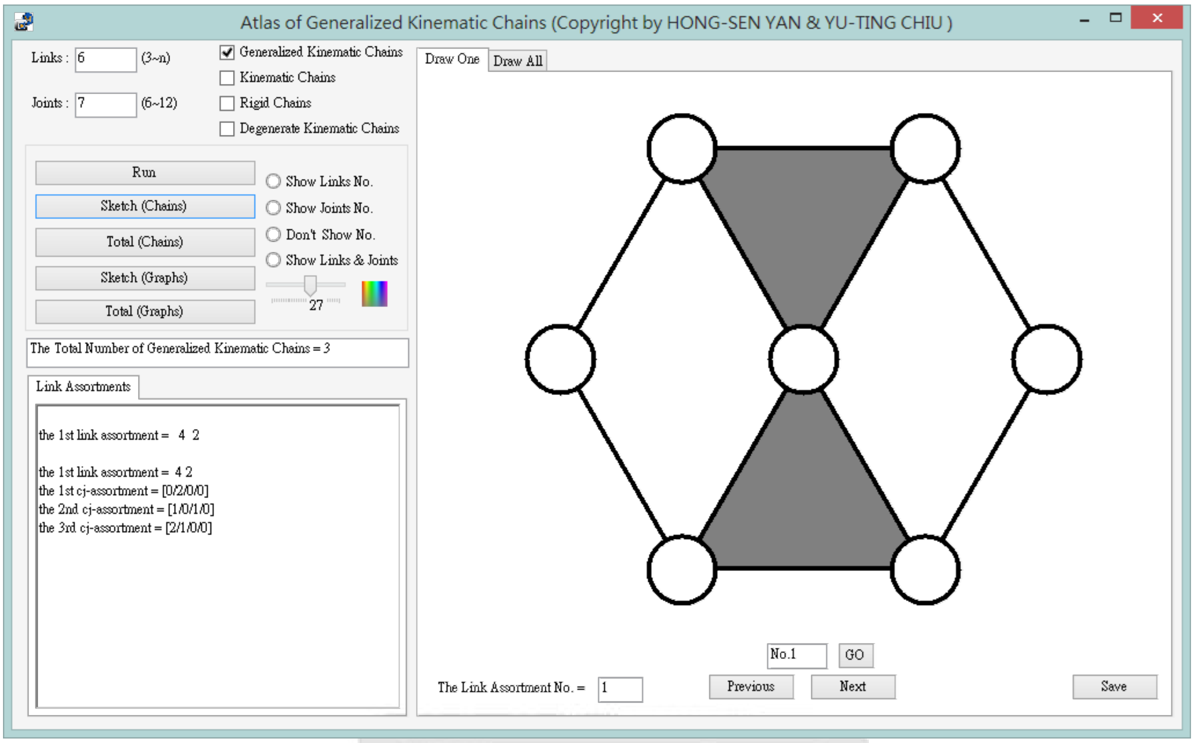


Figure 7.1 User interface of the computer program

assortments are A_{CL}=[0/1] and [2/0]. For the fourth link assortment, A_{L4}=[3/0/3/0], the contracted link assortment is A_{CL}=[3/0]. For the fifth link assortment, A_{L5}=[2/3/0/1], the contracted link assortments are A_{CL}=[0/1] and [2/0]. For the sixth link assortment, A_{L6} =[3/1/1/1], the contracted link assortment is A_{CL}=[3/0]. For the seventh link assortment, A_{L7}=[4/0/0/2], the contracted link assortment is A_{CL}=[4/0].

Based on the link assortments and the contracted link assortments, 13 feasible MLAMs and LAMs are synthesized. By utilizing the computer program, the atlas of (6, 9) generalized kinematic chains are obtained as shown in Figure 7.2. The average computational time is 107 ms (excluding the sketching time).

7.1.2 Example 2: Atlas of (8, 9) generalized kinematic chains

For the input data with Links=8 and Joints=9, the link assortment is A_L=[6/2]. The contracted link assortments are A_{CL}=[0/0/2/0/0], [0/1/0/1/0], [1/0/0/0/1], [0/3/0/0/0], [1/1/1/0/0], and [2/0/0/1/0].

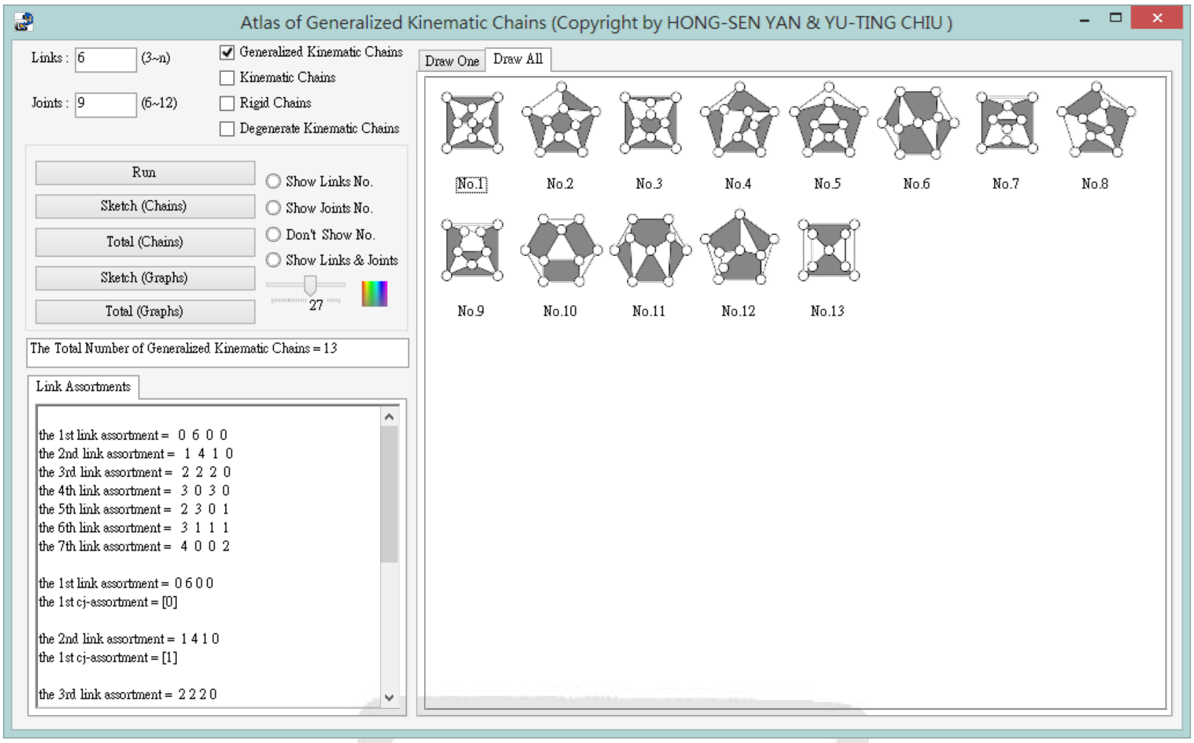


Figure 7.2 Atlas of (6, 9) generalized kinematic chains

Based on the link assortments and contracted link assortments, the 6 feasible MLAMs and LAMs are synthesized. The atlas of (8, 9) generalized kinematic chains are obtained as shown in Figure 7.3 by utilizing the computer program. The average computational time is 52 ms (excluding the sketching time).

Table 7.1 lists the results for the numbers of generalized kinematic chains with 6 to 16 link. The results are also the numbers of planar blocks with 6 to 16 vertices. The results are the same as Gargarin et al. [96]. However, the work in Reference [96] only calculated the number of planar blocks by a generating function, and it is unable to sketch the planar blocks. The proposed computer program in this work not only can enumerate the number of generalized kinematic chains (planar blocks) but also sketch the generalized kinematic chains (planar blocks). Furthermore, some important atlases of generalized kinematic chains are provided in Appendix C.

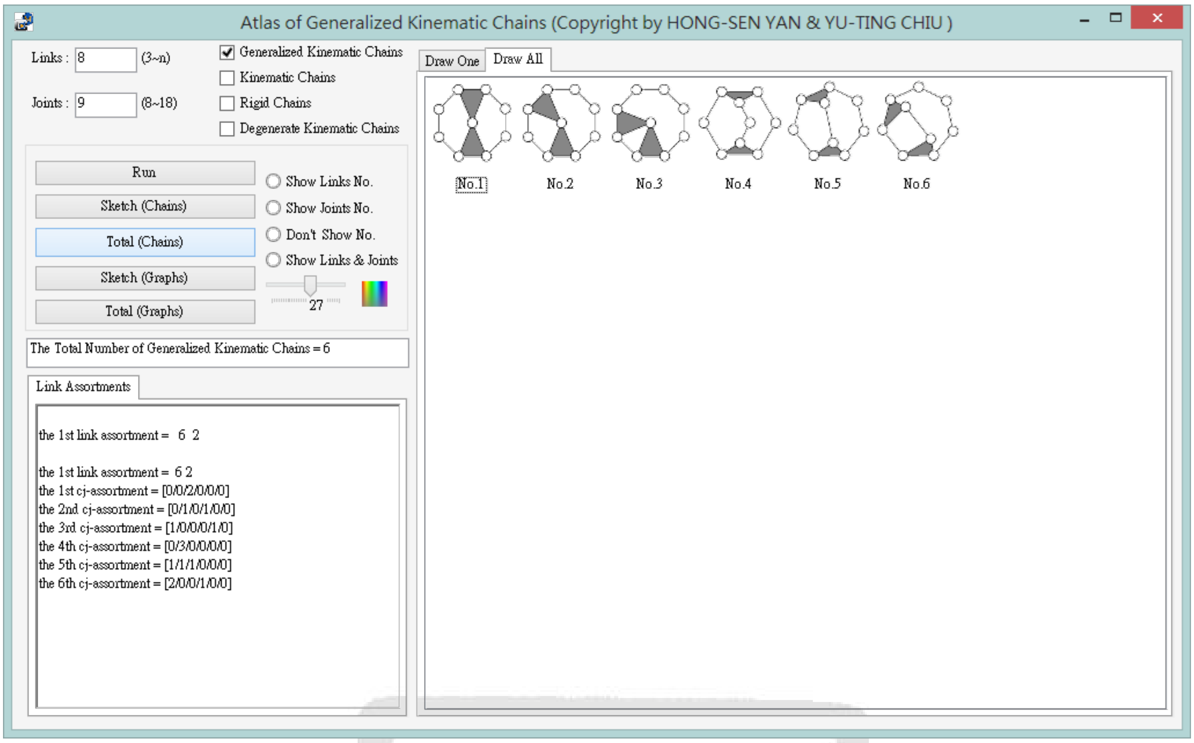


Figure 7.3 Atlas of (8, 9) generalized kinematic chains

Table 7.1 Numbers of generalized kinematic chain with 6 to 16 links

N_L	N_J	N_{GKC}	N_L	N_J	N_{GKC}	N_L	N_J	N_{GKC}
6	6	1	8	13	662	11	13	189
6	7	3	8	14	737	11	14	2,210
6	8	9	8	15	538	11	15	16,650
6	9	13	8	16	259	12	12	1
6	10	11	8	17	72	12	13	13
6	11	5	8	18	14	12	14	292
6	12	2	9	9	1	13	13	1
7	7	1	9	10	7	13	14	15
7	8	4	9	11	70	13	15	428
7	9	20	9	12	426	13	16	8,492
7	10	49	9	13	1,645	14	14	1
7	11	77	9	14	4,176	14	15	18
7	12	75	9	15	7,307	14	16	615
7	13	47	10	10	1	14	17	15,350
7	14	16	10	11	9	15	15	1
7	15	5	10	12	121	15	16	20
8	8	1	10	13	1,018	15	17	852
8	9	6	10	14	5,617	15	18	26,541
8	10	40	10	15	20,515	16	16	1
8	11	158	11	11	1	16	17	23
8	12	406	11	12	11	16	18	1,168

7.2 Kinematic Chains

Two examples are provided to illustrate the computer program for the construction of kinematic chains.

7.2.1 Example 3: Atlas of (8, 10) kinematic chains

For the input data with Links = 8 and Joints = 10, the number of DOF is 1 based on Equation (2.2). The link assortments are determined, A_{L1}=[4/4/0], A_{L2}=[5/2/1], and A_{L3}=[6/0/2]. The contracted link assortments are obtained based on its corresponding link assortments. For the first link assortment, A_{L1}=[4/4/0], the contracted link assortments are A_{CL}=[0/0/0/1], [0/2/0/0], [1/0/1/0], [2/1/0/0], and [4/0/0/0]. For the second link assortment, A_{L2}=[5/2/1], the contracted link assortments are A_{CL}=[0/1/1/0], [1/0/0/1], [1/2/0/0], [2/0/1/0], [3/1/0/0], and [5/0/0/0]. For the third link assortment, A_{L3}=[6/0/2], the contracted link assortments are A_{CL}=[0/3/0/0], [1/1/1/0], [2/0/0/1], [2/2/0/0], and [3/0/1/0].

Based on the link assortments and contracted link assortments, 16 feasible MLAMs and LAMs are obtained. Therefore, the number of (8, 10) kinematic chains are 16. The atlas of (8, 10) kinematic chains are shown in Figure 7.4. The average computational time is 201 ms (excluding the sketching time).

7.2.2 Example 4: Atlas of (10, 13) kinematic chains

For the input data with Links = 10 and Joints = 13, the number of DOF is 1 based on Equation (2.2). The link assortments are determined, A_{L1}=[4/6/0/0], A_{L2}=[5/4/1/0], A_{L3}=[6/2/2/0], A_{L4}=[7/0/3/0], A_{L5}=[6/3/0/1], A_{L6}=[7/1/1/1], and A_{L7}=[8/0/0/2]. For the first link assortment, A_{L1}=[4/6/0/0], the contracted link assortments are A_{CL}=[0/0/0/1/0], [0/2/0/0/0], [1/0/1/0/0], [2/1/0/0/0], and [4/0/0/0/0].

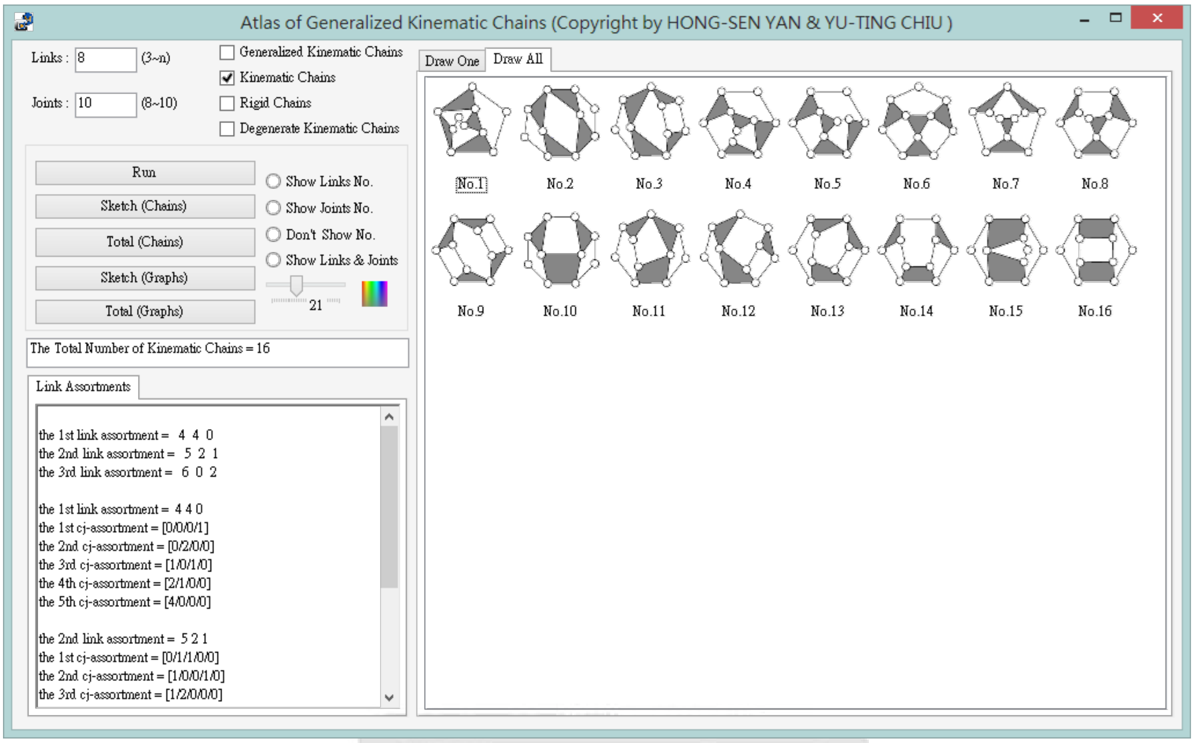


Figure 7.4 Atlas of (8, 10) kinematic chains

For the second link assortment, A_{L2}=[5/4/1/0], the contracted link assortments are A_{CL}=[0/0/0/0/1], [0/1/1/0/0], [1/0/0/1/0], [1/2/0/0/0], [2/0/1/0/0], [3/1/0/0/0], and [5/0/0/0/0]. For the third link assortment, A_{L3}=[6/2/2/0], the contracted link assortments are A_{CL}=[0/0/0/0/0/1], [0/1/1/0/0], [0/0/2/0/0/0], [0/1/0/1/0/0], [1/0/0/0/1/0], [0/3/0/0/0/0], [1/1/1/0/0/0], [2/0/0/1/0/0], [2/2/0/0/0/0], [3/0/1/0/0/0], [4/1/0/0/0/0] and [6/0/0/0/0/0]. For the fourth link assortment, A_{L4}=[7/0/3/0], the contracted link assortments are A_{CL}=[0/2/1/0/0], [1/0/2/0/0], [1/1/0/1/0], [2/0/0/0/1], [1/3/0/0/0], [2/1/1/0/0], [3/0/0/1/0], [3/2/0/0/0], [4/0/1/0/0], [5/1/0/0/0]. For the fifth link assortment, A_{L5}=[6/3/0/1], the contracted link assortments are A_{CL}=[0/0/0/0/0/1], [0/0/2/0/0/0], [0/1/0/1/0/0], [1/0/0/0/1/0], [0/3/0/0/0/0], [1/1/1/0/0/0], [2/0/0/1/0/0], [2/2/0/0/0/0], [3/0/1/0/0/0], [4/1/0/0/0/0], and [6/0/0/0/0/0]. For the sixth link assortment, A_{L6}=[7/1/1/1], the contracted link assortments are A_{CL}=[0/2/1/0/0], [1/0/2/0/0], [1/1/0/1/0], [2/0/0/0/1], [1/3/0/0/0], [2/1/1/0/0], [3/0/0/1/0], [3/2/0/0/0], [4/0/1/0/0], and [5/1/0/0/0]. For the seventh link assortment, A_{L7}=[8/0/0/2], the contracted link assortments are A_{CL}=[0/4/0/0/0], [1/2/1/0/0], [2/0/2/0/0], [2/1/0/1/0], [3/0/0/0/1], [2/3/0/0/0], [3/1/1/0/0], and [4/0/0/1/0].

Based on the link assortments and contracted link assortments, 230 feasible MLAMs and LAMs are obtained. Therefore, the number of (10, 13) kinematic chains is 230. The atlas of (10, 13) kinematic chains is shown in Figure 7.5 via the computer program. The complete atlas of (10, 13) kinematic chains is provided in Appendix D. The average computational time was 5.5 s (excluding the sketching time).

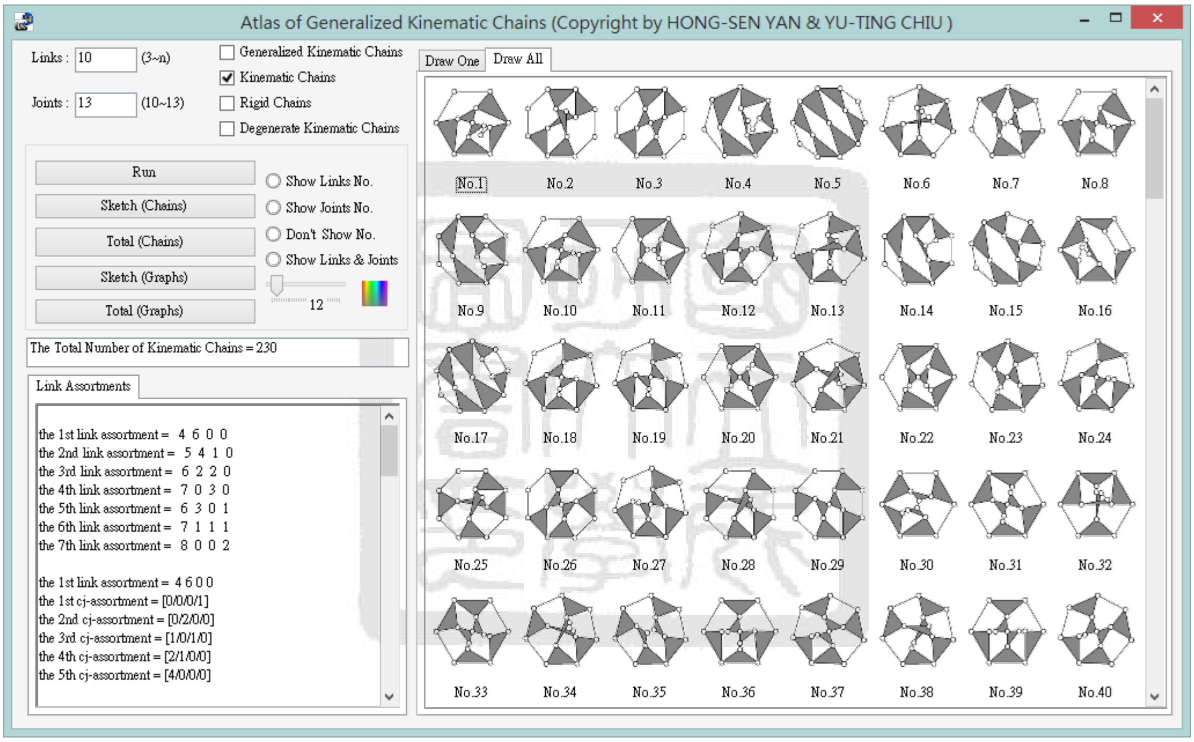


Figure 7.5 Atlas of (10, 13) kinematic chains

Table 7.2 lists the results for the numbers of kinematic chains with 6 to 16 links, and with different DOFs. The atlases of kinematic chains with minimum link crossings are synthesized. The result of the number of (12, 16) kinematic chains with 1 DOF is 6,856 based on the developed computer program. The result is same with in References [20, 21, 38, 68, 69, 71, 72]. However, some of the results are not the same as References [61-63, 69]. The main reason is due to the identification of degenerate kinematic chains. For instance, it might eliminate some of the valid or invalid kinematic chains. Furthermore, some important atlases of kinematic chains are provided in Appendix D.

Table 7.2 Numbers of kinematic chains with 6 to 16 links

N_L	N_J	DOFs	N_{KC}	N_L	N_J	DOFs	N_{KC}
6	6	3	1	12	15	3	1,962
6	7	1	2	12	16	1	6,856
7	7	4	1	13	13	10	1
7	8	2	3	13	14	8	12
8	8	5	1	13	15	6	289
8	9	3	5	13	16	4	4,159
8	10	1	16	13	17	2	27,275
9	9	6	1	14	14	11	1
9	10	4	6	14	15	9	13
9	11	2	35	14	16	7	359
10	10	7	1	14	17	5	6,878
10	11	5	8	15	15	12	1
10	12	3	74	15	16	10	13
10	13	1	230	15	17	8	422
11	11	8	1	15	18	6	9,448
11	12	6	10	16	16	13	1
11	13	4	126	16	17	11	13
11	14	2	753	16	18	9	483
12	12	9	1	16	19	7	11,847
12	13	7	12				
12	14	5	212				

7.3 Rigid Chains

Two examples are provided to illustrate the computer program for the generation of rigid chains.

7.3.1 Example 5: Atlas of (7, 9) rigid chains

For the input data with Links=7 and Joints=9, the number of DOFs is 0 based on Equation

(2.2). The link assortments are determined, A_{L1}=[3/4/0], A_{L2}=[4/2/1], and A_{L3}=[5/0/2]. The contracted link assortments are obtained based on its corresponding link assortments. For the first link assortment, A_{L1}=[3/4/0], the contracted link assortments are A_{CL}=[0/0/1], [1/1/0], and [3/0/0]. For the second link assortment, A_{L2}=[4/2/1], the contracted link assortments are A_{CL}=[0/2/0], [1/0/1], [2/1/0], and [4/0/0]. For the third link assortment, A_{L3}=[5/0/2], the contracted link assortments are A_{CL}=[1/2/0], [2/0/1], and [3/1/0].

Based on the link assortments and contracted link assortments, 20 feasible MLAMs and LAMs are obtained. Therefore, the number of (7, 9) rigid chains are 20. The atlas of (7, 9) rigid chains are shown in Figure 7.6 via the computer program. The average computational time is 65 ms (excluding the sketching time).

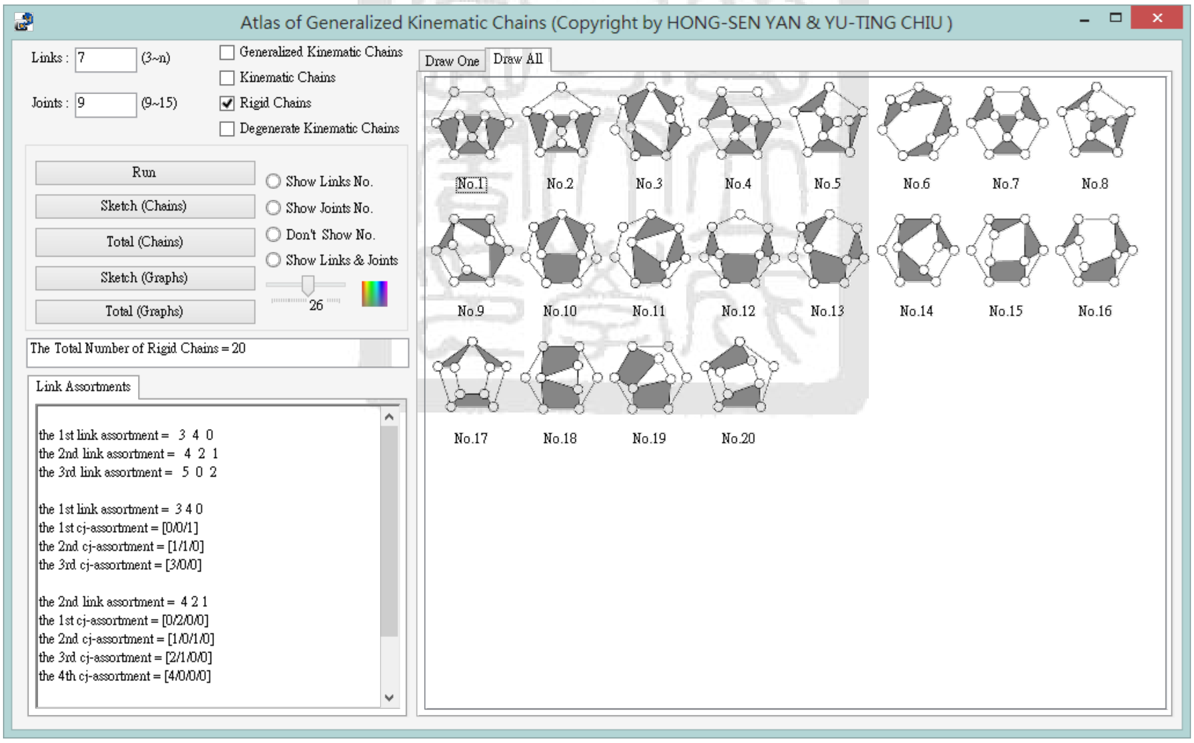


Figure 7.6 Atlas of (7, 9) rigid chains

7.3.2 Example 6: Atlas of (8, 11) rigid chains

For the input data with Links=8 and Joints=11, the number of DOFs is -1 based on Equation (2.2). The link assortments are determined, A_{L1}=[2/6/0/0], A_{L2}=[3/4/1/0], A_{L3}=[4/2/2/0], A_{L4}=[5/0/3/0], A_{L5}=[4/3/0/1], A_{L6}=[5/1/1/1], and A_{L7}=[6/0/0/2]. The contracted link assortments are obtained based on its corresponding link assortments. For the first link assortment, A_{L1}=[2/6/0/0], the contracted link assortments are A_{CL}=[0/1] and [2/0]. For the second link assortment, A_{L2}=[3/4/1/0], the contracted link assortments are A_{CL}=[0/0/1], [1/1/0], and [3/0/0]. For the third link assortment, A_{L3}=[4/2/2/0], the contracted link assortments are A_{CL}=[0/0/0/1], [0/2/0/0], [1/0/1/0], [2/1/0/0], and [4/0/0/0]. For the fourth link assortment, A_{L4}=[5/0/3/0], the contracted link assortments are A_{CL}=[1/2/0/0/0], [2/0/1/0/0], [3/1/0/0/0], and [5/0/0/0/0]. For the fifth link assortment, A_{L5}=[4/3/0/1], the contracted link assortments are A_{CL}=[0/0/0/1], [0/2/0/0], [1/0/1/0], [2/1/0/0], and [4/0/0/0]. For the sixth link assortment, A_{L6}=[5/1/1/1], the contracted link assortments are A_{CL}=[1/2/0], [2/0/1], [3/1/0], and [5/0/0]. For the seventh link assortment, A_{L7}=[6/0/0/2], the contracted link assortments are A_{CL}=[2/2/0], [3/0/1], and [4/1/0].

Based on the link assortments and contracted link assortments, 161 feasible MLAMs and LAMs are obtained. Therefore, the number of (8, 11) rigid chains is 161. The atlas of (8, 11) rigid chains is shown in Figure 7.7. The complete atlas of (8, 11) rigid chains are shown in Appendix E. The average computational time is 438 ms (excluding the sketching time).

Table 7.3 lists the results for the numbers of rigid chains with 6 to 11 links and non-positive DOFs. Furthermore, some important atlases of rigid chains are provided in Appendix E.

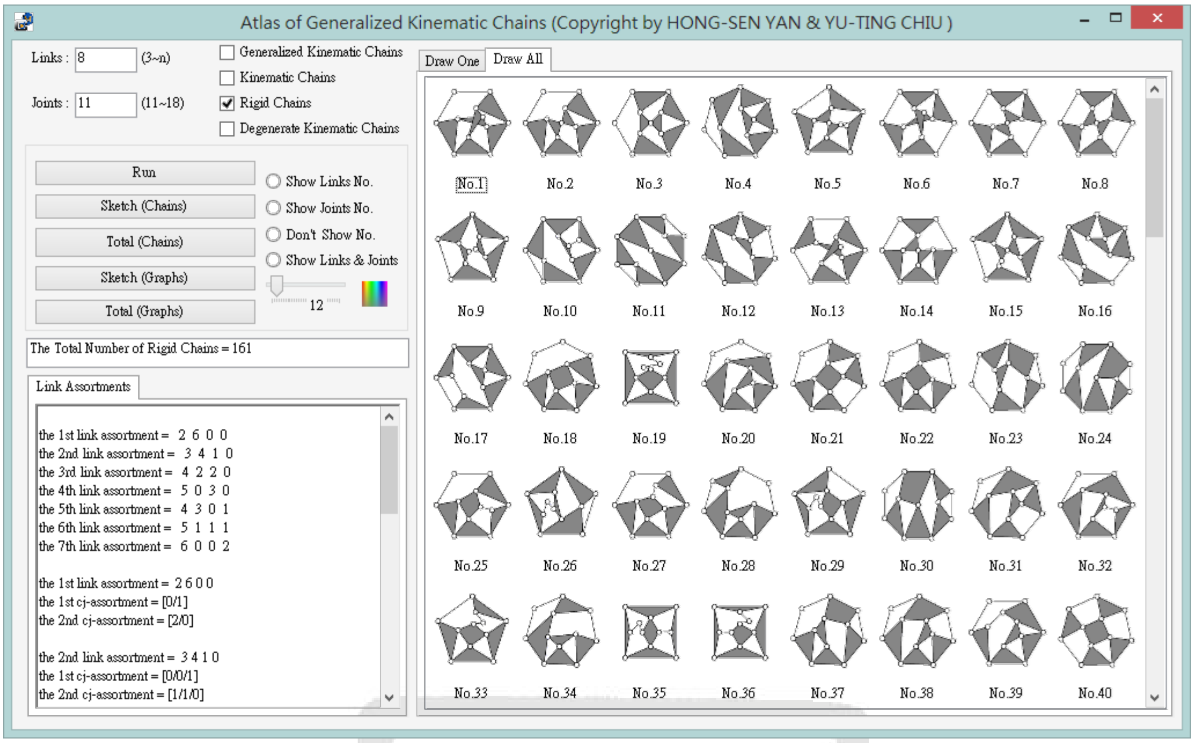


Figure 7.7 Atlas of (8, 11) rigid chains

Table 7.3 Numbers of rigid chains with 6 to 11 links

N_L	N_J	DOFs	N_{RC}	N_L	N_J	DOFs	N_{RC}
6	8	-1	9	8	12	-3	429
6	9	-3	14	8	13	-5	780
6	10	-5	12	8	14	-7	1,076
6	11	-7	8	8	15	-9	1,197
6	12	-9	5	8	16	-11	1,114
7	9	0	20	9	12	0	433
7	10	-2	50	9	13	-2	1,729
7	11	-4	82	9	14	-4	4,796
7	12	-6	94	9	15	-6	9,981
7	13	-8	81	10	14	-1	5,898
7	14	-10	59	10	15	-3	23,370
7	15	-12	38	11	15	0	17,491
8	11	-1	161				

7.4 Summary

A computer program is developed based on the proposed algorithms in Chapters 5 and 6 to generate and sketch atlases of generalized kinematic chains, kinematic chains, and rigid chains. The link assortments, contracted link assortments, and the numbers of generalized kinematic chains, kinematic chains and rigid chains with the given links and joints are displayed in the computer program. Users only need to input the numbers of links and joints, select a check box (choose a specified chain), then click “Run” button, the atlas of specified chains is obtained. Furthermore, this program can also be applied as a useful academic aid in drawing generalized kinematic chains, kinematic chains, and rigid chains for homeworks, ppt files, and even writing articles and books.

Chapter 8 Conclusions and Suggestions

This work proposes the enumeration algorithms for the number synthesis of generalized kinematic chains, kinematic chains, and rigid chains with simple joints. In addition, a computer program based on the proposed enumeration algorithms is developed to generate and sketch the complete atlases of these chains with simple joints. This chapter concludes the contributions of this work, and several topics are suggested for future works.

8.1 Conclusions

The contributions and results of this work are concluded as follows:

1. The terminology and definitions related to graph theory, various types of kinematic chains, and kinematic matrices in various literature are clarified and introduced in this work.
2. An exhaust search regarding literature review on the number synthesis of non-fractionated kinematic chains with simple joints is carried out. Various available methods, such as intuition or visual inspection, Franke's notation, graph theory, Baranov trusses, transformation of binary chains, matrices, group theory, stratified method, and other methods are studied, presented and discussed.
3. In order to enumerate and sketch the complete set of generalized kinematic chains, kinematic chains, and rigid chains for a specified number of links and joints, several algorithms are proposed and applied, e.g., the algorithms for checking cut-links, Kuratowski graphs, three-bar basic rigid chains, degenerate kinematic chains, and for sketching various chains. Each proposed algorithm is illustrated with examples.
4. Three enumeration algorithms are proposed for the synthesis of generalized kinematic chains, kinematic chains, and rigid chains, respectively. It is worth to mention that the

atlas of generalized kinematic chains includes the atlas of kinematic chains for mechanisms and rigid chains for structures (clamping devices). For instance, Figure 2.10 is the atlas of (6, 7) generalized kinematic chains, Figures 2.10(a) and (b) are the atlas of (6, 7) kinematic chains, and Figure 2.10(c) is a rigid chain (a degenerate kinematic chain).

5. Table 8.1 lists the relationship among various chains and regarding some characteristics. The generalized kinematic chains, kinematic chains, and rigid chains cannot have any cut-links in the chains. Furthermore, the kinematic chains should be identified without the degenerate kinematic chains. Besides, the basic difference between kinematic chains and rigid chains is the numbers of degrees of freedom. If the degrees of freedom is more than zero, the chain is a kinematic chain; otherwise, it is a rigid chain. Therefore, the generalized kinematic chains, kinematic chains, and rigid chains have different characteristics and constraints.

Table 8.1 Relationship between the chains and characteristics

Characteristics / **Chains**	**Cut-links**	**Degenerate kinematic chains**	**DOF**
Generalized kinematic chains	x		
Kinematic chains	x	x	> 0
Rigid chains	x		≤ 0

6. A computer program with a human-machine interface is developed based on the proposed algorithms in this work. Users only need to enter the number of links and joints, then the corresponding atlas of generalized kinematic chains, kinematic chains, and rigid chains can be automatically generated. Furthermore, there are some additional functions for users who can label the links and joints with selected color of links.

7. The numbers of generalized kinematic chains, kinematic chains, and rigid chains with

different DOFs have been enumerated based on the computer program, and the results are summarized in Tables 7.1-7.3. In addition, the computer program can enumerate unlisted results in this work.

8. Some different total numbers of kinematic chains are enumerated by different authors in the past years mainly due to different definitions of kinematic chains; such as defining a kinematic chain as a kinematic chain with simple-jointed or multiple-jointed, fractionated or non-fractionated, planar or non-planar, and so forth. Furthermore, since the isomorphic and degenerate chains must be identified and eliminated, the atlas of kinematic chains are reckoned and synthesized.
9. The proposed algorithms apply the multiple link adjacency matrices (MLAMs) as an input data for the synthesis of different types of kinematic chains. Furthermore, the MLAMs are synthesized based on the concept of permutation groups. The motivation of applying the permutation groups is to propose the isomorphism-free enumeration algorithm. Besides, since the dimension of an MLAM is less than link adjacency matrix (LAM) or contracted link adjacency matrix (CLAM), it can increase the computational efficiency and it is easy for programming.
10. The algorithm for checking cut-links is presented for the elimination of the chains with cut-links, i.e., the fractionated chains. Besides, the MLAM is as an input data to perform the algorithm that can successfully delete all chains with cut-links.
11. The algorithm for checking Kuratowski graphs is proposed for the first time to discard the non-planar blocks or graphs. Furthermore, the MLAM is as an input data to execute the algorithm to increase the computational efficiency.
12. Table 8.2 shows the relationship between the algorithms and input matrices. MLAMs are used as input data to perform the algorithms for checking cut-links, Kuratowski graphs, and degenerate kinematic chains. LAMs are used as input data to execute the algorithms

for checking 3-bar basic rigid chains and the sketching algorithm. The three enumeration algorithms need to apply MLAMs and LAMs as the input data to synthesize the generalized kinematic chains, kinematic chains, and rigid chains.

Table 8.2 Relationship between the algorithms and input matrices

Inputs / Algorithms	MLAMs	LAMs	MLAMs & LAMs
Checking algorithms			
Cut-links	v		
Kuratowski graphs	v		
3-bar basic rigid chains		v	
Degenerate kinematic chains	v		
Sketching algorithm			
Sketching algorithm		v	
Enumeration algorithm			
Generalized kinematic chains			v
Kinematic chains			v
Rigid chains			v

13. For the identification of degenerate kinematic chains, the algorithms for checking three-bar basic rigid chains and degenerate kinematic chains are proposed. The LAM is as an input data to execute the algorithm for checking three-bar basic rigid chains. Since the chain with three-bar basic rigid chains is obvious, the algorithm for checking three-bar basic rigid chains is proposed to eliminate the kinematic chains with three-bar basic rigid chains. Then, the algorithm for checking degenerate kinematic chains is put forward to discard the degenerate kinematic chains with any basic rigid chain. Therefore, these two

algorithms are proposed to eliminate all degenerate kinematic chains.

14. The result of this work provides mechanism designers the necessary data bank for the generation of all possible topological structures of mechanical devices (mechanisms or structures) in the conceptual stage.

8.2 Suggestions

Although the number synthesis of generalized kinematic chains, kinematic chains, and rigid chains with simple joints have been explored in this work, there still exists the following works that are worth for further study in the future. Regarding the sketching algorithm, it needs to store the basic contracted graphs and then sketch the various types of kinematic chains from its corresponding basic contracted graphs. The basic contracted graphs are listed with up to five loops. This is the limitation to sketch the kinematic chains with many loops. Therefore, basic contracted graphs with more than five loops can be further investigated.

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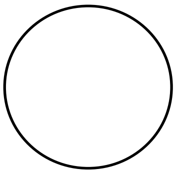
Appendix A: Basic Contracted Graphs with 1, 2, 3, and 4 loops

a. Atlas of basic contracted graphs with 1 loops

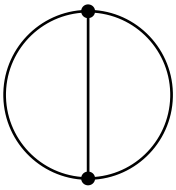
$A_{ML} = [0]$



(01)

b. Atlas of basic contracted graphs with 2 loops

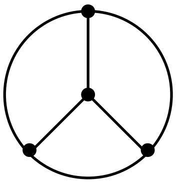
$A_{ML} = [3]$



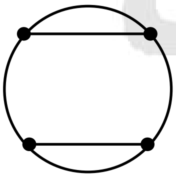
(01)

c. Atlas of basic contracted graphs with 3 loops

$A_{ML} = [4/0]$

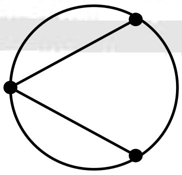


(01)



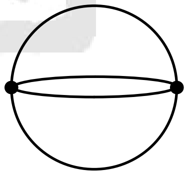
(02)

$A_{ML} = [2/1]$



(03)

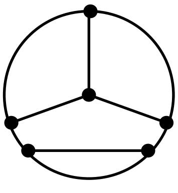
$A_{ML} = [0/2]$



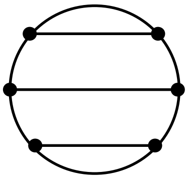
(04)

d. Atlas of basic contracted graphs with 4 loops

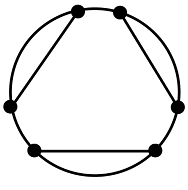
$A_{ML} = [6/0/0]$



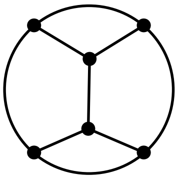
(01)



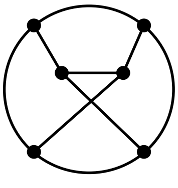
(02)



(03)



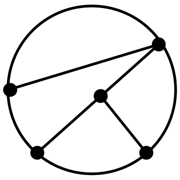
(04)



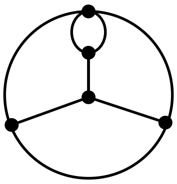
(05)

d. Atlas of basic contracted graphs with 4 loops (cont.)

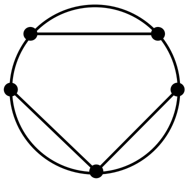
A_{ML} = [4/1/0]



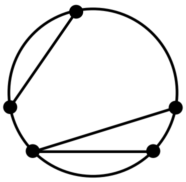
(06)



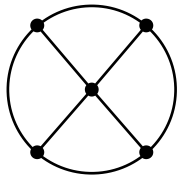
(07)



(08)

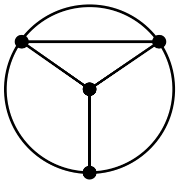


(09)

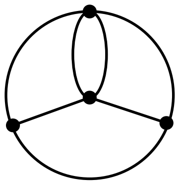


(10)

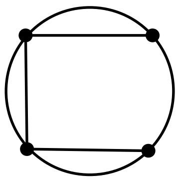
A_{ML} = [2/2/0]



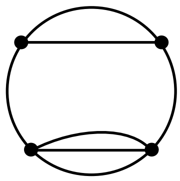
(11)



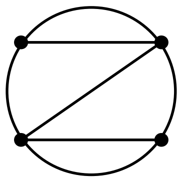
(12)



(13)

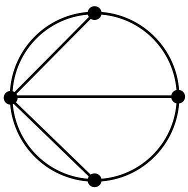


(14)



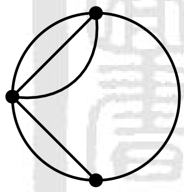
(15)

A_{ML} = [3/0/1]



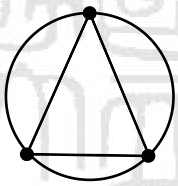
(16)

A_{ML} = [1/1/1]



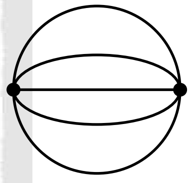
(17)

A_{ML} = [0/3/0]



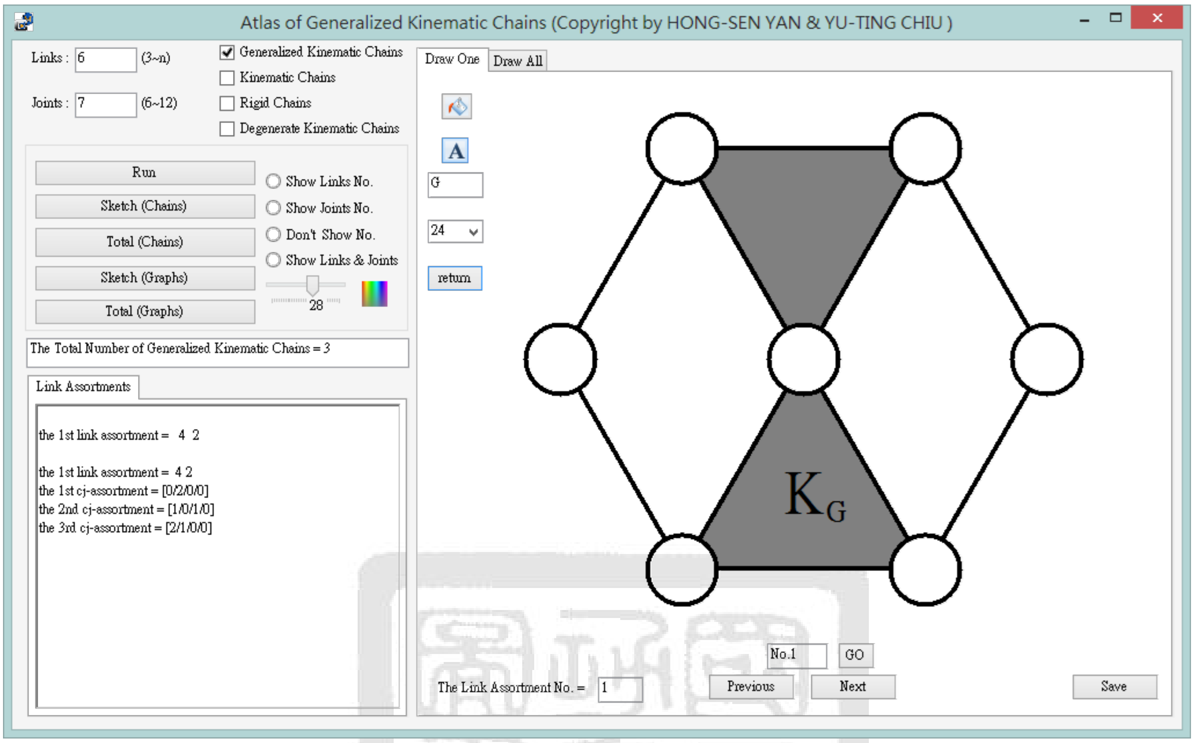
(18)

A_{ML} = [0/0/2]



(19)

Appendix B: User Interface of the Computer Program



The user interface of the computer program has 15 functions as follows:

01. **Links** textbox: input the number of links.

02. **Joints** textbox: input the number of joints.

03. **Run** button: execute the computer program.

04. **Generalized Kinematic Chains, Kinematic Chains, Rigid Chains, and Degenerate Kinematic Chains** check box: choose the chains what the users would like to obtain.

05. **Sketch (Chains)** button: draw the chains with the given links and joints.

06. **Total (Chains)** button: draw all chains with the given links and joints.

07. **Sketch (Graphs)** button: draw the graphs with the given links and joints.

08. **Total (Graphs)** button: draw all graphs with the given links and joints.

09. **Show Links No.** radio button: show the labels of links.

10. **Show Joints No.** radio button: show the labels of joints.

11. **Don’t Show No.** radio button: don’t show any labels of links and joints.

12. **Show Links & Joints** radio button: show the labels of links and joints.

13. **Radius** track bar: change the radius of the circles (joints).

14. **Edit color** button: the users can select the color of multiple links.

15. **Link Assortments** tabpage: show the link assortments with the given links and joints and its corresponding contracted link assortments.

The atlases of generalized kinematic chains, kinematic chains, rigid chains, and graphs are shown in the middle of interface. There are 8 functions in the middle screen and the functions are as follows:

01. **Draw One** tabpage: show a chain or a graph with the given links and joints.

02. **Draw All** tabpage: show all chains or graphs with the given links and joints.

03. **Fill color** button: fill the colors into the multiple links.

04. **Text** button: write words.

05. **Return** button: return to the previous step of writing

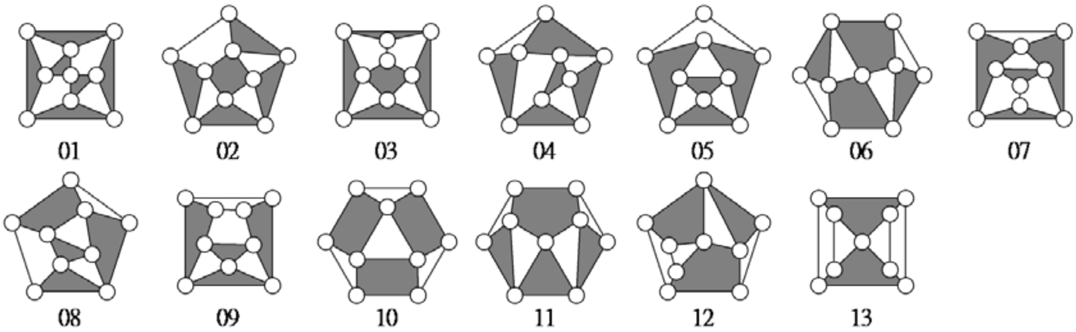
06. **Previous** button: show the previous chain or graph.

07. **Next** button: show the next chain or graph.

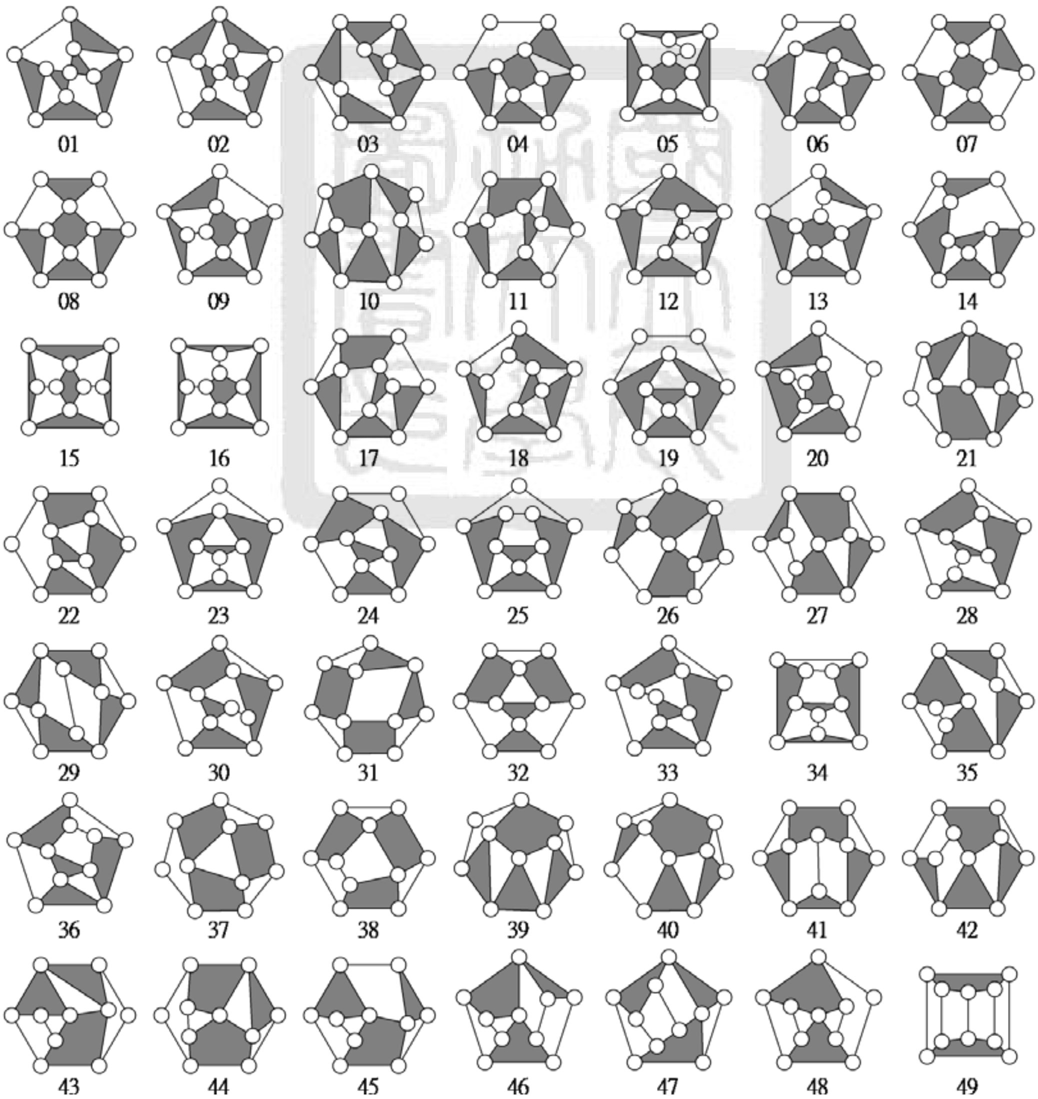
08. **Save** button: save the chains or graphs.

Appendix C: Atlases of Generalized Kinematic Chains

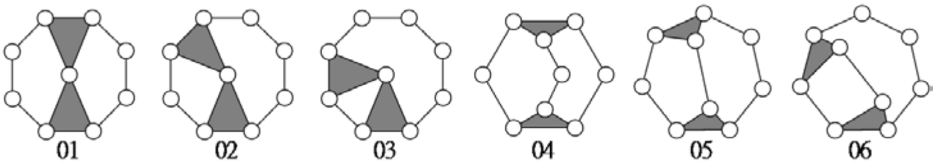
a. Atlas of (6, 9) generalized kinematic chains (13)



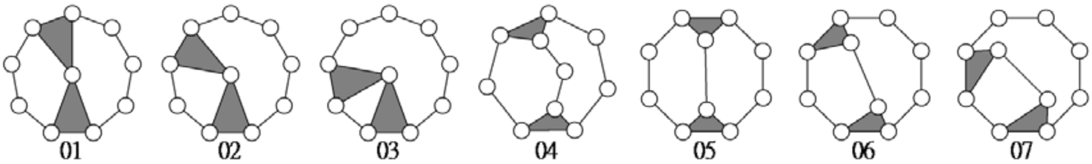
b. Atlas of (7, 10) generalized kinematic chains (49)



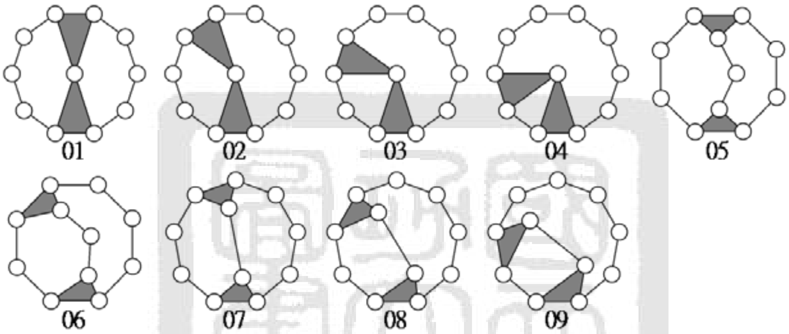
c. Atlas of (8, 9) generalized kinematic chains (06)



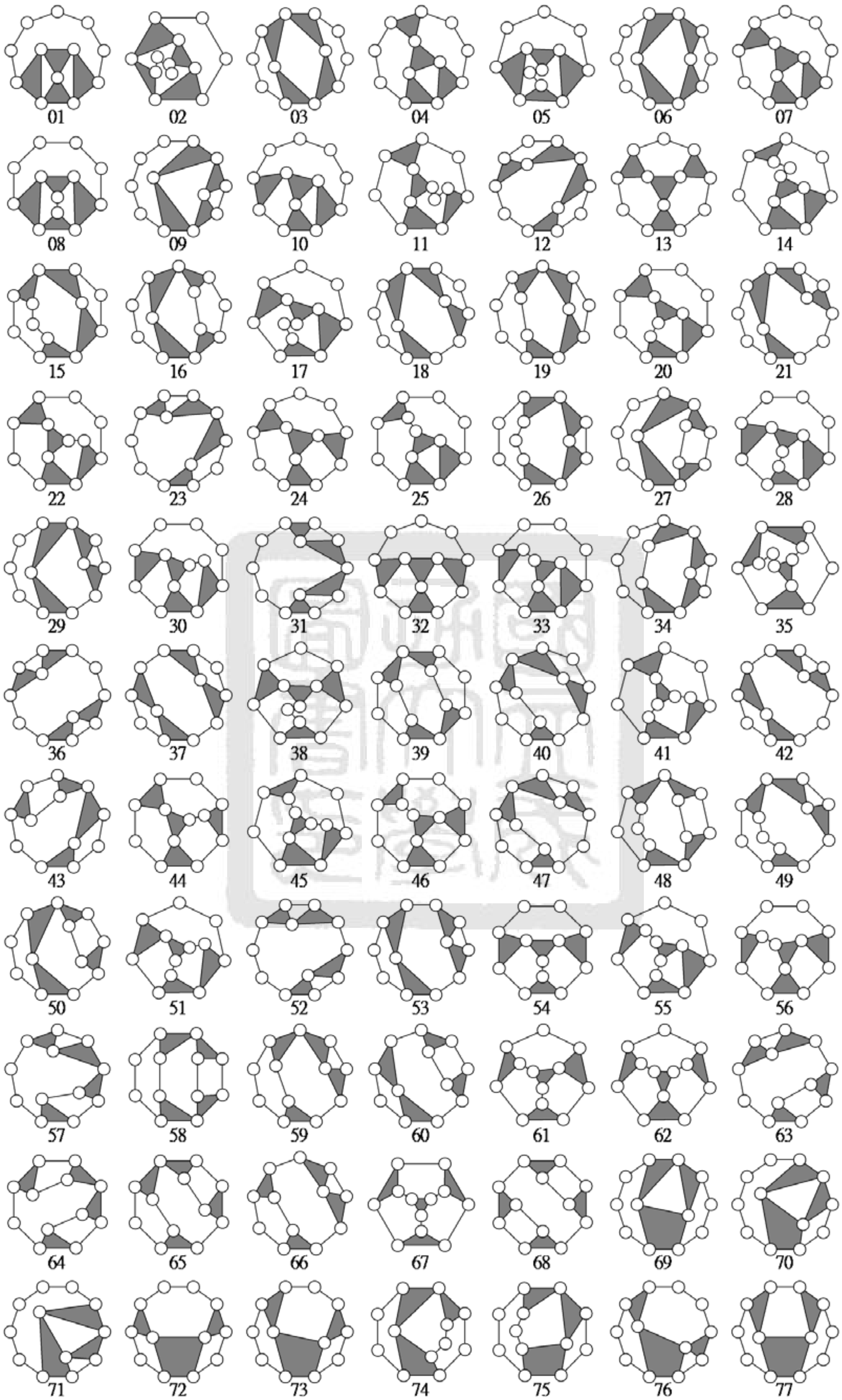
d. Atlas of (9, 10) generalized kinematic chains (07)



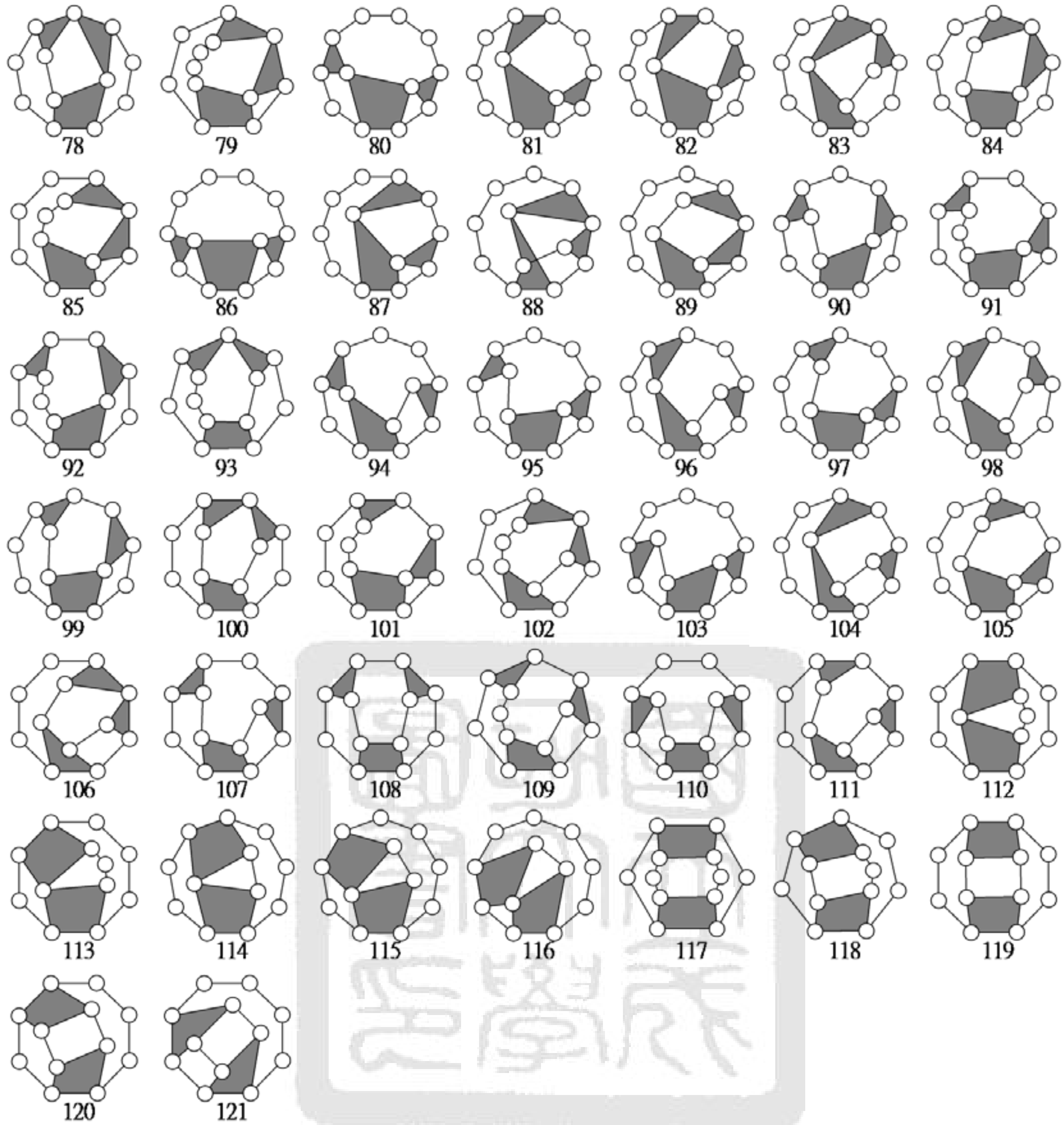
e. Atlas of (10, 11) generalized kinematic chains (09)



f. Atlas of (10, 12) generalized kinematic chains (121)

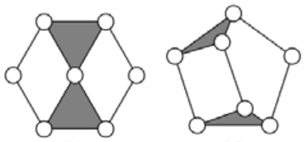


f. Atlas of (10, 12) generalized kinematic chains (cont.) (121)



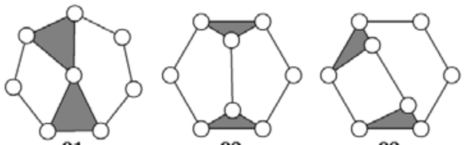
Appendix D: Atlases of Kinematic Chains

a. Atlas of (6, 7) kinematic chains (02)



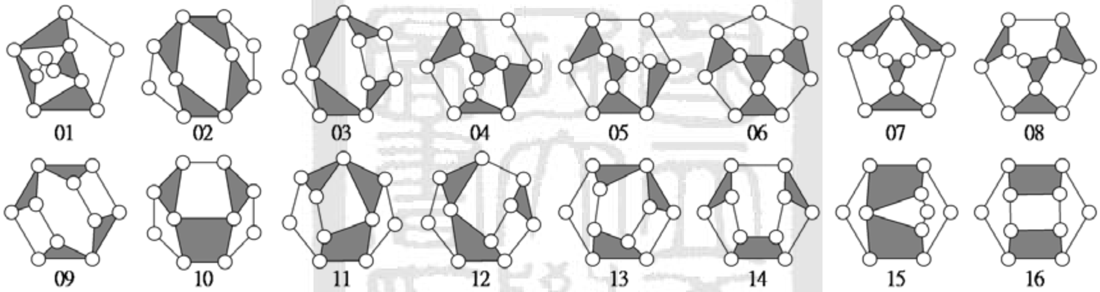
01 02

b. Atlas of (7, 8) kinematic chains (03)

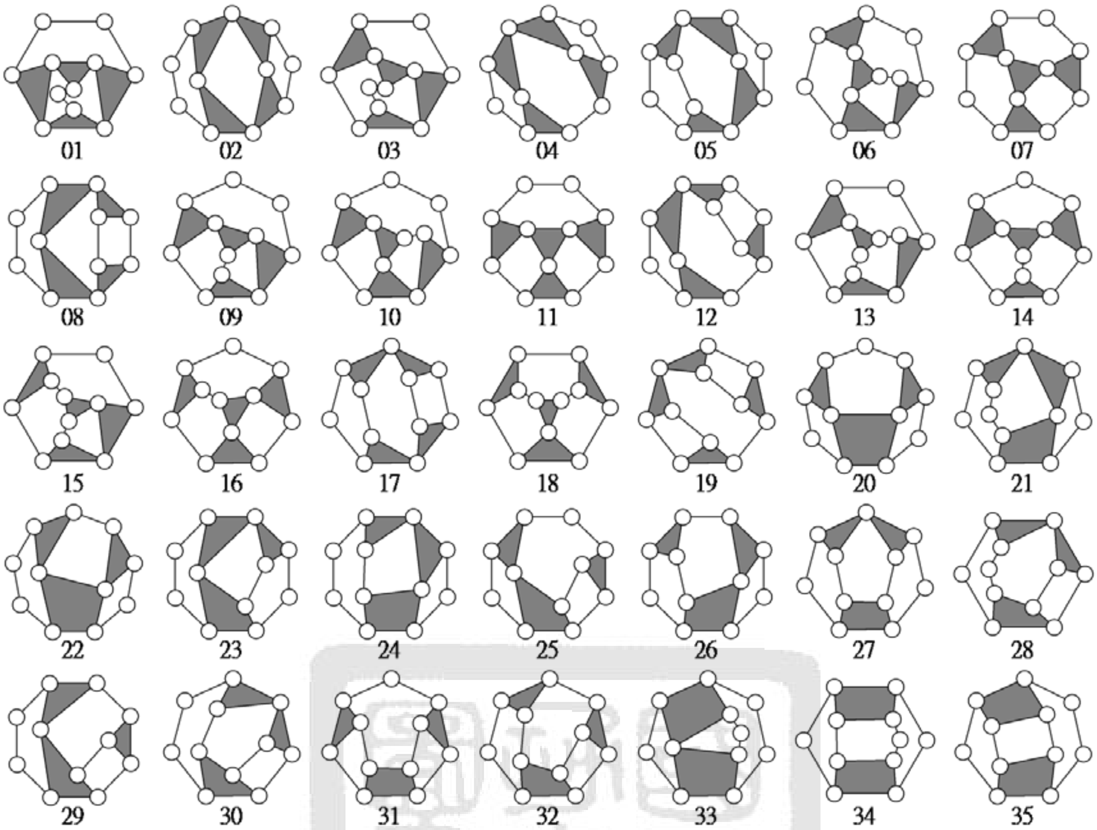


01 02 03

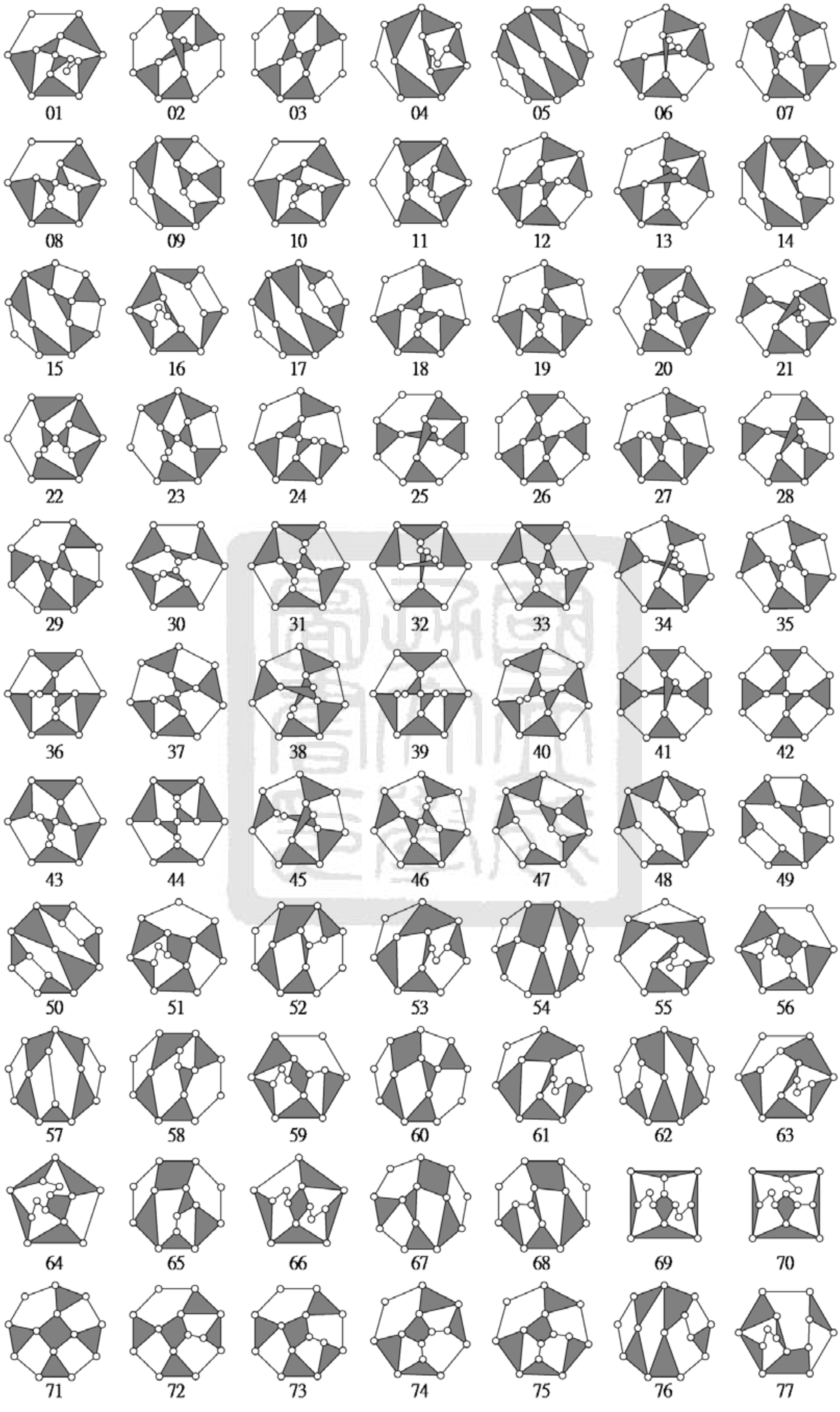
c. Atlas of (8, 10) kinematic chains (16)



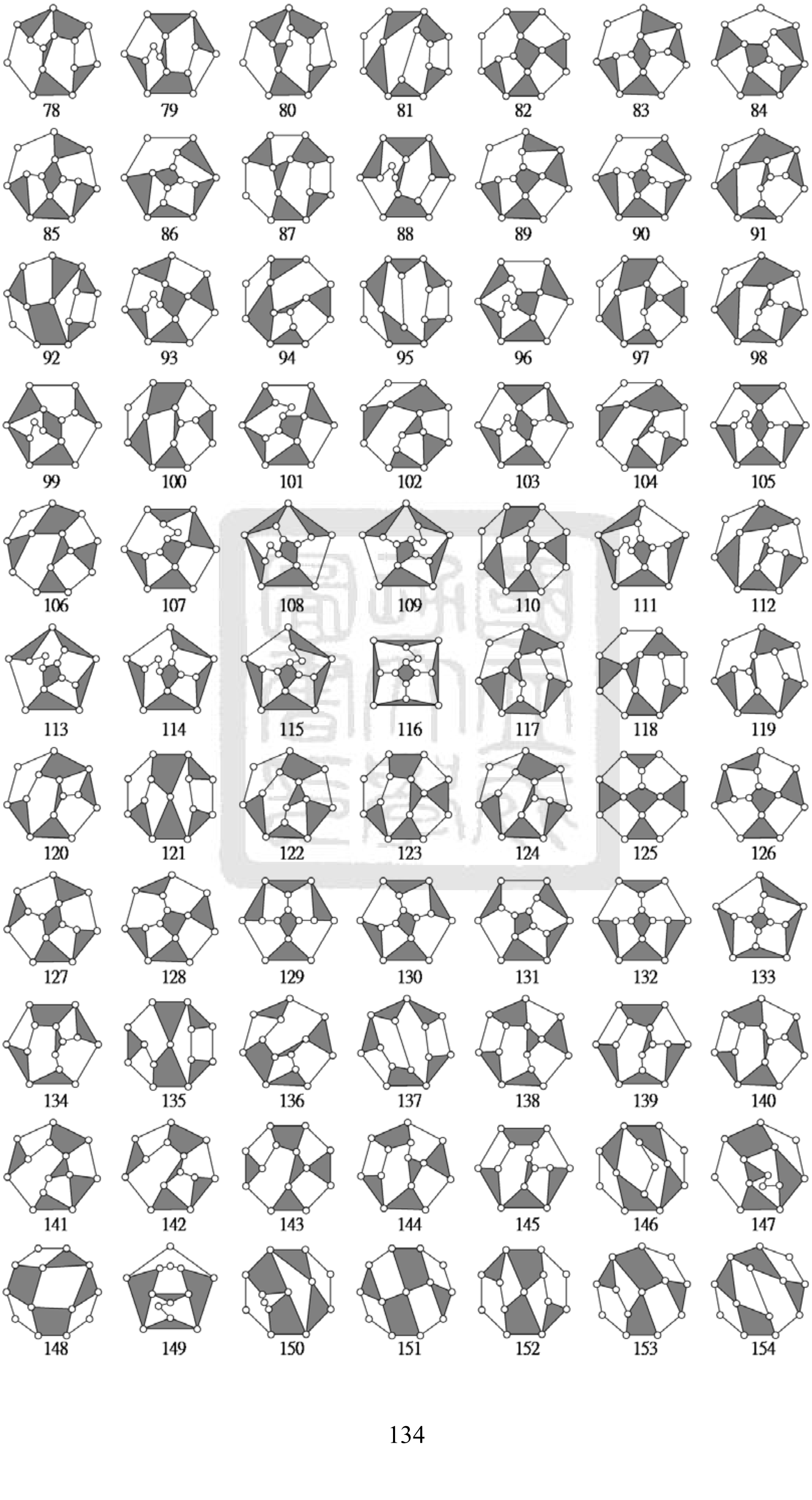
d. Atlas of (9, 11) kinematic chains (35)



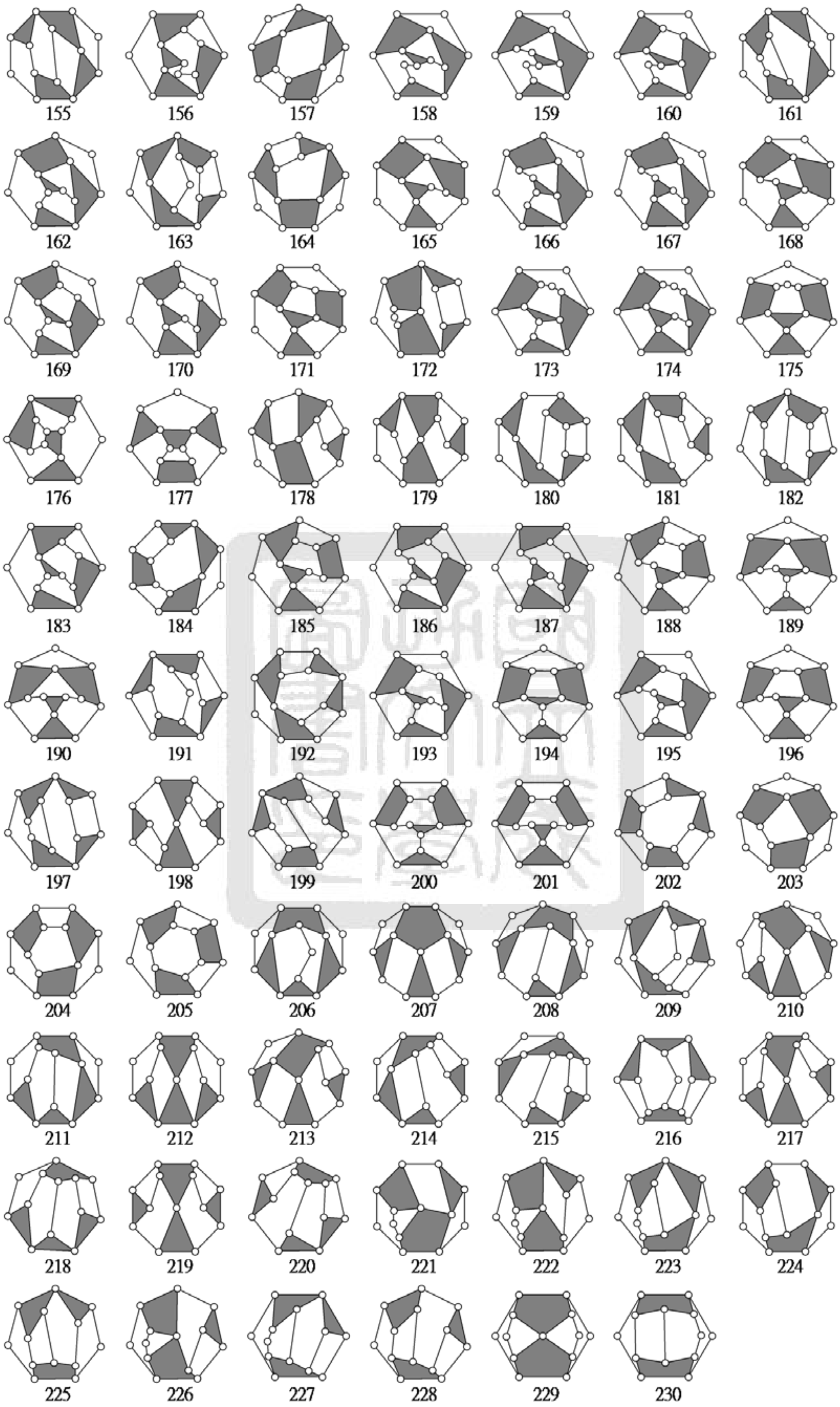
e. Atlas of (10, 13) kinematic chains (230)



e. Atlas of (10, 13) kinematic chains (cont.) (230)

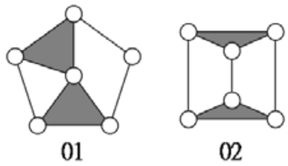


e. Atlas of (10, 13) kinematic chains (cont.) (230)

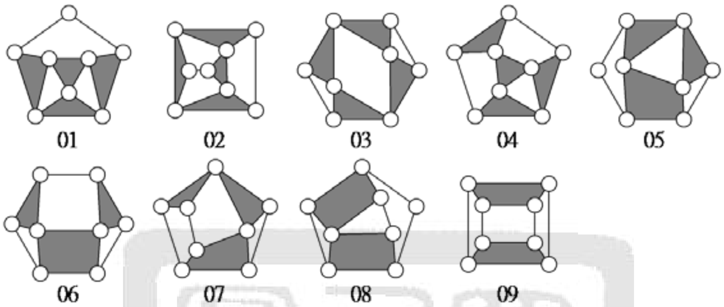


Appendix E: Atlases of Rigid Chains

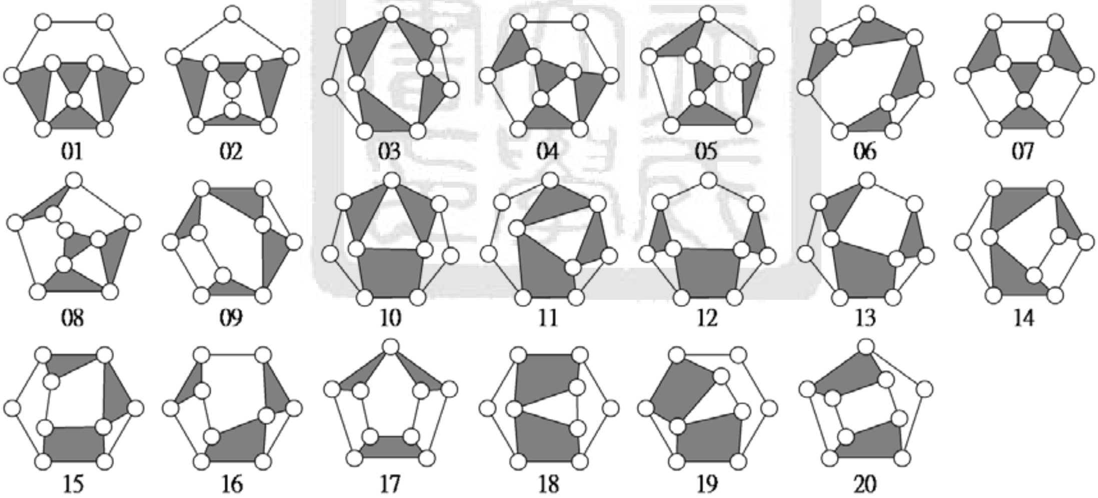
a. Atlas of (5, 6) rigid chains (02)



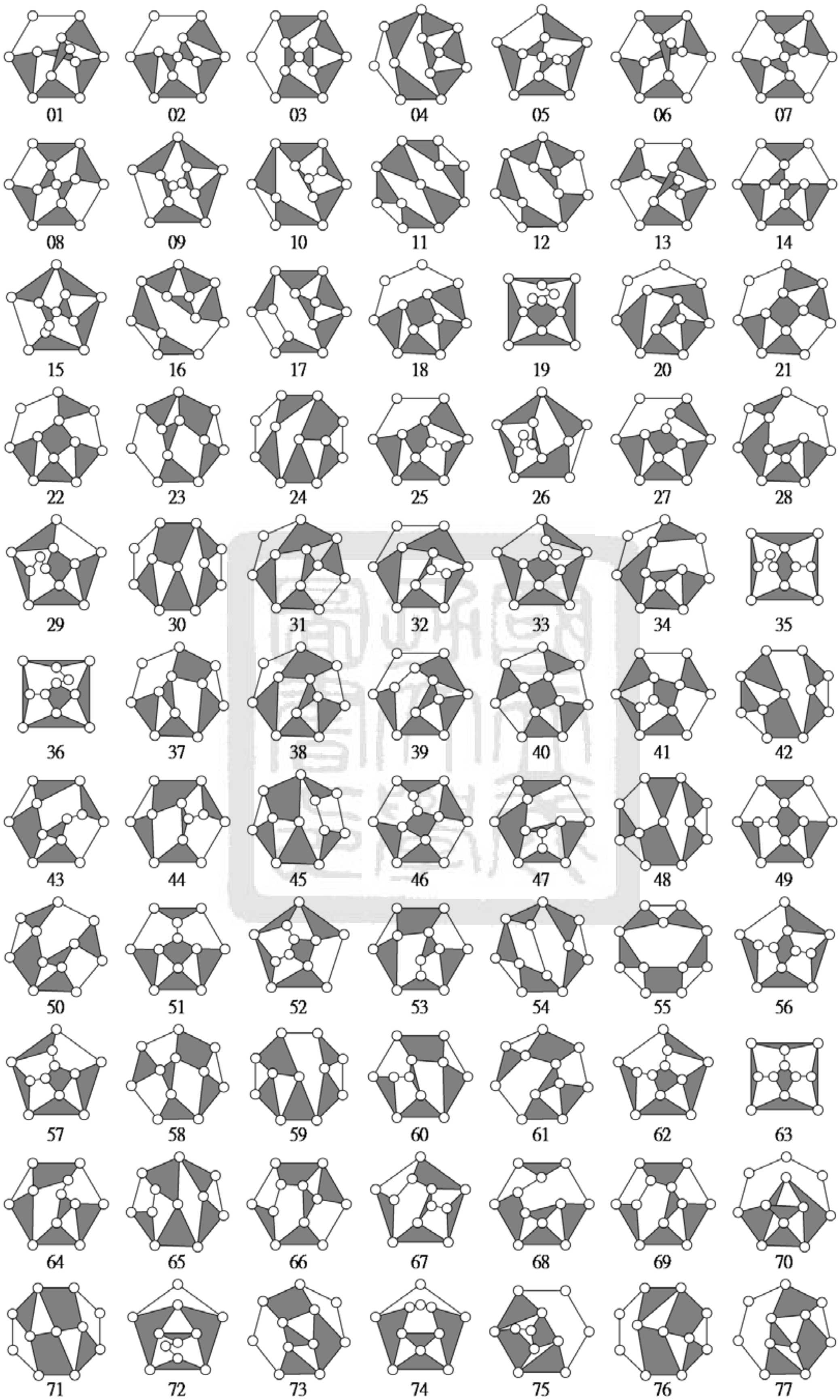
b. Atlas of (6, 8) rigid chains (09)



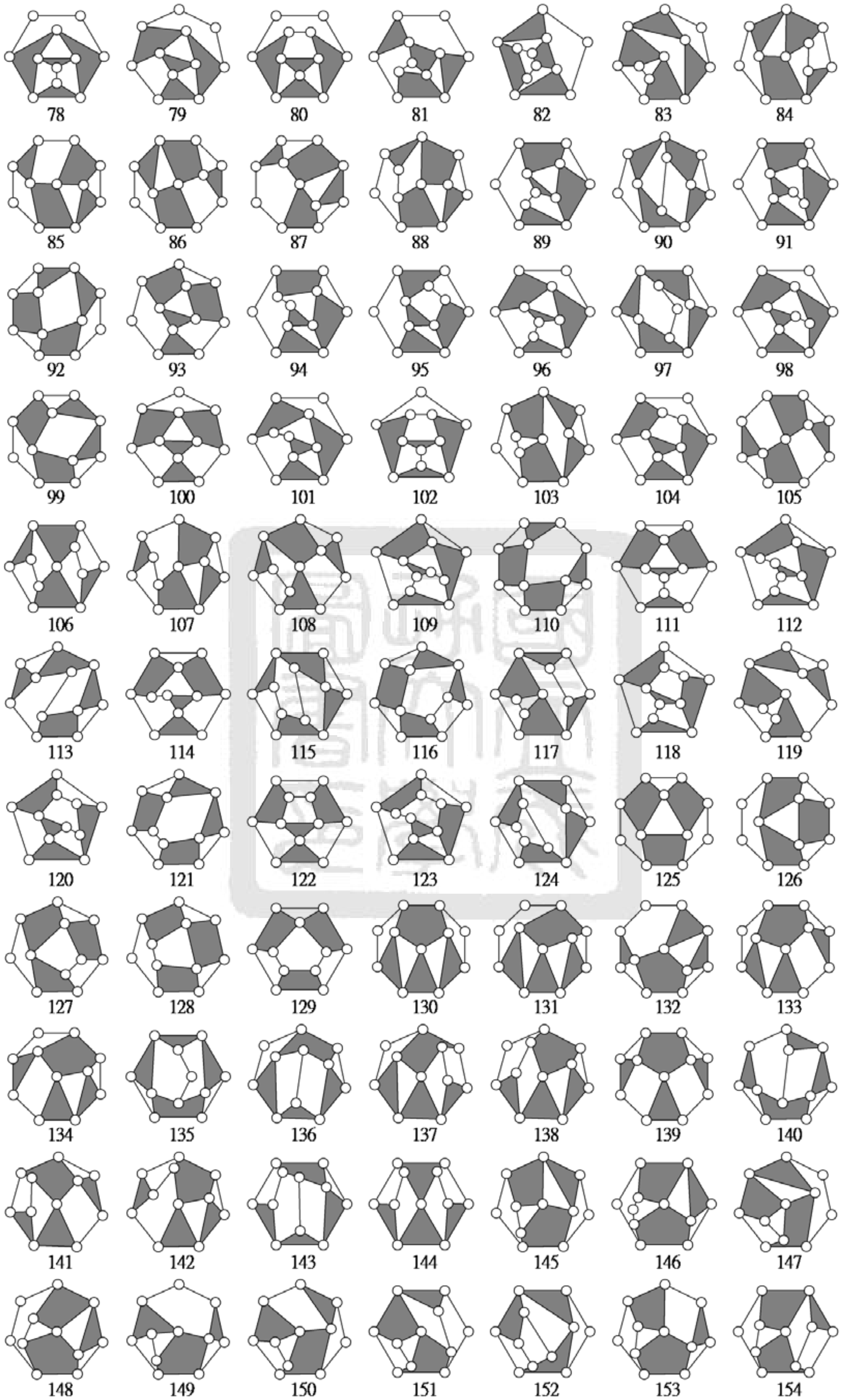
c. Atlas of (7, 9) rigid chains (20)



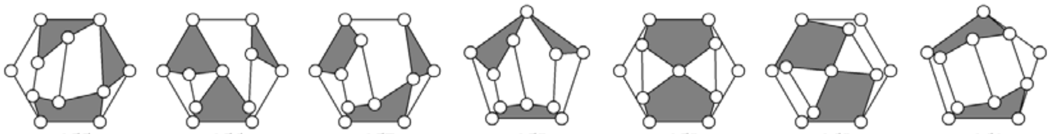
d. Atlas of (8, 11) rigid chains (161)



d. Atlas of (8, 11) rigid chains (cont.) (161)



e. Atlas of (8, 11) rigid chains (cont.) (161)



155 156 157 158 159 160 161

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