

Notes on Firm Investment

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Outline

- 1 Firm Dynamics
- 2 Overview of Investment
 - Basic Optimization Model

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- 3 Models and Estimation
 - Profit Function
 - No Adjustment Costs
 - Q theory
 - Machine Replacement
 - Continuous Choice

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- Profit Function

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- No Adjustment Costs

- Q theory

- *Q theory*

- ### • Making Decisions

- Machine Replacement

- Continuous Choice

Sources of Firm Dynamics

- dynamic factor (capital and labor) demand
- inventories
- prices
- training
- R & D
- advertising
- strategic interactions

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Overview of Investment

- example of dynamic factor demand
- costs of adjustment imply dynamics
- lumpy adjustment of capital and labor at the plant level
- aggregate is smoother
- what are the adjustment costs?
 - what models do we consider?
 - how do we estimate the parameters?
 - what do we find?

$$V(A, K) = \max_{K'} \pi(A, K) - p(K' - (1 - \delta)K) - C(A, K', K) + \beta E_{A'|A} V(A', K') \quad (1)$$

for all (A, K) .

Key Elements

- discount at $\beta(\cdot)$: can allow state dependent SDF
- $\pi(A, K)$ is a reduced form profit function from optimization over flexible factors. This function is strictly increasing and concave. $A = (a, \varepsilon)$.
- $C(\cdot)$ is a general cost of adjustment function
- A plays two roles: influences profits and provides information
- assume problem is bounded and so there exists a solution to the functional equation.

Specifications

- No Adjustment Costs
- Quadratic Adjustment Costs
 - Q - Theory
 - Euler Equation Estimation
 - Cooper-Ejarque on SMM
- Nonconvex Adjustment Costs
 - Dynamic Discrete Choice Problem
 - No Q, No Euler
 - Cooper-Haltiwanger on SMM

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- with CRS production and constant elasticity demand:
 $\Pi(A, K) = AK^\alpha$
- α reflects factor shares and demand elasticity (CH, RES)
- A combines TFP and Demand Shocks and factor prices

- OLS
 - omitted variable bias
 - could use as a moment in SMM
- Quasi-first differences and GMM (Cameron and Trevedi)
- Proxy for Current Productivity

Quasi-first differences and GMM

- Profits in logs

$$\pi_{it} = a_t + \varepsilon_{it} + \alpha k_{it}$$

- ε_{it} has serial correlation ρ
- take difference using $\rho * \pi_{t-1}$

$$\begin{aligned}\pi_{it} &= \rho * \pi_{it-1} + a_t - \rho * a_{t-1} + \varepsilon_{it} - \rho * \varepsilon_{it-1} + \alpha(k_{it} - \rho * k_{it-1}) \\ &= \rho * \pi_{it-1} + a_t - \rho * a_{t-1} + \alpha(k_{it} - \rho * k_{it-1}) + \zeta_{it}\end{aligned}$$

- key is that $\zeta_{it} = \varepsilon_{it} - \rho * \varepsilon_{it-1}$ is a period t innovation uncorrelated with anything prior to period t : $E_{t-1}\zeta_{it} = 0$.

GMM Production Function: No Aggregate Shock

- Orthogonality Condition: firm i , instrument j , time t

$$E z_{jit-1} \times \overbrace{[\pi_{it} - (\rho * \pi_{it-1} + \alpha(k_{it} - \rho k_{it-1}))]}^{error_{it}} = 0$$

- This holds at truth: $\Theta^* = (\alpha^* \rho^*)$
- moment

$$m^{ji}(\Theta) = \frac{\sum_t (z_{jit-1} error_{it})}{T-1}$$

-

$$m^j(\Theta) = \frac{\sum_i m^{ji}(\Theta)}{I}$$

- estimator

$$\hat{\Theta} = \operatorname{argmin}_{\Theta} m' W m'$$

- where

-

$$m(\Theta) = (m^1(\Theta), m^2(\Theta) \dots m^J(\Theta))$$

is $1 \times J$

- W is a $J \times J$ weighting matrix

Proxy Method

- Proxy for Current Productivity:
 - $x_{it} = \phi(\varepsilon_{it})$ from optimal choice
 - invert to obtain $\varepsilon_{it} = \phi^{-1}(x_{it})$
 - approximate $\phi^{-1}(x_{it})$
- Condition on Investment Choices (Olley-Pakes)
- Condition on material demand (Levinson-Petrin)
- Parameterize conditional expectation of productivity using lagged inputs (Wooldridge)
- Estimate parameters with GMM

No Adjustment Costs

- FOC

$$\beta E_{A',p'|A,p} V_k(A', K', p') = p \quad (3)$$

- Euler

$$\beta E_{(A',p'|A,p)} [\Pi_k(A', K') + (1 - \delta)p'] = p. \quad (4)$$

- Specify function $\Pi(A, K) = AK^\alpha$
- All data or Θ
- Estimate Parameters via GMM
- key is $\Pi_k(A', K') = \alpha \frac{\Pi(A', K')}{K'}$ is inferred given α without observing A'

GMM: No Adjustment Costs

- Euler

$$\beta E_{(A', p' | A, p)} [\Pi_k(A', K') + (1 - \delta)p'] = p. \quad (5)$$

- *Ex post*

$$\varepsilon(\Theta) = \beta [\Pi_k(A', K') + (1 - \delta)p'] - p. \quad (6)$$

- using time and firm subscripts

$$\varepsilon_{it+1}(\Theta) = \beta [\Pi_k(A_{it+1}, K_{it+1}) + (1 - \delta)p_{t+1}] - p_t \quad (7)$$

- Theory Implies $E_t[\varepsilon_{it+1}(\Theta)] = 0$ at $\Theta = \Theta^*$
- assume firms are homogenous: i.e. same Θ^* . else estimate by sector

Methodology: (Overview Only)

- impose sample analogue of $E_t[\varepsilon_{it+1}(\Theta^*)] = 0$
- compute *ex post* errors from data given Θ
- $m_T^{ij}(\Theta) \equiv \frac{1}{T} \sum (Z_t^j \times \varepsilon_{it+1}(\Theta))$ for instrument $j = 1, 2, 3, \dots, J$
- solve $J \equiv \min_{\Theta} m_T(\Theta)' W m_T(\Theta)$ where
 - m_T is vector ($J \times 1$) of moments, sample length of T
 - W is a weighting matrix; usually the inverse of the VarCov
 - this minimization yields consistent estimates of Θ^*
 - implement iteratively: $W = I$, estimate parameters, calculate VarCov
- See Hamilton GMM Chpt. or Hansen notes for proofs of these properties

Quadratic Adjustment Costs

- FOC

$$C_{K'}(K', K) + p = \beta E_{(A', p' | A, p)} V_{K'}(A', K', p'). \quad (8)$$

- Euler

$$C_{K'}(K', K) + p = \beta E_{(A', p' | A, p)} \{ \Pi_K(K', A') + p'(1 - \delta) - C_{K'}(K'', K') \}. \quad (9)$$

- Quadratic Adjustment Costs

$$C(\cdot) = \frac{\gamma}{2} \left(\frac{K' - (1 - \delta)K}{K} \right)^2 K \quad (10)$$

Q Theory

Assume

- $\Pi(A, K) = AK$ perfect competition plus CRS
- quadratic adjustment costs
- $V(A, K) = \phi(A)K$ (Guess and Verify from FOC)
 - $V_K(A, K) = \frac{V(A, K)}{K} = \phi(A)$
 -

$$i = \frac{I}{K} = \frac{1}{\gamma}(\beta E_{A'|A} V_K(A', K') - p) = \frac{1}{\gamma}(\beta E_{A'|A} \phi(A') - p) \quad (11)$$

- from this $V(A, K)$ in (1) is proportional to K .

Empirical Implementation Findings

- Many panel studies
- study:

$$i_{it} = \eta_0 + \eta_1 q_{it} + \eta_2 Z_{it} + error_{it}$$

- q_{it} value of firm divided by capital (average Q), Z_{it} are other variables (eg cash flow)
- theory predicts: $\eta_1 = \frac{1}{\gamma}$ and $\eta_2 = 0$ for any instrument
- Find huge adjustment costs: γ big
- **Financial Frictions:** cash flow is significant in explaining investment, contrary to theory

Cooper-Ejarque (RED)

- $\Pi(A, K) = AK^\alpha$
- SMM matching Q regressions, average Q, etc.
- Find:
 - $\alpha < 1$: market power
 - match Q regression coefficients well
 - no evidence of financial frictions
- earlier findings of financial frictions explained by market power: Average Q \neq Marginal Q

GMM Approach

- start with Euler

$$C_{K'}(K', K) + p = \beta E_{(A', p' | A, p)} \{ \Pi_K(K', A') + p'(1 - \delta) - C_{K'}(K'', K') \}. \quad (12)$$

- calculate *ex post* error using form of adjustment cost
- impose orthogonality
- follow the recipe

Simple Discrete Choice: Machine Replacement Problem

$$V_{\Theta}(A, K) = \max\{V_{\Theta}^i(A, K), V_{\Theta}^a(A, K)\}$$

inactive

$$V_{\Theta}^i(A, K) = \Pi(A, K) + \beta E_{A'|A} V(A', K(1 - \delta))$$

and

active

$$V_{\Theta}^a(A, K) = \Pi(A, 1)\lambda - p + \beta E_{A'|A} V(A', (1 - \delta)).$$

for all (A, K)

- λ is an opportunity cost of lumpy investment
- p is the cost of new equipment (fixed cost)
- no time to build
- $A = (a, \varepsilon)$
- value function iteration solution
- parameter vector is Θ

Special Case

- A is not stochastic
- Optimal Solution is K_{Θ}^* such that
 - inactive (no replacement) for $K > K_{\Theta}^*$
 - replace if $K \leq K_{\Theta}^*$
 - interesting case has $1 > K_{\Theta}^* > 0$
- deterministic replacement cycle
- Θ determines K^* : **Identification Problem**

Aggregate Implications: No Shocks

- investment rate determined by initial cross sectional distribution by age
- state space is a ladder
- cycles at one extreme, constant investment rate at the other

More general Problem: Solution and Identification

- A is iid for this part
- Optimal Stopping problem
- $\Delta(A, K) \equiv V^a(A, K) - V^i(A, K)$ is increasing in K given A
- Optimal Solution is $K_{\Theta}^*(A)$ such that replace iff $K \leq K_{\Theta}^*(A)$ in state (A, K)
- identify Θ through dependence of $K^*(A)$ on A .
- $z(a, \varepsilon, K) = 1$ denotes replacement

Estimation

- no continuous choice so no GMM approach
- identify Θ through dependence of $K_{\Theta}^*(A)$ on A .
- Moments for SMM
 - average time to replacement
 - logit: probability of replacement as a function of (A, K)
 - hazard: probability of replacement as a function of capital vintage
 - etc.

Aggregate Implications: Shocks

- I_t is aggregate investment rate in period t
- depends on probability of adjustment and distribution of K .

$$I_t = \sum_K H(A_t, K) \Gamma_t(K). \quad (13)$$

where

$$H(A, K) = \int_{\varepsilon} z(A, \varepsilon, K) dG(\varepsilon)$$

- The evolution of the cross sectional distribution of capital is given by:

$$\Gamma_{t+1}((1 - \delta)K) = (1 - H(A_t, K)) \Gamma_t(K) \quad (14)$$

Continuous Choice

- Cooper-Haltiwanger estimate model with lumpy and continuous choice.
- Choice is:

$$V(A, K) = \max_{K'} \pi(A, K) - p(K' - (1 - \delta)K) - C(A, K', K) + \beta E_{A'|A} V(A', K') \quad (15)$$

for all (A, K) .

- Adjustment costs
 - quadratic
 - fixed cost
 - opportunity cost
 - irreversibility $p^s < p^b = 1$

Find

- opportunity cost model fits moments better
- irreversibility and small quadratic adjustment costs
- $\alpha \approx 0.60$
- tough to match intermediate capital expenditures

General Equilibrium

- What happens to the convexities when there is aggregation across plants?
- Cooper-Haltiwanger:
 - fixed β
 - consider representative firm with quadratic adjustment costs
 - find parameter to match aggregate time series of model with non-convexity at the plant level
 - R^2 about 0.85
- Thomas (JPE)
 - RBC with micro non-convexities
 - indistinguishable from standard RBC
 - Key is countercyclical stochastic discount factor