Consumption, Portfolio Choice and Labor Supply

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Outline

$$v(A,R,y) = max_s u(c) + \beta E_{y',R'|R,y} v(A',R',y') \tag{1}$$

for all (A,R,y)

subject to:

- $c = y + A - \sum_i s^i$
- $A' = \sum_i R^{i\prime} s^i$
- $s^i \geq \underline{s^i}$,

where

- state vector: A is start of period financial wealth, y is exogenous labor income and R is realized return
- control vector: $s = (s^1, ...s^N)$ is the purchase of shares of asset $i = 1, N$
- policy functions: $s^i = \phi^i(A,R,y), c = \zeta(A,R,y)$

Issues

- infinite vs finite horizon
- single vs multiple agents
- income process: (i) aggregate vs idiosyncratic shocks, (ii) transitory vs permanent shocks (iii) unemployment spells and (iv) non-Gaussian stochastic process
- gross or net income: treatment of government taxes and transfers, potential nonlinearities
- exogenous income

Conditions of (Interior) Solution

for $i = 1, 2, ..N$

$$u'(c) = \beta E_{y',R'|R,y}[R^{i\prime} v_A(A', R', y')] \tag{2}$$

Using the Bellman equation, generate Euler Equations

$$u'(c) = \beta E_{y',R'|R,y}[R^{i\prime} u'(c')] \tag{3}$$

Note:

- holds for all assets
- holds for groups of assets
- will fail if borrowing constraint binds

Hall

Assumptions

- $N = 1$
- $\beta R = 1$
- quadratic utility: $u(c) = \mathbf{a}c - (\frac{\mathbf{b}c}{2})^2$
- Euler equation implies consumption follows a random walk

$$Ec' = c \tag{4}$$

- Interpreting this condition
  - changes in consumption are not predictable conditional on current consumption
  - it does not mean that consumption is constant
  - $c_{t+1} = c_t + \epsilon_{t+1}, E_t(\epsilon_{t+1}) = 0$ but $\epsilon_{t+1} \neq 0$

Empirical Implementation

Let z_t be a variable known in period t. Consider

$$c_{t+1} = \alpha_0 + \alpha_1 c_t + \alpha_2 z_t + \varepsilon_{t+1} \tag{5}$$

Theory predicts

- $\alpha_1 = 1$
- $\alpha_2 = 0$ for any z_t: $E z_t \varepsilon_{t+1} = 0$
- so throw in all the variables and test $\alpha_2 = 0$
- test of theory not estimation of structural parameters
    - cannot reject $\alpha_2 = 0$ for consumption and income instruments
    - reject $\alpha_2 = 0$ for stock returns

Hansen-Singleton

Approach

- work with general model: N assets, more general utility
- estimate parameters and test model
- methodology is used in many applications
- particular results less influential
- assume $u(c) = \frac{c^\gamma}{\gamma}$ so $u'(c) = c^\alpha, \alpha \equiv (\gamma - 1)$
- representative agent (not panel data for their application)

Components of Model

- Euler (asset $i = 1, 2, 3, \dots N$)

$$u'(c_t) = \beta E_t[R^i_{t+1} u'(c_{t+1})] \tag{6}$$

- create *ex post* expectational errors for $i = 1, 2, \dots N$ and $t = 1, 2, \dots T - 1$

$$\varepsilon^i_{t+1}(\Theta) = \frac{\beta[R^i_{t+1} u'(c_{t+1})]}{u'(c_t)} - 1 \tag{7}$$

- $\Theta^* = (\beta^*, \alpha^*)$
- THEORY: *ex post* errors not predictable: $E_t(\varepsilon^i_{t+1}(\Theta^*)) = 0$

Methodology: (Overview Only)

- impose sample analogue of $E_t(\varepsilon^i_{t+1}(\Theta^*)) = 0$
- compute *ex post* errors from data given Θ
- $m^{ij}_T(\Theta) \equiv \frac{1}{T}\sum(Z^j_t \times \varepsilon^i_{t+1}(\Theta))$ for instrument $j = 1, 2, 3, \ldots J$
- solve $J \equiv min_{\Theta} m_T(\Theta)' W m_T(\Theta)$ where
  - m_T is vector $(NJX1)$ of moments
  - W is a weighting matrix; usually the inverse of the VarCov
  - this minimization yields consistent estimates of Θ^*
  - implement iteratively: $W = I$, estimate parameters, calculate VarCov
- See Hamilton GMM Chpt. or Hansen notes for proofs of these properties

Findings

- α around -0.9; β around 0.99; some but not all specifications rejected(J too far from zero)
- Errata published in Eca, Jan 1984: much more variability in estimates of utility curvature
- Huge impact on methodology
- concerns over instrument choice and sensitivity

Example

- $N = 1$
- Euler becomes:

$$u'(c_t) = \beta E_t[R_{t+1} u'(c_{t+1})] \tag{8}$$

- *Ex post* error is

$$\varepsilon^i_{t+1}(\Theta) = \frac{\beta[R_{t+1} u'(c_{t+1})]}{u'(c_t)} - 1 \tag{9}$$

- instruments: $Z_t = (1, c_{t-1}, y_{t-1})$
- moment for $Z_t = 1$ is $m^1_T(\Theta) \equiv \frac{1}{T} \sum \varepsilon_{t+1}(\Theta))$
- moment $j > 1$ is: $m^j_T(\Theta) \equiv \frac{1}{T} \sum (Z^j_t \times \varepsilon_{t+1}(\Theta))$
- recall GMM in OLS and IV estimation

SMM Approach

- data on (c_t, y_t, R_t) for $t = 1, 2, 3, \dots. T$.
- estimate process for (y_t, R_t) directly from the data, eg VAR(1).
- estimate $\Theta = (\beta, \alpha)$ using SMM
  - data moments:
  - given Θ solve DPP to obtain policy function: $c = \phi_\Theta(A, y, R)$
  - simulate model to obtain simulated data set
  - calculate moments as you did in the data
  - minimize distance between data and model moments
- much slower than GMM
- use policy function $c = \phi_{\hat{\Theta}}(A, y, R)$ for policy exercises depending on MPC

Extensions

- borrowing constraints
- habit formation
- labor supply
- life cycle model
- portfolio adjustment costs
- durable goods

Borrowing Constraints

- if they bind for asset i: $u'(c_t) > \beta E[R^i_{t+1} u'(c_{t+1})]$
- ignoring them can bias the estimation
- estimate parameters from other Euler equations
- constraints influence choices even if they do not bind (Precautionary savings)
- Sources:
  - natural borrowing constraint (Aiyagari)
  - incentive based: choice over (repay, default) [see additional notes]
  - *ad hoc*: $s^i \geq \underline{s}^i$, constraint could depend on the state

Consumption Habit

$$v(A, y, c_{-1}) = max_{A'} u(c + \gamma c_{-1}) + \beta E_{y'|y} v(A', y', c) \qquad (10)$$

for all (A, y, c_{-1}) with $c = y + A - \frac{A'}{R}$.

- utility depends upon **own** consumption today and last period: internal habit.
- Euler is more complicated: Hansen-Singleton mis-specified
- estimate parameters using GMM
- if $\gamma = 0$, then no habit

Euler Equation

FOC

$$\frac{-1}{R}u'(y+A-\frac{A'}{R}+\gamma c_{-1})+\beta E_{y'|y}[v_1(A',y',c)+\frac{-1}{R}v_3(A',y',c)]=0 \tag{11}$$

Use

$$v_1(A,y,c_{-1})=u'(y+A-\frac{A'}{R}+\gamma c_{-1})+\beta E_{y'|y}v_3(A',y',c) \tag{12}$$

and

$$v_3(A,y,c_{-1})=\gamma u'(y+A-\frac{A'}{R}+\gamma c_{-1}) \tag{13}$$

to get the Euler

$$u'(c+\gamma c_{-1})+\beta E_{y'|y}\gamma u'(c'+\gamma c)=\beta R E_{y'|y}[u'(c'+\gamma c)+\beta\gamma u'(c''+\gamma c')] \tag{14}$$

Labor Supply

- static problem: intensive and extensive margins
- dynamic problem
  - intensive, extensive margins and savings all together
  - dynamics induced by habit
  - estimate wage process
  - interactions with portfolio and borrowing constraints
  - estimation challenge: avoid zero likelihood, use Euler equations

Static Model

- Utility: $u(c, n, \epsilon)$ with $c = \omega n + A$
- if work: $\omega = -\frac{u_n(c,n,\epsilon)}{u_c(c,n,\epsilon)}$
- reservation wage: $\omega^* = -\frac{u_n(A,0,\epsilon)}{u_c(A,0,\epsilon)}$
- work iff $\omega > \omega^*$
- READ
    - Heckman, Eca. July 1974: model
    - Heckman, Eca. July 1979: two-step procedure
    - Killingsworth **Labor Supply**, Cambridge University Press
    - Alternative approaches: propensity score methods

Static Model: Estimation (Killingsworth)

- Utility: $u(c, n, \epsilon) = (\omega(n+\epsilon) + A)^{\alpha}(1 - n - \epsilon)^{\beta}$
- will work iff: $\epsilon < J \equiv (1-b) - b\frac{A}{\omega}$ with $b = \frac{\beta}{\alpha+\beta}$
- Probability no work: $(1 - F(J_i))$ use $i \in \bar{E}$ for variations in $\frac{A_i}{\omega_i}$
- if work: $n_i = J_i - \epsilon_i$, $i \in E$
- likelihood:

$$\Omega(b) = \Pi_{i \in \bar{E}}(1 - F(J_i))\Pi_{i \in E} f(J_i - n_i)$$

- estimation of b combines both intensive and extensive choices
- ASSUMES ω_i is observed even if $n_i = 0$

Static Model: Estimation with selection

- normally observe only accepted wages
- selection bias in the sample of workers: only those with low ϵ participate
- conditional expectation becomes:

$$E(n_i|n_i > 0) = J_i - \kappa_i$$

  with $\kappa_i \equiv E[\epsilon|\epsilon < J_i]$
- regression of n_i without κ_i leads to OVB: selection depends on b
- How can you estimate these parameters?

Static Model: Estimation with selection

- estimate participation and hours choice jointly through MLE

  ...challenge: wages for non-workers not observed
- Heckman (Eca., 1979) two-step procedure (there are alternatives!)
  - need Z_i that impacts participation but not hours worked: see Heckman 1974 re married women's labor supply
  - Step 1: regress participation on Z_i
  - compute inverse Mills ratio (IMR) to capture probability of working
  - Step 2: include IMR in the hours regression to eliminate selection bias
- SMM: $\Theta = (\alpha, \beta, F(\epsilon))$
  - given Θ
  - find workers: $n_\Theta(\epsilon) > 0$
  - match moments
    - participation conditional on observables
    - hours conditional on observables
    - use Heckit two-step as moment
    - impose cross equation restrictions

Labor with Habit

$$v(A, \omega, n_{-1}) = max_{A',n} u(\omega n + A - \frac{A'}{R}, n + \gamma n_{-1}) + \beta E_{\omega'|\omega} v(A', \omega', n) \tag{15}$$

for all (A, ω, n_{-1})

- utility depends upon both labor supply today and last period.
- FOCs will now entail two dynamic equations
- estimate them jointly using GMM
- if $\gamma = 0$, then no dynamic to labor supply

Euler Equations

- intertemporal consumption

$$u_c(c, n + \gamma n_{-1}) = \beta R E_{\omega'|\omega} u_c(c', n' + \gamma n) \tag{16}$$

- intertemporal labor supply

$$\omega u_c(c, n+\gamma n_{-1}) + u_n(c, n+\gamma n_{-1}) + \beta\gamma E_{\omega'|\omega} u_n(c', n'+\gamma n) = 0 \tag{17}$$

Discrete Employment Choice

- $V(A,\omega,\xi) = max\{V^w(A,\omega,\xi), V^n(A,\omega,\xi)\}$ where
- value of work is

$$V^w(A,\omega,\xi) = max_{A'}\, u(A+\omega-\frac{A'}{R})-\xi+\beta E_{\omega',\xi'|\omega,\xi} V(A',\omega',\xi') \tag{18}$$

- value of no work is

$$V^n(A,\omega,\xi) = max_{A'}\, u(A-\frac{A'}{R})+\beta E_{\omega',\xi'|\omega,\xi} V(A',\omega',\xi') \tag{19}$$

Discrete and Continuous Employment Choice

- $V(A, \omega, \xi) = max\{V^w(A, \omega, \xi), V^n(A, \omega, \xi)\}$ where
- value of work is

$$V^w(A, \omega, \xi) = max_{n,A'} u(A + \omega n - \frac{A'}{R}, n) - \xi + \beta E_{\omega', \xi' | \omega, \xi} V(A', \omega', \xi') \qquad (20)$$

- value of no work is

$$V^n(A, \omega, \xi) = max_{A'} u(A - \frac{A'}{R}, 0) + \beta E_{\omega', \xi' | \omega, \xi} V(A', \omega', \xi') \qquad (21)$$

Life Cycle Model

- income: $y_t^i = \alpha_0^i + \alpha_1 age_t^i + \alpha_2 (age_t^i)^2 + \rho y_{t-1}^i + \varepsilon_t^i$
- Bellman Equation

$$v_t(y_t, A_t) = max_{A_{t+1}} u(c_t) + \beta E_{y_{t+1}|y_t} v_{t+1}(y_{t+1}, A_{t+1}) \quad (22)$$

  where $A_{t+1} = (A_t + y_t - c_t)R_{t+1}, t = 1, 2, \ldots T$.
- solve by VFI, starting with $t = T$ and work backwards.
- no stationarity, just an endpoint.

Costly Portfolio Adjustment

- In the data see:
  - majority of households do not hold stocks
  - majority of households do not adjust stocks
- portfolio adjustment is costly
- asset market participation is costly: two period example
- consider dynamic discrete choice problem

Two Period Participation Example

- Period 2: consume and die, no bequest
- Period 1:
  - discrete choice: asset market participation (costly)
  - continuous choice over available assets
- $R = (R^b, R^s)$ for bond and stock returns
- Γ is a cost of participating in stock markets
- F is an adjustment cost

Two Period Participation Example: initial $A^s = 0$

- Period 2

$$V_1(A^b, A^s, R, y) = u(c) \tag{23}$$
$$c = y + A^b R^B + A^s R^s \tag{24}$$

- Period 1
  - Participate

$$\begin{aligned} V_2^p(A^b, A^s, R, y) &= max_{A^{b\prime}, A^{s\prime}} u(c) + \beta E V_1(A^{b\prime}, A^{s\prime}, R', y') \\ c &= y + A^b R^b - A^{b\prime} - A^{s\prime} - \Gamma \end{aligned} \tag{25}$$

  - No Participate

$$\begin{aligned} V_2^n(A^b, A^s, R, y) &= max_{A^{b\prime}} u(c) + \beta E V_1(A^{b\prime}, 0, R', y') \\ c &= y + A^b R^b - A^{b\prime} \end{aligned} \tag{26}$$

  - Discrete Choice

$$V_2(A^b, A^s, R, y) = max\{V_2^p(A^b, A^s, R, y), V_2^n(A^b, A^s, R, y)\} \tag{27}$$

Two Period Adjustment Example: initial $A^s > 0$

- Period 2

$$V_1(A^b, A^s, R, y) = u(c) \tag{28}$$

$$c = y + A^b R^B + A^s R^s \tag{29}$$

- Period 1
  - Adjust

$$\begin{aligned} V_2^a(A^b, A^s, R, y) &= max_{A^{b\prime}, A^{s\prime}} u(c) + \beta E V_1(A^{b\prime}, A^{s\prime}, R', y') \\ c &= y + A^b R^b + A^s R^s - A^{b\prime} - A^{s\prime} - F \end{aligned} \tag{30}$$

  - No Adjust

$$\begin{aligned} V_2^{na}(A^b, A^s, R, y) &= max_{A^{b\prime}} u(c) + \beta E V_1(A^{b\prime}, A^s R^s, R', y') \\ c &= y + A^b R^b - A^{b\prime} \end{aligned} \tag{31}$$

  - Discrete Choice

$$V_2(A^b, A^s, R, y) = max\{V_2^a(A^b, A^s, R, y), V_2^{na}(A^b, A^s, R, y)\} \tag{32}$$

Durables

- see separate notes, Adda-Cooper and Krueger-Villaverde
- preferences: $u(c, D)$
- transition: $D' = D(1 - \delta) + e$
- budget constraint: $A' = R(A + y - c - pe)$