AGGREGATE SHOCKS, TAXES, AND DEBT MANAGEMENT II

Fiscal and Monetary Policy 2023

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PLAN

- We return to the questions posed last week and try to answer them in a different setting.
- We study the problem of a government that wants to maximize welfare of a representative household.
- The government can fully commit to its announced policies. This commitment is credible.

LUCAS AND STOKEY (1983)

SETUP

- We study the following problem based on Lucas and Stokey (1983):
  - the representative household has preferences over consumption and labor (leisure);
  - production technology is linear in labor, there is no capital;
  - the government uses distortionary tax rate τ_t on labor income to finance *stochastic* purchases g_t (we assume g_t is a Markov process);
  - the households and the government can trade a full set of Arrow securities (state-contingent claims).
  - the government wants to finance g_t in a way that maximizes the welfare of the representative household.
- General equilibrium.

SETUP

- Price of securities no longer exogenous.
- The government's cares about welfare of the representative household.
- Think of the problem in two steps:
  1. for given government policies there exists some competitive equilibrium;
  2. the government picks policies that result in the "best" equilibrium.
- Key difference: policies not only have to satisfy the government's budget constraint, but also household's optimality conditions.
- Primal approach: we will look directly for allocations that maximize welfare and then think of policies that implement that.

Competitive equilibrium

A competitive equilibrium given government policies $\tau\left(s^t\right)$ and $g\left(s^t\right)$ is a set of allocations $\left(c\left(s^t\right), \ell\left(s^t\right), a\left(s^{t+1}\right)\right)$ and prices $\left(q\left(s^{t+1} \mid s^t\right), w\left(s^t\right)\right)$ such that:

1. Given prices, allocations solve the household problem;
2. Given prices, allocations solve the firm problem;
3. The government budget constraint is satisfied is each state;
4. All markets clear in each state.

- Note: this is not a proper definition of a competitive equilibrium, but it will do for our purposes.

HOUSEHOLD PROBLEM

- Household chooses consumption, labor supply and portfolio of Arrow securities to maximize expected utility:
- Household problem:

$$
\begin{aligned}
& & \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c\left(s^t\right), \ell\left(s^t\right)\right) \\
\text{s.t.} \quad & \forall s^t & c\left(s^t\right) + \sum_{s^{t+1} \geq s^t} q\left(s^{t+1} \mid s^t\right) a\left(s^{t+1}\right) \\
& & = \left(1 - \tau\left(s^t\right)\right) w\left(s^t\right) \ell\left(s^t\right) + a\left(s^t\right) \\
& & a\left(s^0\right) = a_0
\end{aligned}
$$

where $u(\cdot)$ is the utility function, $c(\cdot)$ consumption and $\ell(\cdot)$ labor.

HOUSEHOLD PROBLEM

- The sequence of household budget constraints can be written as a single constraint

$$\sum_{t=0}^{\infty} \sum_{s^t} q\left(s^t\right) c\left(s^t\right) = \sum_{t=0}^{\infty} \sum_{s^t} q\left(s^t\right)\left(1 - \tau\left(s^t\right)\right) w\left(s^t\right) \ell\left(s^t\right) + q\left(s^0\right) a_0$$

  where $q\left(s^t\right)$ is now the time-0 price of one unit of goods in state s^t.
- Why? Complete markets are equivalent with Arrow securities and time-0 trading.

HOUSEHOLD PROBLEM

- Household problem:

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) u\left(c\left(s^t\right), \ell\left(s^t\right)\right)$$

$$\text{s.t.} \sum_{t=0}^{\infty} \sum_{s^t} q\left(s^t\right) c\left(s^t\right) = \sum_{t=0}^{\infty} \sum_{s^t} q\left(s^t\right) \left(1 - \tau\left(s^t\right)\right) w\left(s^t\right) \ell\left(s^t\right)$$
$$+ q\left(s^0\right) a_0$$

- This is nice because there is only one constraint.
- We could have used the same trick last week.

FIRST ORDER CONDITIONS

- Write down the Lagrangian and differentiate with respect to $c\left(s^t\right)$ and $\ell\left(s^t\right)$ to get the first order conditions:

$$\begin{aligned}
\left[c\left(s^t\right):\right] \quad & \beta^t \pi\left(s^t\right) u_c\left(c\left(s^t\right), \ell\left(s^t\right)\right) = \lambda q\left(s^t\right) \\
\left[\ell\left(s^t\right):\right] \quad & \beta^t \pi\left(s^t\right) u_\ell\left(c\left(s^t\right), \ell\left(s^t\right)\right) = -\lambda q\left(s^t\right)\left(1-\tau\left(s^t\right)\right) w\left(s^t\right)
\end{aligned}$$

- Note: the Lagrange multiplier λ is the same for all states s^t.
- To simplify notation

$$\begin{aligned}
\left[c\left(s^t\right):\right] \quad & \beta^t \pi\left(s^t\right) u_c\left(s^t\right) = \lambda q\left(s^t\right) \\
\left[\ell\left(s^t\right):\right] \quad & \beta^t \pi\left(s^t\right) u_\ell\left(s^t\right) = -\lambda q\left(s^t\right)\left(1-\tau\left(s^t\right)\right) w\left(s^t\right)
\end{aligned}$$

IMPLEMENTABILITY CONSTRAINT

- The first order conditions:

$$\left[c\left(s^t\right):\right]\ \beta^t \pi\left(s^t\right) u_c\left(s^t\right) = \lambda q\left(s^t\right)$$
$$\left[\ell\left(s^t\right):\right]\ \beta^t \pi\left(s^t\right) u_\ell\left(s^t\right) = -\lambda q\left(s^t\right)\left(1-\tau\left(s^t\right)\right) w\left(s^t\right)$$

together with the budget constraint

$$\sum_{t=0}^{\infty}\sum_{s^t} q\left(s^t\right) c\left(s^t\right) = \sum_{t=0}^{\infty}\sum_{s^t} q\left(s^t\right)\left(1-\tau\left(s^t\right)\right) w\left(s^t\right) \ell\left(s^t\right) + q\left(s^0\right) a_0$$

allow us to write the implementability constraint:

$$\sum_{t=0}^{\infty}\sum_{s^t} \beta^t \pi\left(s^t\right)\left[u_c\left(s^t\right) c\left(s^t\right) + u_\ell\left(s^t\right) \ell\left(s^t\right)\right] = u_c\left(s^0\right) a_0$$

IMPLEMENTABILITY CONSTRAINT

- The implementability constraint

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) \left[u_c\left(s^t\right) c\left(s^t\right) + u_\ell\left(s^t\right) \ell\left(s^t\right)\right] = u_c\left(s^0\right) a_0$$

  summarizes household optimality conditions and the budget constraint.
- The government can only choose allocations that satisfy this constraint.
- Captures the notion that the government's choices result in some optimal (given these choices) household behavior.

FIRM PROBLEM

- Output in this economy is produced by a representative price-taking firm.
- Linear production function that uses labor as an input.
- Firm problem is almost trivial – it chooses labor to maximize profits:

$$\max_{\ell(s^t)} A\ell\left(s^t\right) - w\left(s^t\right)\ell\left(s^t\right).$$

- In a competitive equilibrium we must have

$$w\left(s^t\right) = A.$$

GOVERNMENT BUDGET CONSTRAINT

- The implementability constraint is the household budget constraint (+ optimality conditions)
- The government should also be constrained by its budget constraint.
- We can ignore it and focus directly on the resource constraint of the economy:

$$A\ell\left(s^t\right) = g\left(s^t\right) + c\left(s^t\right)$$

  where $A\ell\left(s^t\right)$ is the linear production technology.
- Why? Walras' law – the government budget constraint is redundant.

GOVERNMENT PROBLEM

- The government chooses allocations $c\left(s^t\right), \ell\left(s^t\right)$ to maximize the household's utility

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) u\left(c\left(s^t\right), \ell\left(s^t\right)\right)$$

subject to the implementability constraint:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) \left[u_c\left(s^t\right) c\left(s^t\right) + u_\ell\left(s^t\right) \ell\left(s^t\right)\right] = u_c\left(s^0\right) a_0$$

and resource constraints:

$$\forall s^t \quad A\ell\left(s^t\right) = g\left(s^t\right) + c\left(s^t\right)$$

GOVERNMENT PROBLEM

- Full commitment: the government announces its state-contingent plans at time 0 and cannot change them later.
- Everyone else trusts the government and knows it will stick to its plans.

OPTIMALITY CONDITIONS

- The first order conditions for $s^t > s^0$ (we will return to s^0 later):

$$u_c\left(s^t\right) + \eta\left[u_{cc}\left(s^t\right)c\left(s^t\right) + u_c\left(s^t\right) + u_{c\ell}\left(s^t\right)\ell\left(s^t\right)\right] = \mu\left(s^t\right)$$
$$u_\ell\left(s^t\right) + \eta\left[u_{c\ell}\left(s^t\right)c\left(s^t\right) + u_\ell\left(s^t\right) + u_{\ell\ell}\left(s^t\right)\ell\left(s^t\right)\right] = -\mu\left(s^t\right)A$$

  where η is the Lagrange multiplier on the implementability constraint and $\mu\left(s^t\right)$ is the Lagrange multiplier on the resource constraint.

- There is also a resource constraint for each state:

$$A\ell\left(s^t\right) = g\left(s^t\right) + c\left(s^t\right)$$

- This gives us, for each state, three equations in three unknowns: $c\left(s^t\right), \ell\left(s^t\right), \mu\left(s^t\right)$.

OPTIMALITY CONDITIONS

- We also have a fourth unknown: η and one more equation: the implementability constraint.
- Key: η is the only link between different states.
- Once we know it, $c\left(s^t\right), \ell\left(s^t\right), \mu\left(s^t\right)$ are determined for each s^t from 2 first order conditions and the resource constraint.
- For two different states with the same $g\left(s^t\right)$, the solution must be the same!

IMPLICATIONS

- What are the implications for taxes?
- From household first order condition we have

$$\left(1 - \tau\left(s^t\right)\right) w\left(s^t\right) = -\frac{u_\ell\left(s^t\right)}{u_c\left(s^t\right)}$$

  and linear production technology implies $w\left(s^t\right) = A$ so

$$1 - \tau\left(s^t\right) = -\frac{u_\ell\left(c\left(s^t\right), \ell\left(s^t\right)\right)}{u_c\left(c\left(s^t\right), \ell\left(s^t\right)\right)} \frac{1}{A}.$$

- We have solved for $c\left(s^t\right), \ell\left(s^t\right)$, we can get $\tau\left(s^t\right)$!

IMPLICATIONS

- Since for two different states with the same $g\left(s^t\right)$ consumption and labor are the same – taxes must be the same!
- Conclusion: very different from Barro (1979) – no history dependence.
- If government purchases are i.i.d., taxes are i.i.d. as well.
- Contrast with the Barro (1979) case: taxes were a random walk.
- Contrast with the complete markets case last week: taxes were constant.

IMPLICATIONS

- Why is the result different from the reduced form complete markets case?
- No full tax smoothing (unless with particular household preferences), because of a different objective.
- When g_t is high, the government could want to increase labor supply to finance it without a large drop in consumption.

EXAMPLE

- Let $u(\cdot) = \frac{c^{1-\gamma}}{1-\gamma} - \psi\ell$.
- Then $u_c = c^{-\gamma}$, $u_{cc} = -\gamma c^{-\gamma-1}$, $u_\ell = -\psi$, $u_{\ell\ell} = 0$, $u_{c\ell} = 0$.
- The first order conditions

$$
\begin{aligned}
u_c\left(s^t\right) + \eta\left[u_{cc}\left(s^t\right)c\left(s^t\right) + u_c\left(s^t\right) + u_{c\ell}\left(s^t\right)\ell\left(s^t\right)\right] &= \mu\left(s^t\right) \\
u_\ell\left(s^t\right) + \eta\left[u_{c\ell}\left(s^t\right)c\left(s^t\right) + u_\ell\left(s^t\right) + u_{\ell\ell}\left(s^t\right)\ell\left(s^t\right)\right] &= -\mu\left(s^t\right)A
\end{aligned}
$$

are

$$
\begin{aligned}
\left[1 + \eta\left(1 - \gamma\right)\right]c\left(s^t\right)^{-\gamma} &= \mu\left(s^t\right) \\
\left[1 + \eta\right]\psi &= \mu\left(s^t\right)A
\end{aligned}
$$

EXAMPLE

- For each state $s^t > s^0$ we have:

$$\frac{1+\eta}{[1+\eta(1-\gamma)]A}\psi = c\left(s^t\right)^{-\gamma}$$

  so consumption is constant.

- Taxes satisfy

$$1-\tau\left(s^t\right) = \frac{\psi}{c\left(s^t\right)^{-\gamma}}\frac{1}{A}:$$

  since $c\left(s^t\right)$ is constant, taxes are constant as well.

- With these preferences households supply labor fully elastically, fluctuations in g_t are absorbed by changes in production.

LESSONS

- Rembember: the above result is not general.
- What is general is that here allocations and taxes are history independent – they are fully determined by the current state.
- What is the optimal path of debt? Similar logic as in our example last week: state-contingent debt that depends only on the next period state.

TIME-0

- We ignored one important issue: first order conditions are different for period 0.
- This is because of the $u_c\left(s^0\right) a_0$ term in the implementability constraint.
- First order conditions for s^0 are:

$$u_c\left(s^0\right) + \eta\left[u_{cc}\left(s^0\right) c\left(s^0\right) + u_c\left(s^0\right) + u_{c\ell}\left(s^0\right) \ell\left(s^0\right) - u_{cc}\left(s^0\right) a_0\right] = \mu\left(s^0\right)$$
$$u_\ell\left(s^0\right) + \eta\left[u_{c\ell}\left(s^0\right) c\left(s^0\right) + u_\ell\left(s^0\right) + u_{\ell\ell}\left(s^0\right) \ell\left(s^0\right) - u_{c\ell}\left(s^0\right) a_0\right] = -\mu\left(s^0\right) A$$

- Time-0 allocations will be different from other periods, even with the same level of g_t.

TIME-0

- The government has incentives to play with taxes in period 0 to change $q\left(s^0\right)$ and thus affect the value of debt/assets.
  - For example: it could try to make debt due at time 0 worthless.
  - This is good, because it allows the government to create lower tax distortions in the future.
- No similar effect in other periods, forward looking agents take it into account.

TIME INCONSISTENCY

- Time consistency problem: if the government could re-optimize at future dates, period 1 would be like period 0.
- Same logic applies to all other periods.
- But forward-looking agents would know the government is tempted to do it every period!
- This is actually the essence of Lucas and Stokey (1983).
  - Is there a way to design debt portfolio (maturities etc.) to ensure the government will not be tempted to re-optimize?