DEBT SUSTAINABILITY I

Fiscal and Monetary Policy 2024

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March 17, 2025

PLAN

- We review several approaches to debt sustainability analysis.
- Can the government service its current outstanding debt? Or will a fiscal consolidation be needed?
- Is the outstanding public debt and its projected path consistent with those of the government's revenues and expenditures?

CLASSIC DEBT SUSTAINABILITY ANALYSIS

DEBT TO OUTPUT RATIO

- The government static budget constraint is

$$B_t = (1 + r_t)B_{t-1} + G_t - T_t.$$

  where B_{t-1} is the market value of all outstanding debt (all maturities), G_t is nominal government spending (net of interest expenses), T_t is nominal tax revenue, and r_t is net nominal return on government debt.
- We abstract from money growth (we can treat money as a type of government debt).
- Note: it is often called “flow budget constraint”.
- Note: is it really a “constraint”?

DEBT TO OUTPUT RATIO

- Divide by nominal GDP Y_t and rearrange to get

$$\frac{B_t}{Y_t} = (1 + r_t)\frac{B_{t-1}}{Y_{t-1}}\frac{Y_{t-1}}{Y_t} + \frac{G_t}{Y_t} - \frac{T_t}{Y_t}.$$

- Define

$$R_{t-j,t} := \prod_{k=1}^{j}\left(1 + r_{t-j+k}\right),$$

the cumulative return on debt from $t - j$ to t.

- Define

$$X_{t-j,t} := \prod_{k=1}^{j}\frac{Y_{t-j+k}}{Y_{t-j+k-1}},$$

the cumulative gross growth rate of GDP from $t - j$ to t.

DEBT TO OUTPUT RATIO

- For simplicity assume $B_0 = 0$.
- We can write the debt to output ratio as

$$\frac{B_t}{Y_t} = \sum_{j=0}^{t} \frac{G_{t-j} - T_{t-j}}{Y_{t-j}} \frac{R_{t-j,t}}{X_{t-j,t}}$$

- Debt to output ratio today is determined by:
    1. The past primary deficits to GDP ratios;
    2. The past returns on debt;
    3. The past growth rates of (nominal) GDP.
- We can use the formula above to understand what determines the current debt to output ratio.

CLASSIC DEBT SUSTAINABILITY ANALYSIS

- A version of the formula is often used to assess debt sustainability – "whether the government can service its debt".
- Warning: this is about the future, not the present. The fact that people use the formula to assess the current situation is often a red flag!
- Classic debt sustainability analysis looks at the "long run".

CLASSIC DEBT SUSTAINABILITY ANALYSIS

- Assume that the economy is in a steady state with a constant growth rate of GDP X, a constant rate of return R and a constant primary deficit to GDP ratio.
- What is the debt to output ratio consistent with the above?
- If the observed current debt to output ratio is below this level, it is sustainable.

CLASSIC DEBT SUSTAINABILITY ANALYSIS

- Classic debt sustainability analysis usually analyzes deterministic setups (or perfect foresight).
- In these setups, the appropriate R is the risk-free rate, R^f.
- Assuming the above, the formula in the steady state becomes

$$\frac{B}{Y} = \frac{G - T}{Y} \frac{X}{X - R^f}.$$

- For simplicity define $x := X - 1$ and $r^f := R^f - 1$ so we have

$$\frac{B}{Y} = \frac{G - T}{Y} \frac{1 + x}{x - r^f}.$$

- Hote: here X and R are one period rates (not cumulative).

CLASSIC DEBT SUSTAINABILITY ANALYSIS

$$\frac{B}{Y} = \frac{G - T}{Y} \frac{1 + x}{x - r^f}$$

- Example: the Treaty of Maastricht set the limit of the debt to output ratio at 60% and the deficit to output ratio at 3%. What must be the growth rate of GDP and the risk-free rate for this to be sustainable?

CLASSIC DEBT SUSTAINABILITY ANALYSIS

$$\frac{B}{Y} = \frac{G - T}{Y} \frac{1 + x}{x - r^f}$$

- For $B/Y = 0.6$ and $G - T/Y = 0.03$ we have $\frac{0.6}{0.03} = 20$.
- We get $\frac{1+x}{x-r^f} = 20$.
- For small x we have $x \approx r^f + 0.005$, so the economy has to grow at 0.5% above the risk-free rate per year for the debt at the limit to be sustainable with the largest allowed deficit.
- If the actual growth rate is lower, the debt to output ratio will be larger, even if the deficit is at the limit.

CLASSIC DEBT SUSTAINABILITY ANALYSIS

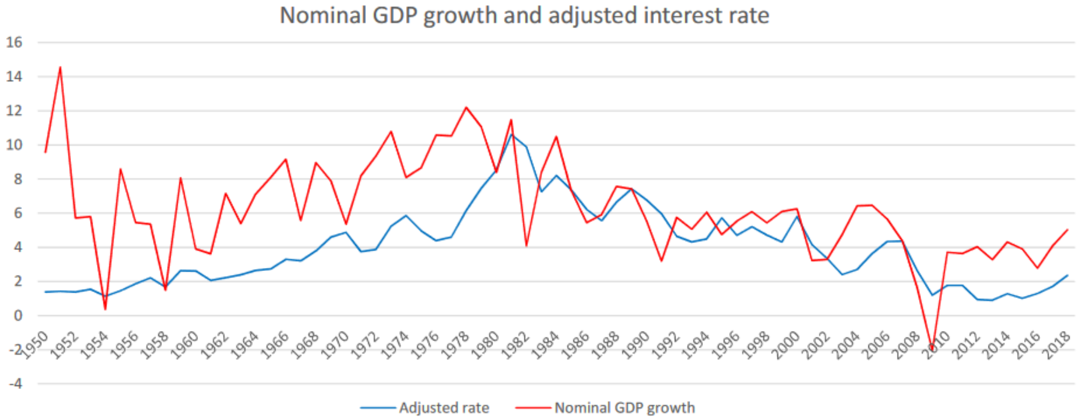
$$\frac{B}{Y} = \frac{G-T}{Y}\frac{1+x}{x-r^f}$$

- The key role of $x - r^f$.
- Depending on the sign of $x - r^f$ you require either surpluses or deficits to keep the debt to output ratio constant.
  - If $x < r^f$ you need surpluses.
  - If $x > r^f$ you can have deficits.
- A big "r versus g" debate (we have x instead of g).

CLASSIC DEBT SUSTAINABILITY ANALYSIS

- If $x > r^f$ it seems there is no fiscal cost to debt.
- Blanchard (2019) argues that $x > r^f$ is a norm, not an exception.
- But what is r^f? How to measure it? Blanchard (2019) looks at the 1-year US Treasury bill rate, the 10-year US Treasury bond rate, adjusts for various maturities...
- Note: it does not necessarily mean that it is optimal to have deficits.

RATES IN THE US



Source: Blanchard (2019)

CLASSIC DEBT SUSTAINABILITY ANALYSIS

- It only defines what long-run debt is for a given long-run primary balance (or vice versa) if stationarity holds, or defines lower bounds on the short-run dynamics of the primary balance.
- It does not connect the outstanding initial debt of a particular period with the steady state.
- There might be multiple paths of debt that do not violate the intertemporal government budget constraint (IGBC), some of them can even go to infinity (but slowly enough)!
- IGBC: the value of debt is equal to the present discounted value of future primary surpluses.

INTERTEMPORAL GOVERNMENT BUDGET CONSTRAINT

IGBC

- We used the government budget constraint by solving it backward (go back in time).
- We can also solve it forward – the valuation approach, the market value of government debt is determined by the discounted value of future government surpluses.
- This idea is often used in finance (e.g., Campbell and Shiller 1988) - what determines stock prices?
- Allows us to think seriously about risk and asset pricing.

IGBC

- Ignore risk for now and rewrite

$$B_{t+1} = (1 + r_{t+1})B_t + G_{t+1} - T_{t+1}$$

  as

$$B_t = \frac{1}{1 + r_{t+1}} \left(B_{t+1} - G_{t+1} + T_{t+1}\right).$$

- Continue this process to get

$$B_t = \sum_{j=1}^{T} \frac{1}{\prod_{k=1}^{j}(1 + r_{t+k})} \left(T_{t+j} - G_{t+j}\right) + \frac{1}{\prod_{k=1}^{T}(1 + r_{t+k})} B_{t+T}.$$

IGBC

- Note: there is a term $\frac{1}{\prod_{k=1}^{T}(1+r_{t+k})}B_{t+T}$.
- If this term does not vanish as $T \to \infty$, there is a bubble.
- Imposing the no-Ponzi game condition ($\lim_{T\to\inf} \frac{1}{\prod_{k=1}^{T}(1+r_{t+k})}B_{t+T} = 0$) on the above budget constraint, we get the IGBC:

$$B_t = \sum_{j=1}^{\infty} \frac{1}{\prod_{k=1}^{j}(1+r_{t+k})} \left(T_{t+j} - G_{t+j}\right).$$

- If the above is satisfied, we say that IGBC holds.

IGBC

- In a more general version IGBC is

$$B_t = \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} \left(T_{t+j} - G_{t+j} \right)$$

- We call $M_{t,t+j}$ the stochastic discount factor (SDF).
- It reflects how holders of government debt value discount future cash flows.
- Generally it is a function of the state of the economy at time t and $t + j$. Recall the first order condition for the household problem in the models we saw.
- We call the formula above the intertemporal government budget constraint (IGBC).

DEBT SUSTAINABILITY

- We can say that debt is sustainable if and only if the IGBC holds.
- Problem: this condition is about the entire future.
- Solution (?): use forecasts of future taxes and spending to compute the present value of future surpluses. Some early papers did this, but they used risk-free rates.
- Valid if one of these conditions holds:
    1. There is perfect foresight;
    2. Investors are risk-neutral;
    3. Primary surpluses do not covary with the SDF.

BOHN (1998)

- Not even debt (or debt to GDP) going to infinity means that the IGBC does not hold, it has to go to infinity slowly enough.
- Bohn (1998): see if the government does something that guarantees the IGBC holds, investigate the fiscal reaction function.
- Allows to sidestep the problem of forecasting future taxes and spending and choosing the correct discount rate.
- Sufficient condition: IGBC might also hold if it is violated, but if it is satisfied, IGBC holds for sure.

BOHN (1998)

- Linear reaction function:

$$\frac{T_t - G_t}{Y_t} = \rho \frac{B_{t-1}}{Y_{t-1}} + Z_t + \epsilon_t$$

- The left hand side is primary surplus.
- Z_t is a vector of exogenous variables that affect the primary surplus.
- Check if $\rho > 0$ – raise surplus if debt is high.
- If $\rho > 0$, then the IGBC holds even if it is below the interest rate (net of x).

Bohn (1998)

- If $\rho > 0$, the IGBC holds for any initial level of debt.
- This analysis works also for $r - x = 0$ – there was division by zero in the classic analysis.
- If $r - x > \rho > 0$, debt explodes, but the IGBC still holds (under certain conditions: see Bohn 2007).

BOHN (1998)

- Bohn (1998) estimates ρ for the US in 1916-1995.
- He includes the level of temporary government spending and business cycle indicator in Z_t.
- He find a positive value of ρ, around 0.05 for the entire sample.

BOHN (1998)

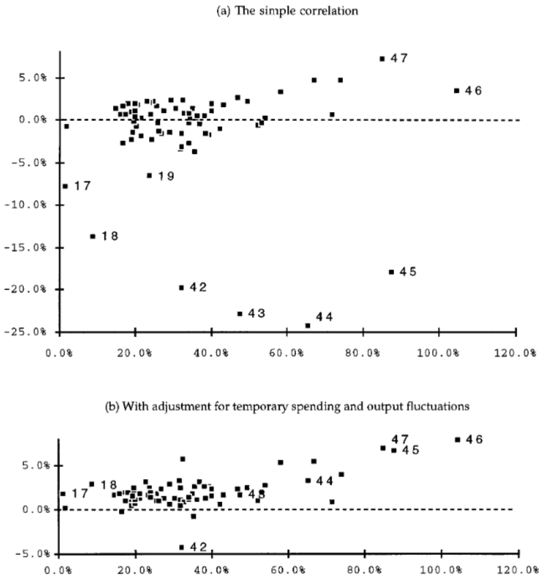


FIGURE I
Primary Surplus versus Initial Debt

BOHN (1998)

TABLE I
DETERMINANTS OF THE BUDGET SURPLUS

Dependent variable primary budget surplus divided by GDP (s_t)							
Sample	Constant	*GVAR*	*YVAR*	d_t	R^2	σ	*DW*
(1) 1916–1995	−0.019	−0.776	−1.450	0.054	0.936	0.014	1.42
	(−5.424)	(−33.001)	(−3.628)	(6.048)			
	[−3.957]	[−20.874]	[−4.075]	[3.787]			
(2) 1920–1995 excl. 1940–1947	−0.009	−0.551	−1.906	0.028	0.618	0.011	1.40
	(−2.030)	(−4.034)	(−4.666)	(2.701)			
	[−2.155]	[−3.721]	[−4.296]	[2.491]			
(3) 1916–1983	−0.018	−0.782	−1.414	0.054	0.942	0.014	1.54
	(−4.903)	(−31.667)	(−3.360)	(5.996)			
	[−3.958]	[−20.943]	[−4.004]	[4.076]			
(4) 1920–1982 excl. 1940–1947	−0.008	−0.520	−1.912	0.030	0.630	0.011	1.56
	(−1.710)	(−3.612)	(−4.441)	(2.815)			
	[−1.932]	[−3.272]	[−3.959]	[2.856]			
(5) 1948–1995	−0.015	−0.593	−2.139	0.037	0.651	0.010	1.54
	(−3.536)	(−4.182)	(−4.361)	(3.589)			
	[−3.496]	[−3.701]	[−3.757]	[2.821]			
(6) 1960–1984	−0.013	−0.410	−2.051	0.044	0.724	0.007	1.43
	(−2.110)	(−2.173)	(−4.174)	(2.028)			
	[−2.174]	[−2.281]	[−3.391]	[2.587]			

The variable d_t is the privately held debt/GDP at the start of the year. *GVAR* and *YVAR* are measures of temporary government spending and of cyclical variations in output, respectively, from Barro [1986a]. All estimates are OLS with annual data; ( ) = ordinary t-statistics; [ ] = heteroskedasticity- and autocorrelation-consistent t-statistics (computed with Newey-West lag window of size 1); σ = standard error; *DW* = Durbin-Watson statistic.

Source: Bohn (1998)

FISCAL REACTION FUNCTIONS

- Bohn (2008) extends the analysis to 1793-2003.
- He finds that $\rho > 0.1$, more than twice as large as in the previous study.
- Mendoza and Ostry (2008) study fiscal reaction functions for a panel of multiple countries – similar results.
- Ghosh et al. (2013) show that ρ is much lower at high levels of debt.
- D'Erasmo et al. (2016):
    1. primary balance adjustment in the US after 2008 was too large to be explained by the fiscal reaction function;
    2. adjustment is slower than before (structural break);
    3. nevertheless, with the estimated ρ, the IGBC holds.

FISCAL REACTION FUNCTIONS

- Leeper (2017) warns against using surplus-debt regressions to assess debt sustainability.
- For the estimator of ρ to be consistent, we must have

$$\mathbb{E}\left(\epsilon_t \mid \frac{B_{t-1}}{Y_{t-1}}\right) = 0.$$

  1. This means that shocks at $t-1$ that affect debt-output ratio in must not affect ϵ_t.
  2. This means that the debt-output ratio cannot depend on the expectation of ϵ_t.

- Since the value of debt depends on the expected value of future surpluses, this is a strong assumption: ϵ_t could be serially correlated.