Notes on Durable Consumption

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Outline

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Basic Optimization Model: Continuous Choice

$$V(A, D, y, p) = \max_{D', A'} u(c, D) + \beta E_{y', p' | y, p} V(A', D', y', p') \qquad (1)$$

for all (A, D, y, p) with

$$c = A + y - (A'/R) - p(D' - (1 - \delta)D) \qquad (2)$$

$$A' = R(A + y - c - pe) \qquad (3)$$

$$D' = D(1 - \delta) + e \qquad (4)$$

FOC and Euler

FOC

$$u_c(c, D) = \beta R E_{y',p'|y,p} V_A(A', D', y', p') \tag{5}$$

and

$$u_c(c, D)p = \beta E_{y',p'|y,p} V_D(A', D', y', p') \tag{6}$$

Euler

$$u_c(c, D) = \beta R E_{y',p'|y,p} u_c(c', D') \tag{7}$$

and

$$p u_c(c, D) = \beta E_{y',p'|y,p}[u_D(c', D') + p'(1-\delta)u_c(c', D')] \tag{8}$$

No Time to Build

$$V(A, D, y, p) = \max_{D', A'} u(c, D') + \beta E_{y', p' | y, p} V(A', D', y', p') \tag{9}$$

Implies

$$u_c(c, D') = \beta R E_{y', p' | y, p} u_c(c', D'') \tag{10}$$

$$p u_c(c, D') = [u_D(c, D') + \beta E_{y', p' | y, p} p'(1 - \delta) u_c(c', D'')] \tag{11}$$

If prices are constant:

$$u_D(c, D') = \beta R E_{y' | y} u_D(c', D''). \tag{12}$$

Mankiw

- $\beta R = 1$
- separable quadratic utility
- implications
  - $E_t D_{t+1} = D_t$
  - $e_{t+1} = a_0 + a_1 e_t + \varepsilon_{t+1} - (1 - \delta)\varepsilon_t$
- empirical evidence implies $\delta = 1$
- Alternatives:
  - adjustment costs
  - alternative preferences
  - discrete choice

Outline

Simple Car Replacement

- keep or replace a car
- service flow falls with car age: $s_i > s_{i+1}$
- deterministic problem
- optimization implies optimal replacement age
- only heterogeneity is in car age

Simple Car Replacement

$V_i = \max[V_i^k, V_i^r]$

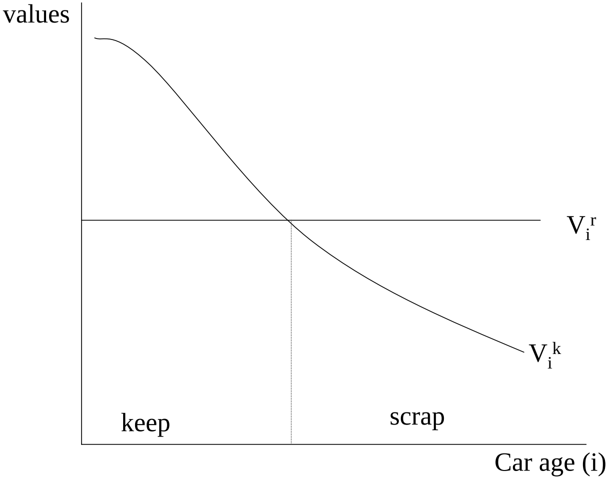
$$V_i^k = u(s_i, y) + \beta V_{i+1} \tag{13}$$

and

$$V_i^r = u(s_1, y - p + \pi) + \beta V_2$$

where $\beta \in (0,1)$. Here y is income, p is the price of a car, s_i is the service flow of age i car and π is the scrap value.

How would you solve this?


values
V_i^r
V_i^k
keep
scrap
Car age (i)

Going to the Data: Micro

- the optimal scrapping time is a critical age, i^*
- this age depends on the vector of parameters, Θ: $i^*(\Theta)$
- Observations on car ownership would then determine the optimal scrapping time
- Θ would not be identified
- zero likelihood problem: not all car scrapped at the same age
- same point we saw with discrete stochastic cake eating problem

Macro Implications: General

$$S_t = \sum_k H(k, Z_t) f_t(k)$$

- aggregate sales: S_t
- age of car: k
- aggregate state: Z_t
- hazard function : $H(k, Z_t)$
- pdf over car ages: $f_k(t)$

This Example

k^* is optimal scrapping age

- $f_k(t)$: fraction of age k cars period t
- $f_{k+1}(t+1) = f_k(t)$ for $k = 1, 2, \ldots k^* - 1$
- $S_t = f_{k^*}(t)$

This Example

What are the aggregate implications?

This Example

What are the aggregate implications?

- Aggregate Car Sales depends on initial distribution
  - smooth if initial distribution is uniform
  - deterministic aggregate cycles if distribution is degenerate
  - intermediate possibilities
- relative variability of aggregate vs idiosyncratic shocks is key

Outline

More Complete Model

$V_i(z, Z) = \max[V_i^k(z, Z), V_i^r(z, Z)]$ where

$$V_i^k(z, Z) = u(s_i, y + Y, \varepsilon) + \beta(1-\delta)EV_{i+1}(z', Z') + \beta\delta EV_1^b(z', Z') \quad (14)$$

and

$$V_i^r(z, Z) = u(s_1, y + Y - p + \pi, \varepsilon) + \beta(1-\delta)EV_2(z', Z') + \beta\delta EV_1^b(z', Z') \quad (15)$$

and

$$V_1^b(z, Z) = u(s_1, y + Y - p + \pi, \varepsilon) + \beta(1-\delta)EV_2(z', Z') + \beta\delta EV_1^b(z', Z') \quad (16)$$

$$u(s_i, c) = \left[i^{-\gamma} + \frac{\varepsilon(c/\lambda)^{1-\xi}}{1-\xi}\right]$$

Aggregate Dynamics

- Aggregate Hazard

$$H_k(Z_t, \theta) = \int h_k(z_t, Z_t, \theta)\phi(z_t)dz_t \tag{17}$$

where $h_k(z_t, Z_t, \theta)$ is individual hazard, including wrecks

- Sales

$$S_t(Z_t, \theta) = \sum_k H_k(Z_t, \theta) f_t(k) \tag{18}$$

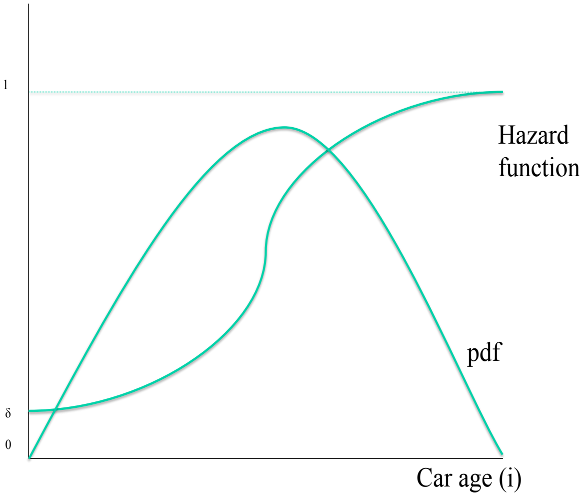
- Distribution

$$f_{t+1}(k, Z_t, \theta) = [1 - H_k(Z_t; \theta)] f_t(k-1) \qquad \text{for } k > 1 \tag{19}$$

$$f_t(1, Z_t, \theta) = S_t(Z_t, \theta)$$

Key Points

- Sales driven by the interaction of the evolution of the distribution and the hazard
- Aggregate Shocks bunch the distribution and leads to echo effects
- Micro Shocks disperse the distribution and smoothes things out


1
Hazard
function
pdf
δ
0
Car age (i)

Estimation

- estimate exogenous processes in the first-stage
- estimate structural parameters using SMM and non-linear least squares in the second stage $\mathcal{L}_N(\theta) = \alpha \mathcal{L}_N^1(\theta) + \mathcal{L}_N^2(\theta)$

$$\mathcal{L}_N^1(\theta) = \frac{1}{T}\sum_{t=1}^{T}\left[(S_t - \bar{S}_t(\theta))^2 - \frac{1}{N(N-1)}\sum_{n=1}^{N}(S_{tn}(\theta) - \bar{S}_t(\theta))^2\right]$$

$$\mathcal{L}_N^2(\theta) = \sum_{i=\{5,10,15,AR,MA\}} \alpha_i(\bar{F}^i - \bar{F}^i(\theta))^2$$

First-Stage Estimation

$$Y_t = \mu_Y + \rho_{YY} Y_{t-1} + \rho_{Yp} p_{t-1} + u_{Yt}$$

$$p_t = \mu_p + \rho_{pY} Y_{t-1} + \rho_{pp} p_{t-1} + u_{pt}$$

$$\varepsilon_t = \mu_\varepsilon + \rho_{\varepsilon Y} Y_{t-1} + \rho_{\varepsilon p} p_{t-1} + u_{\varepsilon t}$$

The covariance matrix of the innovations $u = \{u_{Yt}, u_{pt}, u_{\varepsilon t}\}$ is

$$\Omega = \begin{bmatrix} \omega_Y & \omega_{Yp} & 0 \\ \omega_{pY} & \omega_p & 0 \\ 0 & 0 & \omega_\varepsilon \end{bmatrix}$$

Policy: Adda-Cooper

- use estimated parameters from pre-policy period
- simulate state dependent scrapping subsidies
- study effects on sales and revenues

Outline

Fernandez-Villaverde and Krueger

Key Elements

- life cycle pattern in non-durables and durables
- emphasize role of durables as collateral for borrowing
- dynamic equilibrium model

Individual Optimization builds on F-V and K

$$v(\varepsilon, k, k^d) = \max(v^R(\varepsilon, k, k^d), v^D(\varepsilon, k, k^d)) \qquad \forall(\varepsilon, k, k^d) \quad (20)$$

Here:

- ε is labor productivity
- k is holding of capital
- k^d is the stock of durables
- Borrowing is feasible, i.e., $k < 0$, but is limited by the incentive for repayment.

Value of repayment ($k < 0$):

$$v^R(\varepsilon, k, k^d) = \max_{k'<0, k^{d'}} u(c, k^d) + \beta E_{\varepsilon'|\varepsilon} v(\varepsilon', k', k^{d'}) \tag{21}$$

with

$$c = \omega\varepsilon + k - q(\varepsilon, k', k^{d'})k' + (k^{d'} - (1 - \delta^d)k^d). \tag{22}$$

Value of default:

$$v^D(\varepsilon, k, k^d) = \max_{k' \geq 0, k^{d'}} u(c, \min(k^d, \underline{k}^d)) + \beta E_{\varepsilon'|\varepsilon} v^A(\varepsilon', k', k^{d'}) \quad (23)$$

where

$$k' + k^{d'} + c = \psi\omega\varepsilon + (1 - \delta^d)\min(k^d, \underline{k}^d)). \quad (24)$$

Note: penalty involves loss of durables and income

$$v^A(\varepsilon, k, k^d) = \max_{k' \geq 0, k^{d'}} u(c, k^d) + \beta E_{\varepsilon'|\varepsilon}[\lambda v^A(\varepsilon', k', k^{d'}) + (1-\lambda) v(\varepsilon', k', k^{d'})] \tag{25}$$

$$k' + k^{d'} + c = \omega\varepsilon + (1-\delta^d)k^d + k(R + (1-\delta)) \tag{26}$$

Note:

- save but not borrow under autarky
- possible return to good status