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High-fidelity entanglement routing in quantum networks

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ABSTRACT

Quantum networks, endowed with their distinct quantum properties, are poised to emerge as indispensable platforms for the implementation of quantum information technology applications in the near future. Within the realm of quantum networks, the transmission of remote quantum information presents a pivotal and intricate challenge, commonly referred to as the entanglement routing problem. While previous efforts have delved into the intricacies of entanglement routing, there has been a notable oversight concerning the crucial parameter of entanglement fidelity. Hence, this paper introduces a novel Purification-enabled Entanglement Routing Algorithm (PERA). Initially, PERA utilizes the network throughput as the routing metric and ensures fidelity guarantees on all nodes in the link through a hop-by-hop purification process. Subsequently, acknowledging the probabilistic nature of entanglement purification, a strategy involving multiple attempts at purification is employed. Finally, a comprehensive utility function is deployed to address the path selection problem for multiple source–destination pairs. Numerical simulations confirm that PERA establishes entanglement connections with outstanding fidelity guarantees. Concurrently, PERA outperforms the baseline algorithm in terms of network throughput and resource utilization.

Introduction

In recent years, quantum networks, also known as the quantum Internet [1–3], leveraging quantum teleportation [4], have facilitated the transmission of remote quantum information. Harnessing the properties of quantum entanglement, quantum networks effectively address the limitations of traditional internet systems, enabling some crucial applications such as quantum computing [5], clock synchronization [6], and metrology [7]. It is foreseeable that quantum networks will revolutionize various aspects of our world in the near future.

The qubit, the foundational unit of quantum information, adheres to the no-cloning theorem and cannot be transmitted by store-and-forward [8]. The construction of large-scale and wide-area quantum networks faces the primary challenge of remote quantum communication. To address this problem, quantum teleportation and entanglement swapping [9] offer solutions by extending communication distances through end-to-end entanglement connections. Then, another critical issue for quantum networks involves selecting entangled paths to deliver quantum information between communicating parties (source–destination pairs), known as the entanglement routing problem [10].

Due to the unique characteristics of quantum physics, routing algorithms in quantum networks require redesigning. Recently, several studies have focused on the entanglement routing problem and some innovative approaches have been proposed [11–13]. However, few of these existent works have adequately considered the entanglement fidelity property [14]. In fact, the fidelity of entangled pairs decays hop-by-hop due to the effects of entanglement swapping and environmental noise [15]. To fulfill the fidelity requirements of entanglement connections, entanglement purification [16] techniques become indispensable. Therefore, further research into fidelity-guaranteed entanglement routing algorithms is demanded.

Based on the above considerations, in this paper, we propose a novel Purification-enabled Entanglement Routing Algorithm (PERA). To deal with the complex entanglement routing problem, we first propose a hop-by-hop fidelity-guaranteed entanglement purification method. Following the fidelity decay law, multiple attempts at entanglement purification are made before the fidelity falls below the fidelity threshold, ensuring fidelity on all nodes in the link. Then, in order to make full use of network resources, we design utility function for the path selection problem with multiple source–destination pairs. Finally, we complete the design of PERA under the consideration of maximum network throughput. Numerical simulations are performed to evaluate the proposed algorithm, and PERA can achieve higher network throughput with fidelity guarantee compared to the existing algorithms. Moreover, PERA achieves higher network resource utilization, approximately 10% higher than the conventional algorithms.

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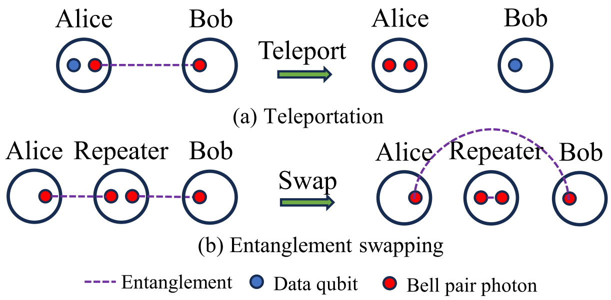


Fig. 1. Concept of teleportation and entanglement swapping.

The contributions of this paper include:

- We propose a hop-by-hop fidelity-guaranteed entanglement routing algorithm, PERA. PERA guarantees that the entanglement fidelity on all the nodes in the link is higher than the fidelity threshold, which not only provides the source–destination nodes with end-to-end fidelity-guaranteed entanglement connections, but also effectively mitigates the decoherence effect during the transmission.
- We consider the probabilistic problem of entanglement purification. Utilizing the decoherence time of quantum memories, we adopt a multiple attempts' purification approach, optimizing the number of attempts to achieve a higher expected network performance.
- We consider optimizing network throughput to obtain routing paths. Meanwhile, a comprehensive utility function is given for the resource allocation problem of multiple source–destination pairs, thereby improving the resource utilization of the whole network.

The rest of this paper is organized as follows. First, Section "Background and motivation" provides background knowledge on quantum networks and outlines the motivation. Then, the network model and the entanglement routing problem are discussed in Section "Network model and problem description". After that, we elaborate our routing algorithm in Section "Routing design in detail" and conduct simulations and evaluations in Section "Performance evaluation". Finally, the conclusions are drawn in Section "Conclusion".

Background and motivation

In this section, we first introduce the basic background of quantum networks, including quantum teleportation, entanglement swapping, network management method and entanglement purification technique. After that, we briefly outline the motivation for writing this paper.

Background on quantum networks

Entanglement swapping and teleportation

In quantum networks, long-distance entanglement connections are indispensable, which are the foundation for realizing remote transmission of qubits. Therefore, the remote transmission of qubits requires two pivotal techniques: quantum teleportation and entanglement swapping. As shown in Fig. 1(a) and (b), Alice and Bob share a pair of entangled qubits, then with the help of quantum measurements and classical networks, Alice realizes the transmission of one data qubit to Bob, which is called teleportation. When the transmission distance increases, a quantum repeater [17] is utilized to generate an entanglement connection through entanglement swapping, then the remote transmission of qubits is completed.

Network management method

Similar to research in [12,18], the quantum network management method operates in a time-slotted mode in this paper. Typically, all quantum nodes are connected through a classical network and they are all controlled by a central controller. The central controller masters global information about the network and processes the requests arriving in the current time slot. For an entanglement routing process, its ultimate task is to establish end-to-end entanglement connection to facilitate the subsequent completion of data qubit transmission. As depicted in Fig. 2, the central controller is responsible for receiving entanglement establishment requests from the application layer, and it executes the entanglement routing scheme to obtain routing paths. Therefore, in each time slot, the following works need to be carried out: entanglement first needs to be generated among the quantum nodes, then the center controller calculates the routing paths for all the requests, and finally the end-to-end entanglement connection will be established through entanglement swapping. Meanwhile, entanglement purification is inevitable due to the quantum decoherence effect, and this process will be performed in quantum repeaters.

Entanglement purification

The main purpose of purification operation is to maintain the quantum state. It produces a higher-fidelity entangled pair at the cost of sacrificing two lower-fidelity entangled pairs. Considering the bit flip error, the resulting fidelity after purification operation can be calculated by [19]:

$$F(f_1, f_2) = \frac{f_1 f_2}{f_1 f_2 + (1 - f_1)(1 - f_2)} \tag{1}$$

where f_1, f_2 is the fidelity of the two Bell pairs used for purification. Considering the probabilistic nature of entanglement swapping, in this paper, we will perform swapping operations hop-by-hop along the selected path. Note that in order to avoid the attenuation of quantum fidelity at each hop, entanglement purification is utilized within the decoherence time of the quantum memory [20]. This ensures that the fidelity of the entangled pair remains above the fidelity threshold F_{th}.

Motivation

The motivation behind this paper lies in establishing fidelity links consistently surpassing the threshold F_{th} to provide end-to-end fidelity guarantee. Despite numerous papers proposing various entanglement routing algorithms, there remains a scarcity of studies addressing the issue of link fidelity guarantee. Consequently, our focus centers on the fidelity on each quantum node within the link while optimizing the routing path. We ensure that the fidelity always exceeds the threshold F_{th} through the implementation of purification methods. Meanwhile, during the design process, considering the probabilistic nature of entanglement purification and the resource competition problem, we supplemented the algorithm with a multiple attempts' purification method and a resource allocation function.

Network model and problem description

In this section, we first provide the network model. After that, we clarify the entanglement routing problem in quantum networks and analyze its property. To facilitate presentation, the notations used in our work are summarized in Table 1.

Network model

Consider the quantum network as a graph $G = (V, E, C)$, where V is the set of quantum nodes, E denotes the edge set, and C is the set of edge capacity. Each edge (u, v) between two nodes is the quantum channel that can transmit quantum information, and entangled pairs

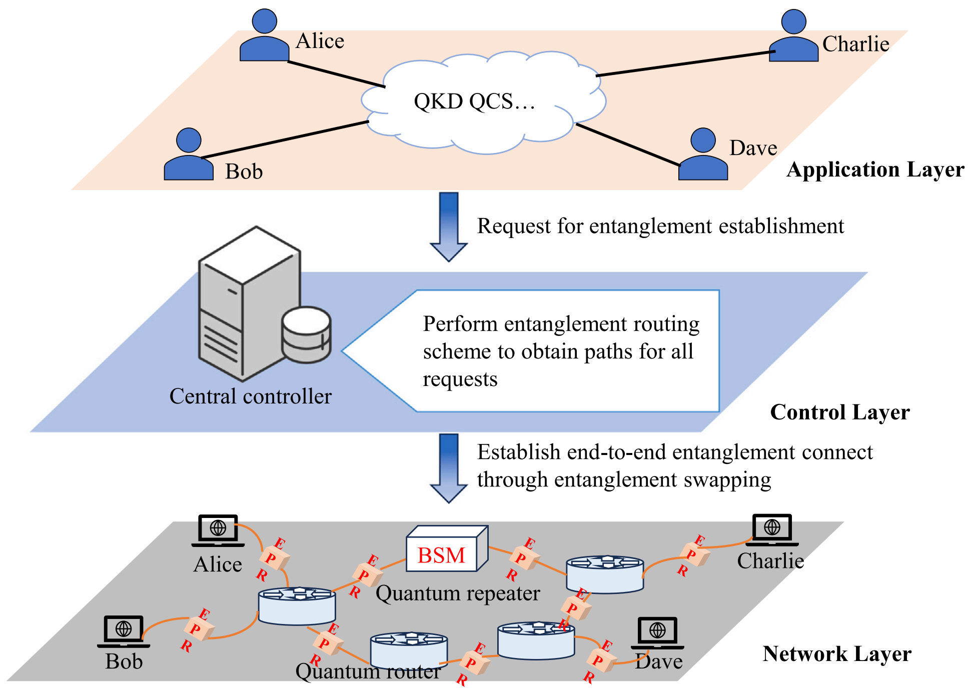


Fig. 2. The architecture of the quantum network management method. The application layer makes multiple requests for entanglement establishment, the control layer performs routing algorithms to allocate paths, and the network layer establishes end-to-end entanglement connections based on routing paths through entanglement swapping.

Table 1
Symbols used in the paper.

Symbol	Description
F_i	The fidelity of the ith node in the network.
F_{th}	The fidelity threshold.
$c(u,v)$	The number of the entangled pairs that can be generated on the channel.
L_k	Maximum hop before each purification for a given routing path.
P_{pur}	Probability of one successful purification.
Γ	The decoherence time of a quantum memory.
M	The number of attempts at entanglement purification during Γ.
F_{suc}	Expected fidelity after successful purification.
$\Psi_j(s_i,d_i)$	The jth routing path of the ith source–destination pair.
R_i	The number of routing requests for the ith source–destination pair.
$t_{i,j}(u,v)$	The expected throughput on any edge (u,v) for the jth routing path of the ith source–destination pair.
N_{ep}	The number of entangled pairs consumed by the purification at the purification node v.
$T_j(s_i,d_i)$	Expected path throughput of $\Psi_j(s_i,d_i)$.
$U_j(s_i,d_i)$	Utility function used for resource allocation of $\Psi_j(s_i,d_i)$.
$N(v)$	The number of one-hop neighbors of node v on the path.

can also be provided between these two nodes. Furthermore, the capacity $c(u,v)$ determines the maximum number of the entangled pairs that can be generated on the channel.

In this paper, we assume that quantum node can play the role of a quantum repeater and holds the information of the network. Besides, at the start of each time slot, the quantum channel is defined as a constant capacity, and the fidelity of generated entangled pairs between initial neighbor nodes is f_0.

Entanglement routing problem

This work considers the entanglement routing problem: given a quantum network with an arbitrary network graph $G = (V,E,C)$, finding the optimal path that guarantees the fidelity $F_i \geq F_{th}$, where F_i is the fidelity of ith node in the network.

Owing to the intricacies of quantum physics, the redesign of routing algorithms becomes imperative in quantum networks. Previous studies have demonstrated that classical routing algorithms are not directly applicable to links reliant on quantum entanglement [21]. Consequently, in quantum networks, it becomes necessary to establish a novel routing metric tailored for entanglement-based algorithms. Moreover, addressing the challenges posed by the decay of quantum fidelity, the probabilistic nature of entanglement purification, and the path selection dilemma for multiple source–destination pairs further compounds the complexity in designing effective entanglement routing algorithms.

Routing design in detail

In this section, at first, we propose a hop-by-hop fidelity-guaranteed entanglement purification method. Then, for the path selection problem with multiple source–destination pairs, a utility function is designed. On these bases, the PERA is proposed to obtain fidelity-guaranteed routing paths for multiple source–destination pairs.

A hop-by-hop fidelity-guaranteed entanglement purification method

In quantum networks, the fidelity of entangled pairs decreases with each entanglement swapping operation. Therefore, to guarantee that the fidelity under each node $F_i \geq F_{th}$, a hop-by-hop entanglement purification method is required. Here, we assume that the entangled pairs are in Werner state, the fidelity is known to degrade exponentially with the path size, satisfying the following equation [22]:

$$F_i = \frac{1}{4} + \frac{3}{4}\left(\frac{4f_0 - 1}{3}\right)^L \tag{2}$$

where L is the number of hops. We require that the fidelity $F_i \geq F_{th}$, substituting this relation into Eq. (2), we will get the following constraint on the hops of the path:

$$L_k = \left\lfloor \frac{\log\left(\frac{4F_{th}-1}{3}\right)}{\log\left(\frac{4f_0-1}{3}\right)} \right\rfloor, k = 1, 2, \ldots \tag{3}$$

where L_k refers to the maximum hop before each purification for a given routing path. Here, we give an example to illustrate the meaning

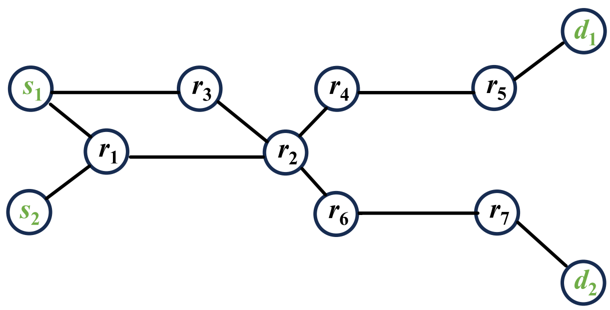


Fig. 3. A routing example.

of L_k. A random network topology is given in Fig. 3. Assuming $f_0 = 0.90$, $F_{th} = 0.80$, and that by the routing algorithm, we get the path of (s_1, d_1) as $s_1 \to r_1 \to r_2 \to r_4 \to r_5 \to d_1$. At the outset, from Eq. (3) we can obtain $L_1 = 1$, then we need to perform the first purification at node r_1. After that, the fidelity will change to 0.95, which will guarantee fidelity for the next 3 hops. This represents that the value of L_2 is 3, which means that we need to perform the second purification at node r_5. From this example, we obtain a fidelity-guaranteed path through two purification operations, and the parameter k only needs to take values 1 and 2. Accordingly, if a path is longer, the parameter k will take more values.

Therefore, Eq. (3) shows that the length of the path will be strictly limited without purification. Between source–destination nodes, we will purify at intermediate nodes before the fidelity falls below the threshold to satisfy that the link fidelity is always above F_{th}. Since entanglement purification is also probabilistic in nature, the probability of one successful purification can be calculated as $P_{pur}(f_1, f_2) = f_1 f_2 + (1 - f_1)(1 - f_2)$ [19].

After considering the probabilistic properties of purification, the resulting fidelity cannot be simply calculated by Eq. (1). For example, when the two entangled pairs used for purification $f_1, f_2 = 0.80$, we note that the probability of successful purification is only 0.68, which is not a very high probability event (obviously the initial fidelity is not low). Therefore, we propose a solution: assume that the decoherence time of a quantum memory is Γ, if the purification fails, we will perform more attempts in the period of time Γ. Since that the probability of one successful attempt is P_{pur}, the successful probability of M attempts can be calculated as $1 - (1 - P_{pur})^M$. Then combining with Eq. (1), we define the expected fidelity after successful purification as:

$$F_{suc}(f_1, f_2) = 1 - (1 - P_{pur})^M \frac{f_1 f_2}{f_1 f_2 + (1 - f_1)(1 - f_2)} \tag{4}$$

Accordingly, a hop-by-hop fidelity-guaranteed purification method is proposed. In the paths from the source node to the destination node, the $F_i \geq F_{th}$ condition will always be satisfied and the end-to-end fidelity will also be guaranteed due to the purification carried out at the intermediate nodes. The proposed purification method is described in Algorithm 1.

Here, in order to evaluate the performance of the network, the definition of network throughput is stated. Similarly, given the probability of failure situation of purification, the expected throughput on any edge (u, v) in path $\Psi_j(s_i, d_i)$ (the jth path of the ith source–destination pair) is defined as follows:

$$t_{i,j}(u, v) = 1 - (1 - P_{pur})^M \min_{v \in \Psi_j} \lfloor \frac{c(u, v)}{N_{ep}} \rfloor \tag{5}$$

where N_{ep} is the number of entangled pairs consumed by the purification at the purification node v. The $\lfloor \frac{c(u,v)}{N_{ep}} \rfloor$ represents the maximum number of end-to-end entanglement connections that can be established. Consequently, the expected path throughput can be calculated by $T_j(s_i, d_i) = \min[t_{i,j}(u, v)]$.

Algorithm 1: The Hop-by-Hop Fidelity-Guaranteed Entanglement Purification Method

Input: Network topology $G = (V, E, C)$, fidelity threshold F_{th}, initial fidelity f_0, the request R_i, source–destination pairs, the number of attempts M;

```
Generate entangled pairs between neighbor nodes;
Execute routing algorithm to discover paths;
if the path is available then
    Source node v_0 ← f_0;
end
while v! = destination node do
    if F_{i+1} < F_th then
        Perform purification process before the next hop
          according to equation (4);
    end
    Perform entanglement swapping;
end
```

The utility function of PERA for multiple source–destination pairs' resource allocation

Due to the finite properties of link resources, in order to achieve higher throughput, determining paths for multiple source–destination pairs has to take the network resource utilization into consideration. Inspired by the idea of [14], we define the following utility function when there is resource competition:

$$U_j(s_i, d_i) = \sum_{v \in \Psi_j} N(v) + \sum_{\Psi_j} L_k \tag{6}$$

where $N(v)$ represents the number of one-hop neighbors of node v on the path.

Analyzing Eq. (6), it can be found that a larger $\sum_{v \in \Psi_j} N(v)$ means more path options for a source–destination pair, and a smaller $\sum_{\Psi_j} L_k$ means that the path consumes less resources. Therefore, when there is resource competition, for different source–destination pairs, we prioritize resource allocation for paths with smaller utility functions (Low selectivity and low resource consumption) so as to achieve higher throughput.

We still take the topology in Fig. 3 as an example. Through the routing algorithm, (s_1, d_1) and (s_2, d_2) determine the routing paths $s_1 \to r_1 \to r_2 \to r_4 \to r_5 \to d_1$ and $s_2 \to r_1 \to r_2 \to r_6 \to r_7 \to d_2$, respectively. Suppose that at this point the resources of the edge (r_1, r_2) are not enough to satisfy the above two routing requests simultaneously, and the resource competition appears. Next, we use the proposed utility function to deal with this problem. Still assuming $f_0 = 0.90$, $F_{th} = 0.80$, for (s_1, d_1), from Eq. (6), we get $\sum_{v \in \Psi_1} N(v) = 14$ and $\sum_{\Psi_1} L_k = 4$. Similarly, for (s_2, d_2), $\sum_{v \in \Psi_1} N(v) = 13$ and $\sum_{\Psi_1} L_k = 4$. Then, since $U_1(s_1, d_1) > U_1(s_2, d_2)$, and we will prioritize assigning the path to (s_2, d_2), obviously there are still other path option for (s_1, d_1). After that, we can get the path $s_1 \to r_3 \to r_2 \to r_4 \to r_5 \to d_1$ for (s_1, d_1) through the re-routing process, so that we have completed the establishment of two entanglement connection requests.

Elaborate design for PERA

Based on above discussions, utilizing the unique properties of quantum networks, we propose the PREA to obtain routing paths with higher network throughput while satisfying the fidelity constraint. The specific PERA scheme is as follows.

The PERA first performs an initialization operation to build the topology graphs and the sets of priority queues for routing (line 1). Then, for all source–destination pairs, we perform the path selection step according to the utility function $U_j(s_i, d_i)$ (line 2–26). To this end, for each source–destination pair, we discover paths based on the possible entangled pair cost (line 3–24), Yen's algorithm [23] is first used to find K paths that satisfy the cost requirement (line 4–7), and then we conduct the purification operation for each link to satisfy the fidelity threshold requirement (line 8–13), after that we calculate the throughput of the fidelity-guaranteed paths based on the link resources, and we will terminate the loop when the total throughput of the network satisfies the request (line 15–24). Finally, we perform resource allocation for multiple source–destination pairs (line 27–36). Similar to the single source–destination pair scenario, the throughput updating process is performed first (line 28), and then since the link resources are limited, an invalid path may occur for a source–destination pair, for this reason, the re-routing process will be executed and the loop will continue to discover the path (line 29–35). The proposed PERA is described in Algorithm 2.

Remark 1. The number of attempts M of the purification operation is crucial. From Eqs. (4) and (5), the expected fidelity and throughput of the link will increase as M increases. However, the increase of M also leads to an increase in the consumption of entangled pairs, which will affect the number of end-to-end entanglement connections and throughput. To address this contradictory issue, we will try to trade-off the value of M in the next section.

Performance evaluation

In this section, we evaluate the performance of PERA through a series of numerical simulations. Considering the practicality of future quantum networks, the network topology in [24] is adopted in our simulation. For a given set of parameters, simulations are run 100 trials and the averaged results are given.

Evaluation setup

In our evaluation, the initial fidelity of the generated entangled pairs are $f_0 = 0.96$. Furthermore, considering that mature quantum memory will become feasible, and then one qubit can be stored with high fidelity for minutes [25], the number of attempts M for purification operations will be guaranteed. As for the baseline algorithm, in a time-slotted mode, since the proposed algorithm performs purification in a hop-by-hop method, similar purification routing algorithms are rare. Therefore, an improved shortest path algorithm (ISPA) is used as the baseline algorithm in our paper. ISPA is based on Dijkstra's algorithm, and adopts the same hop-by-hop purification and resource allocation method as in this paper.

Evaluation results

Effect of the number of attempts M

In order to evaluate the impact of M on the proposed algorithm, in a single source–destination pair scenario, for PERA with different M, we first simulate the change of network throughput when the fidelity threshold is increased.

As shown in Fig. 4, the proposed PERA with different M exhibits different performances. When the required fidelity threshold is low, the link requires little purification, at this moment, the PERA with larger M has a higher throughput; on the contrary, as the fidelity threshold increases, the link starts to be purified, and the PERA with larger M excessively consumes the resources, which leads to a drastic decline in network throughput. This phenomenon also coincides with the analysis in the previous section. It can also be seen from the figure that the change of network throughput is more balanced when $M = 3$.

Based on the above considerations, we will select $M = 3$ and $M = 5$ to conduct all the subsequent simulations. This is because, the PERA with $M = 3$ exhibits superior performance across the board. Although the throughput of PERA with $M = 5$ is not really high at the high-fidelity threshold, considering the high end-to-end fidelity requirements of certain scenarios, we would like to see how it performs in different simulation scenarios to provide a reference for other future studies.

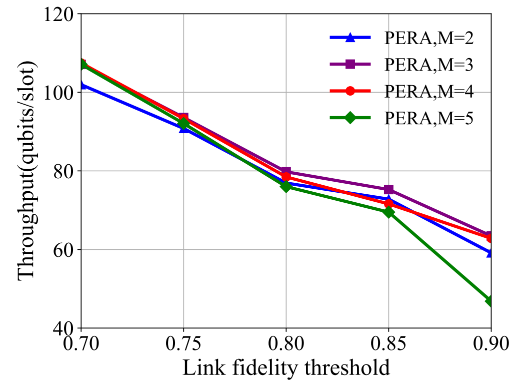


Fig. 4. Throughput performance for PERA with different M when the fidelity threshold increases (channel capacity = 50).

Effect of the link fidelity threshold

In this set of simulations, under different fidelity thresholds, we compare the performance of the proposed algorithms in terms of network throughput, average end-to-end fidelity and network resource utilization for multiple source–destination pairs, respectively.

In Fig. 5(a), both PERA and ISPA exhibit a gradual decrease in throughput as the fidelity threshold increases. This phenomenon stems from the fact that the elevation of the fidelity threshold results in an augmented number of link purifications. The extensive consumption of entangled pairs at purification nodes leads to a reduction in the overall number of end-to-end entangled connections, consequently diminishing the network throughput. Furthermore, as previously analyzed, algorithms with $M = 3$ demonstrate superior throughput compared to those with $M = 5$. Notably, PERA outperforms ISPA, attributed to its higher utilization of link resources.

Examining Fig. 5(b), the algorithms demonstrate excellent end-to-end fidelity guarantees based on the purification method proposed in this paper. Moreover, the algorithms with $M = 5$ achieve higher end-to-end fidelity, a crucial factor in scenarios prioritizing precision. Additionally, ISPA exhibits slightly higher end-to-end fidelity than PERA, given its basis on minimum hops, resulting in smaller fidelity attenuation.

Fig. 5(c) illustrates network resource utilization. In this paper, we characterize the network resource utilization by the ratio of consumed entangled pairs to the total entangled pairs in the network. The ISPA excels in effectively utilizing minimum-hop paths, allowing a superior number of end-to-end entanglement connections for a single path compared to PERA. However, simulations reveal that PERA's multi-path selection is more resource-efficient, even though some of its paths establish fewer end-to-end entanglement connections. With an increase in the fidelity threshold, the total number of end-to-end entanglement connections established by PERA decreases due to the purification operations at certain nodes, despite the number of entangled pairs consumed at these nodes is increased. This impact is less pronounced for PERA, though the contradictory relationship does not ensure a consistent increase in network resource utilization. Furthermore, this contradiction is more noticeable in the series of algorithms with $M = 5$.

Effect of the link capacity

To study how the performance of PERA will be impacted by the link capacity in the network, for different link capacities, we record the performance of network throughput, average end-to-end fidelity and network resource utilization, respectively, and show them in Fig. 6.

Fig. 6(a) demonstrates the relationship between link capacity and network throughput. The higher the link capacity, the higher the network throughput, and the higher link capacity provides more opportunities for the establishment of end-to-end entanglement connections.

Algorithm 2: Purification-enabled Entanglement Routing Algorithm (PERA)

Input: Network topology $G = (V, E, C)$, fidelity threshold F_{th}, initial fidelity f_0, the request R_i, source–destination pairs, the number of attempts M;

Output: The entanglement paths $\Psi_j(s_i, d_i)$, end-to-end fidelity F_e, throughput $T_j(s_i, d_i)$, the total consumed entanglement pairs $N_{sum,ep}$;

1 Construct an update graph $G' = (V, E', C', cost)$ to record purification decision, a residual graph $G^r = (V, E^r, C^r, cost)$ used for re-routing process; Set a priority queue Q to save paths, Q' used for resource allocation;
2 **for** *all source–destination pairs* **do**
3   **for** $cost = minhop : c\,|E|$ **do**
4   For each *cost*, find K paths with Yen's algorithm;
5   **if** *no available path is obtained* **then**
6   break;
7   **end**
8   **for** $v_i \in \Psi_j$ **do**
9   **if** $F_{i+1} < F_{th}$ **then**
10   Calculate the fidelity after purification according to (4); Calculate consumed entanglement pairs N_{ep};
11   **end**
12   **end**
13   **end**
14   $Q \leftarrow \Psi_j(s_i, d_i)$;
15   **while** $Q.pop! = null$ **do**
16   Calculate the $\min\lfloor \frac{c(u,v)}{N_{ep}} \rfloor$ along the path;
17   **if** $min\lfloor \frac{c(u,v)}{N_{ep}} \rfloor \geq 1$ **then**
18   Subtract the total number of entangled pairs consumed each edge on path $\Psi_j(s_i, d_i)$;
19   Calculate the path throughput $T_j(s_i, d_i)$ according to (5);
20   **if** $\sum_j T_j(s_i, d_i) \geq R_i$ **then**
21   Delete all paths from Q;
22   **end**
23   **end**
24   **end**
25   $Q' \leftarrow \Psi_j(s_i, d_i)$ according to (6);
26 **end**
27 **while** Q' *is not empty* **do**
28   Repeat as line 16-23;
29   **else**
30   Perform re-routing process, repeat as line 3-24;
31   **if** $\Psi_{j+1}(s_i, d_i)! = \Psi_j(s_i, d_i)$ **then**
32       $Q' \leftarrow \Psi_{j+1}(s_i, d_i)$ according to (6);
33   **end**
34   Delete $\Psi_j(s_i, d_i)$ from Q';
35   **end**
36 **end**

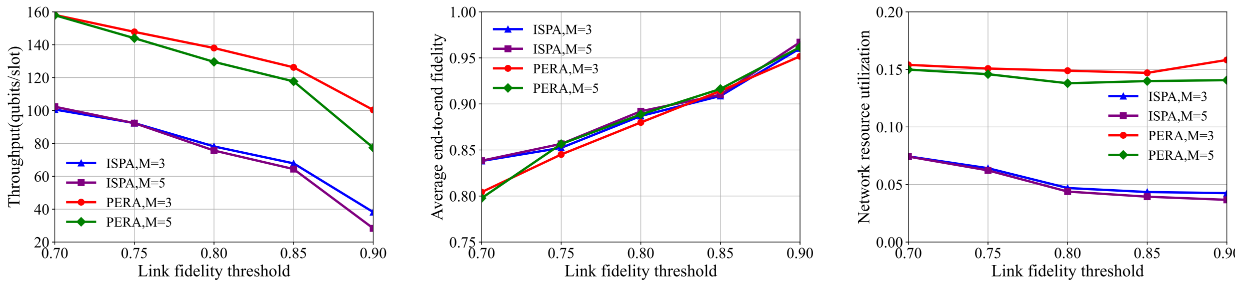
The fidelity of all the algorithms is guaranteed, which is evident in Fig. 6(b). Again, the higher fidelity comes at the cost of reduced network throughput. Regarding the network resource utilization in Fig. 6(c), PERA and ISPA are maintained at around 15% and 5%, respectively, and their proportional relationship is relatively stable, although the increase in link capacity promotes the consumption of more entangled pairs. Due to the contradictory relationship between the number of end-to-end entanglement connections and entanglement purification, the consumption of entangled pairs still does not reach a high standard.

Effect of the number of source–destination pairs

In this set of simulations, for different number of source–destination pairs, we compare the performance of the proposed algorithms in terms of network throughput, average end-to-end fidelity and network resource utilization, respectively.

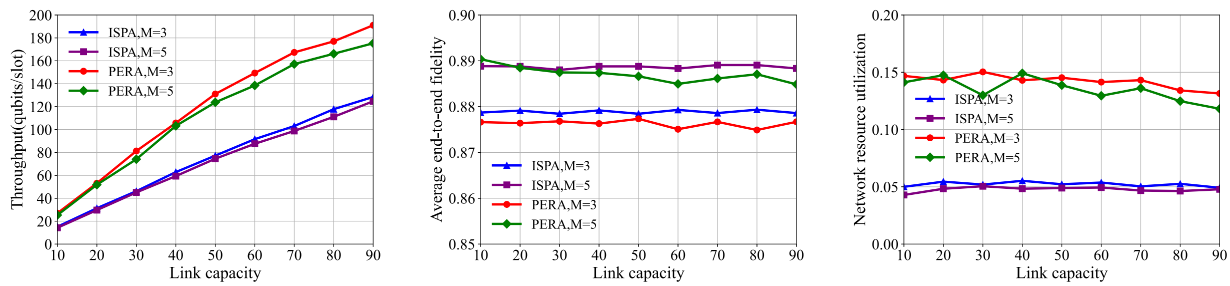
In Fig. 7(a), the network throughput increases with the number of source–destination pairs because more source–destination pairs promote the establishment of more entanglement connections. Fig. 7(b) illustrates the average end-to-end fidelity under this condition, and in general, ISPA based on the minimum hops still provides higher end-to-end fidelity than PERA. Fig. 7(c) demonstrates the relationship between network resource utilization and source–destination pairs. The existence of utility functions effectively solves the problem of path allocation for multiple source–destination pairs. More path options promote the consumption of entangled pairs, which improves the growth of network resource utilization. In addition, the series of algorithms with $M = 3$ and $M = 5$ still have their own advantages and disadvantages according to different problem requirements.

From the comprehensive evaluation results, the algorithm proposed in this paper demonstrates good performance. It plays a role in promoting the construction of large-scale and wide-area quantum Internet [2,3,26] in the future. Considering the above results, some "constraints" are necessary in the quantum Internet setting. First, quantum repeaters are indispensable, which are the basic devices for establishing end-to-end entanglement connections. Then, due to the decoherence property of quantum states, in the process of establishing an end-to-end entanglement connection, i.e., the entanglement routing process, we must take measures to ensure the fidelity of the entangled pairs. That is, entanglement purification needs to be strictly enforced at intermediate



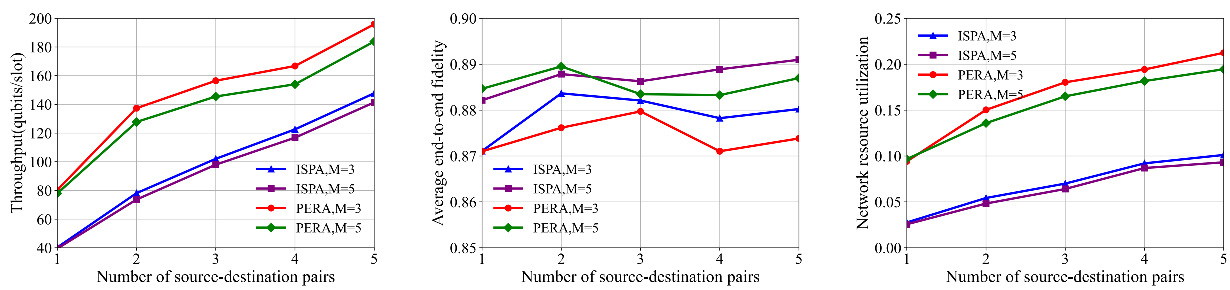
(a) Throughput (b) Average end-to-end fidelity (c) Network resource utilization

Fig. 5. Performance comparison for multiple source–destination pairs under different fidelity thresholds (channel capacity = 50, source–destination pairs = 2).



(a) Throughput (b) Average end-to-end fidelity (c) Network resource utilization

Fig. 6. Performance comparison for multiple source–destination pairs under different link capacities (fidelity threshold = 0.80, source–destination pairs = 2).



(a) Throughput (b) Average end-to-end fidelity (c) Network resource utilization

Fig. 7. Performance comparison under different number of source–destination pairs (fidelity threshold = 0.80, channel capacity = 50).

nodes, or what we can call "short-term destination nodes". In addition, network optimization is also essential in the quantum Internet setting, such as the purification operation optimization and resource allocation method mentioned in this paper, which will further improve the performance of the quantum Internet. As future work, we will further investigate methods to enhance the resource utilization of quantum networks [10,27] and study the nature of free-space channels [28] to consider the associated entanglement routing problem.

Conclusion

In this paper, to provide high-fidelity entanglement routing for quantum networks, we present a novel purification-enabled entanglement routing algorithm. The PERA considers multiple source–destination pairs scenario, discovers paths with possible entangled pair cost, and provides fidelity guarantees for the links in a hop-by-hop purification process. For different problem requirements, alternative number of entanglement purification attempts M increases the flexibility of the algorithm. Meanwhile, an integrated utility function is employed to optimize the allocation of quantum network resources for multiple source–destination pairs. Simulation results demonstrate that PERA outperforms conventional algorithm, providing superior network throughput, resource utilization, and fidelity guarantees.

CRediT authorship contribution statement

HaoRan Hu: Writing – original draft, Software, Methodology, Formal analysis, Data curation. **HuaZhi Lun:** Writing – review & editing, Investigation, Conceptualization. **ZhiFeng Deng:** Visualization, Validation, Conceptualization. **Jie Tang:** Resources, Data curation. **JiaHao Li:** Validation, Investigation. **YueXiang Cao:** Data curation, Conceptualization. **Ya Wang:** Software, Resources. **Ying Liu:** Visualization, Conceptualization. **Dan Wu:** Visualization, Validation. **HuiCun Yu:** Supervision, Project administration, Investigation. **XingYu Wang:** Project administration, Conceptualization. **JiaHua Wei:** Visualization, Project administration, Formal analysis. **Lei Shi:** Writing – review & editing, Supervision, Funding acquisition.

Declaration of competing interest

All the authors of this manuscript certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

Data availability

Data will be made available on request.

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