GOVERNMENT PURCHASES MULTIPLIER: THEORY I

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OLD (?) KEYNESIAN MULTIPLIER

KEYNESIAN MULTIPLIER

- Recall your undergraduate macroeconomics course.
- A simple model of a closed economy

$$
\begin{aligned}
Y_t &= C_t + I_t + G_t \\
C_t &= \alpha_0 + \alpha_1 (Y_t - T_t) \\
I_t &= I_0
\end{aligned}
$$

  where Y_t is output, C_t is consumption, I_t is investment, G_t is government purchases of goods and services and T_t is taxes.
- We *assume* consumption in period t is a function of disposable income in period t and investment is exogenous.
- Here $a_1 \in (0, 1)$ is *the marginal propensity to consume*.

KEYNESIAN MULTIPLIER

- We can solve for equilibrium output Y_t:

$$Y_t = \frac{1}{1 - a_1} \left[\alpha_0 + I_0 + G_t - a_1 T_t \right]$$

- The government spending multiplier is

$$\frac{\partial Y_t}{\partial G_t} = \frac{1}{1 - a_1}.$$

- The size of the multiplier depends on the marginal propensity to consume. Notice the mutliplier is always larger than one because $a_1 > 0$.

KEYNESIAN MULTIPLIER

- The result above relied on the assumption that T_t is constant. What if it is not?
- Suppose $T_t = G_t$ (balanced budget) We then have

$$\frac{\partial Y_t}{\partial G_t} = 1.$$

- Notice that the multiplier is now equal to one regardless of the value of a_1.
- Haavelmo (1945) showed this result.

TAKING STOCK

- Even if the government spends exactly as much as it collects in taxes, output will increase one to one.
- If the government spends more than it collects in taxes, output will increase by more than one to one.
- The multiplier is larger than one because of the marginal propensity to consume greater than zero.

TAKING STOCK

- Too good to be true?
- What are the limitations of this model?
- What determines the key parameter a_1?
- Similar results tend to hold in more complex models (see “The Intertemporal Keynesian Cross” by Auclert et al. 2024).
- We will first study government purchases multiplier in a benchmark model – the Neoclassical Growth Model.

NEOCLASSICAL GROWTH MODEL - RECAP

NGM: HOUSEHOLD

- Representative household chooses $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) - v\left(n_t\right)\right]$$

subject to

$$c_t + \underbrace{k_{t+1} - (1 - \delta)\, k_t}_{i_t} = w_t n_t + r_t^k k_t \quad \text{for all } t \geq 0$$

given k_0.

  - $u(\cdot)$ strictly increasing, C^2, strictly concave, $u'(0) = \infty$, $u'(c)_{c\to\infty} = 0$;
  - $v(\cdot)$ strictly increasing, C^2, strictly convex, $v'(n)_{n\to\infty} = \infty$.

NGM: FIRM

- Representative producer chooses $\{y_t, k_t, n_t\}_{t=0}^{\infty}$ to maximize

$$y_t - r_t^k k_t - w_t n_t \quad \text{for all } t \geq 0$$

subject to

$$y_t = F(k_t, n_t) \quad \text{for all } t \geq 0.$$

  - $F(\cdot,\cdot)$ is CRtS, concave, increasing and satisfies Inada conditions.

NGM: COMPETITIVE EQUILIBRIUM

- **Competitive Equilibrium:** sequence of prices and allocations such that:
  - Households maximize utility subject to their budget constraints given prices;
  - Firms maximize profits subject to their production sets given prices;
  - All markets clear:

$$y_t = c_t + k_{t+1} - (1 - \delta) k_t$$

(+ labor and capital market – by using the same variable for demand and supply).

NGM: CHARACTERIZATION

- Lagrangian for the household problem:

$$\sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) - v\left(n_t\right)\right] + \lambda_t \left[w_t n_t + r_t^k k_t - c_t - k_{t+1} + (1-\delta)\, k_t\right]$$

- First order conditions

$$\begin{aligned}
\beta^t u'\left(c_t\right) &= \lambda_t \\
\beta^t v'\left(n_t\right) &= \lambda_t w_t \\
\lambda_t &= \lambda_{t+1}\left(1-\delta+r_{t+1}^k\right)
\end{aligned}$$

NGM: CHARACTERIZATION

- Solution to the household's problem described by the following:

$$u'(c_t) = \beta u'(c_{t+1}) \left(1 - \delta + r^k_{t+1}\right)$$
$$v'(n_t) = u'(c_t)\, w_t,$$

  the budget constraint

$$c_t + k_{t+1} - (1 - \delta)\, k_t = w_t n_t + r^k_t k_t,$$

  the initial condition (k_0 given), and the transversality condition

$$\lim_{t \to \infty} \beta^t (c_t)\, k_{t+1} = 0.$$

NGM: CHARACTERIZATION

- Solution to the firm's problem described by the following:

$$r_t^k = F_1(k_t, n_t)$$
$$w_t = F_2(k_t, n_t),$$

and the production function

$$y_t = F(k_t, n_t).$$

NGM: CHARACTERIZATION

- **Equilibrium conditions**:

$$u'(c_t) = \beta u'(c_{t+1})\left[1 - \delta + F_1(k_{t+1}, n_{t+1})\right] \tag{EE}$$

$$v'(n_t) = u'(c_t) F_2(k_t, n_t) \tag{LS}$$

$$k_{t+1} = F(k_t, n_t) + (1 - \delta) k_t - c_t \tag{LoM}$$

  + the transversality condition which pins down the solution, and the initial condition (k_0 given).

- Sequences of $\{c_t, n_t, k_t\}$ that satisfy these conditions constitute allocations in a competitive equilibrium.

COMMENTS

- Tradeoffs in the model:
  - consumption today c_t vs. consumption tomorrow c_{t+1};
  - consumption vs. leisure.

NEOCLASSICAL GROWTH MODEL - GOVERNMENT PURCHASES

GOVERNMENT PURCHASES

- We add government purchases, g_t to the model.
- They do not affect utility, they just use up g_t units of goods each period.
- They are financed by lump-sum taxes: $T_t = g_t$ in each period.
- BC of the HH is:

$$c_t + i_t = w_t n_t + r_t^k k_t - T_t \quad \text{for all } t \geq 0.$$

- Market clearing condition is:

$$c_t + i_t + g_t = y_t.$$

- Note: in this model it does not matter if the budget is balanced or g_t is financed by debt, as long as taxes are lump-sum.

MULTIPLIER

- Questions we might want to answer:
  - effects of a permanent change in (constant) $\bar{g}$ in the steady state;
  - effects of an unexpected change and transitory in g_t on impact (i.e. in period t);
  - cumulative effects of a change in g_t;
  - effects of a pre-announced change in g_t...

MULTIPLIER - LONG RUN

- We first discuss the long-run effect of permanent changes (steady state).
- Define $\eta := k/n$.
- By CRtS we have $F(k, n) = F(\eta, 1)\, n$ and $F_1(k, n) = F_1(\eta, 1)$.
- The Euler equation in the steady state is:

$$1 = \beta\left[1 - \delta + F_1(\eta, 1)\right] \tag{EE}$$

which pins down η.

MULTIPLIER - LONG RUN

- The other two equations (labor supply and law of motion for capital) give us a system of two equations in two unknowns, c and n:

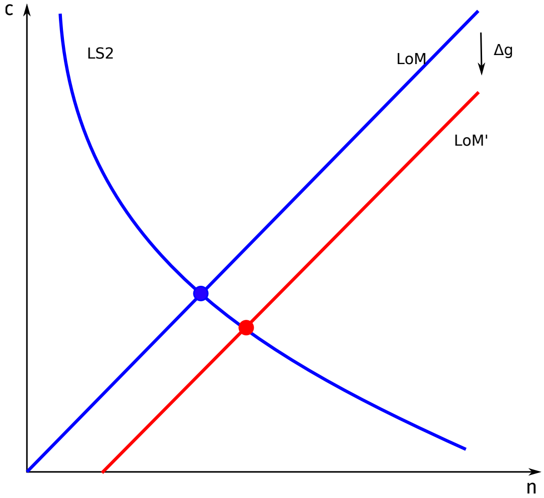
$$v'(n) = u'(c) F_2(\eta, 1) \tag{LS}$$
$$c = (F(\eta, 1) - \eta\delta) n - g \tag{LoM}$$

- Given our assumptions we can rewrite (LS) as (LS2)

$$c = \phi(n, F_2(\eta, 1)) \tag{LS2}$$
$$c = (F(\eta, 1) - \eta\delta) n - g \tag{LoM}$$

- (LS2) is decreasing in n, (LoM) is increasing in n. There exists unique $n = \tilde{n}(g)$ and $c = \tilde{c}(g)$ solving this system.

MULTIPLIER - LONG RUN



Permanent increase in government purchases.

MULTIPLIER - LONG RUN

- Note

$$\frac{dy}{dg} = F(\eta, 1) \frac{\partial n(g)}{\partial g}$$

- Key: response of labor supply to g.
- To get $\partial n(g) / \partial g$ define

$$H(n, g) \equiv \phi(n, F_2(\eta, 1)) - (F(\eta, 1) - \eta\delta) n + g$$

and apply Implicit Function Theorem to obtain

$$\frac{\partial n(g)}{\partial g} = \frac{1}{(F(\eta, 1) - \eta\delta) - \phi_1(n, F_2(\eta, 1))}$$

with $\phi_1(n, F_2(\eta, 1)) = \frac{v''(n)}{u''(c) F_2(\eta, 1)}$

MULTIPLIER - LONG RUN

- We can rewrite the multiplier (verify!) as.

$$\frac{dy}{dg} = \frac{1}{1 - \frac{i}{y} + \frac{c}{y}\frac{1}{\varphi\theta}},$$

  which depends on observable ratios (investment-to-output and consumption-to-output) and the utility function evaluated at the steady state, $\varphi \equiv \frac{v'(n)}{v''(n)n}$, $\theta \equiv -\frac{u''(c)c}{u'(c)}$.
- It is
  - Increasing in φ;
  - Increasing in θ;
  - Increasing in i/y;
  - Decreasing in c/y.

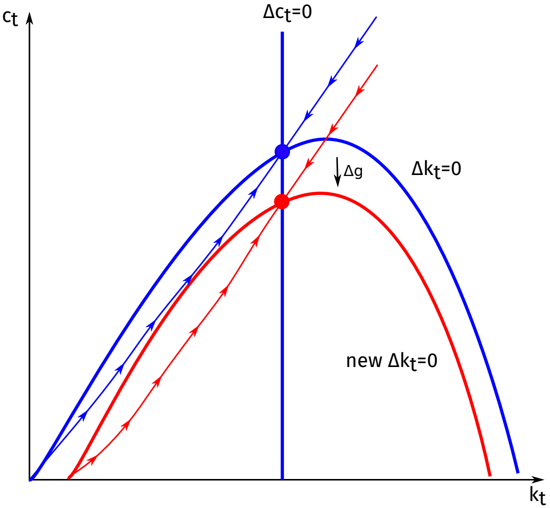
MULTIPLIER - LONG RUN

$$\frac{dy}{dg} = \frac{1}{1 - \frac{i}{y} + \frac{c}{y}\frac{1}{\varphi\theta}},$$

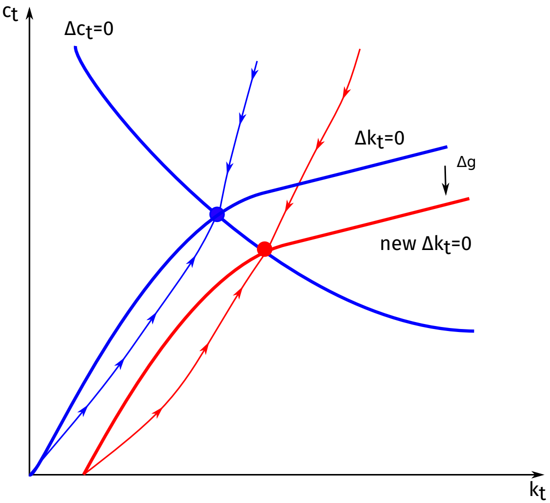
- Some remarks:
  - It **CAN** exceed 1.
  - It equals 0 when labor supply is fixed.
  - It cannot exceed 1 if investment is zero (for example economy with fixed exogenous capital or no capital).
  - It is big when a large increase in labor supply requires a small decrease in consumption.
- Remember: it was financed by lump sum taxes.

PHASE DIAGRAMS

- To visualize the effects of government purchases we can use phase diagrams.
- They are also useful to show short-run effects of government purchases (esp. in continuous time).
- To plot them:
  - Try to eliminate all static variables (in this model you can pick n_t)
  - Think of conditions ensuring that variables do not change over time in terms of other variables and parameters (i.e., find conditions for $c_t = c_{t+1}$ and $k_t = k_{t+1}$).
  - Represent these conditions graphically.
  - Think of dynamics of the system for points (c, k) where these conditions are not satisfied.
  - Locate the saddle path.



Phase diagram: permanent increase in government purchases by Δg in an economy with fixed labor supply.



Phase diagram: permanent increase in government purchases by Δg in an economy with flexible labor supply.

MULTIPLIER - LONG RUN

- Exercise: Suppose $u(c) = \frac{c^{1-\theta}-1}{1-\theta}$, $v(n) = \frac{n^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$, $i/y = 0.2$ and $c/y = 0.6$. What is the size of the multiplier if $\theta = 1$ (log-utility) and $\varphi \in \{0, 0.5, 1, 2, \infty\}$?
- Exercise: Suppose that we have $u(c, n) = \gamma \log c + (1-\gamma) \log(1-n)$. Rederive the formula for the long run multiplier.
- Exercise: Is the adjustment to the new steady state instantaneous or not? What happens on impact?

MULTIPLIER - LONG RUN

- Let's analyze an economy with fixed labor supply.
- In the steady state

$$1 = \beta\left[1 - \delta + F_1(k, 1)\right] \tag{EE}$$

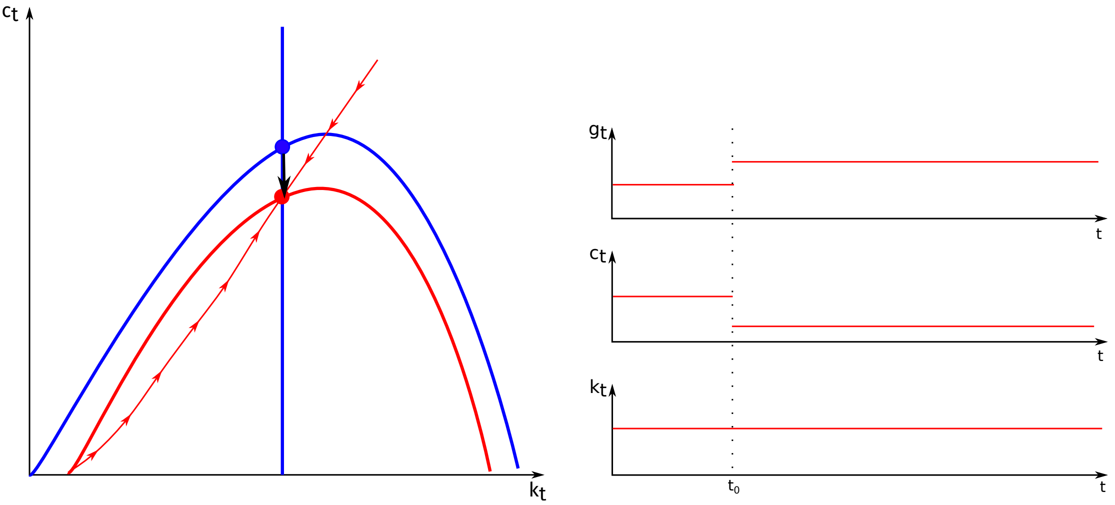
$$c = F(k, 1) - \delta k - g \tag{LoM}$$

- Notice g does not enter (EE). It does not affect incentives to save. Investment is not affected.
- k remains the same and because it is the only factor of production in this economy output does not change. We have $dc = -dg$.
- Government purchases crowd out private consumption one to one.
- Remember that lump-sum tax T has to be raised to finance g. It reduces HH's wealth.

TRANSITION

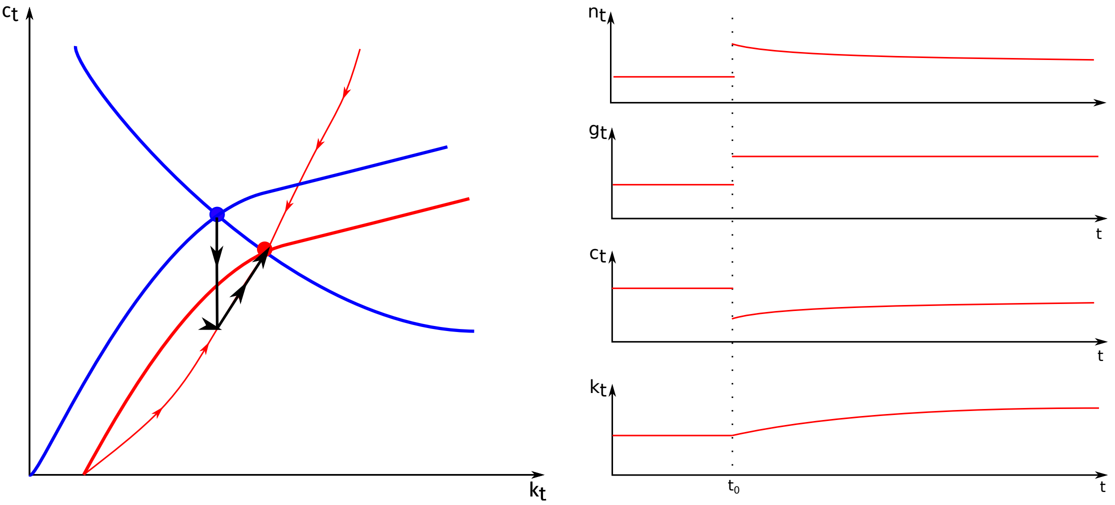
- We will study transition to the new steady state using phase diagrams.
- In discrete time it is a bit awkward, but we still can get some insight.
- Here we analyze unexpected permanent shocks.
- To proceed
  - Locate the new saddle path.
  - Plot arrows representing trajectories.
  - Make sure the economy jumps to the new saddle path on impact.
  - Very important: identify variables that **CANNOT** jump (almost *always* capital).
  - After the shock the economy follows the new saddle path to the new steady state.

TRANSITION



Phase diagram: permanent increase in government purchases by Δg in an economy with fixed labor supply: transition

TRANSITION



Phase diagram: permanent increase in government purchases by Δg in an economy with flexible labor supply: transition

MULTIPLIER - SHORT RUN

- More interesting exercise: temporary increase in g. For simplicity we start in the steady state with $g_t = g$ and consider an unexpected increase in g_0 that lasts for some periods.
- Think of a war or a stimulus package based on government purchases.
- Main result: difficult to generate $dy_0/dg_0 > 1$ in a baseline NGM. Why?
- After discussing theoretical results we will look at empirical findings.

MULTIPLIER - SHORT RUN

- What happens after g_0 goes up?
  - k_0 is fixed.
  - y_0 can grow only if n_0 increases.
- (LS) implies that

$$v'(n_0) = u'(c_0) F_2(k_0, n_0) \tag{LS}$$

so given our assumptions n_0 can increase **only** if c_0 does not increase.

MULTIPLIER - SHORT RUN

- c_0 has to fall (or at least not increase). When is it possible? Recall (EE)

$$\frac{u'(c_0)}{u'(c_1)} = \beta\left[1 - \delta + F_1(k_1, n_1)\right]$$

- We need $u'(c_0)/u'(c_1) > 1$. Intuitively, the economy has to return to the steady state eventually (formally: TVC). This can happen only if

$$\beta\left[1 - \delta + F_1(k_1, n_1)\right] > 1$$

  so the interest rate $r_1 := F_1(k_1, n_1) - \delta$ has to be higher than in the steady state.
- Remember: behavior of consumption determined by the real interest rate.

MULTIPLIER - SHORT RUN

- This is possible if $k_1/n_1 < k/n$. So, unless $n_1 > n$, $k_1 < k_0$.
- This requires fall in investment.
- To have $dy_0/dg_0 > 1$ we need more persistent shocks and high φ. Recall when permanent increase in g does not crowd out consumption.

MULTIPLIER - SHORT RUN

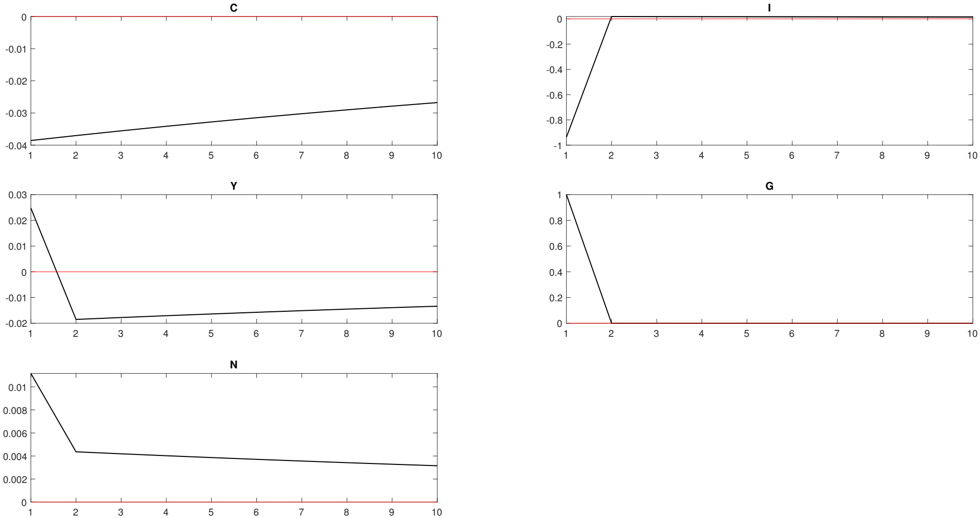
- In a standard NGM it is hard to get multipliers exceeding 1.
- With standard calibration dy_0/dg_0 is rather low.
- Suppose

$$u(c) = \log c, \quad v(n) = \kappa \frac{n^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$$

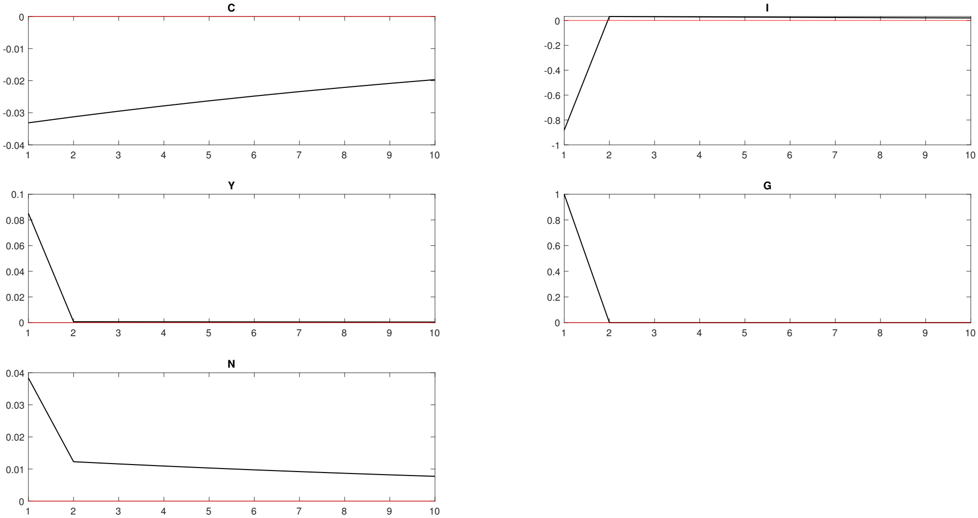
and parameter values be

δ	β	α	$\frac{g}{y}$
0.02	0.99	1/3	0.2

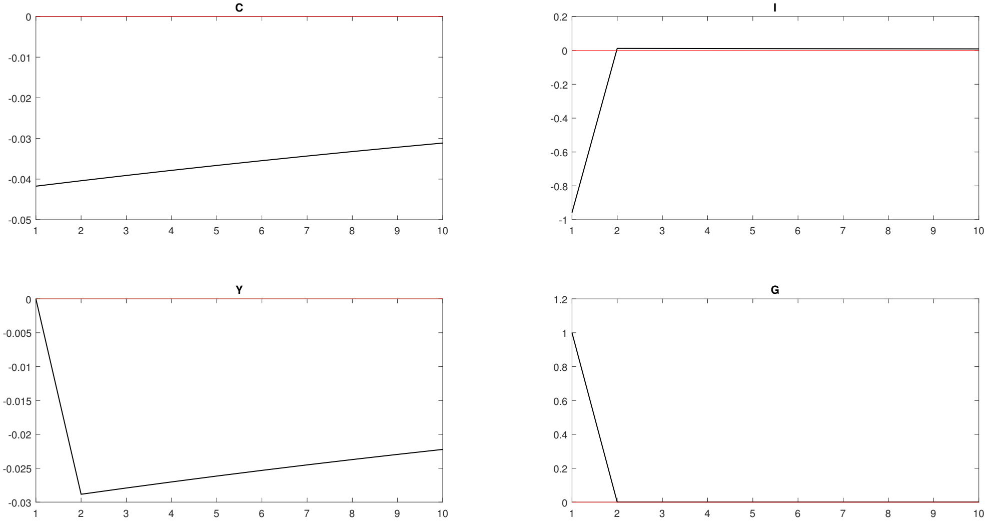
and $n = 1$ in the steady state.



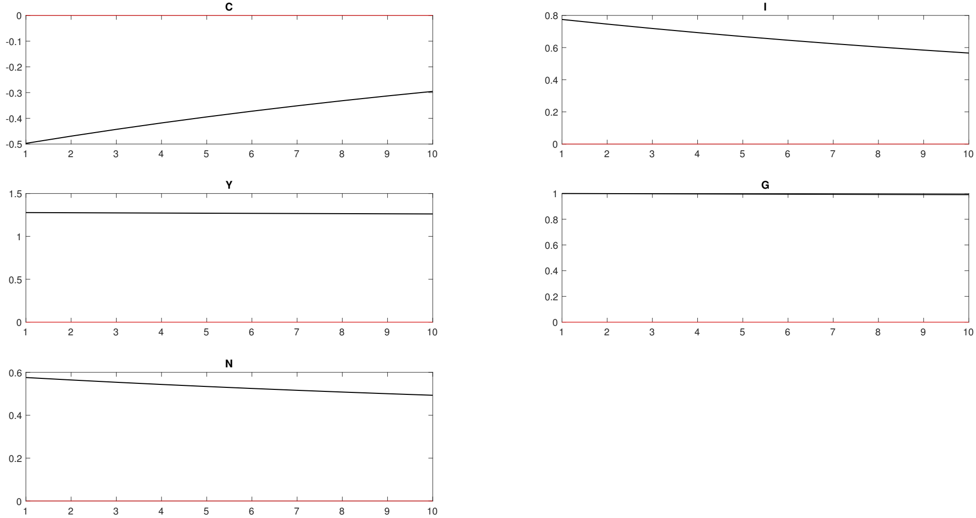
Dynamics after an unexpected increase in g: $\varphi = 1$



Dynamics after an unexpected increase in g: $\varphi = \infty$



Dynamics after an unexpected increase in g: $\varphi = 0$



Dynamics after an unexpected increase in g: $\varphi = \infty$ and very persistent increase in g (almost permanent)