1) CONT. PAUL YUSKA

$$\frac{\Delta x_f^2 - \Delta x_b^2}{\Delta x_f + \Delta x_b} = \frac{\Delta x_f^2}{\Delta x_f + \Delta x_b} - \frac{\Delta x_b^2}{\Delta x_f + \Delta x_b} \approx \boxed{\Delta x_f - \Delta x_b}$$

APPROX.

Hand-wavey, I know.
I'm still missing something here.

c. define $\left[\frac{df}{dx}\right]_{i+1/2} = f'_{i+\frac{1}{2}}$ as $\frac{f'_i + f'_{i+1}}{2}$ ← halfway between grid pts., regardless of spacing

where f'_{i+1} is found by solving the Taylor series about f_{i+1} in a similar matter as above

for $a = x_{i+1}$ and $x = x_i, x_{i+2}$, and $x_{i+2} = x_{i+1} + \Delta x_{f2}$

$$f'_{i+1} = \frac{f_{i+2} - f_i}{\Delta x_f + \Delta x_{f2}} \; ; \; f'_i = \frac{f_{i+1} - f_{i-1}}{\Delta x_f + \Delta x_b}$$

define $\left[\frac{df}{dx}\right]_{i-\frac{1}{2}}$ similarly, finding f'_{i-1} in the same manner as f'_{i+1}:

$$f'_{i-1} = \frac{f_i - f_{i-2}}{\Delta x_b + \Delta x_{b2}} \; ; \; f'_i =$$

define $x_{i\pm\frac{1}{2}}$ as $\frac{x_i + x_{i+1}}{2} = \frac{x_i + (x_i + \Delta x_f)}{2} = x_i + \frac{1}{2}\Delta x_f$ and

$$\frac{x_{i-1} + x_i}{2} = \longrightarrow = x_i - \frac{1}{2}\Delta x_b$$

then, f''_i can be approximated as:

$$f''_i \approx \left[\frac{f'_{i+1} + f'_i}{2} - \frac{f'_i + f'_{i-1}}{2}\right] \frac{1}{\frac{1}{2}(\Delta x_f + \Delta x_b)}$$

$$f''_i \approx \left[\frac{\left(\frac{f_{i+2} - f_i}{\Delta x_f + \Delta x_{f2}} + \frac{f_{i+1} - f_{i-1}}{\Delta x_f + \Delta x_b}\right)}{\Delta x_f + \Delta x_b} - \frac{\left(\frac{f_{i+1} - f_{i-1}}{\Delta x_f + \Delta x_b} + \frac{f_i - f_{i-2}}{\Delta x_b + \Delta x_{b2}}\right)}{\Delta x_f + \Delta x_b}\right]$$