

2023 부산시 양자컴퓨 팅 개발자 교육 프로그램

Lecture 3

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Syllabus

- Lecture 1: 게이트와 양자 회로 기본 작성법
 - Single qubit gate Lecture 1
 - Multiple qubit gate Lecture 2
 - Multiple qubit gate Notebook Demonstration
 - Barriers and Properties of Quantum Circuit
- Lecture 2: 양자 회로의 측정과 OpenQasm
 - Notebook Demonstration
- Lecture 3: 양자 백엔드에 양자회로 실행하기
- Lecture 4: 양자 회로 및 회로의 실행 결과 시각화 및 해석
- Lecture 5: 유용한 기능들

Lecture 3

Lecture 4

Lecture 5

- Lecture 1: 게이트와 양자 회로 기본 작성법
 - Single qubit gate
 - Multiple qubit gate
 - **Multiple qubit gate Notebook Demonstration**
 - **Barriers and Properties of Quantum Circuit**
- **Lecture 2: 양자 회로의 측정과 OpenQasm**
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Jupyternote Demonstration



Quantum circuits



Introduction

This lesson introduces the quantum circuit model of computation, a standard description of quantum computations that we'll use throughout this series.

We'll also introduce a few important mathematical concepts, including inner products between vectors, the notions of orthogonality and orthonormality, and projections and projective measurements, which generalize standard basis measurements. Through these concepts, we'll derive fundamental limitations on quantum

Qiskit Textbook: Quantum circuits

Lecture 2 - 양자 회로의 측정과 OpenQasm

1. 양자 회로의 측정과 비단일 연산자
2. 양자 회로와 데지스터
3. OpenQasm

1. 양자 회로의 측정과 비단일 연산자 (non-unitary operator)

양자 회로에 단일 큐비트 게이트와 다중 큐비트 게이트와 같이 양자 회로에 직접 큐비트에 작용을 시키는 단일 연산자 (unitary operation)이 있는 반면 비단일 연산자에도 접근할 수 있습니다. 비단일 연산자는 주로 아래의 값에 있습니다.

- 측정
- 큐비트의 초기화
- 고정적 조건부 연산자

```
In [1]: from qiskit import *
from qiskit.quantum_info import Statevector
from qiskit.visualization import plot_bloch_multivector
```

측정

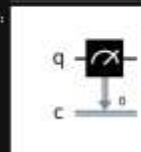
양자 컴퓨터를 측정하기 위해서 모든 정보를 접근할 필요는 없습니다. 양자 상태는 표준 기준으로 내려가게 합니다. Qiskit에서는 두가지의 방법으로 양자상태를 측정할 수 있으며 양자 비트를 고전 비트로 측정한다 라고 표현할 수 있습니다.

- `QuantumCircuit.measure(qubit, cbit)`
- `QuantumCircuit.measure_all`

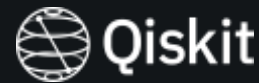
무슨 일을 하기 위해 상태가 1개의 큐비트를 가진 양자 회로를 준비를 해주고 상태가 0인 큐비트를 측정해줍니다.

```
In [2]: qc = QuantumCircuit(3,1) # 3개의 큐비트와 1개의 고전 비트를 생성합니다.
qc.measure(0,0) # 양자 회로에 0번 큐비트를 측정합니다. 큐비트 0번을 고전 비트 0번에 측정합니다.
qc.draw('mpl') # 양자 회로를 그립니다.
```

Out [2]:

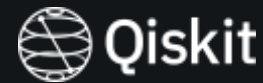


Quantum Circuit and Measurement



1. **Classical Circuit**
2. Quantum Circuit
3. Projective Measurement

Classical Circuit



Circuits

- Wires: information
- Gates: operations

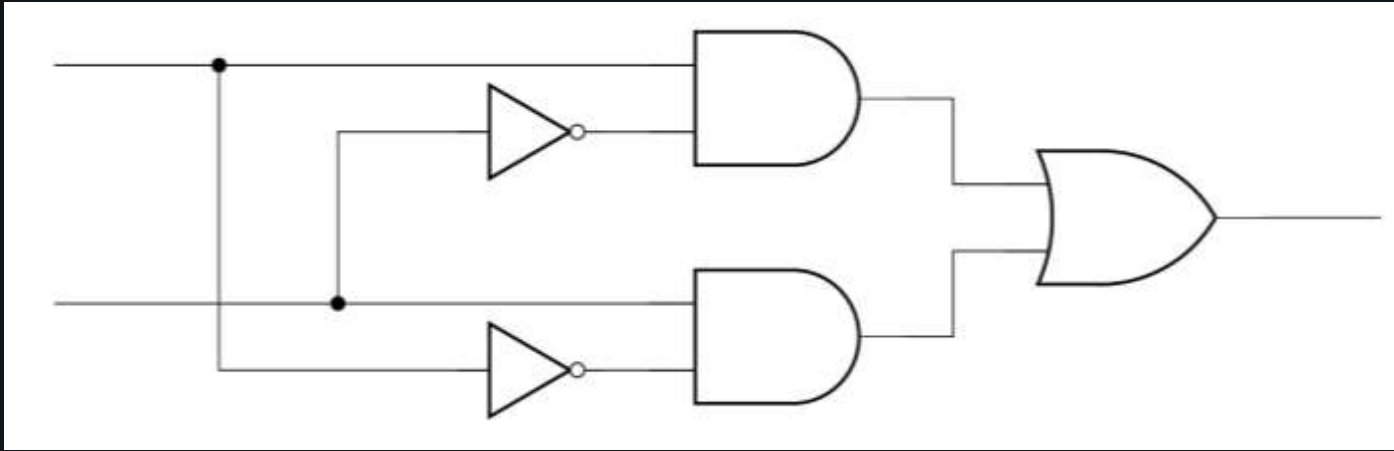
Left → Right

Classical Circuit

Circuits

- Wires: information
- Gates: operations

Left → Right

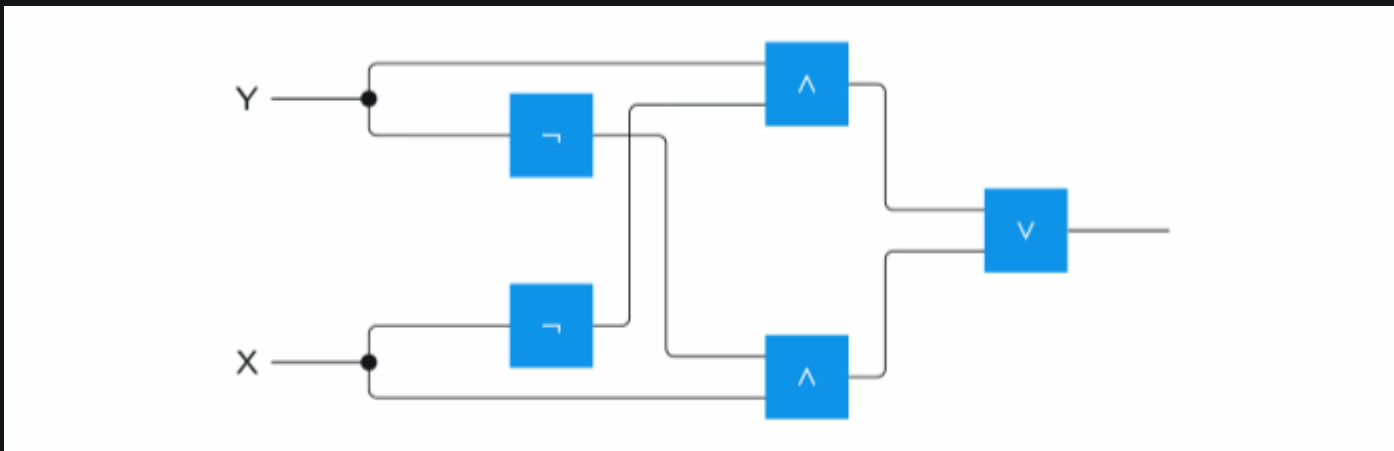


Classical Circuit

Circuits

- Wires: information
- Gates: operations

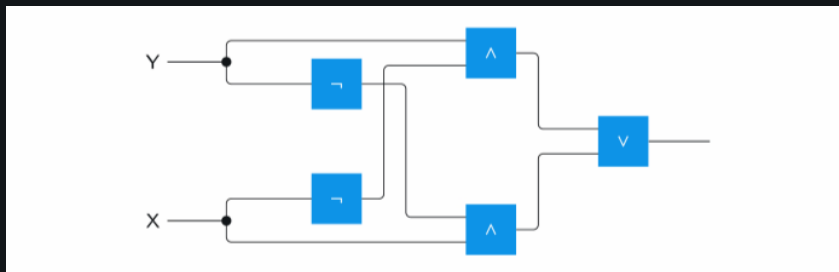
Left → Right



Classical Circuit

Circuits

- Wires: information
- Gates: operations



a	$\neg a$
0	1
1	0

ab	$a \wedge b$
00	0
01	0
10	0
11	1

ab	$a \vee b$
00	0
01	1
10	1
11	1

Classical Circuit

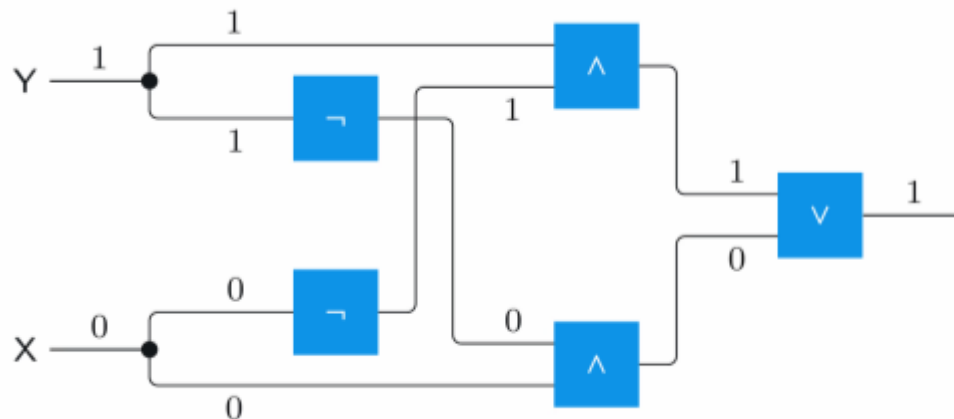
Circuits

- Wires: information
- Gates: operations

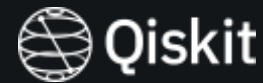
ab	$a \oplus b$
00	0
01	1
10	1
11	0



XOR gate



Quantum Circuit and Measurement



1. Classical Circuit
2. **Quantum Circuit**
3. Projective Measurement

Quantum Circuit

Quantum Circuits

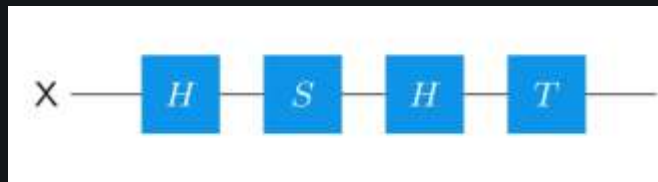
- Wires: qubits
- Gates: unitary operations and measurements



Quantum Circuit

Quantum Circuits

- Wires: qubits
- Gates: unitary operations and measurements



$$H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

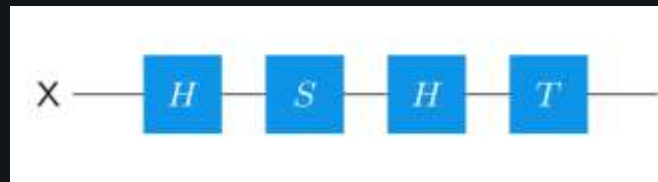
$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1+i}{\sqrt{2}} \end{pmatrix}$$

Quantum Circuit

Quantum Circuits

- Wires: qubits
- Gates: unitary operations and measurements



$$H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1+i}{\sqrt{2}} \end{pmatrix}$$

$$THSH = \begin{pmatrix} \frac{1+i}{2} & \frac{1-i}{2} \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix}$$

Quantum Circuits

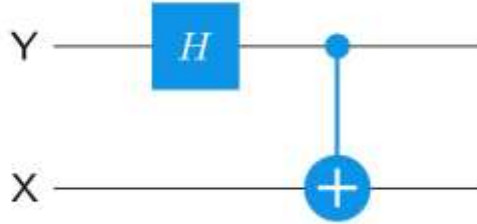
- Wires: qubits
- Gates: unitary operations and measurements

Operations *THSH* on qubit

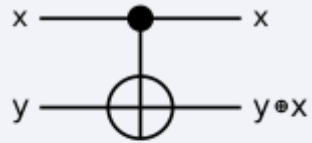


$$THSH = \begin{pmatrix} \frac{1+i}{2} & \frac{1-i}{2} \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix}$$

Quantum Circuit

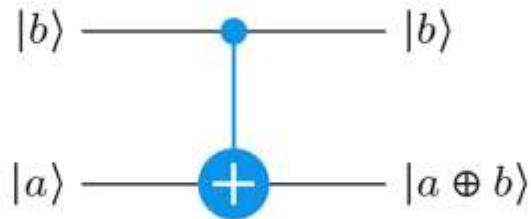
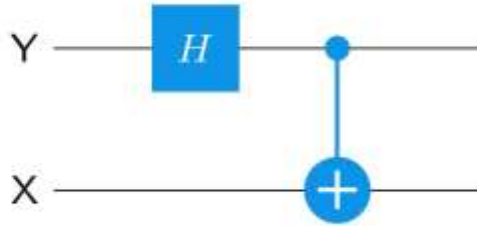


Quantum Circuit

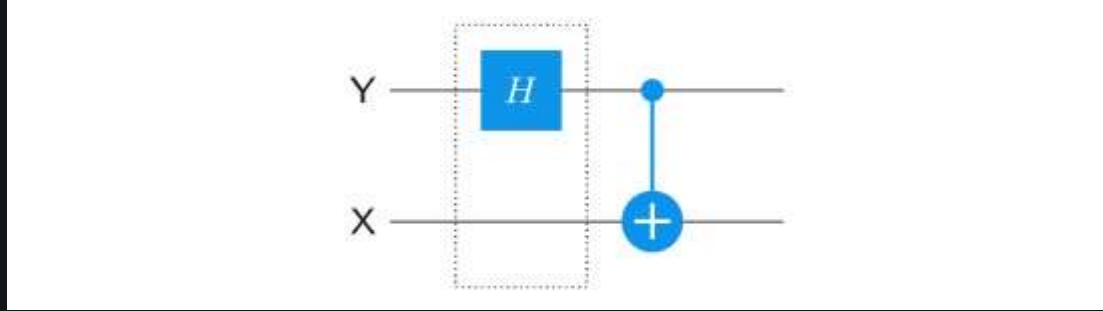


input		output	
x	y	x	y+x
0⟩	0⟩	0⟩	0⟩
0⟩	1⟩	0⟩	1⟩
1⟩	0⟩	1⟩	1⟩
1⟩	1⟩	1⟩	0⟩

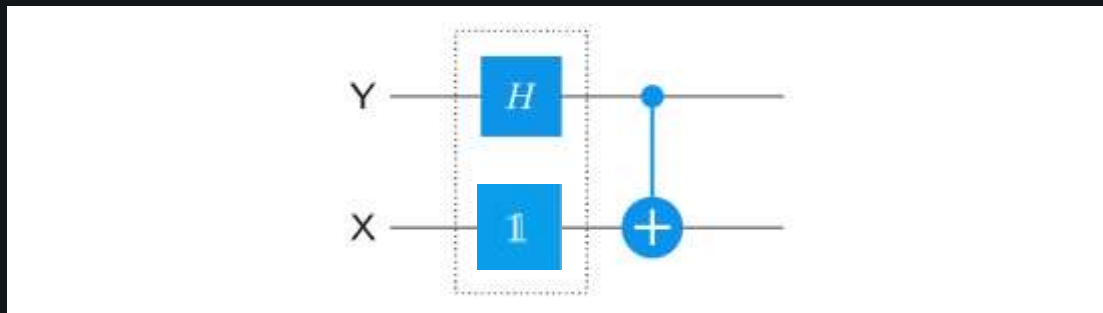
input		output	
x	y	x	y+x
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0



Quantum Circuit

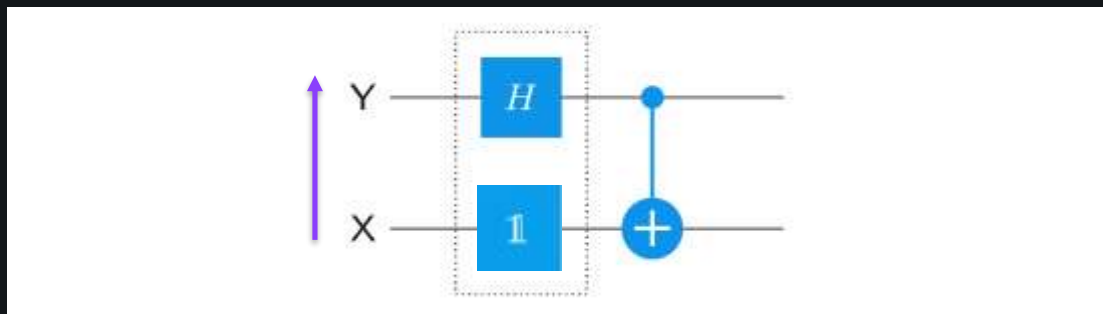


Quantum Circuit



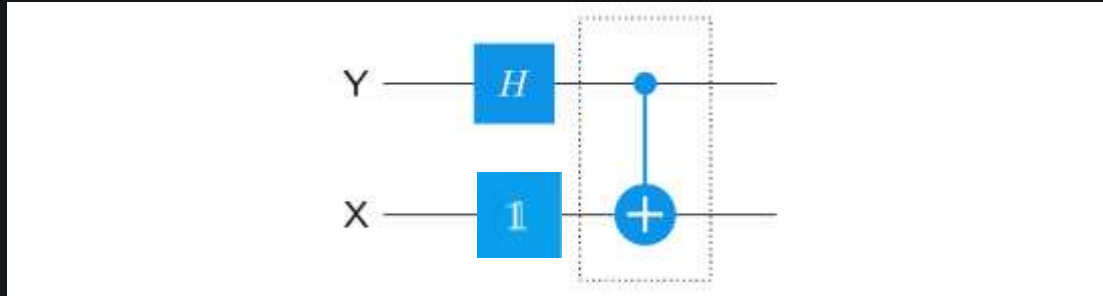
$$\mathbb{I} \otimes H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Quantum Circuit



$$\xrightarrow{\text{purple arrow}} \mathbb{I} \otimes H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

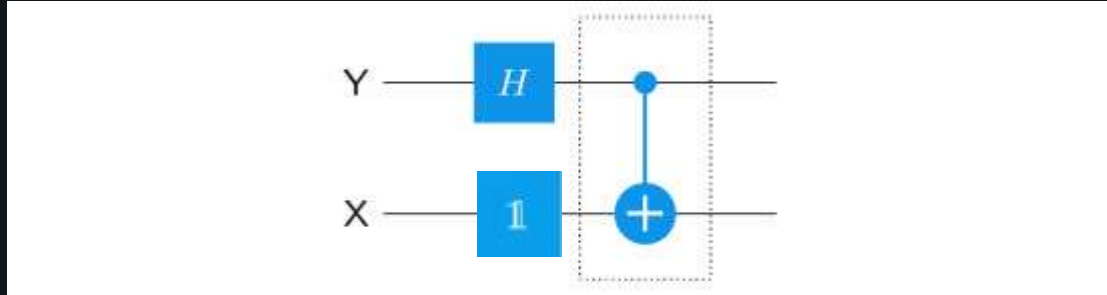
Quantum Circuit



X is the control bit, **Y** is target bit

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

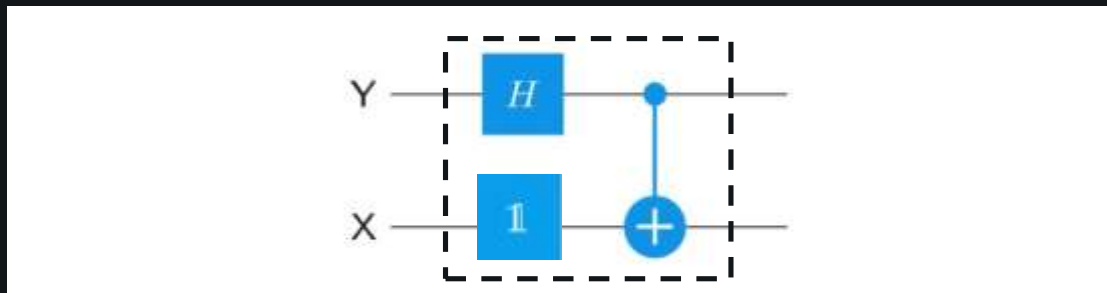
Quantum Circuit



Y is the control bit, **X** is target bit

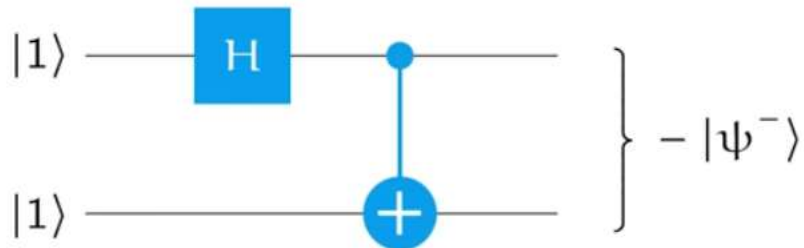
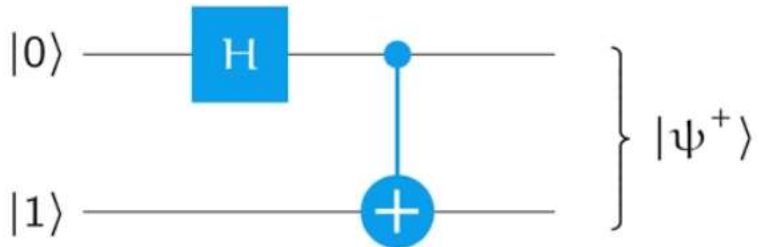
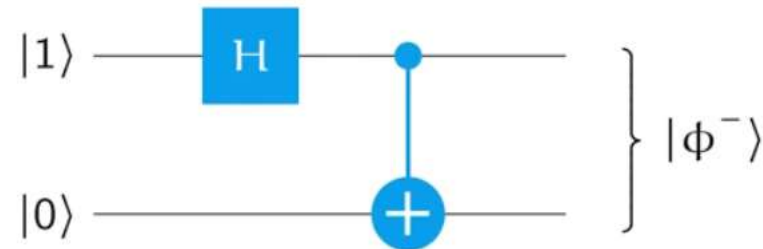
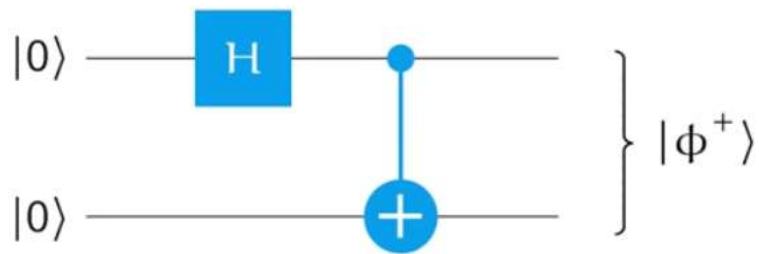
$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Quantum Circuit

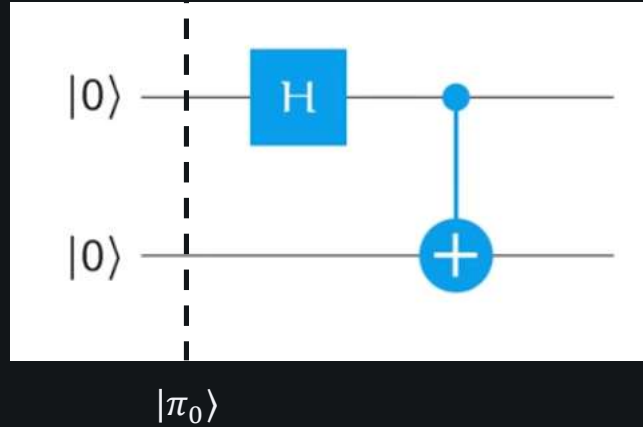


$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix}$$

Quantum Circuit

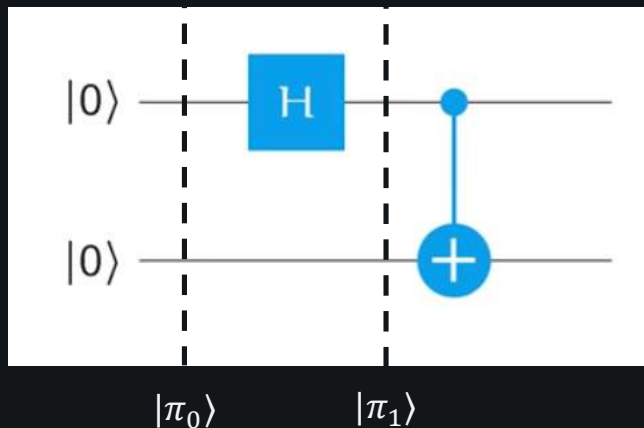


Quantum Circuit



$$|\pi_0\rangle = |0\rangle|0\rangle$$

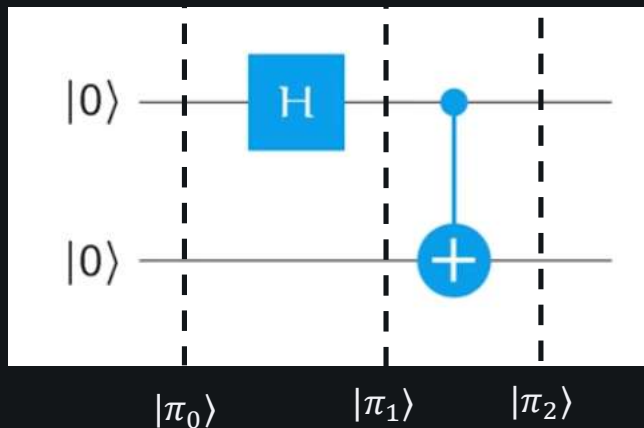
Quantum Circuit



$$|\pi_0\rangle = |0\rangle|0\rangle$$

$$|\pi_1\rangle = |0\rangle|+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle$$

Quantum Circuit

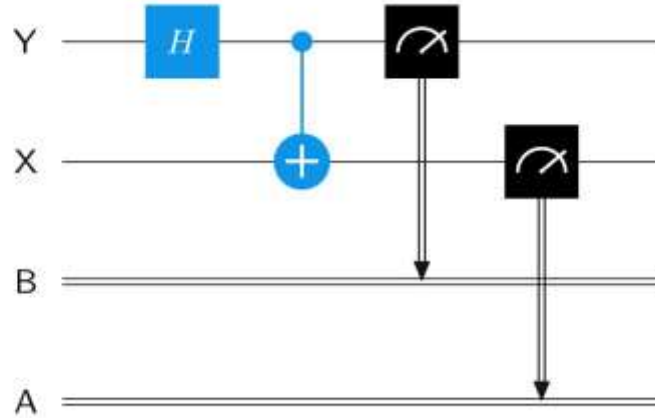


$$|\pi_0\rangle = |0\rangle|0\rangle$$

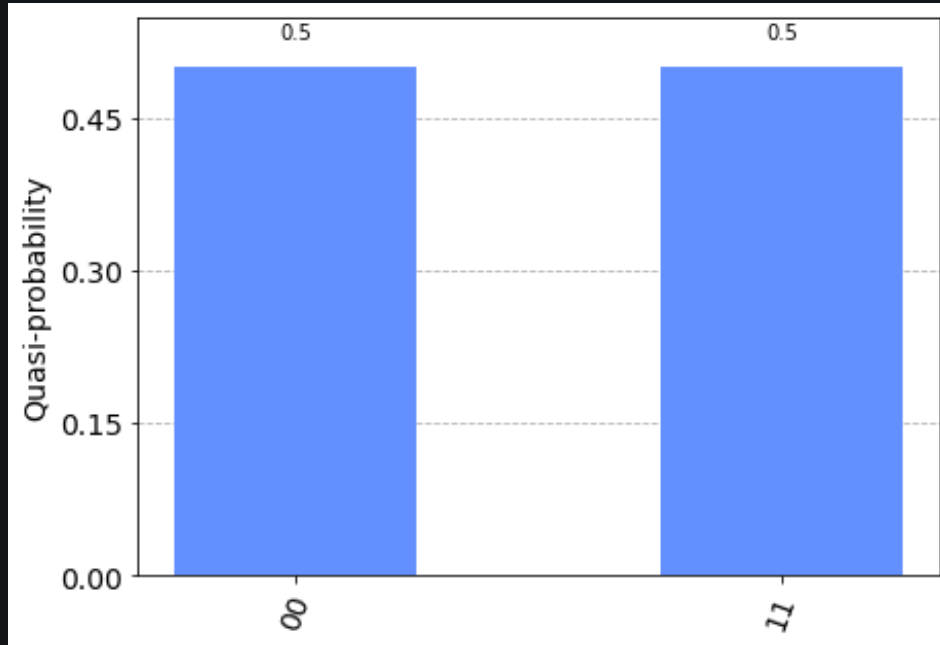
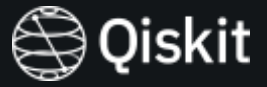
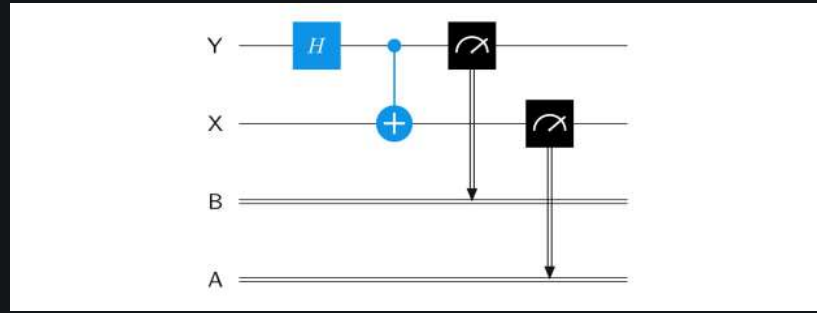
$$|\pi_1\rangle = |0\rangle|+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle$$

$$|\pi_2\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = |\phi^+\rangle$$

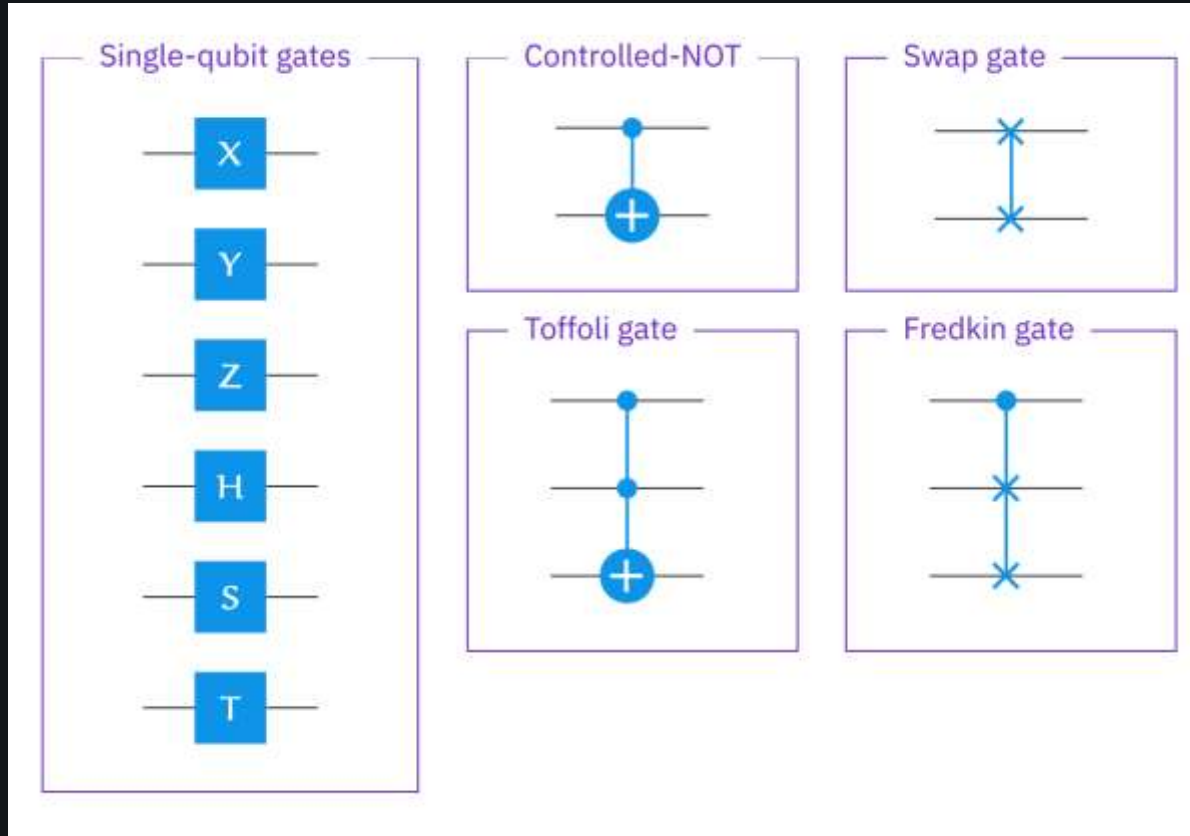
Quantum Circuit



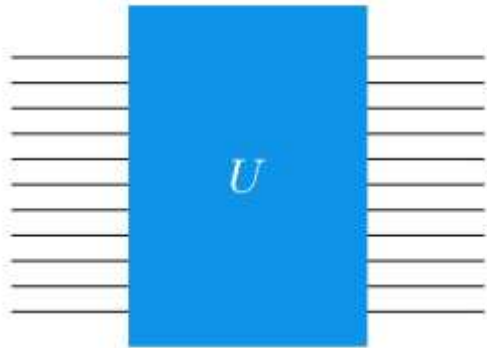
Quantum Circuit



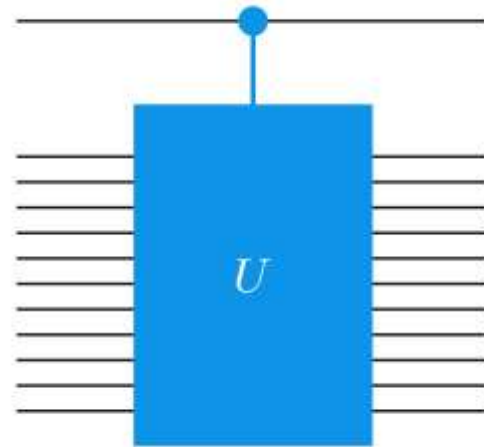
Quantum Circuit



Quantum Circuit

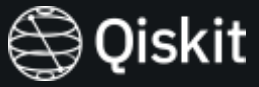


Unitary operation



Controlled-unitary operation

Quantum Circuit and Measurement



1. Classical Circuit
2. Quantum Circuit
3. **Projective Measurement**

Inner product

$$|\psi\rangle = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} \quad |\phi\rangle = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix} \quad \langle\psi|\phi\rangle = (\overline{\alpha_1} \quad \cdots \quad \overline{\alpha_n}) \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix} = \overline{\alpha_1}\beta_1 + \cdots + \overline{\alpha_n}\beta_n$$

Inner product

$$|\psi\rangle = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} \quad |\phi\rangle = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix} \quad \langle\psi|\phi\rangle = (\overline{\alpha_1} \quad \cdots \quad \overline{\alpha_n}) \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix} = \overline{\alpha_1}\beta_1 + \cdots + \overline{\alpha_n}\beta_n$$

$$\begin{aligned} |\psi\rangle &= \sum_{a \in \Sigma} \alpha_a |a\rangle & |\phi\rangle &= \sum_{b \in \Sigma} \beta_b |b\rangle \\ \langle\psi|\phi\rangle &= \left(\sum_{a \in \Sigma} \alpha_a |a\rangle \right) \left(\sum_{b \in \Sigma} \beta_b |b\rangle \right) \\ &= \sum_{a \in \Sigma} \sum_{b \in \Sigma} \overline{\alpha_a} \beta_b \langle a|b\rangle \\ &= \sum_{a \in \Sigma} \overline{\alpha_a} \beta_a \end{aligned}$$

Inner product

$$\langle \psi | \psi \rangle = \sum_{a \in \Sigma} \overline{\alpha_a} \alpha_a = \sum_{a \in \Sigma} |\alpha_a|^2 = ||\psi\rangle||^2$$

Euclidean norm of vector

$$||\psi\rangle|| = \sqrt{\langle \psi | \psi \rangle}$$

Inner product

$$\overline{\langle \psi | \phi \rangle} = \langle \phi | \psi \rangle$$

Orthogonal set

$$\langle \psi_j | \phi_k \rangle = 0 \quad (\text{for all } j \neq k)$$

Orthonormal set

$$\langle \psi_j | \phi_k \rangle = \begin{cases} 1 & j = k \\ 0 & j \neq k \end{cases}$$

Projection

Square matrix Π is projection if

1. $\Pi = \Pi^\dagger$ (Hermitian matrices)
2. $\Pi^2 = \Pi$ (Idempotent matrices)

Projection

Square matrix Π is projection if

$|\psi\rangle$ is a unit vector

1. $\Pi = \Pi^\dagger$
2. $\Pi^2 = \Pi$

$$\Pi = |\psi\rangle\langle\psi|$$

$$\Pi^\dagger = (|\psi\rangle\langle\psi|)^\dagger = (\langle\psi|)^\dagger(|\psi\rangle)^\dagger = |\psi\rangle\langle\psi| = \Pi$$

$$\Pi^2 = (|\psi\rangle\langle\psi|)^2 = |\psi\rangle\langle\psi|\psi\rangle\langle\psi| = |\psi\rangle\langle\psi| = \Pi$$

Projective measurements

Measurement that described by collection of projections.

The sum is:

$$\Pi_1 + \cdots + \Pi_m = \mathbb{I}$$

Projective measurements



Measurement that described by collection of projections.

The sum is:

$$\Pi_1 + \cdots + \Pi_m = \mathbb{I}$$

Measurement on system X ,

$$\Pr(\text{outcome is } k) = \|\Pi_k|\psi\rangle\|^2 = \langle\psi|\Pi_k|\psi\rangle$$

Projective measurements

Measurement that described by collection of projections.

The sum is:

$$\Pi_1 + \cdots + \Pi_m = \mathbb{I}$$

Measurement on system X,

$$\Pr(\text{outcome is } k) = \|\Pi_k|\psi\rangle\|^2 = \langle\psi|\Pi_k|\psi\rangle$$

After outcome measurement produce k, the state of X becomes

$$\frac{\Pi_k|\psi\rangle}{\|\Pi_k|\psi\rangle\|}$$

Projective measurements

Measure

$$|\psi\rangle = \sum_{a \in \Sigma} \alpha_a |a\rangle$$

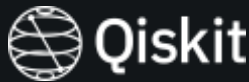
Outcome a appears with probability

$$\| |a\rangle\langle a|\psi\rangle \|^2 = |\alpha_a|^2$$

The state becomes after outcome a

$$\frac{|a\rangle\langle a|\psi\rangle}{\| |a\rangle\langle a|\psi\rangle \|} = \frac{\alpha_a}{|\alpha_a|} |a\rangle$$

Projective measurements



Example

Performing a standard basis measurement on system X , but doing nothing to a system Y is equivalent to perform the projective measurement

$$\{|a\rangle\langle a| \otimes \mathbb{I}_Y : a \in \Sigma\}$$

On the system (X, Y)

Outcome a appears with probability

$$\|(|a\rangle\langle a| \otimes \mathbb{I})|\psi\rangle\|^2 = |\alpha_a|^2$$

The state becomes after outcome a

$$\frac{(|a\rangle\langle a| \otimes \mathbb{I})|\psi\rangle}{\|(|a\rangle\langle a| \otimes \mathbb{I})|\psi\rangle\|}$$

QnA