DEBT SUSTAINABILITY

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PLAN

- We review several approaches to debt sustainability analysis.
- Can the government service its debt?
- Is the outstanding public debt and its projected path consistent with those of the government's revenues and expenditures?

CLASSIC DEBT SUSTAINABILITY ANALYSIS

DEBT TO OUTPUT RATIO

- The government static budget constraint is

$$B_t = (1 + r_t)B_{t-1} + G_t - T_t.$$

  where B_{t-1} is the market value of all outstanding debt (all maturities), G_t is nominal government spending (net of interest expenses), T_t is nominal tax revenue, and r_t is net return on government debt.
- We abstract from money growth.

DEBT TO OUTPUT RATIO

- Divide by nominal GDP Y_t and rearrange to get

$$\frac{B_t}{Y_t} = (1 + r_t)\frac{B_{t-1}}{Y_{t-1}}\frac{Y_{t-1}}{Y_t} + \frac{G_t}{Y_t} - \frac{T_t}{Y_t}.$$

- Define

$$R_{t-j,t} \coloneqq \prod_{k=1}^{j}\left(1 + r_{t-j+k}\right),$$

  the cumulative return on debt from $t - j$ to t.

- Define

$$X_{t-j,t} \coloneqq \prod_{k=1}^{j}\frac{Y_{t-j+k}}{Y_{t-j+k-1}},$$

  the cumulative gross rate of GDP from $t - j$ to t.

DEBT TO OUTPUT RATIO

- For simplicity assume $B_0 = 0$.
- We can write the debt to output ratio as

$$\frac{B_t}{Y_t} = \sum_{j=0}^{t} \frac{G_{t-j} - T_{t-j}}{Y_{t-j}} \frac{R_{t-j,t}}{X_{t-j,t}}$$

- Debt to output ratio today is determined by:
    1. The past primary deficits to GDP ratios;
    2. The past returns on debt;
    3. The past growth rates of (nominal) GDP.
- We saw a similar decomposition when we discussed Hall and Sargent (it was more detailed there).

CLASSIC DEBT SUSTAINABILITY ANALYSIS

- A version of the formula is often used to assess debt sustainability – "whether the government can service its debt".
- Warning: this is about the future, not the present. The fact that people use the formula to assess the current situation is often a red flag!
- Classic debt sustainability analysis looks at the "long run".
- Assume that the economy is in a steady state with a constant growth rate of GDP X, an constant rate of return R and a constant primary deficit to GDP ratio.
- What is the debt to output ratio consistent witht the above?
- If the observed current debt to output ratio is below this level, the debt is sustainable.

CLASSIC DEBT SUSTAINABILITY ANALYSIS

- Classic debt sustainability analysis usually analyzed determinisic setups (or perfect foresight).
- In these setups, the appropriate R is the risk-free rate, R^f.
- Assuming the above, the formula in the steady state becomes

$$\frac{B}{Y} = \frac{G-T}{Y}\frac{X}{X-R^f}.$$

- For simplicity define $x \coloneqq X - 1$ and $r^f \coloneqq R^f - 1$ so we have

$$\frac{B}{Y} = \frac{G-T}{Y}\frac{1+x}{x-r^f}.$$

CLASSIC DEBT SUSTAINABILITY ANALYSIS

$$\frac{B}{Y} = \frac{G - T}{Y} \frac{1 + x}{x - r^f}$$

- Example: the Treaty of Maastricht set the limit of the debt to output ratio at 60% and the deficit to output ratio at 3%. What must be the growth rate of GDP and the risk-free rate for this to be sustainable?
- We get $\frac{1+x}{x-r^f} = 20$, for small x we have $x \approx r^f + 0.005$, so the economy has to grow at 0.5% above the risk-free rate per year for the debt at the limit to be sustainable with the largest allowed deficit.
- If the actual growth rate is lower, the debt to output ratio will be larger, even if the deficit is at the limit.

CLASSIC DEBT SUSTAINABILITY ANALYSIS

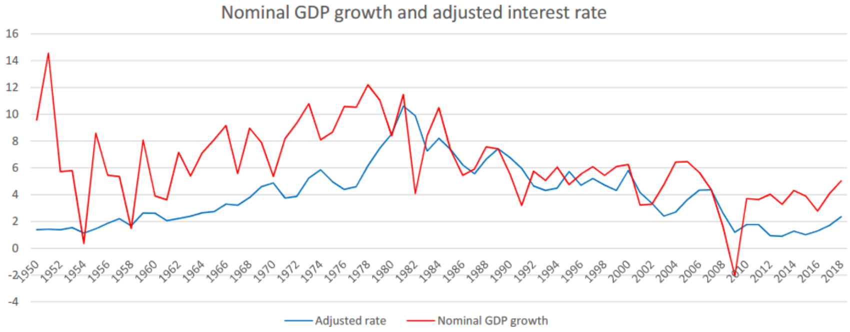
$$\frac{B}{Y} = \frac{G - T}{Y} \frac{1 + x}{x - r^f}$$

- The key role of $x - r^f$.
- Depending on the sign of $x - r^f$ the you require either surpluses or deficits to keep the debt to output ratio constant.
  - If $x < r^f$ you need surpluses.
  - If $x > r^f$ you can have deficits.
- A big "r versus g" debate (we have x instead of g).

CLASSIC DEBT SUSTAINABILITY ANALYSIS

- If $x > r^f$ it seems there is no fiscal cost to debt.
- Blanchard (2019) argues that $x > r^f$ is a norm, not an exception.
- But what is r^f? How to measure it? Blanchard (2019) looks at the 1-year US Treasury bill rate, the 10-year US Treasury bond rate, adjusts for various maturities...
- Note: it does not necessarily mean that it is optimal to have deficits.

RATES IN THE US



Source: Blanchard (2019)

CLASSIC DEBT SUSTAINABILITY ANALYSIS

- It only defines what long-run debt is for a given long-run primary balance (or vice versa) if stationarity holds, or defines lower bounds on the short-run dynamics of the primary balance.
- It does not connect the outstanding initial debt of a particular period with the steady state.
- There might be multiple paths of debt that do not violate the intertemporal government budget constraint (IGBC), some of them can even go to infinity (but slowly enough)!
- IGBC: the value of debt is equal to the present discounted value of future primary surpluses.

INTERTEMPORAL GOVERNMENT BUDGET CONSTRAINT

IGBC

- We used the government budget constraint by going back in time.
- We can also solve it forward – the valuation approach, the market value of government debt is determined by the discounted value of future government surpluses.
- This idea is often used in finance (e.g., Campbell and Shiller 1988).
- Allows us to think seriously about risk and asset pricing.

IGBC

- We want to write something like

$$B_t = \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} \left(T_{t+j} - G_{t+j}\right)$$

- We call $M_{t,t+j}$ the stochastic discount factor (SDF).
- It reflects how holders of government debt value discount future cash flows.
- Generally it is a function of the state of the economy at time t and $t+j$. Recall the first order condition for the household problem in the models we saw.
- We call the formula above the intertemporal government budget constraint IGBC.

DEBT SUSTAINABILITY

- We can say that debt is sustainable if and only if the IGBC holds.
- Problem: this condition is about the entire future.
- Solution (?): use forecasts of future taxes and spending to compute the present value of future surpluses. Some early papers did this, but they used risk-free rates.
- Valid if one of these conditions holds:
  1. There is perfect foresight;
  2. Investors are risk-neutral;
  3. Primary surpluses do not covary with the SDF.

BOHN (1998)

- Recall the Barro (1979) tax smoothing model – debt was a random walk, yet the IGBC held.
- Not even debt (or debt to GDP) going to infinity means that the IGBC does not hold, it has to go to infinity slowly enough.
- Bohn (1998): see if the government does something that guarantees the IGBC holds, investigate the fiscal reaction function.
- Allows to sidestep the problem of forecasting future taxes and spending and choosing the correct discount rate.
- Sufficient condition: IGBC might also hold if it violated, but if it is satisfied, IGBC holds for sure.

BOHN (1998)

- Linear reaction function:

$$\frac{T_t - G_t}{Y_t} = \rho \frac{B_{t-1}}{Y_{t-1}} + Z_t + \epsilon_t$$

- The left hand side is primary surplus.
- Z_t is a vector of exogenous variables that affect the primary surplus.
- Check if $\rho > 0$ – raise surplus if debt is high.
- If $\rho > 0$, then the IGBC holds even if it is below the interest rate (net of x).

BOHN (1998)

- If $\rho > 0$, the IGBC holds for any initial level of debt.
- This analysis works also for $r - x = 0$ – there was division by zero in the classic analysis.
- If $r - x > \rho > 0$, debt explodes, but the IGBC still holds (under certain conditions: see Bohn 2007).

BOHN (1998)

- Bohn (1998) estimates ρ for the US in 1916-1995.
- He includes the level of temporary government spending and business cycle indicator in Z_t.
- He find a positive value of ρ, around 0.05 for the entire sample.

BOHN (1998)

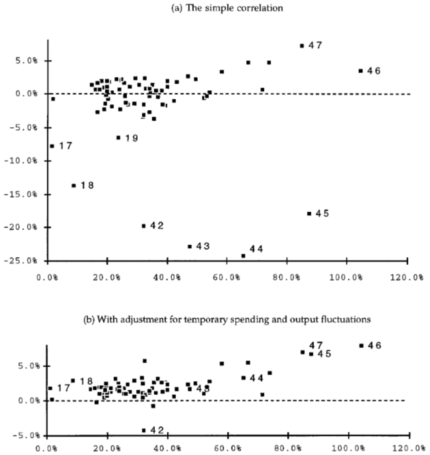


FIGURE I
Primary Surplus versus Initial Debt
The graph shows the privately held government debt/GDP at the start of a period on the horizontal axis against the primary budget surplus/GDP on the vertical axis, for 1916–1995; (a) shows raw data, and (b) shows the adjusted surplus, as explained in the text.

Source: Bohn (1998)

BOHN (1998)

TABLE I
DETERMINANTS OF THE BUDGET SURPLUS

Dependent variable primary budget surplus divided by GDP (s_t)							
Sample	Constant	*GVAR*	*YVAR*	d_t	R^2	σ	*DW*
(1) 1916–1995	−0.019	−0.776	−1.450	0.054	0.936	0.014	1.42
	(−5.424)	(−33.001)	(−3.628)	(6.048)			
	[−3.957]	[−20.874]	[−4.075]	[3.787]			
(2) 1920–1995 excl. 1940–1947	−0.009	−0.551	−1.906	0.028	0.618	0.011	1.40
	(−2.030)	(−4.034)	(−4.666)	(2.701)			
	[−2.155]	[−3.721]	[−4.296]	[2.491]			
(3) 1916–1983	−0.018	−0.782	−1.414	0.054	0.942	0.014	1.54
	(−4.903)	(−31.667)	(−3.360)	(5.996)			
	[−3.958]	[−20.943]	[−4.004]	[4.076]			
(4) 1920–1982 excl. 1940–1947	−0.008	−0.520	−1.912	0.030	0.630	0.011	1.56
	(−1.710)	(−3.612)	(−4.441)	(2.815)			
	[−1.932]	[−3.272]	[−3.959]	[2.856]			
(5) 1948–1995	−0.015	−0.593	−2.139	0.037	0.651	0.010	1.54
	(−3.536)	(−4.182)	(−4.361)	(3.589)			
	[−3.496]	[−3.701]	[−3.757]	[2.821]			
(6) 1960–1984	−0.013	−0.410	−2.051	0.044	0.724	0.007	1.43
	(−2.110)	(−2.173)	(−4.174)	(2.028)			
	[−2.174]	[−2.281]	[−3.391]	[2.587]			

The variable d_t is the privately held debt/GDP at the start of the year. *GVAR* and *YVAR* are measures of temporary government spending and of cyclical variations in output, respectively, from Barro [1986a]. All estimates are OLS with annual data; ( ) = ordinary t-statistics; [ ] = heteroskedasticity- and autocorrelation-consistent t-statistics (computed with Newey-West lag window of size 1); σ = standard error; *DW* = Durbin-Watson statistic.

Source: Bohn (1998)

FISCAL REACTION FUNCTIONS

- Bohn (2008) extends the analysis to 1793-2003.
- He finds that $\rho > 0.1$, more than twice as large as in the previous study.
- Mendoza and Ostry (2008) study fiscal reaction functions for a panel of multiple countries – similar results.
- Ghosh et al. (2013) show that ρ is much lower at high levels of debt.
- D’Erasmo et al. (2016):
    1. primary balance adjustment in the US after 2008 was too large to be explained by the fiscal reaction function;
    2. adjustment is slower than before (structural break);
    3. nevertheless, with the estimated ρ, the IGBC holds.

FISCAL REACTION FUNCTIONS

- Leeper (2017) warns against using surplus-debt regressions to assess debt sustainability.
- For the estimator of ρ to be consistent, we must have

$$\mathbb{E}\left(\epsilon_t \mid \frac{B_{t-1}}{Y_{t-1}}\right) = 0.$$

  1. This means that shocks at $t-1$ that affect debt-output ratio in must not affect ϵ_t.
  2. This means that the debt-output ratio cannot depend on the expectation of ϵ_t.

- Since the value of debt depends on the expected value of future surpluses, this is a strong assumption: ϵ_t could be serially correlated.

VALUATION APPROACH

VALUATION APPROACH

- We go back to the budget constraint and solve it forward as

$$B_t = \mathbb{E}_t \sum_{j=1}^{T} M_{t,t+j} \left(T_{t+j} - G_{t+j}\right) + \mathbb{E}_t M_{t,t+T} B_{t+T}$$

- We obtained the standard IGBC if

$$\lim_{T\to\infty} \mathbb{E}_t M_{t,t+T} B_{t+T} = 0.$$

- The IGBC implies that a higher debt-to-output ratio today can be attributed to higher ex- pected future primary surpluses (cash flows) or lower expected future returns (discount rates).
- The counterpart of the Campbell-Shiller expression for the log of the price-to-dividend ratio in the stock market.

VALUATION APPROACH

- Cochrane (2011) shows that discount rate variation is the main driver of stock valuation ratios.
- Cochrane (2019): half of the variation in the debt-to-GDP ratio to variation in future primary surpluses and half to varying discount rates.
- Jiang et al. (2021) conclude no statistical evidence of a discount rate or cash flow channel.
- Fluctuation in the debt-to-GDP ratio at time t predict fluctuations in the debt-to-GDP ratio at time $t + T$.
- Jiang et al. argue the differences result from small sample bias.

FISCAL CAPACITY

- Jiang et al. in a series of recent papers propose a new approach to debt sustainability analysis.
- Suppose an investor buys the entire stock of government debt and participates in all new issuances.
- How much would that investor be willing to pay for the debt?
- Cash flow is $\{T_t - G_t\}$.
- Use tools from asset pricing to answer this question.
- The price will depend on the riskiness of the cash flows.

ASSET PRICING BASICS

- Before we talk about Jiang et al., let's review some asset pricing basics.
- The general idea dates back to Lucas (1978) who considers asset prices in a general equilibrium model.
- An asset is a claim on a stream of prospective payments.
- Consider an economy with $i = 1, \ldots, N$ assets.
- Each of this assets has an associated stream of real dividends $\left\{d_{i,t}\right\}_{t=0}^{\infty}$.
- Assume the representative investor maximizes $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$.
- Let $p_{i,t}$ be the price of asset i at time t (in goods).

ASSET PRICING BASICS

- For each asset the investor holds, the optimality condition is:

$$p_{i,t} = \beta \, \mathbb{E}_t \frac{u'(c_{t+1})}{u'(c_t)} \left(p_{i,t+1} + d_{i,t+1} \right) .$$

- This is the consumption-based asset pricing equation.
- We can slightly rearrange it as

$$1 = \beta \, \mathbb{E}_t \frac{u'(c_{t+1})}{u'(c_t)} \frac{p_{i,t+1} + d_{i,t+1}}{p_{i,t}} .$$

- Here $R_{i,t,t+1} \coloneqq \frac{p_{i,t+1}+d_{i,t+1}}{p_{i,t}}$ is the (gross) rate of return on asset i from t to $t+1$.

ASSET PRICING BASICS

- Define $M_{t,t+1} \coloneqq \beta \frac{u'(c_{t+1})}{u'(c_t)}$.
- We call it the stochastic discount factor (SDF).
- We can write the asset pricing equation for each asset as

$$1 = \mathbb{E}_t \, M_{t,t+1} R_{i,t,t+1}.$$

- The risk-free rate $R^f_{t,t+1}$ satisfies

$$1 = R^f_{t,t+1} \, \mathbb{E}_t \, M_{t,t+1}.$$

ASSET PRICING BASICS

- Sometimes you will see the asset pricing equation written as

$$v_{i,t} = \mathbb{E}_t \, M_{t,t+1} \frac{d_{i,t+1}}{d_t} \left(1 + v_{i,t+1}\right) .$$

- Here $v_{i,t} := \frac{p_{i,t}}{d_t}$ is the price-dividend ratio of asset i at time t.
- This form is useful when we want to think of an asset that has an ever increasing stream of dividends.

ASSET PRICING BASICS

- Generally $\mathbb{E}_t M_{t,t+1} R_{i,t,t+1} \neq \mathbb{E}_t M_{t,t+1} \cdot \mathbb{E}_t R_{i,t,t+1}$.
- We have

$$\mathbb{E}_t M_{t,t+1} R_{i,t,t+1} = \mathbb{E}_t M_{t,t+1} \cdot \mathbb{E}_t R_{i,t,t+1} + \text{cov}_t \left(M_{t,t+1}, R_{i,t,t+1} \right).$$

- This allows us to write the asset pricing equation as

$$1 = \mathbb{E}_t M_{t,t+1} \cdot \mathbb{E}_t R_{i,t,t+1} + \text{cov}_t \left(M_{t,t+1}, R_{i,t,t+1} \right).$$

- Use $1 = R^f_{t,t+1} \mathbb{E}_t M_{t,t+1}$ to write

$$\mathbb{E}_t R_{i,t,t+1} = R^f_{t,t+1} - \frac{\text{cov}_t \left(M_{t,t+1}, R_{i,t,t+1} \right)}{\mathbb{E}_t M_{t,t+1}}.$$

ASSET PRICING BASICS

- The formula

$$\mathbb{E}_t\, R_{i,t,t+1} = R^f_{t,t+1} - \frac{\mathrm{cov}_t\left(M_{t,t+1}, R_{i,t,t+1}\right)}{\mathbb{E}_t\, M_{t,t+1}}.$$

  tells us that the expected return on asset i is the risk-free rate plus a risk premium.
- The risk premium depends on the covariance between the SDF and the return on asset i.
- When the covariance is negative (SDF is low when the return is high), the risk premium is positive.
- When the covariance is positive (SDF is high when the return is high), the risk premium is negative.

ASSET PRICING BASICS

- To understand it better consider a basic example: let $u(c) = \ln c$. We have

$$M_{t,t+1} = \beta \frac{c_t}{c_{t+1}}.$$

- The SDF is high when c_{t+1} is low.
- If the asset has a low return when c_{t+1} is low, the covariance is negative and the risk premium is positive.
- This is because the asset is risky – it does not pay much when you need it the most.

- We sometimes write the formula as

$$\mathbb{E}_t\, R_{i,t,t+1} = R^f_{t,t+1} - \frac{\operatorname{cov}_t\left(M_{t,t+1}, R_{i,t,t+1}\right)}{\operatorname{var}_t M_{t,t+1}} \times \frac{\operatorname{var}_t M_{t,t+1}}{\mathbb{E}_t\, M_{t,t+1}}.$$

- There are two terms:
  1. The first term is the risk exposure – it is the covariance between the SDF and the return on asset *i* divided by the variance of the SDF.
  2. The second term is the price of risk – it is the variance of the SDF divided by the expected value of the SDF. It does not depend on the asset.

ASSET PRICING BASICS

- We have

$$\mathbb{E}_t\, R_{i,t,t+1} = R^f_{t,t+1} + \beta_{i,t}\lambda_t.$$

- β^i_t is the risk exposure of asset i, $\beta_{i,t} \coloneqq -\frac{\operatorname{cov}_t(M_{t,t+1}, R_{i,t,t+1})}{\operatorname{var}_t M_{t,t+1}}$
- Note: do not confuse $\beta_{i,t}$ with β, the discount factor.
- λ_t is the price of risk, $\lambda_t \coloneqq \frac{\operatorname{var}_t M_{t,t+1}}{\mathbb{E}_t\, M_{t,t+1}}$.
- Risk premium is the product of the risk exposure and the price of risk.

ASSET PRICING BASICS

- So far we assumed that the SDF results from the optimization problem of the representative agent.
- (Some) SDF exists under much weaker conditions: it is enough that there is no arbitrage.
- Once we have a SDF, we can use it to price assets.
- This is the approach of Jiang et al. (2021).

FISCAL CAPACITY

- Return to the formulation of the valuation problem:

$$B_t = \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} \left(T_{t+j} - G_{t+j} \right)$$

- Here we understand everything as nominal and $M_{t,t+j}$ is the SDF used to price nominal claims.
- Jiang et al. call the right hand side the fiscal capacity of the government.

FISCAL CAPACITY

- To simplify the notation, define $S_{t+j} \coloneqq T_{t+j} - G_{t+j}$, the primary surplus at time $t+j$.
- Rewrite the formula as

$$
\begin{aligned}
B_t &= E_t \sum_{j=1}^{\infty} M_{t,t+j} S_{t+j} \\
&= \sum_{j=1}^{\infty} \left(E_t M_{t,t+j} \cdot E_t S_{t+j} \right) + \sum_{j=1}^{\infty} \mathrm{cov}_t \left(M_{t,t+j}, S_{t+j} \right) \\
&= \sum_{j=1}^{\infty} \left(E_t M_{t,t+j} \cdot E_t S_{t+j} \right) \\
&\quad + \sum_{j=1}^{\infty} \mathrm{cov}_t \left(M_{t,t+j}, T_{t+j} \right) - \sum_{j=1}^{\infty} \mathrm{cov}_t \left(M_{t,t+j}, G_{t+j} \right).
\end{aligned}
$$

FISCAL CAPACITY

- Fiscal capacity depends on three terms:
- $\sum_{j=1}^{\infty} \left(E_t M_{t,t+j} \cdot E_t S_{t+j} \right)$ – the expected value of future primary surpluses discounted by the risk-free rate.
- $\sum_{j=1}^{\infty} \mathrm{cov}_t \left(M_{t,t+j}, T_{t+j} \right)$ – the covariance between the SDF and future taxes.
- $\sum_{j=1}^{\infty} \mathrm{cov}_t \left(M_{t,t+j}, G_{t+j} \right)$ – the covariance between the SDF and future government spending.

FISCAL CAPACITY

- In the risk free world, the first term is the only one that matters.
- In the risk free world, fiscal capacity is determined only by the ability to generate current and future surpluses.
- The second and the third term reflect the riskiness of the surplus process.
- If taxes are high when the SDF is low, the second term lowers the fiscal capacity.
- If government spending is high when the SDF is low, the third term lowers the fiscal capacity.
- Tax revenue is usually procyclical, government spending is usually countercyclical – this lowers the fiscal capacity.

FISCAL CAPACITY

- This suggests that the fiscal capacity is most likely lower than the expected value of future primary surpluses discounted by the risk-free rate.
- By how much?
- Jiang et al. (2021) quantify this for the US. They find that the second and the third term matter quantitatively.
- Is there a way to increase the fiscal capacity by financial engineering?
- This would require insuring bondholders against the risk of future taxes and spending. Is it feasible?
- We now follow Jiang et al. (2023) to illustrate it.

FISCAL CAPACITY

- For simplicity assume that taxes to GDP τ and government spending to GDP γ are constant.
- GDP growth is risky, i.i.d. with a mean of x and volatility of σ.
- Let P_t^T and P_t^G denote the present value of future tax revenues and government spending:

$$P_t^T = \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} T_{t+j} = \tau \, \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} Y_{t+j}$$

$$P_t^G = \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} G_{t+j} = \gamma \, \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} Y_{t+j} .$$

FISCAL CAPACITY

- Given the simplifying assumptions, debt to GDP ratio is:

$$\frac{B}{Y} = \frac{\tau - \gamma}{r^f + \text{risk premium on GDP} - x}.$$

FISCAL CAPACITY

- Consider the following parametrization:
  - Taxes to GDP, τ are 25%.
  - Government spending to GDP, γ is 22.5%.
  - Risk-free rate r^f is 1.5%.
  - Mean GDP growth rate x is 2%.
  - GDP risk premium is 3%.
  - Initial GDP is 10 trillion.
- Risk premium on GDP is GDP volatility times the price of risk (3 times 1)
- Stock market acts as a levered claim to the aggregate: the GDP risk premium equals the unlevered equity risk premium.

FISCAL CAPACITY

- The value of the claim to GDP is $10 \cdot \frac{1}{0.015+0.03-0.02} = 400$ trillion.
- The claim on the stream of surpluses is worth $10 \cdot \frac{0.025-0.0225}{0.015+0.03-0.02} = 10$ trillion.
- Fiscal capacity of this economy is 10 trillion.
- It equals 100% of GDP.
- If we evaluated it using the risk-free rate (net of growth), we would get infinity.

FISCAL CAPACITY

- The claim on surpluses is the claim on taxes net of the claim on government spending.
- The government cost of funding r_B can be written as

$$r_B = r_T \frac{P^T}{Y}\frac{Y}{B} - r_G \frac{P^T}{Y}\frac{Y}{B}.$$

- The risk exposure β, the covariance of a return with the SDF divided by the variance of the SDF is

$$\beta_B = \beta_T \frac{P^T}{Y}\frac{Y}{B} - \beta_G \frac{P^G}{Y}\frac{Y}{B}.$$

FISCAL CAPACITY

- The formula

$$\beta_B = \beta_T \frac{P^T}{Y}\frac{Y}{B} - \beta_G \frac{P^G}{Y}\frac{Y}{B}$$

  show that holding the risk exposure of gov. spending constant, if the government insures taxpayers (higher β_T) – lower tax payment in high marginal utility state – there is less insurance of bondholders (higher β_B).

FISCAL CAPACITY

- In this example tax revenue and government purchases are proprtional to GDP.
- The risk exposure of tax revenue is $\beta_T = \beta_{GDP}$.
- The risk exposure of government spending is $\beta_G = \beta_{GDP}$.
- Normalize $\beta_{GDP} = 1$.
- We have

$$\begin{aligned} \beta_B &= \beta_T \frac{P^T}{Y} \frac{Y}{B} - \beta_G \frac{P^G}{Y} \frac{Y}{B} \\ &= \frac{100}{10} - \frac{90}{10} = 1 \end{aligned}$$

- Tax and spending claims are equally risky, but government debt has a positive beta of 1.

FISCAL CAPACITY

- Investors who buy the government debt portfolio are net long a claim to output.
- The output risk in spending does not fully offset the output risk in tax revenue.
- Debt is a constant fraction of GDP, it inherits the risk properties of the GDP claim.
- The government's interest payments are as risky as GDP, because they are a constant fraction of GDP.

FISCAL CAPACITY

- Usually $\beta_T > \beta_Y > \beta_G$.
- This is because tax revenue is more volatile than GDP, and GDP is more volatile than government spending.
- This means that the average tax revenue to output has to be higher to support the same amount of debt.
- See Jiang et al. (2020) for a quantitative analysis.

FISCAL CAPACITY

- Using the risk-free rate to evaluate the fiscal capacity of the government is misleading.
- It requires that the risk exposure of the government debt, β_B, is zero.
- For that to be true we need

$$\begin{aligned}\beta_T &= \left(\frac{P^T}{Y}\frac{Y}{B}\right)^{-1}\left(\frac{P^G}{Y}\frac{Y}{B}\right)\beta_G \\ &= \frac{P^G}{P^G + B}\beta_G\end{aligned}$$

which is lower than β_G if debt is positive.

FISCAL CAPACITY

- Go back to our example: $\beta_G = 1$, $P^G/Y = 90$, $B/Y = 10$.
- We need $\beta_T = 0.9$ to insure bondholders.
- We had $r^f = 1.5\%$ and risk premium on GDP of 3%.
- This meant that $r_Y = r^f + RP = 4.5\%$.
- We have $r_T = 1.5\% + 0.9 \cdot 3\% = 4.2\%$.
- The lower risk premium for the tax process reflects the fact that the tax rate is counter-cyclical.

FISCAL CAPACITY

- What is the average tax revenue to output needed to sustain the debt?
- Recall that $\frac{B}{Y}$ = 100%, $\frac{P^G}{Y}$ = 9.
- We will use the formula

$$\frac{B}{Y} = \frac{T}{Y}\frac{P^T}{T} - \frac{P^G}{Y}$$

- We now have

$$\frac{P^T}{T} = \frac{1}{r_T - x} = \frac{1}{0.042 - 0.2} = 45.45.$$

  so $\frac{T}{Y}$ = 10/45.45 = 0.22.
- The average tax revenue to output ratio is 22%.

FISCAL CAPACITY

- The previous example shows that the government can on average run a deficit of 0.5% of GDP.
- This is because the government provides insurance to bondholders by delivering positive surpluses when GDP growth is lower than average.
- Bondholders pay an insurance premium of 0.5% of GDP to receive relatively larger surplus payments when their marginal utility is high.
- But providing insurance is costly – it requires the government to have surpluses in recessions. Less room for output stabilization.

CONVENIENCE YIELDS

- Sometimes government debt is more valuable than the sum of its discounted cash flows.
- This is because it provides liquidity and safety to investors.
- Similar to cash: we hold it although it has a negative real return, because we need it for transations.
- We call the difference between the return on debt and the risk free rate the convenience yield of government debt.
- Think of it as of some extra benefit that makes investors willing to hold government debt despite low returns.

CONVENIENCE YIELDS

- The convenience yield is nonnegligible, especially for the US.
- Krishnamurthy and Vissing-Jorgensen (2012) estimate convenience yield of 73 basis points per annum on average between 1926 and 2008 in the US.
- This is an important source of seignorage for the US government (0.25% of GDP).
- The convenience yield depends on debt to GDP ratio.

CONVENIENCE YIELDS

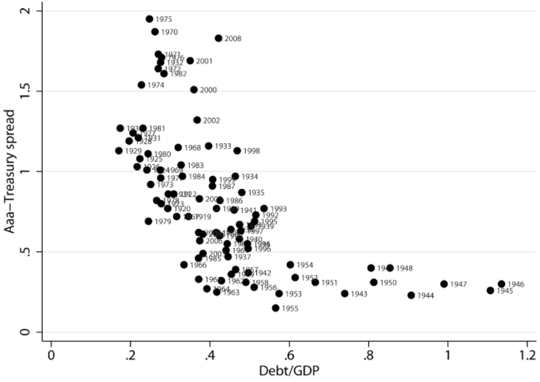


FIG. 1.—Corporate bond spread and government debt. The figure plots the Aaa-Treasury corporate bond spread (*y* axis) against the debt-to-GDP ratio (*x* axis) on the basis of annual observations from 1919 to 2008. The corporate bond spread is the difference between the percentage yield on Moody's Aaa long-maturity bond index and the percentage yield on long-maturity Treasury bonds.

Source: Krishnamurthy and Vissing-Jorgensen (2012)

FISCAL CAPACITY

- We need to modify the IGBC to account for the convenience yield.

$$B_t = \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} \left(T_{t+j} - G_{t+j}\right) + \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} B_{t+j} \left(1 - e^{-\delta_{t+j}}\right)$$

- The new term $K_{t+j} := B_{t+j} \left(1 - e^{-\delta_{t+j}}\right)$ represents the seignorage revenue from issuing debt.
-

FISCAL CAPACITY

- Return to the example with $\beta_T = \beta_G = \beta_Y = 1$, but consider convenience yields.
- Previously we said that the risk-free rate is equal to interest rate on treasuries, 1.5%.
- If the convenience yield is 0.73%, the true risk-free rate is higher – 2.23%.
- The PDV of surpluses is

$$\frac{0.25 - 0.225}{0.0523 - 0.02} = 77\% \text{ of GDP.}$$

- The PDV of seignorage will depend on β_K and the convenience yield.
- Set the convenience yield to 0.73%.

FISCAL CAPACITY

- If $\beta_K = 1$ (seignorage varies proportionally with GDP), then $r_K = 0.523$ and the PDV of seignorage is

$$\frac{0.0073}{0.0523 - 0.02} = 23\% \text{ of GDP.}$$

- This means that fiscal capacity is 100% of GDP.
- Two counteracting forces:
  1. Convenience yield generates seignorage.
  2. For a given interest rate, convenience yield means that the true risk-free rate is higher – this lowers the fiscal capacity.
- Extra surplus increases fiscal capacity by less than without the convenience yield.

FISCAL CAPACITY

- Most likely $\beta_K < 1$.
- In the short run, convenience yields can be counter-cyclical (flight to safety).
- This increases the seignorage term, without affecting the risk-free rate.
- Lowering β_K to 0.584, increases fiscal capacity to 114% of GDP.

FISCAL CAPACITY

- Jiang et al. use two approaches to estimate the fiscal capacity of the US.
- In the first approach they Congressional Budget Office (CBO) projections of tax revenue and non-interest spending for the next 31 years (2022-2052).
- CBO also forecasts interest rates and GDP.
- At the end of the projection horizon debt to GDP is 185%.

FISCAL CAPACITY

- Given r^f = 1.5%, the risk premium on GDP to 3% and the average growth rate 2%, annual surpluses would have to be 4.625% of GDP since 2052 to sustain debt to GDP of 185%.
- The present value of that is 35.2 trillion.
- The present value of the surpluses between 2022 and 2052 is -21.1 trillion (negative).
- The fiscal capacity in 2022 is 35.2-21.1 trillion = 14.1 trillion.
- This is 8.2 trillion below the 22.3 trillion in debt outstanding at the end of 2021.

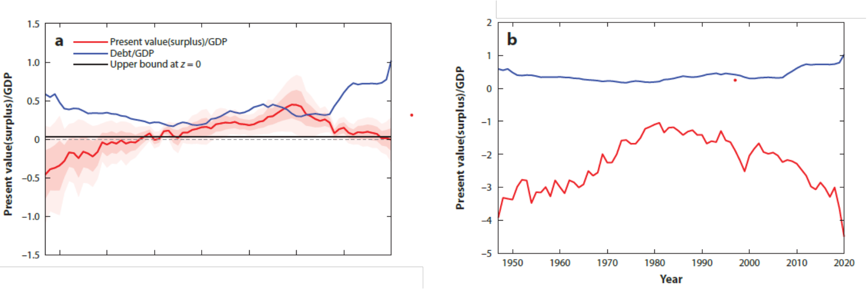
FISCAL CAPACITY

- This is already a generous estimate of the fiscal capacity.
- It assumes that the government can start running surpluses in 2052.
- It assumes acyclicality of taxes and spending.
- Another problem: duration mismatch.
- Surpluses are far in the future, fiscal capacity sensitive to small changes in the risk-free rate.

FISCAL CAPACITY

- The second approach: create forecast of cash flows using a VAR model.
- The model captures the cyclicality of tax and spending ratios.
- It captures multiple aggregate sources of risk: inflation, interest rates, the price-to-dividend ra- tio in the stock market, and shocks to tax and spending rates.
- They calculate total fiscal capacity in two ways: (a) assume the discount rates are the same for taxes and spending, (b) model the SDF.

FISCAL CAPACITY



Source: Jiang et al. (2023)

BOND VALUATION PUZZLE

- These estimates suggest a much lower fiscal capacity than the market value of outstanding debt.
- Possible explanations:
  - Convenience yields?
  - Bubble?
  - Global safe asset supplier?
  - Mispricing?
  - Fiscal correction?
  - Large-scale asset purchases and financial repression?

BOND VALUATION PUZZLE

- Similar calculations for other countries suggest that the US is an outlier.
- For example, for the UK after World War 2 fiscal capacity was 82% of GDP, but the debt to GDP ratio was 53%.
- It was different in 1729-1946 when fiscal capacity was 68% of GDP, but the debt to GDP ratio was 87%.
- Developing countries: procyclical surpluses, debt prices react strongly to funamentals (unlike the US).