

# DEBT SUSTAINABILITY

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FISCAL AND MONETARY POLICY 2023

Piotr Żoch

January 7, 2024

# PLAN

- We review several approaches to debt sustainability analysis.
- Can the government service its debt?
- Is the outstanding public debt and its projected path consistent with those of the government's revenues and expenditures?

# CLASSIC DEBT SUSTAINABILITY ANALYSIS

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## DEBT TO OUTPUT RATIO

- The government static budget constraint is

$$B_t = (1 + r_t)B_{t-1} + G_t - T_t.$$

where  $B_{t-1}$  is the **market value** of all outstanding debt (all maturities),  $G_t$  is nominal government spending (net of interest expenses),  $T_t$  is nominal tax revenue, and  $r_t$  is net return on government debt.

- We abstract from money growth.

## DEBT TO OUTPUT RATIO

- Divide by **nominal** GDP  $Y_t$  and rearrange to get

$$\frac{B_t}{Y_t} = (1 + r_t) \frac{B_{t-1}}{Y_{t-1}} \frac{Y_{t-1}}{Y_t} + \frac{G_t}{Y_t} - \frac{T_t}{Y_t}.$$

- Define

$$R_{t-j,t} := \prod_{k=1}^j \left( 1 + r_{t-j+k} \right),$$

the cumulative return on debt from  $t - j$  to  $t$ .

- Define

$$X_{t-j,t} := \prod_{k=1}^j \frac{Y_{t-j+k}}{Y_{t-j+k-1}},$$

the cumulative gross rate of GDP from  $t - j$  to  $t$ .

## DEBT TO OUTPUT RATIO

- For simplicity assume  $B_0 = 0$ .
- We can write the debt to output ratio as

$$\frac{B_t}{Y_t} = \sum_{j=0}^t \frac{G_{t-j} - T_{t-j}}{Y_{t-j}} \frac{R_{t-j,t}}{X_{t-j,t}}$$

- Debt to output ratio today is determined by:
  1. The past primary deficits to GDP ratios;
  2. The past returns on debt;
  3. The past growth rates of (nominal) GDP.
- We saw a similar decomposition when we discussed Hall and Sargent (it was more detailed there).

## CLASSIC DEBT SUSTAINABILITY ANALYSIS

- A version of the formula is often used to assess **debt sustainability** – "whether the government can service its debt".
- **Warning:** this is about the future, not the present. The fact that people use the formula to assess the current situation is often a red flag!
- Classic debt sustainability analysis looks at the "long run".
- Assume that the economy is in a steady state with a constant growth rate of GDP  $X$ , an constant rate of return  $R$  and a constant primary deficit to GDP ratio.
- What is the debt to output ratio consistent with the above?
- If the observed current debt to output ratio is below this level, the debt is sustainable.

## CLASSIC DEBT SUSTAINABILITY ANALYSIS

- Classic debt sustainability analysis usually analyzed deterministic setups (or perfect foresight).
- In these setups, the appropriate  $R$  is the **risk-free rate**,  $R^f$ .
- Assuming the above, the formula in the steady state becomes

$$\frac{B}{Y} = \frac{G - T}{Y} \frac{X}{X - R^f}.$$

- For simplicity define  $x := X - 1$  and  $r^f := R^f - 1$  so we have

$$\frac{B}{Y} = \frac{G - T}{Y} \frac{1 + x}{x - r^f}.$$



## CLASSIC DEBT SUSTAINABILITY ANALYSIS

$$\frac{B}{Y} = \frac{G - T}{Y} \frac{1 + x}{x - r^f}$$

- Example: the Treaty of Maastricht set the limit of the debt to output ratio at 60% and the deficit to output ratio at 3%. What must be the growth rate of GDP and the risk-free rate for this to be sustainable?
- We get  $\frac{1+x}{x-r^f} = 20$ , for small  $x$  we have  $x \approx r^f + 0.005$ , so the economy has to grow at 0.5% above the risk-free rate per year for the debt at the limit to be sustainable with the largest allowed deficit.
- If the actual growth rate is lower, the debt to output ratio will be larger, even if the deficit is at the limit.

## CLASSIC DEBT SUSTAINABILITY ANALYSIS

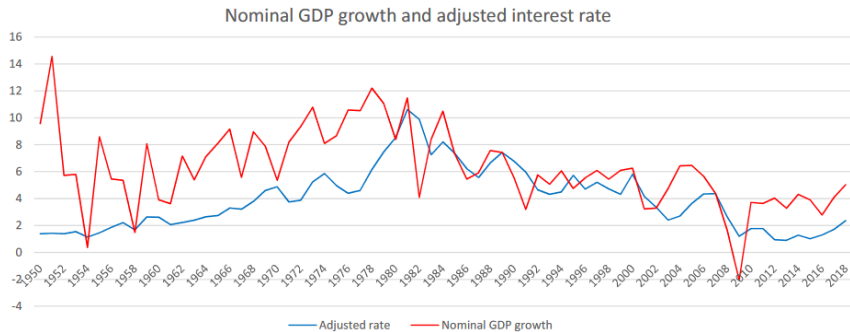
$$\frac{B}{Y} = \frac{G - T}{Y} \frac{1 + x}{x - r^f}$$

- The key role of  $x - r^f$ .
- Depending on the sign of  $x - r^f$  the you require either surpluses or deficits to keep the debt to output ratio constant.
  - If  $x < r^f$  you need surpluses.
  - If  $x > r^f$  you can have deficits.
- A big "r versus g" debate (we have  $x$  instead of  $g$ ).

## CLASSIC DEBT SUSTAINABILITY ANALYSIS

- If  $x > r^f$  it seems there is **no fiscal cost** to debt.
- Blanchard (2019) argues that  $x > r^f$  is a norm, not an exception.
- But what is  $r^f$ ? How to measure it? Blanchard (2019) looks at the 1-year US Treasury bill rate, the 10-year US Treasury bond rate, adjusts for various maturities...
- Note: it does not necessarily mean that it is **optimal** to have deficits.

# RATES IN THE US



Source: Blanchard (2019)

## CLASSIC DEBT SUSTAINABILITY ANALYSIS

- It only **defines** what long-run debt is for a given long-run primary balance (or vice versa) **if** stationarity holds, or defines lower bounds on the short-run dynamics of the primary balance.
- It does not connect the outstanding initial debt of a particular period with the steady state.
- There might be multiple paths of debt that do not violate the **intertemporal government budget constraint** (IGBC), some of them can even go to infinity (but slowly enough)!
- IGBC: the value of debt is equal to the present discounted value of future primary surpluses.

# INTERTEMPORAL GOVERNMENT BUDGET CONSTRAINT

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## IGBC

- We used the government budget constraint by going back in time.
- We can also solve it **forward** – the **valuation approach**, the market value of government debt is determined by the discounted value of future government surpluses.
- This idea is often used in finance (e.g., Campbell and Shiller 1988).
- Allows us to think seriously about risk and asset pricing.

## IGBC

- We want to write something like

$$B_t = \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} (T_{t+j} - G_{t+j})$$

- We call  $M_{t,t+j}$  the **stochastic discount factor** (SDF).
- It reflects how holders of government debt value discount future cash flows.
- Generally it is a function of the state of the economy at time  $t$  and  $t + j$ . Recall the first order condition for the household problem in the models we saw.
- We call the formula above the **intertemporal government budget constraint** IGBC.



## DEBT SUSTAINABILITY

- We can say that debt is sustainable if and only if the IGBC holds.
- **Problem**: this condition is about the entire future.
- **Solution (?)**: use forecasts of future taxes and spending to compute the present value of future surpluses. Some early papers did this, but they used **risk-free** rates.
- Valid if one of these conditions holds:
  1. There is perfect foresight;
  2. Investors are risk-neutral;
  3. Primary surpluses do not covary with the SDF.

## BOHN (1998)

- Recall the Barro (1979) tax smoothing model – debt was a random walk, yet the IGBC held.
- Not even debt (or debt to GDP) going to infinity means that the IGBC does not hold, it has to go to infinity **slowly enough**.
- Bohn (1998): see if the government does something that guarantees the IGBC holds, investigate the **fiscal reaction function**.
- Allows to sidestep the problem of forecasting future taxes and spending and choosing the correct discount rate.
- Sufficient condition: IGBC might also hold if it violated, but if it is satisfied, IGBC holds for sure.

## BOHN (1998)

- Linear reaction function:

$$\frac{T_t - G_t}{Y_t} = \rho \frac{B_{t-1}}{Y_{t-1}} + Z_t + \epsilon_t$$

- The left hand side is **primary surplus**.
- $Z_t$  is a vector of exogenous variables that affect the primary surplus.
- Check if  $\rho > 0$  – raise surplus if debt is high.
- If  $\rho > 0$ , then the **IGBC holds** even if it is **below** the interest rate (net of  $x$ ).

## BOHN (1998)

- If  $\rho > 0$ , the IGBC holds for **any** initial level of debt.
- This analysis works also for  $r - x = 0$  – there was division by zero in the classic analysis.
- If  $r - x > \rho > 0$ , debt explodes, but the IGBC **still** holds (under certain conditions: see Bohn 2007).

## BOHN (1998)

- Bohn (1998) estimates  $\rho$  for the US in 1916-1995.
- He includes the level of temporary government spending and business cycle indicator in  $Z_t$ .
- He find a **positive** value of  $\rho$ , around 0.05 for the entire sample.

# BOHN (1998)

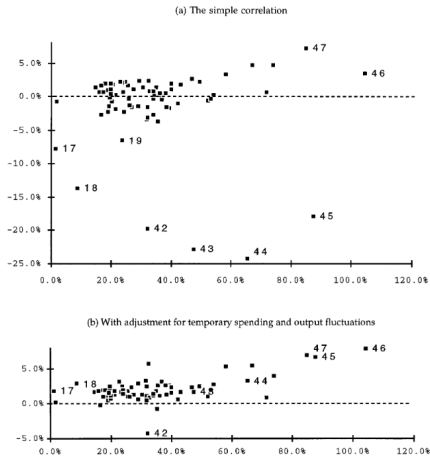


FIGURE I  
Primary Surplus versus Initial Debt

The graph shows the privately held government debt/GDP at the start of a period on the horizontal axis against the primary budget surplus/GDP on the vertical axis, for 1916–1995; (a) shows raw data, and (b) shows the adjusted surplus, as explained in the text.

Source: Bohn (1998)

# BOHN (1998)

TABLE I  
DETERMINANTS OF THE BUDGET SURPLUS

Dependent variable primary budget surplus divided by GDP ( $s_t$ )							
Sample	Constant	GVAR	YVAR	$d_t$	$R^2$	$\sigma$	DW
(1) 1916–1995	–0.019 (–5.424) [–3.957]	–0.776 (–33.001) [–20.874]	–1.450 (–3.628) [–4.075]	0.054 (6.048) [3.787]	0.936	0.014	1.42
(2) 1920–1995 excl. 1940–1947	–0.009 (–2.030) [–2.155]	–0.551 (–4.034) [–3.721]	–1.906 (–4.666) [–4.296]	0.028 (2.701) [2.491]	0.618	0.011	1.40
(3) 1916–1983	–0.018 (–4.903) [–3.958]	–0.782 (–31.667) [–20.943]	–1.414 (–3.360) [–4.004]	0.054 (5.996) [4.076]	0.942	0.014	1.54
(4) 1920–1982 excl. 1940–1947	–0.008 (–1.710) [–1.932]	–0.520 (–3.612) [–3.272]	–1.912 (–4.441) [–3.959]	0.030 (2.815) [2.856]	0.630	0.011	1.56
(5) 1948–1995	–0.015 (–3.536) [–3.496]	–0.593 (–4.182) [–3.701]	–2.139 (–4.361) [–3.757]	0.037 (3.589) [2.821]	0.651	0.010	1.54
(6) 1960–1984	–0.013 (–2.110) [–2.174]	–0.410 (–2.173) [–2.281]	–2.051 (–4.174) [–3.391]	0.044 (2.028) [2.587]	0.724	0.007	1.43

The variable  $d_t$  is the privately held debt/GDP at the start of the year. GVAR and YVAR are measures of temporary government spending and of cyclical variations in output, respectively, from Barro [1986a]. All estimates are OLS with annual data; ( ) = ordinary  $t$ -statistics; [ ] = heteroskedasticity- and autocorrelation-consistent  $t$ -statistics (computed with Newey-West lag window of size 1);  $\sigma$  = standard error; DW = Durbin-Watson statistic.

Source: Bohn (1998)

## FISCAL REACTION FUNCTIONS

- Bohn (2008) extends the analysis to 1793-2003.
- He finds that  $\rho > 0.1$ , more than twice as large as in the previous study.
- Mendoza and Ostry (2008) study fiscal reaction functions for a panel of multiple countries – similar results.
- Ghosh et al. (2013) show that  $\rho$  is much lower at high levels of debt.
- D'Erasmus et al. (2016):
  1. primary balance adjustment in the US after 2008 was **too large** to be explained by the fiscal reaction function;
  2. adjustment is **slower** than before (structural break);
  3. nevertheless, with the estimated  $\rho$ , the IGBC holds.



## FISCAL REACTION FUNCTIONS

- Leeper (2017) warns against using surplus-debt regressions to assess debt sustainability.
- For the estimator of  $\rho$  to be **consistent**, we must have

$$\mathbb{E}\left(\epsilon_t \mid \frac{B_{t-1}}{Y_{t-1}}\right) = 0.$$

1. This means that shocks at  $t - 1$  that affect debt-output ratio in must not affect  $\epsilon_t$ .
  2. This means that the debt-output ratio cannot depend on the expectation of  $\epsilon_t$ .
- Since the value of debt depends on the expected value of future surpluses, this is a strong assumption:  $\epsilon_t$  could be serially correlated.

## VALUATION APPROACH

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## VALUATION APPROACH

- We go back to the budget constraint and solve it forward as

$$B_t = \mathbb{E}_t \sum_{j=1}^T M_{t,t+j} (T_{t+j} - G_{t+j}) + \mathbb{E}_t M_{t,t+T} B_{t+T}$$

- We obtained the standard IGBC if

$$\lim_{T \rightarrow \infty} \mathbb{E}_t M_{t,t+T} B_{t+T} = 0.$$

- The IGBC implies that a higher debt-to-output ratio today can be attributed to higher expected future primary surpluses (cash flows) or lower expected future returns (discount rates).
- The counterpart of the Campbell-Shiller expression for the log of the price-to-dividend ratio in the stock market.

## VALUATION APPROACH

- Cochrane (2011) shows that discount rate variation is the main driver of stock valuation ratios.
- Cochrane (2019): half of the variation in the debt-to-GDP ratio to variation in future primary surpluses and half to varying discount rates.
- Jiang et al. (2021) conclude **no statistical evidence** of a discount rate or cash flow channel.
- Fluctuation in the debt-to-GDP ratio at time  $t$  predict fluctuations in the debt-to-GDP ratio at time  $t + T$ .
- Jiang et al. argue the differences result from small sample bias.

## FISCAL CAPACITY

- Jiang et al. in a series of recent papers propose a new approach to debt sustainability analysis.
- Suppose an investor buys the **entire** stock of government debt and participates in all new issuances.
- How much would that investor be willing to pay for the debt?
- Cash flow is  $\{T_t - G_t\}$ .
- Use tools from asset pricing to answer this question.
- The price will depend on the riskiness of the cash flows.

## ASSET PRICING BASICS

- Before we talk about Jiang et al., let's review some asset pricing basics.
- The general idea dates back to Lucas (1978) who considers asset prices in a general equilibrium model.
- An asset is a claim on a stream of prospective payments.
- Consider an economy with  $i = 1, \dots, N$  assets.
- Each of these assets has an associated stream of real dividends  $\{d_{i,t}\}_{t=0}^{\infty}$ .
- Assume the representative investor maximizes  $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$ .
- Let  $p_{i,t}$  be the price of asset  $i$  at time  $t$  (in goods).

## ASSET PRICING BASICS

- For each asset the investor holds, the optimality condition is:

$$p_{i,t} = \beta \mathbb{E}_t \frac{u'(c_{t+1})}{u'(c_t)} (p_{i,t+1} + d_{i,t+1}) .$$

- This is the **consumption-based asset pricing equation**.
- We can slightly rearrange it as

$$1 = \beta \mathbb{E}_t \frac{u'(c_{t+1})}{u'(c_t)} \frac{p_{i,t+1} + d_{i,t+1}}{p_{i,t}} .$$

- Here  $R_{i,t,t+1} := \frac{p_{i,t+1} + d_{i,t+1}}{p_{i,t}}$  is the (gross) rate of **return** on asset  $i$  from  $t$  to  $t + 1$ .

## ASSET PRICING BASICS

- Define  $M_{t,t+1} := \beta \frac{u'(c_{t+1})}{u'(c_t)}$ .
- We call it the **stochastic discount factor** (SDF).
- We can write the asset pricing equation for **each asset** as

$$1 = \mathbb{E}_t M_{t,t+1} R_{i,t,t+1}.$$

- The **risk-free rate**  $R_{t,t+1}^f$  satisfies

$$1 = R_{t,t+1}^f \mathbb{E}_t M_{t,t+1}.$$



## ASSET PRICING BASICS

- Sometimes you will see the asset pricing equation written as

$$v_{i,t} = \mathbb{E}_t M_{t,t+1} \frac{d_{i,t+1}}{d_t} (1 + v_{i,t+1}) .$$

- Here  $v_{i,t} := \frac{p_{i,t}}{d_t}$  is the **price-dividend ratio** of asset  $i$  at time  $t$ .
- This form is useful when we want to think of an asset that has an ever increasing stream of dividends.

## ASSET PRICING BASICS

- Generally  $\mathbb{E}_t M_{t,t+1} R_{i,t,t+1} \neq \mathbb{E}_t M_{t,t+1} \cdot \mathbb{E}_t R_{i,t,t+1}$ .
- We have

$$\mathbb{E}_t M_{t,t+1} R_{i,t,t+1} = \mathbb{E}_t M_{t,t+1} \cdot \mathbb{E}_t R_{i,t,t+1} + \text{cov}_t (M_{t,t+1}, R_{i,t,t+1}) .$$

- This allows us to write the asset pricing equation as

$$1 = \mathbb{E}_t M_{t,t+1} \cdot \mathbb{E}_t R_{i,t,t+1} + \text{cov}_t (M_{t,t+1}, R_{i,t,t+1}) .$$

- Use  $1 = R_{t,t+1}^f \mathbb{E}_t M_{t,t+1}$  to write

$$\mathbb{E}_t R_{i,t,t+1} = R_{t,t+1}^f - \frac{\text{cov}_t (M_{t,t+1}, R_{i,t,t+1})}{\mathbb{E}_t M_{t,t+1}} .$$

## ASSET PRICING BASICS

- The formula

$$\mathbb{E}_t R_{i,t,t+1} = R_{t,t+1}^f - \frac{\text{cov}_t (M_{t,t+1}, R_{i,t,t+1})}{\mathbb{E}_t M_{t,t+1}}.$$

tells us that the expected return on asset  $i$  is the risk-free rate plus a **risk premium**.

- The risk premium depends on the covariance between the SDF and the return on asset  $i$ .
- When the covariance is negative (SDF is low when the return is high), the risk premium is positive.
- When the covariance is positive (SDF is high when the return is high), the risk premium is negative.

## ASSET PRICING BASICS

- To understand it better consider a basic example: let  $u(c) = \ln c$ . We have

$$M_{t,t+1} = \beta \frac{c_t}{c_{t+1}}.$$

- The SDF is high when  $c_{t+1}$  is low.
- If the asset has a low return when  $c_{t+1}$  is low, the covariance is negative and the risk premium is positive.
- This is because the asset is **risky** – it does not pay much when you need it the most.

## ASSET PRICING BASICS

- We sometimes write the formula as

$$\mathbb{E}_t R_{i,t,t+1} = R_{t,t+1}^f - \frac{\text{cov}_t(M_{t,t+1}, R_{i,t,t+1})}{\text{var}_t M_{t,t+1}} \times \frac{\text{var}_t M_{t,t+1}}{\mathbb{E}_t M_{t,t+1}}.$$

- There are two terms:
  1. The first term is the **risk exposure** – it is the covariance between the SDF and the return on asset  $i$  divided by the variance of the SDF.
  2. The second term is the **price of risk** – it is the variance of the SDF divided by the expected value of the SDF. It does not depend on the asset.

## ASSET PRICING BASICS

- We have

$$\mathbb{E}_t R_{i,t,t+1} = R_{t,t+1}^f + \beta_{i,t} \lambda_t.$$

- $\beta_t^i$  is the risk exposure of asset  $i$ ,  $\beta_{i,t} := -\frac{\text{cov}_t(M_{t,t+1}, R_{i,t,t+1})}{\text{var}_t M_{t,t+1}}$
- Note: do not confuse  $\beta_{i,t}$  with  $\beta$ , the discount factor.
- $\lambda_t$  is the price of risk,  $\lambda_t := \frac{\text{var}_t M_{t,t+1}}{\mathbb{E}_t M_{t,t+1}}$ .
- Risk premium is the product of the risk exposure and the price of risk.

## ASSET PRICING BASICS

- So far we assumed that the SDF results from the optimization problem of the representative agent.
- (Some) SDF **exists** under much weaker conditions: it is enough that there is **no arbitrage**.
- Once we have a SDF, we can use it to price assets.
- This is the approach of Jiang et al. (2021).

## FISCAL CAPACITY

- Return to the formulation of the valuation problem:

$$B_t = \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} (T_{t+j} - G_{t+j})$$

- Here we understand everything as nominal and  $M_{t,t+j}$  is the SDF used to price nominal claims.
- Jiang et al. call the right hand side the **fiscal capacity** of the government.



## FISCAL CAPACITY

- To simplify the notation, define  $S_{t+j} := T_{t+j} - G_{t+j}$ , the primary surplus at time  $t + j$ .
- Rewrite the formula as

$$\begin{aligned} B_t &= E_t \sum_{j=1}^{\infty} M_{t,t+j} S_{t+j} \\ &= \sum_{j=1}^{\infty} (E_t M_{t,t+j} \cdot E_t S_{t+j}) + \sum_{j=1}^{\infty} \text{cov}_t (M_{t,t+j}, S_{t+j}) \\ &= \sum_{j=1}^{\infty} (E_t M_{t,t+j} \cdot E_t S_{t+j}) \\ &\quad + \sum_{j=1}^{\infty} \text{cov}_t (M_{t,t+j}, T_{t+j}) - \sum_{j=1}^{\infty} \text{cov}_t (M_{t,t+j}, G_{t+j}). \end{aligned}$$

## FISCAL CAPACITY

- Fiscal capacity depends on three terms:
- $\sum_{j=1}^{\infty} (E_t M_{t,t+j} \cdot E_t S_{t+j})$  – the expected value of future primary surpluses discounted by the **risk-free rate**.
- $\sum_{j=1}^{\infty} \text{cov}_t (M_{t,t+j}, T_{t+j})$  – the covariance between the SDF and future taxes.
- $\sum_{j=1}^{\infty} \text{cov}_t (M_{t,t+j}, G_{t+j})$  – the covariance between the SDF and future government spending.

## FISCAL CAPACITY

- In the risk free world, the first term is the only one that matters.
- In the risk free world, fiscal capacity is determined only by the ability to generate current and future surpluses.
- The second and the third term reflect the riskiness of the surplus process.
- If taxes are high when the SDF is low, the second term **lowers** the fiscal capacity.
- If government spending is high when the SDF is low, the third term **lowers** the fiscal capacity.
- Tax revenue is usually procyclical, government spending is usually countercyclical – this lowers the fiscal capacity.

## FISCAL CAPACITY

- This suggests that the fiscal capacity is most likely **lower** than the expected value of future primary surpluses discounted by the risk-free rate.
- By how much?
- Jiang et al. (2021) quantify this for the US. They find that the second and the third term matter quantitatively.
- Is there a way to increase the fiscal capacity by financial engineering?
- This would require **insuring** bondholders against the risk of future taxes and spending. Is it feasible?
- We now follow Jiang et al. (2023) to illustrate it.

## FISCAL CAPACITY

- For simplicity assume that taxes to GDP  $\tau$  and government spending to GDP  $\gamma$  are constant.
- GDP growth is risky, i.i.d. with a mean of  $x$  and volatility of  $\sigma$ .
- Let  $P_t^T$  and  $P_t^G$  denote the present value of future tax revenues and government spending:

$$P_t^T = \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} T_{t+j} = \tau \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} Y_{t+j}$$

$$P_t^G = \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} G_{t+j} = \gamma \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} Y_{t+j}.$$

## FISCAL CAPACITY

- Given the simplifying assumptions, debt to GDP ratio is:

$$\frac{B}{Y} = \frac{\tau - \gamma}{r^f + \text{risk premium on GDP} - x}.$$

## FISCAL CAPACITY

- Consider the following parametrization:
  - Taxes to GDP,  $\tau$  are 25%.
  - Government spending to GDP,  $\gamma$  is 22.5%.
  - Risk-free rate  $r^f$  is 1.5%.
  - Mean GDP growth rate  $x$  is 2%.
  - GDP risk premium is 3%.
  - Initial GDP is 10 trillion.
- Risk premium on GDP is GDP volatility times the price of risk (3 times 1)
- Stock market acts as a levered claim to the aggregate: the GDP risk premium equals the unlevered equity risk premium.

## FISCAL CAPACITY

- The value of the claim to GDP is  $10 \cdot \frac{1}{0.015+0.03-0.02} = 400$  trillion.
- The claim on the stream of surpluses is worth  $10 \cdot \frac{0.025-0.0225}{0.015+0.03-0.02} = 10$  trillion.
- Fiscal capacity of this economy is 10 trillion.
- It equals 100% of GDP.
- If we evaluated it using the risk-free rate (net of growth), we would get infinity.



## FISCAL CAPACITY

- The claim on surpluses is the claim on taxes net of the claim on government spending.
- The government cost of funding  $r_B$  can be written as

$$r_B = r_T \frac{P^T}{Y} \frac{Y}{B} - r_G \frac{P^T}{Y} \frac{Y}{B}.$$

- The risk exposure  $\beta$ , the covariance of a return with the SDF divided by the variance of the SDF is

$$\beta_B = \beta_T \frac{P^T}{Y} \frac{Y}{B} - \beta_G \frac{P^G}{Y} \frac{Y}{B}.$$

## FISCAL CAPACITY

- The formula

$$\beta_B = \beta_T \frac{p^T}{Y} \frac{Y}{B} - \beta_G \frac{p^G}{Y} \frac{Y}{B}$$

show that holding the risk exposure of gov. spending constant, if the government insures taxpayers (higher  $\beta_T$ ) – lower tax payment in high marginal utility state – there is less insurance of bondholders (higher  $\beta_B$ ).

## FISCAL CAPACITY

- In this example tax revenue and government purchases are proportional to GDP.
- The risk exposure of tax revenue is  $\beta_T = \beta_{GDP}$ .
- The risk exposure of government spending is  $\beta_G = \beta_{GDP}$ .
- Normalize  $\beta_{GDP} = 1$ .
- We have

$$\begin{aligned}\beta_B &= \beta_T \frac{P^T}{Y} \frac{Y}{B} - \beta_G \frac{P^G}{Y} \frac{Y}{B} \\ &= \frac{100}{10} - \frac{90}{10} = 1\end{aligned}$$

- Tax and spending claims are equally risky, but government debt has a positive beta of 1.

## FISCAL CAPACITY

- Investors who buy the government debt portfolio are **net long** a claim to output.
- The output risk in spending does not fully offset the output risk in tax revenue.
- Debt is a constant fraction of GDP, it inherits the risk properties of the GDP claim.
- The government's interest payments are as risky as GDP, because they are a constant fraction of GDP.

## FISCAL CAPACITY

- Usually  $\beta_T > \beta_Y > \beta_G$ .
- This is because tax revenue is more volatile than GDP, and GDP is more volatile than government spending.
- This means that **the average** tax revenue to output has to be higher to support the same amount of debt.
- See Jiang et al. (2020) for a quantitative analysis.

## FISCAL CAPACITY

- Using the risk-free rate to evaluate the fiscal capacity of the government is misleading.
- It requires that the risk exposure of the government debt,  $\beta_B$ , is zero.
- For that to be true we need

$$\begin{aligned}\beta_T &= \left( \frac{P^T}{Y} \frac{Y}{B} \right)^{-1} \left( \frac{P^G}{Y} \frac{Y}{B} \right) \beta_G \\ &= \frac{P^G}{P^G + B} \beta_G\end{aligned}$$

which is lower than  $\beta_G$  if debt is positive.

## FISCAL CAPACITY

- Go back to our example:  $\beta_G = 1$ ,  $P^G/Y = 90$ ,  $B/Y = 10$ .
- We need  $\beta_T = 0.9$  to insure bondholders.
- We had  $r^f = 1.5\%$  and risk premium on GDP of 3%.
- This meant that  $r_Y = r^f + RP = 4.5\%$ .
- We have  $r_T = 1.5\% + 0.9 \cdot 3\% = 4.2\%$ .
- The lower risk premium for the tax process reflects the fact that the tax rate is counter-cyclical.

## FISCAL CAPACITY

- What is the average tax revenue to output needed to sustain the debt?
- Recall that  $\frac{B}{Y} = 100\%$ ,  $\frac{P^G}{Y} = 9$ .
- We will use the formula

$$\frac{B}{Y} = \frac{T}{Y} \frac{P^T}{T} - \frac{P^G}{Y}$$

- We now have

$$\frac{P^T}{T} = \frac{1}{r_T - x} = \frac{1}{0.042 - 0.2} = 45.45.$$

$$\text{so } \frac{T}{Y} = 10/45.45 = 0.22.$$

- The average tax revenue to output ratio is 22%.



## FISCAL CAPACITY

- The previous example shows that the government can **on average** run a deficit of 0.5% of GDP.
- This is because the government provides insurance to bondholders by delivering positive surpluses when GDP growth is lower than average.
- Bondholders pay an insurance premium of 0.5% of GDP to receive relatively larger surplus payments when their marginal utility is high.
- But providing insurance is costly – it requires the government to have surpluses in recessions. Less room for output stabilization.

## CONVENIENCE YIELDS

- Sometimes government debt is **more** valuable than the sum of its discounted cash flows.
- This is because it provides **liquidity** and **safety** to investors.
- Similar to cash: we hold it although it has a negative real return, because we need it for transactions.
- We call the difference between the return on debt and the risk free rate the **convenience yield** of government debt.
- Think of it as of some extra benefit that makes investors willing to hold government debt despite low returns.

## CONVENIENCE YIELDS

- The convenience yield is nonnegligible, especially for the US.
- Krishnamurthy and Vissing-Jorgensen (2012) estimate convenience yield of 73 basis points per annum on average between 1926 and 2008 in the US.
- This is an important source of [seignorage](#) for the US government (0.25% of GDP).
- The convenience yield depends on debt to GDP ratio.

# CONVENIENCE YIELDS

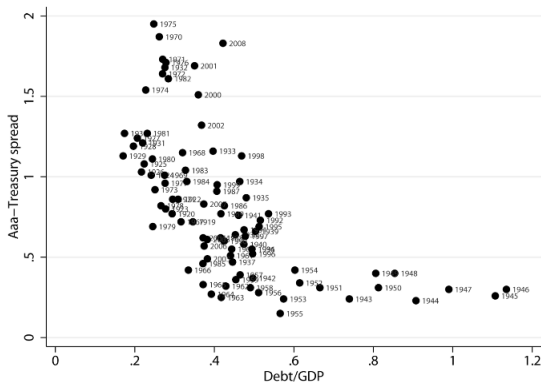


FIG. 1.—Corporate bond spread and government debt. The figure plots the Aaa-Treasury corporate bond spread (y axis) against the debt-to-GDP ratio (x axis) on the basis of annual observations from 1919 to 2008. The corporate bond spread is the difference between the percentage yield on Moody's Aaa long-maturity bond index and the percentage yield on long-maturity Treasury bonds.

Source: Krishnamurthy and Vissing-Jorgensen (2012)

## FISCAL CAPACITY

- We need to modify the IGBC to account for the convenience yield.

$$B_t = \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} (T_{t+j} - G_{t+j}) + \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} B_{t+j} \left(1 - e^{-\delta_{t+j}}\right)$$

- The new term  $K_{t+j} := B_{t+j} \left(1 - e^{-\delta_{t+j}}\right)$  represents the seignorage revenue from issuing debt.
-

## FISCAL CAPACITY

- Return to the example with  $\beta_T = \beta_G = \beta_Y = 1$ , but consider convenience yields.
- Previously we said that the risk-free rate is equal to interest rate on treasuries, 1.5%.
- If the convenience yield is 0.73%, the true risk-free rate is higher – 2.23%.
- The PDV of surpluses is

$$\frac{0.25 - 0.225}{0.0523 - 0.02} = 77\% \text{ of GDP.}$$

- The PDV of seignorage will depend on  $\beta_K$  and the convenience yield.
- Set the convenience yield to 0.73%.

## FISCAL CAPACITY

- If  $\beta_K = 1$  (seignorage varies proportionally with GDP), then  $r_K = 0.523$  and the PDV of seignorage is

$$\frac{0.0073}{0.0523 - 0.02} = 23\% \text{ of GDP.}$$

- This means that fiscal capacity is 100% of GDP.
- Two **counteracting** forces:
  1. Convenience yield generates seignorage.
  2. For a given interest rate, convenience yield means that the true risk-free rate is higher – this lowers the fiscal capacity.
- Extra surplus increases fiscal capacity by **less** than without the convenience yield.

## FISCAL CAPACITY

- Most likely  $\beta_K < 1$ .
- In the short run, convenience yields can be counter-cyclical (flight to safety).
- This increases the seignorage term, without affecting the risk-free rate.
- Lowering  $\beta_K$  to 0.584, increases fiscal capacity to 114% of GDP.



## FISCAL CAPACITY

- Jiang et al. use two approaches to estimate the fiscal capacity of the US.
- In the first approach they Congressional Budget Office (CBO) projections of tax revenue and non-interest spending for the next 31 years (2022-2052).
- CBO also forecasts interest rates and GDP.
- At the end of the projection horizon debt to GDP is 185%.

## FISCAL CAPACITY

- Given  $r^f = 1.5\%$ , the risk premium on GDP to 3% and the average growth rate 2%, annual surpluses would have to be 4.625% of GDP since 2052 to sustain debt to GDP of 185%.
- The present value of that is 35.2 trillion.
- The present value of the surpluses between 2022 and 2052 is -21.1 trillion (negative).
- The fiscal capacity in 2022 is 35.2-21.1 trillion = 14.1 trillion.
- This is 8.2 trillion **below** the 22.3 trillion in debt outstanding at the end of 2021.

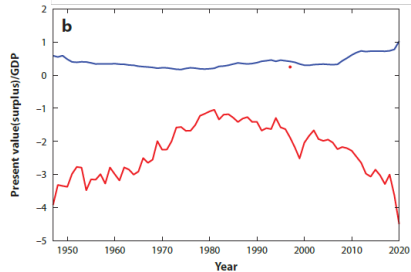
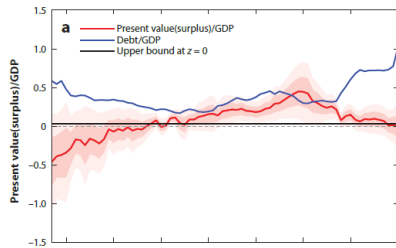
## FISCAL CAPACITY

- This is already a generous estimate of the fiscal capacity.
- It assumes that the government can start running surpluses in 2052.
- It assumes acyclicity of taxes and spending.
- Another problem: duration mismatch.
- Surpluses are far in the future, fiscal capacity sensitive to small changes in the risk-free rate.

## FISCAL CAPACITY

- The second approach: create forecast of cash flows using a VAR model.
- The model captures the cyclicalities of tax and spending ratios.
- It captures multiple aggregate sources of risk: inflation, interest rates, the price-to-dividend ratio in the stock market, and shocks to tax and spending rates.
- They calculate total fiscal capacity in two ways: (a) assume the discount rates are the same for taxes and spending, (b) model the SDF.

# FISCAL CAPACITY



Source: Jiang et al. (2023)

## BOND VALUATION PUZZLE

- These estimates suggest a much lower fiscal capacity than the market value of outstanding debt.
- Possible explanations:
  - Convenience yields?
  - Bubble?
  - Global safe asset supplier?
  - Mispricing?
  - Fiscal correction?
  - Large-scale asset purchases and financial repression?

## BOND VALUATION PUZZLE

- Similar calculations for other countries suggest that the US is an outlier.
- For example, for the UK after World War 2 fiscal capacity was 82% of GDP, but the debt to GDP ratio was 53%.
- It was different in 1729-1946 when fiscal capacity was 68% of GDP, but the debt to GDP ratio was 87%.
- Developing countries: procyclical surpluses, debt prices react strongly to fundamentals (unlike the US).