PRICE LEVEL DETERMINATION IN A MONETARY ECONOMY

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PLAN

- We study how the price level (and inflation) is determined in a monetary economy.
- This will later serve us as a framework to study interactions of fiscal and monetary policy.
- We will start with a very simple environment and then gradually add more features.

ENDOWMENT ECONOMY

ENDOWMENT ECONOMY

- We start with a simple economy in which every period there is an endowment of nonstorable goods y_t.
- For simplicity assume $y_t = y$ for all t.
- We denote the price level in period t by P_t.
- One unit of goods costs P_t units of account.
- There is a representative household that trades contingent claims (Arrow securities).
- The household maximizes $\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t)$.
- No government expenditures,

NOMINAL INTEREST RATES

- Let $q_{t,t+1}$ be the real price (in goods) of a claim that pays one unit of goods in period $t+1$ in particular state of the world.
- The nominal contingent claim price is

$$Q_{t,t+1} = q_{t,t+1} \frac{P_t}{P_{t+1}}.$$

- The nominal interest rate i_t satisfies

$$\frac{1}{1+i_t} = \mathbb{E}_t \, Q_{t,t+1}.$$

NOMINAL INTEREST RATES

- In this economy we have $c_t = y$.
- The SDF is constant and equal to β.
- The real interest rate is also constant r:

$$\frac{1}{1+r} = \beta.$$

- The nominal discount factor is $Q_{t,t+1} = \beta \frac{P_t}{P_{t+1}}$.
- Define $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$.
- We obtain the Fisher equation

$$\frac{1}{1+i_t} = \frac{1}{1+r} E_t \frac{P_t}{P_{t+1}} = \frac{1}{1+r} E_t \frac{1}{\Pi_{t+1}} = \beta E_t \frac{1}{\Pi_{t+1}}.$$

FISHER EQUATION

- The usual formulation of the Fisher equation is obtained by linearization:

$$i_t = r + \mathbb{E}_t \, \pi_{t+1}.$$

- We already used the constancy of the real interest rate.
- In general, the Fisher equation is

$$i_t = r_t + \mathbb{E}_t \, \pi_{t+1}.$$

- This is a no-arbitrage condition: you should be indifferent between two types of investment (real and nominal).

FISHER EQUATION

- In this endowment economy

$$\frac{1}{1+i_t} = \beta E_t \frac{1}{\Pi_{t+1}}$$

  is the only equilibrium condition.
- What does it imply for the sequence of price levels $\{P_t\}_{t=0}^{\infty}$?
- In particular, we say that the level of inflation is unique or determinate in equilibrium if:
    1. There is a unique scalar P_0 in equilibrium.
    2. If $\Pi^p rime$ and Π'' are two sequences that satisfy equilibrium conditions, then $\Pi' = \Pi''$.

DETERMINACY

- The question of determinacy may seem esoteric, but it is important.
- It boils down to the question of whether some policy can control prices.
- In this simple economy, the only available policy is the choice of the nominal interest rate.
- Suppose we have some target for the price level (or inflation rate) – can we achieve it by setting interest rates?

DETERMINACY

- Consider the simplest possible policy: interest rate peg.
- The central bank sets $i_t = i$ for all t.
- We have

$$\frac{1}{1+i} = \beta E_t \frac{1}{\Pi_{t+1}}.$$

- This determines $E_t \frac{1}{\Pi_{t+1}}$ – aproximately the expected inflation rate.
- But it does not determine Π_{t+1} – the actual inflation rate.

DETERMINACY

- What if we have perfect foresight?
- The Π_{t+1} is determined by the peg, but P_0 is not (because Π_0 is not)
- Conclusion: interest rate peg does not ensure price level determinacy.

DETERMINACY

- Consider a interest rate rule $i_t = r + \phi\pi_t$.
- We call these rules Taylor rules.
- Use the linearized Fisher equation $i_t = r + \mathbb{E}_t\, \pi_{t+1}$ to obtain

$$\phi\pi_t = \mathbb{E}_t\, \pi_{t+1}.$$

- If $\phi > 1$ the only solution that does not diverge is $\pi_t = 0$ for all t.
- Taylor principle: $\phi > 1$, increase real rates when inflation increases.

DETERMINACY

- To see it solve

$$\phi \pi_t = \mathbb{E}_t \, \pi_{t+1}.$$

forward as

$$\pi_t = \lim_{T \to \infty} \phi^{-T} \, \mathbb{E}_t \, \pi_{t+T}.$$

- If $\phi > 1$ and $E_t \pi_{t+T}$ does not diverge, $\pi_t = 0$ for all t.
- Inflation is determinate in all periods, even at $t = 0$ so P_0 is determined.
- Where does the assumption $E_t \pi_{t+T}$ does not diverge come from?

DETERMINACY

- The terminal condition is not an optimality condition.
- In this model $\phi > 1$ has nothing to do with the logic like "increase real rates to reduce aggregate demand"
- In a way, the central bank makes a threat: if you do not behave, I will make inflation explode. Is it reasonable? Is it how central banks do it?
- Moreover, nothing bad would happen in this economy if inflation diverges.
- See Cochrane (2011) for a discussion of Taylor rules.
- My advice: you might not like the model (it is obviously simple) – but do not apply some outside logic to understand these results!

DETERMINACY

- The usual justification is that when inflation gets out of control, the central bank will switch to something else.
- But then, it is not a Taylor rule with $\phi > 1$ that ensures determinacy!
- What is this other thing that would ensure determinacy?
- For example, the ECB has a monetary pillar, understood as a commitment to switch to a monetary approach to pin down inflation if inflation starts exploding.
- We will now turn to these monetary approaches.

DETERMINACY

- Assume there is some money demand in this economy:
  $m_t = p_t + y - \eta i_t$, where $\eta > 0$ is a semielasticity of money demand.
- Use the Fisher equation to obtain

$$m_t = p_t + y - \eta \left[r + \mathbb{E}_t \pi_{t+1} \right].$$

- Let the central bank follow some money growth rule $m_t = m_{t-1} + \bar{m}$.
- Is it enough to ensure price level determinacy?

DETERMINACY

- We have a formula

$$(1+\eta)\left(p_t - \bar{m}\cdot t\right) = \eta\left(\mathbb{E}_t\ p_{t+1} - \bar{m}\cdot t\right) - y + p_t + \eta r.$$

  we can solve it forward and there will be a term like
  $\lim_{T\to\infty} \mathbb{E}_t\ p_{t+T} - \bar{m}\cdot(t+T-1)$.
- This term will be equal to 0 by the household's optimality condition (transversality condition).
- Intuition: households do not want to hold too much or too little money.
- We get

$$p_t = \bar{m}\cdot t + \eta\bar{m} + \frac{1}{1+\eta}\sum_{j=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^j(\eta r - y).$$

DETERMINACY

- Money growth rule ensures price level determinacy (in this case).
- This is true even if there are shocks u_t to money demand.
- With different microfoundations for money demand, conclusions might be different.
- We can solve for the implied nominal interest rate:

$$i_t = \frac{1}{\eta}\left(p_t - m_t + y\right)$$

- Note it responds to price level. This is an equilibrium outcome.
- If we instead started with a rule that responds to price levels it would be a Wicksellian rule.