Problem Statement:

Design the following 4-bit ALU (Arithmetic Logic Unit):

S2	S1	S0	Output	Function
1	1	0	$A_i - B_i$	Subtract
0	1	1	$A_i + B_i$	Add
1	1	1	$A_i + B_i + 1$	Add with Carry
0	1	0	$A_i + 1 + 1$	Transfer A with Carry
1	0	X	$A_i . B_i$	AND
0	0	X	$A_i \oplus B_i$	XOR

Function Generation:

S2	S1	S0	Z	X	Y	Output	Function
1	1	0	1	A_i	Bi’	$A_i - B_i$	Subtract
0	1	1	0	A_i	B	$A_i + B_i$	Add
1	1	1	1	A_i	B	$A_i + B_i + 1$	Add with Carry
0	1	0	1	A_i	all(1)	$A_i + 1 + 1$	Transfer A with Carry
1	0	X	0	$A_i . B_i$	0	$A_i . B_i$	AND
0	0	X	0	$A_i \oplus B_i$	0	$A_i \oplus B_i$	XOR

For Function X :

K-Map :

S_2S_1 / S_0	$S_2' S_1'$	$S_2' S_1$	S_2S_1	$S_2 S_1'$
S_0'	$A_i \oplus B_i$	A_i	A_i	$A_i B_i$
S_0	$A_i \oplus B_i$	A_i	A_i	$A_i B_i$

So , From the K-Map –

$= S_1 A_i + S_2 S_1' A_i B_i + S_2' S_1' (A_i \oplus B_i)$

For Function Y :

K-Map :

S_2S_1 / S_0	$S_2' S_1'$	$S_2' S_1$	S_2S_1	$S_2 S_1'$
S_0'		1	Bi'	
S_0		Bi	Bi	

So , From the K-Map –

$= S_2S_1 S_0' B_i' + S_2' S_1 S_0' + S_1 S_0 B_i$

$= S_1 S_0' (B_i' S_2 + S_2') + S_1 S_0 B_i$

$= S_1 S_0' (B_i' + S_2') + S_1 S_0 B_i$

For Function Z :

K-Map :

$S_2 S_1$ / S_0	$S_2' S_1'$	$S_2' S_1$	$S_2 S_1$	$S_2 S_1'$
S_0'		1	1	
S_0			1	

So , From the K-Map –

$= S_1 S_0' + S_2S_1$

$= S_1 (S_0' + S_2)$