# GOVERNMENT PURCHASES MULTIPLIER: THEORY AND EMPIRICAL RESULTS

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# BEYOND THE NGM

# REAL RATES

- Consider the Euler equation

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta\left(1 + r_{t+1}\right).$$

- We said that consumption growth is determined by the real interest rate.
- In NGM the real rate was tied to the marginal product of capital:

$$r_t = F_1(k_t, n_t) - \delta.$$

## REAL RATES

- Suppose that the government can somehow control the real rate and keeps it constant at the steady state level $r = \beta^{-1} - 1$.
- Then the Euler equation becomes

$$\frac{u'(c_t)}{u'(c_{t+1})} = 1,$$

  which implies that consumption is constant.
- In this case the multiplier will depend only on the response of $i_t$ to $g_t$.
- If investment is constant, then the multiplier will be equal to one.

# REAL RATES

- Consider a special case with no capital in this economy (or: fixed capital).
- The multiplier depends only on the behavior of the real rate.
- Nothing else matters.
- This is what happens in a large class of representative agent New Keynesian models (RANK) (which we will discuss later).
- Monetary policy response to government purchases is crucial, because it affects the real rate. See Woodford (2011).

# REAL RATES

- Note: this holds regardless of how the government finances its spending.
- It is not the balanced budget multiplier.
- This result hinges on the presence of the Euler equation for aggregate consumption and

$$y_t = c_t + g_t.$$

## TAKING STOCK

- We saw that multipliers in a NGM seem to be at odds with the usual Keynesian cross analysis.
- We also noticed the key role of real rates in determining the multiplier.
- Does it mean that the Keynesian cross analysis was invalid and we cannot have a simple framework to analyze fiscal policy?
- Not really, but we need to be more careful and take into account the intertemporal nature of consumption-savings decisions.

# AUCLERT, ROGNLIE, AND STRAUB (2024):
# THE INTERTEMPORAL KEYNESIAN CROSS (IKC)

# IKC

- Key insight: intertemporal marginal propesnsities to consume (iMPCs) are sufficient statistics for the effect of fiscal policy.
- Fiscal multipliers depend on the interaction of public deficits and iMPCs.
- If we want to know the effect of fiscal policy, we should measure iMPCs in the data - or at least use models that generate iMPCs roughly consistent with the data.

## HOUSEHOLD

- Household $i$ solves:

$$\max_{\{c_{i,t},b_{i,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u\left(c_{i,t}\right) - \nu\left(h_{i,t}\right)\right)$$

$$b_{i,t} + c_{i,t} = \left(1 + r_t^B\right) b_{i,t-1} + z_{i,t} \left(1 - \tau_t\right) \frac{W_t}{P_t} h_{i,t}$$

$$b_{i,t} \geq \underline{b}$$

- $b_{i,t}$ are real assets, $r_t^B$ is the real interest rate, $P_t$ is the price level, $W_t$ is the nominal wage,
- $z_{i,t}$ is the idiosyncratic shock to income, $\mathbb{E}\left[z_{i,t}\right] = 1$.
- $h_{i,t}$ taken as given - households committed to supply any quantity that is demanded (labor unions in the background).

## PRODUCTION

- A representative firm solves:

$$\max_{h_t} \quad P_t y_t - W_t h_t$$

$$y_t = A_t h_t$$

- $h_t$ is a bundle of labor supplied by labor unions $\ell \in [0,1]$, $h_t = \left(\int_0^1 h_{\ell,t}^{\frac{\sigma_W - 1}{\sigma_W}} d\ell\right)^{\frac{\sigma_W}{\sigma_W - 1}}$.
- Each union supplies $h_{\ell,t} = \int_0^1 z_{i,t} h_{i,\ell,t} d\ell$ with the allocation rule $h_{i,\ell,t} = l(z_{i,t}) h_{\ell,t}$ such that $\int z_{i,t} l(z_{i,t}) di = 1$. We have $h_{i,t} = \int_0^1 h_{i,\ell,t} d\ell$.
- Unions set nominal wage growth to maximize HH welfare subject to adjustment costs → there is some wage Phillips curve.
- What really matters is that households do not make labor supply choices (see Chiang and Zoch, 2024, Appendix D.1).

## GOVERNMENT

- The government sets bounded paths of government purchases $g_t$, tax revenue $T_t$, and government debt $b_t^G$ that satisfy:

$$b_t^G = \left(1 + r_t^B\right) b_{t-1}^G + g_t - T_t$$

- Tax revenue $T_t = \tau_t \cdot y_t$.
- We assume the government controls the real interest rate $r_t^B$ directly.
- Interpretation: there is a nominal interest rate rule that sets $i_t = r_t^B + E_t \pi_{t+1}$, where $\pi_t$ is inflation.
- This allows us to sidestep the discussion of inflation dynamics.

# EQUILIBRIUM

- Sequences of prices and allocations such that agents optimize, government budget constraint holds, and markets clear in every period:

$$\int c_{i,t} di + g_t = y_t,$$
$$\int b_{i,t} di = b_t^G.$$

# INCOME DISTRIBUTION

- Recall that individual income is $z_{i,t}\left(1-\tau_t\right)\frac{W_t}{P_t}h_{i,t}$.
- Optimality condition of the firm implies:

$$\frac{W_t}{P_t} = A_t \rightarrow \frac{W_t}{P_t}h_t = y_t$$

- With symmetric unions:

$$h_{i,t} = l\left(z_{i,t}\right)h_t$$

- Equilibrium after-tax income of household $i$ is:

$$z_{i,t}\left(1-\tau_t\right)\frac{W_t}{P_t}h_{i,t} = z_{i,t}l\left(z_{i,t}\right)\left(y_t - T_t\right).$$

# HOUEHOLD BLOCK

- Each household takes as given:
  - The sequence of real interest rates $\left\{r_t^B\right\}_{t=0}^{\infty}$.
  - The sequence of aggregate income net of taxes $\{y_t - T_t\}_{t=0}^{\infty}$.
  - Process for idiosyncratic productivity $z_{i,t}$.
  - Its initial wealth $b_{i,-1}$.

1. Solve the household problem for each household $i \rightarrow$ policy functions
2. Policy functions + idiosyncratic productivity process → evolution of distribution
3. Evolution of distribution + policy functions → aggregate consumption and savings.

## HOUEHOLD BLOCK

- We obtain aggregate functions

$$\mathcal{B}_t\left(\left\{r_s^B, y_s, T_s\right\}_{s=0}^{\infty}\right), \quad \mathcal{C}_t\left(\left\{r_s^B, y_s, T_s\right\}_{s=0}^{\infty}\right)$$

  such that

$$\int b_{i,t} di = \mathcal{B}_t\left(\left\{r_s^B, y_s, T_s\right\}_{s=0}^{\infty}\right), \quad \int c_{i,t} di = \mathcal{C}_t\left(\left\{r_s^B, y_s, T_s\right\}_{s=0}^{\infty}\right).$$

- We suppress the dependence on the initial conditions.
- Household block: $\boldsymbol{y}, \boldsymbol{T}, \boldsymbol{r} \to \boldsymbol{c}, \boldsymbol{b}$
- These functions embed the notion of household (and firm!) optimization.

## EQUILIBRIUM

- Market clearing conditions can be written as

$$
\begin{aligned}
y_t &= \mathcal{C}_t\left(\left\{r_s^B, y_s, T_s\right\}_{s=0}\right) + g_t, \\
b_t^G &= \mathcal{B}_t\left(\left\{r_s^B, y_s, T_s\right\}_{s=0}\right).
\end{aligned}
$$

- Using the government budget constraint to solve for $b_t^G$ and applying Walras' Law we can find general equilibrium allocations given government policies $\{g_t, T_t\}_{t=0}^{\infty}$ as $\{y_s\}_{t=0}^{\infty}$ that solves the system of equations.

$$
y_t = \mathcal{C}_t\left(\left\{r_s^B, y_s, T_s\right\}_{s=0}\right) + g_t
$$

# EQUILIBRIUM

- In a stacked form:

$$\underbrace{\boldsymbol{y}}_{\text{aggregate income}} = \underbrace{\mathcal{C}\left(\boldsymbol{r^B}, \boldsymbol{y}, \boldsymbol{T}\right) + \boldsymbol{g}}_{\text{aggregate demand}}.$$

# FISCAL POLICY

- We will now consider the effect of fiscal policy on the economy around the steady state.
- Notation: $d\boldsymbol{g} = \{dg_t\}_{t=0}^{\infty}$ and $d\boldsymbol{T} = \{dT_t\}_{t=0}^{\infty}$.
- Assume monetary policy is such that the real rate is constant at $r^B$.
- Focus on policies with PDV = 0, i.e. such that

$$\sum_{t=0}^{\infty} \left( \frac{1}{1 + r^B} \right)^t (dg_t - dT_t) = 0$$

# RESPONSE TO FISCAL POLICY

- Totally differentiate the goods market clearing condition around the steady state to get:

$$dy_t = \sum_{s=0}^{\infty} \left[ \frac{\partial \mathcal{C}_t}{\partial y_s} dy_t + \frac{\partial \mathcal{C}_t}{\partial T_s} dT_s \right] + dg_t$$

- Let $\mathbf{C}_y$ be the Jacobian of $\mathcal{C}$ with respect to $\boldsymbol{y}$ and $\mathbf{C}_T$ the Jacobian of $\mathcal{C}$ with respect to $\boldsymbol{T}$: $\mathbf{C}_{y\,t,s} = \frac{\partial \mathcal{C}_t}{\partial y_s}$, $\mathbf{C}_{T\,t,s} = \frac{\partial \mathcal{C}_t}{\partial T_s}$:

$$d\boldsymbol{y} = \mathbf{C}_y d\boldsymbol{y} + \mathbf{C}_T d\boldsymbol{T} + d\boldsymbol{g}$$

- Recall that after-tax income of each household was proportional to $y_t - T_t$. We thus have $\mathbf{M} := \mathbf{C}_y = -\mathbf{C}_T$.

# INTERTEMPORAL KEYNESIAN CROSS

- We can write the system of linearized equilibrium conditions as

$$d\boldsymbol{y} = \mathbf{M}\left(d\boldsymbol{y} - d\boldsymbol{T}\right) + d\boldsymbol{g},$$

- Auclert, Rognlie, and Straub (2024) call this system the Intertemporal Keynesian Cross (IKC).
- We need to know "only" $\mathbf{M}$ to solve for the effect of fiscal policy $\boldsymbol{g}, \boldsymbol{T}$ on the economy.
- The matrix $\mathbf{M}$ is a sufficient statistic for the effect of fiscal policy.
- Example: two economies with different processes for idiosyncratic income, but with the same $\mathbf{M}$ $\rightarrow$ fiscal multipliers are the same.
- Challenge: $\mathbf{M}$ is an infinite-dimensional object.

# INTERTEMPORAL MARGINAL PROPENSITY TO CONSUME

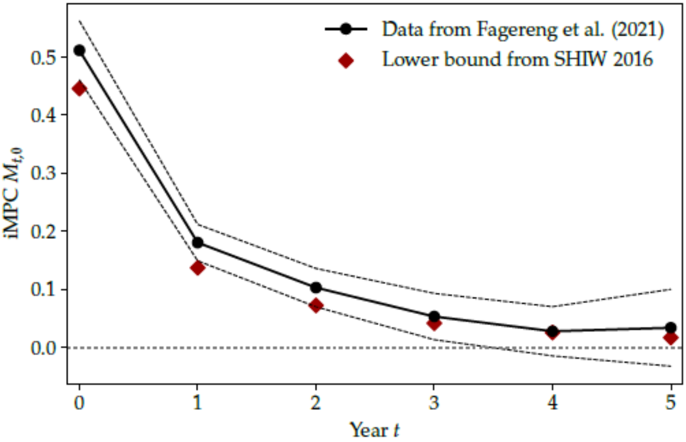
- $\mathbf{M}$ is a matrix of intertemporal marginal propensities to consume (iMPCs).
- It describes how aggregate consumption at $t$ responds to changes in after-tax income at $s$.
- We established it is a sufficient statistic for the effect of fiscal policy in this class of models...
- ...but it is an infinite-dimensional object - how can we even measure it?

# INTERTEMPORAL MARGINAL PROPENSITY TO CONSUME

- Some evidence on responses of individual consumption to income shocks e.g. Fagereng et al. (2021) (lottery winners)
- The first column of $\mathbf{M}$: average of individual responses to an unexpected income shock, weighted by after-tax income in the year of the income shock.
- Very little evidence on other entries of $\mathbf{M}$.
- We need to impose more structure to estimate the entire matrix.

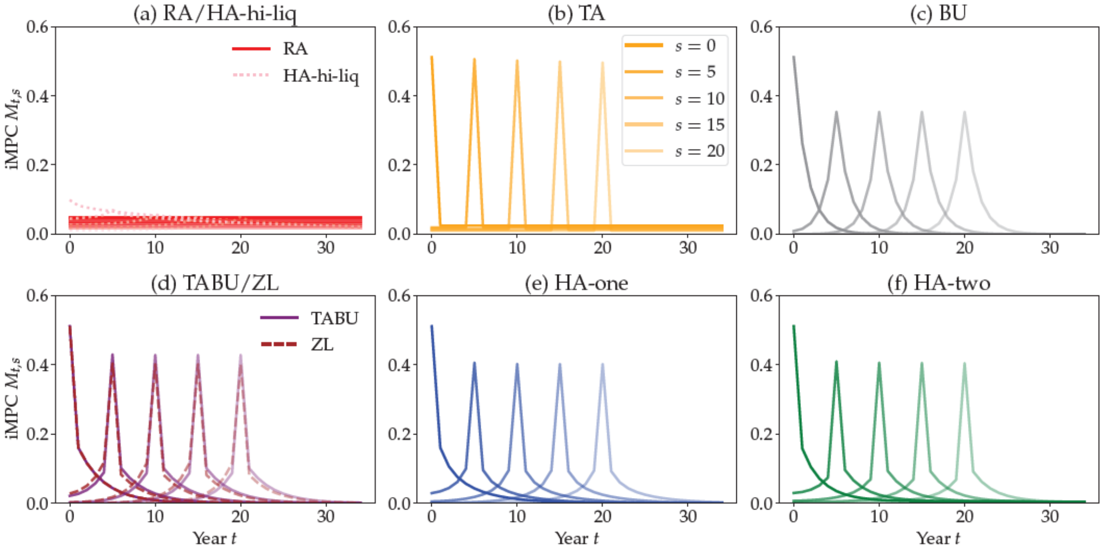
# INTERTEMPORAL MARGINAL PROPENSITY TO CONSUME

Figure 1: iMPCs in the Norwegian and Italian data



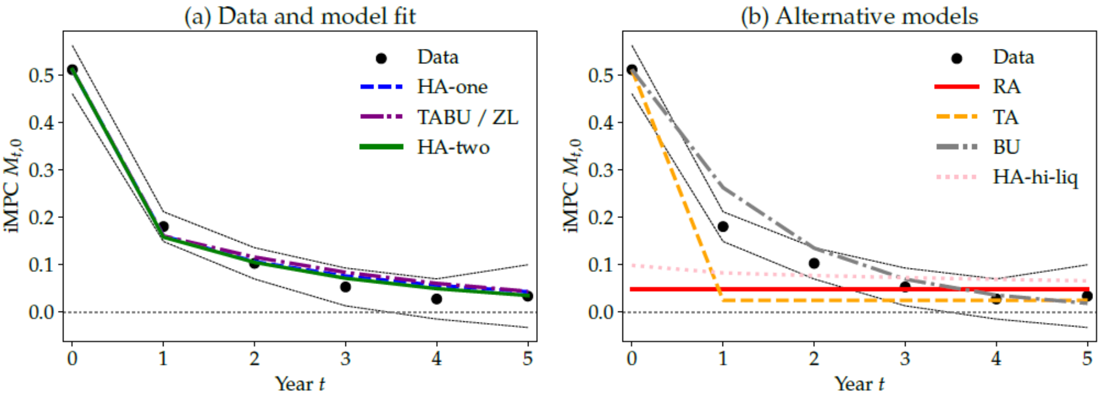

Source: Auclert et al. (2024)

## IMPCS VISUALLY




Source: Auclert et al. (2024)

# IMPCS VISUALLY




Source: Auclert et al. (2024)

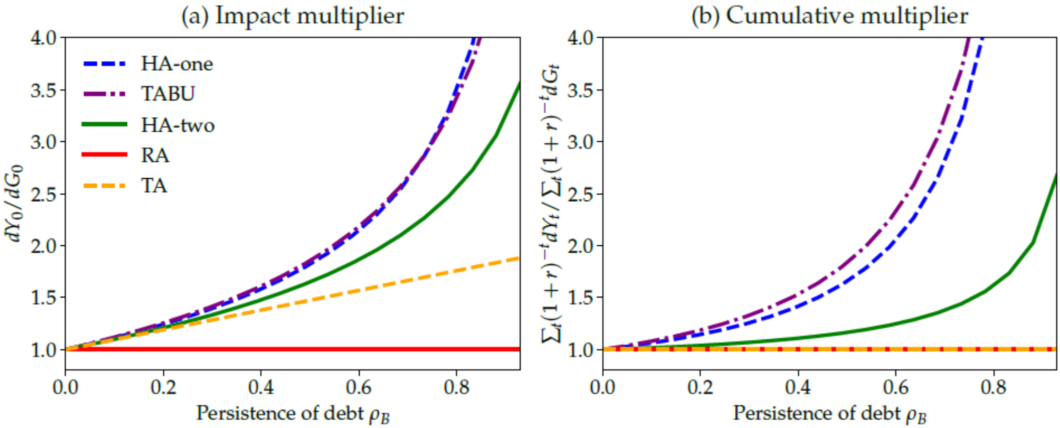
# BALANCED BUDGET

- In a old-fashioned Keynesian model, for $dg = dT$ we have $dy = dg$.
- Same holds in the IKC with $d\boldsymbol{g} = d\boldsymbol{T}$:

$$
\begin{aligned}
d\boldsymbol{y} &= d\boldsymbol{g} + \mathbf{M}d\boldsymbol{y} - \mathbf{M}d\boldsymbol{T} \\
&= d\boldsymbol{g} + \mathbf{M}d\boldsymbol{y} - \mathbf{M}d\boldsymbol{g} \\
&= (\mathbf{I} - \mathbf{M})\, d\boldsymbol{g} + \mathbf{M}d\boldsymbol{y} \\
\implies d\boldsymbol{y} &= d\boldsymbol{g}
\end{aligned}
$$

- This is a general property of the IKC. Holds irrespective of the details of the model, as long as we can write it in this form.

# MULTIPLIERS




Source: Auclert et al. (2024)

# EMPIRICAL RESULTS

# USUAL APPROACHES

- How to measure the multiplier?
- Ramey (2019) distinguishes three main approaches:
  1. aggregate country-level time series or panel estimates
  2. calibrated or estimated structural models
  3. subnational geographic cross-section or panel estimates
- Common problem - identificiation: the time series approaches requires exogenous variation in policy.
  - structural vector autoregressions and natural experiment methods, combined with narrative methods.
  - instruments often have low correlation with the fiscal variable they are trying to explain

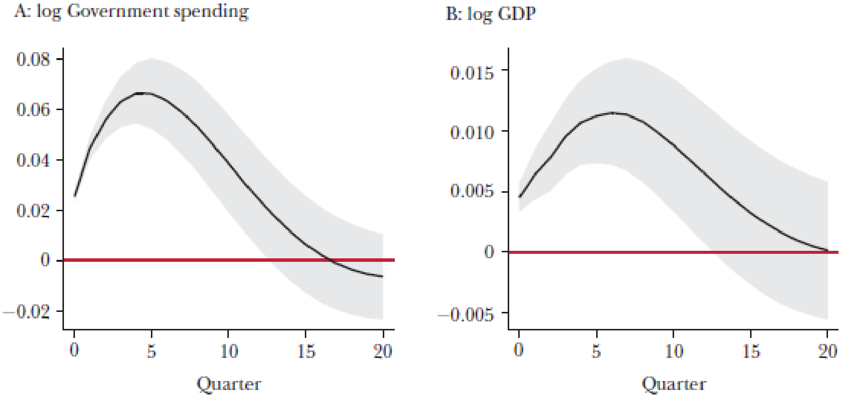
## USUAL APPROACHES

- The third approach estimates across subnational units (states or provinces).
- These analyses have stronger identification due to more plausible assumptions and relevant instruments.
- This approach does not directly lead to macroeconomic estimates.
- Cross-sectional estimating equations include a constant term, netting out macroeconomic effects. Parameters estimated are only relative effects: *"If State A is awarded \$1 more in defense prime contracts than the average state, by how much does its employment change relative to the average state?"*
- To infer national-level effects, researchers must use macroeconomic New Keynesian DSGE models.

# USUAL APPROACHES

- What even is “the” multiplier?
- The effects of government spending on output are dynamic. Which ratio to use?
- Blanchard and Perotti (2002) calculate the ratio of output response at horizon $h$ to the initial change of government spending.
- Mountford and Uhlig (2009) calculate the ratio of the cumulative response of output to the cumulative response of government spending, discounted.
- Another issue: regressions are often in logs, need to convert it to levels. What is the right ratio?

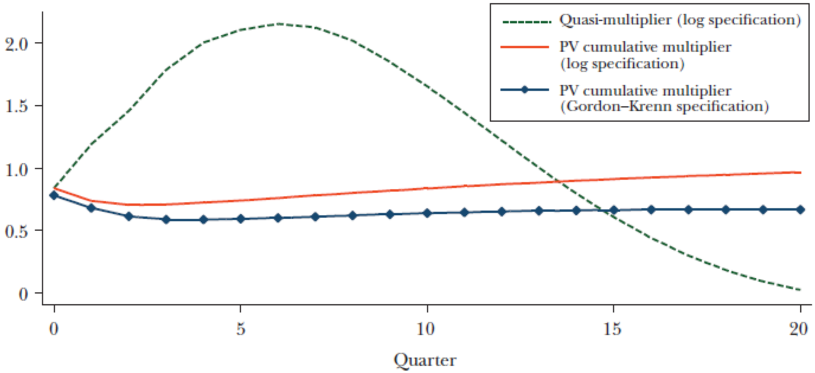
**Estimated Impulse Response Functions for a Shock to Government Purchases**




*Source:* Author.
*Note:* Estimated impulse responses based on structural vector autoregression (SVAR) estimates using quarterly data from 1939Q1– 2015Q4. The shaded area shows the 95-percent confidence bands. See the text and online appendix available with this paper at the journal website for more detail.

Ramey (2019)

*Figure 2*
**Alternative Definitions of Multipliers: Multipliers by Horizon**




*Source:* Author.
*Note:* The dotted and solid lines show multipliers calculated based on the log impulse responses shown in Figure 1. The dashed line shows the multiplier given by Blanchard-Perotti's (2002) method, which I call a *quasi-multiplier.* The solid line shows the the Mountford and Uhlig (2009) *present value (PV) cumulative multiplier.* The line with diamonds shows the PV cumulative multiplier using the impulse responses estimated using the Gordon–Krenn specification. See text and online appendix available with this paper at the journal website for more details.

Ramey (2019)

## SHOCK IDENTIFICATION

- Narrative methods
  - Military spending is often used. Defense spending is usually driven by political events and not macroeconomic events. Big increases in $g$ due to miliary buildups (eg. WW2).
  - Regress variables of interest on changes in miliary spending (Barro and Redlick 2009) or “war dates”.
  - For example Ramey and Shapiro (1998) used narrative techniques to create a dummy variable capturing major military buildups. They read through Business Week in order to isolate the political events that led to the buildups in order to create a series that was exogenous to the current state of the economy.
  - Potential problem with military events: other policies might be implemented at the same time, for example rationing or increases in taxes.
  - Timing of news vs spending?

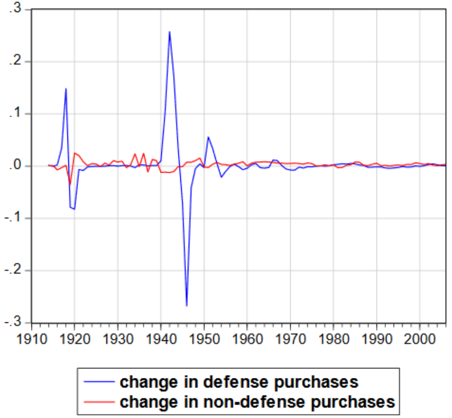



**Figure 1**

**Changes in Defense and Non-Defense Government Purchases, 1914-2006**

**(expressed as ratios to the previous year's GDP)**

Note: The figure shows the change in per capita real government purchases (nominal purchases divided by the GDP deflator), expressed as a ratio to the prior year's per capita real GDP. The blue graph is for defense purchases, and the red graph is for non-defense purchases by all levels of government. The government purchases data are from Bureau of Economic Analysis since 1929 and, before that, from Kendrick (1961). The GDP data are described at http://www.economics.harvard.edu/faculty/barro/data_sets_barro

Barro, Redlick (2009)

# SHOCK IDENTIFICATION

- Structural Vector Autoregressions
  - We will talk about in detail when we discuss monetary policy. Now just the basic idea.
  - Run regression (VAR)

    $$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + U_t$$

    where $Y_t$ is a vector of variables at time $t$. For example: government purchases, taxes and GDP.
  - $U_t$ is a vector of residuals. We have to relate them to “structural shocks”. To do so we impose restrictions. For example: unexpected changes in GDP do not lead to changes in purchases in the same quarter (it takes time to pass stimulus bill).
  - Then we track response of variables to “structural shocks”
  - Blanchard and Perotti (2002): gov. purchases not explained by some lagged variables $\rightarrow$ shocks.
  - This can make use of “narrative approach” - just add “war dummies”!

*Table 1*

**Estimates of Government Spending Multipliers Using Aggregate Data, No State Dependence**

*(almost all are cumulative multipliers, typically over horizons between 0 to 20 quarters)*

| *Method/Sample* | *Multipliers* | *Comments* |
|---|---|---|
| **A: Time series analysis** | | |
| Updated implementation of Blanchard and Perotti (2002) identified SVAR | | The tax response is positive for the 1939Q1–2015Q4 period, but is essentially 0 for the later periods. |
| 1939Q1–2015Q4 | 0.6 to 0.8 | |
| 1947Q1–2015Q4 | 0.6 to 0.7 | |
| Military news shocks, local projections Ramey and Zubairy (2018) military news | | Tax response is positive for 1939Q1–2015Q4 period. |
| 1889Q1–2015Q4 | 0.6 to 0.8 | SE from 0.04 to 0.06 |
| 1939Q1–2015Q4 | 0.7 to 0.8 | SE from 0.05 to 0.1 |
| 1947Q1–2015Q4 | 0.5 to 0.7 | SE from 0.15 to 0.2 |
| Ben Zeev and Pappa (2017) news, 1947Q1–2007Q4[a] | 1.1 to 2 | SE from 0.6 to 1 |
| Hall (2019), Barro and Redlick (2011)—based on regressions using annual defense spending. | 0.6 to 0.7 | The Barro–Redlick analysis nets out effects of changes in tax rates. |
| Mountford and Uhlig (2009), SVAR with sign restrictions | 0.65 | Deficit-financed increase in government spending. |
| Iltzetzki, Mendoza, and Végh (2013), Blanchard–Perotti identification in SVAR, quarterly data, 1960–2007, 44 countries high-income countries | 0.3 to 0.7 | |
| Corsetti, Meier, and Müller (2012) | 0.7 | Based on unconditional model results reported in their Figure 1. |
| Leigh et al. (2010), Guajardo, Leigh, and Pescatori (2014), 17 OECD countries, 1978–2009, narrative method identification of spending-based fiscal consolidations | 0.3 | |
| Alesina, Favero, and Giavazzi (forthcoming). Narrative analysis of austerity plans, 16 OECD economies from 1978–2014. | 0.3 | |

Ramey (2019)

*Table 1*

**Estimates of Government Spending Multipliers Using Aggregate Data, No State Dependence**

*(almost all are cumulative multipliers, typically over horizons between 0 to 20 quarters)*

| *Method/Sample* | *Multipliers* | *Comments* |
|---|---|---|
| **B: Estimated New Keynesian DSGE models** | | |
| Cogan et al. (2010), estimated Smets–Wouters DSGE model on US data | 0.6 to 0.7 | Based on my visual inspection of figures 2, 3, and 4. |
| Coenen et al. (2012), large-scale macro models used by central banks and IMF, United States and Europe | 0.7 to 1 | Based on the two-year cumulative multipliers shown in the upper left graph in figure 6. |
| Zubairy (2014), estimated medium-scale DSGE model on US data | 0.7 to 1.05 | Deficit financed, model features deep habits. |
| Leeper, Traum, and Walker (2017), estimated DSGE model on US data | 0.7 to 1.36 | Active monetary policy, table 7 |
| Sims and Wolff (2018a) | 1.07 | The multiplier above 1 is due to estimated complementarity of government spending with private consumption. |

Ramey (2019)

*Table 3*
**Multipliers for the American Recovery and Reinvestment Act (ARRA)**

| *Method/Sample* | *Peak cumulative multipliers within first 5 years* | *Comments* |
|---|---|---|
| Cogan et al. (2010) | 0.6 to 0.7 | |
| Coenen et al. (2012), large-scale macro models used by central banks and IMF, US, and Europe | | From figure 7. These are the peak instantaneous multipliers. |
| No monetary accommodation | 0.3 to 0.5 | |
| 1 year monetary accommodation | 0.4 to 0.6 | |
| 2 years monetary accommodation | 0.5 to 1.8 | |
| Drautzburg and Uhlig (2015), medium-scale New Keynesian DSGE model, with ZLB, credit constraints | 0.5 | Multipliers become negative in the long run because of the necessary increase in taxation. |
| Chodorow-Reich (forthcoming), based on cross-state estimates and theoretical arguments about the relationship between subnational and national multipliers at the ZLB. | | |
| Gross State Product multiplier | 1.7 to 2 | |
| Cost per job year | 2 job-years per $100K | |

Ramey (2019)

*Table 4*
**Conversion of Chodorow-Reich Estimates to Nationally Representative Estimates**

| | *Cumulative employment multiplier estimates (number of job-years created per $100K of ARRA spending)* |
|---|---|
| Chodorow-Reich (forthcoming) headline estimates (his table 1, column 4) | 2.01<br>(0.59) |
| Weighted estimates (using December 2008 population of state) | 1.15<br>(0.72) |
| Weighted estimates based on total spending, including induced spending by states | 0.89<br>(0.45) |

*Note: ARRA* is American Recovery and Reinvestment Act. Estimates presented in the last two rows are the author's estimates, based on Chodorow-Reich's programs and data in his forthcoming paper. See the text and online appendix for more detail and programs.

Ramey (2019)