

In statistics, **confidence interval** and **margin of error** are key concepts used to express the uncertainty or precision of an estimate, especially in the context of survey results, experiments, or any form of sampling.

## 1. Confidence Interval (CI):

A confidence interval is a range of values, derived from sample data, that is likely to contain the true population parameter (like a mean or proportion) with a certain level of confidence.

- **Interpretation:** If you have a 95% confidence interval, it means that if you were to take 100 different random samples and compute a confidence interval for each, about 95 of them would contain the true population parameter.
- **Formula for Confidence Interval:**  $CI = \text{Sample Statistic} \pm (\text{Critical Value} \times \text{Standard Error})$   
 $\text{CI} = \text{Sample Statistic} \pm (\text{Critical Value} \times \text{Standard Error})$ 
  - The **sample statistic** could be a sample mean, proportion, etc.
  - The **critical value** depends on the desired confidence level (e.g., 1.96 for a 95% CI if using a normal distribution).
  - The **standard error** measures the variability of the sample statistic.

### Example:

Let's say the average weight of 100 students is 70 kg with a standard deviation of 5 kg. A 95% confidence interval for the population mean weight might be:

$$70 \pm 1.96 \times \frac{5}{\sqrt{100}} = 70 \pm 0.98$$

So the 95% CI is (69.02, 70.98) kg.

## 2. Margin of Error (MoE):

The margin of error is the amount added or subtracted from the sample statistic to create the confidence interval. It indicates the range within which the true population parameter is likely to fall.

- **Interpretation:** A margin of error of  $\pm 3\%$  in an election poll means that the true percentage of voters for a candidate could be 3% higher or lower than the estimated percentage.
- **Formula for Margin of Error:**  $MoE = \text{Critical Value} \times \text{Standard Error}$   
 $MoE = \text{Critical Value} \times \text{Standard Error}$ 
  - The **critical value** corresponds to the desired confidence level (e.g., 1.96 for 95% CI).
  - The **standard error** depends on the variability of the data and the sample size.

**Example:**

In the previous weight example, the margin of error was calculated as:

$$\text{MoE} = 1.96 \times 5100 = 0.98 \quad \text{MoE} = 1.96 \times \frac{5}{\sqrt{100}} = 0.98 \quad \text{MoE} = 1.96 \times 1005 = 0.98$$

**Relationship between CI and MoE:**

- The **confidence interval** is centered around the sample statistic and extends a certain distance on either side, determined by the **margin of error**.
- A **higher confidence level** (e.g., 99%) will result in a **larger margin of error**, making the confidence interval wider, while a **lower confidence level** (e.g., 90%) will result in a smaller margin of error, making the interval narrower.

In summary:

- **Confidence Interval** tells us the range in which we expect the population parameter to lie.
- **Margin of Error** quantifies the uncertainty in the sample estimate.