2023 부산시 양자컴퓨팅 개발자 교육 프로그램

Lecture 3

Inho Choi

Qiskit Advocate

Qiskit

Syllabus

- Lecture 1: 게이트와 양자 회로 기본 작성법
  - Single qubit gate — Lecture 1
  - Multiple qubit gate — Lecture 2
  - Multiple qubit gate Notebook Demonstration
  - Barriers and Properties of Quantum Circuit
- Lecture 2: 양자 회로의 측정과 OpenQasm

Lecture 3

  - Notebook Demonstration
- Lecture 3: 양자 백엔드에 양자회로 실행하기

Lecture 4

- Lecture 4: 양자 회로 및 회로의 실행 결과 시각화 및 해석
- Lecture 5: 유용한 기능들

Lecture 5

Syllabus

- Lecture 1: 게이트와 양자 회로 기본 작성법
  - Single qubit gate
  - Multiple qubit gate
  - **Multiple qubit gate Notebook Demonstration**
  - **Barriers and Properties of Quantum Circuit**
- **Lecture 2: 양자 회로의 측정과 OpenQasm**
  - Notebook Demonstration
- Lecture 3: 양자 백엔드에 양자회로 실행하기
- Lecture 4: 양자 회로 및 회로의 실행 결과 시각화 및 해석
- Lecture 5: 유용한 기능들

Jupyternote Demonstration

Qiskit Textbook: Quantum circuits

Github: Qiskit-Dev-Cert-lectures, 2 양자 회로의 측정과 OpenQasm

Quantum Circuit and Measurement

1. **Classical Circuit**
2. Quantum Circuit
3. Projective Measurement

Classical Circuit

Qiskit

Circuits

- Wires: information
- Gates: operations

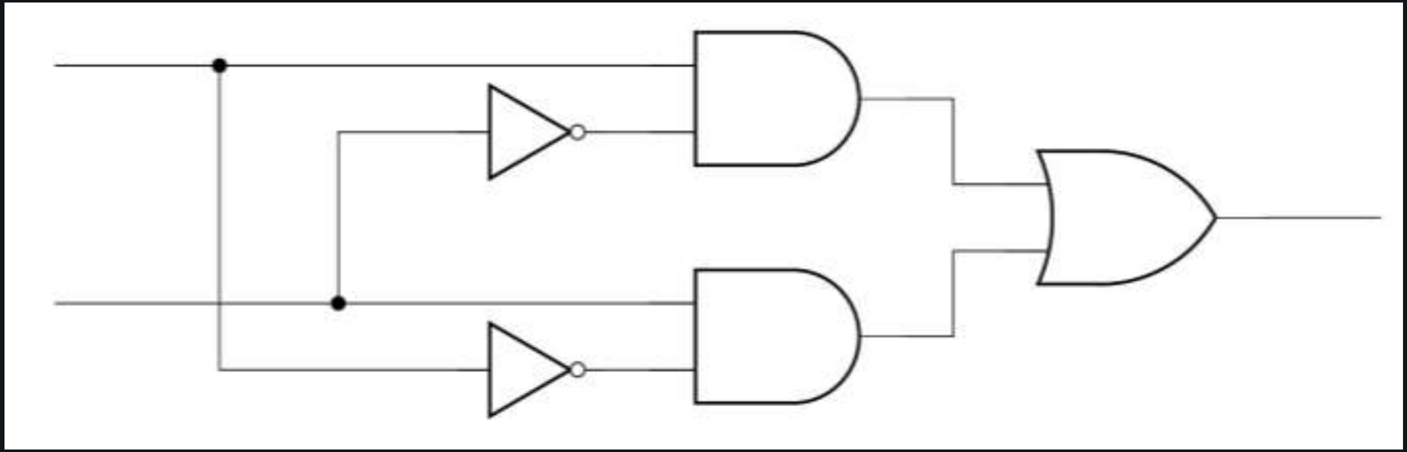
Left → Right

Classical Circuit

Circuits

Left → Right

- Wires: information
- Gates: operations

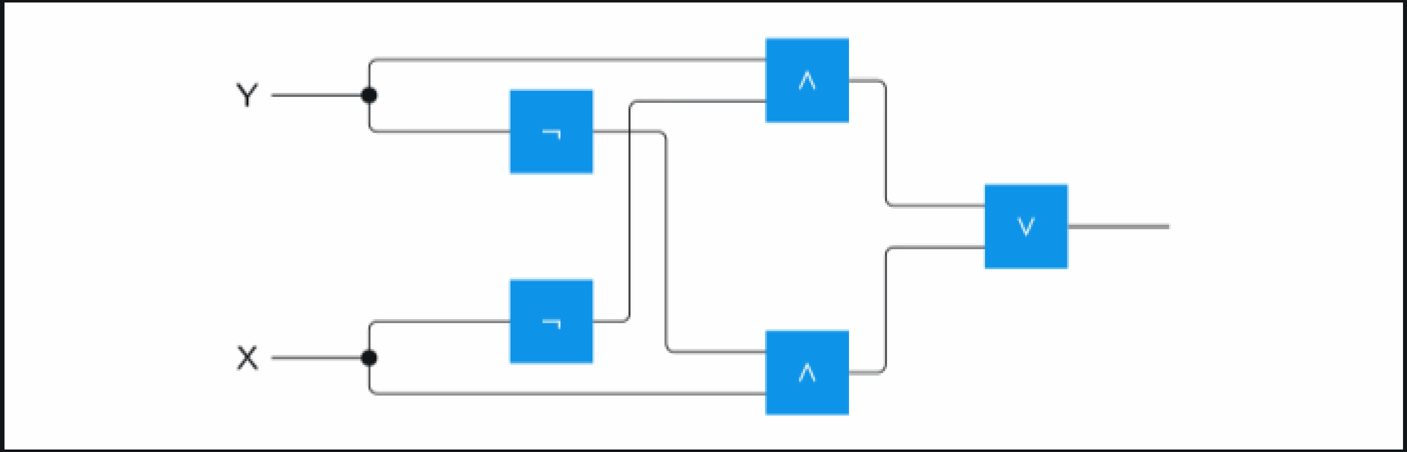


Classical Circuit

Circuits

- Wires: information
- Gates: operations

Left → Right



Classical Circuit

Circuits

- Wires: information
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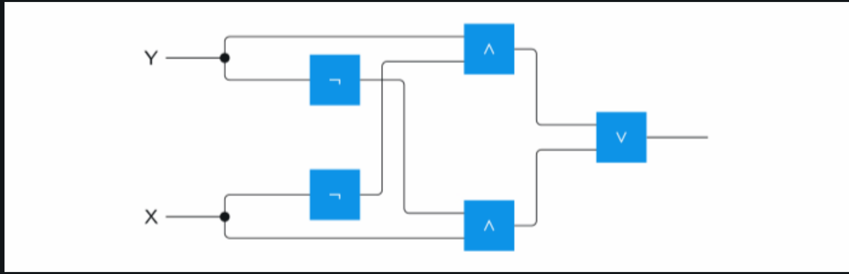


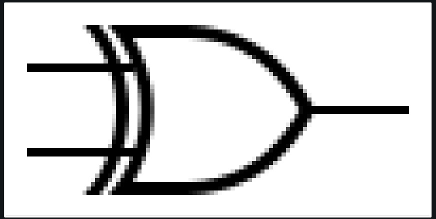
a	$\neg a$
0	1
1	0

ab	$a \wedge b$
00	0
01	0
10	0
11	1

ab	$a \vee b$
00	0
01	1
10	1
11	1

Classical Circuit

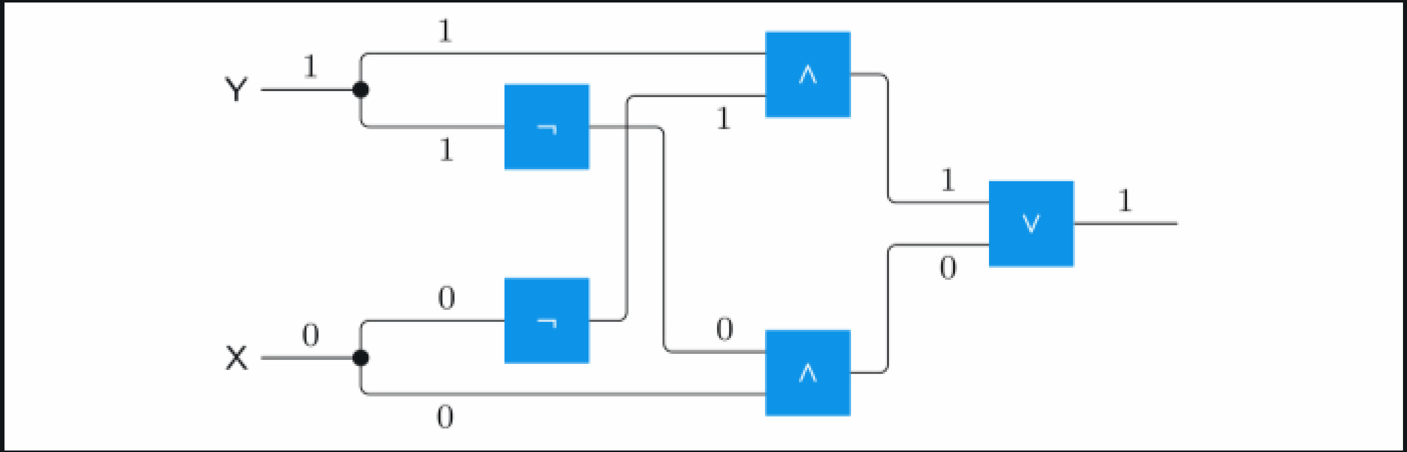
ab	$a \oplus b$
00	0
01	1
10	1
11	0



XOR gate

<u>Circuits</u>

- Wires: information
- Gates: operations



Quantum Circuit and Measurement

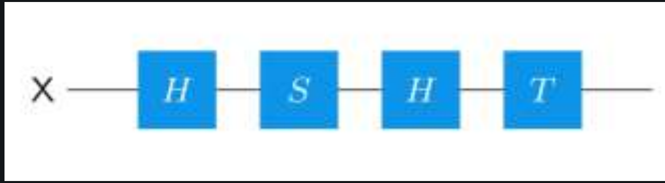
1. Classical Circuit
2. **Quantum Circuit**
3. Projective Measurement

Quantum Circuit

Quantum Circuits

- Wires: qubits
- Gates: unitary operations and measurements

Quantum Circuit



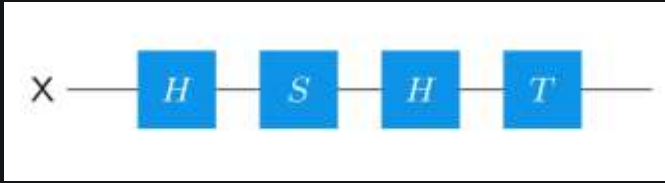
Quantum Circuits

- Wires: qubits
- Gates: unitary operations and measurements

$$H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \qquad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1+i}{\sqrt{2}} \end{pmatrix}$$

Quantum Circuit

Quantum Circuits



- Wires: qubits
- Gates: unitary operations and measurements

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$$THSH = \begin{pmatrix} \frac{1+i}{2} & \frac{1-i}{2} \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix}$$

Quantum Circuit

Qiskit

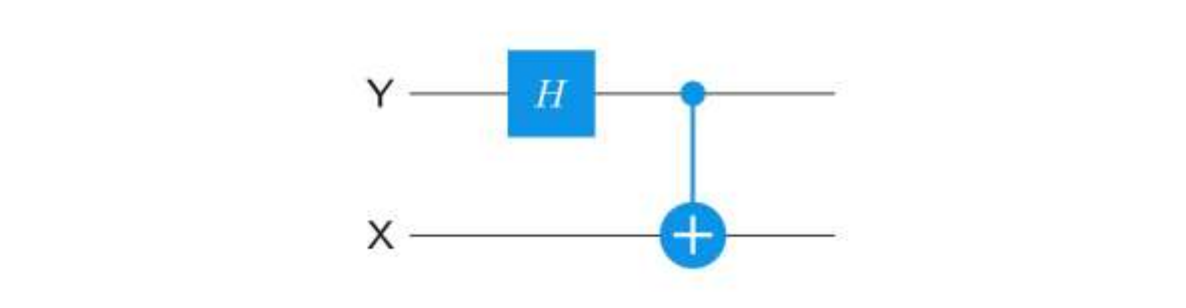
Quantum Circuits

- Wires: qubits
- Gates: unitary operations and measurements

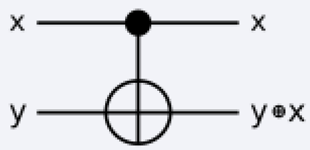
Operations $TSHS$ on qubit

$$THSH = \begin{pmatrix} \frac{1+i}{2} & \frac{1-i}{2} \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix}$$

Quantum Circuit

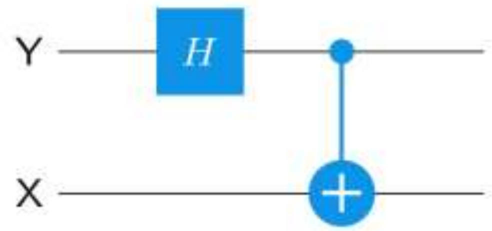


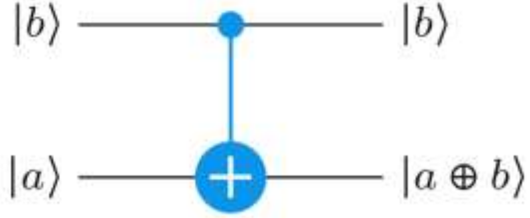
Quantum Circuit



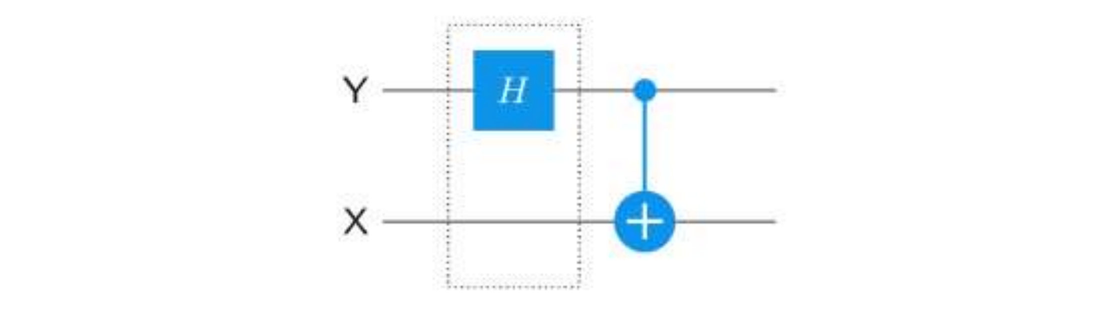
input		output	
x	y	x	y+x
$\vert 0\rangle$	$\vert 0\rangle$	$\vert 0\rangle$	$\vert 0\rangle$
$\vert 0\rangle$	$\vert 1\rangle$	$\vert 0\rangle$	$\vert 1\rangle$
$\vert 1\rangle$	$\vert 0\rangle$	$\vert 1\rangle$	$\vert 1\rangle$
$\vert 1\rangle$	$\vert 1\rangle$	$\vert 1\rangle$	$\vert 0\rangle$

input		output	
x	y	x	y+x
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

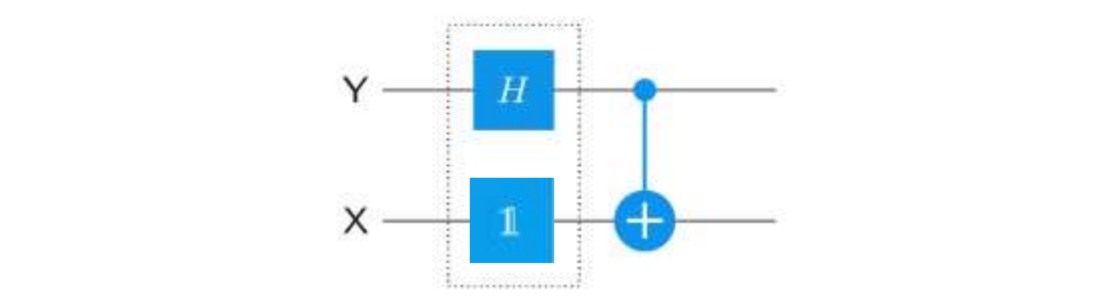




Quantum Circuit

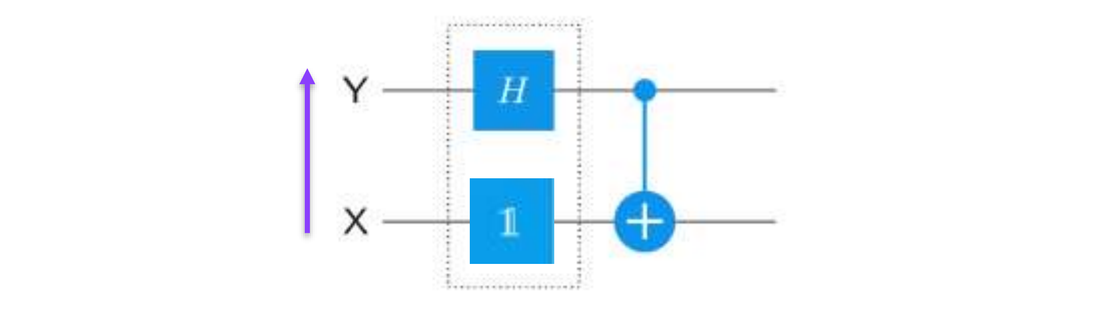


Quantum Circuit



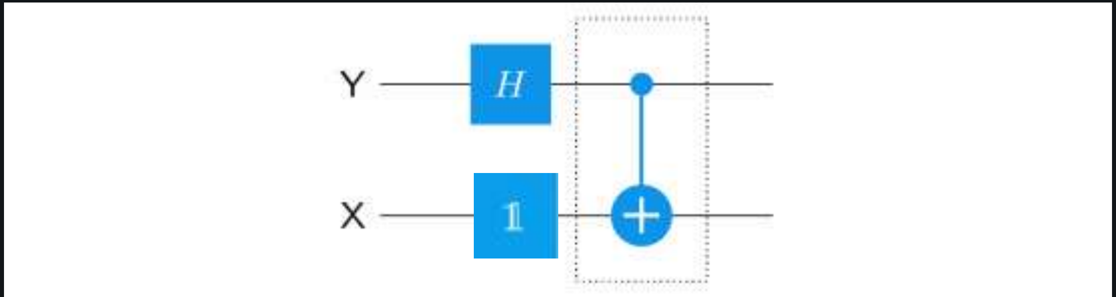
$$\mathbb{I} \otimes H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Quantum Circuit



$$\mathbb{I} \otimes H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

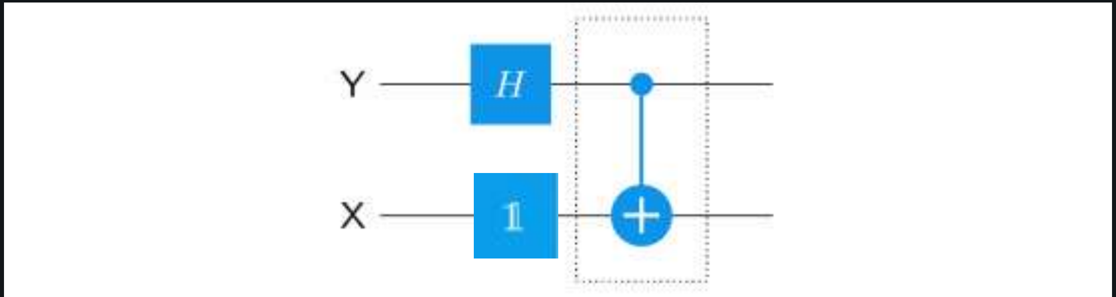
Quantum Circuit



X is the control bit, **Y** is target bit

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Quantum Circuit

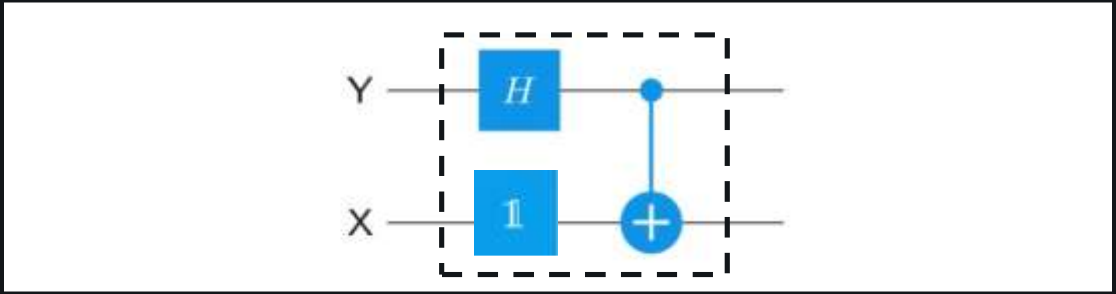


Y is the control bit, **X** is target bit

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

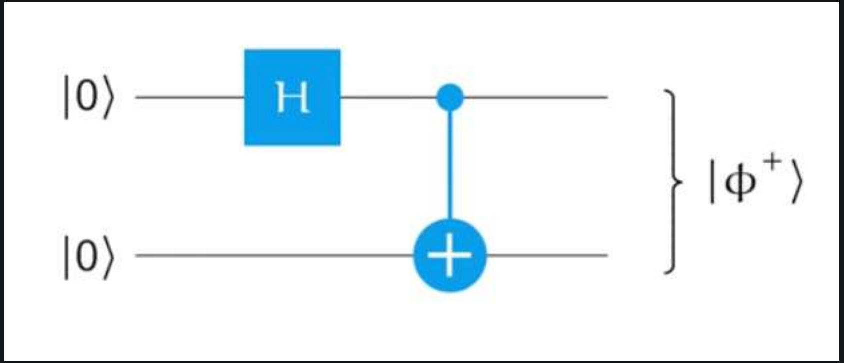
Quantum Circuit

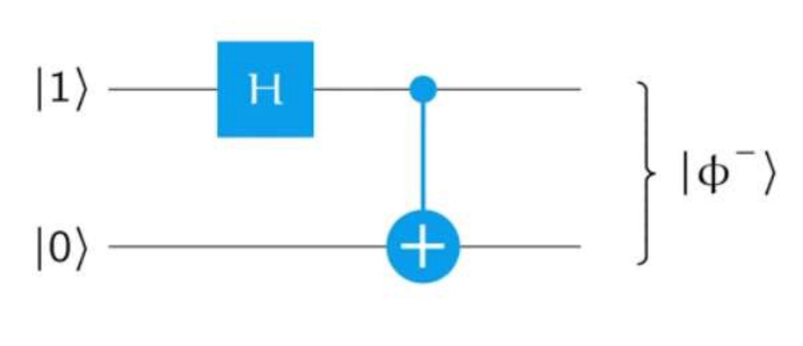
Qiskit

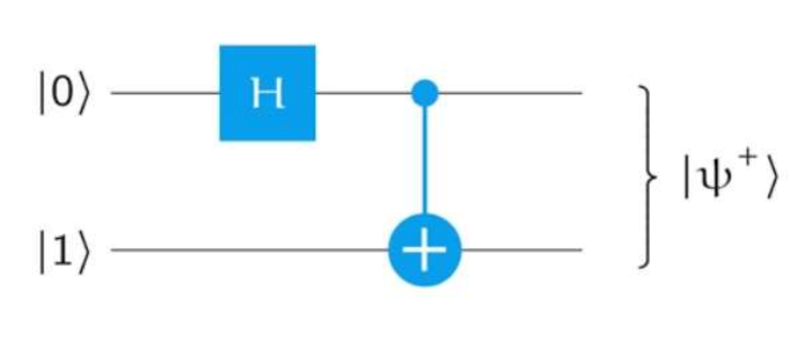


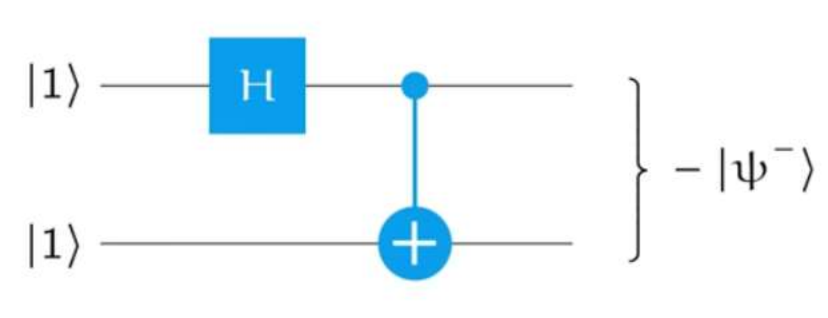
$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix}$$

Quantum Circuit



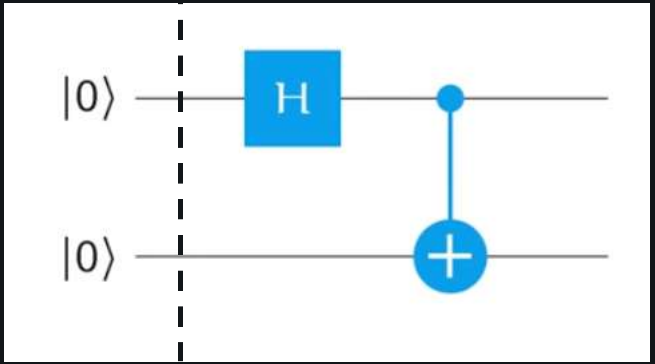






Quantum Circuit

Qiskit

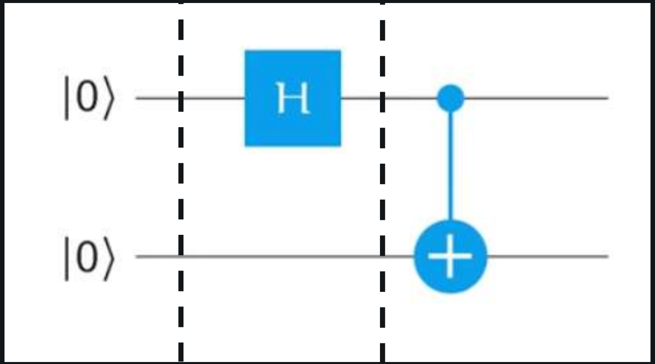


$|\pi_0\rangle$

$$|\pi_0\rangle = |0\rangle|0\rangle$$

Quantum Circuit

Qiskit



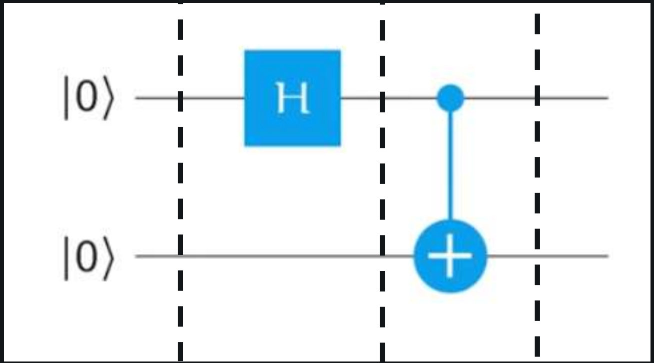
$|\pi_0\rangle$ $|\pi_1\rangle$

$$|\pi_0\rangle = |0\rangle|0\rangle$$

$$|\pi_1\rangle = |0\rangle|+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle$$

Quantum Circuit

Qiskit



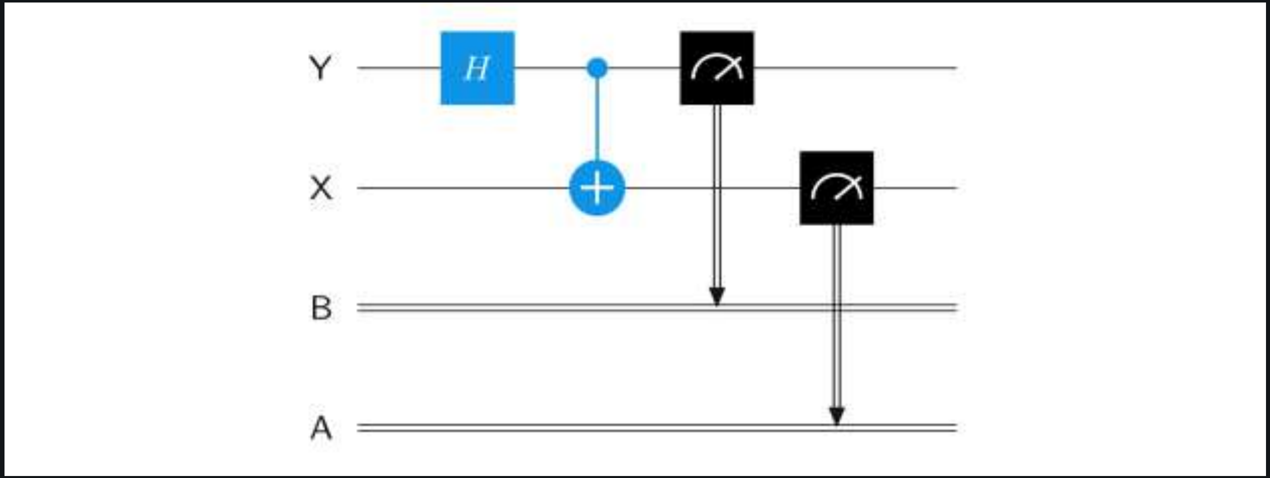
$|\pi_0\rangle$ $|\pi_1\rangle$ $|\pi_2\rangle$

$$|\pi_0\rangle = |0\rangle|0\rangle$$

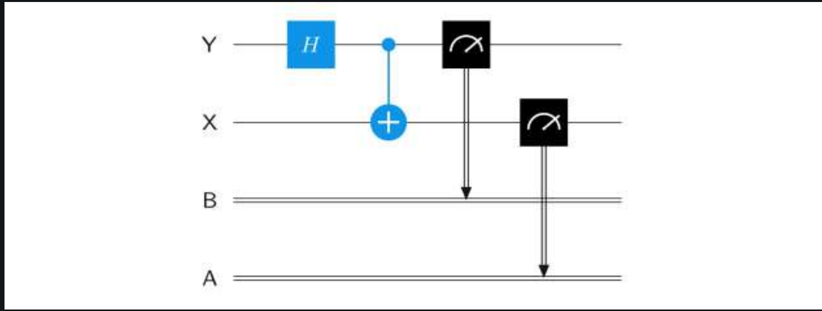
$$|\pi_1\rangle = |0\rangle|+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle$$

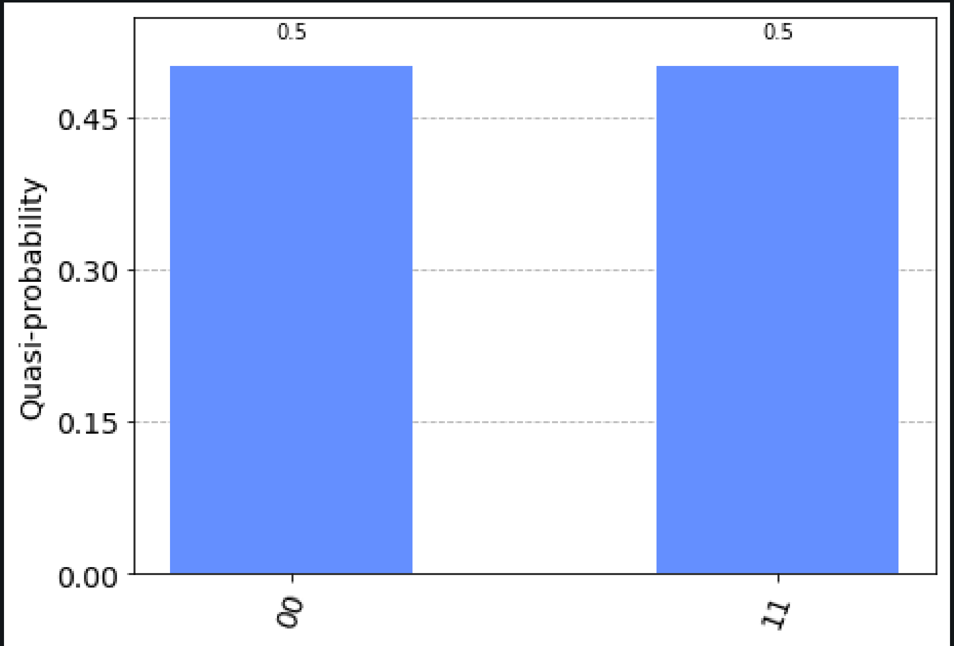
$$|\pi_2\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = |\phi^+\rangle$$

Quantum Circuit

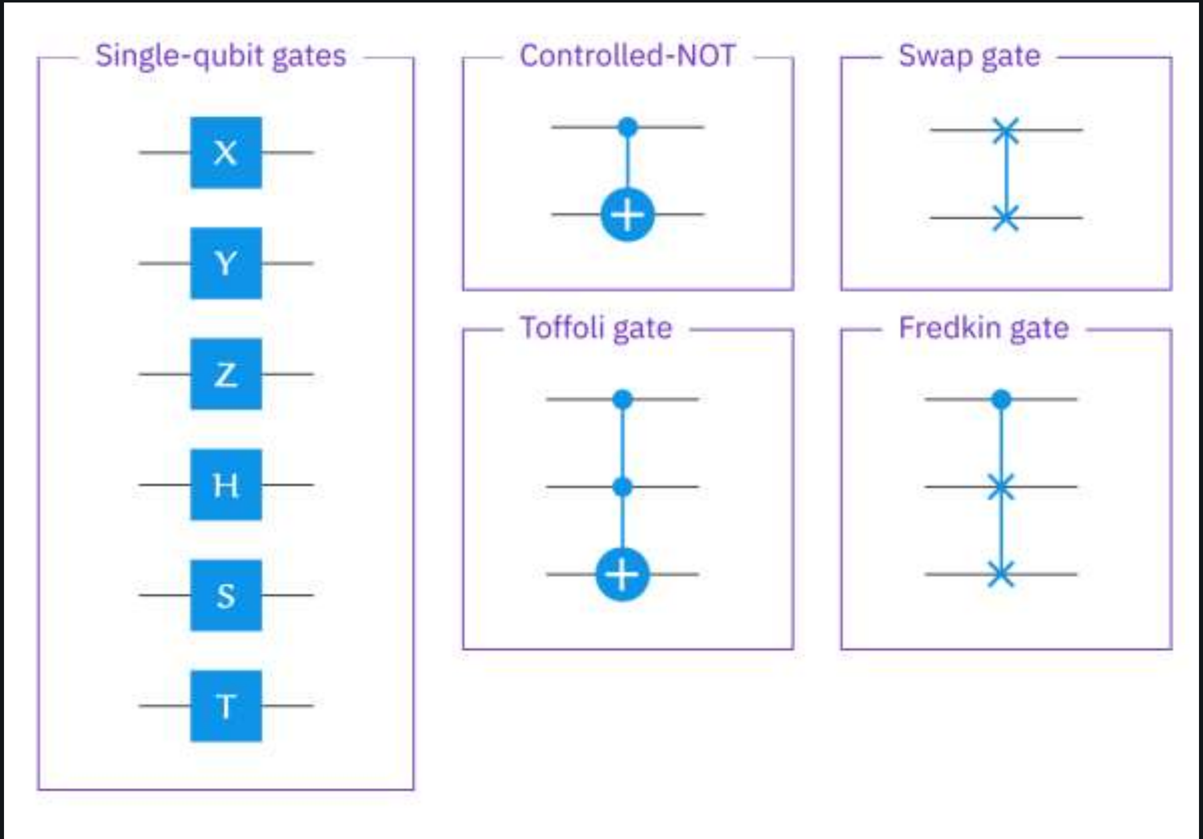


Quantum Circuit

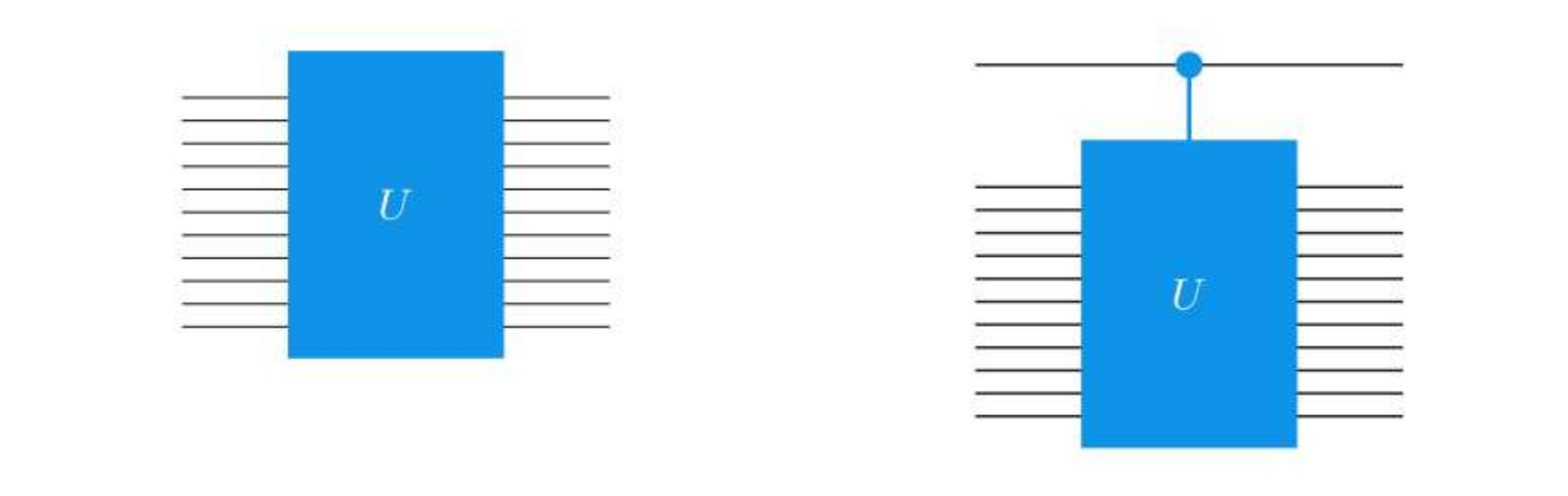




Quantum Circuit



Quantum Circuit



Unitary operation

Controlled-unitary operation

Quantum Circuit and Measurement

1. Classical Circuit
2. Quantum Circuit
3. **Projective Measurement**

Inner product

$$|\psi\rangle = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} \quad |\phi\rangle = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix}$$

$$\langle\psi|\phi\rangle = \begin{pmatrix} \overline{\alpha_1} & \cdots & \overline{\alpha_n} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix} = \overline{\alpha_1}\beta_1 + \cdots + \overline{\alpha_n}\beta_n$$

Inner product

$$|\psi\rangle = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} \quad |\phi\rangle = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix}$$

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$$|\psi\rangle = \sum_{a\in\Sigma} \alpha_a |a\rangle \quad |\phi\rangle = \sum_{b\in\Sigma} \beta_b |b\rangle$$

$$\langle\psi|\phi\rangle = \left(\sum_{a\in\Sigma} \alpha_a |a\rangle\right)\left(\sum_{b\in\Sigma} \beta_b |b\rangle\right)$$

$$= \textstyle\sum_{a\in\Sigma}\sum_{b\in\Sigma} \overline{\alpha_a}\beta_b\langle a|b\rangle$$

$$= \textstyle\sum_{a\in\Sigma} \overline{\alpha_a}\beta_b$$

Inner product

$$\langle\psi|\psi\rangle = \sum_{a\in\Sigma} \overline{\alpha_a}\alpha_a = \sum_{a\in\Sigma} |\alpha_a|^2 = \||\psi\rangle\|^2$$

Euclidean norm of vector

$$\||\psi\rangle\| = \sqrt{\langle\psi|\psi\rangle}$$

Inner product

$$\overline{\langle \psi | \phi \rangle} = \langle \phi | \psi \rangle$$

Orthogonal set

$$\langle \psi_j | \phi_k \rangle = 0 \qquad \text{(for all } j \neq k\text{)}$$

Orthonormal set

$$\langle \psi_j | \phi_k \rangle = \begin{cases} 1 & j = k \\ 0 & j \neq k \end{cases}$$

Projection

Square matrix Π is projection if

1. $\Pi = \Pi^\dagger$ (Hermitian matrices)
2. $\Pi^2 = \Pi$ (Idempotent matrices)

Projection

Square matrix Π is projection if

$|\psi\rangle$ is a unit vector

1. $\Pi = \Pi^\dagger$
2. $\Pi^2 = \Pi$

$$\Pi = |\psi\rangle\langle\psi|$$
$$\Pi^\dagger = (|\psi\rangle\langle\psi|)^\dagger = (\langle\psi|)^\dagger(|\psi\rangle)^\dagger = |\psi\rangle\langle\psi| = \Pi$$
$$\Pi^2 = (|\psi\rangle\langle\psi|)^2 = |\psi\rangle\langle\psi|\psi\rangle\langle\psi| = |\psi\rangle\langle\psi| = \Pi$$

Projective measurements

Measurement that described by collection of projections.

The sum is:

$$\Pi_1 + \cdots + \Pi_m = \mathbb{I}$$

Projective measurements

Measurement that described by collection of projections.

The sum is:

$$\Pi_1 + \cdots + \Pi_m = \mathbb{I}$$

Measurement on system X,

$$\mathrm{Pr}(outcome\ is\ k) = \|\Pi_k|\psi\rangle\|^2 = \langle\psi|\Pi_k|\psi\rangle$$

Projective measurements

Qiskit

Measurement that described by collection of projections.

The sum is:

$$\Pi_1 + \cdots + \Pi_m = \mathbb{I}$$

Measurement on system X,

$$\Pr(outcome\ is\ k) = \|\Pi_k|\psi\rangle\|^2 = \langle\psi|\Pi_k|\psi\rangle$$

After outcome measurement produce k, the state of X becomes

$$\frac{\Pi_k|\psi\rangle}{\|\Pi_k|\psi\rangle\|}$$

Projective measurements

Measure

$$|\psi\rangle = \sum_{a\in\Sigma} \alpha_a |a\rangle$$

Outcome a appears with probability

$$\| |a\rangle\langle a|\psi\rangle \|^2 = |\alpha_a|^2$$

The state becomes after outcome a

$$\frac{|a\rangle\langle a|\psi\rangle}{\| |a\rangle\langle a|\psi\rangle \|} = \frac{\alpha_a}{|\alpha_a|} |a\rangle$$

Projective measurements

Example

Performing a standard basis measurement system X, but doing nothing to a system Y is equivalent to perform the projective measurement

$$\{|a\rangle\langle a| \otimes \mathbb{I}_Y : a \in \Sigma\}$$

On the system (X, Y)
Outcome a appears with probability

$$\|(|a\rangle\langle a| \otimes \mathbb{I})|\psi\rangle\|^2 = |\alpha_a|^2$$

The state becomes after outcome a

$$\frac{(|a\rangle\langle a| \otimes \mathbb{I})|\psi\rangle}{\|(|a\rangle\langle a| \otimes \mathbb{I})|\psi\rangle\|}$$

QnA