Problem Set 1

Fiscal and Monetary Policy, Spring 2025

March 27, 2025

This problem set consists of two problems. Submit your solutions until April 13th 11:59 PM. You can work in teams of up to three students.

Problem 1: Government purchases multiplier

In this problem you will calculate government purchases multiplier in a simple economy without capital.

The representative agent's preferences are represented by

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta} - 1}{1-\theta} - \frac{n_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right]$$

and her budget constraint is

$$c_t + b_{t+1} = w_t n_t - T_t + R_t b_t,$$

where c_t is consumption. b_t is real government bonds held at the beginning of period, w_t is real wage, n_t is labor supply, T_t is lump sum tax and R_t is (gross) real interest rate. We assume $\beta \in (0,1)$, and $\varphi > 0$, $\theta > 0$. The agent chooses sequences of bond holdings $\{b_{t+1}\}_{t=0}^{\infty}$, consumption $\{c_t\}_{t=0}^{\infty}$ and labor supply $\{n_t\}_{t=0}^{\infty}$ to maximize her utility subject to the budget constraint and initial bond holdings b_0.

Each period the government finances its purchases $g_t \geq 0$ by levying lump sum taxes - the budget is balanced and for all t $b_t = 0$. There is also a representative firm using production function

$$F(n_t) = n_t$$

to produce its output y_t. The resource constraint in this economy is thus

$$y_t = g_t + c_t.$$

Observe that there is no storage technology, so this economy is essentially static.

1. Write down the Lagrangian and solve the household optimization problem (i.e., show first order conditions). Interpret the Euler equation and the equation linking labor supply with wage and consumption in terms of marginal rates of substitution.

2. Real wage w_t must equal 1 in equilibrium. Explain why.

3. Use the above fact and $b_t = 0$, $g_t = T_t$ together with the budget constraint of the household to express equilibrium consumption c_t as a function of n_t and g_t.

4. Substitute the expression derived in 3) for c_t in the equation linking labor supply with wage (=1) and consumption. You should obtain an expression consisting of n_t, g_t and parameters of the model.

5. Apply the implicit function theorem to derive dn_t/dg_t. It should contain parameters of the model and the ratio g_t/n_t, equal to g_t/y_t.

6. Can the multiplier derived above ever exceed 1? Can it be negative? How does it depend on the parameters of the model and g_t/y_t?

7. Does R_{t+1} increase when g_t goes up? Why/why not? Use the Euler equation together with your answer to 6). Interpret this finding by describing the household's incentives to save/borrow after an increase in g_t together with the fact that bonds are in zero net supply in equilibrium.

8. How is the size of change in R_{t+1} related to the size of the multiplier?

Problem 2: IGBC and Bohn (1998)

There is a budget constraint of the government:

$$\frac{B_t}{Y_t} = \frac{G_t - T_t}{Y_t} + (1+r)\,\frac{B_{t-1}}{Y_{t-1}}\frac{Y_{t-1}}{Y_t}.$$

Let $X_{t-1,t} := \frac{Y_t}{Y_{t-1}}$. Assume that $X_{t-1,t}$ is always 1. Assume that $r > 0$. For simplicity, define $b_t := \frac{B_t}{Y_t}$, $g_t := \frac{G_t}{Y_t}$, $\tau_t := \frac{T_t}{Y_t}$. These are ratios of various variables to GDP. We can write (given the assumption that $X_{t-1,t} = 1$):

$$b_t = g_t - \tau_t + (1+r)\,b_{t-1}.$$

Suppose the government follows a policy rule of the form

$$\tau_t - g_t = \rho b_{t-1} + \bar{s}.$$

The parameter ρ is the strength of the response. Assume that the stochastic discount factor is equal to $\frac{1}{1+r}$. We say that the IGBC holds if the following condition is satisfied:

$$b_t = \sum_{j=1}^{\infty} \left(\frac{1}{1+r}\right)^j \mathbb{E}_t\left(g_{t+j} - \tau_{t+j}\right).$$

This is derived from solving the budget constraint forward and using $\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^T \mathbb{E}_t\, b_T = 0$.

Let $b_0 > 0$ be the initial debt to GDP. Assume perfect foresight (so that you can drop $\mathbb{E}_t$).

1. Use the policy rule to express b_t as a function of b_{t-1} and constants.
2. Use the policy rule to express b_t as a function of initial debt b_0 and constants.
3. Express primary surplus $\tau_t - g_t$ as a function of b_0 and constants.
4. What is $\lim_{t\to\infty} b_t$ if $\rho > r$? What if $\rho < r$?
5. What is $\lim_{t\to\infty}\left(\frac{1}{1+r}\right)^t b_t$ if $\rho > r$? What if $\rho < r$? Does the answer depend on b_0?
6. Use your answers to the previous question to show conditions under which the IGBC holds.