Probability

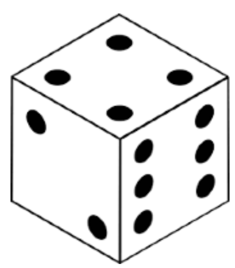
8.11.17

Probability

- Given an experiment with a random outcome, the “sample space” of the experiment is the set of all possibilities

Sets notation is represented as follows:
S = {1,2,3}

Notation for number of items in a set or it’s “cardinality” is represented as follows:
| S | = 3



- The outcome of rolling a single dice:
  - Sample space: D = {1,2,3,4,5,6}
  - Cardinality: I D | = 6

- Example: Tossing a coin:
  - Sample space C = {H, T},
  - Cardinality = | C | = 2

Tossing a Coin

- All possible outcomes of flipping two coins in sequence:
  - C2 = {HT, TH, HH, TT}
  - | C2 | = 4

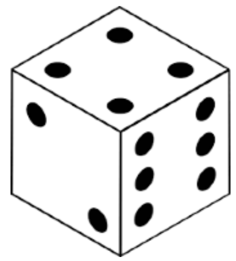
Rolling Two Dice

- The sum of the outcome of rolling two dice:
  - D2 = {2*, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
  - | D2 | = 11

N.B. Not the total number of possible outcomes - see later for that…

* the smallest possible outcome is 1 + 1 = 2

Flipping a Coin & Rolling a Dice



- The outcome of flipping a coin then rolling a dice:
    - F = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}
    - | F | = 12

Definitions

Intersection:	all common elements	$A \cap B =$	{5, 7}
		$C \cap D =$	∅ (null set symbol)
		$A \cap B \cap C =$	∅
Union:	the set of all elements of two sets	$C \cup D =$	{1, 2, 4, 6, 9, 10}
		$A \cup B =$	{3, 4, 5, 6, 7, 9, 10}
Compliment:	all elements that are not in a set	$A^c =$	{1,2,3,4,6,8}
		$A \cup A^c =$	this would equal the universe of all numbers
Subset:	a set is a subset of another set if each element of the first set is contained in the second set	$D \subseteq B$ $1 \in C$ $D \not\subseteq A$	$\subseteq$ = is an element of $\in$ = is an element of* $\not\subseteq$ = is not an element of

* only used for single integers

Definitions in Action

- Sets:
  - A = {5, 7, 9, 10}
  - B = {3, 4, 5, 6, 7}
  - C = {1, 2, 9, 10}
  - D = {4, 6}

- $1 \in C$ = 1 is a member of C (we use this for a single element only)
- {1,2} = C is not correct as this is a set. For a set, we do as follows:
  - $\{1,2\} \subseteq C$

$P(A^c) = 1 - P(A)$ the probability of something not happening is equal to 1 minus the probability of something happening

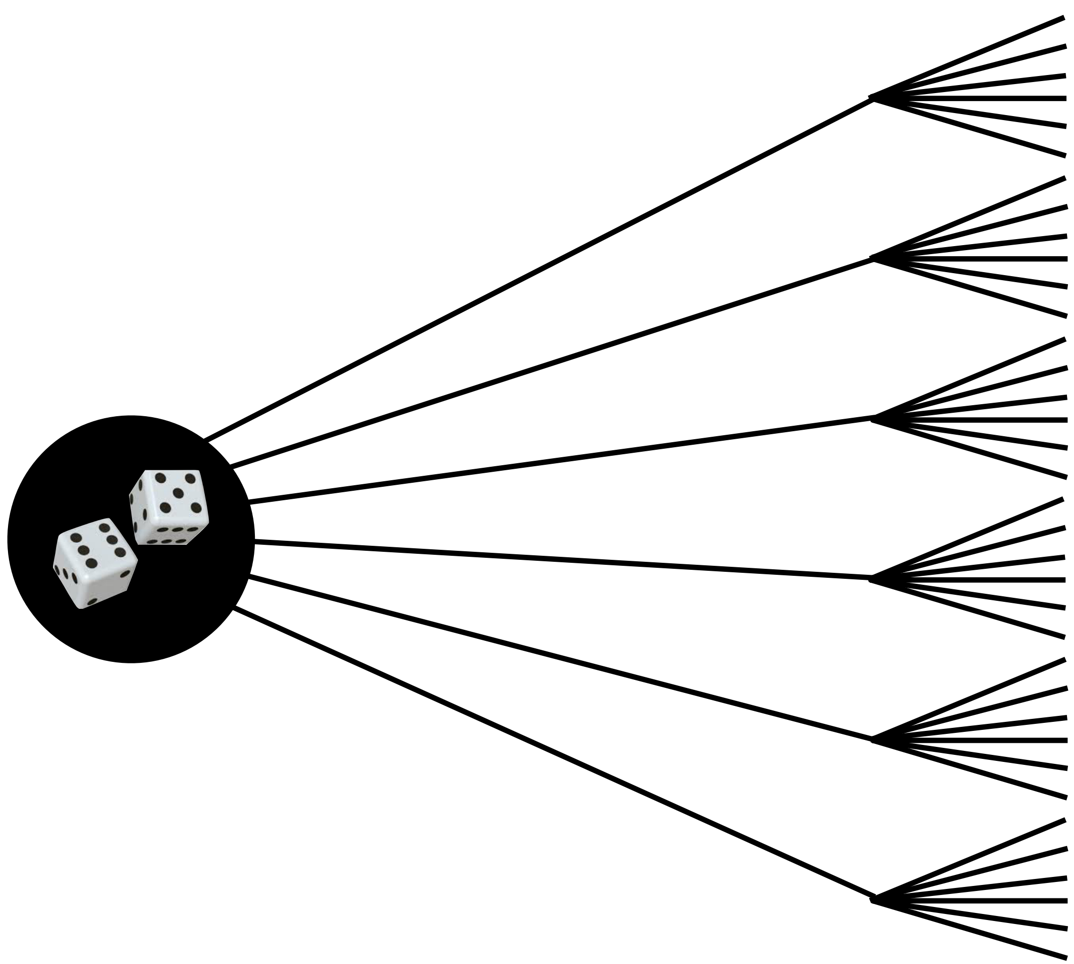
Probability Axioms

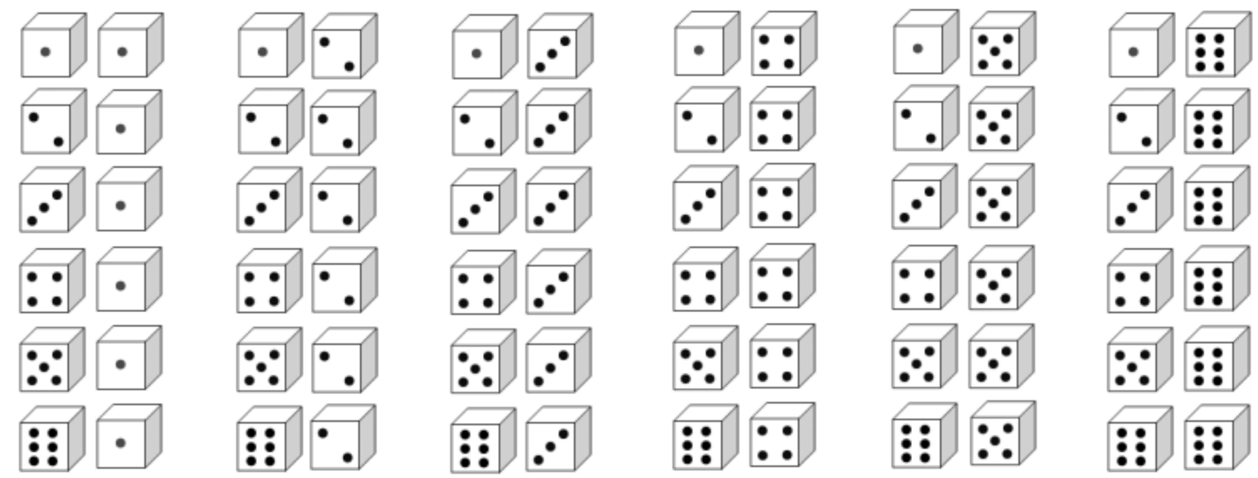
- **Definitions:**
  - A subset of probabilities from the sample space is called an "**event**":
    - Example: in tossing 2 coins, the following captures the event of having at least a single head (H): Q = {HH, HT, TH} - this is called an event as it captures all the possibilities
    - Example: both coins landing on the same side: B = {TT, HH}
  - **Probability**: given and event E, the probability of the event P(E) is the likelihood of the event happening:
    - R = {HH, TT}
      - P(R) represents the probability of R happening
- **3 axioms for probability:**
  - 1) For all events E: 0 <= P (E) <= 1 (the probability of this happening is between 0 and 1)
  - 2) P(U) = 1 (the probability of U always being equal to 1) i.e. something will always happen
  - 3) Given n disjoint events: E^1, E^2….E^n then the probability $P(E^1 \cup E^2 \cup \dots \cup E^n) = \sum_{i}^{n} P(E^2)$*

* what this means is that for this axiom to work, there can be no commonality between the events

Experiment of Rolling Two Dice

When rolling 2 dice, the maximum possibilities are 36 (6*6)





	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Experiment of Rolling Two Dice continued…

When rolling 2 dice, the maximum possibilities are 36 (6*6)

- E^1 : Both of them have an even number:
  - E^1 ={22, 24, 26, 42, 44, 46, 62, 64, 66} **Probability: 9/36**
- E^2 : The first number is a 2:
  - E^2 ={21, 22, 23, 24, 25, 26} **Probability: 6/36**
- E^3 : The first number is a 3:
  - E^3 ={31, 32, 33, 34, 35, 36} **Probability: 6/36**
- E^4 : The sum of the dice is less than 4:
  - E^4 ={11,12,21} **Probability: 3/36**
- E^5 : The sum is more than 10:
  - E^5 ={56, 65, 66} **Probability: 3/36**
- E^6 : The sum of the dice is equal to 11:
  - E^6 ={56, 65} **Probability: 2/36**

- **P($E^4 \cup E^5$):**
  - P({11,12,21} ∪ {56,65,66}) or;
  - P({11,12,21,56,65,66})
  - = 6/36 (16.7% probability)
- **E^4 is disjoint from E^5 because:**
  - $E^4 \cap E^5 = \varnothing$
  - Because they are disjoint, we must apply the 3rd axiom
  - Using the 3rd axiom of probability we have P(E^4) + P(E^5) = 6/36 (16.7% probability)
- **P(E^5 U E^6):**
  - P({56,65,66} u {56,65})
  - P({56,65,66}) or 3/36 (8.3% probability)

In Summary…

1 = P(U) *(axiom 2)*

P(U) = P(A u A^c) *(by definition)*

P(Au Ac) = P(A) + P(Ac) *(axiom 3)*

Therefore:
1 = P(A) + P(A^c) or 1-P(A) = P(A^c)

I.e. the probability of something not happening is equal to 1 minus the probability of something happening!

P(A^c) = 1- P(A)

U = Universe