CSE 462 | ALGORITHM ENGINEERING SESSIONAL

The Subset Sum Problem

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INTRODUCTION

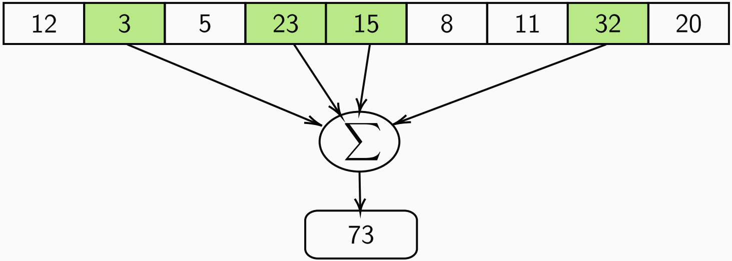
The Subset Sum Problem

Let's assume we're given a set of integers. Can we find a subset that sums up to a given target integer?

Input Set:

12	3	5	23	15	8	11	32	20

Selection of Subset:



The Subset Sum Problem

Formal definition of the problem: (Decision Version)
Given a Multiset of integers, $S = \{x_1, x_2, x_3, \cdots, x_n\}$ and a target sum W, does there exist a subset $S' \subseteq S$ such that $\sum_{x \in S'} x = W$?

The Subset Sum Problem

Formal definition of the problem: (Optimization Version)
Given a Multiset of integers, $S = \{x_1, x_2, x_3, \cdots, x_n\}$ and a target sum W, find a subset $S' \subseteq S$ so as to

maximize $Z = \sum_{x \in S'} x$,

subject to $\sum_{x \in S'} x \leq W$.

The Algorithms We Discussed

- **Exact Algorithms** - Brute-Force, Backtracking, Branch and Bound, Dynamic Programming
- **Approximation Algorithms** - A PTAS for Subset Sum Problem, An FPTAS for Subset Sum Problem
- **Heuristics and Metaheuristics** - A competitive local search based heuristic, Hill Climbing, Simulated Annealing, Genetic Algorithm

IMPLEMENTATION OF THE ALGORITHMS

The Algorithms We Implemented

- An FPTAS for Subset Sum Problem [1]
- Simulated Annealing
- Genetic Algorithm [2]

We also implemented the Dynamic Programming Algorithm for determining the optimal solution.

The Data used

- For running the algorithms, we used three categories of data (of different sizes). And each such type of set was used 10 times. This is shown in the form of a table below.

Set Size	Number of Sets
10	10
100	10
1000	10

Table: The Data

We used Python Programming language for running the algorithms.

The metrics we evaluated

For comparing the algorithms, we evaluated the following metrics.

- Average elapsed time in seconds.
- The average accuracy.

For each input sent to each algorithm, the accuracy was considered to be the ratio of obtained solution and optimal solution, i.e. Accuracy $= \frac{Solution_{obtained}}{Solution_{optimal}}$

FPTAS for Subset Sum Optimization Problem

- Let L be a list of integers. We will use the concept of 'trimming' a list.
- The idea is that if two values in L are close to each other, then for the purpose of finding an approximate solution there is no reason to maintain both of them explicitly.
- δ is a trimming parameter such that $0 < \delta < 1$.
- TRIM(L,δ) reduces L to L' such that for any $y \in L$, there exists some $z \in L'$ such that $\frac{y}{1+\delta} \leq z \leq y$.
- This procedure takes as input a set $S = \{x_1, x_2, ..., x_n\}$ of n integers, a target integer W, and an approximation parameter ϵ where $0 < \epsilon < 1$. Here, $S + x = \{x_1 + x, x_2 + x, ..., x_n + x\}$ and MERGE(L, L') returns union of L and L'.

FPTAS for Subset Sum Optimization Problem

Algorithm 1 TRIM(L,δ)

Input: A Sorted List $L =< y_1, y_2, ..., y_m >$, Trimming Parameter ($\delta$).

Output: A Trimmed List L'.

$L' =< y_1 >$

$last = y_1$

for j=$2...m$ **do**

  if $y_j > last \cdot (1 + \delta)$ **then**

    append y_j to L'

    last $= y_j$

end

return L'

FPTAS for Subset Sum Optimization Problem

Algorithm 2 APPROX-SUBSET-SUM(S,W,ϵ)

Input: A Set ($S = \{x_1, x_2, ..., x_n\}$), Target Sum (W), Approximation parameter (ϵ).

Output: An Approximate Solution (Weight closest to but not exceeding W).

$n = |S|$

$L_0 =< 0 >$

for i=$1, 2...n$ **do**

    L_i = MERGE($L_{i-1}, L_{i-1} + x_i$)

    L_i = TRIM($L_i, \frac{\epsilon}{2n}$)

    remove from L_i any values strictly greater than W

end

return largest value in L_n

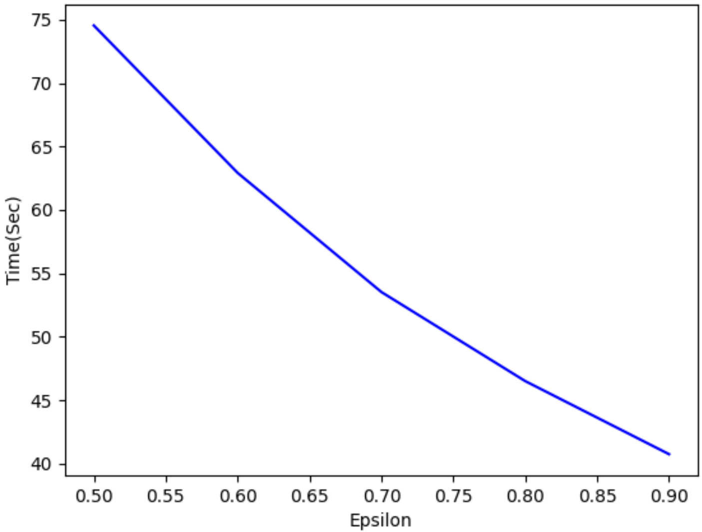


Figure: FPTAS Elapsed Time(Sec) vs. Epsilon

Simulated Annealing I

- Simulated Annealing [3] is a metaheuristic algorithm.
- It makes use of Hill Climbing while addressing the problem of local optima.

Simulated Annealing II

- Simulated Annealing keeps a probability of accepting a solution that is worse than the previous one.
- This allows the algorithm to explore more paths that might lead to the global optimum.

Simulated Annealing III

- However, if the algorithm keeps on accepting bad solutions, it might move about a lot without bringing any improvement.
- To address this, a **temperature** variable is kept. Its value is lowered with the iterations of the algorithm.
- The probability of accepting bad solutions is made proportional to the temperature.
- Therefore, as the algorithm proceeds, bad solutions are accepted less and optimization is done more.

Simulated Annealing for SSP-Optimization

Algorithm 3 Simulated-Annealing-SSP(S, W, r)

Input: Set $S = \{x_1, x_2, ..., x_n\}$, Target Sum W, integer r.
Output: An Approximate Solution (Weight closest to W).

```
S' = initial_random_subset(S)
current_residue = residue(S', W)
for i = 1...r do
    T = random_neighbor_selection(S')
    neighbor_residue = residue(T, W)
    if (neighbor_residue < current_residue) then
        S' = T
    if (residue ≥ current_residue) then
        P = calculate_probability(residue, current_residue, i)
        with probability P do S' = T
end
return (W − current_residue)
```

A few Implementation Details I

- The probability P of accepting a worse neighbor is set to e^{-X} where X is given by:

$$X = \frac{(neighbor\_residue - current\_residue)}{10^{10} * 0.8^{\frac{i}{300}}} \tag{1}$$

.

- In this case, $0.8^{\frac{i}{300}}$ is the **temperature** for this algorithm.
- Initially the value of P will be high, so the algorithm accepts worse neighbors and explores the solution space more.
- As more iterations are run, P will decrease, and bad neighbors will be explored less.

A few Implementation Details II

- While exploring bad neighbors, we allow neighbors with sum greater than the target W.
- For such a neighbor, the residue is set to be the total sum of that subset, instead of the difference with the target W.

A few Implementation Details III

- If the target W is reached before the maximum iteration r, the program breaks and returns the exact solution.

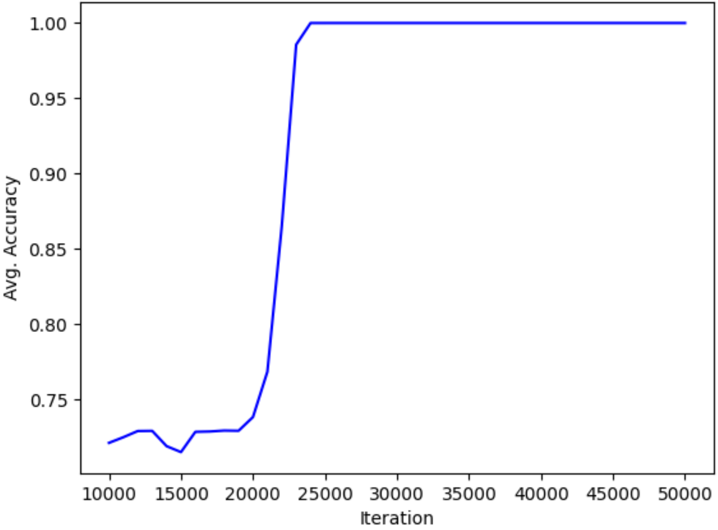


Figure: Simulated Annealing Average Accuracy vs. Iteration

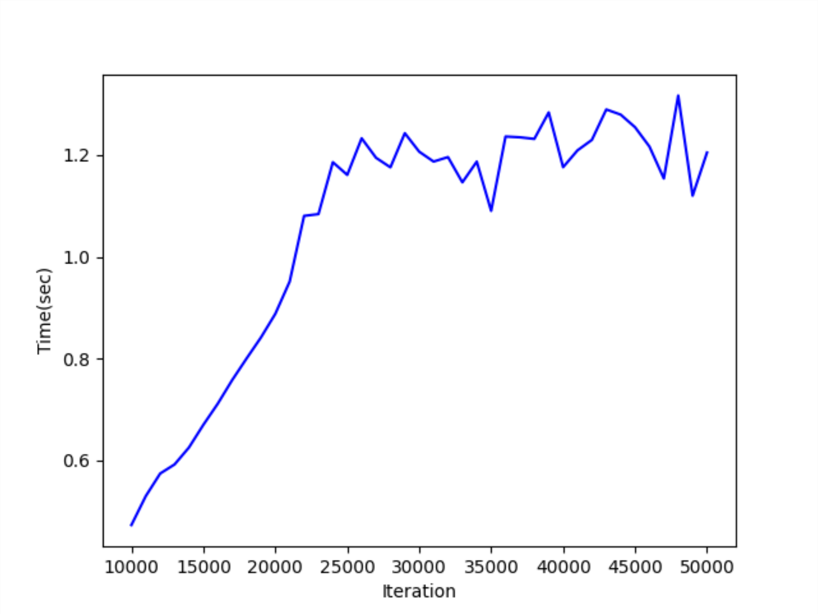


Figure: Simulated Annealing Elapsed Time(Sec) vs. Iteration

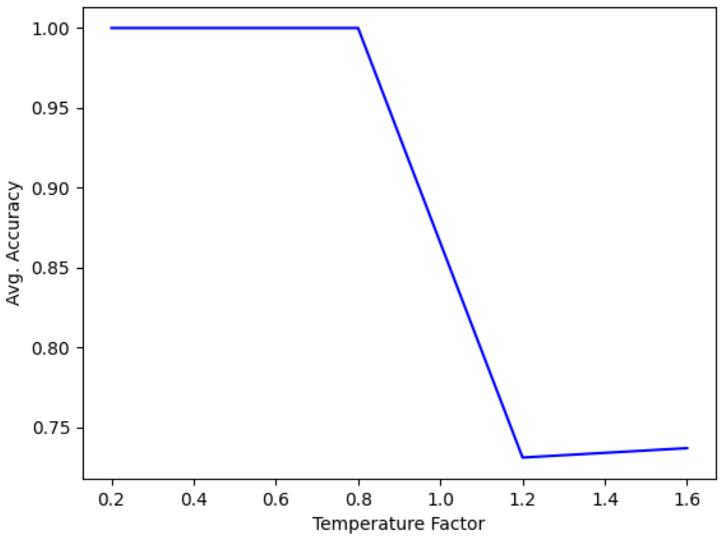
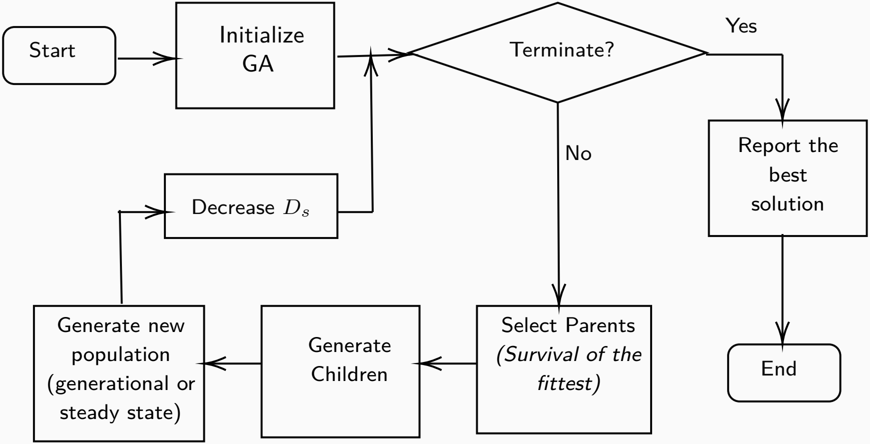


Figure: Simulated Annealing Accuracy vs. Temperature Factor

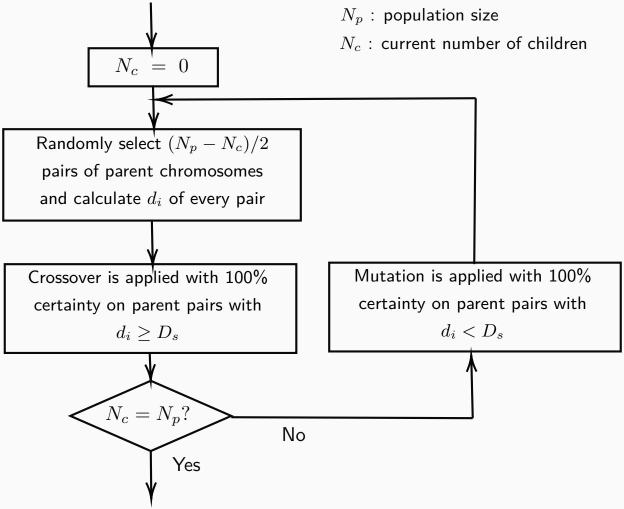
Genetic Algorithm: Key Ideas

- Global search heuristic for optimization and search problems
- Inspired by evolutionary biology concept of "Survival of the fittest"
- A population of abstract representations (genotype) of candidate solutions (individuals) evolves toward better solutions

Genetic Algorithm



Genetic Algorithm: Crossover and Mutation



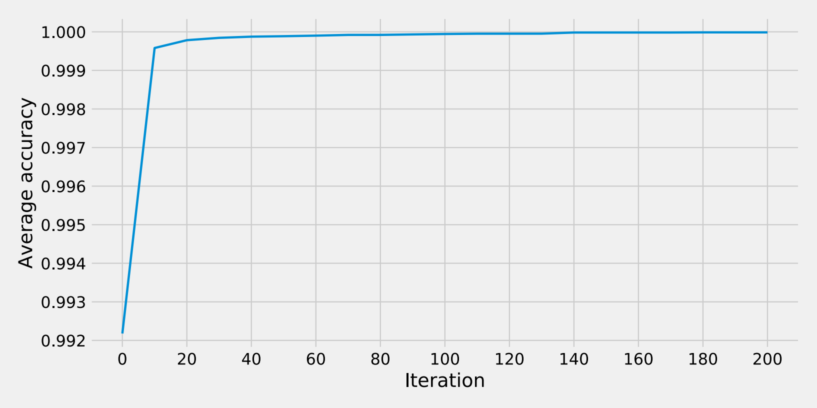


Figure: Genetic Algorithm Average Accuracy vs. Iteration

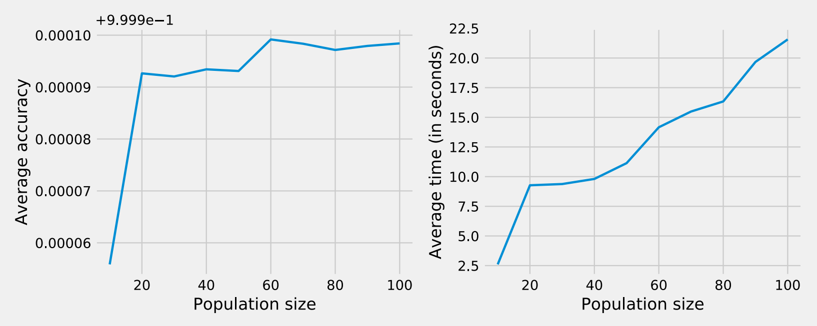


Figure: Varying the population size of Genetic Algorithm

IMPLEMENTATION AND COMPARISON

Run-time Comparison

- The average run-times (in seconds) of the algorithms were compared on the 3 categories of sets.

Set Size	FPTAS	Simulated Annealing	Genetic Algorithm
10	0.0011	0.1857	0.6841353
100	0.2604	0.0878	0.9269712
1000	41.8134	1.1680	22.178837

Table: Average Run-time Comparison

Accuracy Comparison

- The average accuracy of the algorithms were compared on the 3 categories of sets.

Set Size	FPTAS	Simulated Annealing	Genetic Algorithm
10	0.9674	1	1
100	0.9976	1	1
1000	0.9997	1	1

Table: Average Accuracy Comparison

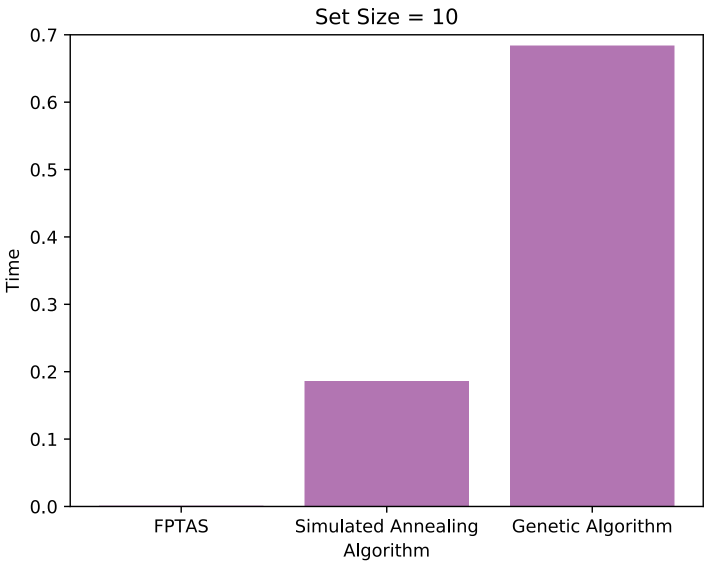


Figure: Elapsed time (sec) for three algorithms (Input Set Size = 10)

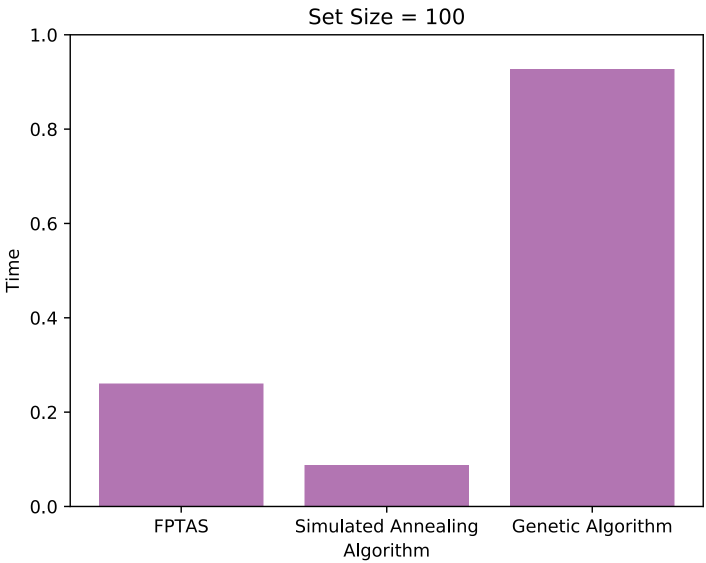


Figure: Elapsed time (sec) for three algorithms (Input Set Size = 100)

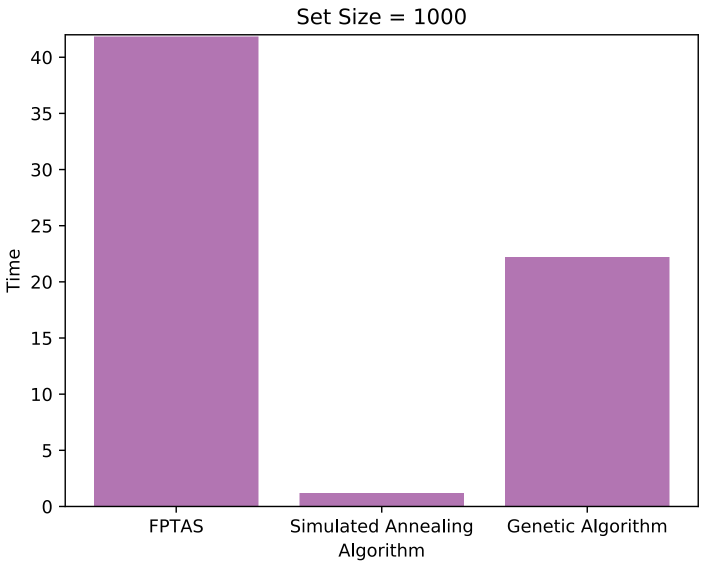


Figure: Elapsed time (sec) for three algorithms (Input Set Size = 1000)

REFERENCES

References I

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- S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi, "Optimization by simulated annealing," *science*, vol. 220, no. 4598, pp. 671–680, 1983.

THANK YOU!