AGGREGATE SHOCKS, TAXES, AND DEBT MANAGEMENT I

Fiscal and Monetary Policy 2023

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PLAN

- How should the government change taxes and debt in response to changes in purchases?
- For example: should it finance war spending in the same way as public education?
- This is a normative question, but to the extent real world governments behave according to prescriptions of models we will see in class, it will also be a positive theory of government debt and taxes.

PLAN

- We will study optimal taxation and debt policy in two environments:
  - incomplete markets: the government can buy or sell only a limited set of securities, often only a single risk-free security.
  - complete markets: the government can buy or sell claims contingent on all possible states of the world.
- We will first study some reduced form models and then proceed to optimal taxation in a competitive equilibrium (Lucas and Stokey, 1983).

REDUCED FORM MODELS

INCOMPLETE MARKETS

- We study the following problem based on Barro (1979):
  - the government uses taxes τ_t to finance *stochastic* purchases g_t;
  - $D(\tau_t)$ is the deadweight loss of taxes;
  - the government wants to finance g_t in a way that minimizes the expected value of present discounted deadweight loss of taxes:

$$\min_{\{\tau_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left[\left(\frac{1}{1+r} \right)^t D(\tau_t) \right]$$

  where $1/(1+r)$ is the (gross) rate at which the government discounts the future.

INCOMPLETE MARKETS

- Assume that g_t is governed by an N state Markov chain.
- The state of the world is given by s_t that follows a Markov chain with a transition probability matrix P:

$$P_{ij} = P\left\{s_{t+1} = \bar{s}_j \mid s_t = \bar{s}_i\right\}.$$

- Government spending $\{g_t\}$ obeys

$$g_t = \begin{cases} g_1 & \text{if } s_t = \bar{s}_1 \\ \vdots & \\ g_N & \text{if } s_t = \bar{s}_N. \end{cases}$$

INCOMPLETE MARKETS

- The government can issue only one period, non-state contingent debt (incomplete markets) at price $1/(1+r)$, equal to the discount rate.
- Government problem:

$$\min_{\{\tau_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left[\left(\frac{1}{1+r} \right)^t D(\tau_t) \right]$$

$$\text{s.t.} \quad g_t + b_t = \frac{1}{1+r} b_{t+1} + \tau_t$$

and, in addition, we assume

$$\mathbb{E}_0 \left[\left(\frac{1}{1+r} \right)^t b_t^2 \right] \leq \infty$$

DIGRESSION ON BELLMAN EQUATIONS

- How to solve the above problem?
- My recommendation is to write it in a *recursive* form:

$$V(b,g) = \min_{\tau} D(\tau) + \frac{1}{1+r}\mathbb{E}\left[V\left(b',g'\right) \mid g\right]$$
$$\text{s.t.} \quad g + b = \frac{1}{1+r}b' + \tau$$

  where x denotes current variables (x_t) and x' denotes variables in the next period (x_{t+1}).
- It looks like a two period problem (instead of infinite horizon) – the "only" issue is that we do not know $V(b,g)$ (and it appears on the right hand side).
- More on that in, for example, my Quantitative Economics course.

DIGRESSION ON BELLMAN EQUATIONS

- Write

$$V(b, g) = \min_{\tau} D(\tau) + \frac{1}{1+r} \mathbb{E}\left[V\left((1+r)(g+b-\tau), g'\right) \mid g\right]$$

and differentiate with respect to τ to get the first order condition:

$$D'(\tau) = \mathbb{E}\left[\frac{\partial V\left(b', g'\right)}{\partial b'} \,\middle|\, g\right]$$

- How to get $V'\left(b', g'\right) \coloneqq \partial V\left(b', g'\right) / \partial b'$?

DIGRESSION ON BELLMAN EQUATIONS

- Write

$$V(b, g) = D\left(g + b - \frac{1}{1+r} b'\right) + \frac{1}{1+r} \mathbb{E}\left[V\left(b', g'\right) \mid g\right]$$

(note: there is no $\min_\tau$) and differentiate with respect to b:

$$V'(b, g) = D'(\tau) .$$

This is the envelope condition and it implies

$$V'\left(b', g'\right) = D'\left(\tau'\right) .$$

- Important: do not write $b' = b'(b, g)$ or anything like that, these terms drop out anyway (why?).

DIGRESSION ON BELLMAN EQUATIONS

- The first order condition

$$D'(\tau) = \mathbb{E}\left[\frac{\partial V(b', g')}{\partial b'} \,\Big|\, g\right]$$

together with the envelope condition

$$V'(b', g') = D'(\tau')$$

give us the Euler equation

$$D'(\tau_t) = \mathbb{E}_t\left[D'(\tau_{t+1})\right]$$

INCOMPLETE MARKETS

- Solution to the government's problem is characterized by the first order condition

$$D'(\tau) = \mathbb{E}_t\left[D'\left(\tau'\right)\right].$$

  - Tax smoothing: spread out distortionary effects of taxes across time.

- There are two important special cases:
  - Special case 1 - no uncertainty (Barro, 1979):

$$\tau_t = \tau_{t+1}$$

  - Special case 2 - quadratic deadweight loss $\left(D(\tau_t) = a\tau_t^2 + \gamma\right)$:

$$\tau_t = \mathbb{E}_t\left(\tau_{t+1}\right)$$

SPECIAL CASE 1: PERFECT FORESIGHT

- With no uncertainty about future g_t (perfect foresight), taxes are constant.
- τ is chosen to satisfy the budget constraint:

$$\tau = \frac{r}{1+r}\left[\sum_{t=0}^{\infty}(1+r)^t g_t + b_0\right]$$

- Debt absorbs all fluctuations in government spending:

$$b_{t+1} - b_t = r\,(b_t - b_0) + (1+r)\,g_t - r\sum_{s=0}^{\infty}(1+r)^s g_s.$$

SPECIAL CASE 1: PERFECT FORESIGHT

- If $g_t = g$ is constant, then

$$\tau = g + \frac{r}{1+r} b_0, \quad b_t = b_0;$$

  debt remains constant at its initial level.

- If $g_0 = g + \epsilon$ and $g_t = g$ for all $t \geq 1$ then

$$\tau = g + \frac{r}{1+r} (\epsilon + b_0), \quad b_t = b_0 + \epsilon;$$

  debt increases by ϵ and stays elevated forever.

- The level of debt b_t is irrelevant. What matters for debt is transitory fluctuations in g_t, not the average level of g_t.
- Intuition valid also with trend growth of GDP and government spending (see Barro's paper).

SPECIAL CASE 2 - QUADRATIC DEADWEIGHT LOSS

- What does the condition

$$\tau_t = \mathbb{E}_t\left[\tau_{t+1}\right]$$

  tell us about the optimal tax policy?

- τ_t is a random walk:

$$\begin{aligned} \tau_{t+1} - \tau_t &= \tau_{t+1} - \mathbb{E}_t\left[\tau_{t+1}\right] \\ &= \tau_{t+1} - (\tau_{t+1} - \epsilon_{t+1}) \\ &= \epsilon_{t+1} \end{aligned}$$

  where ϵ_{t+1} is white noise.

SPECIAL CASE 2 - QUADRATIC DEADWEIGHT LOSS

- Taxes tomorrow expected to be the same as today, τ_t moves only because of changes in expected future g_t.
- Whenever new information is revealed, taxes immediately jump to a new level.
- To see this, use the budget constraint together with $\mathbb{E}_t[\tau_{t+1}] = \tau_t$ to get

$$\tau_t = \frac{r}{1+r} b_t + \frac{r}{1+r} \sum_{s=0}^{\infty} \left(\frac{1}{1+r} \right)^s \mathbb{E}_t [g_{t+s}] .$$

so

$$\tau_t - \mathbb{E}_{t-1}[\tau_t] = \frac{r}{1+r} \sum_{s=0}^{\infty} \left(\frac{1}{1+r} \right)^s (\mathbb{E}_t [g_{t+s}] - \mathbb{E}_{t-1}[g_{t+s}])$$

SPECIAL CASE 2 - QUADRATIC DEADWEIGHT LOSS

- The size of the jump in τ_t depends on the size of the change in expected future g_t:
  - If there is a permanent shift in g_t, $\mathbb{E}_t[g_{t+s}] - \mathbb{E}_{t-1}[g_{t+s}] = \epsilon$ for all s, taxes change by exactly the same amount ϵ.
  - If the change is temporary, $\mathbb{E}_t[g_{t+s}] - \mathbb{E}_{t-1}[g_{t+s}] = \epsilon$ only for $s = 0$, taxes change by $\epsilon r/(1+r)$.
- Remaining needed resources raised through debt issuance:
  - Debt remains constant for a permanent change in g_t.
  - Debt changes by $\epsilon/(1+r)$ for a temporary change in g_t.
- Implication: finance education by taxes, finance war by debt.

SPECIAL CASE 2 - QUADRATIC DEADWEIGHT LOSS

- Debt also inherits the random walk property of taxes:

$$\mathbb{E}_t\left[b_{t+1}\right] = b_t$$

- There is nothing that acts as a force that would push debt to a particular level:
  - No debt target;
  - No "raise taxes if debt is too high" rule;
  - Debt is a random walk, so its variance goes to infinity unless all shocks to g_t are fully permanent.

COMPLETE MARKETS

- In the complete markets case, the government can issue state-contingent securities.
- This means that the government can issue a security that pays 1 unit of consumption in state s in the next period, and it can do it for all possible states.
- We call such a security an Arrow security.
- Let s_t denote the state in period t and $s^t := (s_0, s_1, \cdots, s_t)$ denote the history of states up to period t.
- Let $q\left(s^{t+1} \mid s^t\right)$ denote the period t price of a security that pays 1 unit of goods for a particular history s^{t+1} in period $t+1$ (and zero in other states).

COMPLETE MARKETS

- To facilitate comparison with the incomplete markets case we will assume

$$q\left(s^{t+1} \mid s^t\right) = \frac{1}{1+r}\pi\left(s^{t+1} \mid s^t\right)$$

  where $\pi\left(s^{t+1} \mid s^t\right)$ is the conditional probability of history s^{t+1} given history s^t.

- The above assumption implies:

$$\sum_{s^{t+1} \geq s^t} q\left(s^{t+1} \mid s^t\right) = \frac{1}{1+r},$$

  the price of a portfolio that pays one unit of goods for sure is equal to $(1+r)^{-1}$.

- The problem of the government is now

$$\min_{\{\tau\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \sum_{s^t} \pi\left(s^t\right) \left(\frac{1}{1+r}\right)^t D\left(\tau\left(s^t\right)\right)$$

$$\text{s.t.} \quad \forall s^t \quad g\left(s^t\right) + b\left(s^t\right) = \tau\left(s^t\right) + \sum_{s^{t+1} \geq s^t} q\left(s^{t+1} \mid s^t\right) b\left(s^{t+1}\right)$$

$$b\left(s_0\right) = b_0$$

where $\pi\left(s^t\right)$ is the probability of history s^t, and $b\left(s^t\right)$ is the quantitity of (negative) Arrow securities that pay in history s^t.

COMPLETE MARKETS

- We can write a Lagrangian:

$$\sum_{t=0}^{\infty} \sum_{s^t} \pi\left(s^t\right) \left(\frac{1}{1+r}\right)^t D\left(\tau\left(s^t\right)\right)$$
$$+ \sum_{t=0}^{\infty} \sum_{s^t} \lambda\left(s^t\right) \left[g\left(s^t\right) + b\left(s^t\right) - \tau\left(s^t\right) - \sum_{s^{t+1} \geq s^t} q\left(s^{t+1} \mid s^t\right) b\left(s^{t+1}\right)\right]$$

COMPLETE MARKETS

- First order condition with respect to $\tau\left(s^t\right)$:

$$\pi\left(s^t\right)\left(\frac{1}{1+r}\right)^t D'\left(\tau\left(s^t\right)\right) = \lambda\left(s^t\right)$$

- First order condition with respect to $b\left(s^{t+1}\right)$:

$$q\left(s^{t+1} \mid s^t\right)\lambda\left(s^t\right) = \lambda\left(s^{t+1}\right)$$

- Given the assumption

$$q\left(s^{t+1} \mid s^{t}\right) = \frac{1}{1+r}\pi\left(s^{t+1} \mid s^{t}\right)$$

we have

$$\frac{1}{1+r}\pi\left(s^{t+1} \mid s^{t}\right)\lambda\left(s^{t}\right) = \lambda\left(s^{t+1}\right)$$

so

$$\begin{aligned}\frac{1}{1+r}\pi\left(s^{t+1} \mid s^{t}\right)\pi\left(s^{t}\right)\left(\frac{1}{1+r}\right)^{t} D'\left(\tau\left(s^{t}\right)\right) \\ = \pi\left(s^{t+1}\right)\left(\frac{1}{1+r}\right)^{t+1} D'\left(\tau\left(s^{t+1}\right)\right)\end{aligned}$$

COMPLETE MARKETS

- It simplifies to

$$D'\left(\tau\left(s^t\right)\right) = D'\left(\tau\left(s^{t+1}\right)\right)$$

  i.e.

$$\tau\left(s^t\right) = \tau\left(s^{t+1}\right).$$

- This is the key result: perfect smoothing of taxes across states.
- The result holds regardless of the form of $D(\tau)$ or the nature of risk.
- Compare it with incomplete markets case: it was true only with perfect foresight.

COMPLETE MARKETS

- What about debt?
- Guess that $b\left(s^{t+1}\right)$ depends only on the future state s_{t+1}, and not on the history s^t.
- Intuitively: the government cares only about the future state, not about the history (why?).
- Use this guess together with $\tau\left(s^t\right) = \tau$ and the initial condition $b(s_0) = b_0$ to solve for τ and optimal debt policy.
- This also verifies the guess.

COMPLETE MARKETS

- Since $b\left(s^{t+1}\right) = b\left(s_{t+1}\right)$, debt neither accumulates, nor decumulates, nor drifts – it switches between different levels.

COMPARISON

- Incomplete: taxes drift over time as a random walk; the level of taxes at time t depends on the level of debt that the government brings into the period as well as the expected discounted present value of government purchases.
- Complete: taxes are constant, regardless of the state of the world; the level of taxes at time t is independent of the level of debt that the government brings into the period.

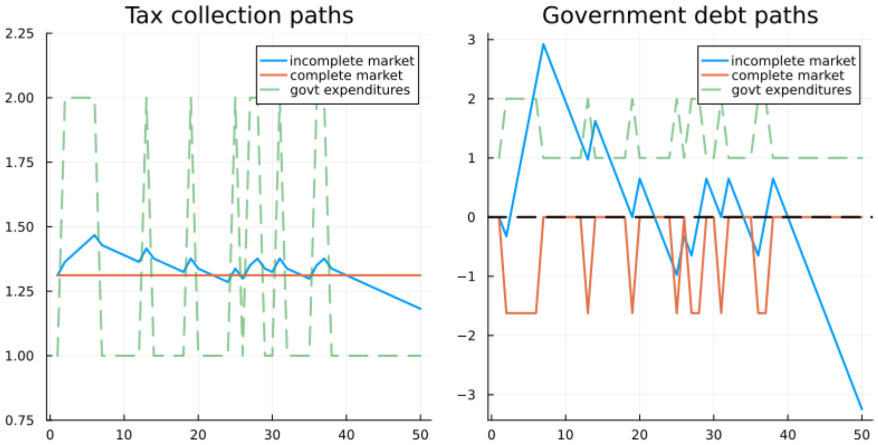
COMPARISON

- Incomplete: debt drifts upward over time in response to large purchases and drifts downward over time in response to low purchases.
- Complete: debt osciallates between several levelsl; this is akin to the government purchasing insurance that protects against the need to raise taxes too high or issue too much debt in the high government expenditure event.

NUMERICAL EXAMPLE

- Let $(1 + r)^{-1} = 0.96$ and there are two levels of government purchases: $g_1 = 1$ and $g_2 = 2$ with with transition probabilities $P_{12} = 0.2$ and $P_{21} = 0.4$.
- The initial condition is $b_0 = 0$, and we start with g_1.

NUMERICAL EXAMPLE



Tax and debt policy in complete and incomplete markets. Computed using QuantEcon Julia package.