DEBT SUSTAINABILITY II

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VALUATION APPROACH

VALUATION APPROACH

- Today we will look at debt sustainability from a different perspective: asset pricing.
- Idea: use tools from asset pricing to value government debt.

VALUATION APPROACH

- We go back to the budget constraint and solve it forward as

$$B_t = \mathbb{E}_t \sum_{j=1}^{T} M_{t,t+j} \left(T_{t+j} - G_{t+j} \right) + \mathbb{E}_t \, M_{t,t+T} B_{t+T}$$

- We obtained the standard IGBC if

$$\lim_{T \to \infty} \mathbb{E}_t \, M_{t,t+T} B_{t+T} = 0.$$

- The IGBC implies that a higher debt-to-output ratio today can be attributed to higher expected future primary surpluses (cash flows) or lower expected future returns (discount rates).
- The counterpart of the Campbell-Shiller expression for the log of the price-to-dividend ratio in the stock market.

VALUATION APPROACH

- Cochrane (2011) shows that discount rate variation is the main driver of stock valuation ratios.
- Cochrane (2019): half of the variation in the debt-to-GDP ratio to variation in future primary surpluses and half to varying discount rates.
- Jiang et al. (2021) conclude no statistical evidence of a discount rate or cash flow channel.
- Fluctuation in the debt-to-GDP ratio at time t predict fluctuations in the debt-to-GDP ratio at time $t + T$.
- Jiang et al. argue the differences result from small sample bias.

FISCAL CAPACITY

- Jiang et al. in a series of recent papers propose a new approach to debt sustainability analysis.
- Suppose an investor buys the entire stock of government debt and participates in all new issuances.
- How much would that investor be willing to pay for the debt?
- Cash flow is $\{T_t - G_t\}$.
- Use tools from asset pricing to answer this question.
- The price will depend on the riskiness of the cash flows.

ASSET PRICING REVIEW

ASSET PRICING BASICS

- Before we talk about Jiang et al., let's review some asset pricing basics.
- The general idea dates back to Lucas (1978) who considers asset prices in a general equilibrium model.
- An asset is a claim on a stream of prospective payments.
- Consider an economy with $i = 1, \ldots, N$ assets.
- Each of this assets has an associated stream of real dividends $\left\{d_{i,t}\right\}_{t=0}^{\infty}$.
- Assume the representative investor maximizes $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$.
- Let $p_{i,t}$ be the price of asset i at time t (in goods).

ASSET PRICING BASICS

- For each asset the investor holds, the optimality condition is:

$$p_{i,t} = \beta \, \mathbb{E}_t \frac{u'(c_{t+1})}{u'(c_t)} \left(p_{i,t+1} + d_{i,t+1}\right).$$

- This is the consumption-based asset pricing equation.
- We can slightly rearrange it as

$$1 = \beta \, \mathbb{E}_t \frac{u'(c_{t+1})}{u'(c_t)} \frac{p_{i,t+1} + d_{i,t+1}}{p_{i,t}}.$$

- Here $R_{i,t,t+1} := \frac{p_{i,t+1}+d_{i,t+1}}{p_{i,t}}$ is the (gross) rate of return on asset i from t to $t+1$.

ASSET PRICING BASICS

- Define $M_{t,t+1} := \beta \frac{u'(c_{t+1})}{u'(c_t)}$; we call it the stochastic discount factor (SDF).
- We can write the asset pricing equation for each asset as

$$1 = \mathbb{E}_t \, M_{t,t+1} R_{i,t,t+1}.$$

- The risk-free rate is defined as $R^f_{t,t+1}$ that satisfies

$$1 = R^f_{t,t+1} \, \mathbb{E}_t \, M_{t,t+1}.$$

- Sometimes you will see the asset pricing equation written as

$$v_{i,t} = \mathbb{E}_t \, M_{t,t+1} \frac{d_{i,t+1}}{d_t} \left(1 + v_{i,t+1}\right).$$

- Here $v_{i,t} := \frac{p_{i,t}}{d_t}$ is the price-dividend ratio of asset i at time t.
- This form is useful when we want to think of an asset that has an ever increasing stream of dividends.

ASSET PRICING BASICS

- Generally $\mathbb{E}_t\, M_{t,t+1} R_{i,t,t+1} \neq \mathbb{E}_t\, M_{t,t+1} \cdot \mathbb{E}_t\, R_{i,t,t+1}$, and we have

$$\mathbb{E}_t\, M_{t,t+1} R_{i,t,t+1} = \mathbb{E}_t\, M_{t,t+1} \cdot \mathbb{E}_t\, R_{i,t,t+1} + \mathrm{cov}_t \left(M_{t,t+1}, R_{i,t,t+1} \right).$$

- This allows us to write the asset pricing equation as

$$1 = \mathbb{E}_t\, M_{t,t+1} \cdot \mathbb{E}_t\, R_{i,t,t+1} + \mathrm{cov}_t \left(M_{t,t+1}, R_{i,t,t+1} \right).$$

- Use $1 = R^f_{t,t+1}\, \mathbb{E}_t\, M_{t,t+1}$ to write

$$\mathbb{E}_t\, R_{i,t,t+1} = R^f_{t,t+1} - \frac{\mathrm{cov}_t \left(M_{t,t+1}, R_{i,t,t+1} \right)}{\mathbb{E}_t\, M_{t,t+1}}.$$

ASSET PRICING BASICS

- The formula

$$\mathbb{E}_t \, R_{i,t,t+1} = R^f_{t,t+1} - \frac{\operatorname{cov}_t \left(M_{t,t+1}, R_{i,t,t+1} \right)}{\mathbb{E}_t \, M_{t,t+1}}.$$

  tells us that the expected return on asset i is the risk-free rate plus a risk premium.
- The risk premium depends on the covariance between the SDF and the return on asset i.
- When the covariance is negative (SDF is low when the return is high), the risk premium is positive.
- When the covariance is positive (SDF is high when the return is high), the risk premium is negative.

ASSET PRICING BASICS

- Consider a basic example: let $u(c) = \ln c$. We have

$$M_{t,t+1} = \beta \frac{c_t}{c_{t+1}}.$$

- The SDF is high when c_{t+1} is low.
- If the asset has a low return when c_{t+1} is low, the covariance is negative and the risk premium is positive.
- This is because the asset is risky – it does not pay much when you need it the most.

- We sometimes write the formula as

$$\mathbb{E}_t R_{i,t,t+1} = R^f_{t,t+1} - \frac{\mathrm{cov}_t\left(M_{t,t+1}, R_{i,t,t+1}\right)}{\mathrm{var}_t\, M_{t,t+1}} \times \frac{\mathrm{var}_t\, M_{t,t+1}}{\mathbb{E}_t\, M_{t,t+1}}.$$

- There are two terms:
    1. The first term is the risk exposure – it is the covariance between the SDF and the return on asset i divided by the variance of the SDF.
    2. The second term is the price of risk – it is the variance of the SDF divided by the expected value of the SDF. It does not depend on the asset.

ASSET PRICING BASICS

- We have

$$\mathbb{E}_t R_{i,t,t+1} = R^f_{t,t+1} + \beta_{i,t}\lambda_t.$$

- β^i_t is the risk exposure of asset i, $\beta_{i,t} := -\frac{\mathrm{cov}_t(M_{t,t+1}, R_{i,t,t+1})}{\mathrm{var}_t M_{t,t+1}}$
- Note: do not confuse $\beta_{i,t}$ with β, the discount factor.
- λ_t is the price of risk, $\lambda_t := \frac{\mathrm{var}_t M_{t,t+1}}{\mathbb{E}_t M_{t,t+1}}$.
- Risk premium is the product of the risk exposure and the price of risk.

ASSET PRICING BASICS

- So far we assumed that the SDF results from the optimization problem of the representative agent.
- (Some) SDF exists under much weaker conditions: it is enough that there is no arbitrage.
- Once we have a SDF, we can use it to price assets.
- This is the approach of Jiang et al. (2021).

FISCAL CAPACITY

- Return to the formulation of the valuation problem:

$$B_t = \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} \left(T_{t+j} - G_{t+j} \right)$$

- Here we understand everything as nominal and $M_{t,t+j}$ is the SDF used to price nominal claims.
- Jiang et al. call the right hand side the fiscal capacity of the government.

FISCAL CAPACITY

- To simplify the notation, define $S_{t+j} := T_{t+j} - G_{t+j}$, the primary surplus at time $t + j$.
- Rewrite the formula as

$$
\begin{aligned}
B_t &= E_t \sum_{j=1}^{\infty} M_{t,t+j} S_{t+j} \\
&= \sum_{j=1}^{\infty} \left(E_t M_{t,t+j} \cdot E_t S_{t+j} \right) + \sum_{j=1}^{\infty} \operatorname{cov}_t \left(M_{t,t+j}, S_{t+j} \right) \\
&= \sum_{j=1}^{\infty} \left(E_t M_{t,t+j} \cdot E_t S_{t+j} \right) \\
&\quad + \sum_{j=1}^{\infty} \operatorname{cov}_t \left(M_{t,t+j}, T_{t+j} \right) - \sum_{j=1}^{\infty} \operatorname{cov}_t \left(M_{t,t+j}, G_{t+j} \right).
\end{aligned}
$$

FISCAL CAPACITY

- Fiscal capacity depends on three terms:
  - $\sum_{j=1}^{\infty}\left(E_t M_{t,t+j} \cdot E_t S_{t+j}\right)$ – the expected value of future primary surpluses discounted by the risk-free rate.
  - $\sum_{j=1}^{\infty} \mathrm{cov}_t\left(M_{t,t+j}, T_{t+j}\right)$ – the covariance between the SDF and future taxes.
  - $\sum_{j=1}^{\infty} \mathrm{cov}_t\left(M_{t,t+j}, G_{t+j}\right)$ – the covariance between the SDF and future government spending.

FISCAL CAPACITY

- In the risk free world, the first term is the only one that matters.
- In the risk free world, fiscal capacity is determined only by the ability to generate current and future surpluses.
- The second and the third term reflect the riskiness of the surplus process.
- If taxes are high when the SDF is low, the second term lowers the fiscal capacity.
- If government spending is low when the SDF is low, the third term lowers the fiscal capacity.
- Tax revenue is usually procyclical, government spending is usually countercyclical – this lowers the fiscal capacity.

FISCAL CAPACITY

- This suggests that the fiscal capacity is most likely lower than the expected value of future primary surpluses discounted by the risk-free rate.
- By how much?
- Jiang et al. (2021) quantify this for the US. They find that the second and the third term matter quantitatively.
- Is there a way to increase the fiscal capacity by financial engineering?
- This would require insuring bondholders against the risk of future taxes and spending. Is it feasible?
- We now follow Jiang et al. (2023) to illustrate it.

FISCAL CAPACITY

- For simplicity assume that taxes to GDP τ and government spending to GDP γ are constant.
- GDP growth is risky, i.i.d. with a mean of x and volatility of σ.
- Let P_t^T and P_t^G denote the present value of future tax revenues and government spending:

$$
\begin{aligned}
P_t^T &= \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} T_{t+j} = \tau \, \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} Y_{t+j} \\
P_t^G &= \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} G_{t+j} = \gamma \, \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} Y_{t+j}.
\end{aligned}
$$

FISCAL CAPACITY

- Given the simplifying assumptions, debt to GDP ratio is:

$$\frac{B}{Y} = \frac{\tau - \gamma}{r^f + \text{risk premium on GDP } - x}.$$

- Notice no time subscripts: this is because of the i.i.d. on growth rates assumption + constant ratios τ, γ.

FISCAL CAPACITY

- Consider the following parametrization:
  - Taxes to GDP, τ are 25%.
  - Government spending to GDP, γ is 22.5%.
  - Risk-free rate r^f is 1.5%.
  - Mean GDP growth rate x is 2%.
  - GDP risk premium is 3%.
  - Initial GDP is 10 trillion.
- Risk premium on GDP is GDP volatility times the price of risk (3 times 1)
- Stock market acts as a levered claim to the aggregate: the GDP risk premium equals the unlevered equity risk premium.

FISCAL CAPACITY

- The value of the claim to GDP is $10 \cdot \frac{1}{0.015+0.03-0.02} = 400$ trillion.
- The claim on the stream of surpluses is worth $10 \cdot \frac{0.25-0.225}{0.015+0.03-0.02} = 10$ trillion.
- Fiscal capacity of this economy is 10 trillion.
- It equals 100% of (initial) GDP.
- If we evaluated it using the risk-free rate (net of growth), we would get infinity.

FISCAL CAPACITY

- The claim on surpluses is the claim on taxes net of the claim on government spending.
- The government cost of funding r_B can be written as

$$r_B = r_T \frac{P^T}{Y} \frac{Y}{B} - r_G \frac{P^G}{Y} \frac{Y}{B}.$$

- The risk exposure β, the covariance of a return with the SDF divided by the variance of the SDF is

$$\beta_B = \beta_T \frac{P^T}{Y} \frac{Y}{B} - \beta_G \frac{P^G}{Y} \frac{Y}{B}.$$

FISCAL CAPACITY

- The formula

$$\beta_B = \beta_T \frac{P^T}{Y} \frac{Y}{B} - \beta_G \frac{P^G}{Y} \frac{Y}{B}$$

  show that holding the risk exposure of gov. spending constant, if the government insures taxpayers (higher β_T) – lower tax payment in high marginal utility state – there is less insurance of bondholders (higher β_B).

FISCAL CAPACITY

- In this example tax revenue and government purchases are proportional to GDP.
- The risk exposure of tax revenue is $\beta_T = \beta_{GDP}$.
- The risk exposure of government spending is $\beta_G = \beta_{GDP}$.
- Normalize $\beta_{GDP} = 1$.
- We have

$$\begin{aligned} \beta_B &= \beta_T \frac{P^T}{Y}\frac{Y}{B} - \beta_G \frac{P^G}{Y}\frac{Y}{B} \\ &= \frac{100}{10} - \frac{90}{10} = 1 \end{aligned}$$

- Tax and spending claims are equally risky, but government debt has a positive beta of 1.

FISCAL CAPACITY

- Investors who buy the government debt portfolio are net long a claim to output.
- The output risk in spending does not fully offset the output risk in tax revenue.
- Debt is a constant fraction of GDP, it inherits the risk properties of the GDP claim.
- The government's interest payments are as risky as GDP, because they are a constant fraction of GDP.

FISCAL CAPACITY

- Usually $\beta_T > \beta_Y > \beta_G$.
- This is because tax revenue is more volatile than GDP, and GDP is more volatile than government spending.
- This means that the average tax revenue to output has to be higher to support the same amount of debt.
- See Jiang et al. (2020) for a quantitative analysis.

FISCAL CAPACITY

- Using the risk-free rate to evaluate the fiscal capacity of the government is misleading.
- It requires that the risk exposure of the government debt, β_B, is zero.
- For that to be true we need

$$\begin{aligned}\beta_T &= \left(\frac{P^T}{Y}\frac{Y}{B}\right)^{-1}\left(\frac{P^G}{Y}\frac{Y}{B}\right)\beta_G \\ &= \frac{P^G}{P^G+B}\beta_G\end{aligned}$$

which is lower than β_G if debt is positive.

FISCAL CAPACITY

- Go back to our example: $\beta_G = 1$, $P^G/Y = 90$, $B/Y = 1$.
- We need $\beta_T = 0.9$ to insure bondholders.
- We had $r^f = 1.5\%$ and risk premium on GDP of 3%.
- This meant that $r_Y = r^f + RP = 4.5\%$.
- We have $r_T = 1.5\% + 0.9 \cdot 3\% = 4.2\%$.
- The lower risk premium for the tax process reflects the fact that the tax rate is counter-cyclical.

FISCAL CAPACITY

- What is the average tax revenue to output needed to sustain the debt?
- Recall that $\frac{B}{Y} = 100\%$, $\frac{P^G}{Y} = 9$.
- We will use the formula

$$\frac{B}{Y} = \frac{T}{Y}\frac{P^T}{T} - \frac{P^G}{Y}$$

- We now have

$$\frac{P^T}{T} = \frac{1}{r_T - x} = \frac{1}{0.042 - 0.2} = 45.45.$$

  so $\frac{T}{Y} = 10/45.45 = 0.22$.
- The average tax revenue to output ratio is 22%.

FISCAL CAPACITY

- The previous example shows that the government can on average run a deficit of 0.5% of GDP.
- This is because the government provides insurance to bondholders by delivering positive surpluses when GDP growth is lower than average.
- Bondholders pay an insurance premium of 0.5% of GDP to receive relatively larger surplus payments when their marginal utility is high.
- But providing insurance is costly – it requires the government to have surpluses in recessions. Less room for output stabilization.

CONVENIENCE YIELDS

- Sometimes government debt is more valuable than the sum of its discounted cash flows.
- This is because it provides liquidity and safety to investors.
- Similar to cash: we hold it although it has a negative real return, because we need it for transations.
- We call the difference between the return on debt and the risk free rate the convenience yield of government debt.
- Think of it as of some extra benefit that makes investors willing to hold government debt despite low returns.

CONVENIENCE YIELDS

- The convenience yield is nonnegligible, especially for the US.
- Krishnamurthy and Vissing-Jorgensen (2012) estimate convenience yield of 73 basis points per annum on average between 1926 and 2008 in the US.
- This is an important source of seignorage for the US government (0.25% of GDP).
- The convenience yield depends on debt to GDP ratio.

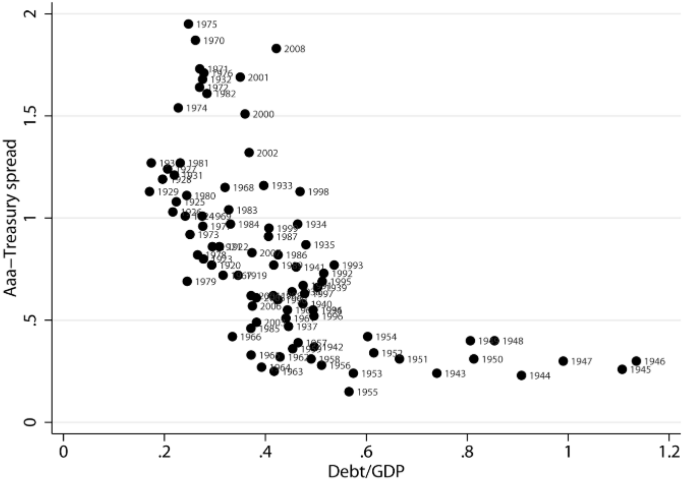


FIG. 1.—Corporate bond spread and government debt. The figure plots the Aaa-Treasury corporate bond spread (y axis) against the debt-to-GDP ratio (x axis) on the basis of annual observations from 1919 to 2008. The corporate bond spread is the difference between the percentage yield on Moody's Aaa long-maturity bond index and the percentage yield on long-maturity Treasury bonds.

FISCAL CAPACITY

- We need to modify the IGBC to account for the convenience yield.

$$B_t = \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} \left(T_{t+j} - G_{t+j}\right) + \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,t+j} B_{t+j} \left(1 - e^{-\delta_{t+j}}\right)$$

- The new term $K_{t+j} := B_{t+j}\left(1 - e^{-\delta_{t+j}}\right)$ represents the seignorage revenue from issuing debt.

FISCAL CAPACITY

- Return to the example with $\beta_T = \beta_G = \beta_Y = 1$, but consider convenience yields.
- Previously we said that the risk-free rate is equal to interest rate on treasuries, 1.5%.
- If the convenience yield is 0.73%, the true risk-free rate is higher: 2.23%.
- The PDV of surpluses is

$$\frac{0.25 - 0.225}{0.0523 - 0.02} = 77\% \text{ of GDP.}$$

- The PDV of seignorage will depend on β_K and the convenience yield.
- Set the convenience yield to 0.73%.

FISCAL CAPACITY

- If $\beta_K = 1$ (seignorage varies proportionally with GDP), then $r_K = 0.523$ and the PDV of seignorage is

$$\frac{0.0073}{0.0523 - 0.02} = 23\% \text{ of GDP.}$$

- This means that fiscal capacity is 100% of GDP.
- Two counteracting forces:
  1. Convenience yield generates seignorage.
  2. For a given interest rate, convenience yield means that the true risk-free rate is higher – this lowers the fiscal capacity.
- Extra surplus increases fiscal capacity by less than without the convenience yield.

FISCAL CAPACITY

- Most likely $\beta_K < 1$.
- In the short run, convenience yields can be counter-cyclical (flight to safety).
- This increases the seignorage term, without affecting the risk-free rate.
- Lowering β_K to 0.584, increases fiscal capacity to 114% of GDP.

FISCAL CAPACITY

- Jiang et al. use two approaches to estimate the fiscal capacity of the US.
- In the first approach they Congressional Budget Office (CBO) projections of tax revenue and non-interest spending for the next 31 years (2022-2052).
- CBO also forecasts interest rates and GDP.
- At the end of the projection horizon debt to GDP is 185%.

FISCAL CAPACITY

- Given $r^f = 1.5\%$, the risk premium on GDP to 3% and the average growth rate 2%, annual surpluses would have to be 4.625% of GDP since 2052 to sustain debt to GDP of 185%.
- The present value of that is 35.2 trillion.
- The present value of the surpluses between 2022 and 2052 is -21.1 trillion (negative).
- The fiscal capacity in 2022 is 35.2-21.1 trillion = 14.1 trillion.
- This is 8.2 trillion below the 22.3 trillion in debt outstanding at the end of 2021.

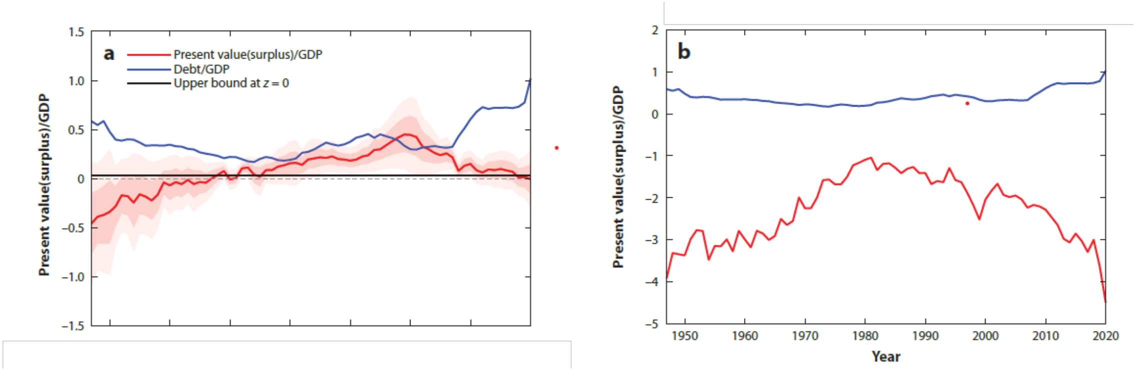
FISCAL CAPACITY

- This is already a generous estimate of the fiscal capacity.
- It assumes that the government can start running surpluses in 2052.
- It assumes acyclicality of taxes and spending.
- Another problem: duration mismatch.
- Surpluses are far in the future, fiscal capacity sensitive to small changes in the risk-free rate.

FISCAL CAPACITY

- The second approach: create forecast of cash flows using a VAR model.
- The model captures the cyclicality of tax and spending ratios.
- It captures multiple aggregate sources of risk: inflation, interest rates, the price-to-dividend ra- tio in the stock market, and shocks to tax and spending rates.
- They calculate total fiscal capacity in two ways: (a) assume the discount rates are the same for taxes and spending, (b) model the SDF.

FISCAL CAPACITY



Source: Jiang et al. (2023)

BOND VALUATION PUZZLE

- These estimates suggest a much lower fiscal capacity than the market value of outstanding debt.
- Possible explanations:
  - Convenience yields?
  - Bubble?
  - Global safe asset supplier?
  - Mispricing?
  - Fiscal correction?
  - Large-scale asset purchases and financial repression?

BOND VALUATION PUZZLE

- Similar calculations for other countries suggest that the US is an outlier.
- For example, for the UK after World War 2 fiscal capacity was 82% of GDP, but the debt to GDP ratio was 53%.
- It was different in 1729-1946 when fiscal capacity was 68% of GDP, but the debt to GDP ratio was 87%.
- Developing countries: procyclical surpluses, debt prices react strongly to funamentals (unlike the US).