CSC-421 Applied Algorithms and Structures
Fall 2021-22

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Assignment 4
(Due November 15)

1. (50 points) The greedy algorithm we described for the class scheduling problem is not the only greedy strategy we could have tried. For each of the following alternative greedy strategies, either prove that the resulting algorithm always constructs an optimal schedule, or describe a small input example for which the algorithm does not produce an optimal schedule. Assume that all algorithms break ties arbitrarily (that is, in a manner that is completely out of your control).

   (a) Choose the course x that ends last, discard classes that conflict with x, and recurse.

   (b) Choose the course x that starts first, discard all classes that conflict with x, and recurse.

   (c) Choose the course x that starts last, discard all classes that conflict with x, and recurse.

   (d) Choose the course x with shortest duration, discard all classes that conflict with x, and recurse.

   (e) If no classes conflict, choose them all. Otherwise, discard the course with longest duration and recurse.

   (f) If no classes conflict, choose them all. Otherwise, discard a course that conflicts with the most other courses and recurse.

(g) If any course x completely contains another course, discard x and recurse. Otherwise, choose the course y that ends last, discard all classes that conflict with y, and recurse.

2. (15 points) Construct a Huffman code for the following set of characters and their frequencies that are based on the first 8 Fibonacci numbers:
   $a : 1$; $b : 1$; $c : 2$; $d : 3$; $e : 5$; $f : 8$; $g : 13$; $h : 21$.

3. (20 points) An *independent set* in a graph G is a set of vertices I in G such that no two vertices in I are adjacent (neighbors). The *maximum independent set problem* is, given a graph G, to compute an independent set in G of maximum size (maximum number of vertices). Pinocchio claims that he has a greedy algorithm that solves the maximum independent set problem. Pinocchio's algorithm works as follows. The algorithm initializes the set I to the empty set, and repeats the following steps: Pick a vertex in the graph with the minimum degree, add it to the set I, and remove it and all the vertices adjacent to it from the graph. The algorithm stops when the graph is empty. Does Pinocchio's greedy algorithm always produces a maximum independent set? Prove your answer (if it does, give a proof; if it does not, give a counter example, that is, a graph on which Pinocchio's algorithm does not produce a maximum independent set).

4. (15 points) Professor Gekko has always dreamed of inline skating across North Dakota. He plans to cross the state on highway U.S. 2, which runs from Grand Forks, on the eastern border with Minnesota, to Williston, near the western border with Montana.

   The professor can carry two liters of water, and he can skate m miles before running out of water. (Because North Dakota is relatively flat, the professor does not have to worry about drinking water at a greater rate on uphill sections than on flat or downhill sections.) The professor will start in Grand Forks with two full liters of water. His official North Dakota state map shows all the places along U.S. 2 at which he can refill his water and the distances between these locations.

   The professor's goal is to minimize the number of water stops along his route across the state. Give an efficient algorithm by which he can determine which water stops he should make. Prove that your strategy yields an optimal solution, and give its running time.