LIST OF CHANGES MADE TO THE ARTICLE "IRREDUCIBLE SUBQUOTIENTS OF GENERIC GELFAND-TSETLIN MODULES OVER $U_q(\mathfrak{gl}_n)$"

We are grateful to the referee for the careful reading and the comments made. The changes made to the article "Irreducible subquotients of generic Gelfand-Tsetlin modules over $U_q(\mathfrak{gl}_n)$" are listed below. The referee's comments are cited in italic font followed by our responses in regular font.

(1) *Page 1, line 3. The phrase 'a classical Gelfand-Tsetlin basis' should say 'classical Gelfand-Tsetlin bases'*
Done!

(2) *Page 1, line 10. Delete the 'the' in the phrase 'among the others'.*
Done!

(3) *Page 1, lines 14-17. Reference [11] is unpublished and yet is spoken of in the past tense. In any case it is unclear whether the authors mean [11] or the paper under review by the phrase 'this paper'.*
The reference [11] is deleted since it is not used.

(4) *Page 2, lines 7-9. The authors fix the proverbial $q \in \mathbb{C}$, and set $1(q) = \{x \in \mathbb{C} | q^x = 1\}$. Since complex exponentiation is not uniquely defined, this set is not well-defined and yet the authors make no comment regarding it. This is a common omission in the literature on quantum groups, and whole books have been written on the subject without giving this problem any thought, since one usually considers only integral powers of q in which case there is no ambiguity. However the set $1(q)$ plays a central role in this article, and several statements are made obscure by this omission, in particular Definition 6.1 and Theorem 6.2 (the main results!). This is not really a problem if, as in [22], one first fixes $h \in \mathbb{C}$ and sets $q^x = exp(hx)$, in particular $q = exp(h)$. Only once this is done does the definition of $1(q)$ makes sense, and $1(q) = \{\frac{2k\pi i}{h} | k \in \mathbb{Z}\}$. Notice in particular that unless q is a root of unity, a different choice of h will change the set $1(q)$. Since most of the contents of this article hinge on the property that certain numbers lie in $\frac{1(q)}{2} + \mathbb{Z}$, this should be addressed at some point. See also the comment about Definition 6.1 and Theorem 6.2.*

We added the sentence "Let $1(q)$ be the set of all complex x such that $q^x = 1$, where $q = e^{\mathfrak{h}}$, that is $1(q) = \{\frac{2k\pi i}{\mathfrak{h}} | k \in \mathbb{Z}\}$."

(5) *Page 2, line 10. The 'n' after 'rank' should be between \$'s.*
Done!

(6) *Page 2, line 14. U_q is 'the' algebra and not 'a' algebra defined by the relations (1)-(7). Also the index i should only go up to $n-1$.*
Done!

(7) *Page 2, line 25. The word 'representation' should be 'presentation'.*
Done!

(8) *NOTE: Reference [13] is only available in Russian, or in an awful English translation where the numeration has been changed and statements abbreviated.*

Is there any alternative source, or alternative translation?

We added Reference [5].

(9) *Page 3, lines 3-4. There should be no point between '$\mathfrak{gl}_m$' and 'We'. The word 'denotes' should be 'denote'.*
Done!

(10) *Page 3, line 6. Theorem 3.2. As far as I can tell the statement in [11] is about $\mathfrak{sl}_n$ and not $\mathfrak{gl}_n$, and $k = 1\ldots, n-1$ there, so the statement does not seem to follow immediately. Also, what does it mean when $l^+_{i,j}$ or $l^-_{j,i}$ with $i > j$ appear in this formula? Finally, does the result hold with q a root of unity? (This is the first of several instances in which this question is ignored in statements, but used in the proofs.)*

$l^+_{ij} = l^-_{ji} = 0$ for $1 \leq j < i \leq n$. Remark 3.1 was included.

(11) *Page 3, line 16. Proof of Lemma 3.4. The proof in [9] is short and the idea is quite simple: the algebra G separates elements with different characters. A short comment, or better yet a copy of the short proof, would make this part of the article five lines longer and much more readable.*

This lemma was deleted since it was not used.

(12) *Page 3, line 22. The choice of notation is a bit unfortunate, as we now have $l^{\pm}_{i,j}$ and $l_{i,j}$, which mean completely different things.*

We changed l_{ij} for v_{ij}.

(13) *Page 4, line 1. As stated in [22], the theorem was originally proved in M. Jimbo, "Quantum R matrix related to the generalized Toda system: an algebraic approach", section 5. Also the formulation found here is more like the one found in [19] than the one found in [25]. Finally in this case q must certainly not be a root of unity, and this has to be clearly stated. (More on this later.)*

The references have been adjusted. Also, we assume q is not root of unity for section 4.

(14) *Page 4, line 9. Proposition 4.3 and proof. At some point $\sigma(i)$ becomes σ_i without any clarification. The permutations described in the statement are usually called $(k, n-k)$-shuffles. Let us denote the set of $(k, n-k)$-shuffles by Sh. The last string of equalities would be easier to follow with a comment such as "Since $\sum_{\sigma\in S_n} q^{l(\sigma)} = (n)_q$ and since $S_n = \cup_{\tau\in Sh}(S_k \times S_{n-k})\tau$, the following equality holds $\sum_{\sigma\in S_n} q^{-2l(\sigma)} = \sum_{\sigma'\in S_k} q^{-2l(\sigma')} \cdot \sum_{\sigma''\in S_{n-k}} q^{-2l(\sigma'')} \cdot \sum_{\tau\in Sh} q^{-2l(\tau)} = (k)_{q^{-2}}!(n-k)_{q^{-2}}! \sum_{\tau\in Sh} q^{-2l(\tau)}$ so... "*

The following sequence of equalities have been included in the proof:

$$\begin{aligned}
&\sum_{\sigma\in S_m} q^{-2l(\sigma)} q^{\mu_{\sigma(1)}+\cdots+\mu_{\sigma(k)}-\mu_{\sigma(k+1)}-\cdots-\mu_{\sigma(n)}}\\
&=\sum_{\sigma\in S_k} q^{-2l(\sigma)} \sum_{\sigma'\in S_{m-k}} q^{-2l(\sigma')} \sum_{\tau\in Sh} q^{-2l(\tau)} q^{\mu_{\tau(1)}+\cdots+\mu_{\tau(k)}-\mu_{\tau(k+1)}-\cdots-\mu_{\tau(n)}}\\
&=(k)_{q^{-2}}!(n-k)_{q^{-2}}! \sum_{\tau\in Sh} q^{-2l(\tau)} q^{\mu_{\tau(1)}+\cdots+\mu_{\tau(k)}-\mu_{\tau(k+1)}-\cdots-\mu_{\tau(n)}}\\
&=(k)_{q^{-2}}!(n-k)_{q^{-2}}! \sum_{\tau\in Sh} q^{-2\sum_{i=1}^k(\tau(i)-i)+\sum_{i=1}^k(v_{m\tau(i)}+\tau(i))-\sum_{i=k+1}^m(v_{n\tau(i)}+\tau(i))}\\
&=(k)_{q^{-2}}!(n-k)_{q^{-2}}! q^{k(k+1)-\frac{m(m+1)}{2}} \sum_{\tau\in Sh} q^{\sum_{i=1}^k v_{m\tau(i)}-\sum_{i=k+1}^n v_{m\tau(i)}}.
\end{aligned}$$

(15) *Page 5, Corollary 4.4. It is not clear a priori that the eigenvalues of c_{mk} depend only on the m-th row, or that c_{mk} is diagonalizable at all, since c_{mk} involves l_{ij} with $1\le i,j\le m$. I believe a more correct statement would be that $c_{mk}\in (U_m)_q \subset Uq$, and the action of $(U_m)_q$ on the bottom m-rows is given by the Gelfand-Tsetlin formulas, so c_{mk} acts as if the table $T(R)$ was of height m, ignoring all rows above m.*

The corollary was included in the statements of Theorem 4.3.

(16) *Page 5, Definition 5.1. As stated above, this depends not just on q but on h, and equivalently states that $l_{ij}-l_{ik}\notin\{\frac{2k\pi i}{h}+s$ with $r,s\in\mathbb{Z}\}$.*

Done!

(17) *Page 5, Theorem 5.2. q should not be a root of unity.*

We fixed q being not root of unity for section 5.

(18) *Page 5, Proposition 5.3. 'separate' should be 'separates'. Also the proof assumes that q is not a root of unity.*

We fixed q being not root of unity for section 5.

(19) *Page 6, Definition 5.5. Add 'irreducible' after the word 'unique'.*

Done!

(20) *Page 6, Notation 5.6. In the statement $L\in\mathbb{C}^{\frac{n(n+1)}{2}}$ but $z\in\mathbb{C}^{\frac{n(n-1)}{2}}$, so what is $L+z$?.*

We consider $z\in\mathbb{Z}^{\frac{n(n-1)}{2}}$ with $z_{ni}=0$, $1\le i\le n$.

(21) *Page 6, Definition 5.7. 'exist' should be 'exists'.*

Done!

(22) *Page 6, Theorem 5.9. Point (i) is equivalent to saying that $W(T(R))$ is a submodule of $V(T(R))$, and this seems clearer. In page 7 line 4, the proof begins*

by assuming $e_kT(S) \notin W(T(R))$ *but ends proving that* $e_kT(S) \in W(T(R))$ *without using the hypothesis. Finally, at the bottom of page 7, the inductive step is almost incomprehensible and should be rewritten.*

We changed the proof to make it more clear.

(23) *Page 8, Definition 6.1. Since the last few results are only true if* q *is not a root of unity, I am assuming that this hypothesis is also in place here. Notice that this implies that* $1(q) + \mathbb{Z} = \mathbb{Z}\frac{\pi i}{h} \oplus \mathbb{Z}$*. Now fix* $1 \leq u < p \leq n$*. By the definition of a* q*-generic tableaux there is at most one* $1 \leq s \leq p$ *such that* $(p, s, u) \in W(T(R))$*, and since the sum above is direct* $d_{p,u}$ *is either* 1 *or* 0*, depending on whether* $W(T(R))$ *contains an element of the form* (p, s, u) *or not.*

For section 6 we assume q not root of unity. Also, if $p = n$ is it possible $d_{p,u} > 1$, because the definition of q-generic does not includes row n.

(24) *Page 8, Theorem 6.2. In the statement, there is an 'a' missing before 'q-generic tableau'. Also item (i) is not really clear, since by Theorem 5.9* $I(T(R))$ *does not generate a submodule of* $V(T(L))$ *in general; the proof makes it clear that one should see tableaux* $T(S)$ *with* $\Omega^+(T(R)) \subsetneq \Omega^+(T(S))$ *as zero. Finally, by the previous comment the number of irreducible modules in the block associated with* $T(L)$ *is* 2^ω*, with* $\omega = |\Omega(T(L))|$*.*

We added the sentence "(note that one should see tableaux $T(S)$ with $\Omega^+(T(R)) \subsetneq \Omega^+(T(S))$ as zero)". When $p = n$ it is possible to have $d_{p,u} > 1$, because the definition of q-generic does not includes row n

(25) *Page 9, Remark 7.2. The first claim only holds if* $T(L)$ *is standard. The claim that Theorem 5.2 holds when* q *is a root of unity deserves at least a separate statement as a lemma and a proof. The argument that generic Gelfand-Tsetlin modules are indeed modules in the non-root-of-unity case is done by arguing that every standard tableaux appears in a finite dimensional representation, which does not happen in the present context.*

Remark 7.2 was added. To prove that generic Gelfand-Tsetlin modules are indeed modules when q is a root of unity we use that this is true for any q non root of unity.

(26) *Page 9, Theorem 7.3. The equality should be a congruence. In the first line of the proof, delete the word 'two'. The last line of the proof states that since two numbers are different modulo e, they are equal.*

Done!

(27) *Page 9, Proposition 7.4. Is* $T(L)$ *generic? Also, the second to last line of the proof mentions* $W(T(R))$*, which is not defined yet.*
We added the hypothesis of $T(L)$ being a generic tableau and avoid the use of $W(T(R))$ in the proof.

(28) *Page 10, line 23.* $W_{ij}(R)$ *is defined as a submodule, but it is never stated a submodule of what.*

It is a submodule of $V(T(v))$.

(29) *Page 10, proof of Theorem 7.7. There is no Proposition 7.3.*

It is Theorem 7.3 instead of Proposition 7.3.

(30) *Page 10, Definition 7.9. What is* $v_{i+1,s}$*? Without this definition I was unable to understand Definition 7.10 and Theorem 7.11.*

$v_{i+1,s}$ is replaced by $\beta_{i+1,s,j}$, the definition of $\beta_{i+1,s,j}$ is given in Definition 7.5

(31) *Page 11, Example. Set* $n = 2$*. This should also be made explicit in Theorem 7.12.*

Done!