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MATH 309 Assignment 1

6/6 1. We can prove the contrapositive for the forward implication. That is, "if $(0 \le m$ and $0 \le n)$ and $(m \le 0$ and $n \le 0)$ then $0 \le mn$".

Suppose $0 \le m$ and $0 \le n$. Then, by definition, we have $0 \le mn$, so we are done.

Suppose $m \le 0$ and $n \le 0$. Then $m + (-m) \le 0 + (-m)$ and therefore $0 \le -m$. Similarly, $0 \le -n$. Then $0 \le (-m)(-n) = (-1)m(-1)n = (-1)(-1)mn = mn$. So we are done.

For the remaining direction, we wish to prove "if $0 < m$ and $n < 0$ or if $m < 0$ and $0 < n$ then $mn < 0$."

Suppose $0 < m$ and $n < 0$. ✓ Then $n + (-n) < 0 + (-n) = -n$. Thus $0 < m(-n)$. Thus $0 + mn < m(-n) + mn = m(-1)n + mn = -mn + mn = 0$. Therefore $mn < 0$.

The proof for $m < 0$ ✓ and $0 < n$ is identical by relabeling which was m and which was n. As such, the biconditional is proved.

2. We need to prove "if $mn = 1$ for some m, n then $m = n = 1$ or $m = n = -1$" and also "if $m = n = 1$ or $m = n = -1$ then $mn = 1$."

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Starting with first direction, suppose $m, n \in \mathbb{Z}$ and $m = n = 1$. By multiplication, we have $m \cdot n = 1 \cdot 1 = 1$

Omitted the case m=n=-1

Secondly, suppose $mn = 1$, $m, n \in \mathbb{Z}$. Then $m = \frac{1}{n}$. But for n an integer, then m is a fraction. Then only way both are integers is if $n = 1$ so that $m = 1$, or if $n = -1$ so that $m = -1$. ✓

Thus, ~~th~~ both directions are shown and biconditional is proved.

3. We can provide a counter example. Let $a=0$ and b, c be integers such that $b \neq c$ (by example $b=1$, $c=2$). Then $ab = 0 \cdot 1 = 0$, $ac = 0 \cdot 2 = 0$ and $ab = ac$, but $b \neq c$.
As such, by proving negation true, we proved original statement false.

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4. A composite number is a positive integer that can be expressed by multiplying two smaller positive integers, by definition.

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Let $n > 1$ be a composite integer such that it does not have a prime less than or to $\sqrt{n}$. Since n is a composite integer, there are integers m, k such that $n = mk$. In addition, if $m > \sqrt{n}$, $k > \sqrt{n}$. Then

$$n = mk > \sqrt{n}\sqrt{n} = n$$

Still need to show one of them is a prime or has prime factors.

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Thus we have $n > n$ which is impossible so our assumption is not true.

5.
$$2475 = 3 \cdot 825 = 3^2 \cdot 275 = 3^2 5^1 55 = 3^2 5^2 11 = 3^2 5^2 11^1 23^0$$

$$-1035 = 3 \cdot (-345) = 3^2 (-115) = 3^2 5 (-23) = 3^2 5^1 11^0 (-23^1)$$

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Then $\gcd(2475, -1035) = \gcd(2475, +1035) = 3^2 5 \cdot 11^0 \cdot 23^0 = 3^2 \cdot 5 = 45$

MATH 309 Assignment 1 Part 2

6. "If $\gcd(v,m)=1$, then $1 = vs + mt$ for some integers s, t"
"If $v \mid mn$, then $vs = mn$ for some integer s".

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For each condition, different $s = s_1$ and $s = s_2$ should be selected, which do not neccessarily have to be the same.

7. Let $v=2$, $m=3$ and $n=4$. Then $mn=12$ and $v \mid mn$,

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$\gcd(v,m) = \gcd(2,3) = 1$ but $m \neq \pm 1$.

8. The statement $P(n)$ is incorrect. Observe

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$2+4+\dots 2n = 2(1+2\dots n) = 2\frac{(n+1)}{2}n = (n+1)n = n^2+n \neq (n+2)(n+1)$

You still need to verify to see if it is true or not.

The student also did not test the base case. I would give student 2/6 as he did the correct transformation of expressions

9. Let $P(n): 2n+1 \leq 2^n$. Show $P(3)$ is true

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Base. $P(3): 2\cdot 3+1 = 7 < 8 = 2^3$ ✓

IH: Show for $k \geq 3$, if $P(k)$ is true then $P(k+1)$ is true.

$\boxed{P(k): 2k+1 \leq 2^k}$, $p(k+1): 2(k+1) \leq 2^{k+1}$. Then

+1 ?

$2(k+1) = 2k+2+1$

$\leq 2^k + 2$ (hypothesis)

$\leq 2^k + 2^k$

$= 2\cdot 2^k$

$= 2^{k+1}$

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10. $p \wedge (p \to q) \wedge r \equiv p \wedge (\neg p \vee q) \wedge r \equiv ((p \wedge \neg p) \vee (p \wedge q)) \wedge r$

$\equiv \cancel{F} \vee (p \wedge q) \wedge r \equiv \boxed{p \wedge q \wedge r}$

The condition "$[p \wedge (p \to q) \wedge r] \to [(p \vee q) \to r]$ is false" is possible when $p \wedge (p \to q) \wedge r$ is true and $(p \vee q) \to r$ is false. But if $p \wedge (p \to q) \wedge r$ is true ($p \wedge q \wedge r$ is true) then p, q, and r must be true. Then $p \vee q$ is true and so $(p \vee q) \to r$ is true. Need to state clearly and conclude that you get a contradiction.

Hence, ~~for any~~ p, q, and r, the logical statement

$[p \wedge (p \to q) \wedge r] \to [(p \vee q) \to r]$ always is true.

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11. a) The argument can be expressed as

$p \to q$
$\neg q$
$\therefore \neg p$

This is a valid argument with the rule "modus tollens".

b) The argument is not valid as $p \to q \not\equiv \neg p \to \neg q$. Inverse of a statement does not necessarily have same truth value as the original conditional by definition. Thus if p, then q, we cannot conclude if not p, then not q.

MATH 309 Assignment 1 Part 3

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1.	$p \wedge q$	(premise)
2.	p	(conjunctive simplification)
3.	$p \to (r \wedge q)$	
4.	~~$\neg p \vee (r \wedge q)$~~	~~(definition)~~
5.	$(r \wedge q)$	(disjunctive syllogism, step 2)
6.	r	(conjunctive simplification)
7.	$r \to (s \vee t)$	(premise)
8.	~~$\neg r \vee (s \vee t)$~~	~~definition~~
9.	$s \vee t$	(disjunctive syllogism, step 6)
10.	$\neg s$	premise
11.	$\therefore t$	(disjunctive syllogism, step 10)

−1 Two steps not needed

13. Let $L(x,y)$ be relation 'x loves y'. Then we have

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a) $\forall x, \exists y, L(x,y)$.

b) $\exists x, \forall y, L(x,y)$.

14. The statement is true

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Since $\gcd(5,7)=1$, we have $s,t \in \mathbb{Z}$ such that $1 = 5s + 7t$.
Then for any $x \in \mathbb{Z}$, we have $x = 5sx + 7tx$. Let $y = sx$ and $z = tx$.
Therefore, for any $x \in \mathbb{Z}$, there are $y, z \in \mathbb{Z}$ such that

$$x = 5y + 7z.$$

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15. "There is a unique element x in U with property P."

There are two ways this is false; either no element in U with property P or more than one does. We can express the first as

Still need to combine as one statement.

$$\forall x \; [x \in U \rightarrow (\neg P(x))]$$

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The second can be parsed as

$$\exists x \, \exists y \; (x \in U \wedge y \in U \wedge x \neq y \wedge P(x) \wedge P(y))$$

So together we have

$$\boxed{[\forall x \; [x \in U \rightarrow (\neg P(x))] \vee [\exists x \, \exists y \; (x \in U \wedge y \in U \wedge x \neq y \wedge P(x) \wedge P(}$$

16. ~~Let C(x) be proposition "x is composite" and P(x) be proposition "x is prime".~~

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~~Then we have $\forall n \in \mathbb{Z}^+ \; \exists p \; [C(n) \wedge P(p) \rightarrow p | n]$.~~

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Need to check my emails, also read news forums. It shoul be P38, theorem 1.33. You may resubmit this one again for marks.