GEOL 415 Assignment 4

1. The three main solar system characteristics an acceptable model should account for are: distinction between terrestrial and gas giants, spacing of these planets orbits and how the planetary mass is distributed with heliocentric distance.

2. a) The paper outlines the conditions in which dust particles will stick or not to an accretion nucleus. The nucleus captures all dust particles which cross its orbit and plus also dust particles whose orbits fall in an unstable region related to the accretion nucleus's gravitational cross section. The equation for this radial extent region around the accretion nucleus is given by $x = r\mu^{1/4}$ where r is distance of nucleus from centre of nebula and μ is the reduced mass of the nucleus with respect to the Sun. The authors further specify $\mu = m/(1+m)$, m being the nuclear mass expressed in solar masses

   b) Initially, the accretion nucleus only collects dust particles. Once their masses and escape velocities are high enough, they are able to collect gas well with some additional requirements. To start, we require $v_e > v_g$, escape velocity greater than gas velocity. This translates to $C_1 m_c^{1/3} = C_2 r^{-1/4}$ where m_c is critical mass and r is perihelion distance of planetoid's orbit. m_c can be expressed as $C_3 r^{-3/4}$ where all Cs are constants of proportionality and C_3 in specific was found to be $\sim 10^{-5}$. Once these conditions are met i.e. critical mass is reached, the nucleus will accrete some gas along with dust.

3. a) First, calculate μ which is given by m(1+m). Substitute for m as $3 \times 10^{-8}\ M_\odot$ gives us $\sim 3 \times 10^{-8}$ for μ (notice μ is unit-less). Then, $x = r\mu^{1/4}$ where we substitute 1 and 3 AU for r. For r = 1 AU, we get x = 0.01316 AU and for r = 3 AU, we get 0.03948 AU.

   b) Recall that x is a radial distance from the centre of the accretion nucleus that 'sweeps' dust particles. When x is small, i.e. when a value of 1 AU is used for r, that means that the gravitational cross section of the unstable region around the accretion nucleus is small. In the small x case, it is more likely that terrestrial planets will form in very low mass solar nebulae. However, in the larger x case, terrestrial planets as well as Jovian planets will form. As a matter of fact, the converse never occurs; Jovian planets cannot solely exist whereas terrestrial planets can. This is why a larger r value, which results in a larger x value, is needed in the formation of Jovian planets since they are required to sweep up a larger volume of dust and gas.

4. The gas to dust mass ratio is given by $K = p_g/p_d$ where p_g and p_d are radially dependant densities of gas and dust. The K was a constant in the model that could vary and change in gas to dust mass ratio were looked at. For smaller K values i.e. decreased amounts of gas, the terrestrial planets (black circles on figure 3) have smaller masses. The Jovian planets for small K are smaller themselves in radius and mass. For large K such as K = 100, there is actually an additional terrestrial planet that has formed and the Jovian planets are much larger as well. The terrestrial planets are also just slightly more bigger for large K in the same figure.

5. a) To understand what the constant A represents in that expression, we must understand what the expression itself represents. ρ_d represents the density of dust with respect to distance, r. Changing r has a direct relationship with how density of dust falls. As such, the parameter A in specific controls scaling the mass in the nebulous cloud.

   b) These are the bounds (0.02 - 0.2 $M_\odot$) for the nebular mass because they create models of the solar system that are accurate and relevant to the ones we are interested in. If we use a value of A below 0.02, there will only be terrestrial planets generated and eventually if we go even lower, only asteroids. This is not of interest to us. Conversely, if we try to use values over 0.2, the program model will generate double or even multi star systems. The original author of the program, Dole, states that this pushes the ACRETE software beyond the limits and it was not intended to model multi star systems.

   c) The explanation given by Isaacman and Sagan is related to the software they are using, appropriately named ACRETE. ACRETE assumes that all of the dust in the solar nebula is accreted whereas in reality that is not necessarily the case. Because of this, models are able to be generated using smaller nebular mass values ranging from 0.02 - 0.2 $M_\odot$ since it is assumed all of that mass is used. Another reason is because Isaacman and Sagan use their nebular mass values *after* the generation of the initial accretion nuclei and don't include that. This is a key difference with how other researchers evaluate their solar nebula mass. As such, this also provides an explanation to why their nebular masses are lower.

Therefore, this discrepancy is not problematic given how they evaluate their total solar nebula mass is slightly different than other astronomers.

6. Changing the eccentricity of the dust particles to higher values will result in more cross orbits with the nucleus which results in more accumulation of dust particles. This leads to the generation of more massive planets. However, while there will be more massive planets with higher ε , fewer of them will be in the given system. On the other hand, smaller eccentricity values yield more planets that are relatively of smaller bodies.

7. The model is extremely sensitive to the changes in distribution of density in the nebula. As per Isaacman and Sagan's conclusions, we have seen that greater distribution of density in the nebula leads to more massive planets. Terrestrial and Jovian planets are more massive given higher density distributions. As a matter of fact, given an even higher density distribution, it will yield multi star systems. However, if the density distribution of the nebula is low, the accretion nucleus will not be able to 'sweep' as much. This results in smaller/fewer amounts of planets being formed. If the density distribution is small enough, Jovian planets will not even form and only terrestrial planets will. If the value is even smaller, only asteroids will appear. As such, the model is very sensitive to changes in the distribution density of the initial nebula. It is one of the defining characteristics after all of how a solar system model is to be.

8. The authors explain this by a sort of 'collisional natural selection'. What is happening is the accreting nucleus has continuous collisions to the point where accreting bodies with interacting orbits emerge. Accreting bodies are expected to alter the orbits of nearby dust particles within a certain radius, in a way can be visualized as 'sweeping' the dust. As such, there are spacings created since the dust is 'swept' and accreting bodies i.e. planets form the orbital spacing between each other. This happens even for unusual planetary systems by what was mentioned previously, self correcting 'natural selection collision' type behaviour.

   The final configuration then shows that the planets naturally space themselves apart from one another 'nicely' which is essentially what the Titius-Bode, or a similar type of law, states.

Reference material:

ICARUS 31, 510–533 (1977)

Computer Simulations of Planetary Accretion Dynamics: Sensitivity to Initial Conditions

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Received November 11, 1976; revised January 3, 1977