

2) a. Start w/ $\underline{A}x = \underline{b}$

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Perturb the system: $\underline{A} \rightarrow \underline{A} + \underline{\delta A}$, $\underline{b} \rightarrow \underline{b} + \underline{\delta b}$

$$(\underline{A} + \underline{\delta A})x = \underline{b} + \underline{\delta b}$$

$$\underline{A}x + \underline{\delta A}x = \underline{b} + \underline{\delta b} \quad \text{subtract unperturbed system to get}$$

$$\underline{\delta A}x = \underline{\delta b} \rightarrow \underbrace{\underline{A} \underline{A}^{-1}}_{\underline{I}} \underline{\delta A}x = \underline{\delta b}$$

$$\|\underline{A}\| \|\underline{A}^{-1}\| \geq \|\underline{I}\| = \|\underline{A} \underline{A}^{-1}\|$$

$$\underbrace{\|\underline{A}\| \|\underline{A}^{-1}\|}_{C(A)} \|\underline{\delta A}\| \|x\| \geq \|\underline{\delta b}\| \rightarrow C(A) \|\underline{\delta A}\| \|x\| \geq \|\underline{\delta b}\|$$
$$C(A) \frac{\|\underline{\delta A}\|}{\|\underline{A}\|} \frac{\|x\|}{\|x\|} \geq \frac{\|\underline{\delta b}\|}{\|\underline{b}\|}$$

b. The condition # is a measure of sensitivity of changes in the solution of a system to perturbation of the input. High condition numbers ($C \gg 1$) are bad. C on the order of 1 is desirable.