Conditional Probability & Bayes' Theorem

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Conditional probability

- The probability of event A happening given that event B happened, is denoted by P(A |* B)
  - A: event of rolling two dice and the sum of each roll adds up to 2
    - P(A) = 1/36 (2.7%)
  - B: is one of the die landing on a 1
    - P(B) = 11/36 (30.5%)
  - Therefore:
    - P(A | B) = 1/11 (9%)
    - P(B | A) = 6/6 (100%)
  - C: one of the die lands on 5
    - P(A | C) = ∅ (null-set or impossible event)
    - P(C | A) = ∅ (null-set or impossible event)

*| = given event / the event following | is a given

Conditional probability

- P(B) possible outcomes (is 11/36 OR 30.5%):
  - B: is one of the die landing on a 1

1,1	1,2	1,3	1,4	1,4	1, 5	1, 6	2,1	3,1	4,1	5, 1	6, 1

- If the sum of 2 dice must equal 4, then the only outcomes are:

1,3	2,2	3,1

- The probability of a 1 appearing is 2/3 or 75%

Conditional probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

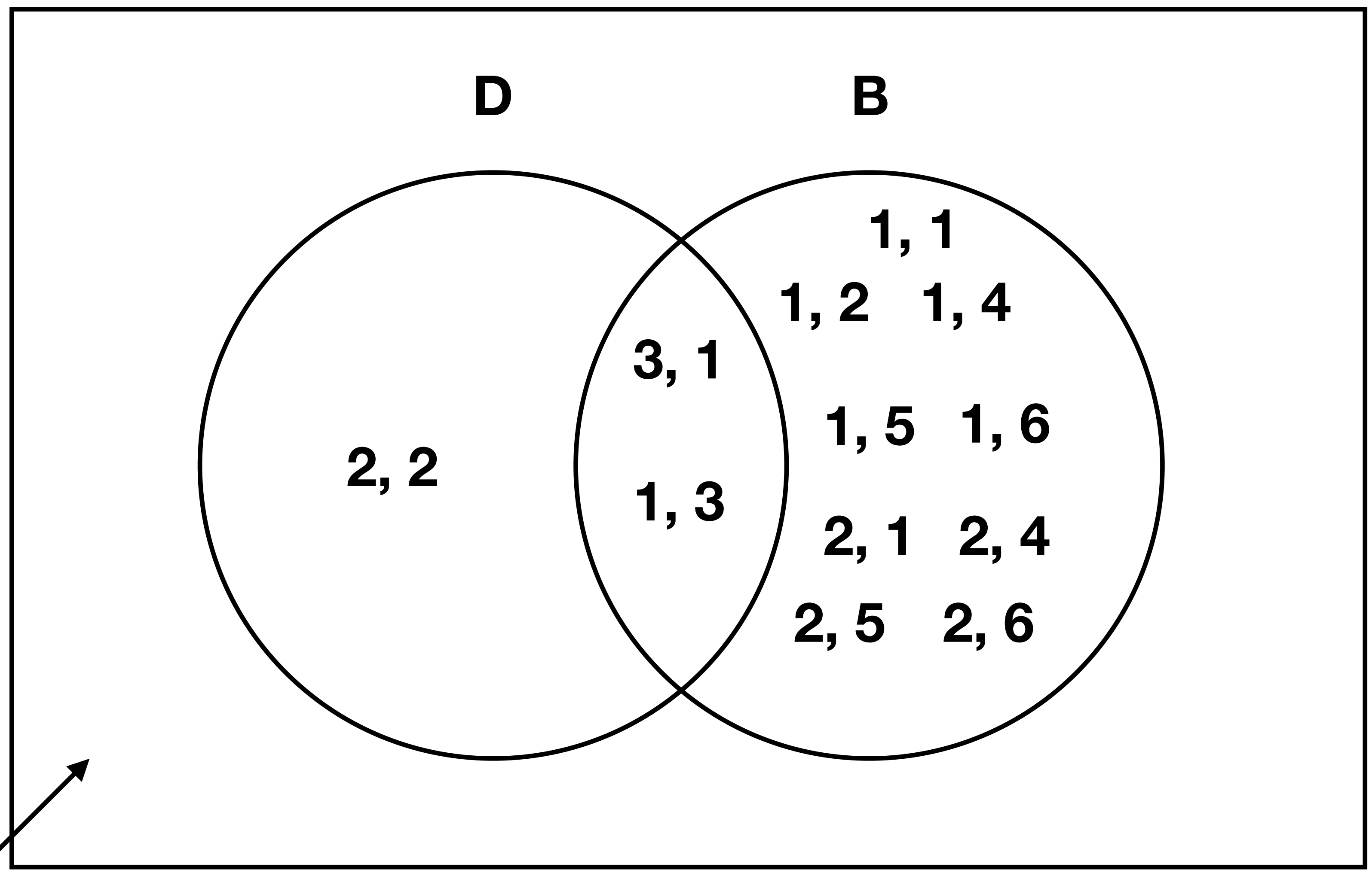
Because B is | or 'given', it becomes the denominator

$$P(A \mid C) = \frac{P(A \cap C)}{P(C)} = 0$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1/36}{11/36} = 1/11^{*}$$

$$^{*}1/36 * 36/11 = 1/11$$

Conditional probability



The remaining 24 (out of 36) outcomes occupy this space

P(B | D)

Is 2/3 but why?

- D is a given
- The intersection is of B & D = 3,1 and 1,3
- The total outcomes are 3
- So it's 2/3 (66%)

$$P(B \mid D) = \frac{P(B \cap D)}{P(D)}$$

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Conditional probability

- A: The eldest child is a girl {GB, GG}
- B: Both of them are a girl {GG}
- C: one of the children is a girl {GB, GG, BG}

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{P(\{GG\})}{P(\{GB, GG)} = \frac{1/4}{1/2} = 1/2 \text{ or } 50\%$$

$$P(B \mid C) = \frac{P(B \cap C)}{P(C)} = \frac{P(\{GG\})}{P(\{GB, GG, BG)} = \frac{1/4}{3/4} = 1/3 \text{ or } 33\%$$

Conditional probability

The possibility of an event happening is the weighted sum of two theories:

$$P(A) = P(A \mid B) * P(B) + P(A \mid B^c)P(B^c)$$

For example: 80% * P(B) + 35% * P(B^c)

Bayes' Theorem (explained)

$$P(A \mid B) = \frac{P(B \mid A) * P(A)}{P(B)}$$

Example:

- 1% of the population has D (a disease)
- There is a medical test that returns a 'positive' for those who have D, 90% of the time
- However 5% of those who do not have D, also return a 'positive' test

- A: Person A has disease D
- B: Person A returns a 'positive' test result for D

Step 1: $P(B) = P(B \mid A) * P(A) + P(B \mid A^c)P(A^c)$

Step 2: $.9 * P(A) + .05 * P(A^c)$

Step 3: $.9 * .01 + .05 * .99 \quad = .058\ (5\%)$

Step 4: $\frac{.9 \ * .01}{.058} = .15\ (15\%)$

In other words, if you tested positive for this test, there is only a 15% chance you have D. Given these odds, no medical professional or institution would sanction this test.

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