OPTIMAL LEVEL OF DEBT

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PLAN

- We investigate what simple models can say about the optimal level of debt.
- Previously we either had results that say that the long run level of debt should be equal to the initial condition, or there should be time 0 default.

NEOCLASSICAL GROWTH MODEL

SETUP

- There is a representative household that maximizes utility from consumption.
- There is no risk – everything is deterministic.
- The household supplies 1 unit of labor inelastically.
- The household can trade assets that pay return $1 + r_{t+1}$ next period. These assets are capital and government debt.
- The household pays taxes / receives transfers T_t. These are lump sum.
- There is a borrowing constraint: $a_{t+1} \geq -\bar{A}_t$, but $\bar{A}_t$ is very large.
- $\bar{A}_t$ is such that is equals the present discounted value of all future labor income.
- We call it the natural debt limit.

SETUP

- The government purchases goods G_t, finances them with lump sum taxes T_t and issues debt B_{t+1}.
- The law of motion for debt is

$$B_{t+1} = (1 + r_{t+1})B_t + G_t - T_t.$$

- The representative firm uses production function $F(K_t, L_t)$. It is increasing, concave and homogeneous of degree 1 in inputs.
- Capital depreciates at rate δ.
- There is a competitive labor market with the wage rate w_t.
- There is a competitive capital market with the rental rate $r_t + \delta$.

SETUP

- The household utility maximization problem is

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t. } c_t + a_{t+1} = (1 + r_t)\, a_t + w_t \cdot 1 - T_t$$

$$a_{t+1} \geq -\bar{\mathcal{A}}_t$$

and a_0 given.

SETUP

- The firm maximization problem is

$$\max_{L_t, K_t} \quad F(K_t, L_t) - w_t L_t - (r_t + \delta) K_t \quad \text{for all } t \geq 0.$$

SETUP

- The government is fully characterized by sequences $\{G_t, T_t, B_{t+1}\}_{t=0}^{\infty}$ that satisfy

$$B_{t+1} = (1 + r_{t+1})B_t + G_t - T_t \quad \text{for all } t \geq 0.$$

given B_0.

SETUP

- All markets must clear for all $t \geq 0$:

$$L_t = 1,$$

$$a_t = K_t + B_t,$$

$$c_t + G_t + K_{t+1} = F(K_t, L_t) + (1 - \delta) K_t.$$

COMPETITIVE EQUILIBRIUM

- A **competitive equilibrium** given government policies $\{G_t, T_t, B_{t+1}\}_{t=0}^{\infty}$ is sequences of prices $(w_t, r_t)_{t=0}^{\infty}$ (that satisfy the government budget constraint) and allocations $\{c_t, K_t, L_t, a_{t+1}\}_{t=0}^{\infty}$ such that
    1. Given $\{w_t, r_t\}_{t=0}^{\infty}$ and $\{T_t\}_{t=0}^{\infty}$, $\{c_t, a_{t+1}\}_{t=0}^{\infty}$ solves the household problem.
    2. Given $\{w_t, r_t\}_{t=0}^{\infty}$, $\{K_t, L_t\}_{t=0}^{\infty}$ solves the firm problem.
    3. Markets clear.
- Implicit in this definition is that the government budget constraint is satisfied.

EQUILIBRIUM CHARACTERIZATION

- First order conditions for the household problem give us the Euler equation:

$$u'(c_t) = \beta\,(1 + r_{t+1})\,u'(c_{t+1})$$

  Note: the constraint is not binding.

- First order conditions for the firm problem give us:

$$w_t = F_L(K_t, 1)$$

$$r_t = F_K(K_t, 1) - \delta$$

- Homogeneity of degree 1 implies that $F(K_t, 1) = (r_t + \delta)\,K_t + w_t \cdot 1$.

EQUILIBRIUM CHARACTERIZATION

- Use the asset market clearing condition and the budget constraint of the government in the budget constraint of the household:

$$c_t + (K_{t+1} + B_{t+1}) = (1 + r_t)(K_t + B_t) + w_t \cdot 1 - (G_t + (1 + r_t) B_t - B_{t+1}).$$

  and reorganize to get

$$c_t + K_{t+1} = (1 + r_t) K_t + w_t \cdot 1 - G_t.$$

- Finally, use the FOCs of the firm problem to get

$$c_t + K_{t+1} = F(K_t, 1) + (1 - \delta) K_t - G_t.$$

- This is the resource constraint.

EQUILIBRIUM CHARACTERIZATION

- Equilibrium is characterized by the Euler equation and the resource constraint:

$$u'(c_t) = \beta\,(1 + F_K(K_{t+1}, 1) - \delta)\,u'(c_{t+1})$$

$$c_t + K_{t+1} = F(K_t, 1) + (1 - \delta)\,K_t - G_t \text{ for all } t \geq 0.$$

- In other words, if we know $K_0 = a_0 - B_0$ and $\{G_t\}_{t=0}^{\infty}$, we can solve for $\{c_t, K_{t+1}\}_{t=0}^{\infty}$.
- If we know $\{c_t, K_{t+1}\}_{t=0}^{\infty}$, we can solve for $\{w_t, r_t\}_{t=0}^{\infty}$.

RICARDIAN EQUIVALENCE

- The quantity of government debt does not matter!
- The only thing related to the government that matters is the sequence of government purchases $\{G_t\}_{t=0}^{\infty}$.
- It does not matter how the government finances it: whether by taxes or debt.
- This is called Ricardian equivalence (see Barro (1974)).
- Note: you might think B_0 matters – but it is only because it is part of the initial condition $K_0 = a_0 - B_0$. If we pinned down K_0 directly, B_0 would not matter.

RICARDIAN EQUIVALENCE

- We have a very stark answer to the question of how much debt should the government issue: does not matter.
- This is because taxes were lump-sum: they did not distort the household's decision.
- The household does not care whether taxes are high or low now – it only cares about the present discounted value of taxes, determined by the sequence of government purchases.
- If the government issues debt, it will have to raise taxes in the future to pay it off.

STEADY STATE

- We will now focus on the steady state of this economy.
- We set $G_t = G$, $c_t = c$, $K_t = K$ and $r_t = r$ for all $t \geq 0$.

$$1 = \beta \left(1 + F_K(K, 1) - \delta\right)$$

$$c + \delta K = F(K, 1) - G.$$

- The first equation pins down K.
- Given K and G, the second equation pins down c.

STEADY STATE

- Government policy does not affect the steady state level of capital.
- No crowding out in the steady state of this economy!
- Government policy (specifically, government purchases) does affect the steady state level of consumption.

STEADY STATE

- It sometimes helps to think of the Euler equation as a demand for assets.
- What is the elasticity of demand for assets with respect to the interest rate?
- In the long run (steady state) it is infinity!
- The household is willing to absorb any amount of assets at the interest rate $\beta^{-1} - 1$.
- If the rate is lower, it wants to hold zero assets, if it is higher, it wants to hold infinite assets.

ENDOGENOUS LABOR SUPPLY

- What if the household can choose how much to work, but it causes some disutility?
- The steady state Euler equation is now

$$1 = \beta \left(1 + F_K(K, L) - \delta\right).$$

- If the production function is homogeneous of degree 1 (constant returns to scale) $F_K(K, L) = F(K/L, 1)$.
- In this case the Euler equation pins down the ratio of capital to labor. This determines the wage rate and the rate of return on capital.
- The level of capital and labor are determined by the labor supply decision. It can only work through wealth effect.
- Crowding out (or in!) is possible.

AIYAGARI AND MCGRATTAN (1998)

FROM THEIR INTRODUCTION...

- Aiyagari and McGrattan (1998) model has a different role for government debt.
- Government debt enhances the liquidity of households by providing an additional means of smoothing consumption (in addition to claims to capital) and by effectively loosening borrowing constraints
- When the interest rate is raised, government debt makes assets both less costly to hold and more effective in smoothing consumption.
- The implied taxes have adverse wealth distribution and incentive effects.
- Government debt crowds out capital via higher interest rates, and it lowers per capita consumption.

SETUP

- We modify the household side of the model (Bewley-Imrohoroglu-Huggett-Aiyagari).
- There is now a continuum of households indexed by $i \in [0, 1]$.
- Agents receive idiosyncratic shocks to their labor income $e_{i,t}$. $e_{i,t}$ is i.i.d. across agents and follows a Markov process. $\mathbb{E}\left(e_{i,t}\right) = 1$.
- Assets are risk-free (not state-contingent!) – incomplete markets.
- There are borrowing constraints: $a_{i,t+1} \geq -\bar{\mathcal{A}}_t$. For simplicity set $\bar{\mathcal{A}}_t = 0$.
- We will focus on the stationary equilibrium of this economy.
- This is an equilibrium where all prices and the distribution of agents are constant over time.
- We allow for individual allocations of agent i to vary over time.

SETUP

- The household utility maximization problem is

$$\max_{\{c_{i,t},a_{i,t+1}\}_{t=0}^{\infty}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}\right)$$

$$\text{s.t. } c_{i,t} + a_{i,t+1} = (1 + r)\, a_{i,t} + we_{i,t} - T$$

$$a_{i,t+1} \geq 0$$

$$\text{and } a_{i,0}, e_{i,0} \text{ given.}$$

SETUP

- All markets must clear:

$$L = \int_0^1 e_i di,$$

$$\int_0^1 a_i di = K + B$$

$$\int_0^1 c_i di + G + K = F(K, L) + (1 - \delta) K.$$

ANALYSIS

- From the budget constraint of the government we get

$$T = rB + G.$$

- From the firm problem we get

$$K = \kappa(r),$$
$$w = \omega(r).$$

- The solution to the household problem gives a decision rule for asset accumulation:

$$a_{i,t+1} = \alpha\left(a_{i,t}, e_{i,t}; r, B, G\right).$$

ANALYSIS

- The policy function $\alpha(a, e; r, B, G)$ together with the Markov process for e allows to calculate the stationary distribution $\mu(a, e; r, B, G)$.
- Intuitively: find the joint distribution of assets and shocks that reporoduces itself.
- The aggregate demand for assets is

$$\bar{\alpha}(r, B, G) = \int \int \alpha(a, e; r, B, G)\mu(a, e; r, B, G)dade.$$

ANALYSIS

- To find the equilibrium interest rate we need to solve

$$\bar{\alpha}(r, B, G) = \kappa(r) + B.$$

- Then we can back out all other prices and allocations.
- In principle this is not different from what we saw earlier. We had $\bar{\alpha}(r, B, G)$ that was special.
- The point is that idiosyncratic income risk together with borrowing constraints affects the shape of $\bar{\alpha}(r, B, G)$.

DEMAND FOR ASSETS

- Look at the Euler equation (notice the constraint can be binding)

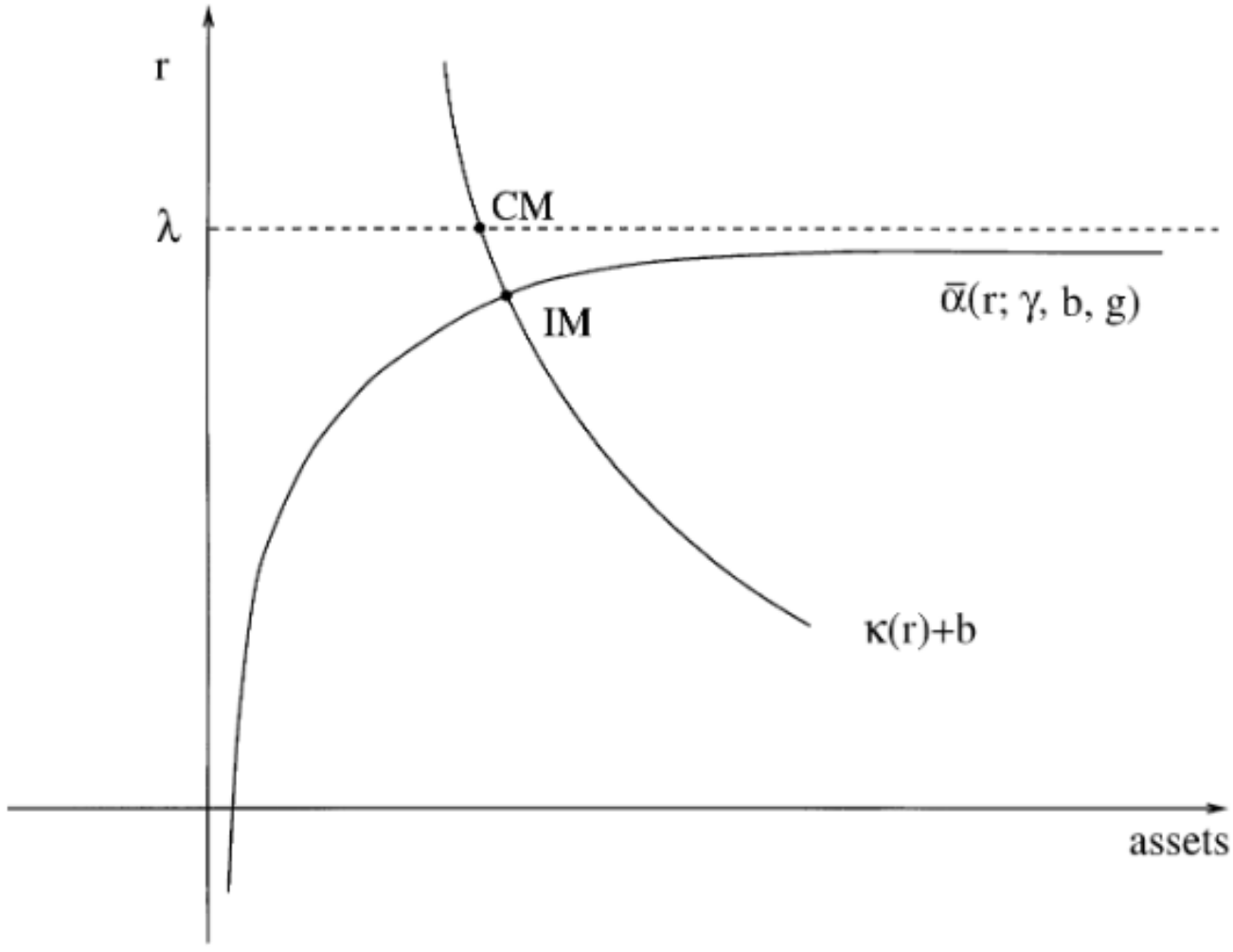
$$u'\left(c_{i,t}\right) \geq \beta\left(1+r\right)\mathbb{E}_t\, u'\left(c_{i,t+1}\right).$$

- Suppose that $r = \beta^{-1} - 1$ as we had in NGM (or we would have in a model without risk).
- The agent would like to have a smooth profile of marginal utility of consumption. Utility is concave: lower consumption hurts more than higher consumption helps.
- It is possible that there is a very long sequence of bad income shocks.
- The only way to insure against this is to accumulate assets.

DEMAND FOR ASSETS

- How much to accumulate? Infinity! So we know that $r = \beta^{-1} - 1$ is not an equilibrium. It must be lower!
- We have precautionary savings. For any r the demand for assets is larger than in the NGM / complete markets.

EQUILIBRIUM



Source: Aiyagari and McGrattan (1998)

EQUILIBRIUM

- Aiyagari and McGrattan (1998) argue that savings are too high in this economy (relative to the complete markets benchmark).
- Hence the policy recommendation is to reduce savings.
- This can be done by reducing the need for precautionary savings.
- How can debt help with that?

DEBT LEVEL

- Aiyagari and McGrattan (1998) approach: define $a^*_{i,t} := a_{i,t} - B$.
- Rewrite the budget constraint of the household as

$$c_{i,t} + a^*_{i,t+1} + B = (1 + r)\left(a^*_{i,t} + B\right) + we_{i,t} - T$$

  and use $T = rB + G$ to get

$$c_{i,t} + a^*_{i,t+1} = (1 + r)\, a^*_{i,t} + we_{i,t} - G.$$

- The borrowing constraint $a^*_{i,t+1} \geq 0$ is now

$$a_{i,t+1} \geq -B.$$

DEBT LEVEL

- In this formulation government debt B only enters the con- sumer's borrowing constraint.
- Higher levels of B in effect loosen the borrowing constraint and reduce the average asset holdings (net of government debt).
- Intuition: no need to save as much when the borrowing constraint is looser.
- The solution $a^*_{i,t+1} = \alpha^* \left(a^*_{i,t}, e_{i,t}; r, B, G \right)$ is decreasing in B.
- The amount saved in capital is decreasing in B – crowding out.

WELFARE

- How does it all affect welfare?

$$\Omega = \int \int V(a,e)\mu(a,e)dade$$

- An increase in debt increases the return on assets, thus making them less costly for the consumer to hold. Assets are cheaper in enabling the consumer to smooth consumption.
- Lump-sum taxes levied to pay interest on government debt:
  1. more onerous for individuals with low assets and low earnings than for individuals with high assets and high earnings;
  2. exacerbate the percentage variability in after-tax earnings.
- Crowding out of capital and the consequent reduction in per capita consumption.

QUANTITATIVE RESULTS

- Aiyagari and McGrattan (1998) extend and calibrate their model to the US economy.
- They allow for elastic labor supply, distortionary income taxes and a more general borrowing constraint.

$$u(c, l) = \frac{\left(c^{\eta} l^{1-\eta}\right)^{1-\mu}}{1-\mu}$$

$$\text{Profit}_t = (1-\tau_t)\left[Z_t F(K_t, L_t) - \delta K_t\right] - w_t L_t - r_t K_t$$

$$B_{t+1} = (1 + r_{t+1})B_t + G_t + \text{Transfer}_t - \tau_t\left[Z_t F(K_t, L_t) - \delta K_t\right].$$

where $l \in (0, 1)$ is leisure and Z_t is technology growing at a constant rate.

QUANTITATIVE RESULTS

- Technology growth rate 1.85% per year.
- G/Y is 21.7%.
- Transfers to GDP ratio is 8.2%.
- Labor share 70%.
- Capital to GDP ratio is 2.5.
- Earnings process from Aiyagari (1994).
- In the baseline set debt to GDP to 2/3.
- Exercise: vary B/Y together with τ_y and see what happens.

QUANTITATIVE RESULTS

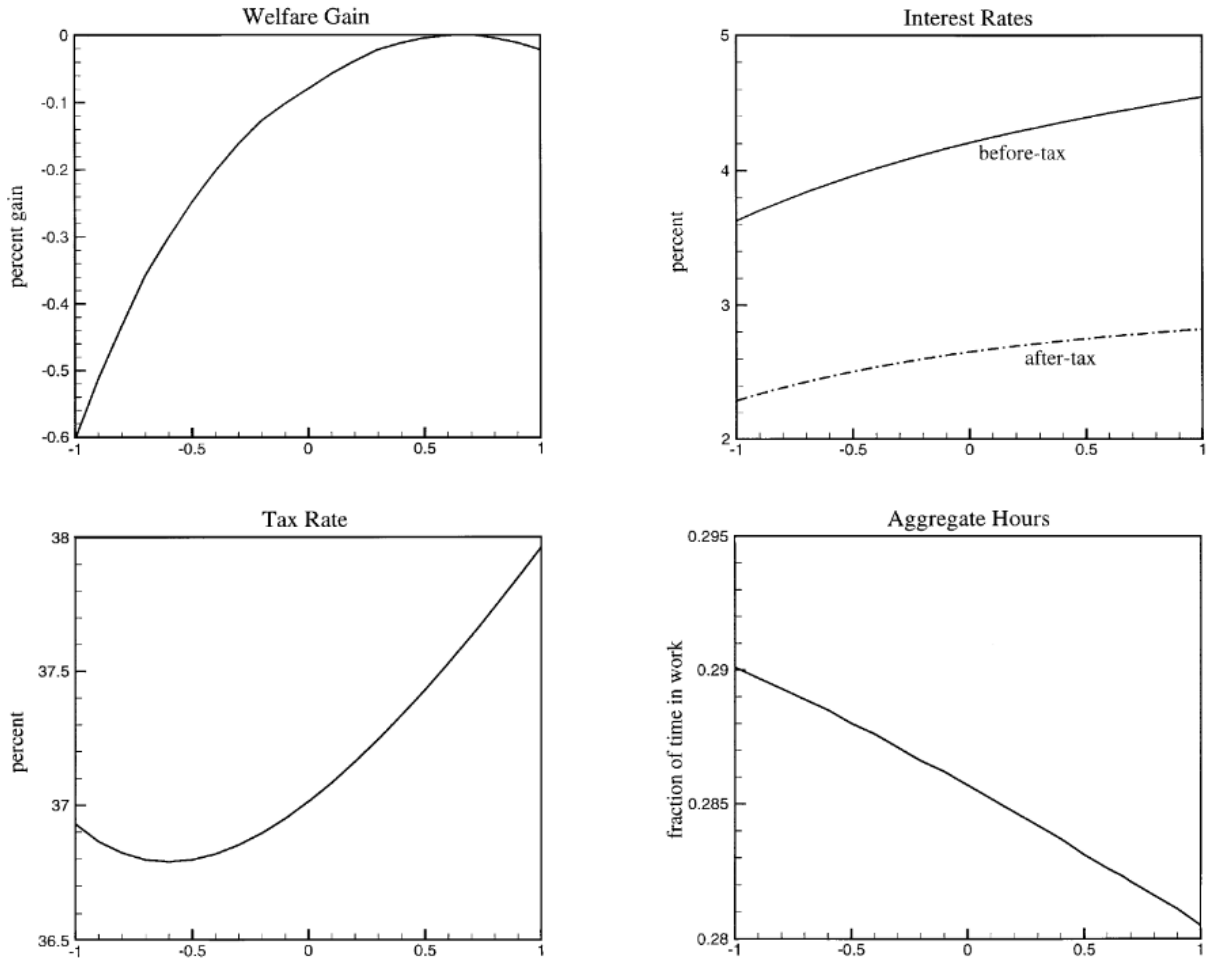


Fig. 2. Welfare gain, interest rates, tax rate, and aggregate hours versus debt/GDP ratio (x-axis) for the benchmark economy.

Source: Aiyagari and McGrattan (1998)

TAKING STOCK

- Is Aiyagari and McGrattan (1998) intuition about pushing the interest rate closer to the complete markets case really valid?
- What can we say about the optimal level of borrowing constraints?

BHANDARI, EVANS, GOLOSOV AND SARGENT (2017)

BEGS (2017)

If, indeed, the debt were distributed in exact proportion to the taxes to be paid so that every one should pay out in taxes as much as he received in interest, it would cease to be a burden. . . . if it were possible, there would be no need of incurring the debt. For if a man has money to loan the Government, he certainly has money to pay the Government what he owes it.
Simon Newcomb (1865, p.85)

- Seek features of government debt that affect continuation allocations and prices.
- Economy with incomplete markets and distortionary taxes.
- Beautiful paper: easy to read, full of subtle insights.

BEGS (2017)

- How does the level of the initial government debt B_{-1} affect welfare in an optimal equilibrium?
- What determines properties of an optimal path of government debt B.
- Answer: the role of government debt depends on how well private debt contracts are enforced.

BEGS (2017)

- Bhandari et al. (2017) study equilibria in economies that differ only in the borrowing constraint.
- Natural debt limit versus ad-hoc borrowing constraint.
- Interpretation: ad hoc borrowing constraints arise when the government is unable to enforce private debt contracts.
- They allow tax schemes to be $\mathcal{T}(y_t) = -T_t + \tau_t y_t$, here y_t is individual income that depens on labor supply and idiosyncratic productivity.
- Key here: natural debt limits are endogenous – they depend on the tax scheme.

BEGS (2017) - NATURAL DEBT LIMITS

- Result I: if there is a competitive equilibrium with natural debt limits, it is possible to find a different path of government debt, T_t and private asset holdings that supports the same consumption, labor supply and so on as a competitive equilibrium.
- Idea: just give everyone the same extra amount of debt and change T_t accordingly.
- Implication: the aggregate debt is irrelevant, what matters is who holds it.
- Result II: Suppose there is an optimal competitive equilibrium given initial aggregate and individual debt holdings. Then any path of aggregate debt supports it.

BEGS (2017) - NATURAL DEBT LIMITS

- The presence or absence of distorting taxes or incomplete markets is by itself insufficient to imply anything about the level public debt or its welfare costs.
- Here both τ_t and T_t are distorting, but the path of debt in the optimal allocation is indeterminate.
- Bad ideas:
  - comparing debt burdens across countries based purely on aggregate quantities like debt to GDP ratios;
  - lumping together explicit debt and implicit into one aggregate number without adjusting for heterogeneity across holdings of the various types of debts.

BEGS (2017) - AD HOC BORROWING CONSTRAINTS

- Result I: if there is a competitive equilibrium with some consumption and labor allocations with natural borrowing limits, it is possible to find tax policies that support these allocations with any ad hoc borrowing constraints.
- Result II: same as above but between two different ad hoc borrowing constraints.
- Result III: Welfare in an optimum equilibrium with ad hoc debt limits is higher than welfare in the optimum equilibrium with natural debt limits. This is true for any debt limit.
- Result IV: The optimal continuation level of government debt is determined, the initial level of government debt is irrelevant for welfare.

BEGS (2017) - AD HOC BORROWING CONSTRAINTS

- Intuition: *While the lower bound on debt is endogenous and depends on the government tax-transfer policy Section 3 discussion of natural debt limits, it is exogenous with the ad hoc debt limits of this section. The presence of a policy invariant debt limit here implies that changing the timing of transfers can change the set of agents who are up against their borrowing limits. This power lets the government increase welfare.*

BEGS (2017) - AD HOC BORROWING CONSTRAINTS

- Intuition: If the government issues debt in equilibrium, it is generally better off if interest payments on that debt are lower.
- In the economy with natural debt limits, equilibrium interest rates are determined implicitly by a competition between the government and private borrowers to supply assets ("liquidity") to savers.
- When private debt contracts are unenforceable, borrowers cannot issue riskless debt, so the government becomes a monopoly supplier of liquidity to agents. The government can use its monopoly power to extract additional surplus from savers by issuing debt at a lower interest rate.
-

TAKING STOCK

- Without exogenous restrictions on transfers, the level of government debt does not matter. What matters is how ownership of government debt is distributed
- An optimal path of government debt is determined when agent's abilities to borrow are restricted because that allows prospective public debt issues to affect interest rates.

DAVILA, HONG, KRUSELL, RIOS-RULL (2012)

TOO MUCH CAPITAL?

- Recall the picture from Aiyagari and McGrattan (1998).
- The interest rate was lower than with complete markets.
- The capital stock was higher than with complete markets.
- Implication: does it mean that there is too much capital in the economy?
- Seems consistent with the logic of Aiyagari and McGrattan (1998): crowd out capital and boost interest rate to reduce the opportunity cost of holding assets.
- What would the planner do?

TOO MUCH CAPITAL?

- Davila, Hong, Krusell, Rios-Rull (2012) is not about government debt.
- It studies a different, but related question.
- Incomplete markets economy with an ad hoc borrowing constraint.
- idiosyncratic productivity shocks.
- Fixed labor supply.
- Savings in capital only.
- Is there too much capital in the economy?

TOO MUCH CAPITAL?

- Davila, Hong, Krusell, Rios-Rull (2012) look at the constrained optimum.
- Constrained = the utilitarian planner cannot "complete markets".
- Allocations with zero net transfers between agents.
- Whether the is overaccumulation or underaccumulation of capital depends on income risk.

TWO FORCES

- Income risk together with incomplete markets is the source of problems in this economy.
- If wages are high, shocks to idiosyncratic productivity are amplified.
- Solution (?): lower real wages, increase return on assets to make income less sensitive to shocks (reduce capital accumulation)
- Downside: high returns on assets make wealth inequality more persistent.
- Whether there is too much or too little capital depends on factor composition of the income of the poor agents.

CONCLUSION

- The key factor behind whether the constrained optimum should have higher or lower capital than the laissez-faire equilibrium is the factor income of the consumption-poor – have a high weight for the planner.
- The comparison between their relative labor income and their relative asset income guides how they are affected.
- In conclusion, in economies where the unlucky consumers do relatively better in terms of their labor income, there is too little aggregate capital accumulation.