

# Task 6

for Advanced Methods for Regression and Classification

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## Contents

<b>Regression on Splines</b>	<b>1</b>
B-splines on: . . . . .	18
Natural Cubic splines on: . . . . .	21
Smoothing spline . . . . .	24
<b>Regression on the whole data with splines</b>	<b>27</b>
Stepwise variable selection . . . . .	30
<b>Calculate the RMSE on the test set</b>	<b>31</b>

## Regression on Splines

In this exercise we will see the different splines methods and how we can regress this data as non-linear.

```
library(ISLR)
library(splines)

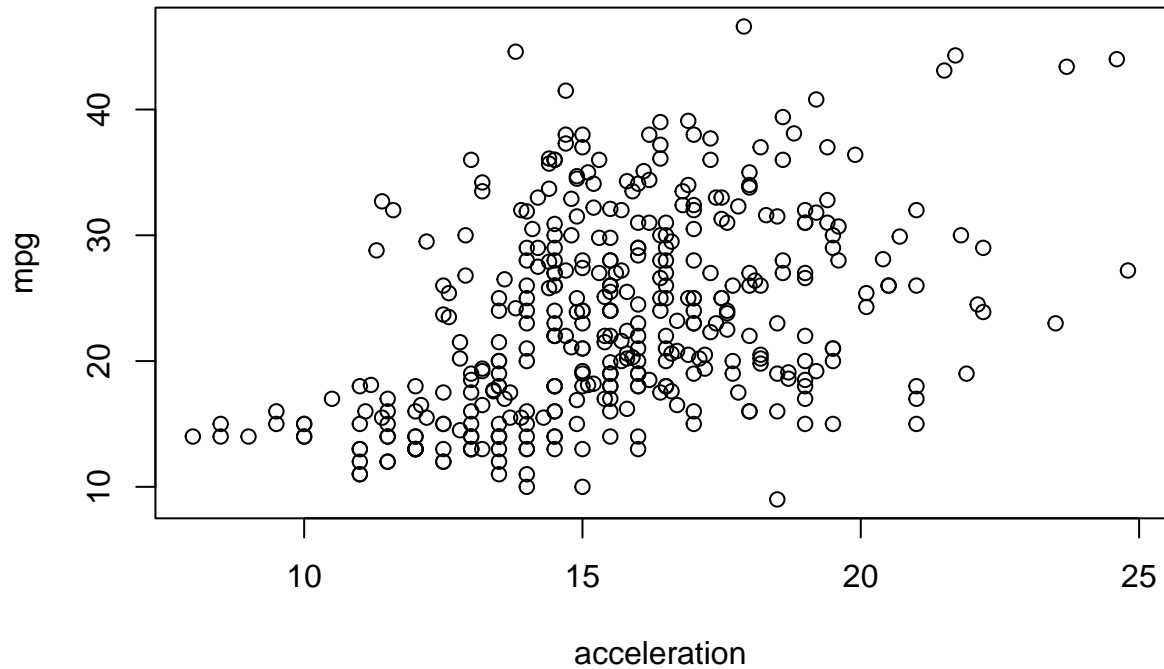
data(Auto ,package="ISLR")

df <- Auto

str(df)
```

```
## 'data.frame':   392 obs. of  9 variables:
##  $ mpg      : num  18 15 18 16 17 15 14 14 15 ...
##  $ cylinders : num   8  8  8  8  8  8  8  8  8 ...
##  $ displacement: num  307 350 318 304 302 429 454 440 455 390 ...
##  $ horsepower  : num  130 165 150 150 140 198 220 215 225 190 ...
##  $ weight      : num  3504 3693 3436 3433 3449 ...
##  $ acceleration: num   12 11.5 11 12 10.5 10 9 8.5 10 8.5 ...
##  $ year        : num   70 70 70 70 70 70 70 70 70 ...
##  $ origin      : num    1  1  1  1  1  1  1  1  1 ...
##  $ name       : Factor w/ 304 levels "amc ambassador brougham",...: 49 36 231 14 161 141 54 223 241 ...
```

```
plot(mpg~acceleration, data = df)
```



This scatter plot is showing non-linearity of those two attributes.

```
df
```

##	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin
## 1	18.0	8	307.0	130	3504	12.0	70	1
## 2	15.0	8	350.0	165	3693	11.5	70	1
## 3	18.0	8	318.0	150	3436	11.0	70	1
## 4	16.0	8	304.0	150	3433	12.0	70	1
## 5	17.0	8	302.0	140	3449	10.5	70	1
## 6	15.0	8	429.0	198	4341	10.0	70	1
## 7	14.0	8	454.0	220	4354	9.0	70	1
## 8	14.0	8	440.0	215	4312	8.5	70	1
## 9	14.0	8	455.0	225	4425	10.0	70	1
## 10	15.0	8	390.0	190	3850	8.5	70	1
## 11	15.0	8	383.0	170	3563	10.0	70	1
## 12	14.0	8	340.0	160	3609	8.0	70	1
## 13	15.0	8	400.0	150	3761	9.5	70	1
## 14	14.0	8	455.0	225	3086	10.0	70	1
## 15	24.0	4	113.0	95	2372	15.0	70	3
## 16	22.0	6	198.0	95	2833	15.5	70	1
## 17	18.0	6	199.0	97	2774	15.5	70	1
## 18	21.0	6	200.0	85	2587	16.0	70	1
## 19	27.0	4	97.0	88	2130	14.5	70	3

## 20	26.0	4	97.0	46	1835	20.5	70	2
## 21	25.0	4	110.0	87	2672	17.5	70	2
## 22	24.0	4	107.0	90	2430	14.5	70	2
## 23	25.0	4	104.0	95	2375	17.5	70	2
## 24	26.0	4	121.0	113	2234	12.5	70	2
## 25	21.0	6	199.0	90	2648	15.0	70	1
## 26	10.0	8	360.0	215	4615	14.0	70	1
## 27	10.0	8	307.0	200	4376	15.0	70	1
## 28	11.0	8	318.0	210	4382	13.5	70	1
## 29	9.0	8	304.0	193	4732	18.5	70	1
## 30	27.0	4	97.0	88	2130	14.5	71	3
## 31	28.0	4	140.0	90	2264	15.5	71	1
## 32	25.0	4	113.0	95	2228	14.0	71	3
## 34	19.0	6	232.0	100	2634	13.0	71	1
## 35	16.0	6	225.0	105	3439	15.5	71	1
## 36	17.0	6	250.0	100	3329	15.5	71	1
## 37	19.0	6	250.0	88	3302	15.5	71	1
## 38	18.0	6	232.0	100	3288	15.5	71	1
## 39	14.0	8	350.0	165	4209	12.0	71	1
## 40	14.0	8	400.0	175	4464	11.5	71	1
## 41	14.0	8	351.0	153	4154	13.5	71	1
## 42	14.0	8	318.0	150	4096	13.0	71	1
## 43	12.0	8	383.0	180	4955	11.5	71	1
## 44	13.0	8	400.0	170	4746	12.0	71	1
## 45	13.0	8	400.0	175	5140	12.0	71	1
## 46	18.0	6	258.0	110	2962	13.5	71	1
## 47	22.0	4	140.0	72	2408	19.0	71	1
## 48	19.0	6	250.0	100	3282	15.0	71	1
## 49	18.0	6	250.0	88	3139	14.5	71	1
## 50	23.0	4	122.0	86	2220	14.0	71	1
## 51	28.0	4	116.0	90	2123	14.0	71	2
## 52	30.0	4	79.0	70	2074	19.5	71	2
## 53	30.0	4	88.0	76	2065	14.5	71	2
## 54	31.0	4	71.0	65	1773	19.0	71	3
## 55	35.0	4	72.0	69	1613	18.0	71	3
## 56	27.0	4	97.0	60	1834	19.0	71	2
## 57	26.0	4	91.0	70	1955	20.5	71	1
## 58	24.0	4	113.0	95	2278	15.5	72	3
## 59	25.0	4	97.5	80	2126	17.0	72	1
## 60	23.0	4	97.0	54	2254	23.5	72	2
## 61	20.0	4	140.0	90	2408	19.5	72	1
## 62	21.0	4	122.0	86	2226	16.5	72	1
## 63	13.0	8	350.0	165	4274	12.0	72	1
## 64	14.0	8	400.0	175	4385	12.0	72	1
## 65	15.0	8	318.0	150	4135	13.5	72	1
## 66	14.0	8	351.0	153	4129	13.0	72	1
## 67	17.0	8	304.0	150	3672	11.5	72	1
## 68	11.0	8	429.0	208	4633	11.0	72	1
## 69	13.0	8	350.0	155	4502	13.5	72	1
## 70	12.0	8	350.0	160	4456	13.5	72	1
## 71	13.0	8	400.0	190	4422	12.5	72	1
## 72	19.0	3	70.0	97	2330	13.5	72	3
## 73	15.0	8	304.0	150	3892	12.5	72	1
## 74	13.0	8	307.0	130	4098	14.0	72	1

## 75	13.0	8	302.0	140	4294	16.0	72	1
## 76	14.0	8	318.0	150	4077	14.0	72	1
## 77	18.0	4	121.0	112	2933	14.5	72	2
## 78	22.0	4	121.0	76	2511	18.0	72	2
## 79	21.0	4	120.0	87	2979	19.5	72	2
## 80	26.0	4	96.0	69	2189	18.0	72	2
## 81	22.0	4	122.0	86	2395	16.0	72	1
## 82	28.0	4	97.0	92	2288	17.0	72	3
## 83	23.0	4	120.0	97	2506	14.5	72	3
## 84	28.0	4	98.0	80	2164	15.0	72	1
## 85	27.0	4	97.0	88	2100	16.5	72	3
## 86	13.0	8	350.0	175	4100	13.0	73	1
## 87	14.0	8	304.0	150	3672	11.5	73	1
## 88	13.0	8	350.0	145	3988	13.0	73	1
## 89	14.0	8	302.0	137	4042	14.5	73	1
## 90	15.0	8	318.0	150	3777	12.5	73	1
## 91	12.0	8	429.0	198	4952	11.5	73	1
## 92	13.0	8	400.0	150	4464	12.0	73	1
## 93	13.0	8	351.0	158	4363	13.0	73	1
## 94	14.0	8	318.0	150	4237	14.5	73	1
## 95	13.0	8	440.0	215	4735	11.0	73	1
## 96	12.0	8	455.0	225	4951	11.0	73	1
## 97	13.0	8	360.0	175	3821	11.0	73	1
## 98	18.0	6	225.0	105	3121	16.5	73	1
## 99	16.0	6	250.0	100	3278	18.0	73	1
## 100	18.0	6	232.0	100	2945	16.0	73	1
## 101	18.0	6	250.0	88	3021	16.5	73	1
## 102	23.0	6	198.0	95	2904	16.0	73	1
## 103	26.0	4	97.0	46	1950	21.0	73	2
## 104	11.0	8	400.0	150	4997	14.0	73	1
## 105	12.0	8	400.0	167	4906	12.5	73	1
## 106	13.0	8	360.0	170	4654	13.0	73	1
## 107	12.0	8	350.0	180	4499	12.5	73	1
## 108	18.0	6	232.0	100	2789	15.0	73	1
## 109	20.0	4	97.0	88	2279	19.0	73	3
## 110	21.0	4	140.0	72	2401	19.5	73	1
## 111	22.0	4	108.0	94	2379	16.5	73	3
## 112	18.0	3	70.0	90	2124	13.5	73	3
## 113	19.0	4	122.0	85	2310	18.5	73	1
## 114	21.0	6	155.0	107	2472	14.0	73	1
## 115	26.0	4	98.0	90	2265	15.5	73	2
## 116	15.0	8	350.0	145	4082	13.0	73	1
## 117	16.0	8	400.0	230	4278	9.5	73	1
## 118	29.0	4	68.0	49	1867	19.5	73	2
## 119	24.0	4	116.0	75	2158	15.5	73	2
## 120	20.0	4	114.0	91	2582	14.0	73	2
## 121	19.0	4	121.0	112	2868	15.5	73	2
## 122	15.0	8	318.0	150	3399	11.0	73	1
## 123	24.0	4	121.0	110	2660	14.0	73	2
## 124	20.0	6	156.0	122	2807	13.5	73	3
## 125	11.0	8	350.0	180	3664	11.0	73	1
## 126	20.0	6	198.0	95	3102	16.5	74	1
## 128	19.0	6	232.0	100	2901	16.0	74	1
## 129	15.0	6	250.0	100	3336	17.0	74	1

## 130 31.0	4	79.0	67	1950	19.0	74	3
## 131 26.0	4	122.0	80	2451	16.5	74	1
## 132 32.0	4	71.0	65	1836	21.0	74	3
## 133 25.0	4	140.0	75	2542	17.0	74	1
## 134 16.0	6	250.0	100	3781	17.0	74	1
## 135 16.0	6	258.0	110	3632	18.0	74	1
## 136 18.0	6	225.0	105	3613	16.5	74	1
## 137 16.0	8	302.0	140	4141	14.0	74	1
## 138 13.0	8	350.0	150	4699	14.5	74	1
## 139 14.0	8	318.0	150	4457	13.5	74	1
## 140 14.0	8	302.0	140	4638	16.0	74	1
## 141 14.0	8	304.0	150	4257	15.5	74	1
## 142 29.0	4	98.0	83	2219	16.5	74	2
## 143 26.0	4	79.0	67	1963	15.5	74	2
## 144 26.0	4	97.0	78	2300	14.5	74	2
## 145 31.0	4	76.0	52	1649	16.5	74	3
## 146 32.0	4	83.0	61	2003	19.0	74	3
## 147 28.0	4	90.0	75	2125	14.5	74	1
## 148 24.0	4	90.0	75	2108	15.5	74	2
## 149 26.0	4	116.0	75	2246	14.0	74	2
## 150 24.0	4	120.0	97	2489	15.0	74	3
## 151 26.0	4	108.0	93	2391	15.5	74	3
## 152 31.0	4	79.0	67	2000	16.0	74	2
## 153 19.0	6	225.0	95	3264	16.0	75	1
## 154 18.0	6	250.0	105	3459	16.0	75	1
## 155 15.0	6	250.0	72	3432	21.0	75	1
## 156 15.0	6	250.0	72	3158	19.5	75	1
## 157 16.0	8	400.0	170	4668	11.5	75	1
## 158 15.0	8	350.0	145	4440	14.0	75	1
## 159 16.0	8	318.0	150	4498	14.5	75	1
## 160 14.0	8	351.0	148	4657	13.5	75	1
## 161 17.0	6	231.0	110	3907	21.0	75	1
## 162 16.0	6	250.0	105	3897	18.5	75	1
## 163 15.0	6	258.0	110	3730	19.0	75	1
## 164 18.0	6	225.0	95	3785	19.0	75	1
## 165 21.0	6	231.0	110	3039	15.0	75	1
## 166 20.0	8	262.0	110	3221	13.5	75	1
## 167 13.0	8	302.0	129	3169	12.0	75	1
## 168 29.0	4	97.0	75	2171	16.0	75	3
## 169 23.0	4	140.0	83	2639	17.0	75	1
## 170 20.0	6	232.0	100	2914	16.0	75	1
## 171 23.0	4	140.0	78	2592	18.5	75	1
## 172 24.0	4	134.0	96	2702	13.5	75	3
## 173 25.0	4	90.0	71	2223	16.5	75	2
## 174 24.0	4	119.0	97	2545	17.0	75	3
## 175 18.0	6	171.0	97	2984	14.5	75	1
## 176 29.0	4	90.0	70	1937	14.0	75	2
## 177 19.0	6	232.0	90	3211	17.0	75	1
## 178 23.0	4	115.0	95	2694	15.0	75	2
## 179 23.0	4	120.0	88	2957	17.0	75	2
## 180 22.0	4	121.0	98	2945	14.5	75	2
## 181 25.0	4	121.0	115	2671	13.5	75	2
## 182 33.0	4	91.0	53	1795	17.5	75	3
## 183 28.0	4	107.0	86	2464	15.5	76	2

## 184	25.0	4	116.0	81	2220	16.9	76	2
## 185	25.0	4	140.0	92	2572	14.9	76	1
## 186	26.0	4	98.0	79	2255	17.7	76	1
## 187	27.0	4	101.0	83	2202	15.3	76	2
## 188	17.5	8	305.0	140	4215	13.0	76	1
## 189	16.0	8	318.0	150	4190	13.0	76	1
## 190	15.5	8	304.0	120	3962	13.9	76	1
## 191	14.5	8	351.0	152	4215	12.8	76	1
## 192	22.0	6	225.0	100	3233	15.4	76	1
## 193	22.0	6	250.0	105	3353	14.5	76	1
## 194	24.0	6	200.0	81	3012	17.6	76	1
## 195	22.5	6	232.0	90	3085	17.6	76	1
## 196	29.0	4	85.0	52	2035	22.2	76	1
## 197	24.5	4	98.0	60	2164	22.1	76	1
## 198	29.0	4	90.0	70	1937	14.2	76	2
## 199	33.0	4	91.0	53	1795	17.4	76	3
## 200	20.0	6	225.0	100	3651	17.7	76	1
## 201	18.0	6	250.0	78	3574	21.0	76	1
## 202	18.5	6	250.0	110	3645	16.2	76	1
## 203	17.5	6	258.0	95	3193	17.8	76	1
## 204	29.5	4	97.0	71	1825	12.2	76	2
## 205	32.0	4	85.0	70	1990	17.0	76	3
## 206	28.0	4	97.0	75	2155	16.4	76	3
## 207	26.5	4	140.0	72	2565	13.6	76	1
## 208	20.0	4	130.0	102	3150	15.7	76	2
## 209	13.0	8	318.0	150	3940	13.2	76	1
## 210	19.0	4	120.0	88	3270	21.9	76	2
## 211	19.0	6	156.0	108	2930	15.5	76	3
## 212	16.5	6	168.0	120	3820	16.7	76	2
## 213	16.5	8	350.0	180	4380	12.1	76	1
## 214	13.0	8	350.0	145	4055	12.0	76	1
## 215	13.0	8	302.0	130	3870	15.0	76	1
## 216	13.0	8	318.0	150	3755	14.0	76	1
## 217	31.5	4	98.0	68	2045	18.5	77	3
## 218	30.0	4	111.0	80	2155	14.8	77	1
## 219	36.0	4	79.0	58	1825	18.6	77	2
## 220	25.5	4	122.0	96	2300	15.5	77	1
## 221	33.5	4	85.0	70	1945	16.8	77	3
## 222	17.5	8	305.0	145	3880	12.5	77	1
## 223	17.0	8	260.0	110	4060	19.0	77	1
## 224	15.5	8	318.0	145	4140	13.7	77	1
## 225	15.0	8	302.0	130	4295	14.9	77	1
## 226	17.5	6	250.0	110	3520	16.4	77	1
## 227	20.5	6	231.0	105	3425	16.9	77	1
## 228	19.0	6	225.0	100	3630	17.7	77	1
## 229	18.5	6	250.0	98	3525	19.0	77	1
## 230	16.0	8	400.0	180	4220	11.1	77	1
## 231	15.5	8	350.0	170	4165	11.4	77	1
## 232	15.5	8	400.0	190	4325	12.2	77	1
## 233	16.0	8	351.0	149	4335	14.5	77	1
## 234	29.0	4	97.0	78	1940	14.5	77	2
## 235	24.5	4	151.0	88	2740	16.0	77	1
## 236	26.0	4	97.0	75	2265	18.2	77	3
## 237	25.5	4	140.0	89	2755	15.8	77	1

## 238 30.5	4	98.0	63	2051	17.0	77	1
## 239 33.5	4	98.0	83	2075	15.9	77	1
## 240 30.0	4	97.0	67	1985	16.4	77	3
## 241 30.5	4	97.0	78	2190	14.1	77	2
## 242 22.0	6	146.0	97	2815	14.5	77	3
## 243 21.5	4	121.0	110	2600	12.8	77	2
## 244 21.5	3	80.0	110	2720	13.5	77	3
## 245 43.1	4	90.0	48	1985	21.5	78	2
## 246 36.1	4	98.0	66	1800	14.4	78	1
## 247 32.8	4	78.0	52	1985	19.4	78	3
## 248 39.4	4	85.0	70	2070	18.6	78	3
## 249 36.1	4	91.0	60	1800	16.4	78	3
## 250 19.9	8	260.0	110	3365	15.5	78	1
## 251 19.4	8	318.0	140	3735	13.2	78	1
## 252 20.2	8	302.0	139	3570	12.8	78	1
## 253 19.2	6	231.0	105	3535	19.2	78	1
## 254 20.5	6	200.0	95	3155	18.2	78	1
## 255 20.2	6	200.0	85	2965	15.8	78	1
## 256 25.1	4	140.0	88	2720	15.4	78	1
## 257 20.5	6	225.0	100	3430	17.2	78	1
## 258 19.4	6	232.0	90	3210	17.2	78	1
## 259 20.6	6	231.0	105	3380	15.8	78	1
## 260 20.8	6	200.0	85	3070	16.7	78	1
## 261 18.6	6	225.0	110	3620	18.7	78	1
## 262 18.1	6	258.0	120	3410	15.1	78	1
## 263 19.2	8	305.0	145	3425	13.2	78	1
## 264 17.7	6	231.0	165	3445	13.4	78	1
## 265 18.1	8	302.0	139	3205	11.2	78	1
## 266 17.5	8	318.0	140	4080	13.7	78	1
## 267 30.0	4	98.0	68	2155	16.5	78	1
## 268 27.5	4	134.0	95	2560	14.2	78	3
## 269 27.2	4	119.0	97	2300	14.7	78	3
## 270 30.9	4	105.0	75	2230	14.5	78	1
## 271 21.1	4	134.0	95	2515	14.8	78	3
## 272 23.2	4	156.0	105	2745	16.7	78	1
## 273 23.8	4	151.0	85	2855	17.6	78	1
## 274 23.9	4	119.0	97	2405	14.9	78	3
## 275 20.3	5	131.0	103	2830	15.9	78	2
## 276 17.0	6	163.0	125	3140	13.6	78	2
## 277 21.6	4	121.0	115	2795	15.7	78	2
## 278 16.2	6	163.0	133	3410	15.8	78	2
## 279 31.5	4	89.0	71	1990	14.9	78	2
## 280 29.5	4	98.0	68	2135	16.6	78	3
## 281 21.5	6	231.0	115	3245	15.4	79	1
## 282 19.8	6	200.0	85	2990	18.2	79	1
## 283 22.3	4	140.0	88	2890	17.3	79	1
## 284 20.2	6	232.0	90	3265	18.2	79	1
## 285 20.6	6	225.0	110	3360	16.6	79	1
## 286 17.0	8	305.0	130	3840	15.4	79	1
## 287 17.6	8	302.0	129	3725	13.4	79	1
## 288 16.5	8	351.0	138	3955	13.2	79	1
## 289 18.2	8	318.0	135	3830	15.2	79	1
## 290 16.9	8	350.0	155	4360	14.9	79	1
## 291 15.5	8	351.0	142	4054	14.3	79	1

## 292 19.2	8	267.0	125	3605	15.0	79	1
## 293 18.5	8	360.0	150	3940	13.0	79	1
## 294 31.9	4	89.0	71	1925	14.0	79	2
## 295 34.1	4	86.0	65	1975	15.2	79	3
## 296 35.7	4	98.0	80	1915	14.4	79	1
## 297 27.4	4	121.0	80	2670	15.0	79	1
## 298 25.4	5	183.0	77	3530	20.1	79	2
## 299 23.0	8	350.0	125	3900	17.4	79	1
## 300 27.2	4	141.0	71	3190	24.8	79	2
## 301 23.9	8	260.0	90	3420	22.2	79	1
## 302 34.2	4	105.0	70	2200	13.2	79	1
## 303 34.5	4	105.0	70	2150	14.9	79	1
## 304 31.8	4	85.0	65	2020	19.2	79	3
## 305 37.3	4	91.0	69	2130	14.7	79	2
## 306 28.4	4	151.0	90	2670	16.0	79	1
## 307 28.8	6	173.0	115	2595	11.3	79	1
## 308 26.8	6	173.0	115	2700	12.9	79	1
## 309 33.5	4	151.0	90	2556	13.2	79	1
## 310 41.5	4	98.0	76	2144	14.7	80	2
## 311 38.1	4	89.0	60	1968	18.8	80	3
## 312 32.1	4	98.0	70	2120	15.5	80	1
## 313 37.2	4	86.0	65	2019	16.4	80	3
## 314 28.0	4	151.0	90	2678	16.5	80	1
## 315 26.4	4	140.0	88	2870	18.1	80	1
## 316 24.3	4	151.0	90	3003	20.1	80	1
## 317 19.1	6	225.0	90	3381	18.7	80	1
## 318 34.3	4	97.0	78	2188	15.8	80	2
## 319 29.8	4	134.0	90	2711	15.5	80	3
## 320 31.3	4	120.0	75	2542	17.5	80	3
## 321 37.0	4	119.0	92	2434	15.0	80	3
## 322 32.2	4	108.0	75	2265	15.2	80	3
## 323 46.6	4	86.0	65	2110	17.9	80	3
## 324 27.9	4	156.0	105	2800	14.4	80	1
## 325 40.8	4	85.0	65	2110	19.2	80	3
## 326 44.3	4	90.0	48	2085	21.7	80	2
## 327 43.4	4	90.0	48	2335	23.7	80	2
## 328 36.4	5	121.0	67	2950	19.9	80	2
## 329 30.0	4	146.0	67	3250	21.8	80	2
## 330 44.6	4	91.0	67	1850	13.8	80	3
## 332 33.8	4	97.0	67	2145	18.0	80	3
## 333 29.8	4	89.0	62	1845	15.3	80	2
## 334 32.7	6	168.0	132	2910	11.4	80	3
## 335 23.7	3	70.0	100	2420	12.5	80	3
## 336 35.0	4	122.0	88	2500	15.1	80	2
## 338 32.4	4	107.0	72	2290	17.0	80	3
## 339 27.2	4	135.0	84	2490	15.7	81	1
## 340 26.6	4	151.0	84	2635	16.4	81	1
## 341 25.8	4	156.0	92	2620	14.4	81	1
## 342 23.5	6	173.0	110	2725	12.6	81	1
## 343 30.0	4	135.0	84	2385	12.9	81	1
## 344 39.1	4	79.0	58	1755	16.9	81	3
## 345 39.0	4	86.0	64	1875	16.4	81	1
## 346 35.1	4	81.0	60	1760	16.1	81	3
## 347 32.3	4	97.0	67	2065	17.8	81	3



## 348	37.0	4	85.0	65	1975	19.4	81	3
## 349	37.7	4	89.0	62	2050	17.3	81	3
## 350	34.1	4	91.0	68	1985	16.0	81	3
## 351	34.7	4	105.0	63	2215	14.9	81	1
## 352	34.4	4	98.0	65	2045	16.2	81	1
## 353	29.9	4	98.0	65	2380	20.7	81	1
## 354	33.0	4	105.0	74	2190	14.2	81	2
## 356	33.7	4	107.0	75	2210	14.4	81	3
## 357	32.4	4	108.0	75	2350	16.8	81	3
## 358	32.9	4	119.0	100	2615	14.8	81	3
## 359	31.6	4	120.0	74	2635	18.3	81	3
## 360	28.1	4	141.0	80	3230	20.4	81	2
## 361	30.7	6	145.0	76	3160	19.6	81	2
## 362	25.4	6	168.0	116	2900	12.6	81	3
## 363	24.2	6	146.0	120	2930	13.8	81	3
## 364	22.4	6	231.0	110	3415	15.8	81	1
## 365	26.6	8	350.0	105	3725	19.0	81	1
## 366	20.2	6	200.0	88	3060	17.1	81	1
## 367	17.6	6	225.0	85	3465	16.6	81	1
## 368	28.0	4	112.0	88	2605	19.6	82	1
## 369	27.0	4	112.0	88	2640	18.6	82	1
## 370	34.0	4	112.0	88	2395	18.0	82	1
## 371	31.0	4	112.0	85	2575	16.2	82	1
## 372	29.0	4	135.0	84	2525	16.0	82	1
## 373	27.0	4	151.0	90	2735	18.0	82	1
## 374	24.0	4	140.0	92	2865	16.4	82	1
## 375	36.0	4	105.0	74	1980	15.3	82	2
## 376	37.0	4	91.0	68	2025	18.2	82	3
## 377	31.0	4	91.0	68	1970	17.6	82	3
## 378	38.0	4	105.0	63	2125	14.7	82	1
## 379	36.0	4	98.0	70	2125	17.3	82	1
## 380	36.0	4	120.0	88	2160	14.5	82	3
## 381	36.0	4	107.0	75	2205	14.5	82	3
## 382	34.0	4	108.0	70	2245	16.9	82	3
## 383	38.0	4	91.0	67	1965	15.0	82	3
## 384	32.0	4	91.0	67	1965	15.7	82	3
## 385	38.0	4	91.0	67	1995	16.2	82	3
## 386	25.0	6	181.0	110	2945	16.4	82	1
## 387	38.0	6	262.0	85	3015	17.0	82	1
## 388	26.0	4	156.0	92	2585	14.5	82	1
## 389	22.0	6	232.0	112	2835	14.7	82	1
## 390	32.0	4	144.0	96	2665	13.9	82	3
## 391	36.0	4	135.0	84	2370	13.0	82	1
## 392	27.0	4	151.0	90	2950	17.3	82	1
## 393	27.0	4	140.0	86	2790	15.6	82	1
## 394	44.0	4	97.0	52	2130	24.6	82	2
## 395	32.0	4	135.0	84	2295	11.6	82	1
## 396	28.0	4	120.0	79	2625	18.6	82	1
## 397	31.0	4	119.0	82	2720	19.4	82	1
##								
##					name			
## 1					chevrolet chevelle malibu			
## 2					buick skylark 320			
## 3					plymouth satellite			
## 4					amc rebel sst			

```

## 5         ford torino
## 6         ford galaxie 500
## 7         chevrolet impala
## 8         plymouth fury iii
## 9         pontiac catalina
## 10        amc ambassador dpl
## 11        dodge challenger se
## 12        plymouth 'cuda 340
## 13        chevrolet monte carlo
## 14        buick estate wagon (sw)
## 15        toyota corona mark ii
## 16        plymouth duster
## 17        amc hornet
## 18        ford maverick
## 19        datsun pl510
## 20        volkswagen 1131 deluxe sedan
## 21        peugeot 504
## 22        audi 100 ls
## 23        saab 99e
## 24        bmw 2002
## 25        amc gremlin
## 26        ford f250
## 27        chevy c20
## 28        dodge d200
## 29        hi 1200d
## 30        datsun pl510
## 31        chevrolet vega 2300
## 32        toyota corona
## 33        amc gremlin
## 34        plymouth satellite custom
## 35        chevrolet chevelle malibu
## 36        ford torino 500
## 37        amc matador
## 38        chevrolet impala
## 39        pontiac catalina brougham
## 40        ford galaxie 500
## 41        plymouth fury iii
## 42        dodge monaco (sw)
## 43        ford country squire (sw)
## 44        pontiac safari (sw)
## 45        amc hornet sportabout (sw)
## 46        chevrolet vega (sw)
## 47        pontiac firebird
## 48        ford mustang
## 49        mercury capri 2000
## 50        opel 1900
## 51        peugeot 304
## 52        fiat 124b
## 53        toyota corolla 1200
## 54        datsun 1200
## 55        volkswagen model 111
## 56        plymouth cricket
## 57        toyota corona hardtop
## 58        dodge colt hardtop

```

```

## 60          volkswagen type 3
## 61          chevrolet vega
## 62          ford pinto runabout
## 63          chevrolet impala
## 64          pontiac catalina
## 65          plymouth fury iii
## 66          ford galaxie 500
## 67          amc ambassador sst
## 68          mercury marquis
## 69          buick lesabre custom
## 70          oldsmobile delta 88 royale
## 71          chrysler newport royal
## 72          mazda rx2 coupe
## 73          amc matador (sw)
## 74          chevrolet chevelle concours (sw)
## 75          ford gran torino (sw)
## 76          plymouth satellite custom (sw)
## 77          volvo 145e (sw)
## 78          volkswagen 411 (sw)
## 79          peugeot 504 (sw)
## 80          renault 12 (sw)
## 81          ford pinto (sw)
## 82          datsun 510 (sw)
## 83          toyouta corona mark ii (sw)
## 84          dodge colt (sw)
## 85          toyota corolla 1600 (sw)
## 86          buick century 350
## 87          amc matador
## 88          chevrolet malibu
## 89          ford gran torino
## 90          dodge coronet custom
## 91          mercury marquis brougham
## 92          chevrolet caprice classic
## 93          ford ltd
## 94          plymouth fury gran sedan
## 95          chrysler new yorker brougham
## 96          buick electra 225 custom
## 97          amc ambassador brougham
## 98          plymouth valiant
## 99          chevrolet nova custom
## 100         amc hornet
## 101         ford maverick
## 102         plymouth duster
## 103         volkswagen super beetle
## 104         chevrolet impala
## 105         ford country
## 106         plymouth custom suburb
## 107         oldsmobile vista cruiser
## 108         amc gremlin
## 109         toyota carina
## 110         chevrolet vega
## 111         datsun 610
## 112         maxda rx3
## 113         ford pinto

```

```

## 114             mercury capri v6
## 115             fiat 124 sport coupe
## 116             chevrolet monte carlo s
## 117             pontiac grand prix
## 118             fiat 128
## 119             opel manta
## 120             audi 100ls
## 121             volvo 144ea
## 122             dodge dart custom
## 123             saab 99le
## 124             toyota mark ii
## 125             oldsmobile omega
## 126             plymouth duster
## 128             amc hornet
## 129             chevrolet nova
## 130             datsun b210
## 131             ford pinto
## 132             toyota corolla 1200
## 133             chevrolet vega
## 134             chevrolet chevelle malibu classic
## 135             amc matador
## 136             plymouth satellite sebring
## 137             ford gran torino
## 138             buick century luxus (sw)
## 139             dodge coronet custom (sw)
## 140             ford gran torino (sw)
## 141             amc matador (sw)
## 142             audi fox
## 143             volkswagen dasher
## 144             opel manta
## 145             toyota corona
## 146             datsun 710
## 147             dodge colt
## 148             fiat 128
## 149             fiat 124 tc
## 150             honda civic
## 151             subaru
## 152             fiat x1.9
## 153             plymouth valiant custom
## 154             chevrolet nova
## 155             mercury monarch
## 156             ford maverick
## 157             pontiac catalina
## 158             chevrolet bel air
## 159             plymouth grand fury
## 160             ford ltd
## 161             buick century
## 162             chevroelt chevelle malibu
## 163             amc matador
## 164             plymouth fury
## 165             buick skyhawk
## 166             chevrolet monza 2+2
## 167             ford mustang ii
## 168             toyota corolla

```

```

## 169             ford pinto
## 170             amc gremlin
## 171             pontiac astro
## 172             toyota corona
## 173             volkswagen dasher
## 174             datsun 710
## 175             ford pinto
## 176             volkswagen rabbit
## 177             amc pacer
## 178             audi 100ls
## 179             peugeot 504
## 180             volvo 244dl
## 181             saab 99le
## 182             honda civic cvcc
## 183             fiat 131
## 184             opel 1900
## 185             capri ii
## 186             dodge colt
## 187             renault 12tl
## 188     chevrolet chevelle malibu classic
## 189             dodge coronet brougham
## 190             amc matador
## 191             ford gran torino
## 192             plymouth valiant
## 193             chevrolet nova
## 194             ford maverick
## 195             amc hornet
## 196             chevrolet chevette
## 197             chevrolet woody
## 198             vw rabbit
## 199             honda civic
## 200             dodge aspen se
## 201             ford granada ghia
## 202             pontiac ventura sj
## 203             amc pacer d/l
## 204             volkswagen rabbit
## 205             datsun b-210
## 206             toyota corolla
## 207             ford pinto
## 208             volvo 245
## 209     plymouth volare premier v8
## 210             peugeot 504
## 211             toyota mark ii
## 212             mercedes-benz 280s
## 213             cadillac seville
## 214             chevy c10
## 215             ford f108
## 216             dodge d100
## 217             honda accord cvcc
## 218             buick opel isuzu deluxe
## 219             renault 5 gtl
## 220             plymouth arrow gs
## 221             datsun f-10 hatchback
## 222             chevrolet caprice classic

```

```

## 223         oldsmobile cutlass supreme
## 224             dodge monaco brougham
## 225             mercury cougar brougham
## 226             chevrolet concours
## 227             buick skylark
## 228             plymouth volare custom
## 229             ford granada
## 230             pontiac grand prix lj
## 231         chevrolet monte carlo landau
## 232             chrysler cordoba
## 233             ford thunderbird
## 234             volkswagen rabbit custom
## 235             pontiac sunbird coupe
## 236             toyota corolla liftback
## 237             ford mustang ii 2+2
## 238             chevrolet chevette
## 239             dodge colt m/m
## 240             subaru dl
## 241             volkswagen dasher
## 242             datsun 810
## 243             bmw 320i
## 244             mazda rx-4
## 245         volkswagen rabbit custom diesel
## 246             ford fiesta
## 247             mazda glc deluxe
## 248             datsun b210 gx
## 249             honda civic cvcc
## 250         oldsmobile cutlass salon brougham
## 251             dodge diplomat
## 252             mercury monarch ghia
## 253             pontiac phoenix lj
## 254             chevrolet malibu
## 255             ford fairmont (auto)
## 256             ford fairmont (man)
## 257             plymouth volare
## 258             amc concord
## 259             buick century special
## 260             mercury zephyr
## 261             dodge aspen
## 262             amc concord d/l
## 263             chevrolet monte carlo landau
## 264         buick regal sport coupe (turbo)
## 265             ford futura
## 266             dodge magnum xe
## 267             chevrolet chevette
## 268             toyota corona
## 269             datsun 510
## 270             dodge omni
## 271         toyota celica gt liftback
## 272             plymouth sapporo
## 273             oldsmobile starfire sx
## 274             datsun 200-sx
## 275             audi 5000
## 276             volvo 264gl

```

```

## 277          saab 99gle
## 278          peugeot 604sl
## 279          volkswagen scirocco
## 280          honda accord lx
## 281          pontiac lemans v6
## 282          mercury zephyr 6
## 283          ford fairmont 4
## 284          amc concord dl 6
## 285          dodge aspen 6
## 286          chevrolet caprice classic
## 287          ford ltd landau
## 288          mercury grand marquis
## 289          dodge st. regis
## 290          buick estate wagon (sw)
## 291          ford country squire (sw)
## 292          chevrolet malibu classic (sw)
## 293 chrysler lebaron town @ country (sw)
## 294          vw rabbit custom
## 295          maxda glc deluxe
## 296          dodge colt hatchback custom
## 297          amc spirit dl
## 298          mercedes benz 300d
## 299          cadillac eldorado
## 300          peugeot 504
## 301          oldsmobile cutlass salon brougham
## 302          plymouth horizon
## 303          plymouth horizon tc3
## 304          datsun 210
## 305          fiat strada custom
## 306          buick skylark limited
## 307          chevrolet citation
## 308          oldsmobile omega brougham
## 309          pontiac phoenix
## 310          vw rabbit
## 311          toyota corolla tercel
## 312          chevrolet chevette
## 313          datsun 310
## 314          chevrolet citation
## 315          ford fairmont
## 316          amc concord
## 317          dodge aspen
## 318          audi 4000
## 319          toyota corona liftback
## 320          mazda 626
## 321          datsun 510 hatchback
## 322          toyota corolla
## 323          mazda glc
## 324          dodge colt
## 325          datsun 210
## 326          vw rabbit c (diesel)
## 327          vw dasher (diesel)
## 328          audi 5000s (diesel)
## 329          mercedes-benz 240d
## 330          honda civic 1500 gl

```

```

## 332          subaru dl
## 333          vokswagen rabbit
## 334          datsun 280-zx
## 335          mazda rx-7 gs
## 336          triumph tr7 coupe
## 338          honda accord
## 339          plymouth reliant
## 340          buick skylark
## 341          dodge aries wagon (sw)
## 342          chevrolet citation
## 343          plymouth reliant
## 344          toyota starlet
## 345          plymouth champ
## 346          honda civic 1300
## 347          subaru
## 348          datsun 210 mpg
## 349          toyota tercel
## 350          mazda glc 4
## 351          plymouth horizon 4
## 352          ford escort 4w
## 353          ford escort 2h
## 354          volkswagen jetta
## 356          honda prelude
## 357          toyota corolla
## 358          datsun 200sx
## 359          mazda 626
## 360          peugeot 505s turbo diesel
## 361          volvo diesel
## 362          toyota cressida
## 363          datsun 810 maxima
## 364          buick century
## 365          oldsmobile cutlass ls
## 366          ford granada gl
## 367          chrysler lebaron salon
## 368          chevrolet cavalier
## 369          chevrolet cavalier wagon
## 370          chevrolet cavalier 2-door
## 371          pontiac j2000 se hatchback
## 372          dodge aries se
## 373          pontiac phoenix
## 374          ford fairmont futura
## 375          volkswagen rabbit l
## 376          mazda glc custom l
## 377          mazda glc custom
## 378          plymouth horizon miser
## 379          mercury lynx l
## 380          nissan stanza xe
## 381          honda accord
## 382          toyota corolla
## 383          honda civic
## 384          honda civic (auto)
## 385          datsun 310 gx
## 386          buick century limited
## 387          oldsmobile cutlass ciera (diesel)

```



```
## 388          chrysler lebaron medallion
## 389          ford granada l
## 390          toyota celica gt
## 391          dodge charger 2.2
## 392          chevrolet camaro
## 393          ford mustang gl
## 394          vw pickup
## 395          dodge rampage
## 396          ford ranger
## 397          chevy s-10
```

```
df <- df[order(df$acceleration),]
```

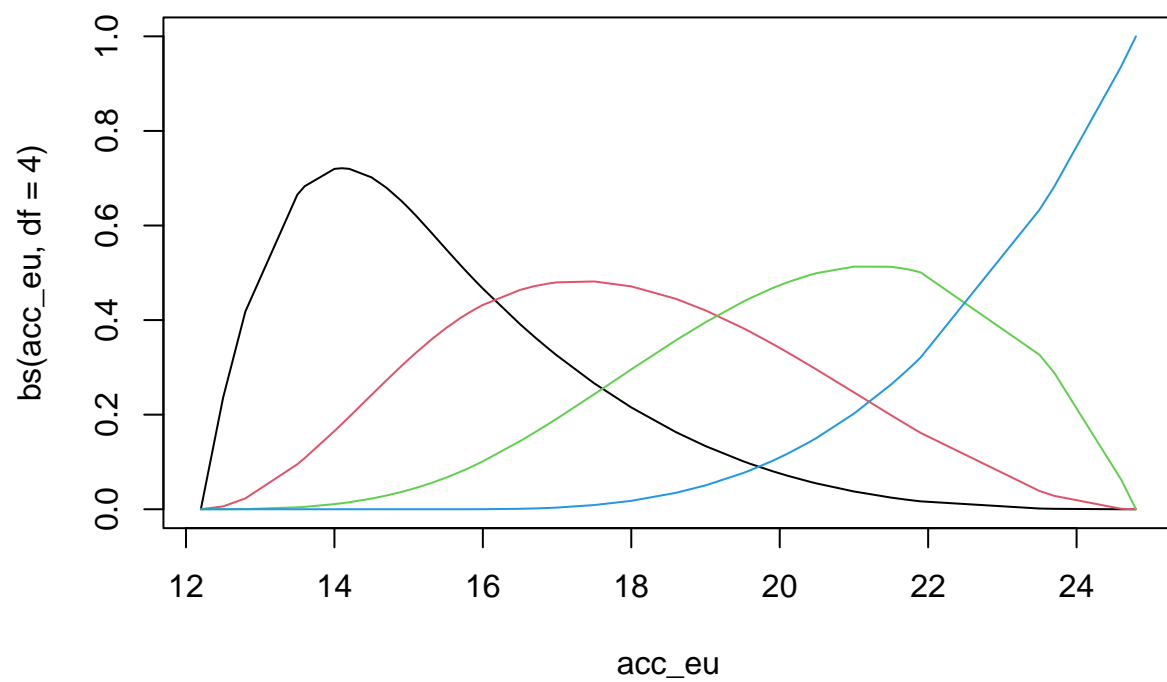
Splitting the data based on the origin group:

```
american <- subset(df, origin==1)
european <- subset(df, origin==2)
japan <- subset(df, origin==3)

mpg_eu <- european[, which(names(european) %in% c("mpg"))]
acc_eu <- european[, which(names(european) %in% c("acceleration"))]
mpg_us <- american[, which(names(american) %in% c("mpg"))]
acc_us <- american[, which(names(american) %in% c("acceleration"))]
mpg_jp <- japan[, which(names(japan) %in% c("mpg"))]
acc_jp <- japan[, which(names(japan) %in% c("acceleration"))]
```

Lets see the linear functions we create with B-splines with degrees of freedom = 4

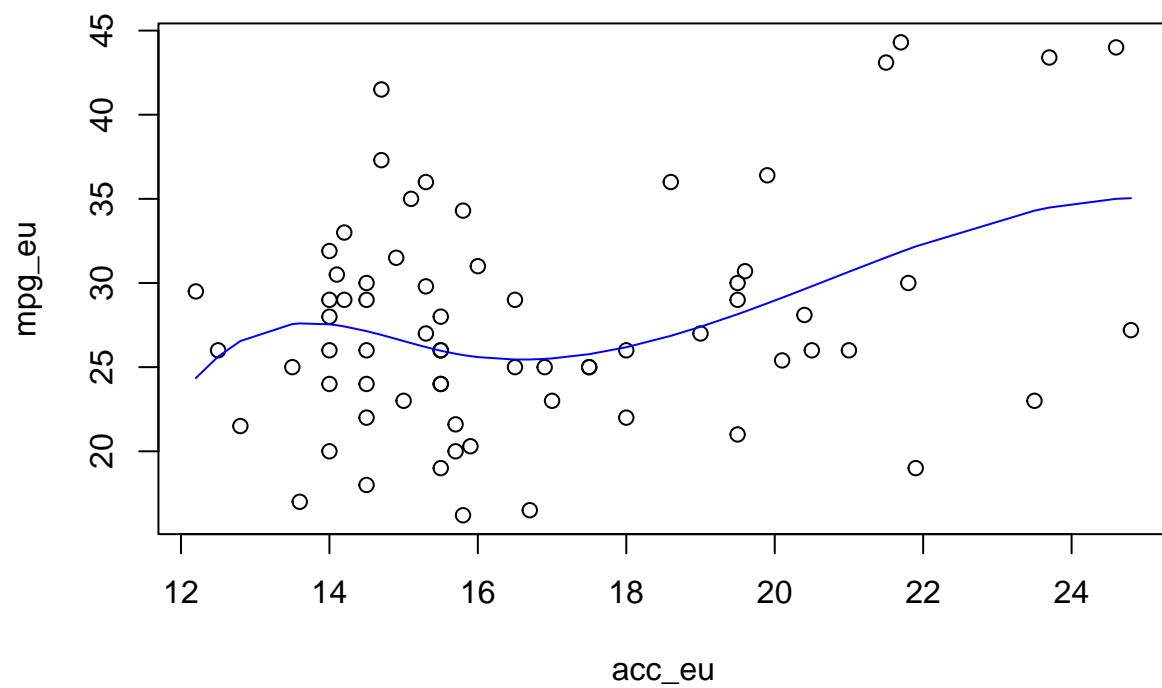
```
matplot(acc_eu, bs(acc_eu, df = 4), type="l",lty=1)
```



**B-splines on:**

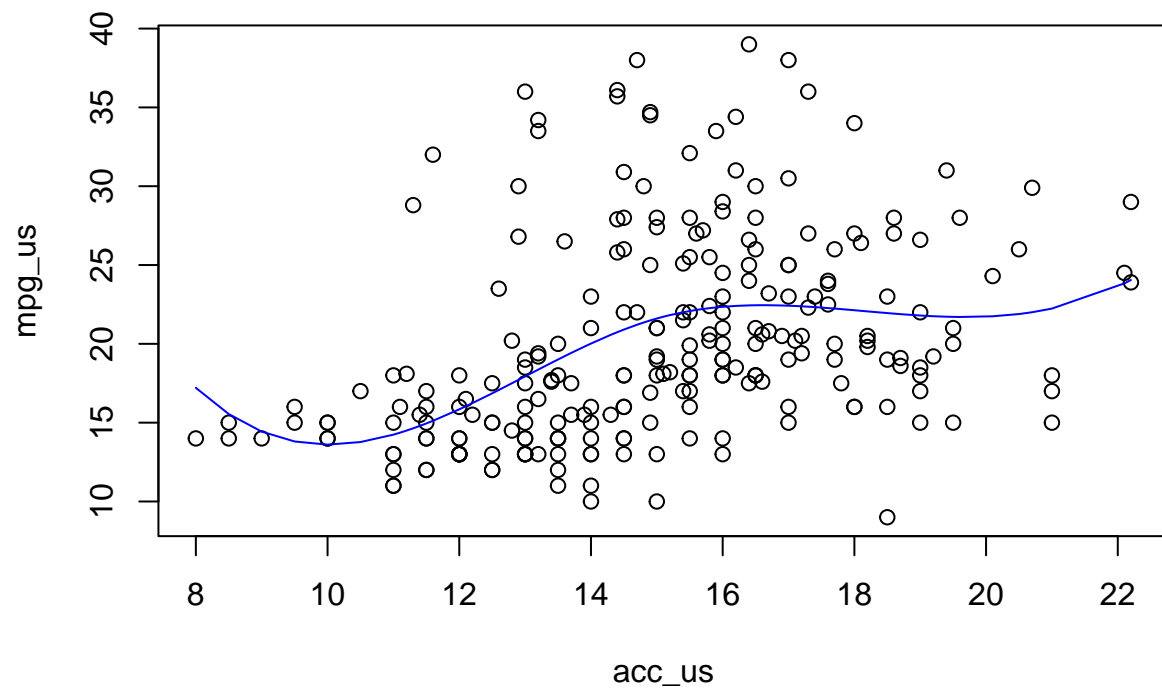
- European cars:

```
plot(acc_eu, mpg_eu)
lm1 <- lm(mpg_eu ~ bs(acc_eu, df=4))
lines(acc_eu, predict.lm(lm1, list(x=acc_eu)), col="blue")
```



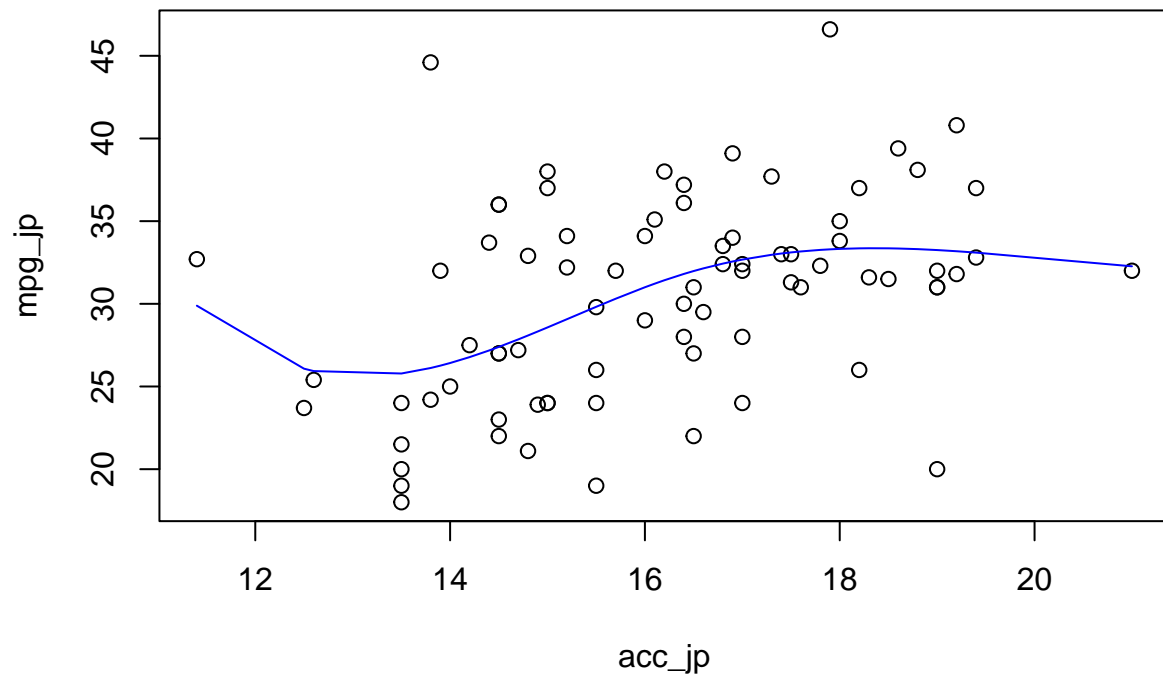
- United states cars:

```
plot(acc_us, mpg_us)
lm1 <- lm(mpg_us ~ bs(acc_us, df=4))
lines(acc_us, predict.lm(lm1, list(x=acc_us)), col="blue")
```



- Japanese cars:

```
plot(acc_jp, mpg_jp)
lm1 <- lm(mpg_jp ~ bs(acc_jp, df=4))
lines(acc_jp, predict.lm(lm1, list(x=acc_jp)), col="blue")
```

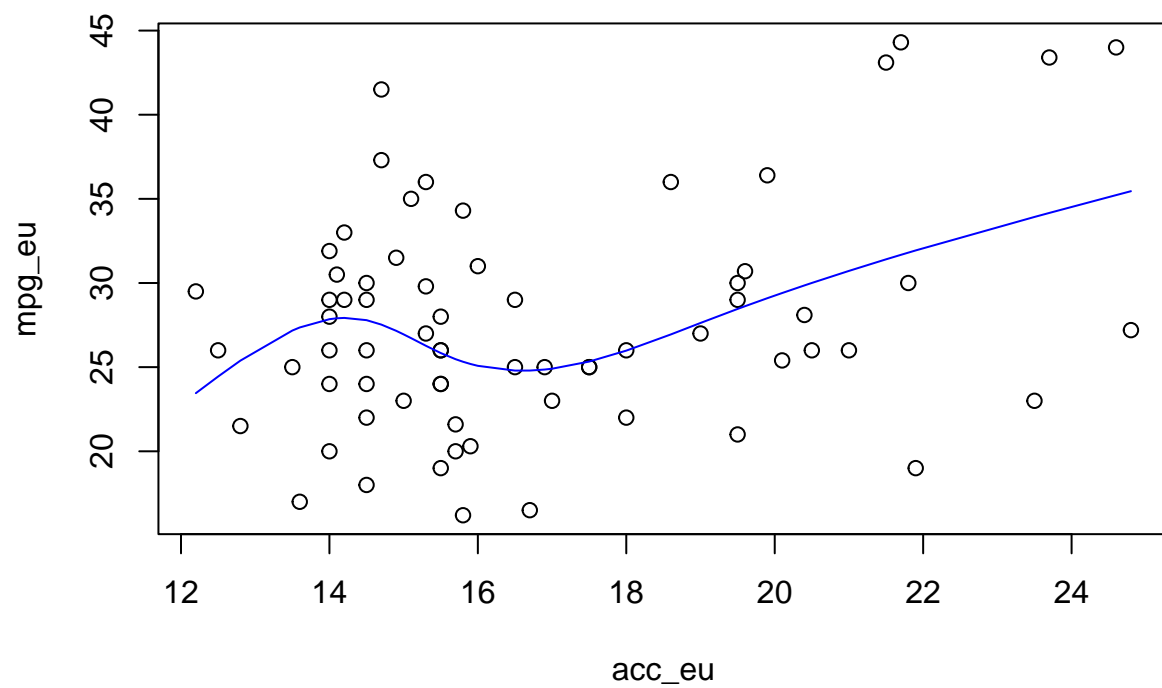


As we saw, we created separate linear regression models for each category of cars based on the 3 categories we have. As predictive values we used acceleration and for dependent value we use the mpg. We can see this time we don't have a single straight line but a curved spline which tries to describe the best way the given values.

### Natural Cubic splines on:

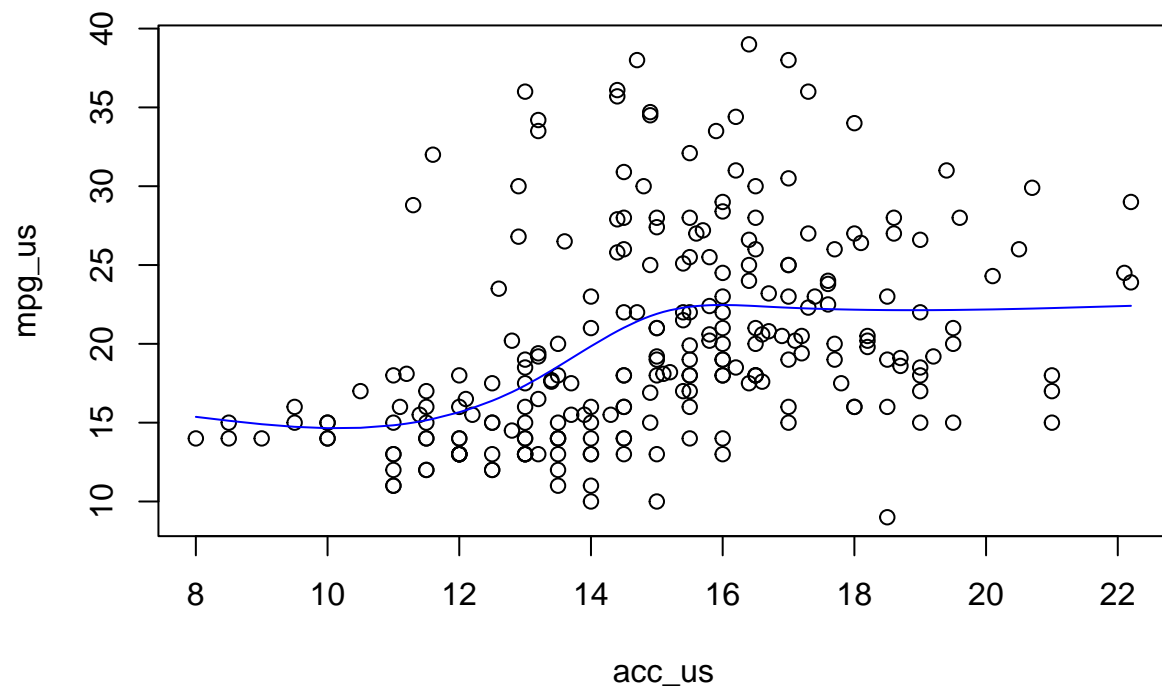
- European cars:

```
plot(acc_eu, mpg_eu)
lm1 <- lm(mpg_eu ~ ns(acc_eu, df=4))
lines(acc_eu, predict.lm(lm1, list(x=acc_eu)), col="blue")
```



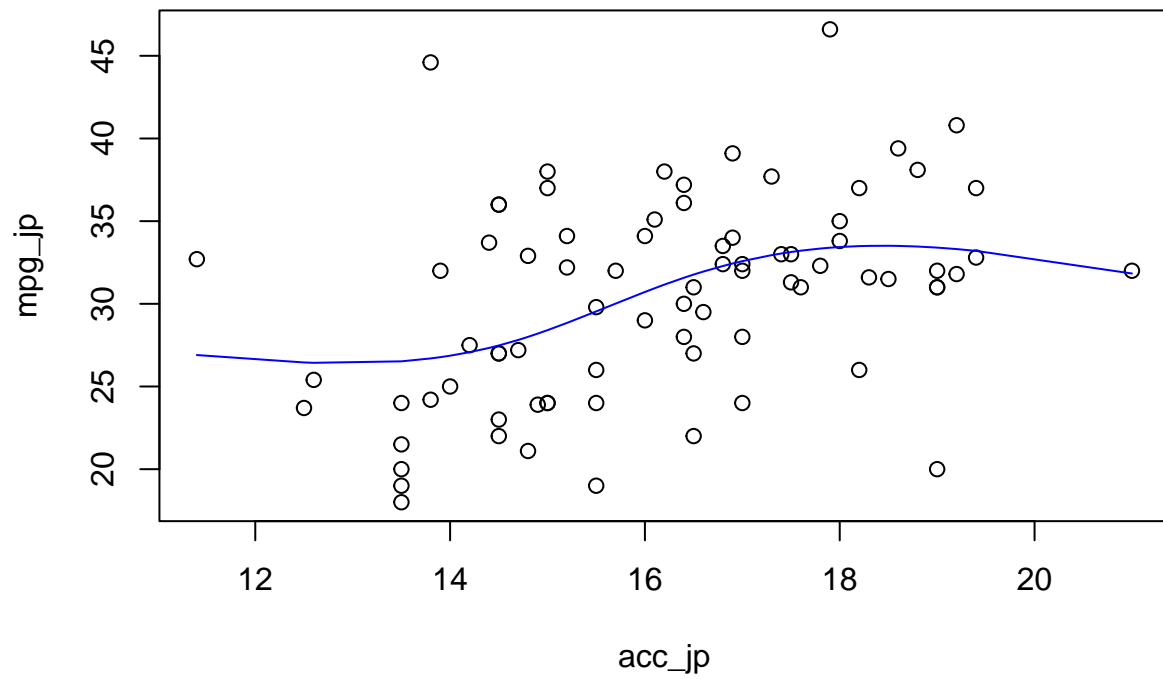
- United states cars:

```
plot(acc_us, mpg_us)
lm1 <- lm(mpg_us ~ ns(acc_us, df=4))
lines(acc_us, predict.lm(lm1, list(x=acc_us)), col="blue")
```



- Japanese cars:

```
plot(acc_jp, mpg_jp)
lm1 <- lm(mpg_jp ~ ns(acc_jp, df=4))
lines(acc_jp, predict.lm(lm1, list(x=acc_jp)), col="blue")
```

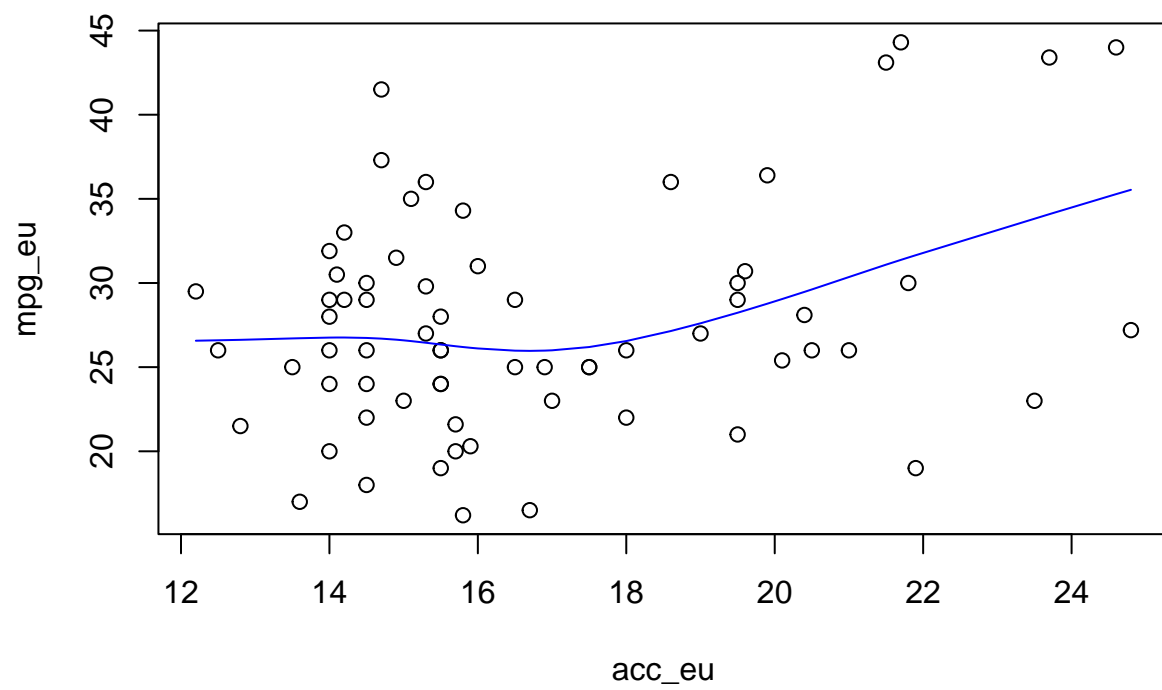


## Smoothing spline

- European cars:

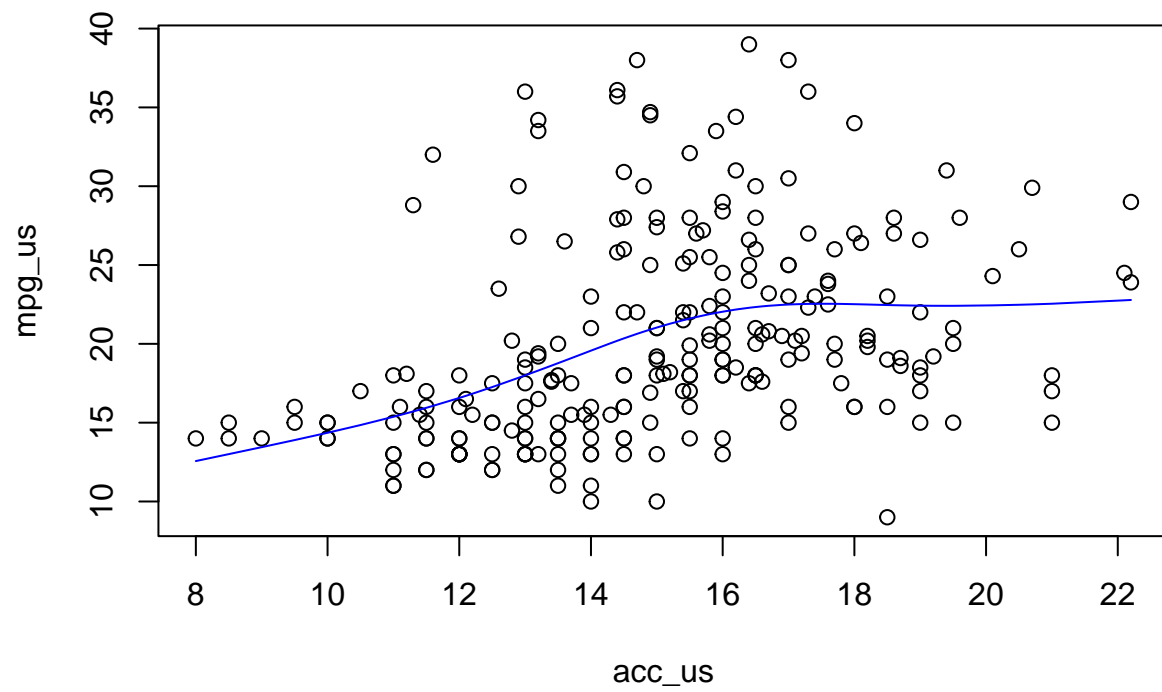
```
plot(mpg_eu ~ acc_eu)
spline_eu <- smooth.spline(acc_eu, mpg_eu, df=4)
lines(spline_eu, col="blue")
```





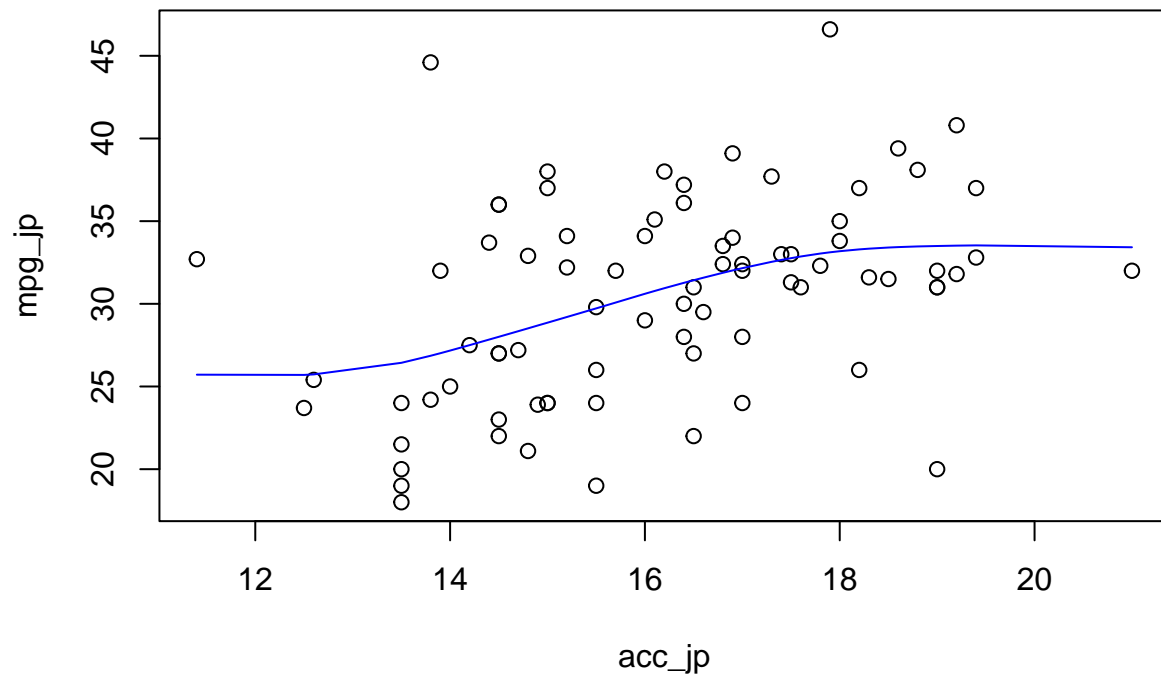
- United states cars:

```
plot(mpg_us ~ acc_us)
spline_us <- smooth.spline(acc_us, mpg_us, df=4)
lines(spline_us, col="blue")
```



- Japanese cars:

```
plot(mpg_jp ~ acc_jp)
spline_us <- smooth.spline(acc_jp, mpg_jp, df=4)
lines(spline_us, col="blue")
```



We can see the different types of splines. In general they are following the same curves though the data with all using degrees of freedom = 4.

## Regression on the whole data with splines

```
df1 <- df[ , -which(names(df) %in% c("name"))]
```

```
str(df1)
```

```
## 'data.frame':  392 obs. of  8 variables:
## $ mpg      : num  14 14 15 14 15 16 15 14 15 14 ...
## $ cylinders : num   8  8  8  8  8  8  8  8  8  8 ...
## $ displacement: num  340 440 390 454 400 400 429 455 383 455 ...
## $ horsepower  : num  160 215 190 220 150 230 198 225 170 225 ...
## $ weight      : num 3609 4312 3850 4354 3761 ...
## $ acceleration: num   8 8.5 8.5  9  9.5 9.5 10 10 10 10 ...
## $ year       : num  70 70 70 70 70 73 70 70 70 70 ...
## $ origin     : num   1  1  1  1  1  1  1  1  1  1 ...
```

```
set.seed(16)
```

```
## 2/3 of the sample size
```

```
smp_size <- floor(round(nrow(df1)*2/3))
```

```
train_ind <- sample(seq_len(nrow(df1)), size = smp_size)
smp_size
```

```
## [1] 261
```

```
train <- df1[train_ind, ]
test <- df1[-train_ind, ]
```

```
# Setting the y to be "Apps"
```

```
y_train = train[ , which(names(train) %in% c("mpg"))]
y_test = test[ , which(names(test) %in% c("mpg"))]
```

```
# Removing the predictive variable from the training and testing sets.
```

```
x_train = train[ , -which(names(train) %in% c("mpg"))]
x_test = test[ , -which(names(test) %in% c("mpg"))]
```

```
str(y_train)
```

```
## num [1:261] 33.5 25.8 17.5 22 20.6 18 33.5 27 13 31.5 ...
```

```
str(y_test)
```

```
## num [1:131] 15 14 14 17 13 12 13 11 32.7 14 ...
```

```
str(x_train)
```

```
## 'data.frame': 261 obs. of 7 variables:
## $ cylinders : num 4 4 6 6 6 6 4 4 8 4 ...
## $ displacement: num 98 156 258 225 225 225 85 112 350 98 ...
## $ horsepower : num 83 92 95 100 110 105 70 88 145 68 ...
## $ weight : num 2075 2620 3193 3233 3360 ...
## $ acceleration: num 15.9 14.4 17.8 15.4 16.6 16.5 16.8 18.6 13 18.5 ...
## $ year : num 77 81 76 76 79 74 77 82 73 77 ...
## $ origin : num 1 1 1 1 1 1 3 1 1 3 ...
```

```
str(x_test)
```

```
## 'data.frame': 131 obs. of 7 variables:
## $ cylinders : num 8 8 8 8 8 8 8 8 6 8 ...
## $ displacement: num 390 454 455 302 440 455 360 350 168 400 ...
## $ horsepower : num 190 220 225 140 215 225 175 180 132 175 ...
## $ weight : num 3850 4354 3086 3449 4735 ...
## $ acceleration: num 8.5 9 10 10.5 11 11 11 11 11.4 11.5 ...
## $ year : num 70 70 70 70 73 73 73 73 80 71 ...
## $ origin : num 1 1 1 1 1 1 1 1 3 1 ...
```

```
# Create the linear model with natural cubic splines
```

```
model1 <- lm(train$mpg ~ ns(horsepower, df = 4) + ns(displacement, df = 4) + ns(weight, df = 4) + ns(ac
```

```
# Summarize the linear model
```

```
summary(model1)
```

```
##
## Call:
## lm(formula = train$mpg ~ ns(horsepower, df = 4) + ns(displacement,
##     df = 4) + ns(weight, df = 4) + ns(acceleration, df = 4) +
##     year + cylinders + origin, data = train)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -7.3208 -1.6088  0.1443  1.4202  8.3130
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -20.21801     5.07438  -3.984 8.97e-05 ***
## ns(horsepower, df = 4)1     -6.38007     1.94102  -3.287 0.001164 **
## ns(horsepower, df = 4)2    -10.54480     2.58567  -4.078 6.17e-05 ***
## ns(horsepower, df = 4)3    -15.08432     4.34554  -3.471 0.000614 ***
## ns(horsepower, df = 4)4     -9.08901     3.05745  -2.973 0.003251 **
## ns(displacement, df = 4)1   -3.42039     2.17303  -1.574 0.116796
## ns(displacement, df = 4)2   -6.22686     2.95816  -2.105 0.036329 *
## ns(displacement, df = 4)3   -4.94774     4.19365  -1.180 0.239237
## ns(displacement, df = 4)4   -3.85467     3.65490  -1.055 0.292639
## ns(weight, df = 4)1        -6.06396     2.16338  -2.803 0.005475 **
## ns(weight, df = 4)2        -7.75344     2.52441  -3.071 0.002375 **
## ns(weight, df = 4)3       -10.87782     4.32894  -2.513 0.012631 *
## ns(weight, df = 4)4        -8.78335     2.83696  -3.096 0.002193 **
## ns(acceleration, df = 4)1   -3.17206     2.29715  -1.381 0.168599
## ns(acceleration, df = 4)2   -4.31654     1.69888  -2.541 0.011688 *
## ns(acceleration, df = 4)3   -5.21680     4.73366  -1.102 0.271532
## ns(acceleration, df = 4)4  -0.72205     2.38957  -0.302 0.762785
## year              0.74107     0.05241  14.139 < 2e-16 ***
## cylinders          0.84874     0.49382   1.719 0.086947 .
## origin            0.19921     0.31289   0.637 0.524943
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.728 on 241 degrees of freedom
## Multiple R-squared:  0.8819, Adjusted R-squared:  0.8725
## F-statistic: 94.67 on 19 and 241 DF, p-value: < 2.2e-16
```

We have two different types of attributes fitted in this model: 1) splines with degrees of freedom = 4, so in the summary all spline attributes exist 4 times. 2) second group of attributes are the categorical ones. We just passed them in the model as we do in other linear models.

We can see as well how significant each one is for the model (acceleration, weight, horsepower and year)

```
y_pred <- predict(model1, x_test)
RMSE <- sqrt(mean((y_test - y_pred)^2))
RMSE
```

```
## [1] 3.17864
```

For RMSE score we have 2.87. Let's now try the stepwise model.

## Stepwise variable selection

```
fit2 <- step(lm(train$mpg ~ ns(horsepower, df = 4) + ns(displacement, df = 4) + ns(weight, df = 4) + ns
```

```
## Start: AIC=543.04
## train$mpg ~ ns(horsepower, df = 4) + ns(displacement, df = 4) +
##      ns(weight, df = 4) + ns(acceleration, df = 4) + year + cylinders +
##      origin
##
```

	Df	Sum of Sq	RSS	AIC
## - origin	1	3.02	1796.4	541.47
## - ns(displacement, df = 4)	4	47.49	1840.9	541.86
## <none>			1793.4	543.04
## - cylinders	1	21.98	1815.4	544.22
## - ns(acceleration, df = 4)	4	71.42	1864.8	545.23
## - ns(weight, df = 4)	4	111.19	1904.6	550.74
## - ns(horsepower, df = 4)	4	157.32	1950.7	556.98
## - year	1	1487.53	3280.9	698.68

```
## Step: AIC=541.47
## train$mpg ~ ns(horsepower, df = 4) + ns(displacement, df = 4) +
##      ns(weight, df = 4) + ns(acceleration, df = 4) + year + cylinders
##
```

	Df	Sum of Sq	RSS	AIC
## <none>			1796.4	541.47
## + origin	1	3.02	1793.4	543.04
## - cylinders	1	25.19	1821.6	543.11
## - ns(displacement, df = 4)	4	70.90	1867.3	543.58
## - ns(acceleration, df = 4)	4	71.79	1868.2	543.70
## - ns(weight, df = 4)	4	108.69	1905.1	548.81
## - ns(horsepower, df = 4)	4	154.50	1950.9	555.01
## - year	1	1484.62	3281.0	696.69

```
summary(fit2)
```

```
##
## Call:
## lm(formula = train$mpg ~ ns(horsepower, df = 4) + ns(displacement,
##      df = 4) + ns(weight, df = 4) + ns(acceleration, df = 4) +
##      year + cylinders, data = train)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -7.3449 -1.6081  0.1861  1.4331  8.4453
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -19.74931     5.01452  -3.938 0.000107 ***
## ns(horsepower, df = 4)1    -6.27836     1.93206  -3.250 0.001320 **
## ns(horsepower, df = 4)2   -10.47571     2.58021  -4.060 6.63e-05 ***
## ns(horsepower, df = 4)3   -15.01083     4.33866  -3.460 0.000639 ***
## ns(horsepower, df = 4)4    -8.97944     3.04885  -2.945 0.003542 **
```

```
## ns(displacement, df = 4)1 -4.00800    1.96487 -2.040 0.042454 *
## ns(displacement, df = 4)2 -6.93165    2.73985 -2.530 0.012044 *
## ns(displacement, df = 4)3 -5.87030    3.93052 -1.494 0.136604
## ns(displacement, df = 4)4 -4.49638    3.50886 -1.281 0.201266
## ns(weight, df = 4)1      -6.01149    2.15916 -2.784 0.005790 **
## ns(weight, df = 4)2      -7.63595    2.51456 -3.037 0.002654 **
## ns(weight, df = 4)3      -10.87435    4.32361 -2.515 0.012549 *
## ns(weight, df = 4)4      -8.72778    2.83213 -3.082 0.002296 **
## ns(acceleration, df = 4)1 -3.11202    2.29239 -1.358 0.175873
## ns(acceleration, df = 4)2 -4.31640    1.69679 -2.544 0.011586 *
## ns(acceleration, df = 4)3 -5.16044    4.72701 -1.092 0.276054
## ns(acceleration, df = 4)4 -0.68882    2.38606 -0.289 0.773071
## year                      0.73985    0.05232 14.142 < 2e-16 ***
## cylinders                  0.89759    0.48722  1.842 0.066659 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.725 on 242 degrees of freedom
## Multiple R-squared:  0.8817, Adjusted R-squared:  0.8729
## F-statistic: 100.2 on 18 and 242 DF,  p-value: < 2.2e-16
```

The final model excludes of course cylinders and displacement but keeps the origin attribute.

## Calculate the RMSE on the test set

```
ypred <- predict(fit2, x_test)
RMSE2 <- sqrt(mean((y_test - ypred)^2))
RMSE2
```

```
## [1] 3.209117
```

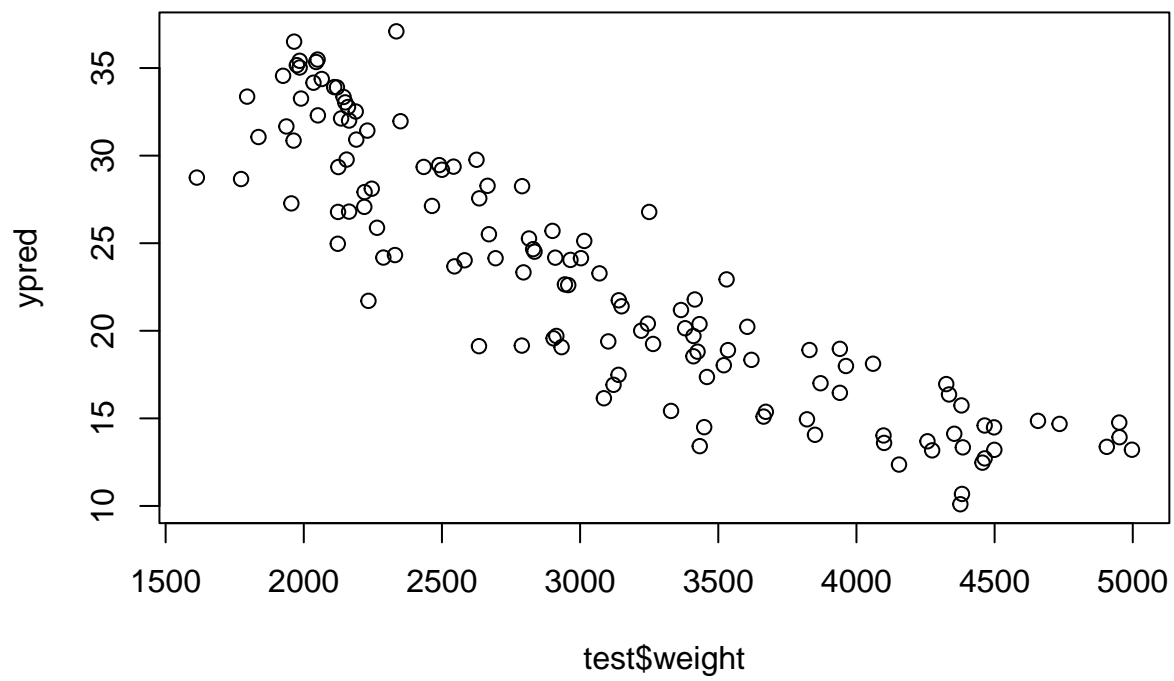
We have little bit more error bit with 2 attributes less. Lets plot the variables now.

```
plot(test$horsepower, ypred)
```

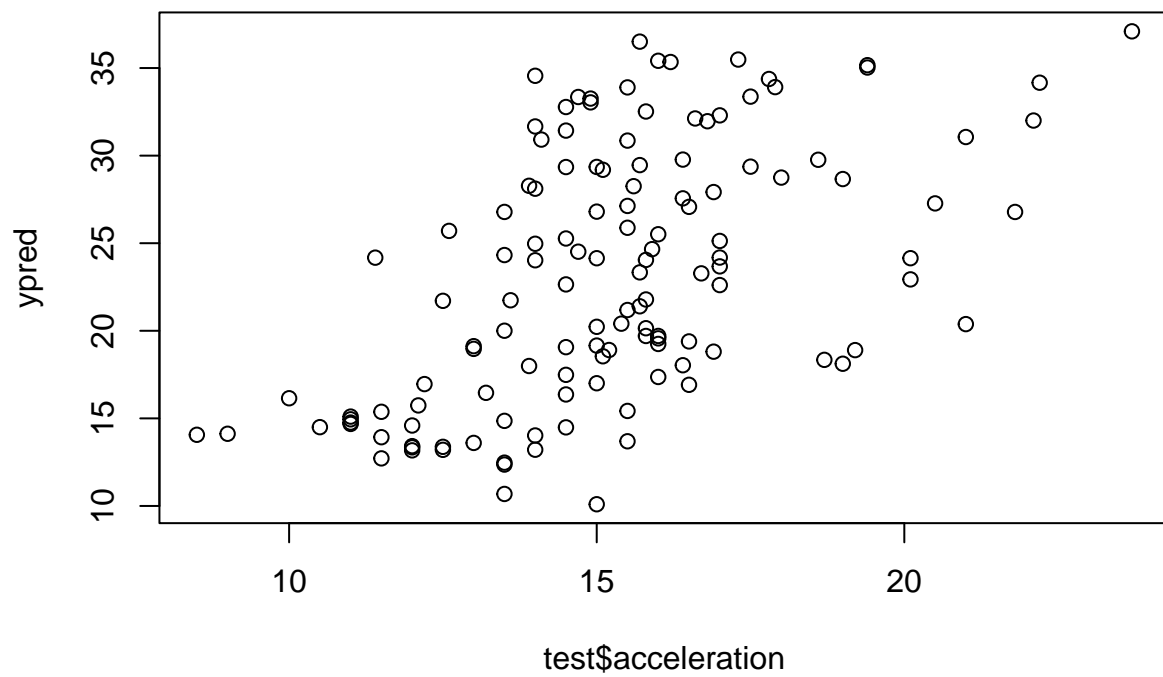


```
plot(test$weight, ypred)
```

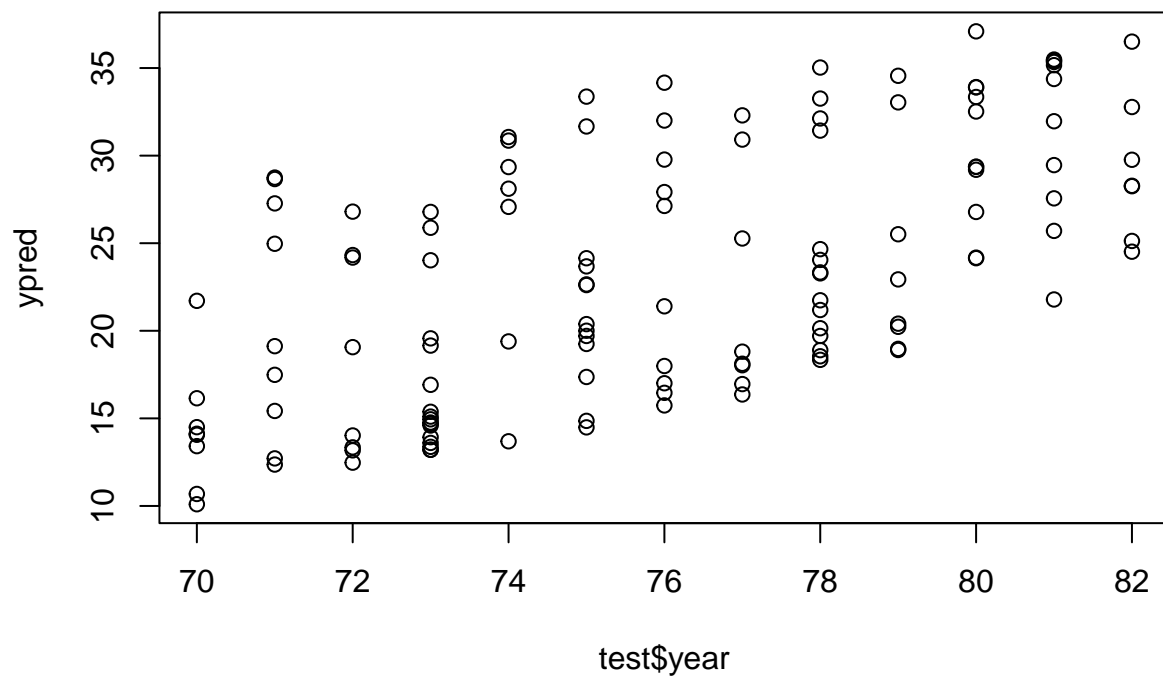




```
plot(test$acceleration, ypred)
```



```
plot(test$year, ypred)
```



```
plot(test$origin, ypred)
```

