

# PRICE LEVEL DETERMINATION IN A MONETARY ECONOMY

---

FISCAL AND MONETARY POLICY 2023

Piotr Żoch

December 18, 2023

# PLAN

- We study how the price level (and inflation) is determined in a monetary economy.
- This will later serve us as a framework to study interactions of fiscal and monetary policy.
- We will start with a very simple environment and then gradually add more features.

# ENDOWMENT ECONOMY

---

## ENDOWMENT ECONOMY

- We start with a simple economy in which every period there is an endowment of nonstorable goods  $y_t$ .
- For simplicity assume  $y_t = y$  for all  $t$ .
- We denote the price level in period  $t$  by  $P_t$ .
- One unit of goods costs  $P_t$  units of account.
- There is a representative household that trades contingent claims (Arrow securities).
- The household maximizes  $\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t)$ .
- No government expenditures,

## NOMINAL INTEREST RATES

- Let  $q_{t,t+1}$  be the real price (in goods) of a claim that pays one unit of goods in period  $t + 1$  in particular state of the world.
- The nominal contingent claim price is

$$Q_{t,t+1} = q_{t,t+1} \frac{P_t}{P_{t+1}}.$$

- The **nominal interest rate**  $i_t$  satisfies

$$\frac{1}{1 + i_t} = \mathbb{E}_t Q_{t,t+1}.$$

## NOMINAL INTEREST RATES

- In this economy we have  $c_t = y$ .
- The SDF is constant and equal to  $\beta$ .
- The **real interest rate** is also constant  $r$ :

$$\frac{1}{1+r} = \beta.$$

- The nominal discount factor is  $Q_{t,t+1} = \beta \frac{P_t}{P_{t+1}}$ .
- Define  $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ .
- We obtain the **Fisher equation**

$$\frac{1}{1+i_t} = \frac{1}{1+r} E_t \frac{P_t}{P_{t+1}} = \frac{1}{1+r} E_t \frac{1}{\Pi_{t+1}} = \beta E_t \frac{1}{\Pi_{t+1}}.$$

## FISHER EQUATION

- The usual formulation of the Fisher equation is obtained by linearization:

$$i_t = r + \mathbb{E}_t \pi_{t+1}.$$

- We already used the constancy of the real interest rate.
- In general, the Fisher equation is

$$i_t = r_t + \mathbb{E}_t \pi_{t+1}.$$

- This is a no-arbitrage condition: you should be indifferent between two types of investment (real and nominal).

## FISHER EQUATION

- In this endowment economy

$$\frac{1}{1+i_t} = \beta E_t \frac{1}{\Pi_{t+1}}$$

is the **only** equilibrium condition.

- What does it imply for the sequence of price levels  $\{P_t\}_{t=0}^{\infty}$ ?
- In particular, we say that the level of inflation is unique or determinate in equilibrium if:
  1. There is a unique scalar  $P_0$  in equilibrium.
  2. If  $\Pi^{prime}$  and  $\Pi''$  are two sequences that satisfy equilibrium conditions, then  $\Pi' = \Pi''$ .



## DETERMINACY

- The question of determinacy may seem esoteric, but it is important.
- It boils down to the question of whether some policy can control prices.
- In this simple economy, the only available policy is the choice of the nominal interest rate.
- Suppose we have some target for the price level (or inflation rate) – can we achieve it by setting interest rates?

## DETERMINACY

- Consider the simplest possible policy: **interest rate peg**.
- The central bank sets  $i_t = i$  for all  $t$ .
- We have

$$\frac{1}{1+i} = \beta E_t \frac{1}{\Pi_{t+1}}.$$

- This determines  $E_t \frac{1}{\Pi_{t+1}}$  – approximately the **expected** inflation rate.
- But it does **not** determine  $\Pi_{t+1}$  – the **actual** inflation rate.

## DETERMINACY

- What if we have perfect foresight?
- The  $\Pi_{t+1}$  is determined by the peg, but  $P_0$  is not (because  $\Pi_0$  is not)
- Conclusion: interest rate peg does not ensure price level determinacy.

## DETERMINACY

- Consider a interest rate rule  $i_t = r + \phi \pi_t$ .
- We call these rules **Taylor rules**.
- Use the linearized Fisher equation  $i_t = r + \mathbb{E}_t \pi_{t+1}$  to obtain

$$\phi \pi_t = \mathbb{E}_t \pi_{t+1}.$$

- If  $\phi > 1$  the only solution that does not diverge is  $\pi_t = 0$  for all  $t$ .
- **Taylor principle**:  $\phi > 1$ , increase real rates when inflation increases.

## DETERMINACY

- To see it solve

$$\phi \pi_t = \mathbb{E}_t \pi_{t+1}.$$

forward as

$$\pi_t = \lim_{T \rightarrow \infty} \phi^{-T} \mathbb{E}_t \pi_{t+T}.$$

- If  $\phi > 1$  and  $E_t \pi_{t+T}$  does not diverge,  $\pi_t = 0$  for all  $t$ .
- Inflation is determinate in all periods, even at  $t = 0$  so  $P_0$  is determined.
- Where does the assumption  $E_t \pi_{t+T}$  does not diverge come from?

## DETERMINACY

- The terminal condition is **not** an optimality condition.
- In this model  $\phi > 1$  has nothing to do with the logic like "increase real rates to reduce aggregate demand"
- In a way, the central bank makes a threat: if you do not behave, I will make inflation explode. Is it reasonable? Is it how central banks do it?
- Moreover, nothing bad would happen in this economy if inflation diverges.
- See Cochrane (2011) for a discussion of Taylor rules.
- My advice: you might not like the model (it is obviously simple) – but do not apply some outside logic to understand these results!

## DETERMINACY

- The usual justification is that when inflation gets out of control, the central bank will switch to something else.
- But then, it is not a Taylor rule with  $\phi > 1$  that ensures determinacy!
- What is this other thing that would ensure determinacy?
- For example, the ECB has a monetary pillar, understood as a commitment to switch to a monetary approach to pin down inflation if inflation starts exploding.
- We will now turn to these monetary approaches.

## DETERMINACY

- Assume there is some money demand in this economy:  
 $m_t = p_t + y - \eta i_t$ , where  $\eta > 0$  is a semielasticity of money demand.
- Use the Fisher equation to obtain

$$m_t = p_t + y - \eta [r + \mathbb{E}_t \pi_{t+1}].$$

- Let the central bank follow some money growth rule  $m_t = m_{t-1} + \bar{m}$ .
- Is it enough to ensure price level determinacy?



## DETERMINACY

- We have a formula

$$(1 + \eta) (p_t - \bar{m} \cdot t) = \eta (\mathbb{E}_t p_{t+1} - \bar{m} \cdot t) - y + p_t + \eta r.$$

we can solve it forward and there will be a term like

$$\lim_{T \rightarrow \infty} \mathbb{E}_t p_{t+T} - \bar{m} \cdot (t + T - 1).$$

- This term will be equal to 0 by the household's optimality condition (transversality condition).
- **Intuition:** households do not want to hold too much or too little money.
- We get

$$p_t = \bar{m} \cdot t + \eta \bar{m} + \frac{1}{1 + \eta} \sum_{j=0}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^j (\eta r - y).$$

## DETERMINACY

- Money growth rule ensures price level determinacy (in this case).
- This is true even if there are shocks  $u_t$  to money demand.
- With different microfoundations for money demand, conclusions might be different.
- We can solve for the implied nominal interest rate:

$$i_t = \frac{1}{\eta} (p_t - m_t + y)$$

- Note it responds to **price level**. This is an equilibrium outcome.
- If we instead started with a rule that responds to price levels it would be a **Wicksellian rule**.