2023 부산시 양자컴퓨팅 개발자 교육 프로그램

Lecture 2

Inho Choi

Qiskit Advocate

Qiskit

Lecture2	downgrade lectures to qiskit==0.29.1	last month
Lecture3	remove ipynb_checkpoints	last month
Lecture4	downgrade lectures to qiskit==0.29.1	last month
Lecture5	downgrade lectures to qiskit==0.29.1	last month
.gitignore	ignore .ipynb_checkpoints	last month
HOWTO.md	turns out, rust is not required :(	3 weeks ago
LICENSE	Initial commit	last year
README.md	Update README.md	4 days ago
environment.yml	turns out, rust is not required :(	3 weeks ago

README.md

Qiskit-Dev-Cert-lectures

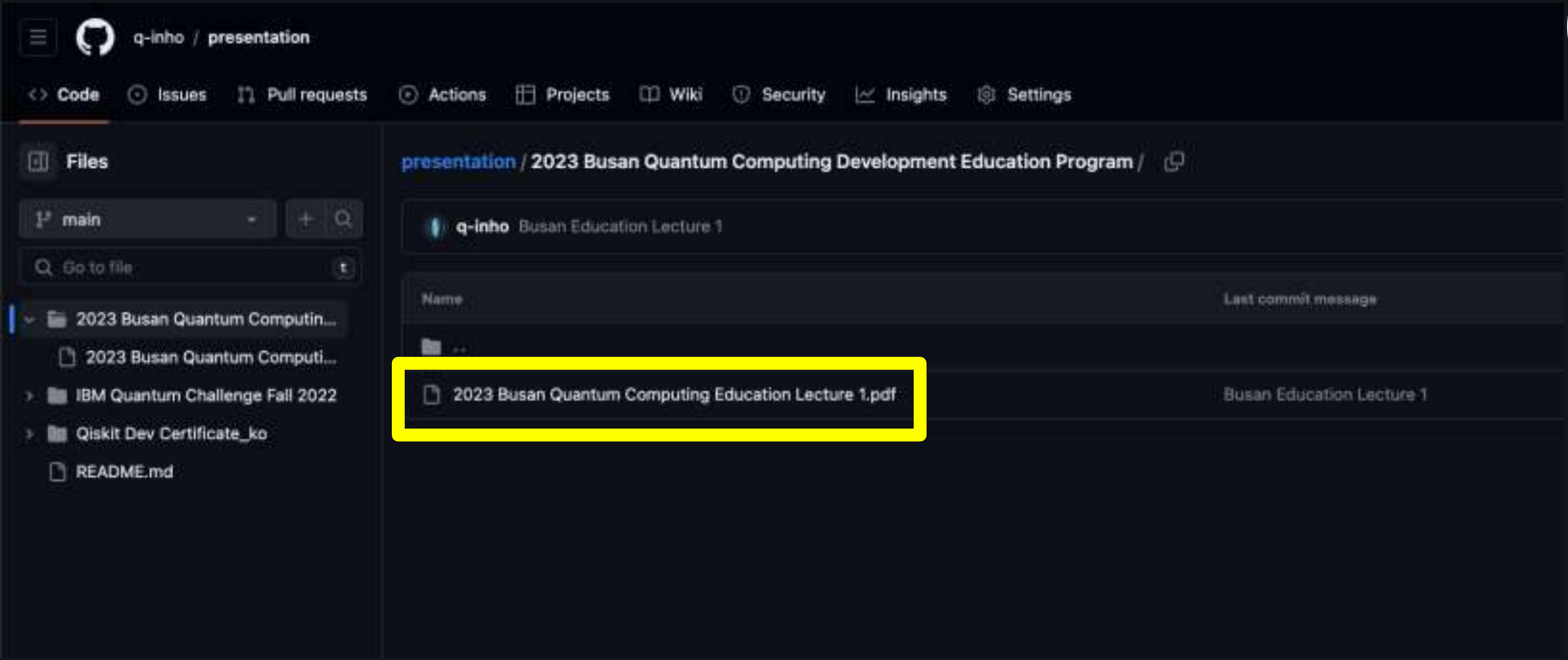
이 노트들은 한국의 Qiskit Community 멤버들이 Qiskit 개발자 자격 시험을 준비할때 도움이 되기 위해 만들어졌습니다. 각 강의 영상들은 개방된 오픈소스 오픈튜토리얼스에서 확인할 수 있습니다.

이 강의 노트를 커뮤니티 여러분과 함께 만들어가기 원합니다! 기여를 원하시는 분들은 Readme의 "제작 및 검수에 참여한 사람들"에 여러분의 이름을 추가하신 후 컨텐츠를 추가, 수정하여 PR을 보내주세요.

양자 컴퓨팅 개발자 교육을 위한 강의에서 사용된 추가적인 발표 자료입니다.

- 2023 부산 양자컴퓨팅 개발자 교육 프로그램

Qiskit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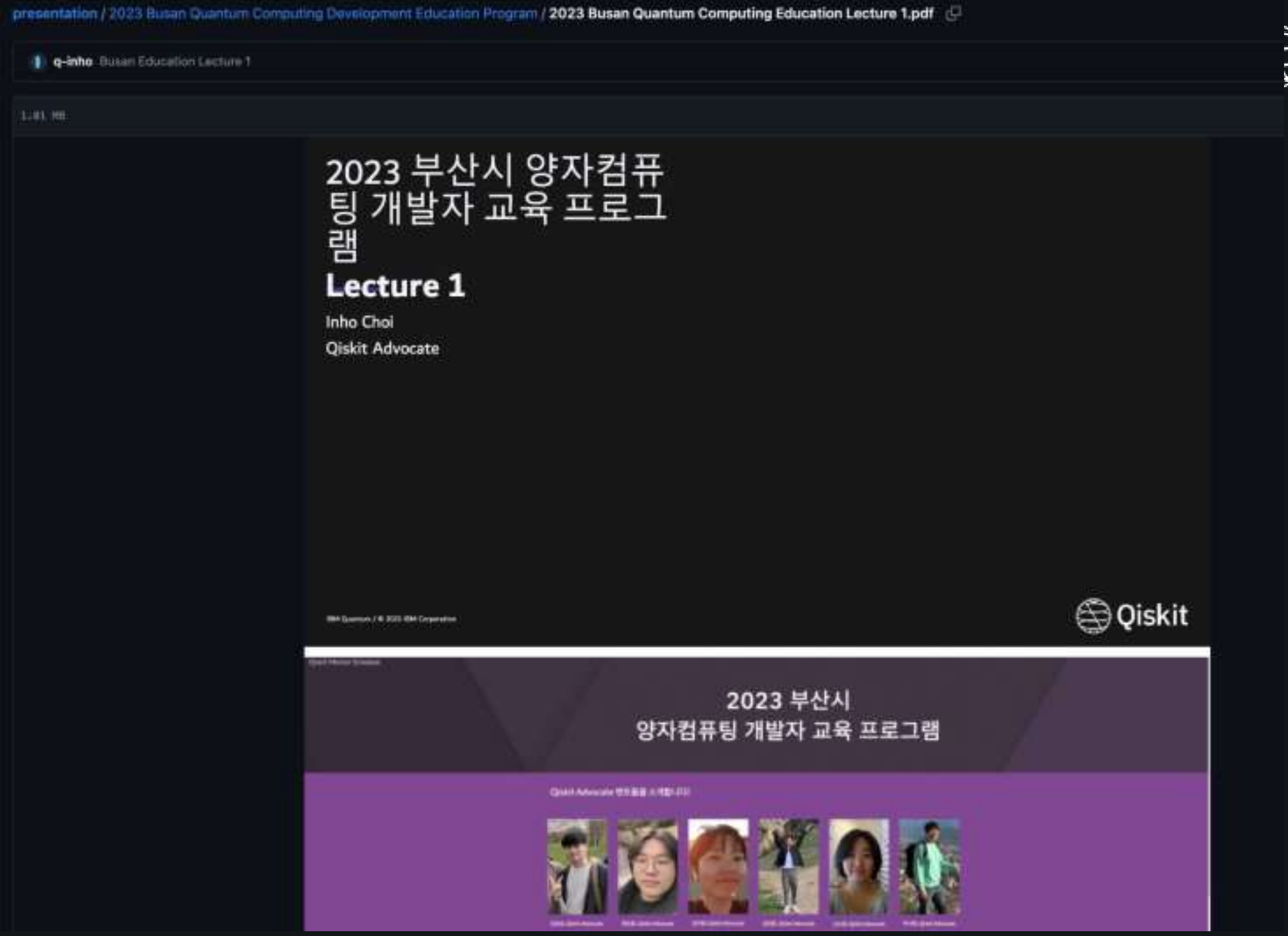
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2023 부산시 양자컴퓨팅 개발자 교육 프로그램
Lecture 1
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운영자가 맨위에 고정함

고정 취소

Soyoung Shin • 1촌

Development Strategist for Education & Workforce, IBM Quantum

운영자 • 1주

참고 자료 목록:

1. 프로그램 안내 웹: https://lnkd.in/gF5fWQvC

2. 이수 필수 요건 두 가지:

2.1 파이썬 강좌: https://lnkd.in/gU6s8ST4

2.2 오픈튜토리얼스 강좌: https://lnkd.in/gWqt3CtZ

2.3 깃헙 링크: https://lnkd.in/gDyWF2pC

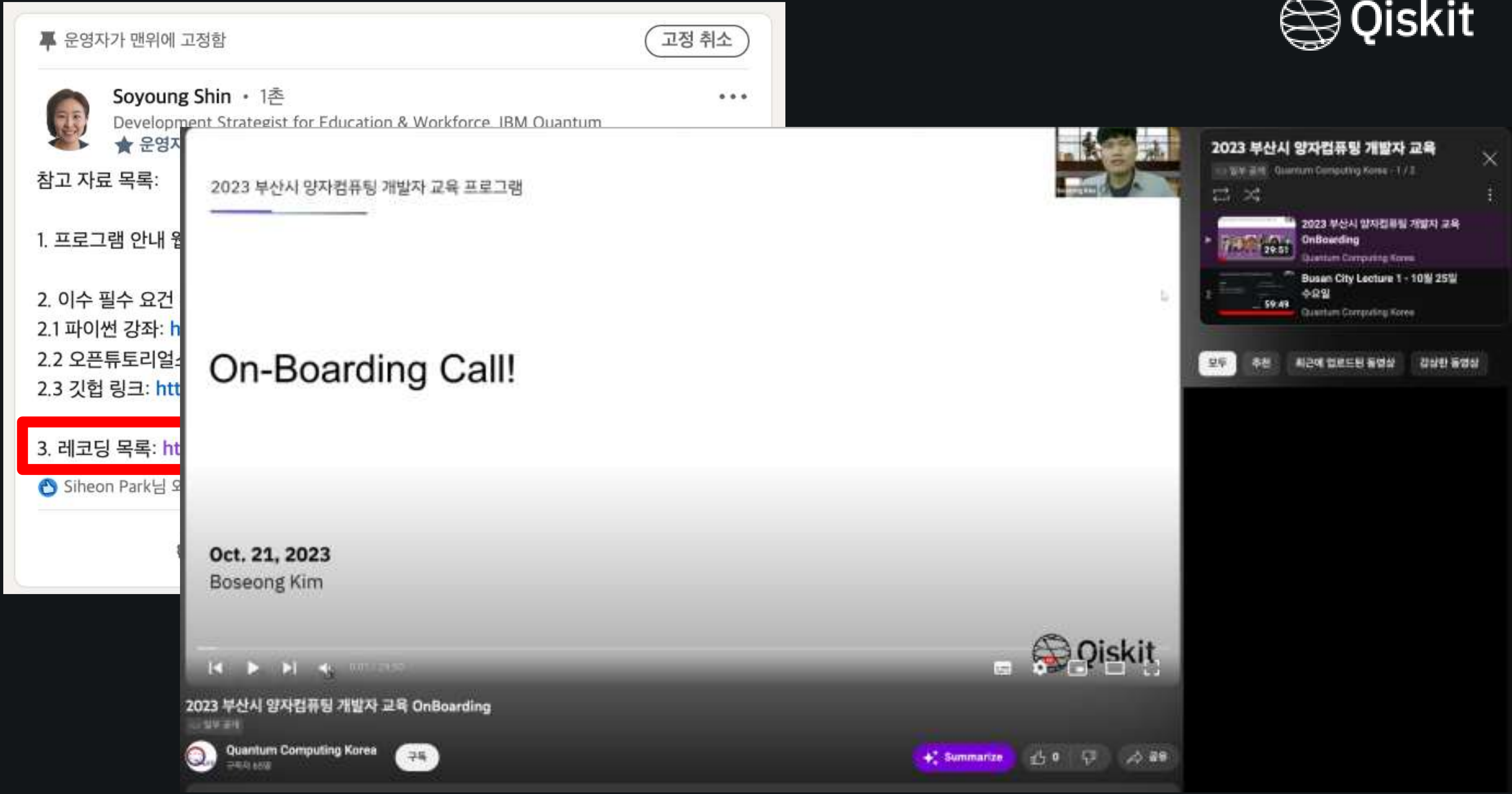
3. 레코딩 목록: https://lnkd.in/guQQziYC

Siheon Park님 외 9명

추천 댓글

Qiskit

Qiskit



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운영자
참고 자료 목록:
1. 프로그램 안내 웹
2. 이수 필수 요건
2.1 파이썬 강좌: h
2.2 오픈튜토리얼
2.3 깃헙 링크: htt
3. 레코딩 목록: ht
Siheon Park님 외
2023 부산시 양자컴퓨팅 개발자 교육 프로그램
On-Boarding Call!
Oct. 21, 2023
Boseong Kim
Qiskit
2023 부산시 양자컴퓨팅 개발자 교육 OnBoarding
Quantum Computing Korea
구독
Summarize
공유
2023 부산시 양자컴퓨팅 개발자 교육
2023 부산시 양자컴퓨팅 개발자 교육 OnBoarding
29:51
Busan City Lecture 1 - 10월 25일 수요일
59:49
모두
추천
최근에 업로드된 동영상
감상한 동영상

Syllabus

- Lecture 1: 게이트와 양자 회로 기본 작성법
    - Single qubit gate Lecture 1
    - Multiple qubit gate Lecture 2
    - Barriers and Properties of Quantum Circuit
- Lecture 2: 양자 회로의 측정과 OpenQasm

Lecture 3

- Lecture 3: 양자 백엔드에 양자회로 실행하기 Lecture 4
- Lecture 4: 양자 회로 및 회로의 실행 결과 시각화 및 해석
- Lecture 5: 유용한 기능들

Lecture 5

Syllabus

- Lecture 1: 게이트와 양자 회로 기본 작성법
  - Single qubit gate
  - **Multiple qubit gate**
  - Barriers and Properties of Quantum Circuit
- Lecture 2: 양자 회로의 측정과 OpenQasm
- Lecture 3: 양자 백엔드에 양자회로 실행하기
- Lecture 4: 양자 회로 및 회로의 실행 결과 시각화 및 해석
- Lecture 5: 유용한 기능들

Qiskit Textbook: Multiple systems

Lecture 1 - 게이트와 양자 회로

1-1 다중 큐비트 상태와 다중 큐비트 게이트

1. 다중 큐비트 상태

한개의 큐비트가 두가지의 상태를 나타낼 수 있는것처럼 두개의 큐비트는 아래와 같이 4가지의 상태를 나타낼 수 있습니다. 따라서 n개의 큐비트가 있다면 2^n가지의 상태를 나타낼 수 있습니다.

`00 01 10 11`

우선 두개의 큐비트를 설명하려면 4개의 복소 진폭(complex amplitude)가 필요합니다. 이러한 진폭들은 4차원의 벡터에 아래와 같이 표현할 수 있습니다.

$$|a\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle = \begin{bmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{bmatrix}$$

만약 두개의 큐비트가 분리되서 표현이 되었을때 이 두개의 큐비트를 크로네커 곱(kronecker product)을 사용하여 표현할 수 있습니다.

$$|a\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \quad |b\rangle = \begin{bmatrix} b_0 \\ b_1 \end{bmatrix}$$

$$|ba\rangle = |b\rangle \otimes |a\rangle = \begin{bmatrix} b_0 \times \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \\ b_1 \times \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} b_0a_0 \\ b_0a_1 \\ b_1a_0 \\ b_1a_1 \end{bmatrix}$$

위의 표현과 같이 크로네커 곱을 이용하여 여러개의 큐비트를 표현할 수 있습니다. 아래는 3개의 큐비트를 설명하는 벡터입니다.

$$|cba\rangle = \begin{bmatrix} c_0b_0a_0 \\ c_0b_0a_1 \\ c_0b_1a_0 \\ c_0b_1a_1 \\ \vdots \end{bmatrix}$$

Github: Qiskit-Dev-Cert-lectures, 1-1 다중 큐비트 게이트

Multiple Qubit Gate

1. **Classical Information**
2. Quantum Information
3. Multiple Qubit Gate with Qiskit

Multiple Classical states

X is a system with classical set Σ

Y is a system with classical state Γ

$$(X, Y)$$

Cartesian proudct

$$\Sigma \times \Gamma = \{(a, b): \ a \in \Sigma \text{ and } b \in \Gamma$$

Multiple Classical states

X is a system with classical set Σ

Y is a system with classical state Γ

$$\Sigma \times \Gamma = \{(a, b)\colon\ a \in \Sigma \text{ and } b \in \Gamma$$

If $\Sigma = \{0,1\}$ and $\Gamma = \{a, b, c\}$

$$\Sigma \times \Gamma = \{(0, a), (0, b), (0, c), (1, a), (1, b), (1, c)\}$$

Multiple Classical states

X is a system with classical set Σ

Y is a system with classical state Γ

$$\Sigma_1 \times \cdots \times \Sigma_n = \{(a_1, \cdots, a_n)\colon\ a_1 \in \Sigma_1, \cdots, a_n \in \Sigma_n$$

If $\Sigma_1 = \Sigma_2 = \Sigma_3 = \{0,1\}$

$$\Sigma_1 \times \Sigma_2 \times \Sigma_3 = \{(0,0,0), (0,0,1), (0,1,0), (0,1,1),$$
$$(1,0,0), (1,0,1), (1,1,0), (1,1,1)\}$$

Multiple Classical states

X is a system with classical set Σ

Y is a system with classical state Γ

$$\Sigma_1 \times \cdots \times \Sigma_n = \{(a_1, \cdots, a_n)\colon\ a_1 \in \Sigma_1, \cdots, a_n \in \Sigma_n$$

If $\Sigma_1 = \Sigma_2 = \Sigma_3 = \{0,1\}$

$$\Sigma_1 \times \Sigma_2 \times \Sigma_3 = \{0,1\}^3$$

Probabilistic states

$$\Pr\big((X,Y) = (0,0)\big) = \frac{1}{2}$$

$$\Pr\big((X,Y) = (0,1)\big) = \frac{1}{4}$$

$$\Pr\big((X,Y) = (1,0)\big) = \frac{1}{6}$$

$$\Pr\big((X,Y) = (1,1)\big) = \frac{1}{12}$$

$$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{4} \\ \frac{1}{6} \\ \frac{1}{12} \end{pmatrix} \begin{array}{l} \longleftarrow \text{Probability for state 00} \\ \longleftarrow \text{Probability for state 01} \\ \longleftarrow \text{Probability for state 10} \\ \longleftarrow \text{Probability for state 11} \end{array}$$

Probabilistic states

Probabilistic state of (X,Y),

X and Y are independent if

$$\Pr\big((X,Y) = (a,b)\big) = \Pr(X = a)\Pr(X = b)$$

Probabilistic state as vector

$$|\pi\rangle = \sum_{(a,b)\in\Sigma\times\Gamma} p_{ab}|ab\rangle \xrightarrow{\text{independent}} \begin{matrix} |\phi\rangle = \sum_{a\in\Sigma} q_a|a\rangle \\ |\psi\rangle = \sum_{b\in\Gamma} r_b|b\rangle \end{matrix}$$

Probabilistic states (Independent)

$$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{4} \\ \frac{1}{6} \\ \frac{1}{12} \end{pmatrix}$$

$$|\pi\rangle = \frac{1}{2}|00\rangle + \frac{1}{4}|01\rangle + \frac{1}{6}|10\rangle + \frac{1}{12}|11\rangle$$

$$\Big\downarrow \text{ independent}$$

$$|\phi\rangle = \frac{3}{4}|0\rangle + \frac{1}{4}|1\rangle \quad \text{and} \quad |\psi\rangle = \frac{2}{3}|0\rangle + \frac{1}{3}|1\rangle$$

Tensor products of vectors

$$|\phi\rangle = \sum_{a\in\Sigma} \alpha_a |a\rangle \qquad |\psi\rangle = \sum_{b\in\Gamma} \beta_b |b\rangle$$

$$|\pi\rangle = |\phi\rangle \otimes |\psi\rangle = \sum_{(a,b)\in\Sigma\times\Gamma} \alpha_a \beta_b |ab\rangle$$

$$|\phi\rangle \otimes |\psi\rangle = |\phi\rangle|\psi\rangle = |\phi\psi\rangle$$

Tensor products of vectors

$$\begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_m \end{pmatrix} \otimes \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_m \end{pmatrix} = \begin{pmatrix} \alpha_1\beta_1 \\ \vdots \\ \alpha_1\beta_k \\ \alpha_2\beta_1 \\ \vdots \\ \alpha_2\beta_k \\ \vdots \\ \alpha_m\beta_1 \\ \vdots \\ \alpha_m\beta_k \end{pmatrix}$$

Tensor products of vectors

$$\begin{pmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{pmatrix}\otimes\begin{pmatrix}\beta_1\\\beta_2\\\beta_3\\\beta_4\end{pmatrix}=\begin{matrix}\alpha_1\begin{pmatrix}\beta_1\\\beta_2\\\beta_3\\\beta_4\end{pmatrix}\\\alpha_2\begin{pmatrix}\beta_1\\\beta_2\\\beta_3\\\beta_4\end{pmatrix}\\\alpha_3\begin{pmatrix}\beta_1\\\beta_2\\\beta_3\\\beta_4\end{pmatrix}\end{matrix}$$

Tensor products of vectors

$$\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{pmatrix} \otimes \begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\\ \beta_4\end{pmatrix} = \begin{pmatrix}\alpha_1\beta_1\\ \alpha_1\beta_2\\ \alpha_1\beta_3\\ \alpha_1\beta_4\end{pmatrix} \begin{pmatrix}\alpha_2\beta_1\\ \alpha_2\beta_2\\ \alpha_2\beta_3\\ \alpha_2\beta_4\end{pmatrix} \begin{pmatrix}\alpha_3\beta_1\\ \alpha_3\beta_2\\ \alpha_3\beta_3\\ \alpha_3\beta_4\end{pmatrix}$$

Tensor products of vectors

$$\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{pmatrix} \otimes \begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\\ \beta_4\end{pmatrix} = \begin{pmatrix}\alpha_1\beta_1\\ \alpha_1\beta_2\\ \alpha_1\beta_3\\ \alpha_1\beta_4\\ \alpha_2\beta_1\\ \alpha_2\beta_2\\ \alpha_2\beta_3\\ \alpha_2\beta_4\\ \alpha_3\beta_1\\ \alpha_3\beta_2\\ \alpha_3\beta_3\\ \alpha_3\beta_4\end{pmatrix}$$

Probabilistic states (Correlated)

$$\frac{1}{2}|00\rangle + \frac{1}{2}|11\rangle$$

$$q_0 r_0 = \frac{1}{2}$$

$$\boxed{\begin{aligned} q_0 r_1 &= 0 \\ q_1 r_0 &= 0 \end{aligned}}$$

$$q_1 r_1 = \frac{1}{2}$$

Lack of independnce

Probabilistic state measurement

$$\frac{1}{2}|00\rangle + \frac{1}{2}|11\rangle$$

$\frac{1}{2}$ probability of 00

$\frac{1}{2}$ probability of 11

Probabilistic state measurement

Two systems (X,Y)

What happens when we measure X and but not in Y?

$$\sum_{(a,b)\in\Sigma\times\Gamma} p_{ab}|ab\rangle = \sum_{(a,b)\in\Sigma\times\Gamma} p_{ab}|a\rangle \otimes |b\rangle = \sum_{a\in\Sigma} |a\rangle \otimes \sum_{b\in\Gamma} p_{ab}|b\rangle$$

$$\Pr(X = a) = \sum_{b\in\Gamma} p_{ab}$$

Measurement X yields on $a \in \Sigma$

$$\frac{\sum_{b\in\Gamma} p_{ab}|b\rangle}{\sum_{c\in\Gamma} p_{ac}}$$

Probabilistic state of Y in given condition

Probabilistic state measurement

$$|\pi\rangle = \frac{1}{2}|00\rangle + \frac{1}{4}|01\rangle + \frac{1}{6}|10\rangle + \frac{1}{12}|11\rangle$$

$$|0\rangle \otimes \left(\frac{1}{2}|0\rangle + \frac{1}{4}|1\rangle\right) + |1\rangle \otimes \left(\frac{1}{6}|0\rangle + \frac{1}{12}|1\rangle\right)$$

If the measurement of X is 0

$$\Pr(\mathit{outcome\ is}\ 0) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

The probabilistic state of Y

$$\frac{\frac{1}{2}|0\rangle + \frac{1}{4}|1\rangle}{\frac{3}{4}} = \frac{2}{3}|0\rangle + \frac{1}{3}|1\rangle$$

Probabilistic state measurement

$$|\pi\rangle = \frac{1}{2}|00\rangle + \frac{1}{4}|01\rangle + \frac{1}{6}|10\rangle + \frac{1}{12}|11\rangle$$

$$|0\rangle \otimes \left(\frac{1}{2}|0\rangle + \frac{1}{4}|1\rangle\right) + |1\rangle \otimes \left(\frac{1}{6}|0\rangle + \frac{1}{12}|1\rangle\right)$$

If the measurement of X is 1

$$\Pr(outcome\ is\ 1) = \frac{1}{6} + \frac{1}{12} = \frac{1}{4}$$

The probabilistic state of Y

$$\frac{\frac{1}{6}|0\rangle + \frac{1}{12}|1\rangle}{\frac{1}{4}} = \frac{2}{3}|0\rangle + \frac{1}{3}|1\rangle$$

Multiple Qubit Gate

1. Classical Information
2. **Quantum Information**
3. Multiple Qubit Gate with Qiskit

Multiple Quantum states

Qiskit

Represented by column vectors

Indices correspond to the Cartesian product of the individual system's classical state sets

Tensor products of quantum state vectors are also quantum state vectors

If X and Y are qubits,

$$\{0,1\} \times \{0,1\} = \{00,01,10,11\}$$

Quantum state vectors of pair (X, Y):

$$\frac{1}{2}|00\rangle + \frac{i}{2}|01\rangle - \frac{1}{2}|10\rangle - \frac{i}{2}|11\rangle$$

$$\frac{3}{5}|00\rangle - \frac{4}{5}|11\rangle$$

$$|01\rangle$$

Multiple Quantum states

$|\phi\rangle$ be quantum state vector of system X

$|\psi\rangle$ be quantum state vector of system Y

$$|\phi\rangle \otimes |\psi\rangle$$

Proudct States **Independence**

$$|\psi_1\rangle \otimes \cdots \otimes |\psi_n\rangle$$

For n number of systems, n qubits

Multiple Quantum states

Qiskit

$$\frac{1}{2}|00\rangle + \frac{i}{2}|01\rangle - \frac{1}{2}|10\rangle - \frac{i}{2}|11\rangle = \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle\right)$$

Proudct States **Independence**

Multiple Quantum states

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = |\phi\rangle \otimes |\psi\rangle$$

$$\langle 0|\phi\rangle\langle 1|\psi\rangle = \langle 01|\phi \otimes \psi\rangle = 0$$

$\langle 0|\phi\rangle = 0$ or/and $\langle 1|\psi\rangle = 0$

Contradiction occurs!

$$\langle 0|\phi\rangle\langle 0|\psi\rangle = \langle 00|\phi \otimes \psi\rangle = \frac{1}{\sqrt{2}}$$

$$\langle 1|\phi\rangle\langle 1|\psi\rangle = \langle 11|\phi \otimes \psi\rangle = \frac{1}{\sqrt{2}}$$

Multiple Quantum states

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = |\phi\rangle \otimes |\psi\rangle$$

$$\langle 0|\phi\rangle\langle 1|\psi\rangle = \langle 01|\phi \otimes \psi\rangle = 0$$

ENTANGLEMENT

$\langle 0|\phi\rangle = 0$ or/and $\langle 1|\psi\rangle = 0$

Contradiction occurs!

$$\langle 0|\phi\rangle\langle 0|\psi\rangle = \langle 00|\phi \otimes \psi\rangle = \frac{1}{\sqrt{2}}$$

$$\langle 1|\phi\rangle\langle 1|\psi\rangle = \langle 11|\phi \otimes \psi\rangle = \frac{1}{\sqrt{2}}$$

Entangled state

Qiskit

Bell states

$|+\rangle$ **state**

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$|-\rangle$ **state**

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle$$

Entangled state

GHZ state

$$\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|111\rangle$$

W state

$$\frac{1}{\sqrt{3}}|001\rangle + \frac{1}{\sqrt{3}}|010\rangle + \frac{1}{\sqrt{3}}|100\rangle$$

Quantum State Measurement

Same way measurements of single systems

$|\psi\rangle$ quantum state of system and every one of the systems is measured

$$|\langle a_1 \cdots a_n|\psi\rangle|^2$$

$$\frac{1}{2}|00\rangle + \frac{i}{2}|01\rangle - \frac{1}{2}|10\rangle - \frac{i}{2}|11\rangle$$

$(\frac{1}{2})^2 = \frac{1}{4}$ probability of outcome (0,0)

$(\frac{i}{2})^2 = \frac{1}{4}$ probability of outcome (0,1)

$(\frac{-1}{2})^2 = \frac{1}{4}$ probability of outcome (1,0)

$(\frac{-i}{2})^2 = \frac{1}{4}$ probability of outcome (1,1)

Quantum State Measurement

Two Quantum systems (X,Y)

What happens when we measure X and but not in Y?

$$|\psi\rangle = \sum_{(a,b)\in\Sigma\times\Gamma} \alpha_{ab}|ab\rangle$$

$$= \sum_{a\in\Sigma}|a\rangle \otimes \underbrace{\sum_{b\in\Gamma} \alpha_{ab}|b\rangle}_{} \rightarrow |\phi_a\rangle$$

To obtain each outcome $a \in \Sigma$

$$\Pr(outcome\ is\ a) = \sum_{b\in\Gamma}|\alpha_{ab}|^2 = \||\phi_a\rangle\|^2$$

After X measurement giving outcome a

$$|a\rangle \otimes \frac{|\phi_a\rangle}{\||\phi_a\rangle\|}$$

Quantum state of (X,Y)

Quantum State Measurement

Qiskit

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{2}|01\rangle + \frac{i}{2\sqrt{2}}|10\rangle - \frac{1}{2\sqrt{2}}|11\rangle$$

System X is measured

$$|\psi\rangle = |0\ \rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|1\rangle\right) + |1\rangle \otimes \frac{i}{2\sqrt{2}}|0\rangle - \frac{1}{2\sqrt{2}}|1\rangle$$

Probability for measurement result is 0

$$\left\|\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|1\rangle\right\|^2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

Then state of (X,Y) becomes

$$|0\rangle \otimes \frac{\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|1\rangle}{\sqrt{\frac{3}{4}}} = |0\rangle \otimes \left(\sqrt{\frac{2}{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle\right)$$

Quantum State Measurement

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{2}|01\rangle + \frac{i}{2\sqrt{2}}|10\rangle - \frac{1}{2\sqrt{2}}|11\rangle$$

System X is measured

$$|\psi\rangle = |0\,\rangle \otimes \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|1\rangle\right) + |1\rangle \otimes \frac{i}{2\sqrt{2}}|0\rangle - \frac{1}{2\sqrt{2}}|1\rangle$$

Probability for measurement result is 1

$$\left\|\frac{i}{2\sqrt{2}}|0\rangle - \frac{1}{2\sqrt{2}}|1\rangle\right\|^2 = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

Then state of (X,Y) becomes

$$|1\rangle \otimes \frac{\frac{i}{2\sqrt{2}}|0\rangle - \frac{1}{2\sqrt{2}}|1\rangle}{\sqrt{\frac{1}{4}}} = |1\rangle \otimes \left(\frac{i}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$$

Unitary Operations on multiple systems

Qiskit

Quantum operations are represented by **unitary matrices**

Unitary operations U on a system X do **notihng** to a system Y

Combined actions of unitary operations represented by tensor product

$$U_1 \otimes \cdots \otimes U_n$$

Tensor products of matrices

Qiskit

$$M = \sum_{a,b\in\Sigma} \alpha_{ab} \, |a\rangle\langle b| \qquad\qquad N = \sum_{c,d\in\Gamma} \beta_{cd} \, |c\rangle\langle d|$$

$$M \otimes N = \sum_{a,b\in\Sigma} \sum_{c,d\in\Gamma} \alpha_{ab}\beta_{cd} \, |ac\rangle\langle bd|$$

$$(M \otimes N)|\phi \otimes \psi\rangle = M|\phi\rangle \otimes N|\psi\rangle$$

Tensor products of matrices

$$\begin{pmatrix} \alpha_{11} & \cdots & \alpha_{1m} \\ \vdots & \ddots & \vdots \\ \alpha_{m1} & \cdots & \alpha_{mm} \end{pmatrix} \otimes \begin{pmatrix} \beta_{11} & \cdots & \beta_{1k} \\ \vdots & \ddots & \vdots \\ \beta_{k1} & \cdots & \beta_{kk} \end{pmatrix}$$

$$= \begin{pmatrix} \alpha_{11}\beta_{11} & \cdots & \alpha_{11}\beta_{1k} & & \alpha_{1m}\beta_{11} & \cdots & \alpha_{1m}\beta_{1k} \\ \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ \alpha_{11}\beta_{k1} & \cdots & \alpha_{11}\beta_{kk} & & \alpha_{1m}\beta_{k1} & \cdots & \alpha_{1m}\beta_{kk} \\ & \vdots & & \ddots & & \vdots & \\ \alpha_{m1}\beta_{11} & \cdots & \alpha_{m1}\beta_{1k} & & \alpha_{mm}\beta_{11} & \cdots & \alpha_{mm}\beta_{1k} \\ \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ \alpha_{m1}\beta_{k1} & \cdots & \alpha_{m1}\beta_{kk} & & \alpha_{mm}\beta_{k1} & \cdots & \alpha_{mm}\beta_{kk} \end{pmatrix}$$

Tensor products of matrices

$$\begin{pmatrix} \alpha_{00} & \alpha_{01} \\ \alpha_{10} & \alpha_{11} \end{pmatrix} \otimes \begin{pmatrix} \beta_{00} & \beta_{01} \\ \beta_{10} & \beta_{11} \end{pmatrix}$$

$$= \begin{matrix} \alpha_{00} \begin{pmatrix} \beta_{00} & \beta_{01} \\ \beta_{10} & \beta_{11} \end{pmatrix} & \alpha_{01} \begin{pmatrix} \beta_{00} & \beta_{01} \\ \beta_{10} & \beta_{11} \end{pmatrix} \\ \alpha_{10} \begin{pmatrix} \beta_{00} & \beta_{01} \\ \beta_{10} & \beta_{11} \end{pmatrix} & \alpha_{11} \begin{pmatrix} \beta_{00} & \beta_{01} \\ \beta_{10} & \beta_{11} \end{pmatrix} \end{matrix}$$

Tensor products of matrices

$$\begin{pmatrix} \alpha_{00} & \alpha_{01} \\ \alpha_{10} & \alpha_{11} \end{pmatrix} \otimes \begin{pmatrix} \beta_{00} & \beta_{01} \\ \beta_{10} & \beta_{11} \end{pmatrix}$$

$$= \begin{matrix} \begin{pmatrix} \alpha_{00}\beta_{00} & \alpha_{00}\beta_{01} \\ \alpha_{00}\beta_{10} & \alpha_{00}\beta_{11} \end{pmatrix} & \begin{pmatrix} \alpha_{01}\beta_{00} & \alpha_{01}\beta_{01} \\ \alpha_{01}\beta_{10} & \alpha_{01}\beta_{11} \end{pmatrix} \\ \begin{pmatrix} \alpha_{10}\beta_{00} & \alpha_{10}\beta_{01} \\ \alpha_{10}\beta_{10} & \alpha_{10}\beta_{11} \end{pmatrix} & \begin{pmatrix} \alpha_{11}\beta_{00} & \alpha_{11}\beta_{01} \\ \alpha_{11}\beta_{10} & \alpha_{11}\beta_{11} \end{pmatrix} \end{matrix}$$

Tensor products of matrices

$$\begin{pmatrix} \alpha_{00} & \alpha_{01} \\ \alpha_{10} & \alpha_{11} \end{pmatrix} \otimes \begin{pmatrix} \beta_{00} & \beta_{01} \\ \beta_{10} & \beta_{11} \end{pmatrix}$$

$$= \begin{pmatrix} \alpha_{00}\beta_{00} & \alpha_{00}\beta_{01} & \alpha_{01}\beta_{00} & \alpha_{01}\beta_{01} \\ \alpha_{00}\beta_{10} & \alpha_{00}\beta_{11} & \alpha_{01}\beta_{10} & \alpha_{01}\beta_{11} \\ \alpha_{10}\beta_{00} & \alpha_{10}\beta_{01} & \alpha_{11}\beta_{00} & \alpha_{11}\beta_{01} \\ \alpha_{10}\beta_{10} & \alpha_{10}\beta_{11} & \alpha_{11}\beta_{10} & \alpha_{11}\beta_{11} \end{pmatrix}$$

Unitary Operations on multiple systems

X and Y are qubits

Unitary operations that perform Hadamard operation on X but nothing on Y

$$H \otimes I = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Unitary Operations on multiple systems

X and Y are qubits

Unitary operations that perform Hadamard operation on Y but nothing on X

$$\mathrm{I} \otimes H = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Multiple Qubit Gate

1. Classical Information
2. Quantum Information
3. **Multiple Qubit Gate with Qiskit**

Controlled-U

For every unitary operation U on Y

$$|0\rangle\langle 0| \otimes I_Y + |1\rangle\langle 1| \otimes U = \begin{pmatrix} I_Y & 0 \\ 0 & U \end{pmatrix}$$

Controlled-NOT operation where first qubit is control

Operate when control qubit is 1

$$|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes \sigma_x = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Controlled-U

- CX-gate (Controlled Not gate)
- CY-gate
- CZ-gate
- CH-gate
- CP-gate
- CRX-gate
- CRY-gate
- CRZ-gate

SWAP gate

Exchange the contents of the two systems

$$SWAP|\phi \otimes \psi\rangle = |\psi \otimes \phi\rangle$$

$$SWAP = \sum_{a,b\in\Sigma} |a\rangle\langle b| \otimes |b\rangle\langle a|$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

SWAP gate

$$\mathrm{SWAP}|\phi^+\rangle = |\phi^+\rangle$$

$$\mathrm{SWAP}|\phi^-\rangle = |\phi^-\rangle$$

$$\mathrm{SWAP}|\psi^+\rangle = |\psi^+\rangle$$

$$\mathrm{SWAP}|\psi^-\rangle = -|\psi^-\rangle$$

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle$$

Toffoli gate

It is controlled-controlled-NOT gate

$$Toffoli\ Gate = |0\rangle\langle 0| \otimes I \otimes I + |1\rangle\langle 1| \otimes (|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes \sigma_x)$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Demonstation Multiple Qubit Gate

Take-away message

Qiskit

- What multiple quantum gate does exist?
- How do they operate and change quantum state?
- How to use multiple quantum gates in Qiskit?

QnA