Problem Set 2

Quantitative Economics, Fall 2023

December 5, 2023

This problem set consists of two problems. You have two weeks to solve them and submit your solutions (Dec. 18th 11:59 PM). You can work in teams of up to three students.

Problem 1: Job search and human capital accumulation

Consider a modified version of the job search model from the lecture. There are N possible levels of human capital: $H_1, \dots, H_N$. The worker's labor income depends both on the level of human capital H and the wage rate w. It equals $w \cdot H$.

At the beginning of period t an unemployed worker gets a wage offer w_t and decides whether to accept it or not. If she accepts, she starts working and gets $w_t \cdot H_t$. She might gain some experience: if her current human capital is lower than H_N, she will enter the next period with human capital higher by one level with probability α. It is also possible, that she loses her job. If she works, she will enter the next period unemployed with probability ϱ. She then gets to draw a new wage offer w_{t+1}. If she does not lose her job she gets the same wage next period and cannot decide to quit.

If she rejects, she stays unemployed and gets the unemployment benefit c. Because she is not working, her skills might become obsolete. If her human capital is not equal to H_1, it might decrease with probability ℓ. The worker's problem is to maximize the expected discounted sum of future earnings.

The Bellman equation for the unemployed agent with human capital H_n and the current wage offer w is

$$
\begin{aligned}
v^U(w, H_n) = \max\Bigg\{ & wH_n + \beta\,(1-\varrho)\left[\alpha v^E(w, \min\{H_{n+1}, H_N\}) + (1-\alpha)\, v^E(w, H_n)\right], \\
& + \beta\varrho \sum_{w' \in W} \phi(w')\left[\alpha v^U(w', \min\{H_{n+1}, H_N\}) + (1-\alpha)\, v^U(w', H_n)\right], \\
& c + \beta \sum_{w' \in W} \phi(w')\left[\ell v^U(w', \max\{H_{n-1}, H_1\}) + (1-\ell)\, v^U(w', H_n)\right] \Bigg\}
\end{aligned}
$$

The Bellman equation for the employed agent with human capital H_n and the current wage w is

$$
\begin{aligned}
v^E(w, H_n) = {} & wH_n + \beta\,(1-\varrho)\left[\alpha v^E(w, \min\{H_{n+1}, H_N\}) + (1-\alpha)\, v^E(w, H_n)\right] \\
& + \beta\varrho \sum_{w' \in W} \phi(w')\left[\alpha v^U(w', \min\{H_{n+1}, H_N\}) + (1-\alpha)\, v^U(w', H_n)\right].
\end{aligned}
$$

I want to you explore reservation wages for agents with various levels of human capital. Are the agents with low human capital more willing to accept low wage offers? Or is it the other way around?

Use the following parameters: $\beta = 0.96, \alpha = 0.1, \varrho = 0.02, \ell = 0.1, c = 0.1$. Let there be 7 human capital levels, with $H_1 = 1, H_7 = 4$ and other 5 points equally spaced. Assume there are 51 levels of wages, equally spaced from 10 to 100, with ϕ corresponding to the probability mass function of a beta-binomial distribution with $a = 200, b = 100, n = 50$.[1]

[1] $\phi(10)$ is equal to the probability of zero successes, $\phi(50)$ is equal to the probability of 50 successes, etc.

Calculate and plot reservation wages for agents with various levels of human capital. How do they change when α equals 0.2 (so that agents are more likely to increase their human capital when they are employed)?

Problem 2: Markov dynamics

Consider a model with two state variables: X_t and Z_t. Z_t is exogenous and follows a Markov process with transition matrix P.

Let $Z_t \in \mathrm{Z} = \{z_1, z_2, z_3\}$ and

$$P = \begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.7 & 0.1 \\ 0.3 & 0.3 & 0.4 \end{pmatrix}.$$

$X_t \in \mathrm{X} = \{0, 1, 2, 3, 4, 5\}$ is endogenous. The policy function $X_{t+1} = \sigma(X_t, Z_t)$ is given by

$$\sigma(X_t, Z_t) = \begin{cases} 0 & \text{if } Z_t = z_1 \\ X_t & \text{if } Z_t = z_2 \\ X_t + 1 & \text{if } Z_t = z_3 \text{ and } X_t \leq 4 \\ 3 & \text{if } Z_t = z_3 \text{ and } X_t = 5 \end{cases}.$$

Obtain the transition matrix for $\{X_t, Z_t\}$ (jointly). Is the Markov chain irreducible? Find the stationary distribution. Find the marginal distribution of X_t. Calculate the expected value of X_t using the marginal stationary distribution.