Problem Set 3

Quantitative Economics, Fall 2023

February 6, 2024

This problem set consists of a single problem. Please submit your solutions by Feb. 18th if you need to have your grade in the first session or by March 15th. You can work in teams of up to three students.

Problem 1: Marginal propensity to consume

Recall your introduction to macro. You learned about the consumption function and the marginal propensity to consume (MPC). The MPC is the change in consumption resulting from a change in disposable income. It it a crucial parameter in a Keynesian cross model. For example, it determines the size of the multiplier. It remains true in more sophisticated modern models as well – see Auclert, Rognlie and Straub (2024) for a disucssion of the intertemporal MPC and its role in the propagation of shocks and policies. In this exercise you will compute MPC in a standard heterogeneous agent model.

The Bellman equation is

$$V(a,z) = \max_{c,a'} \left\{ u(c) + \beta \sum_{z' \in Z} P(z,z') V(a',z') \right\}$$
$$\text{subject to } c + a' = zw + (1+r)\,a,$$
$$a' \geq -\phi.$$

where a us the level of assets, z is the level of productivity, c is the level of consumption, a' is the level of assets in the next period, w is the wage rate, r is the interest rate, β is the discount factor, $P(z,z')$ is the transition probability from state z to state z', and ϕ is the borrowing limit. The utility function is $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, where $\gamma \geq 0$ is the coefficient of relative risk aversion and, at the same time, the inverse of the intertemporal elasticity of substitution.

After solving the Bellman equation, we can obtain the policy function $c(a,z)$, which gives the optimal consumption level for a given level of assets and productivity. The MPC out of a transfer of size τ can be calculated as

$$\text{MPC}(a,z;\tau) = \frac{c\left(a + (1+r)^{-1}\tau, z\right) - c(a,z)}{(1+r)^{-1}\tau}.$$

This means that you need to evaluate the policy function at two points: $c(a,z)$ and $c\left(a + (1+r)^{-1}\tau, z\right)$. The first argument of the policy function differs by $(1+r)^{-1}\tau$. This means that beginning of period wealth differs by τ, the size of the transfer. Observe that usually you will have to interpolate the policy function to obtain the consumption level for a given level of assets and productivity.

Your task is to calculate i) the average MPC (weighted using the stationary distribution – you need to obtain in) and ii) the MPC for agents in the 10th, 50th, and the 90th percentile of the wealth distribution, the iii) the ratio of wealth (aggregate assets) to income (aggregate labor income) in this economy.

Use the following parameters: $\beta = 0.97, \gamma = 1, r = 0.01, \phi = 0.1, w = 1$. One period in the model is meant to represent one quarter. Let there be 7 productivity levels, obtained by discretization of an AR(1) process with $\rho = 0.95, \sigma = 0.1$. Ensure that the expected productivity level is equal to 1.

Note that the average wage in this economy is 1. We are usually interested in values expressed in the unit of account, for example US dollars. Choose τ so that it corresponds to 1000$. You will need to find out the current average wage in the US.

I suggest that you use grid that has more points in the region of the borrowing constraint. Use this exercise as the opportunity to experiment with the grid.