

# GOVERNMENT PURCHASES MULTIPLIER: THEORY I

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OLD (?) KEYNESIAN MULTIPLIER

## KEYNESIAN MULTIPLIER

- Recall your undergraduate macroeconomics course.
- A simple model of a closed economy

$$Y_t = C_t + I_t + G_t$$

$$C_t = \alpha_0 + \alpha_1 (Y_t - T_t)$$

$$I_t = I_0$$

where  $Y_t$  is output,  $C_t$  is consumption,  $I_t$  is investment,  $G_t$  is government purchases of goods and services and  $T_t$  is taxes.

- We *assume* consumption in period  $t$  is a function of disposable income in period  $t$  and investment is exogenous.
- Here  $\alpha_1 \in (0, 1)$  is *the marginal propensity to consume*.

## KEYNESIAN MULTIPLIER

- We can solve for equilibrium output  $Y_t$ :

$$Y_t = \frac{1}{1 - a_1} [\alpha_0 + I_0 + G_t - a_1 T_t]$$

- The **government spending multiplier** is

$$\frac{\partial Y_t}{\partial G_t} = \frac{1}{1 - a_1}.$$

- The size of the multiplier depends on the marginal propensity to consume. Notice the multiplier is **always** larger than one because  $a_1 > 0$ .

## KEYNESIAN MULTIPLIER

- The result above relied on the assumption that  $T_t$  is constant. What if it is not?
- Suppose  $T_t = G_t$  (balanced budget) We then have

$$\frac{\partial Y_t}{\partial G_t} = 1.$$

- Notice that the multiplier is now equal to one regardless of the value of  $a_1$ .
- Haavelmo (1945) showed this result.

## TAKING STOCK

- Even if the government spends exactly as much as it collects in taxes, output will increase one to one.
- If the government spends more than it collects in taxes, output will increase by more than one to one.
- The multiplier is larger than one because of the marginal propensity to consume greater than zero.

## TAKING STOCK

- Too good to be true?
- What are the limitations of this model?
- What determines the key parameter  $a_1$ ?
- Similar results tend to hold in more complex models (see “The Intertemporal Keynesian Cross” by Auclert et al. 2024).
- We will first study government purchases multiplier in a benchmark model – the Neoclassical Growth Model.

## NEOCLASSICAL GROWTH MODEL - RECAP



## NGM: HOUSEHOLD

- Representative household chooses  $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$  to maximize

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - v(n_t)]$$

subject to

$$\underbrace{c_t + k_{t+1} - (1 - \delta) k_t}_{i_t} = w_t n_t + r_t^k k_t \quad \text{for all } t \geq 0$$

given  $k_0$ .

- $u(\cdot)$  strictly increasing,  $C^2$ , strictly concave,  $u'(0) = \infty$ ,  $u'(c)_{c \rightarrow \infty} = 0$ ;
- $v(\cdot)$  strictly increasing,  $C^2$ , strictly convex,  $v'(n)_{n \rightarrow \infty} = \infty$ .

## NGM: FIRM

- Representative producer chooses  $\{y_t, k_t, n_t\}_{t=0}^{\infty}$  to maximize

$$y_t - r_t^k k_t - w_t n_t \quad \text{for all } t \geq 0$$

subject to

$$y_t = F(k_t, n_t) \quad \text{for all } t \geq 0.$$

- $F(\cdot, \cdot)$  is CRtS, concave, increasing and satisfies Inada conditions.

## NGM: COMPETITIVE EQUILIBRIUM

- **Competitive Equilibrium:** sequence of prices and allocations such that:
  - Households maximize utility subject to their budget constraints given prices;
  - Firms maximize profits subject to their production sets given prices;
  - All markets clear:

$$y_t = c_t + k_{t+1} - (1 - \delta) k_t$$

(+ labor and capital market – by using the same variable for demand and supply).

## NGM: CHARACTERIZATION

- Lagrangian for the household problem:

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - v(n_t)] + \lambda_t [w_t n_t + r_t^k k_t - c_t - k_{t+1} + (1 - \delta) k_t]$$

- First order conditions

$$\beta^t u'(c_t) = \lambda_t$$

$$\beta^t v'(n_t) = \lambda_t w_t$$

$$\lambda_t = \lambda_{t+1} (1 - \delta + r_{t+1}^k)$$

## NGM: CHARACTERIZATION

- Solution to the household's problem described by the following:

$$u'(c_t) = \beta u'(c_{t+1}) (1 - \delta + r_{t+1}^k)$$

$$v'(n_t) = u'(c_t) w_t,$$

the budget constraint

$$c_t + k_{t+1} - (1 - \delta) k_t = w_t n_t + r_t^k k_t,$$

the initial condition ( $k_0$  given), and the transversality condition

$$\lim_{t \rightarrow \infty} \beta^t (c_t) k_{t+1} = 0.$$

## NGM: CHARACTERIZATION

- Solution to the firm's problem described by the following:

$$r_t^k = F_1(k_t, n_t)$$

$$w_t = F_2(k_t, n_t),$$

and the production function

$$y_t = F(k_t, n_t).$$

## NGM: CHARACTERIZATION

- **Equilibrium conditions:**

$$u'(c_t) = \beta u'(c_{t+1}) [1 - \delta + F_1(k_{t+1}, n_{t+1})] \quad (\text{EE})$$

$$v'(n_t) = u'(c_t) F_2(k_t, n_t) \quad (\text{LS})$$

$$k_{t+1} = F(k_t, n_t) + (1 - \delta) k_t - c_t \quad (\text{LoM})$$

+ the transversality condition which pins down the solution, and the initial condition ( $k_0$  given).

- Sequences of  $\{c_t, n_t, k_t\}$  that satisfy these conditions constitute allocations in a competitive equilibrium.

## COMMENTS

- Tradeoffs in the model:
  - consumption today  $c_t$  vs. consumption tomorrow  $c_{t+1}$ ;
  - consumption vs. leisure.



## NEOCLASSICAL GROWTH MODEL - GOVERNMENT PURCHASES

## GOVERNMENT PURCHASES

- We add government purchases,  $g_t$  to the model.
- They do not affect utility, they just use up  $g_t$  units of goods each period.
- They are financed by lump-sum taxes:  $T_t = g_t$  in each period.
- BC of the HH is:

$$c_t + i_t = w_t n_t + r_t^k k_t - T_t \quad \text{for all } t \geq 0.$$

- Market clearing condition is:

$$c_t + i_t + g_t = y_t.$$

- Note: in this model it does not matter if the budget is balanced or  $g_t$  is financed by debt, as long as taxes are lump-sum.

## MULTIPLIER

- Questions we might want to answer:
  - effects of a permanent change in (constant)  $\bar{g}$  in the steady state;
  - effects of an unexpected change and transitory in  $g_t$  on impact (i.e. in period  $t$ );
  - cumulative effects of a change in  $g_t$ ;
  - effects of a pre-announced change in  $g_t$ ...

## MULTIPLIER - LONG RUN

- We first discuss the long-run effect of permanent changes (steady state).
- Define  $\eta := k/n$ .
- By CRtS we have  $F(k, n) = F(\eta, 1)n$  and  $F_1(k, n) = F_1(\eta, 1)$ .
- The Euler equation in the steady state is:

$$1 = \beta [1 - \delta + F_1(\eta, 1)] \quad (\text{EE})$$

which pins down  $\eta$ .

## MULTIPLIER - LONG RUN

- The other two equations (labor supply and law of motion for capital) give us a system of two equations in two unknowns,  $c$  and  $n$ :

$$v'(n) = u'(c) F_2(\eta, 1) \quad (\text{LS})$$

$$c = (F(\eta, 1) - \eta\delta) n - g \quad (\text{LoM})$$

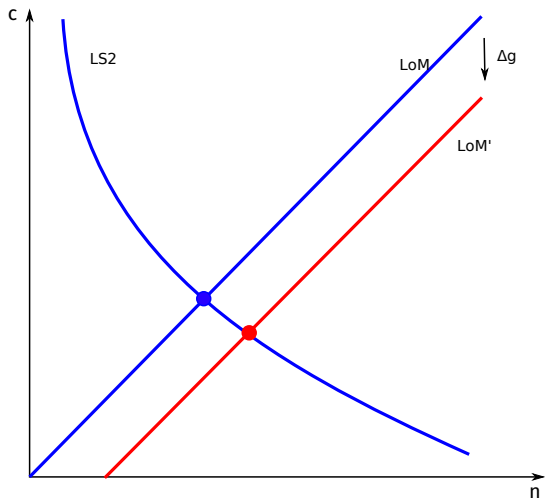
- Given our assumptions we can rewrite (LS) as (LS2)

$$c = \phi(n, F_2(\eta, 1)) \quad (\text{LS2})$$

$$c = (F(\eta, 1) - \eta\delta) n - g \quad (\text{LoM})$$

- (LS2) is decreasing in  $n$ , (LoM) is increasing in  $n$ . There exists unique  $n = \tilde{n}(g)$  and  $c = \tilde{c}(g)$  solving this system.

## MULTIPLIER - LONG RUN



Permanent increase in government purchases.

## MULTIPLIER - LONG RUN

- Note

$$\frac{dy}{dg} = F(\eta, 1) \frac{\partial n(g)}{\partial g}$$

- Key: response of labor supply to  $g$ .
- To get  $\partial n(g) / \partial g$  define

$$H(n, g) \equiv \phi(n, F_2(\eta, 1)) - (F(\eta, 1) - \eta\delta)n + g$$

and apply Implicit Function Theorem to obtain

$$\frac{\partial n(g)}{\partial g} = \frac{1}{(F(\eta, 1) - \eta\delta) - \phi_1(n, F_2(\eta, 1))}$$

$$\text{with } \phi_1(n, F_2(\eta, 1)) = \frac{v''(n)}{u''(c)F_2(\eta, 1)}$$

## MULTIPLIER - LONG RUN

- We can rewrite the multiplier (verify!) as.

$$\frac{dy}{dg} = \frac{1}{1 - \frac{i}{y} + \frac{c}{y} \frac{1}{\varphi\theta}},$$

which depends on observable ratios (investment-to-output and consumption-to-output) and the utility function evaluated at the steady state,  $\varphi \equiv \frac{v'(n)}{v''(n)n}$ ,  $\theta \equiv -\frac{u''(c)c}{u'(c)}$ .

- It is
  - Increasing in  $\varphi$ ;
  - Increasing in  $\theta$ ;
  - Increasing in  $i/y$ ;
  - Decreasing in  $c/y$ .



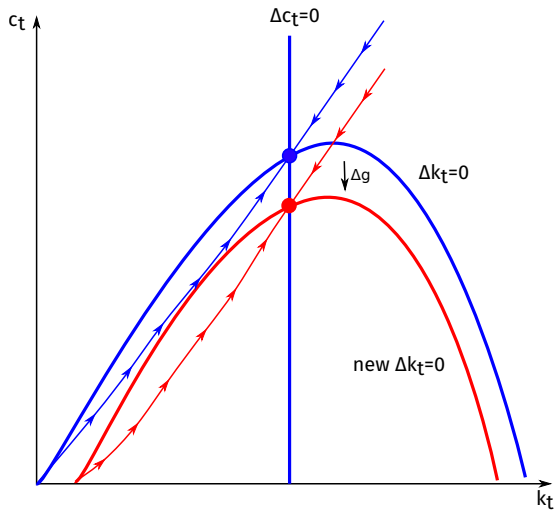
## MULTIPLIER - LONG RUN

$$\frac{dy}{dg} = \frac{1}{1 - \frac{i}{y} + \frac{c}{y} \frac{1}{\varphi\theta}},$$

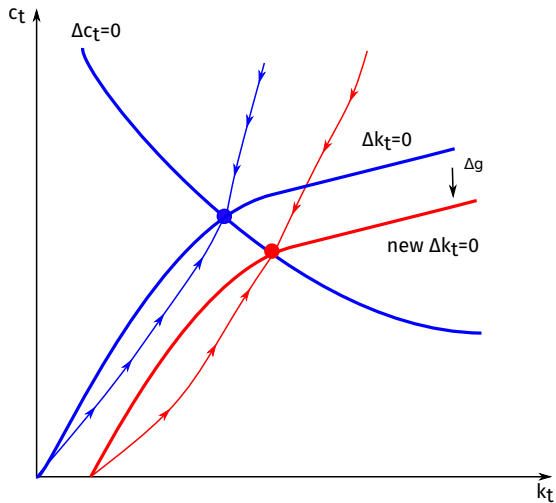
- Some remarks:
  - It **CAN** exceed 1.
  - It equals 0 when labor supply is fixed.
  - It cannot exceed 1 if investment is zero (for example economy with fixed exogenous capital or no capital).
  - It is big when a large increase in labor supply requires a small decrease in consumption.
- Remember: it was financed by lump sum taxes.

## PHASE DIAGRAMS

- To visualize the effects of government purchases we can use phase diagrams.
- They are also useful to show short-run effects of government purchases (esp. in continuous time).
- To plot them:
  - Try to eliminate all static variables (in this model you can pick  $n_t$ )
  - Think of conditions ensuring that variables do not change over time in terms of other variables and parameters (i.e., find conditions for  $c_t = c_{t+1}$  and  $k_t = k_{t+1}$ ).
  - Represent these conditions graphically.
  - Think of dynamics of the system for points  $(c, k)$  where these conditions are not satisfied.
  - Locate the saddle path.



Phase diagram: permanent increase in government purchases by  $\Delta g$  in an economy with fixed labor supply.



Phase diagram: permanent increase in government purchases by  $\Delta g$  in an economy with flexible labor supply.

## MULTIPLIER - LONG RUN

- Exercise: Suppose  $u(c) = \frac{c^{1-\theta}-1}{1-\theta}$ ,  $v(n) = \frac{n^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$ ,  $i/y = 0.2$  and  $c/y = 0.6$ . What is the size of the multiplier if  $\theta = 1$  (log-utility) and  $\varphi \in \{0, 0.5, 1, 2, \infty\}$ ?
- Exercise: Suppose that we have  $u(c, n) = \gamma \log c + (1 - \gamma) \log(1 - n)$ . Rederive the formula for the long run multiplier.
- Exercise: Is the adjustment to the new steady state instantaneous or not? What happens on impact?

## MULTIPLIER - LONG RUN

- Let's analyze an economy with fixed labor supply.
- In the steady state

$$1 = \beta [1 - \delta + F_1(k, 1)] \quad (\text{EE})$$

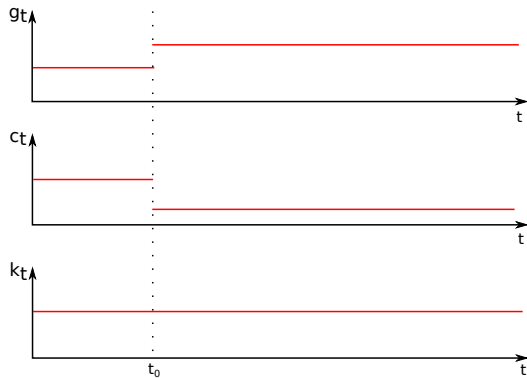
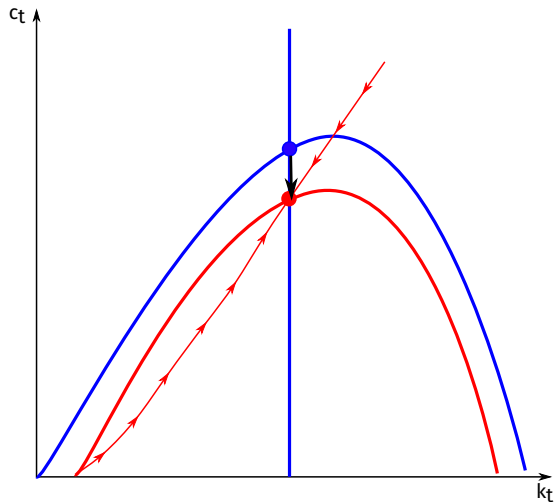
$$c = F(k, 1) - \delta k - g \quad (\text{LoM})$$

- Notice  $g$  does not enter (EE). It does not affect incentives to save. Investment is not affected.
- $k$  remains the same and because it is the only factor of production in this economy output does not change. We have  $dc = -dg$ .
- Government purchases crowd out private consumption one to one.
- Remember that lump-sum tax  $T$  has to be raised to finance  $g$ . It reduces HH's wealth.

## TRANSITION

- We will study transition to the new steady state using phase diagrams.
- In discrete time it is a bit awkward, but we still can get some insight.
- Here we analyze unexpected permanent shocks.
- To proceed
  - Locate the new saddle path.
  - Plot arrows representing trajectories.
  - Make sure the economy jumps to the new saddle path on impact.
  - Very important: identify variables that **CANNOT** jump (almost *always* capital).
  - After the shock the economy follows the new saddle path to the new steady state.

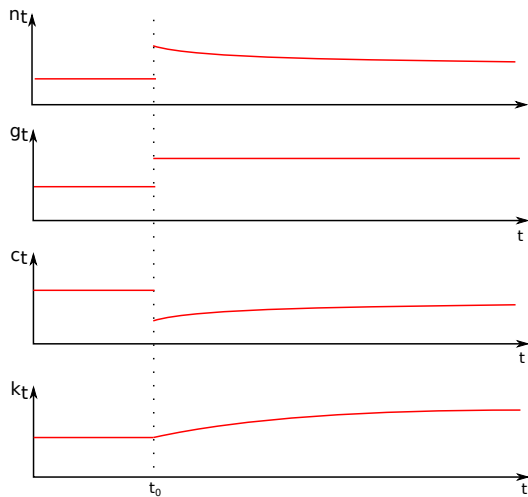
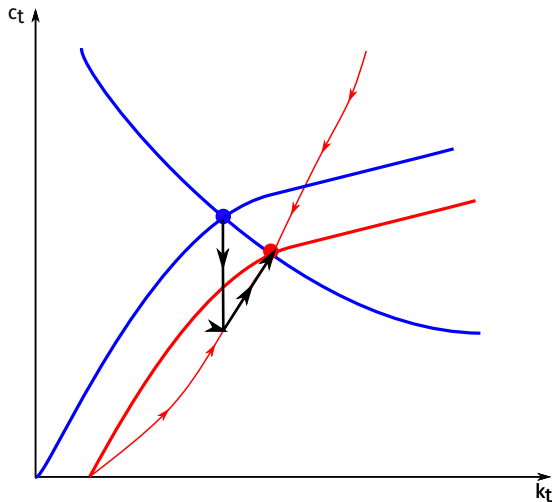
## TRANSITION



Phase diagram: permanent increase in government purchases by  $\Delta g$  in an economy with fixed labor supply: transition



## TRANSITION



Phase diagram: permanent increase in government purchases by  $\Delta g$  in an economy with flexible labor supply: transition

## MULTIPLIER - SHORT RUN

- More interesting exercise: temporary increase in  $g$ . For simplicity we start in the steady state with  $g_t = g$  and consider an unexpected increase in  $g_0$  that lasts for some periods.
- Think of a war or a stimulus package based on government purchases.
- Main result: difficult to generate  $dy_0/dg_0 > 1$  in a baseline NGM. Why?
- After discussing theoretical results we will look at empirical findings.

## MULTIPLIER - SHORT RUN

- What happens after  $g_0$  goes up?
  - $k_0$  is fixed.
  - $y_0$  can grow only if  $n_0$  increases.
- (LS) implies that

$$v'(n_0) = u'(c_0) F_2(k_0, n_0) \quad (\text{LS})$$

so given our assumptions  $n_0$  can increase **only** if  $c_0$  does not increase.

## MULTIPLIER - SHORT RUN

- $c_0$  has to fall (or at least not increase). When is it possible? Recall (EE)

$$\frac{u'(c_0)}{u'(c_1)} = \beta [1 - \delta + F_1(k_1, n_1)]$$

- We need  $u'(c_0) / u'(c_1) > 0$ . Intuitively, the economy has to return to the steady state eventually (formally: TVC). This can happen only if

$$\beta [1 - \delta + F_1(k_1, n_1)] > 1$$

so the interest rate  $r_1 := F_1(k_1, n_1) - \delta$  has to be higher than in the steady state.

- Remember: behavior of consumption determined by the real interest rate.

## MULTIPLIER - SHORT RUN

- This is possible if  $k_1/n_1 < k/n$ . So, unless  $n_1 > n$ ,  $k_1 < k_0$ .
- This requires fall in investment.
- To have  $dy_0/dg_0 > 1$  we need more persistent shocks and high  $\varphi$ . Recall when permanent increase in  $g$  does not crowd out consumption.

## MULTIPLIER - SHORT RUN

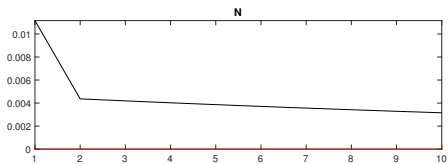
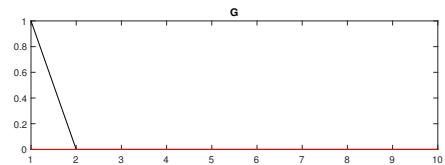
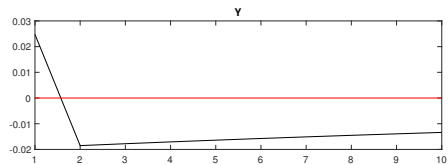
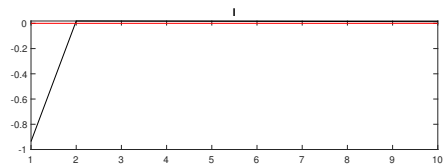
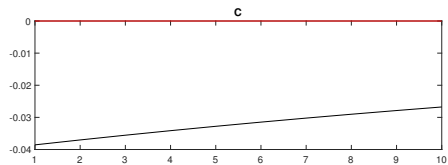
- In a standard NGM it is hard to get multipliers exceeding 1.
- With standard calibration  $dy_0/dg_0$  is rather low.
- Suppose

$$u(c) = \log c, \quad v(n) = \kappa \frac{n^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$$

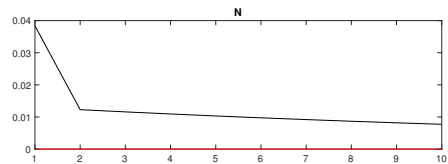
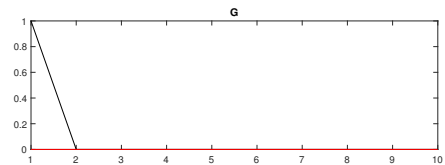
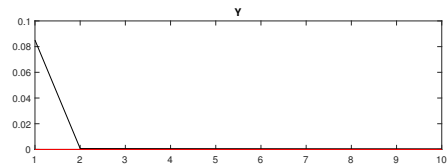
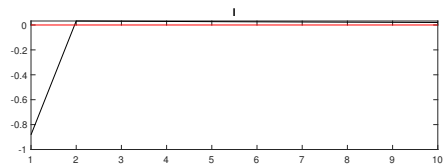
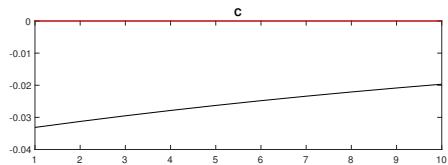
and parameter values be

$\delta$	$\beta$	$\alpha$	$\frac{g}{y}$
0.02	0.99	1/3	0.2

and  $n = 1$  in the steady state.

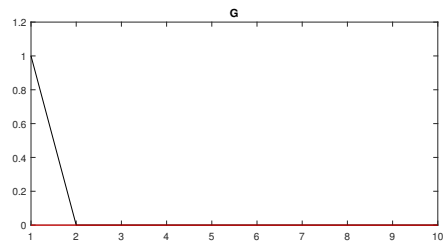
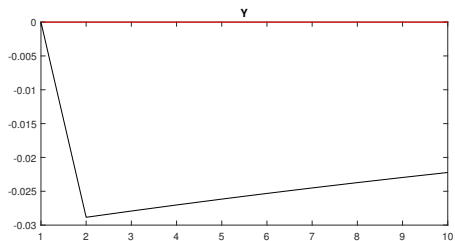
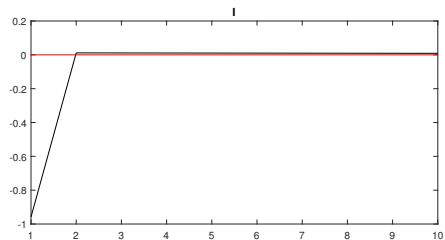
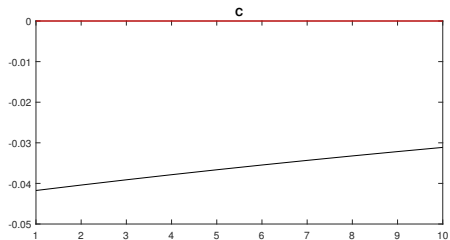


Dynamics after an unexpected increase in  $g$ :  $\varphi = 1$

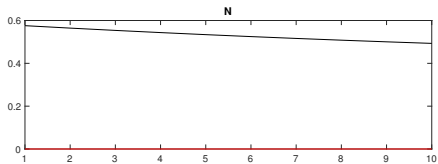
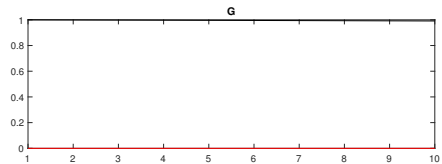
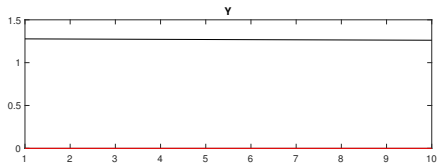
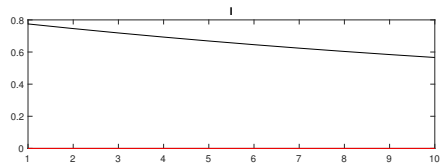
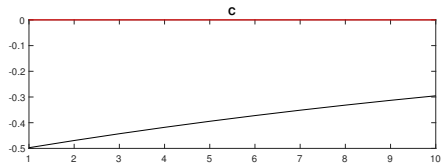


Dynamics after an unexpected increase in  $g$ :  $\varphi = \infty$





Dynamics after an unexpected increase in  $g$ :  $\varphi = 0$



Dynamics after an unexpected increase in  $g$ :  $\varphi = \infty$  and very persistent increase in  $g$  (almost permanent)

BEYOND THE NGM

## REAL RATES

- Let us deviate from the standard NGM for a while.
- Consider the Euler equation

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta (1 + r_{t+1}).$$

- We said that consumption growth is determined by the real interest rate.

## REAL RATES

- Suppose that the government can somehow control the real rate and keeps it constant at the steady state level  $r = \beta^{-1} - 1$ .
- Then the Euler equation becomes

$$\frac{u'(c_t)}{u'(c_{t+1})} = 1,$$

which implies that consumption is constant.

- In this case the multiplier will depend only on the response of  $i_t$  to  $g_t$ .
- If investment is constant, then the multiplier will be equal to one.

## REAL RATES

- Consider a special case with no capital in this economy (or: fixed capital).
- The multiplier depends **only** on the behavior of the real rate.
- Nothing else matters.
- This is what happens in a large class of New Keynesian models (which we will discuss later).
- Monetary policy response to government purchases is crucial, because it affects the real rate. See Woodford (2011).

## KEYNESIAN CROSS

- So: is there anything wrong with the standard old Keynesian logic?
- Not really, but we need to be more careful and take into account the intertemporal nature of consumption-savings decisions.
- We will discuss it in more detail next time