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UNIVERSITY OF TORONTO SCARBOROUGH
Department of Computer and Mathematical Sciences

AUGUST 2021 EXAMINATIONS

CSC C63H3

Duration — 3 hours

INSTRUCTOR: Eric Corlett

Student Number: |_|_|_|_|_|_|_|_|_|_|_|_|

Last (Family) Name: _____

First (Given) Name(s): _____

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*Do **not** turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and carefully read the instructions below.)

This final examination consists of 7 questions on 14 pages (including this one). *When you receive the signal to start, please make sure that your copy of the examination is complete.*

Answer each question directly on the examination paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of one or more page(s) and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any fact covered in lecture, tutorial, or assignments, as long as you clearly mention the fact you are using. You must justify all other facts required for your solutions.

When writing your polytime reductions, you can assume the any of the problems in your polytime reductions handout are **NP**-complete. These languages include, but are not limited to:

3-SAT:

Input: A Boolean formula ϕ in 3CNF.

Question: Does ϕ have a satisfying assignment?

3DM:

Input: Three sets A, B, and C, where $|A| = |B| = |C|$, and a set $T \subseteq A \times B \times C$.

Question: Is there a subset $M \subseteq T$ such that:

i) $|M| = |A|$,

ii) For any distinct (a, b, c) and $(a', b', c') \in M, a \neq a', b \neq b'$, and $c \neq c'$?

CLIQUE:

Input: A graph G and a number $k \in \mathbb{N}$.

Question: Does G have a clique of size k?

VC:

Input: A graph G and a number $k \in \mathbb{N}$.

Question: Does G have a vertex cover of size k?

HAM-PATH:

Input: A directed graph G and two nodes s and t.

Question: Does G have a Hamiltonian path from s to t?

3COL:

Input: A graph G.

Question: Is G 3-colourable?

SUBSET-SUM:

Input: A multiset S of non-negative integers, an integer t.

Question: Is there a subset $S' \subseteq S$ such that $\sum_{x \in S'} = t$?

1. **(15 pts)** Consider the language THREE-CLIQUE-DECOMPOSITION:

   Input: An undirected graph $G = (V, E)$.
   Question: Can G be partitioned into three disjoint cliques?

   (a) **(5 pts)**
   Show THREE-CLIQUE-DECOMPOSITION is in NP. Do not use an NTM.

(b) **(10 pts)**
Show THREE-CLIQUE-DECOMPOSITION is NP-hard.

2. **(10 pts)**

   Let f be the polytime reduction from **TQBF** to **GG** (generalized geography) as described in class.

   Let $\phi = \forall x_1, \exists x_2, x_3, (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg(x_2 \vee x_3)) \wedge (\neg x_1 \vee \neg(\neg x_2 \wedge \neg x_3))$.

   Draw $f(\phi)$. Label your graph appropriately and indicate the starting node. Do not simplify ϕ.

   A bonus of up to five marks will be given if the output is not only a valid encoding of ϕ, but is also planar.

3. **(15 pts)**

Consider the language HALF-PATH:

Instance: A directed graph $G = (V, E)$, and two nodes s and t.
Question: Is there a path of length $|V|/2$ from s to t?

(a) **(5 pts)**
Show HALF-PATH is in NP. Do not use an NTM.

(b) **(10 pts)**
Show HALF-PATH is NP-hard.

4. **(15 pts)**

Consider the language MINIMUM-COVER:

Instance: A collection $\mathcal{C}$ of subsets of a finite set S, a positive integer $k \leqslant |C|$

Question: Does $\mathcal{C}$ contain a cover C' for S of size k or less, i.e., a subset $C' \subseteq \mathcal{C}$ with $|C'| \leqslant k$ such that every element of S belongs to at least one element of C'?

(a) **(5 pts)**

Show MINIMUM-COVER is in NP. Do not use an NTM.

(b) **(10 pts)**
Show MINIMUM-COVER is NP-hard.

5. **(10 pts)**

We can define the language DOMINATING-SET as follows:

> **Instance:** A graph $G = (V, E)$ and a number $k \in \mathbb{N}$.
>
> **Question:** Does G contain a dominating set of size k or less (i.e., a set D of k vertices in V such that, for every vertex $u \in V \setminus V$, there is some $v \in D$ such that $\{u, v\} \in E$)?

It turns out that this language is NP-complete. Show how you could use an oracle **DS** for DOMINATING-SET to find a certificate for it: that is, use **DS** to build a polytime program that, given an instance $\langle G = (V, E), k\rangle$ of DOMINATING-SET, will return a size-k dominating set of of G if one exists, and will return *null* otherwise.

6. **(20 pts)**

   Let the language $L_6 = \{\langle M \rangle \mid M \text{ is a TM such that } \forall n \in \mathbb{N}, 0^n 1^n \in L(M)\}$.

   (a) **(10 pts)**

   Is L_6 recognizable? Prove your answer.

(b) **(10 pts)**
Is L_6 co-recognizable? Prove your answer.

7. **(5 pts)**

   We have said in class that for any NP-complete language L, we can use an oracle for L to find a certificate for any yes-instance of it in polynomial time. But suppose that L is just in NP, but not necessarily NP-complete. Suppose further that we have an oracle **3S** for 3SAT. Is it reasonable to believe that we can use **3S** to find a certificate for any yes-instance of L in polytime? Justify your answer.

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