# AGGREGATE SHOCKS AND DEBT MANAGEMENT II

FISCAL AND MONETARY POLICY 2024

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April 14, 2025

### **PLAN**

- We look at how the U.S. financed its wars over the last 200 years.
- We confront these patterns with the predictions of two theories: Barro (1979) and Lucas and Stokey (1983).
- We follow the chapter of the Handbook of Historical Economics by Hall and Sargent (2021) but we omit many interesting institutional details.



- Recall the two theories we discussed earlier:
  - Barro (1979): incomplete markets
  - Lucas and Stokey (1983): complete markets
- These two theories have different recommendations regarding the optimal mix of taxes and debt to finance wars.
- We will focus on a simplified version of Lucas and Stokey (1983): assume the same reduced form objective function as in the previous lecture.

- The only difference is the budget constraint.
- In the Barro (1979) case the government can only use one-period debt:

$$\tau_t + a_t = \frac{1}{1+r} a_{t+1} + g_t,$$

where  $a_t$  is a risk free claim on time t goods that the government purchased at time t-1; debt is  $-a_t$ .

 In the simplified version of Lucas and Stokey (1983) case the government can use state-contingent debt:

$$\tau_t + a_{t-1}(s_t) = \int q_t(s_{t+1} | s_t) a_t(s_{t+1}) ds_{t+1} + g_t,$$

where  $a_{t-1}\left(s_{t}\right)$  is Arrow security that pays in state  $s_{t}$  and  $q_{t}\left(\cdot\right)$  are prices.

• Let  $PVG_t$  be the expected present value of future government expenditures:

$$PVG_t = \mathbb{E}\left[\sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^j g_{t+j} \mid s_t\right] = H(s_t).$$

H satisfies the functional equation

$$H(s_t) = G(s_t) + \frac{1}{1+r} \int H(s_{t+1}) \, \varphi(s_{t+1} \mid s_t) \, ds_{t+1}$$
$$= G(s_t) + \frac{1}{1+r} \mathbb{E} [H(s_{t+1}) \mid s_t].$$

• In Barro (1979) the optimal plan is

$$\tau_t = \frac{r}{1+r} \left[ H(s_t) + a_t \right]$$

and has the implication that taxes are a random walk

$$\mathbb{E}_t\left[\tau_{t+1}\right] = \tau_t.$$

Government assets are also a random walk.

This simplified version of Lucas and Stokey (1983) has the optimal plan that

$$\tau_t = \tau_0$$
.

• The state-contingent asset-purchasing strategy supports this tax collection policy:

$$a_{t-1}(s_t) = H(s_t) - \frac{1+r}{r}\tau_0.$$

• To facilitate comparison we assume

$$q(s_{t+1} | s_t) = \frac{1}{1+r} \phi(s_{t+1} | s_t).$$

• This results in the budget constraint for the Lucas and Stokey (1983) case:

$$\tau_t + a_{t-1}(s_t) = \frac{1}{1+r} \mathbb{E}[a_{t+1} \mid s_t] + g_t,$$

where we define  $\mathbb{E}[a_{t+1} | s_t] = \int a_t(s_{t+1}) \, \phi(s_{t+1} | s_t) \, ds_{t+1}$ .

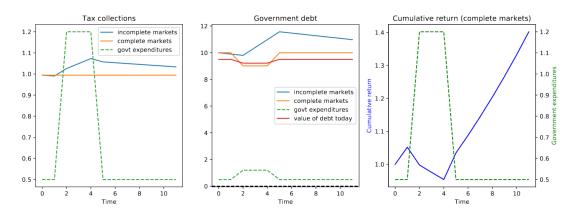
### **EX-POST RETURNS**

- Useful to distinguish between ex-ante and ex-post returns.
- Ex-post return on a portfolio of Arrow securities is

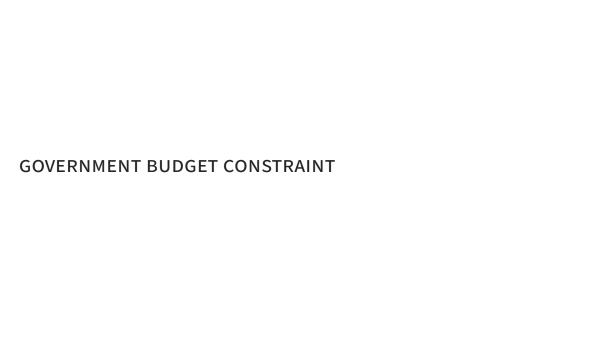
$$R_t(s_{t+1} | s_t) = \frac{a_t(s_{t+1})}{\mathbb{E}[a_{t+1} | s_t]}$$

• The conditional expectation of the return on the government portfolio (ex-ante) is then

$$\mathbb{E}\left[R_t\left(s_{t+1} \mid s_t\right) \mid s_t\right] = 1 + r$$



Tax and debt policy in complete and incomplete markets.



- We need to map the model into the data.
- Let  $B_{t-1} = \sum_{j=1}^{n} B_{t-1}^{j}$  be the total nominal value of interest bearing government debt at t-1, where  $B_{t-1}^{j}$  is the nominal value of zero coupon bonds of maturity j at t-1.
- The government budget constraint at time t is

$$B_t = B_{t-1} + r_{t-1,t}B_{t-1} + G_t - T_t - (M_t - M_{t-1}),$$

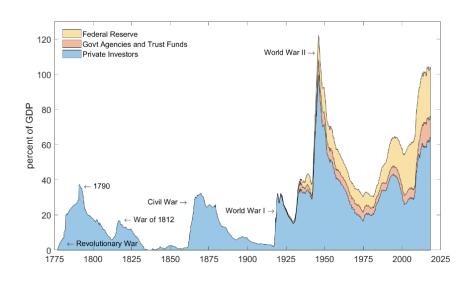
where  $M_t$  is the nominal money supply and  $r_{t-1,t}$  is implicitly defined by

$$r_{t,t-1}B_{t-1} = \sum_{j=1}^{n} r_{t-1,t}^{j} B_{t-1}^{j}.$$

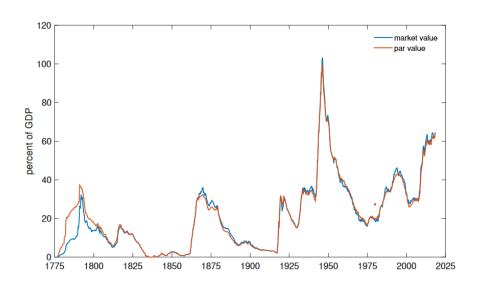
All variables above are nominal.

- Hall and Sargent make two adjustments to the U.S. Treasury's record of total public debt outstanding:
  - 1. net out holdings by the Federal Reserve and Government Agencies and Trust Funds;
  - 2. measure Treasury debt by its market value rather than its par value.

# **DEBT OWNERSHIP**



# **DEBT VALUE**



We can rewrite the budget constraint as

$$\begin{aligned} \frac{G_t}{Y_t} + r_{t-1,t} \frac{B_{t-1}}{Y_{t-1}} &= \frac{T_t}{Y_t} + \left(\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}}\right) + \frac{M_t - M_{t-1}}{Y_t} \\ &+ g_{t-1,t} \frac{B_{t-1}}{Y_{t-1}} + \pi_{t-1,t} \frac{B_{t-1}}{Y_{t-1}} \\ &+ r_{t-1,t} \left(g_{t-1,t} + \pi_{t-1,t}\right) \frac{B_{t-1}}{Y_{t-1}} \end{aligned}$$

where  $g_{t-1,t}$  denotes the growth rate of real GDP, and  $\pi_{t-1,t}$  denotes the inflation rate.

- The first three terms on the right side record sources of government revenue as shares of GDP: taxes, new borrowing and money creation.
- The next two terms record the diminution of the debt/GDP ratio due to real GDP growth and inflation.

A "peacetime baseline" version of the constraint

$$\left(\frac{G}{\gamma}\right)^{\text{base}} + \left(r_{-1,0}\frac{B_{-1}}{\gamma_{-1}}\right)^{\text{base}} = \left(\frac{T}{\gamma}\right)^{\text{base}} + \left(\frac{B}{\gamma} - \frac{B_{-1}}{\gamma_{-1}}\right)^{\text{base}}$$

$$+ \left(\frac{M - M_{-1}}{\gamma}\right)^{\text{base}}$$

$$+ \left(g_{-1,0}\frac{B_{-1}}{\gamma_{-1}}\right)^{\text{base}} + \left(\pi_{-1,0}\frac{B_{-1}}{\gamma_{-1}}\right)^{\text{base}}$$

$$+ \left(r_{-1,t}\left(g_{-1,0} + \pi_{-1,0}\right)\frac{B_{-1}}{\gamma_{-1}}\right)^{\text{base}}$$

- In each period we can calculate deviation of "wartime" budget constraint from the "peacetime baseline".
- Hall and Sargent sum from the beginning of the war to the end of the war to get the total wartime deviation.

$$\sum_{t=T_{1}}^{T_{2}} \left[ \frac{G_{t}}{\gamma_{t}} - \left( \frac{G}{\gamma} \right)^{\text{base}} \right] + \sum_{t=T_{1}}^{T_{2}} \left[ r_{t-1,t} \frac{B_{t-1}}{\gamma_{t-1}} - \left( r_{-1,0} \frac{B_{-1}}{\gamma_{-1}} \right)^{\text{base}} \right]$$

$$= \sum_{t=T_{1}}^{T_{2}} \left[ \frac{T_{t}}{\gamma_{t}} - \left( \frac{T}{\gamma} \right)^{\text{base}} \right] + \sum_{t=T_{1}}^{T_{2}} \left[ \left( \frac{B_{t}}{\gamma_{t}} - \frac{B_{t-1}}{\gamma_{t-1}} \right) - \left( \frac{B}{\gamma} - \frac{B_{-1}}{\gamma_{-1}} \right)^{\text{base}} \right]$$

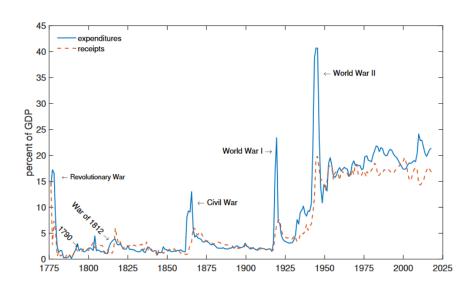
$$+ \sum_{t=T_{1}}^{T_{2}} \left[ \frac{M_{t} - M_{t-1}}{\gamma_{t}} - \left( \frac{M - M_{-1}}{\gamma} \right)^{\text{base}} \right] + \cdots$$

War Start - End (U.S. entry - )	(1) government spending	(2) return on debt	(3) (1)+(2)	(4) tax revenue	(5) debt growth	(6) money growth	(7) GDP growth	(8) inflation	(9) cross term	(10) residual
War of 1812 1812:6 - 1815:2	7.34	-0.20	7.14	-2.35 -32.9	10.60 148.5	0.00	-0.16 -2.2	0.06 0.8	-0.39 -5.5	-0.62 -8.7
Mexican War 1846:5 - 1848:2	2.26	0.20	2.47	-0.06 -2.4	2.72 110.4	0.00	-0.06 -2.5	-0.01 -0.5	-0.00 -0.1	-0.12 -4.8
Civil War (Union) 1861:4 - 1865:4	31.04	2.10	33.14	2.26 6.8	19.74 59.6	6.49 19.6	1.08 3.2	3.95 11.9	$0.40 \\ 1.2$	-0.77 -2.3
Spanish-American War 1898:4 - 1898:8	0.78	0.11	0.90	0.45 50.0	-0.26 -28.9	0.07 7.3	0.67 74.3	0.13 14.6	0.03 3.2	-0.18 -20.4
World War I 1914:7 - 1918:11	36.11	0.43	36.54	6.83 18.7	26.76 73.2	3.41 9.3	0.52 1.4	1.22 3.4	0.03	-2.24 -6.1
(1917:4 -)	36.93	0.30	37.23	7.76 20.8	27.79 $74.6$	2.59 7.0	$0.05 \\ 0.1$	$0.76 \\ 2.1$	0.00	-1.73 -4.6
World War II 1939:9 - 1945:8	129.50	0.10	129.60	49.91 38.5	54.78 42.3	11.32 8.7	15.42 11.9	9.62 7.4	0.26	-11.71 -9.0
(1941:12 - )	116.48	2.00	118.48	35.80 30.2	54.53 46.0	11.96 10.1	8.99 7.6	6.05 5.1	$0.43 \\ 0.4$	$0.71 \\ 0.6$
Korean War 1950:6 - 1953:6	15.43	-0.71	14.73	5.42 36.8	4.17 28.3	2.53 17.2	10.99 74.6	-10.12 -68.7	0.05	1.70 11.5
Vietnam War 1964:8 - 1973:6	5.53	-2.13	3.41	1.39 40.8	0.44 12.9	-0.60 -17.8	-5.55 -163.0	3.91 114.9	0.19 5.7	3.63 106.5

### **OPTIMAL RESPONSE**

- Suppose r = 0.06, the net real interest rate in our model is 6%.
- Barro (1979) implies that a purely transitory increse in  $g_t$  should be financed in 6% by taxes and in 94% by debt.
- We see it for the Civil War only.
- Caveat: were these increases in government purchases really transitory?

# **RECEIPTS AND EXPENDITURES**



# **OPTIMAL RESPONSE**

- Lucas and Stokey (1983) implies that unexpected increases in government spending should be absorbed by wartime decreases in returns to government creditors.
- With the exception of the Mexican War and the Korean War, the contribution of inflation is greater than the contribution of the nominal return on the debt.
- Negative wartime real returns.

	100 × Debt/GDP			Contributions						
War postwar period	(1) end of war	(2) 15 years postwar	(3) change	(4) nominal returns	(5) real gdp growth	(6) inflation	(7) primary deficit	(8) cross term	(9) seignorage	(10) residual
War for Independence										
1791-1806	33.3	9.6	-23.8	11.3 47	-15.4 -65	-7.6 -32	-17.3 -73	-1.0 -4	_	6.3 26
War of 1812										
1815-1830	11.6	3.4	-8.2	$10.5 \\ 128$	-5.6 -68	4.9 60	-19.4 -237	0.1 1	_	1.3 16
Mexican War										
1848-1860 <sup>†</sup>	2.7	1.2	-1.5	0.8 53	-0.9 -60	-0.1 -7	-1.5 -100	-0.1 -7	_	0.3 20
Civil War (Union)										
1865-1880	22.1	15.6	-6.5	$\frac{21.4}{329}$	-14.5 -223	13.5 208	-29.5 -454	0.1	1.2 18	1.3 20
Spanish-American War										
1898-1913	4.6	2.2	-2.4	0.9 38	-1.2 -50	-1.1 -46	-1.9 -79	-0.1 -4	0.8 33	$\frac{0.1}{4}$
World War I										
1919-1929‡	28.6	20.2	-8.4	$\frac{12.5}{149}$	-6.4 -76	2.4 29	-20.3 -242	0.3 4	2.0 24	1.0 12
World War II										
1945-1960	90.1	35.7	-54.4	$\frac{14.3}{26}$	-15.8 -29	-38.9 -71	-13.0 -24	-0.6 - 1	-0.3 -1	-0.2 0
Korean War										
1953-1968	49.9	21.8	-28.1	14.0 50	-20.3 -72	-10.8 -38	4.0 14	-0.8 -3	-5.8 -21	-8.5 -30
Vietnam War										
1973-1988	16.4	34.7	+18.3	32.3	-12.2	-19.2	19.7	-2.7	-6.0	6.3

177

-67

-105

108

-15

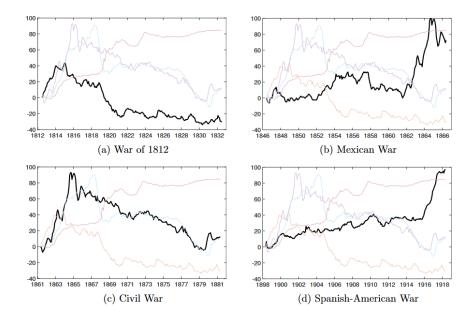
-33

35

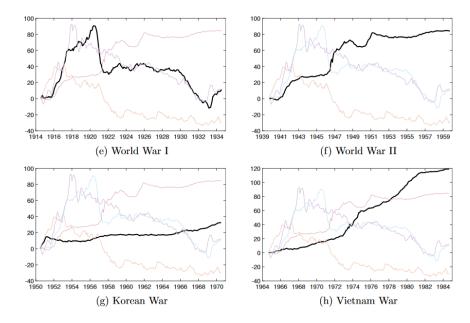
Contributions

100 v Debt/CDP

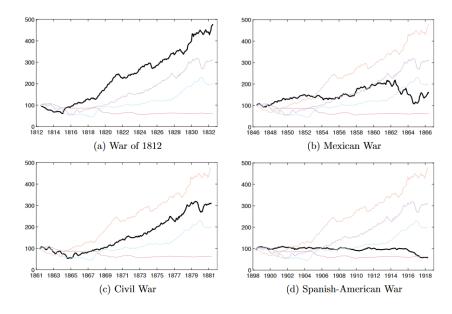
# **PRICES**



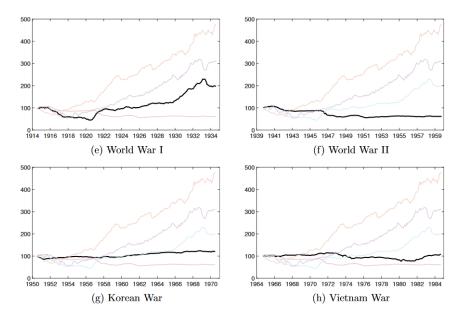
# **PRICES**



### **RETURNS**



### **RETURNS**



### CONCLUSIONS

- Evidence of Barro tax smoothing in most wars, but only during the Civil War the split between taxes and debt that the model recommends for a purely temporary expenditure surge.
- Negative wartime bond returns followed by positive postwar returns in the War of 1812, the Civil War, World War I and the Korean War as prescribed by the Lucas-Stokey model
- However, this model directs that bondholders should receive an immediate capital loss at the outbreak of a war. This happens only in the Korean War.
- The U.S. had little debt at the outbreak of most wars, the Lucas-Stokey action would not help the government's financial situation.