

AGGREGATE SHOCKS, TAXES, AND DEBT MANAGEMENT II

FISCAL AND MONETARY POLICY 2023

Piotr Żoch

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PLAN

- We return to the questions posed last week and try to answer them in a different setting.
- We study the problem of a government that wants to maximize welfare of a representative household.
- The government can **fully commit** to its announced policies. This commitment is credible.

LUCAS AND STOKEY (1983)

SETUP

- We study the following problem based on Lucas and Stokey (1983):
 - the representative household has preferences over consumption and labor (leisure);
 - production technology is linear in labor, there is no capital;
 - the government uses **distortionary** tax rate τ_t on **labor income** to finance *stochastic* purchases g_t (we assume g_t is a Markov process);
 - the households and the government can trade a full set of **Arrow securities** (state-contingent claims).
 - the government wants to finance g_t in a way that maximizes the welfare of the representative household.
- General equilibrium.

SETUP

- Price of securities no longer exogenous.
- The government's cares about welfare of the representative household.
- Think of the problem in two steps:
 1. for **given** government policies there exists some **competitive equilibrium**;
 2. the government picks policies that result in the "best" equilibrium.
- Key difference: policies not only have to satisfy the government's budget constraint, but also household's optimality conditions.
- **Primal approach**: we will look directly for allocations that maximize welfare and then think of policies that implement that.

COMPETITIVE EQUILIBRIUM

Competitive equilibrium

A competitive equilibrium given government policies $\tau(s^t)$ and $g(s^t)$ is a set of allocations $(c(s^t), \ell(s^t), a(s^{t+1}))$ and prices $(q(s^{t+1} | s^t), w(s^t))$ such that:

1. Given prices, allocations solve the household problem;
 2. Given prices, allocations solve the firm problem;
 3. The government budget constraint is satisfied in each state;
 4. All markets clear in each state.
- Note: this is not a proper definition of a competitive equilibrium, but it will do for our purposes.

HOUSEHOLD PROBLEM

- Household chooses consumption, labor supply and portfolio of Arrow securities to maximize expected utility:
- Household problem:

$$\begin{aligned} \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u \left(c \left(s^t \right), \ell \left(s^t \right) \right) \\ \text{s.t. } \forall s^t \quad c \left(s^t \right) + \sum_{s^{t+1} \geq s^t} q \left(s^{t+1} \mid s^t \right) a \left(s^{t+1} \right) \\ = \left(1 - \tau \left(s^t \right) \right) w \left(s^t \right) \ell \left(s^t \right) + a \left(s^t \right) \\ a \left(s^0 \right) = a_0 \end{aligned}$$

where $u(\cdot)$ is the utility function, $c(\cdot)$ consumption and $\ell(\cdot)$ labor.

HOUSEHOLD PROBLEM

- The sequence of household budget constraints can be written as a single constraint

$$\sum_{t=0}^{\infty} \sum_{s^t} q(s^t) c(s^t) = \sum_{t=0}^{\infty} \sum_{s^t} q(s^t) (1 - \tau(s^t)) w(s^t) \ell(s^t) + q(s^0) a_0$$

where $q(s^t)$ is now the time-0 price of one unit of goods in state s^t .

- Why? **Complete markets** are **equivalent** with Arrow securities and time-0 trading.

HOUSEHOLD PROBLEM

- Household problem:

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c(s^t), \ell(s^t)) \\ \text{s.t.} \quad & \sum_{t=0}^{\infty} \sum_{s^t} q(s^t) c(s^t) = \sum_{t=0}^{\infty} \sum_{s^t} q(s^t) (1 - \tau(s^t)) w(s^t) \ell(s^t) \\ & + q(s^0) a_0 \end{aligned}$$

- This is nice because there is only one constraint.
- We could have used the same trick last week.

FIRST ORDER CONDITIONS

- Write down the Lagrangian and differentiate with respect to $c(s^t)$ and $\ell(s^t)$ to get the first order conditions:

$$\left[c(s^t) : \right] \beta^t \pi(s^t) u_c(c(s^t), \ell(s^t)) = \lambda q(s^t)$$

$$\left[\ell(s^t) : \right] \beta^t \pi(s^t) u_\ell(c(s^t), \ell(s^t)) = -\lambda q(s^t) (1 - \tau(s^t)) w(s^t)$$

- Note: the Lagrange multiplier λ is **the same** for all states s^t .
- To simplify notation

$$\left[c(s^t) : \right] \beta^t \pi(s^t) u_c(s^t) = \lambda q(s^t)$$

$$\left[\ell(s^t) : \right] \beta^t \pi(s^t) u_\ell(s^t) = -\lambda q(s^t) (1 - \tau(s^t)) w(s^t)$$

IMPLEMENTABILITY CONSTRAINT

- The first order conditions:

$$\begin{aligned} \left[c \left(s^t \right) : \right] \quad & \beta^t \pi \left(s^t \right) u_c \left(s^t \right) = \lambda q \left(s^t \right) \\ \left[\ell \left(s^t \right) : \right] \quad & \beta^t \pi \left(s^t \right) u_\ell \left(s^t \right) = -\lambda q \left(s^t \right) \left(1 - \tau \left(s^t \right) \right) w \left(s^t \right) \end{aligned}$$

together with the budget constraint

$$\sum_{t=0}^{\infty} \sum_{s^t} q \left(s^t \right) c \left(s^t \right) = \sum_{t=0}^{\infty} \sum_{s^t} q \left(s^t \right) \left(1 - \tau \left(s^t \right) \right) w \left(s^t \right) \ell \left(s^t \right) + q \left(s^0 \right) a_0$$

allow us to write the **implementability constraint**:

$$\sum_{t=0}^{\infty} \sum_{s^t} \pi \left(s^t \right) \left[u_c \left(s^t \right) c \left(s^t \right) + u_\ell \left(s^t \right) \ell \left(s^t \right) \right] = u_c \left(s^0 \right) a_0$$

IMPLEMENTABILITY CONSTRAINT

- The **implementability constraint**

$$\sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) \left[u_c(s^t) c(s^t) + u_\ell(s^t) \ell(s^t) \right] = u_c(s^0) a_0$$

summarizes household optimality conditions and the budget constraint.

- The government can only choose allocations that satisfy this constraint.
- Captures the notion that the government's choices result in some **optimal** (given these choices) household behavior.

FIRM PROBLEM

- Output in this economy is produced by a representative price-taking firm.
- Linear production function that uses labor as an input.
- Firm problem is almost trivial – it chooses labor to maximize profits:

$$\max_{\ell(s^t)} A \ell(s^t) - w(s^t) \ell(s^t).$$

- In a competitive equilibrium we must have

$$w(s^t) = A.$$

GOVERNMENT BUDGET CONSTRAINT

- The **implementability constraint** is the household budget constraint (+ optimality conditions)
- The government should also be constrained by its budget constraint.
- We can ignore it and focus **directly** on the **resource constraint** of the economy:

$$A\ell(s^t) = g(s^t) + c(s^t)$$

where $A\ell(s^t)$ is the linear production technology.

- Why? **Walras' law** – the government budget constraint is redundant.

GOVERNMENT PROBLEM

- The government chooses **allocations** $c(s^t), \ell(s^t)$ to maximize the household's utility

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c(s^t), \ell(s^t))$$

subject to the **implementability constraint**:

$$\sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) [u_c(s^t) c(s^t) + u_\ell(s^t) \ell(s^t)] = u_c(s^0) a_0$$

and **resource constraints**:

$$\forall s^t \quad A\ell(s^t) = g(s^t) + c(s^t)$$

GOVERNMENT PROBLEM

- **Full commitment:** the government announces its state-contingent plans at time 0 and cannot change them later.
- Everyone else trusts the government and knows it will stick to its plans.

OPTIMALITY CONDITIONS

- The **first order conditions** for $s^t > s^0$ (we will return to s^0 later):

$$\begin{aligned}u_c(s^t) + \eta \left[u_{cc}(s^t) c(s^t) + u_{cl}(s^t) \ell(s^t) \right] &= \mu(s^t) \\ u_\ell(s^t) + \eta \left[u_{cl}(s^t) c(s^t) + u_{\ell\ell}(s^t) \ell(s^t) \right] &= -\mu(s^t) A\end{aligned}$$

where η is the Lagrange multiplier on the **implementability constraint** and $\mu(s^t)$ is the Lagrange multiplier on the **resource constraint**.

- There is also a resource constraint for each state:

$$A\ell(s^t) = g(s^t) + c(s^t)$$

- This gives us, for each state, three equations in three unknowns:
 $c(s^t), \ell(s^t), \mu(s^t)$.

OPTIMALITY CONDITIONS

- We also have a fourth unknown: η and one more equation: the [implementability constraint](#).
- Key: η is the only link between different states.
- Once we know it, $c(s^t)$, $\ell(s^t)$, $\mu(s^t)$ are determined for each s^t from 2 first order conditions and the resource constraint.
- For two different states with the same $g(s^t)$, the solution must be the same!

IMPLICATIONS

- What are the implications for taxes?
- From household first order condition we have

$$\left(1 - \tau \left(s^t\right)\right) w \left(s^t\right) = -\frac{u_{\ell} \left(s^t\right)}{u_c \left(s^t\right)}$$

and linear production technology implies $w \left(s^t\right) = A$ so

$$1 - \tau \left(s^t\right) = -\frac{u_{\ell} \left(c \left(s^t\right), \ell \left(s^t\right)\right)}{u_c \left(c \left(s^t\right), \ell \left(s^t\right)\right)} \frac{1}{A}.$$

- We have solved for $c \left(s^t\right), \ell \left(s^t\right)$, we can get $\tau \left(s^t\right)$!

IMPLICATIONS

- Since for two different states with the same $g(s^t)$ consumption and labor are the same – taxes must be the same!
- **Conclusion:** very different from Barro (1979) – no history dependence.
- If government purchases are i.i.d., taxes are i.i.d. as well.
- Contrast with the **Barro (1979) case**: taxes were a random walk.
- Contrast with the **complete markets case last week**: taxes were constant.

IMPLICATIONS

- Why is the result different from the reduced form complete markets case?
- No full tax smoothing (unless with particular household preferences), because of a different objective.
- When g_t is high, the government could want to **increase** labor supply to finance it without a large drop in consumption.

EXAMPLE

- Let $u(\cdot) = \frac{c^{1-\gamma}}{1-\gamma} - \psi \ell$.
- Then $u_c = c^{-\gamma}$, $u_{cc} = -\gamma c^{-\gamma-1}$, $u_\ell = -\psi$, $u_{\ell\ell} = 0$, $u_{c\ell} = 0$.
- The first order conditions

$$u_c(s^t) + \eta \left[u_{cc}(s^t) c(s^t) + u_c(s^t) + u_{c\ell}(s^t) \ell(s^t) \right] = \mu(s^t)$$
$$u_\ell(s^t) + \eta \left[u_{c\ell}(s^t) c(s^t) + u_\ell(s^t) + u_{\ell\ell}(s^t) \ell(s^t) \right] = -\mu(s^t) A$$

are

$$[1 + \eta(1 - \gamma)] c(s^t)^{-\gamma} = \mu(s^t)$$
$$[1 + \eta] \psi = \mu(s^t) A$$

EXAMPLE

- For each state $s^t > s^0$ we have:

$$\frac{1 + \eta}{[1 + \eta (1 - \gamma)] A} \psi = c (s^t)^{-\gamma}$$

so consumption is constant.

- Taxes satisfy

$$1 - \tau (s^t) = \frac{\psi}{c (s^t)^{-\gamma}} \frac{1}{A} :$$

since $c (s^t)$ is constant, taxes are **constant** as well.

- With these preferences households supply labor fully elastically, fluctuations in g_t are absorbed by changes in production.

LESSONS

- **Remember:** the above result is **not** general.
- What is general is that here allocations and taxes are **history independent** – they are fully determined by the **current** state.
- What is the optimal path of debt? Similar logic as in our example last week: state-contingent debt that depends only on the next period state.

TIME-0

- We ignored one important issue: **first order conditions** are different for period 0.
- This is because of the $u_c(s^0) a_0$ term in the **implementability constraint**.
- First order conditions for s^0 are:

$$\begin{aligned} u_c(s^0) + \eta \left[u_{cc}(s^0) c(s^0) + u_c(s^0) + u_{c\ell}(s^0) \ell(s^0) - u_{cc}(s^0) a_0 \right] &= \mu(s^0) \\ u_\ell(s^0) + \eta \left[u_{c\ell}(s^0) c(s^0) + u_\ell(s^0) + u_{\ell\ell}(s^0) \ell(s^0) - u_{c\ell}(s^0) a_0 \right] &= -\mu(s^0) A \end{aligned}$$

- Time-0 allocations will be different from other periods, even with the same level of g_t .

TIME-0

- The government has incentives to play with taxes in period 0 to change $q(s^0)$ and thus affect the value of debt/assets.
 - For example: it could try to make debt due at time 0 worthless.
 - This is **good**, because it allows the government to create **lower** tax distortions in the future.
- No similar effect in other periods, forward looking agents take it into account.

TIME INCONSISTENCY

- **Time consistency problem**: if the government could re-optimize at future dates, period 1 would be like period 0.
- Same logic applies to all other periods.
- But forward-looking agents would know the government is tempted to do it every period!
- This is actually the essence of Lucas and Stokey (1983).
 - Is there a way to design debt portfolio (maturities etc.) to ensure the government will **not be tempted** to re-optimize?