

# Implicit Diffusion: Efficient Optimization through Stochastic Sampling

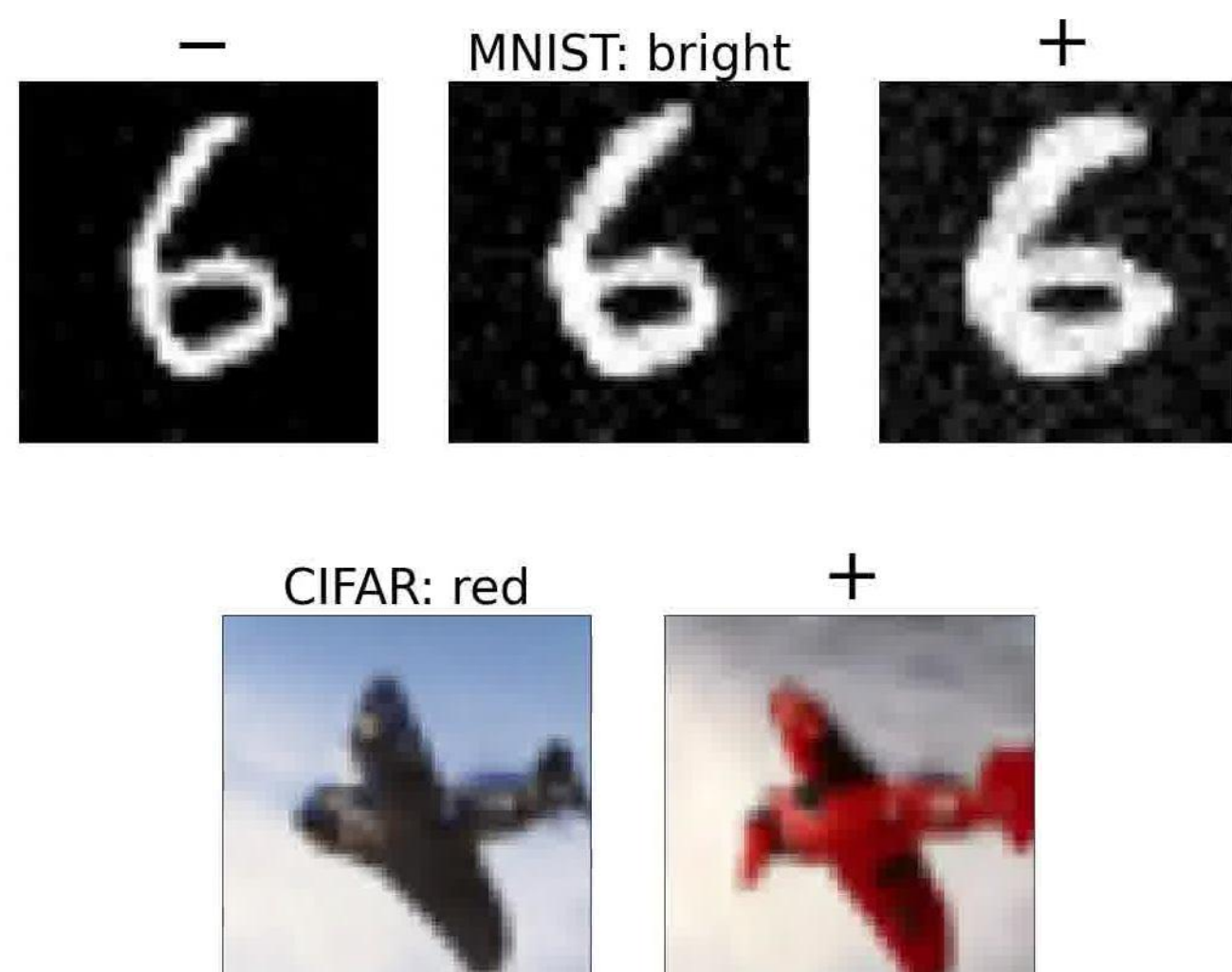
P. Marion, A. Korba, P. Bartlett, M. Blondel, V. De Bortoli, A. Doucet,  
F. Linares-López, C. Paquette, Q. Berthet

Introduction

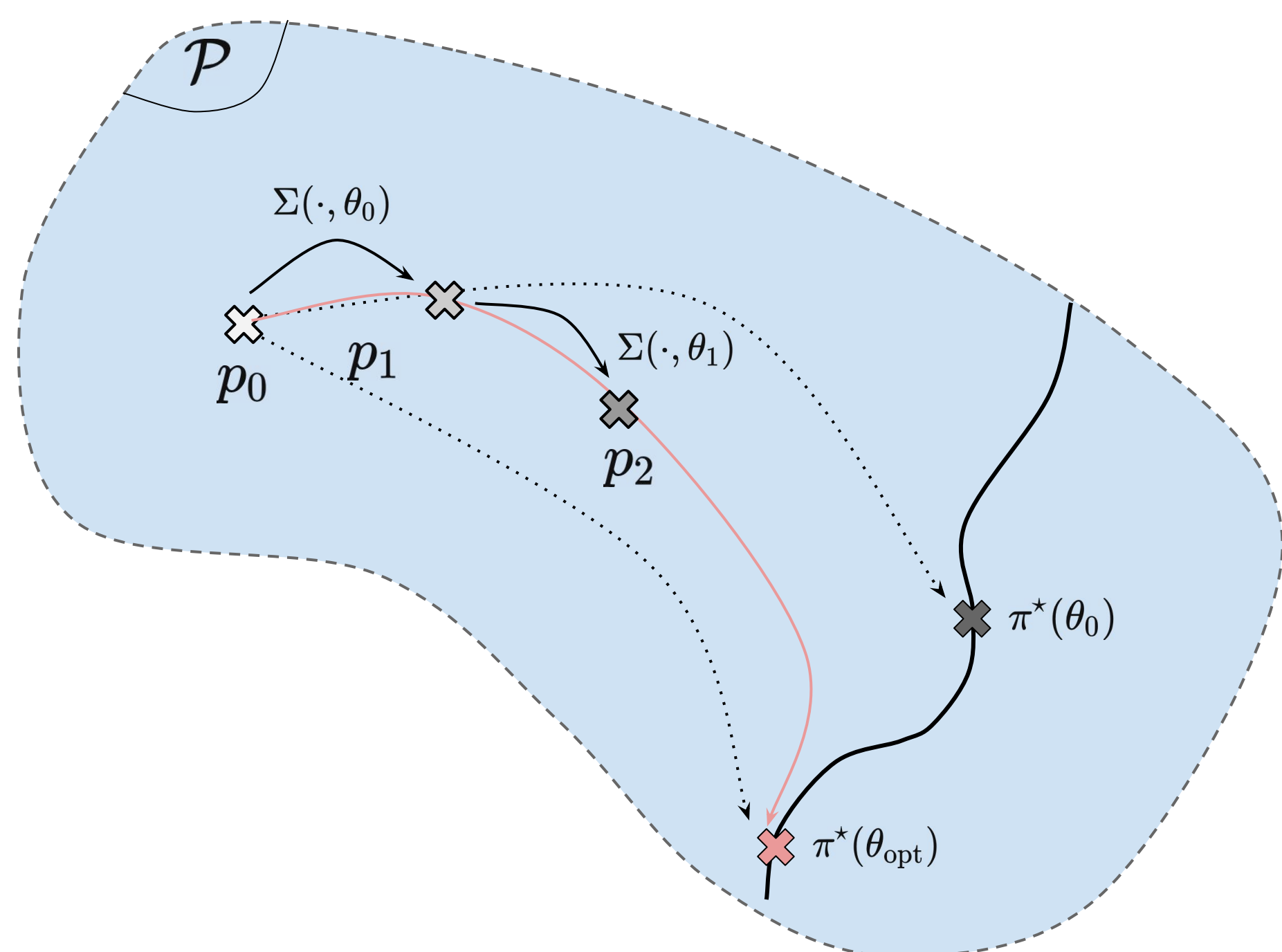
How can we optimize through sampling?:

Modify a sampling process to steer outcomes

Can be used to finetune generative models



Sampling / Optimization as bilevel optimization



Optimization - objective function defined on probabilities

$$\min_{\theta \in \mathbb{R}^p} \ell(\theta) := \mathcal{F}(\pi^*(\theta))$$

Example: regularized reward

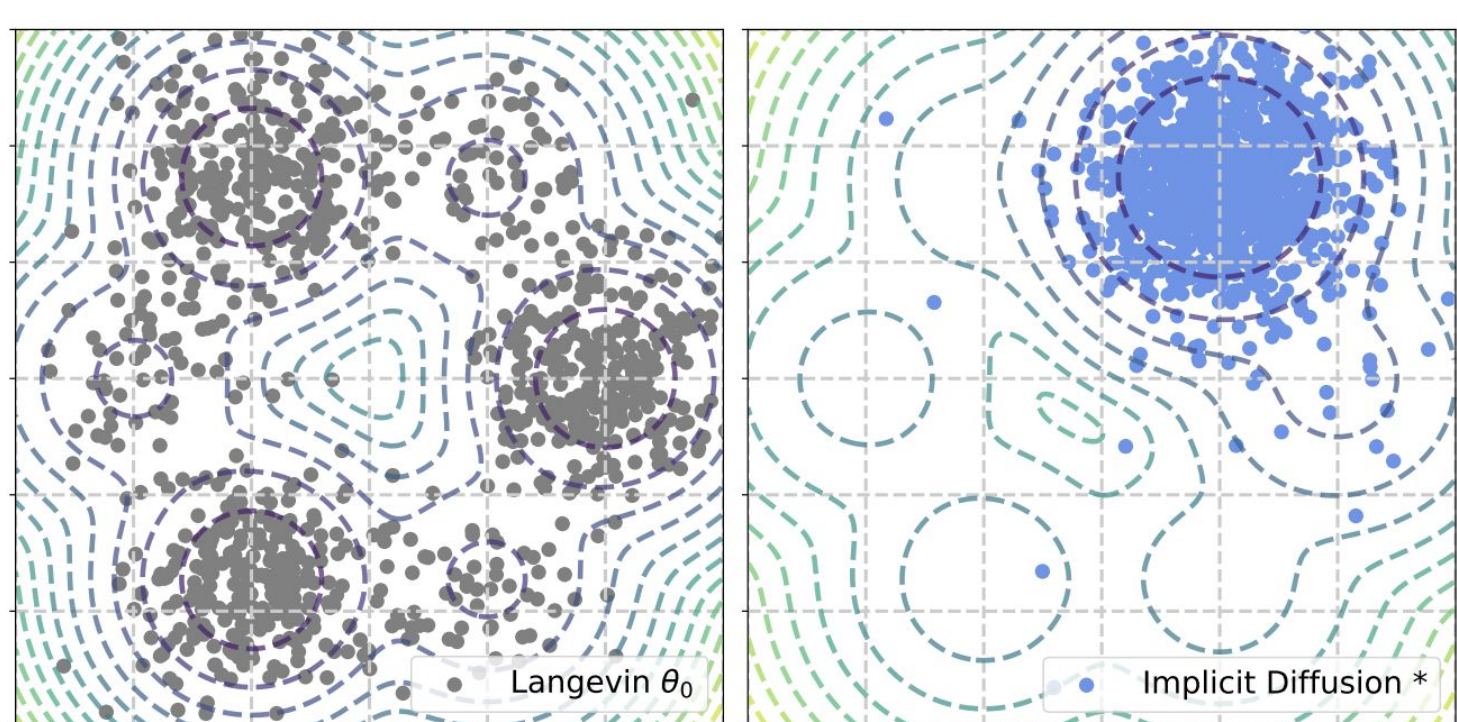
$$\mathcal{F}(p) = -\lambda \mathbf{E}_{x \sim p}[R(x)] + \beta \text{KL}(p \parallel \pi^*(\theta_0))$$

Langevin - EBM

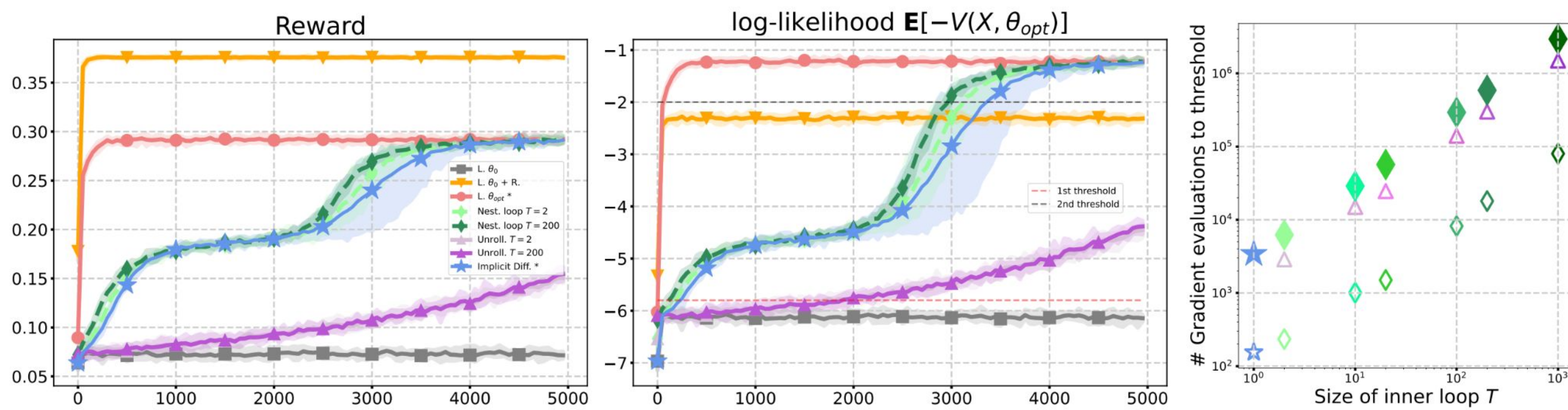
Energy-based models

Sampling with a Langevin diffusion

$$\pi^*(\theta) \propto \exp(-V(x, \theta))$$



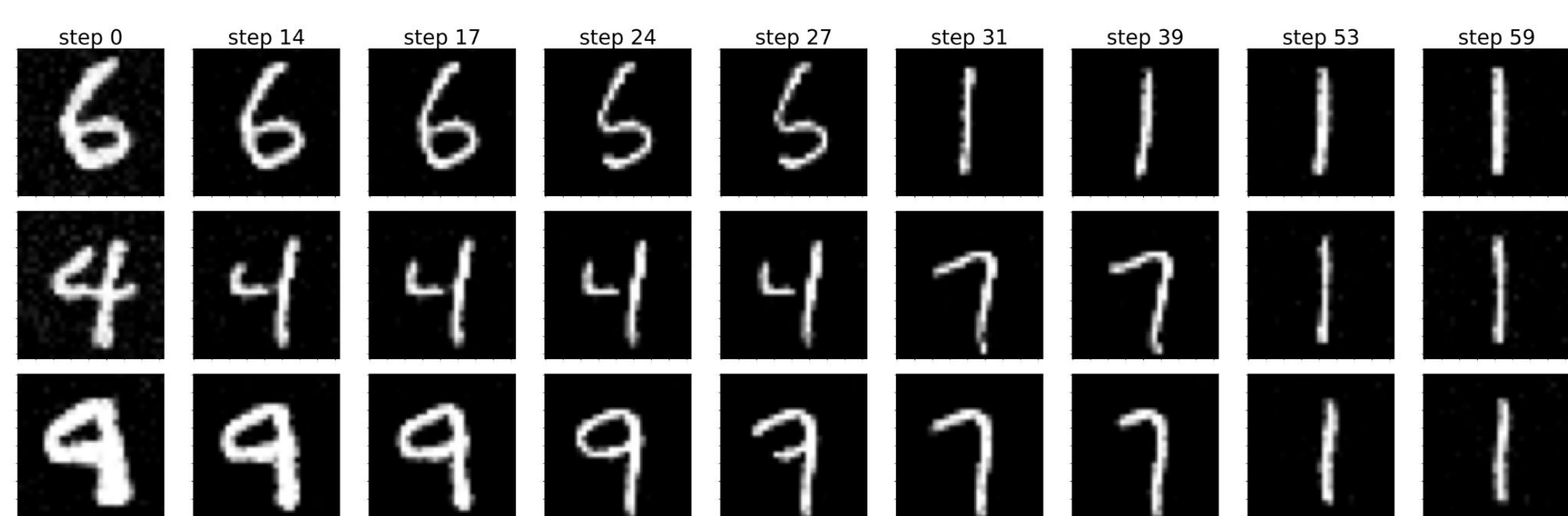
Comparison: Strong advantage of single-loop methods.



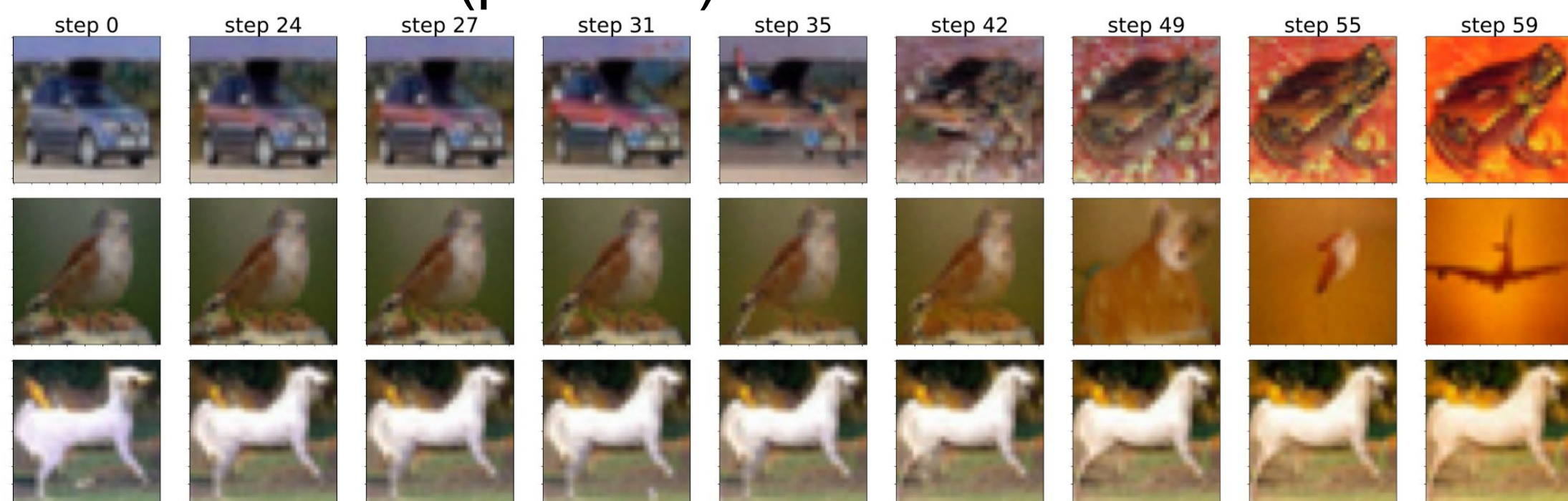
Diffusions

Diffusion denoising finetuning

MNIST - brightness (negative)

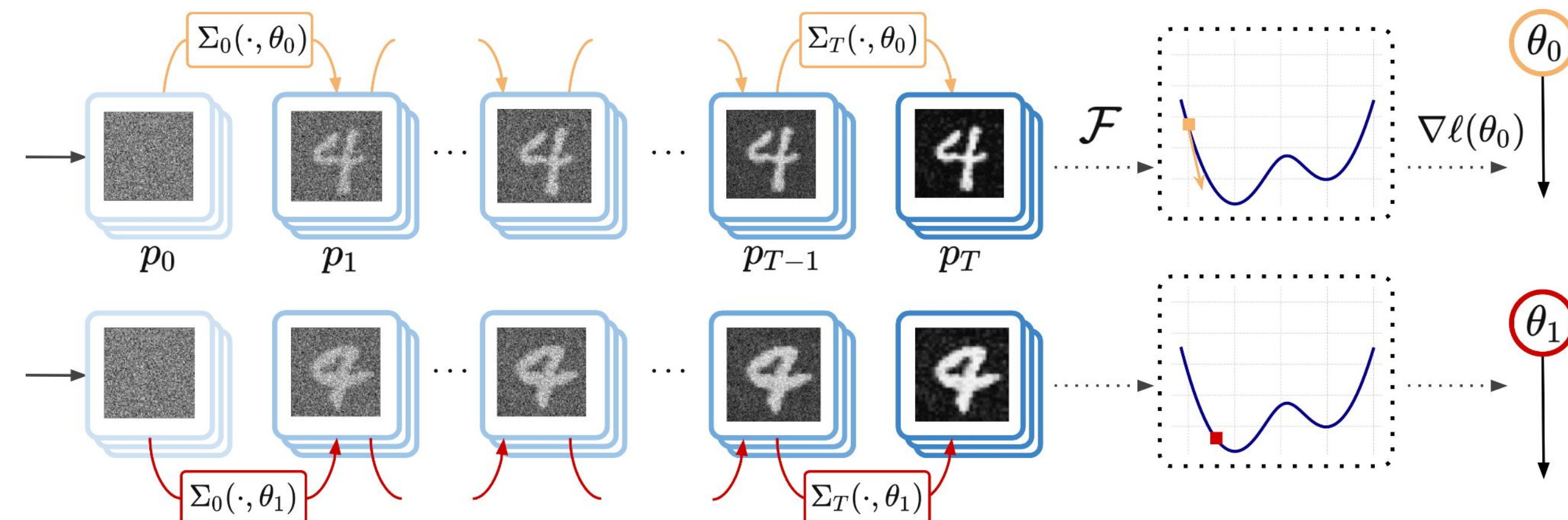


CIFAR10 - red (positive)



First-order optimization on sampling

Example: Diffusion denoising with brightness reward



Challenges

No closed forms for outcome distributions

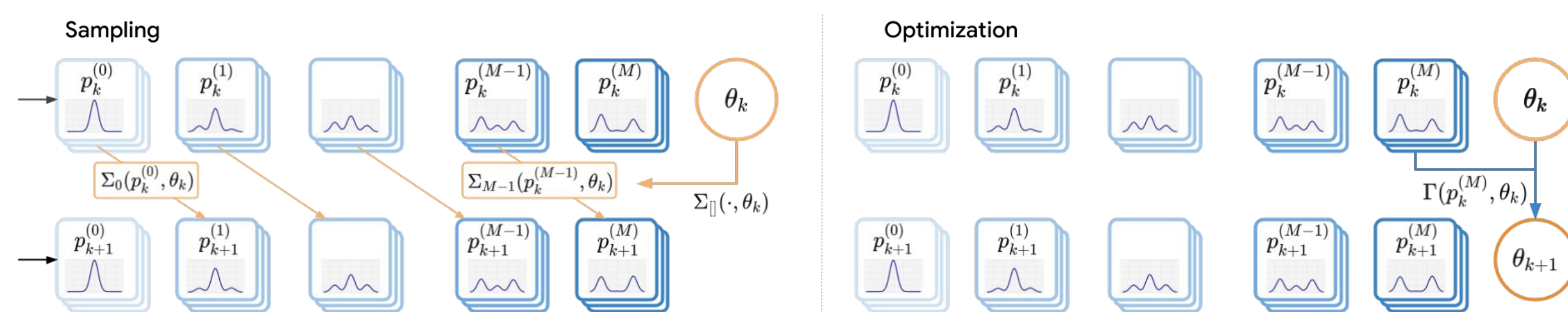
Methodology problem: how can we evaluate gradients?

Loss / Gradient evaluation require iterative sampling (nested loops)

Implicit Diffusion

Single loop algorithm - based on implicit differentiation

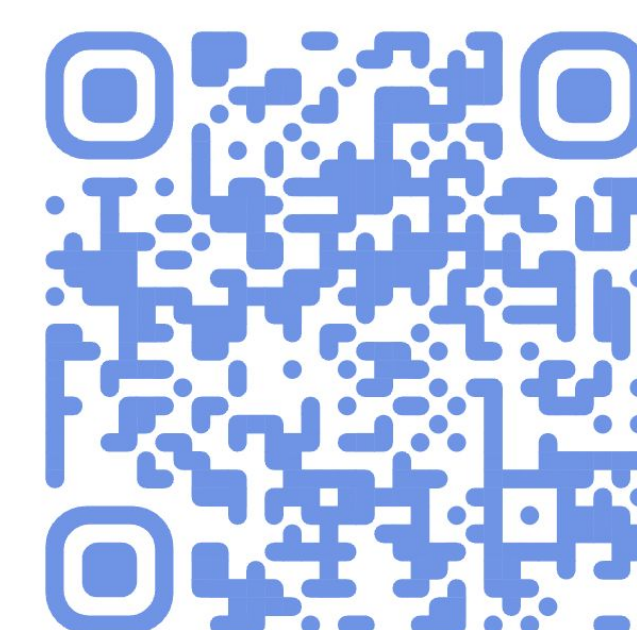
Inspired by literature on bilevel optimization



**Algorithm 2** Implicit Diff. optimization, infinite time

**input**  $\theta_0 \in \mathbb{R}^p, p_0 \in \mathcal{P}$   
**for**  $k \in \{0, \dots, K-1\}$  (joint single loop) **do**  
 $p_{k+1} \leftarrow \Sigma_k(p_k, \theta_k)$   
 $\theta_{k+1} \leftarrow \theta_k - \eta \Gamma(p_k, \theta_k)$  (or another optimizer)  
**output**  $\theta_K$

Paper



Code

