

Calculus I, section 10: syllabus

Time and location: TR 4:10 - 5:25 PM, 407 Mathematics

Instructor: Avi Zeff, email avizeff@math.columbia.edu

Office hours: TBD

TA: TBD

Welcome to calculus! In this course we'll study the language of functions, motion, and change. This language is fundamental to fields like physics, economics, engineering, and of course mathematics itself, as well as many others. You are all probably coming from very different backgrounds and with different goals for this class; the good news is that you can all succeed. (There is no bad news, because we are doing math and so everything is good.)

Summary

There are four main areas we will cover:

- Functions and limits
- Derivatives
- Applications of derivatives
- Antiderivatives, integrals, and the fundamental theorem of calculus

By the end of the course, students should have a good working understanding of mathematical functions; be able to compute limits and derivatives of various kinds of functions, and apply these skills to concrete problems such as optimization; and understand the relationship between integration and differentiation, and be able to compute simple integrals and apply integrals to problems such as finding areas or average values.

A more detailed course outline can be found at the end of this document.

Prerequisites

The prerequisite for this class is precalculus or equivalent: in other words, you should be comfortable manipulating algebraic expressions (for example to solve equations) and be familiar with the general notions of functions and graphs, as well as examples of functions such as polynomials, exponential and logarithmic functions, and trigonometric functions such as sine and cosine, although we will take a little time at the beginning of the course to review these notions. If you are not sure whether you have the prerequisites, email me or come to my office hours.

Textbook

The nominal textbook for this course is *Calculus: Early Transcendentals*, by James Stewart. If you feel that access to a textbook will help you learn, you are free to purchase it; it can also be found online or in the library. If you would like access to the textbook and are having trouble getting it, let me know.

That said, use of the textbook in any form is not required: although problems may be drawn from it, their statements will always be included in the assignment, and no readings from the book will be assigned. Lecture notes will be posted before each class; you will not be responsible for any material not included in these.

For those who find textbooks useful, in addition to the lecture notes there exist many good calculus textbooks, including Strang's *Calculus*, which the author has made freely available online.

We will not use WebAssign. (You also do not need a calculator for this course, and one will not be permitted on exams.)

Course structure

This course will be taught via “standards-based learning,” the central idea of which is that there is a set of objectives which you are here to learn, and the class should be taught in such a way as to optimize the amount of these (both in number and degree) that you learn by the end of the semester, and graded based on how many you have learned to a satisfactory standard. Concretely, this means that both the class structure and grading may be different from what you are used to:

- Your grade will be determined by the number of objectives for which you have demonstrated achievement (see below for a detailed table).
- In order to demonstrate achievement of each objective, you need to solve a problem (or set of smaller problems) on that topic **both** the homework **and** on the exams.
- However, if you don't manage to solve a problem correctly on the first attempt, **you can re-attempt it without penalty**, both on homeworks and on exams.

Each problem will be graded either S (satisfactory), M (minor revisions needed), or N (not yet satisfactory), with comments to help you revise as needed. Your level of achievement for each objective will be the highest of these that you have earned on every homework (dropping the lowest score) and exam problem testing that objective. In general, only S is considered as demonstrating achievement of an objective. This is a high standard, and one which I believe all of you can achieve.

Examples of S-level, M-level, and N-level work are posted on the website.

To keep the workload manageable for all of us, **you are limited to re-attempting at most three (either homework or exam) problems per week**, where weeks end at class time on Tuesdays. (You cannot save up re-attempts for later in the semester, so there is no reason to not re-attempt early on.)

Classes

Classes will be mostly lecture, with some time set aside for Q&A sessions and group work. Attendance is not mandatory but is highly encouraged; empirically, students tend to do better when they come to class.

Homework

Homework will be weekly, generally due on Tuesdays in (or by, if submitted digitally) class (i.e. at 4:10 PM). You should expect to spend a while on these, in the neighborhood of 3-9 hours; much of learning is done through exercises. Collaboration is strongly encouraged, but everyone should write their own solutions; **write on your homeworks anyone you have worked with.** (Note, though, that contributions from different collaborators should be roughly equal; if you find that you are typically doing more or less than your collaborators, consider finding a different group.)

Unless otherwise specified, you may use any and all resources (again, **citing any sources you have used**), including any textbooks you have access to, your classmates and friends, the help room, office hours (you should come to these!), or the internet, with the following exceptions:

- do not post the problem on any website to be answered by someone else;
- do not use computer algebra systems (e.g. WolframAlpha, SageMath, Mathematica, integral-calculator.com, etc.) to do your computations unless otherwise specified. That said, these are all useful resources I encourage you to familiarize yourself with for any purpose other than homework for this class (and naturally any other classes with similar policies).

The reason for these exceptions is that while using other resources does not prevent you from learning from doing computations, outsourcing those computations entirely does.

If you are unable to turn in your homework by the official deadline, please contact me **at least 24 hours in advance**; extensions will be given on a case-by-case basis.

For any homework problem on which you are dissatisfied with your grade, you should carefully review the feedback and reattempt the problem, and resubmit it as soon as possible.

Exams

There will be three midterms and a final exam. The third midterm can be replaced by a project, group or individual; if you want to do this let me know by the second midterm so that we can discuss a suitable topic. (For the purpose of demonstrating achievement of objectives, the project counts like an exam, and can also be revised.) If you choose to do a project, you will give a short presentation on it to the class near the end of the semester. I'll talk more about projects and guidelines for them further into the semester.

For any exam problem on which you are dissatisfied with your grade, you should carefully review the feedback and, when ready, schedule a time to reattempt an analogous problem (preferably during my office hours if possible).

There will be no homework the weeks of the midterms (or, of course, the final exam).

Grading

Your grade will be determined by the number of objectives you achieve. There are 16 objectives, listed below. All of them are important (or else we would not spend time on them!), but some are crucial that you know in order to be able to progress to future classes; these have been marked as (CORE) below. In order to pass the class (i.e. with a C-) you must achieve all eight of these core objectives; achieving all 16 objectives is the most I can

ask for and so receives an A+. Thus the remaining grades interpolate these possibilities as follows:

| Grade | Objectives achieved |
|--------------|---|
| A+ | All 16 objectives |
| A | 15 objectives, including all 8 core objectives |
| A- | 14 objectives, including all 8 core objectives |
| B+ | 13 objectives, including all 8 core objectives |
| B | 12 objectives, including all 8 core objectives |
| B- | 11 objectives, including all 8 core objectives |
| C+ | 10 objectives, including all 8 core objectives |
| C | 9 objectives, including all 8 core objectives |
| C- | All 8 core objectives |
| D | 7 core objectives and an M on the remaining core objective OR 6 core objectives, an M on the remaining core objectives, and at least two S's on other objectives |
| F | Have not fully achieved any of the above. |

The objectives are as follows.

- (1) You understand and can manipulate and use algebraic expressions and functions. (CORE)
- (2) You can calculate and use limits of various kinds of functions, including infinite limits, and explain their meaning. (CORE)
- (3) You can determine whether a function is continuous at each point and describe its discontinuities.
- (4) You can describe the asymptotes of a given function in relationship to limits.
- (5) You understand the meaning of the derivative and the relationship between the limit definition and the geometric picture. (CORE)
- (6) You can directly compute examples via the limit definition and determine whether a function is differentiable at each point.
- (7) You understand and can apply linearity, the product, quotient, and chain rules, and implicit differentiation to calculate derivatives. (CORE)
- (8) You can combine multiple rules in order to compute derivatives of complicated functions, including combinations of polynomials, trigonometric functions, exponentials, or their inverses. (CORE)
- (9) You can apply the concept of the derivative and the second derivative test to find extrema of functions. (CORE)

- (10) You can apply this method to real-world optimization problems.
- (11) You can apply derivatives to computing limits via L'Hopital's rule.
- (12) You can apply derivatives to other problems such as related rates problems, graphing functions, or numerically solving equations via Newton's method.
- (13) You understand the meanings of the antiderivative and definite integral, and can compute simple examples from the definitions. (CORE)
- (14) You understand the relationship between integration and differentiation through the fundamental theorem of calculus. (CORE)
- (15) You can apply *u*-substitution in combination with differentiation to compute certain kinds of integrals.
- (16) You can apply integration to real-world problems such as computing areas and average values.

COVID-19 policies

Classes will be in person, but please do not come to class if you are feeling sick or test positive: lecture notes will be available, class can be streamed or recorded as needed, and I will be happy to help you make up the material. If you are sick and unable to do the work for a prolonged period, contact me to work out a way to make up the work: it is important to do the work for each section of the class, since that is the way to learn the material and the later portions of the class will build on the earlier ones, but when necessary we can figure out how to reduce the workload to be manageable without having to work through illness. Similarly, please do not attend exams if you are ill or have recently tested positive for COVID-19: we will figure out solutions if these situations arise.

Academic Honesty Policy

Please read (and follow) the Columbia University Undergraduate Guide to Academic Integrity. If you are ever unsure whether something is allowed, please ask me first—you will never be penalized for asking.

Accessibility and accommodations

Please let me know if there is anything I can do to make this course more accessible to you, or if aspects of the course are excluding you, and we can work together to develop strategies to improve the class. If you think you may need official accommodations, such as extended time on exams, I encourage you to contact the Office of Disability Services for an accommodation letter.

Feedback

There will be some formal opportunities to give feedback, such as on homeworks. In addition, feel free to give me comments on the class at any time, via email or in person; if you are not comfortable doing either, you can give me anonymous feedback via this form.

Tentative course outline

As promised:

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|--------------|--|
| September 6 | Class overview, review of algebra and functions |
| September 8 | More functions: examples, inverses, iteration |
| September 13 | Introduction to limits |
| September 15 | Limit laws and the squeeze theorem |
| September 20 | Continuity and discontinuities |
| September 22 | Asymptotes and limiting behavior of functions |
| September 27 | Introduction to derivatives |
| September 29 | Midterm 1 |
| October 4 | Product and quotient rules |
| October 6 | Trigonometric functions |
| October 11 | Chain rule and implicit differentiation |
| October 13 | Exponentials and logarithms |
| October 18 | Differentiating general functions |
| October 20 | Midterm 2 |
| October 25 | Related rates and inverse functions |
| October 27 | Maximization, extrema, and the second derivative test |
| November 1 | Optimization |
| November 3 | The mean value theorem and L'Hopital's rule |
| November 8 | <i>Election Day</i> |
| November 10 | Graphs and Newton's method |
| November 15 | Midterm 3 |
| November 17 | Antiderivatives |
| November 22 | Definite integrals and the fundamental theorem of calculus |
| November 24 | <i>Thanksgiving</i> |
| November 29 | Examples and u -substitution |
| December 1 | Areas between curves and average value |
| December 6 | Presentations (as needed) |
| December 8 | Review |
| TBD | Final exam |