On Gelfand-Zetlin modules over $U_q(gl_n)$

Volodymyr Mazorchuk and Lyudmyla Turowska

Descriptive title: Gelfand-Zetlin modules
1991 Mathematics Subject Classification: 17B37, 81R50, 17B10

Abstract

We construct and investigate a new large family of simple modules over $U_q(gl_n)$.

1 Introduction and setup

The Gelfand-Zetlin formal construction of simple finite-dimensional modules over the groups of unimodular and orthogonal matrices was developed in the celebrated papers [GZ1, GZ2] in 1950 (see [BR] for more details). Later this construction was studied from different points of view, for example, the main result was reobtained using lowering operators method ([Z]). In the last fifteen years this construction has been generalized on different quantum analogues for Lie algebras, see for example [J1, J2, UTS1, UTS2, C, NT1, NT2, GI, GK]. On the other hand, this method was used to obtain the classification of unitarizable modules for several algebras (see [O1, O2, GK]), to construct and investigate the structure of a large family of simple modules over classical algebras (see [DFO1, DFO2, M1, M3, MO]) or to define and study new classes of algebras (see [M2]). Recently some deep results in this theory were obtained in [O], and an analogue of Gelfan-Zetlin construction for symplectic algebras was obtained in [Mo].

The aim of this paper is to analyze the Gelfand-Zetlin construction of simple finite-dimensional modules over the quantum algebra $U_q(gl_n)$, where q is a non-zero complex non root of unity in order to construct and investigate a new large family of simple $U_q(gl_n)$-modules.

We will work over the complex field and fix q to be a non-zero complex non root of unity. For any complex x we set $[x]_q = (q^x - q^{-x})/(q - q^{-1}) = (e^{xh} - e^{-xh})/(e^h - e^{-h})$, where $q = \exp h$. All the notions that will be used without preliminary definition can be found in [KS].

In Section 2 we recall the Gelfand-Zetlin construction of simple modules over $U_q(gl_n)$. In Section 3 we present a new large family of simple $U_q(gl_n)$-modules. In Section 4 we give an abstract definition of Gelfand-Zetlin modules over $U_q(gl_n)$ and present some examples. Finally, in Section 5 we construct an extension of $U_q(gl_n)$ inspired by modules constructed in Section 3.

2 $U_q(gl_n)$ and Gelfand-Zetlin basis for finite-dimensional modules

We define $U_q(gl_n)$ as a unital associative complex algebra generated by E_i, F_i, $i = 1, 2, \ldots, n-1$, K_j, K_j^{-1}, $j = 1, 2, \ldots, n$ subject to the relations

$$\begin{gathered}
K_i K_j = K_j K_i, \quad K_i K_i^{-1} = K_i^{-1} K_i = 1, \\
K_i E_j K_i^{-1} = q^{\delta_{ij}/2} q^{-\delta_{i,j+1}/2} E_j, \\
K_i F_j K_i^{-1} = q^{-\delta_{ij}/2} q^{\delta_{i,j+1}/2} F_j, \\
[E_i, F_j] = \delta_{ij} \frac{K_i^2 K_{i+1}^{-2} - K_i^{-2} K_{i+1}^2}{q - q^{-1}}, \\
[E_i, E_j] = [F_i, F_j] = 0, \quad |i-j| \geqslant 2, \\
E_i^2 E_{i\pm1} - (q + q^{-1}) E_i E_{i\pm1} E_i + E_{i\pm1} E_i^2 = 0, \\
F_i^2 F_{i\pm1} - (q + q^{-1}) F_i F_{i\pm1} F_i + F_{i\pm1} F_i^2 = 0
\end{gathered}$$

(see, for example [KS, UTS1, UTS2]).

The following Theorem describes the Gelfand-Zetlin approach for simple finite-dimensional $U_q(gl_n)$ modules with a given highest weight. It was obtained in [J1, J2], then reobtained by lowering operators method in [UTS1, UTS2]. We present it in the most general situation (for q which is any non-zero non root of unity), as stated in [KS, Section 7.3.3].

Theorem 1. *Let $V(m)$ be a simple $U_q(gl_n)$-module with a highest weight $m = (m_{n,1}, m_{n,2}, \ldots, m_{n,n})$, $m_{n,i} \geqslant m_{n,i+1}$. Then $V(m)$ possesses a basis consisting of all tableaux $[s] = (s_{ij})_{i=1,2,\ldots,n}^{j=1,2,\ldots,i}$ such that $s_{n,j} = m_{n,j}$, $j = 1, 2, \ldots, n$ and $s_{i+1,j} \geqslant s_{i,j} \geqslant s_{i+1,j+1}$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, i$ and the action of generators of $U_q(gl_n)$ are given by the following formulae:*

$$K_k[s] = q^{a_k/2}[s], \quad a_k = \sum_{i=1}^{k} s_{k,i} - \sum_{i=1}^{k-1} s_{k-1,i}, \quad k = 1, 2, \ldots, n,$$

$$E_k[s] = \sum_{j=1}^{k} a_{kj}^{+}([s])([s] + [\delta^{k,j}]), \quad F_k[s] = \sum_{j=1}^{k} a_{kj}^{-}([s])([s] - [\delta^{k,j}]),$$

where $\delta^{k,j}$ is the Kronecker tableau and for $l_{rt} = s_{rt} - t$ we have

$$a_{kj}^{\pm}([s]) = \mp \frac{\prod_i [l_{k\pm1,i} - l_{k,j}]_q}{\prod_{i\neq j} [l_{k,i} - l_{k,j}]_q}.$$

Remark 1. *It is worth to note, that the highest weight over the corresponding $U_q(sl_n)$ algebra can be expressed as $(m_{n,1} - m_{n,2}, m_{n,2} - m_{n,3}, \ldots, m_{n,n-1} - m_{n,n})$.*

Remark 2. *Originally the Gelfand-Zetlin formulae were obtained for a positive real $q \neq 1$ and had the following form:*

$$K_k[s] = q^{a_k/2}[s], \quad a_k = \sum_{i=1}^{k} s_{k,i} - \sum_{i=1}^{k-1} s_{k-1,i}, \quad k = 1, 2, \ldots, n,$$

$$E_k[s] = \sum_{j=1}^{k} A_{kj}([s])([s] + [\delta^{k,j}]), \quad F_k[s] = \sum_{j=1}^{k} A_{kj}([s] - [\delta^{k,j}])([s] - [\delta^{k,j}])$$

with

$$A_{kj}([s]) = \left(-\frac{\prod_i [l_{k+1,i} - l_{k,i}]_q \prod_i [l_{k-1,i} - l_{k,i} - 1]_q}{\prod_{i \neq j} [l_{k,i} - l_{k,j}]_q \prod_{i \neq j} [l_{k,i} - l_{k,j} - 1]_q} \right)^{1/2}.$$

One can obtain these formulae from those above multiplying the basis elements by appropriate factors.

3 Generic Gelfand-Zetlin modules

Let $1(q)$ be the set of all complex x such that $q^x = 1$. Fix a tableau $[m]$ with complex entries $m_{i,j}$, $1 \leqslant i \leqslant n$ and $1 \leqslant j \leqslant i$ satisfying the following defining condition:

- $2(m_{i,j} - m_{i,k}) \notin 1(q) + 2\mathbb{Z}$ for all $1 \leqslant i \leqslant n-1$ and all $j \neq k$.

We will call such $[m]$ admissible. Consider the set $B([m])$ consisting of all tableaux $[l]$ such that

- $l_{n,j} = m_{n,j}$ for all j;
- $l_{i,j} - m_{i,j}$ is an integer for all $1 \leqslant i \leqslant n-1$ and all j.

Let $V([m])$ be the vector space with a basis $B([m])$. For $[l] \in B([m])$ set

$$K_k[l] = q^{a_k/2}[l], \quad a_k = \sum_{i=1}^{k} l_{k,i} - \sum_{i=1}^{k-1} l_{k-1,i} + k, \quad k = 1, 2, \ldots, n,$$

$$E_k[l] = \sum_{j=1}^{k} a_{kj}^{+}([l])([l] + [\delta^{k,j}]), \quad F_k[l] = \sum_{j=1}^{k} a_{kj}^{-}([l])([l] - [\delta^{k,j}]),$$

where

$$a_{kj}^{\pm}([l]) = \mp \frac{\prod_i [l_{k\pm1,i} - l_{k,j}]_q}{\prod_{i \neq j} [l_{k,i} - l_{k,j}]_q}.$$

We will call the formulae above the Gelfand-Zetlin (GZ) formulae.

Theorem 2. *GZ formulae define on $V([m])$ the structure of a $U_q(gl_n)$-module of finite length.*

Proof. First we show that GZ formulae define on $V([\,m\,])$ the structure of a $U_q(gl_n)$-module. Let $u = 0$ be a relation in $U_q(gl_n)$. It is enough to show that this relation holds in $V([\,m\,])$. For this it is enough to show that $u[l] = 0$ for any $[l] \in B([m])$. Clearly, using GZ formulae we can write $u[l] = \sum_{[t]\in I(u,[l])} f([t])[t]$, where the set $I(u,[l]) - [l]$ depends only on u and for any fixed u each $f([t])$ is a rational function in $q^{l_{i,j}}$. Thus, it is enough to show that each $f([t])$ is identically zero. Hence, we have only to show that some polynomials in $q^{l_{i,j}}$ are zero. Let p be such a polynomial, k be its degree and s be the degree of u. Clearly, there exists a tableau $[\tilde{l}]$ such that all $\tilde{l}_{i,j}$ are positive integers and for any integer $-k-s \leqslant \tilde{t}_{i,j} \leqslant k+s$ the tableau $[\tilde{l}+\tilde{t}]$ occurs as a basis element in a finite-dimensional $U_q(gl_n)$-module (this means that the entries of it satisfy the conditions presented in Section 2). Taking into account that $q^a \neq q^b$, if $a \neq b$ are positive integers, we conclude that p is identically zero, since GZ formulae really define finite-dimensional $U_q(gl_n)$-modules as in Theorem 1 (thus for tableaux from them $p = 0$ holds). This completes the proof of the first part of our theorem.

Let $A([m])$ be a subalgebra of $U_q(gl_n)$ consisting of elements, which are diagonalizable in the basis $B([m])$. It is non-empty, because it contains at least the quantized Cartan subalgebra, generated by K_i. Let $U_q(gl_k)$, $1 \leqslant k \leqslant n$ be a subalgebra of $U_q(gl_n)$ generated by K_i, $1 \leqslant i \leqslant k$, E_i, F_i, $1 \leqslant i \leqslant k-1$. Denote by Z_k the center of this $U_q(gl_k)$. Since Z_k is diagonalizable in the GZ basis of any finite-dimensional $U_q(gl_n)$-module, it follows that it is diagonalizable in the basis $B([m])$. Thus Z_k is a subalgebra in $A([m])$. Let Γ be a subalgebra of $A([m])$ generated by all Z_k. To complete our proof it is enough to show that for any $[l(1)] \neq [l(2)] \in B([m])$ there exists an element $u \in \Gamma$ such that the eigenvalues of u on $[l(1)]$ and $[l(2)]$ are different (see also [M3, Theorem 1]). Indeed, having this we easily obtain that any subquotient of $V([m])$ is determined by the corresponding subset of basis elements from $B([m])$. Form a non-oriented graph with a vertex set $B([m])$ in the following way: we say $[a]$ and $[b]$ to be connected by an edge if $[a]$ occurs with a non-zero multiplicity in $E_i[b]$ and $[b]$ occurs with a non-zero multiplicity in $F_i[a]$ for some i. Now the subquotients of $B([m])$ are determined by the connected components of this graph, and it is trivial, that there are only finitely many of them.

Therefore, we have only to check that Γ separates the elements of $B([m])$. It is easy to see that for $z \in Z_k$ the eigenvalue of z on $[l]$ can be expressed as a rational function on $q^{l_{k,j}}$, where only j varies. Hence, we need only to show that two tableaux in $B([m])$ having different k-th rows can be separated by an element from Z_k. Without loss of generality we can assume $k = n$. Now the last statement is equivalent to the following fact: the central characters of $V([m(1)])$ and $V([m(2)])$, $m(i)_{n,j} - m(i)_{n,s} \notin \mathbb{Z}$ for all $i = 1, 2,\ j, s = 1, 2, \ldots, n$, where the difference between the upper rows of $[m(1)]$ and $[m(2)]$ is a non-zero vector with integer entries, do not coincide. To prove this we have to compute the central character of $V([m])$. Let $z \in Z_n$ and $[t]$ be a tableau determining the highest weight of a finite-dimensional simple $U_q(gl_n)$-module. Denote by π the Harish-Chandra homomorphism from Z_n to the subalgebra U^0 generated by $K_i^{\pm 1}$, $i = 1, \ldots, n$

([J, Section 6.2]). According to [J, Lemma 6.3] the eigenvalue of z on $[t]$ equals $\lambda(\pi(z))$, where λ is the highest weight of $[t]$. Since we work with $U_q(gl_n)$, its center is generated by a (unique up to inverse) monomial in U^0, whose eigenvalue can be computed directly, and the center Z'_n of $U_q(sl_n)$. Since q is not a root of unity, we can apply [J, Theorem 6.25] and [J, Section 6.26] to obtain the eigenvalues of the elements from Z'_n. According to these results, the eigenvalues are invariant (Laurent) polynomials in $q^{t_{n,i}-t_{n,i+1}}$ under the natural action of the Weyl group (here $t_{n,i} - t_{n,i+1}$ appears as components of the highest weight with respect to $U_q(sl_n)$).

Now we see, that the central character of $V([m])$ depends only on entries in the upper row of $[m]$. Moreover, it coincides with the central character of a Verma module with the highest weight $(m_{n,1}+1, m_{n,2}+2, \ldots, m_{n,n}+n)$ (with respect to $U_q(gl_n)$). By [J, Claim 6.26] two Verma modules over $U_q(sl_n)$ have the same central character if and only if their highest weights (shifted by a half-sum of all positive roots) lie on the same orbit of the Weyl group. A shift by a half-sum of all positive roots corresponds to the substitution of $(m_{n,1}+1, m_{n,2}+2, \ldots, m_{n,n}+n)$ with $(m_{n,1}, m_{n,2}, \ldots, m_{n,n})$. The Weyl group acts on $U_q(gl_n)$-space of weights by permutation of the vector entries. Clearly, the restriction of this action on $U_q(sl_n)$-space of weights coincides with the standard action of the Weyl group on it. We remark, that the stable complement of $U_q(sl_n)$-space of weights in $U_q(gl_n)$-space of weights determines the eigenvalue of the additional central element of $U_q(gl_n)$. Hence [J, Claim 6.26] can be extended to $U_q(gl_n)$-case. To complete the proof now we have only to note that under our choice of $[m(i)]$, $i = 1, 2$ their upper rows can not be conjugated by a permutation. □

Remark 3. *From the discussion above it follows that $A([m])$ does not coincide with Γ. Since Z_n is diagonalizable in $B([m])$ it follows that $V([m])$ has a central character and thus $A([m])$ contains the two-sided ideal of $U_q(gl_n)$ generated by the kernel of this central character. We conjecture that Γ coincides with the intersection of all $A([m])$, where $[m]$ varies. This is equivalent to the fact that Γ is a maximal commutative subalgebra in $U_q(gl_n)$. The last is the case for a non-quantum situation ([O, M3]).*

Corollary 1. *$V([m])$ is simple if and only if $2(m_{i+1,j} - m_{i,k}) \notin 1(q) + 2\mathbb{Z}$ for all i, j, k.*

Proof. Clearly this is a necessary and sufficient condition for the graph described in the proof of Theorem 2 to have a unique connected component, which completes the proof. □

Remark 4. *Since $[x]_q \to x$, when $q \to 1$, we obtain that $V([m])$ is a quantum deformation of the generic Gelfan-Zetlin modules constructed in [DFO2, Section 2.3]*

4 Gelfand-Zetlin subalgebra and abstract Gelfand - Zetlin modules

The central arguments used in the proof of Theorem 2 motivate to give the following abstract definition. A $U_q(gl_n)$-module V will be called Gelfand-Zetlin module (GZ-module) if it decomposes into a direct sum of finite-dimensional Γ-modules. Since Γ is comutative, this means that $V=\oplus_{\chi\in\Gamma^*}V_\chi$, where V_χ is a root subspace of V corresponding to the character χ (see [MO, Section 3]).

Clearly, any weight $U_q(gl_n)$-module with finite-dimensional weight spaces is a GZ-module. Thus finite-dimensional modules, Verma modules, highest weight modules are GZ-modules. It follows also from the proof of Theorem 2, that $V([m])$ is a GZ-module. Following [DFO2], one can easily prove the following standard results:

Proposition 1. *Γ is a Harish-Chandra subalgebra in $U_q(gl_n)$.*

Proof. Analogous to that of [DFO2, Corollary 26]. □

Proposition 2. *Let $\chi\in\Gamma^*$, and $[m]$ be such that $V([m])_\chi\neq 0$. Set $S([m])$ to be a submodule of $V([m])$, generated by $V([m])_\chi$. Then there exists a unique maximal submodule $K([m])$ in $S([m])$ and the quotient $S([m])/K([m])$ is the unique simple GZ-module having a non-trivial χ-root subspace. In fact the last is a χ-weight subspace of dimension one.*

Proof. Analogous to that of [DFO2, Theorem 30]. □

Theorem 3. *Suppose that $V([m])$ is simple. Then the category of all GZ-modules decomposes into a direct sum of two full subcategories $A\oplus B$ such that A contains the unique simple object, namely $V([m])$.*

Proof. Analogous to that of [DFO2, Corollary 33]. □

5 Extending $U_q(gl_n)$

Consider a field $\mathbb{F}$ of rational functions with complex coefficients in $n(n+1)/2$ variables $q^{\mathfrak{l}_{i,j}}$, $1\leqslant i\leqslant n$, $1\leqslant j\leqslant i$. Let $[T]$ be a tableau (i.e. $n(n+1)$-dimensional doubly-indexed vector) with entries $T_{i,j}=\mathfrak{l}_{i,j}$. Consider a set B consisting of all $[t]$ satisfying the following conditions:

- $t_{n,i}=T_{n,i}$ for all i,
- $T_{k,i}-t_{k,i}\in\mathbb{Z}$ for all k,i.

Let $V(B)$ be an $\mathbb{F}$-vectorspace with the basis B. Define $\mathbb{F}$-linear operators $X^{\pm}_{k,j}$, $H^{\pm}_{k,j}$, $1\leqslant i\leqslant n-1$, $1\leqslant j\leqslant i$ as follows: for $[t]\in B$ set

$$X^{\pm}_{k,j}[t]=\mp\frac{\displaystyle\prod_i[t_{k\pm 1,i}-t_{k,j}]_q}{\displaystyle\prod_{i\neq j}[t_{k,i}-t_{k,j}]_q}([t]\pm[\delta^{k,j}]),\quad H^{\pm}_{k,j}[t]=q^{(\pm(t_{k,j}+j)/2)}[t]$$

and extend this action on $V(B)$ by $\mathbb{F}$-linearity. Consider a complex associative algebra $\mathfrak{A}=\mathfrak{A}(q,n)$, generated by all $X^{\pm}_{k,j}$ and $H^{\pm}_{k,j}$.

The following property of $\mathfrak{A}$ follows immediately from the definition.

Proposition 3. *Let $[l]$ be an admissible tableau. Then the specialization $\mathfrak{l}_{i,j} = l_{i,j}$ in the above formulae defines on $V(B)_{[l]}$ a structure of an $\mathfrak{A}$-module.*

Theorem 4. *The subalgebra A of $\mathfrak{A}$, generated by*

$$e_k = \sum_{j=1}^{k} X_{k,j}^{+}, \quad f_k = \sum_{j=1}^{k} X_{k,j}^{-}, \quad 1 \leqslant k \leqslant n-1, \quad h_k = \prod_{j=1}^{k} H_{k,j}^{+} \prod_{s=1}^{k-1} H_{k-1,j}^{-}, \quad 1 \leqslant k \leqslant n$$

is canonically isomorphic to $U_q(gl_n)$.

Proof. It follows from the proof of Theorem 2, that the natural map $\psi : U_q(gl_n) \to A$, defined by $\psi(E_i) = e_i$, $\psi(F_i) = f_i$ and $\psi(K_i) = h_i$ can be extended to an algebra homomorphism, which should be an epimorphism since the image contains all generators of A. To prove the injectivity of ψ compose it with the specialization $s_{[l]}$, for an admissible tableau $[l]$. From the construction of $V([l])$ we deduce that under the map $s_{[l]} \circ \psi$ the space $V(B)_{[l]}$ becomes an $U_q(gl_n)$-module, isomorphic to $V([l])$. Thus the kernel of $s_{[l]} \circ \psi$ coincides with the annihilator of $V([l])$. To complete the proof we have only to show that the intersection of annihilators of all $V([l])$ is trivial. Suppose not and $u \neq 0$ annihilates all $V([l])$ with admissible $[l]$. Since the set of admissible $[l]$ is large enough, Gelfand-Zetlin formulae defining $V([l])$ are rational and also valid for finite-dimensional modules, we deduce, that u annihilates all finite-dimensional modules, which contradicts [J, Proposition 5.11]. Theorem is proved. $\square$

We outline, that a part of the proof of Theorem 4 can be formulated in the following statement.

Corollary 2. *The restriction of $V(B)_{[l]}$ on A canonically isomorphic to $V([l])$.*

6 Acknowledgments

The work was done during the stay of the first author in Bielefeld University as an Alexander von Humboldt fellow. The financial support of Humboldt foundation and the hospitality of the Bielefeld University are gratefully acknowledged.

References

[BR] Barut A.O. and Raczka R., Theory of group representations and applications. PWN - Polish Scientific Publisher, Warsaw, 1977.

[C] I.V.Cherednik, A new interpretation of Gelfand-Tzetlin bases, Duke Math. J., 54 (1987), 563-577.

[D] J.Dixmier, Algebres enveloppantes, Gauthier-Villarrd, Paris, 1974.

[DFO1] Drozd Yu.A., Ovsienko S.A., Futorny V.M., On Gelfand-Zetlin modules, Suppl. Rend. Circ. Mat.Palermo, 26 (1991), 143-147.

[DFO2] Drozd Yu.A., Futorny V.M., Ovsienko S.A., Harish-Chandra subalgebras and Gelfand-Zetlin modules, in: Finite-dimensional algebras and related topics (Ottawa, ON, 1992), 79-93, NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., 424, Kluwer Acad. Publ., Dordrecht, 1994.

[GI] A.M.Gavrilik and N.Z.Iorgov, q-deformed algebras $U_q(so_n)$ and their representations, Preprint

[GK] A.M.Gavrilik and A.U.Klimyk, q-deformed orthogonal and pseudo-orthogonal algebras and their representations, Letters Math. Phys, 21 (1991), 215-220.

[GZ1] I.M.Gelfand and M.L.Zetlin, Finite-dimensional representations of the group of unimodular matrices, Dokl. Akad. Nauk SSSR, 71 (1950), 825-828.

[GZ2] I.M.Gelfand and M.L.Zetlin, Finite-dimensional representations of the group of orthogonal matrices, Dokl. Akad. Nauk SSSR, 71 (1950), 1017-1020.

[J] J.C.Jantzen, Lectures on Quantum Groups, AMS Graduate Studies in Mathematics, v. 6.

[J1] Jimbo M., Quantum $R-$matrix for the generalized Toda system, Commun. Math. Phys., 102 (1986), 537-534.

[J2] Jimbo M., Quantum $R-$matrix for the generalized Toda system: an algebraic approach, Field theory, quantum gravity and strings (Meudon/Paris, 1984/1985), Lecture notes in Phys., 246, Springer, Berlin-New York, 1986, 335-361.

[KS] A.Klimyk, K.Schmüdgen, Quantum groups and their representations, Texts and Monographs in Physics, Springer-Verlag, Berlin, 1997.

[M1] V.Mazorchuk, Tableaux realization of generalized Verma modules, Canad. J. Math.. 50 (1998), 816-828.

[M2] V.Mazorchuk, Orthogonal Gelfand-Zetlin algebras, I, Preprint 96-114, Bielefeld Univerity, to appear in Beiträge zur Algebra und Geometrie.

[M3] V.Mazorchuk, On Gelfan-Zetlin modules over orthogonal Lie algebras, Preprint 98-106, Bielefeld University.

[MO] V.Mazorchuk and S.Ovsienko, Submodule structure of Generalized Verma Modules induced from generic Gelfand-Zetlin modules, Alg. and Repr. Theory, 1 (1998), 3-26.

[Mo] A.Molev, A basis for representations of symplectic Lie algebras, Preprint math.QA /9804127.

[NT1] M.Nazorov and V.Tarasov, Yangians and Gelfand-Zetlin bases, Publ. RIMS. Kyoto Univ., 30 (1994), 459-478.

[NT2] M.Nazorov and V.Tarasov, Representations of Yangians with Gelfand-Zetlin bases, J. reine angew. Math., 496 (1998), 181-212.

[O1] U.Ottoson, A classicfication of the unitary irreducible representations of $SO_0(N,1)$, Commun. Math. Phys. 8 (1968), 228-244.

[O2] U.Ottoson, A classicfication of the unitary irreducible representations of $SU(N,1)$, Commun. Math. Phys. 10 (1968), 114-131.

[O] S.Ovsienko, Some finiteness statements for Gelfand-Zetlin modules, to appear

[UTS1] K.Ueno, T.Takebayashi and Y.Shibukawa, Gelfand-Zetlin Basis for $U_q(gl(N+1))$-modules, Letters Math. Phys. 18 (1989), 215-221.

[UTS2] K.Ueno, T.Takebayashi and Y.Shibukawa, Construction of Gelfand-Zetlin Basis for $U_q(gl(N+1))$-modules, Publ. RIMS Kyoto Univ. 26 (1990), 667-679.

[Z] Zhelobenko D.P., Compact Lie groups and their representations, Nauka, Moscow, 1970, Translations of mathematical monographs, 40, AMS, Providence, Rhode Island, 1973.

V.Mazorchuk:

Mechanics and Mathematics Department, Kyiv Taras Shevchenko University, 64, Volodymyrska st., 252033, Kyiv, Ukraine, e-mail: `mazorchu@uni-alg.kiev.ua`

Address until 31 December 1999:

Fakultät für Mathematik, Universität Bielefeld, Postfach 100131, D-33501, Bielefeld FRG, e-mail: `mazor@mathematik.uni-bielefeld.de`

L.Turowska:

Institute of Mathematics, 3, Tereshchenkivska st., 252601, Kyiv, Ukraine, e-mail: `turowska@imath.kiev.ua`

Address until 30 June 1999:

Chalmers Tekniska Högskola, Mathematik, 412 96, Göteborg, Sweden, e-mail: `turowska@math.chalmers.se`