OPTIMAL LEVEL OF DEBT

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PLAN

- We investigate what simple models can say about the optimal level of debt.
- Previously we either had results that say that the long run level of debt should be equal to the initial condition, or there should be time 0 default.

NEOCLASSICAL GROWTH MODEL

SETUP

- There is a representative household that maximizes utility from consumption.
- There is no risk – everything is deterministic.
- The household supplies 1 unit of labor inelastically.
- The household can trade assets that pay return $1 + r_{t+1}$ next period. These assets are capital and government debt.
- The household pays taxes / receives transfers T_t. These are lump sum.
- There is a borrowing constraint: $a_{t+1} \geq -\bar{A}_t$, but $\bar{A}_t$ is very large.
- $\bar{A}_t$ is such that is equals the present discounted value of all future labor income.
- We call it the natural debt limit.

SETUP

- The government purchases goods G_t, finances them with lump sum taxes T_t and issues debt B_{t+1}.
- The law of motion for debt is

$$B_{t+1} = (1 + r_{t+1})B_t + G_t - T_t.$$

- The production function is $F(K_t, L_t)$. It is increasing, concave and homogeneous of degree 1 in inputs.
- There is a competitive labor market with the wage rate w_t.
- There is a competitive capital market with the rental rate r_t.
- Capital depreciates at rate δ.

SETUP

- The household utility maximization problem is

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t.} c_t + a_{t+1} = (1 + r_t)\, a_t + w_t \cdot 1 - T_t$$

$$a_{t+1} \geq -\bar{\mathcal{A}}_t$$

and a_0 given.

SETUP

- The firm maximization problem is

$$\max_{L_t, K_t} \quad F(K_t, L_t) - w_t L_t - (r_t + \delta) K_t \quad \text{for all } t \geq 0.$$

SETUP

- The government is fully characterized by sequences $\{G_t, T_t, B_{t+1}\}_{t=0}^{\infty}$ that satisfy

$$B_{t+1} = (1 + r_{t+1})B_t + G_t - T_t \qquad \text{for all } t \geq 0.$$

given B_0.

SETUP

- All markets must clear for all $t \geq 0$:

$$L_t = 1,$$

$$a_t = K_t + B_t,$$

$$c_t + G_t + K_{t+1} = F(K_t, L_t) + + (1 - \delta) K_t.$$

COMPETITIVE EQUILIBRIUM

- A **competitive equilibrium** given government policies $\{G_t, T_t, B_{t+1}\}_{t=0}^{\infty}$ is sequences of prices $(w_t, r_t)_{t=0}^{\infty}$ (that satisfy the government budget constraint) and allocations $\{c_t, K_t, L_t, a_{t+1}\}_{t=0}^{\infty}$ such that
    1. Given $\{w_t, r_t\}_{t=0}^{\infty}$ and $\{T_t\}_{t=0}^{\infty}$, $\{c_t, a_{t+1}\}_{t=0}^{\infty}$ solves the household problem.
    2. Given $\{w_t, r_t\}_{t=0}^{\infty}$, $\{K_t, L_t\}_{t=0}^{\infty}$ solves the firm problem.
    3. Markets clear.
- Implicit in this definition is that the government budget constraint is satisfied.

EQUILIBRIUM CHARACTERIZATION

- First order conditions for the household problem give us the Euler equation:

$$u'(c_t) = \beta\,(1 + r_{t+1})\,u'(c_{t+1})$$

  Note: the constraint is not binding.

- First order conditions for the firm problem give us:

$$w_t = F_L(K_t, 1)$$
$$r_t = F_K(K_t, 1) - \delta$$

- Homogeneity of degree 1 implies that $F(K_t, 1) = r_t K_t + w_t \cdot 1$.

EQUILIBRIUM CHARACTERIZATION

- Use the asset market clearing condition and the budget constraint of the government in the budget constraint of the household:

$$c_t + (K_{t+1} + B_{t+1}) = (1 + r_t)\,(K_t + B_t) + w_t \cdot 1 - (G_t + (1 + r_t)\,B_t - B_{t+1})\,.$$

  and reorganize to get

$$c_t + K_{t+1} = (1 + r_t)\,K_t + w_t \cdot 1 - G_t.$$

- Finally, use the FOCs of the firm problem to get

$$c_t + K_{t+1} = F(K_t, 1) + (1 - \delta)\,K_t - G_t.$$

- This is the resource constraint.

EQUILIBRIUM CHARACTERIZATION

- Equilibrium is characterized by the Euler equation and the resource constraint:

$$u'(c_t) = \beta\,(1 + F_K(K_{t+1}, 1) - \delta)\,u'(c_{t+1})$$
$$c_t + K_{t+1} = F(K_t, 1) + (1 - \delta)\,K_t - G_t \text{ for all } t \geq 0.$$

- In other words, if we know $K_0 = a_0 - B_0$ and $\{G_t\}_{t=0}^{\infty}$, we can solve for $\{c_t, K_{t+1}\}_{t=0}^{\infty}$.
- If we know $\{c_t, K_{t+1}\}_{t=0}^{\infty}$, we can solve for $\{w_t, r_t\}_{t=0}^{\infty}$.

RICARDIAN EQUIVALENCE

- The quantity of government debt does not matter!
- The only thing related to the government that matters is the sequence of government purchases $\{G_t\}_{t=0}^{\infty}$.
- It does not matter how the government finances it: whether by taxes or debt.
- This is called Ricardian equivalence (see Barro (1974)).
- Note: you might think B_0 matters – but it is only because it is part of the initial condition $K_0 = a_0 - B_0$. If we pinned down K_0 directly, B_0 would not matter.

RICARDIAN EQUIVALENCE

- We have a very stark answer to the question of how much debt should the government issue: does not matter.
- This is because taxes were lump-sum: they did not distort the household's decision.
- The household does not care whether taxes are high or low now – it only cares about the present discounted value of taxes, determined by the sequence of government purchases.
- If the government issues debt, it will have to raise taxes in the future to pay it off.

STEADY STATE

- We will now focus on the steady state of this economy.
- We set $G_t = G$, $c_t = c$, $K_t = K$ and $r_t = r$ for all $t \geq 0$.

$$
\begin{aligned}
1 &= \beta \left(1 + F_K(K, 1) - \delta\right) \\
c + \delta K &= F(K, 1) - G.
\end{aligned}
$$

- The first equation pins down K.
- Given K and G, the second equation pins down c.

STEADY STATE

- Government policy does not affect the steady state level of capital.
- No crowding out in the steady state of this economy!
- Government policy (specifically, government purchases) does affect the steady state level of consumption.

STEADY STATE

- It sometimes helps to think of the Euler equation as a demand for assets.
- What is the elasticity of demand for assets with respect to the interest rate?
- In the long run (steady state) it is infinity!
- The household is willing to absorb any amount of assets at the interest rate $\beta^{-1} - 1$.
- If the rate is lower, it wants to hold zero assets, if it is higher, it wants to hold infinite assets.

ENDOGENOUS LABOR SUPPLY

- What if the household can choose how much to work, but it causes some disutility?
- The steady state Euler equation is now

$$1 = \beta\left(1 + F_K(K, L) - \delta\right).$$

- If the production function is homogeneous of degree 1 (constant returns to scale) $F_K(K, L) = F(K/L, 1)$.
- In this case the Euler equation pins down the ratio of capital to labor. This determines the wage rate and the rate of return on capital.
- The level of capital and labor are determined by the labor supply decision. It can only work through wealth effect.
- Crowding out (or in!) is possible.

AIYAGARI AND MCGRATTAN (1998)

FROM THEIR INTRODUCTION...

- Aiyagari and McGrattan (1998) model has a different role for government debt.
- Government debt enhances the liquidity of households by providing an additional means of smoothing consumption (in addition to claims to capital) and by effectively loosening borrowing constraints
- When the interest rate is raised, government debt makes assets both less costly to hold and more effective in smoothing consumption.
- The implied taxes have adverse wealth distribution and incentive effects.
- Government debt crowds out capital via higher interest rates, and it lowers per capita consumption.

SETUP

- We modify the household side of the model (Bewley-Imrohoroglu-Huggett-Aiyagari).
- There is now a continuum of households indexed by $i \in [0, 1]$.
- Agents receive idiosyncratic shocks to their labor income $e_{i,t}$. $e_{i,t}$ is i.i.d. across agents and follows a Markov process. $\mathbb{E}(e_{i,t}) = 1$.
- Assets are risk-free (not state-contingent!) – incomplete markets.
- There are borrowing constraints: $a_{i,t+1} \geq -\bar{\mathcal{A}}_t$. For simplicity set $\bar{\mathcal{A}}_t = 0$.
- We will focus on the stationary equilibrium of this economy.
- This is an equilibrium where all prices and the distribution of agents are constant over time.
- We allow for individual allocations of agent i to vary over time.

SETUP

- The household utility maximization problem is

$$
\begin{aligned}
\max_{\{c_{i,t}, a_{i,t+1}\}_{t=0}^{\infty}} \; & \mathbb{E} \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}\right) \\
\text{s.t. } & c_{i,t} + a_{i,t+1} = (1+r)\, a_{i,t} + w e_{i,t} - T \\
& a_{i,t+1} \geq 0 \\
\text{and } & a_{i,0}, e_{i,0} \text{ given.}
\end{aligned}
$$

SETUP

- All markets must clear:

$$L = \int_0^1 e_i di,$$

$$\int_0^1 a_i di = K + B$$

$$\int_0^1 c_i di + G + K = F(K, L) + + (1 - \delta) K.$$

ANALYSIS

- From the budget constraint of the government we get

$$T = rB + G.$$

- From the firm problem we get

$$K = \kappa(r),$$
$$w = \omega(r).$$

- The solution to the household problem gives a decision rule for asset accumulation:

$$a_{i,t+1} = \alpha\left(a_{i,t}, e_{i,t}; r, B, G\right).$$

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ANALYSIS

- The policy function $\alpha(a, e; r, B, G)$ togerther with the Markov process for e allows to calculate the stationary distribution $\mu(a, e; r, B, G)$.
- Intuitively: find the joint distribution of assets and shocks that reporoduces itself.
- The aggregate demand for assets is

$$\bar{\alpha}(r, B, G) = \int \int \alpha(a, e; r, B, G)\mu(a, e; r, B, G)dade.$$

ANALYSIS

- To find the equilibrium interest rate we need to solve

$$\bar{\alpha}(r, B, G) = \kappa(r) + B.$$

- Then we can back out all other prices and allocations.
- In principle this is not different from what we saw earlier. We had $\bar{\alpha}(r, B, G)$ that was special.
- The point is that idiosyncratic income risk together with borrowing constraints affects the shape of $\bar{\alpha}(r, B, G)$.

DEMAND FOR ASSETS

- Look at the Euler equation (notice the constraint can be binding)

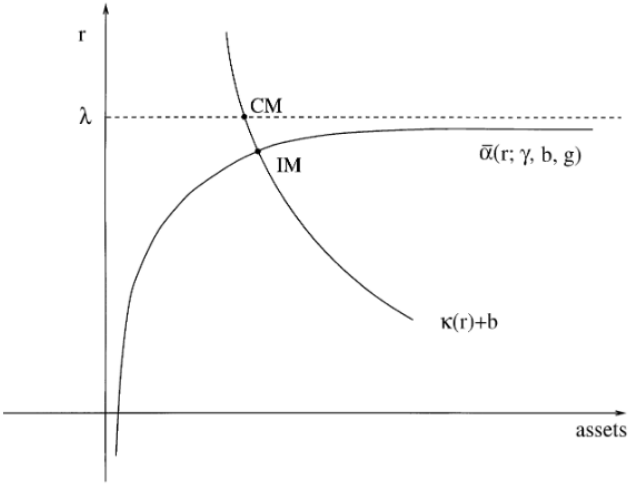
$$u'\left(c_{i,t}\right) \geq \beta\left(1+r\right) \mathbb{E}_t\, u'\left(c_{i,t+1}\right).$$

- Suppose that $r = \beta^{-1} - 1$ as we had in NGM (or we would have in a model without risk).
- The agent would like to have a smooth profile of marginal utility of consumption. Utility is concave: lower consumption hurts more than higher consumption helps.
- It is possible that there is a very long sequence of bad income shocks.
- The only way to insure against this is to accumulate assets.

DEMAND FOR ASSETS

- How much to accumulate? Infinity! So we know that $r = \beta^{-1} - 1$ is not an equilibrium. It must be lower!
- We have precautionary savings. For any r the demand for assets is larger than in the NGM / complete markets.

EQUILIBRIUM



Source: Aiyagari and McGrattan (1998)

EQUILIBRIUM

- Aiyagari and McGrattan (1998) argue that savings are too high in this economy (relative to the complete markets benchmark).
- Hence the policy recommendation is to reduce savings.
- This can be done by reducing the need for precautionary savings.
- How can debt help with that?

DEBT LEVEL

- Aiyagari and McGrattan (1998) approach: define $a^*_{i,t} \coloneqq a_{i,t} - B$.
- Rewrite the budget constraint of the household as

$$c_{i,t} + a^*_{i,t+1} + B = (1 + r)\left(a^*_{i,t} + B\right) + we_{i,t} - T$$

and use $T = rB + G$ to get

$$c_{i,t} + a^*_{i,t+1} = (1 + r)\, a^*_{i,t} + we_{i,t} - G.$$

- The borrowing constraint $a_{i,t+1} \geq 0$ is now

$$a^*_{i,t+1} \geq -B.$$

DEBT LEVEL

- In this formulation government debt B only enters the con- sumer's borrowing constraint.
- Higher levels of B in effect loosen the borrowing constraint and reduce the average asset holdings (net of government debt).
- Intuition: no need to save as much when the borrowing constraint is looser.
- The solution $a^*_{i,t+1} = \alpha^*\left(a^*_{i,t}, e_{i,t}; r, B, G\right)$ is decreasing in B.
- The amount saved in capital is decreasing in B – crowding out.

WELFARE

- How does it all affect welfare?

$$\Omega = \int \int V(a, e)\mu(a, e) dade$$

- An increase in debt increases the return on assets, thus making them less costly for the consumer to hold. Assets are cheaper in enabling the consumer to smooth consumption.
- Lump-sum taxes levied to pay interest on government debt:
    1. more onerous for individuals with low assets and low earnings than for individuals with high assets and high earnings;
    2. exacerbate the percentage variability in after-tax earnings.
- Crowding out of capital and the consequent reduction in per capita consumption.

QUANTITATIVE RESULTS

- Aiyagari and McGrattan (1998) extend and calibrate their model to the US economy.
- They allow for elastic labor supply, distrtionary income taxes and a more general borrowing constraint.
- They find that the optimal level of debt to GDP is around 2/3.
- This is close to the average level of debt to GDP in the US economy after WWII (until the 2000s).
- At this level the positive (liquidity) role of debt is balancing the negative (crowding out and distortionary taxes) role.

QUANTITATIVE RESULTS

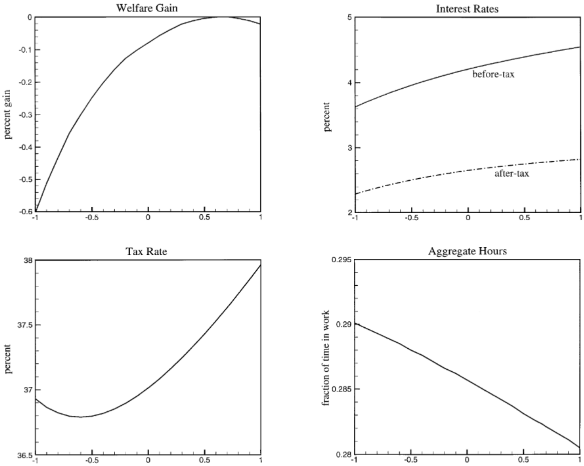


Fig. 2. Welfare gain, interest rates, tax rate, and aggregate hours versus debt/GDP ratio (x-axis) for the benchmark economy.

Source: Aiyagari and McGrattan (1998)

NEXT TIME

- Is Aiyagari and McGrattan (1998) intuition about pushing the interest rate closer to the complete markets case really valid?
- What can we say about the optimal level of borrowing constraints?