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UNIVERSITY OF TORONTO SCARBOROUGH
Department of Computer and Mathematical Sciences

APRIL 2021 EXAMINATIONS

CSC C63H3

Duration — 3 hours

NO AIDS.

INSTRUCTOR: Eric Corlett

Student Number: |_|_|_|_|_|_|_|_|_|_|_|_|_|

Last (Family) Name: _____

First (Given) Name(s): _____

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*Do **not** turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and carefully read the instructions below.)

This final examination consists of 8 questions on 5 pages (including this one), printed on one side of the paper. *When you receive the signal to start, please make sure that your copy of the examination is complete.*

Answer each question directly on the examination paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of one or more page(s) and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any fact covered in lecture, tutorial, or assignments, as long as you clearly mention the fact you are using. You must justify all other facts required for your solutions.

# 1: _____ /	5
# 2: _____ /	15
# 3: _____ /	15
# 4: _____ /	15
# 5: _____ /	15
# 6: _____ /	15
# 7: _____ /	10
# 8: _____ /	10
TOTAL: _____ /	100

When writing your polytime reductions, you can assume the following problems are **NP**-complete:

SAT:

Input: A Boolean formula ϕ.

Question: Does ϕ have a satisfying assignment?

3-SAT:

Input: A Boolean formula ϕ in 3CNF.

Question: Does ϕ have a satisfying assignment?

CLIQUE:

Input: A graph G and a number $k \in \mathbb{N}$.

Question: Does G have a clique of size k?

IS:

Input: A graph G and a number $k \in \mathbb{N}$.

Question: Does G have an independent set of size k?

VC:

Input: A graph G and a number $k \in \mathbb{N}$.

Question: Does G have a vertex cover of size k?

HAM-PATH:

Input: A directed graph G and two nodes s and t.

Question: Does G have a Hamiltonian path from s to t?

3COL:

Input: A graph G.

Question: Is G 3-colourable?

SUBSET-SUM:

Input: A multiset S of non-negative integers, an integer t.

Question: Is there a subset $S' \subseteq S$ such that $\sum_{x \in S'} = t$?

Question 1. [5 MARKS]

In order to better understand what you're taking away from this course, let me know:

- What did you learn in this course? What were your favourite topics?
- Which topics were the most challenging?
- How will you transfer what you have learned here into your professional life?

Question 2. [15 MARKS]

For the following short-answer question, no formal proof of your answer is required, but you are required to show some justification for your answers.

Part (a) [5 MARKS]

Suppose a language L is in NP. Is L necessarily co-recognizable?

Part (b) [5 MARKS]

Assuming that P$\neq$NP, is it reasonable to say that if $L \in$P, then $\#L \in$FP?

Part (c) [5 MARKS]

Recall the following two languages:

HAM-CYCLE:

Input: A directed graph G.

Question: Does G have a Hamiltonian cycle?

and

UHAM-CYCLE:

Input: An undirected graph G.

Question: Does G have a Hamiltonian cycle?

Is it possible to find a parsimonious 1-1 reduction from HAM-CYCLE to UHAM-CYCLE? Why or Why not?

Question 3. [15 MARKS]

Let LONGEST-PATH be defined as follows:

IN: A directed graph G, two nodes s and t, and a number k.

QUESTION: Is there a path from s to t in G of length $\geq k$?

Part (a) [5 MARKS]

Show LONGEST-PATH $\in$ **NP**.

Part (b) [10 MARKS]

Show LONGEST-PATH is **NP**-hard.

Question 4. [15 MARKS]

Let COMMON-CLIQUE be defined as follows:

IN: Two undirected graphs G and H.

QUESTION: Do G and H share a maximal clique of the same size?

Remember, a clique is maximal if it can't be extended by adding more nodes. A clique is a maximum clique if there is no other clique of larger size.

Part (a) [5 MARKS]

Show COMMON-CLIQUE $\in$ **NP**.

Part (b) [10 MARKS]

Show COMMON-CLIQUE is **NP**-hard.

Question 5. [15 MARKS]

Let DOMINATING-SET be defined as follows:

IN: An undirected graph G, and a number k.

QUESTION: Does G have a dominating set of size k?

A dominating set is a set $S \subseteq V$ such that for all $v \in V \setminus S$, v is adjacent to some node in S.

Part (a) [5 MARKS]

Show DOMINATING-SET $\in$ **NP**.

Part (b) [10 MARKS]

Show DOMINATING-SET is **NP**-hard.

Question 6. [15 MARKS]

Let f be the polytime reduction from **TQBF** to **GG** (generalized geography) as described in class.

Let $\phi = \exists x_1, x_2, \forall x_3, \exists x_4, \forall x_5, \big[(x_1 \vee x_2) \wedge \neg(x_2 \vee (x_3 \vee \neg x_4))\big] \vee x_5$.

Part (a) [5 MARKS]

Is $\phi \in$TQBF? Why or why not?

Part (b) [10 MARKS]

Draw $f(\phi)$. Label your graph appropriately and indicate the starting node.

Question 7. [10 MARKS]

Show that the language L is not recognizable, where

$$L = \big\{ \langle M \rangle \,\big|\, L(M) \in \mathrm{P} \big\}.$$

Question 8. [10 MARKS]

Show how you could use a polytime solver M_{VC} for VC to find a certificate for it: that is, use M_{VC} to build a polytime program that, given an instance $\langle G = (V, E), k \rangle$ of VC, will return a size-k vertex covering of G if one exists, and will return *null* otherwise.

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