Review of "The Groupoid Galois Extension and the Partial Isomorphisms Groupoid"

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I am recommending the article to be rejected, both on mathematical grounds and for a poor exposition of the results. The results as stated in the introduction seem to be outside the aim and scope of the Journal of Pure and Applied Algebra. I can make no claims regarding the validity of said results, since the exposition of the proofs was obscure and several of the statements made no obvious sense. Let me give four examples.

First, it was very dificult to make sense of the definitions in section 2. The condition that the morphisms $\beta_g : E_{g^{-1}} \to E_g$ between ideals be "isomorphisms of K-algebras" (page 3, middle of the page) and the presence of a mysterious element 1_g in condition (ii) of the definition of a *Galois extension* only made sense after skipping them and reading section 3, where it is stated that the ideals must be generated by a local unit, a restriction not present in [1] (where algebras are allowed to be non-unital).

Second, I was unable to understand the statement of Proposition 3.1. The R-algebra S is a direct sum of the form $\bigoplus_{e \in G_0} E_e$, but in that statement an element $s \in S$ is decomposed as $\sum_{g \in G} s_g$, without clarification. Also the statement claims that it is possible to calculate a certain image $\phi_g(s_g)$ in terms of images of $s_{g^{-1}}$, which only makes sense if s_g determines $s_{g^{-1}}$. Since no explanation is given as to what exactly is $s_{g^{-1}}$, it is impossible to decide if this assertion is correct. Reading the proof did not help clarify this matter. A similar statement appears in the proof of Corollary 3.2.

Third, right after the proof of Proposition 3.1 the authors begin discussing the star product $S \star G$ without ever defining it, and look at a map $j : S \star G \to End_R(S)$. These definitions are again taken from [1], without clarification, but one may assume that this star product is an analogue of the star product between an algebra and a group acting on it, which begs the question of what the authors mean by $E_h \star {}_g\mathcal{G}_g$ since the set on the right is not a groupoid unless $g \in G_0$.

Fourth, the proof of Theorem 3.5 states that a groupoid can be written as a disjoint union of sets ${}_g\mathcal{G}_g$ over $g \in G$. However if g and h have the same source and target in G_0 then ${}_h\mathcal{G}_h = {}_g\mathcal{G}_g$, so either there must be some extra hypothesis on the groupoid or the proof needs to be ammended.

[1] D. Bagio; A. Paques, *Partial groupoid actions: globalization, Morita theory and Galois theory*, Comm. Algebra (2012).