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MATH 309 Assignment 2

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1. a) $A \subseteq B$ b) $A \cap \bar{B} = \emptyset$ c) $\bar{A} \cup B = \mathcal{U}$

These statements are equivalent if they imply eachother.

a) $\Rightarrow$ b)

Assume a) is true. Then let x be an arbitrary element in A. Then $x \in B$ since A is a subset. This implies $x \notin \bar{B}$. As such, by definition, the set $A \cap \bar{B}$ is always empty and $A \cap \bar{B} = \emptyset$.

b) $\Rightarrow$ c)

Assume b) is true. We use de Morgan's Laws.

$$\bar{A} \cup B = \overline{\overline{\bar{A} \cup B}} = \overline{A \cap \bar{B}} = \overline{A \cap \bar{B}} = \bar{\emptyset} = U$$

c) $\Rightarrow$ a)

Assume c) is true. We have $\bar{A} \cup B = U$. Let x be an arbitrary element in A. Then $x \notin \bar{A}$. Since $\bar{A} \cup B = U$, x must be in B, $x \in B$.

Hence $A \subseteq B$.

Since all statements imply eachother, they are equivalent.

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2. Recall power set of any set S is the set of all subsets of S.

Let $X \in P(A \cap B)$. Then $X \subseteq A \cap B$. Thus $X \subseteq A$ and $X \subseteq B$. Then $X \in P(A)$ and $X \in P(B)$. Hence, $X \in P(A) \cap P(B)$. ✓

Let $X \in P(A) \cap P(B)$. Then $X \in P(A)$ and $X \in P(B)$. Thus ✓ $X \subseteq A$ and $X \subseteq B$. Then $X \subseteq A \cap B$. Hence, $X \in P(A \cap B)$.

Hence, $P(A \cap B) = P(A) \cap P(B)$ as shown.

3. This statement is false. Consider the counter example:

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Let $A = \{a\}$, $U = \{a, b\}$. Then $P(A) = \{\emptyset, \{a\}\}$, $\bar{A} = \{b\}$, $\boxed{P(\bar{A}) = \{\emptyset, \{b\}\}}$, $P(U) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ and then

$\boxed{P(U) - P(A) = \{\{b\}, \{a, b\}\}}$ Hence, $P(\bar{A}) \neq P(U) - P(A)$.

4. This statement is also false. Consider the counter example:

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Let $A = \{a, b\}$ and $B = \{b, c\}$. Then $A - B = \{a\}$ and $\boxed{P(A - B) = P(\{a\}) = \{\emptyset, \{a\}\}}$. Then we have

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$, $P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$ and

$\boxed{P(A) - P(B) = \{\{a\}, \{a, b\}\}}$. Thus, $P(A - B) \neq P(A) - P(B)$.

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5. We create a diagram to visualize

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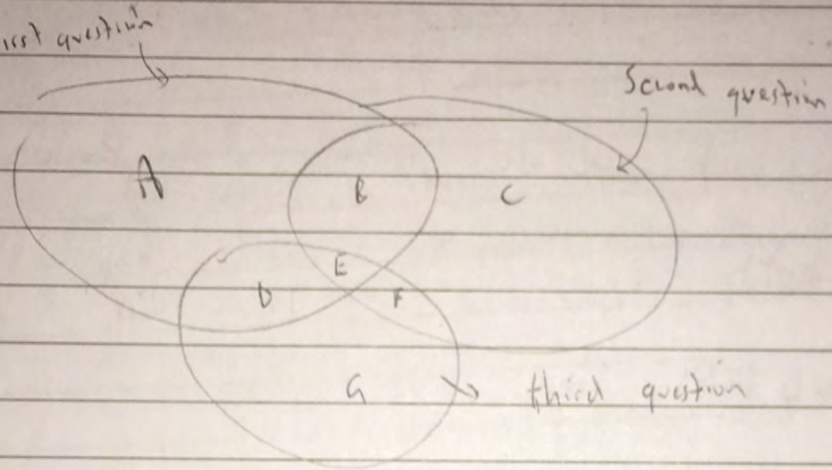

It is given that

$C + F + G = 10$ (did not answer first)

$A + D + G = 14$ (did not answer second)

$A + B + C = 12$ (did not answer third)

Adding equations : $\boxed{2A + 2C + 2G + F + D + B = 36 \quad (1)}$

We also have : $A + C + G + F + B + D + E = 40$

So then $\boxed{A + C + G + F + B + D = 22 \quad (2)}$ because $E = 18$.

Then we subtract (2) from (1) to get

$A + C + G = \boxed{14}$ ✓

Thus, 14 students answered only one question, be it either only 1st, 2nd or 3rd question but only 1.

6. Let $A = \{a, b\}$. List four elements of power set $P(A)$.

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$P(A) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

a) To explain why we use axioms of Boolean Algebra.

B1: identity

Let $x \in P(A)$. Then

$x + 0 = x \cup \emptyset = x$, $x \cdot 1 = x \cap A = x$

B2: Complement

Let $x \in P(A)$. Then

$x + \bar{x} = x \cup \bar{x} = A = 1$, $x \cdot \bar{x} = x \cap \bar{x} = \emptyset = 0$

B3: Commutativity

Let $x, y \in P(A)$. Then

$x + y = x \cup y = y \cup x = y + x$, $x \cdot y = x \cap y = y \cap x = y \cdot x$

B4: Distributivity

Let $x, y, z \in P(A)$. Then

$x \cdot (y + z) = x \cap (y \cup z) = (x \cap y) \cup (x \cap z) = (x \cdot y) + (x \cdot z)$

$x + (y \cdot z) = x \cup (y \cap z) = (x \cup y) \cap (x \cup z) = (x + y) \cdot (x + z)$

Hence this description defines Boolean Algebra.

6. i) Let $x = \{a\}$ and $y = \{b\}$.

We have then $x \neq \emptyset$ and $y \neq \emptyset$.

We also have $xy = x \cap y = \{a\} \cap \{b\} = \emptyset$

7. There are four ways to choose three aces. If aces chosen, 8/8 can not form pair. Then, it is possible for 12 kinds of pairs. For each suit, there are $\binom{4}{2} = 6$ ways to select pairs. Hence, we have

$$4 \cdot 12 \cdot \binom{4}{2} = 48 \cdot 6 = \boxed{288}$$

$\boxed{288}$ ways to select three aces and a pair.

b. There are 13 three of kind. For each suit, 4 ways to select three of kind. If the kind is chosen, can not form pair. Therefore, 12 possible kinds of pairs. For each suit, $\binom{4}{2} = 6$ ways to select pairs. Hence, we have

$$13 \cdot 4 \cdot 12 \cdot \binom{4}{2} = 13 \cdot 4 \cdot 12 \cdot 6 = \boxed{3744}$$

$\boxed{3744}$ ways to ~~sel~~ select three of a kind and a pair.

8. $(2w - x + y - 2z)^8 = \sum_{k=0}^{8} \binom{8}{k} (2w-x)^{8-k} (y-2z)^k$

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$$= \sum_{k=0}^{8} \binom{8}{k} \left(\sum_{i=0}^{8-k} \binom{8-k}{i} (2w)^{8-k-i} (-x)^i \right) \left(\sum_{j=0}^{k} \binom{k}{j} y^{k-j} (-2z)^j \right)$$

$$= \sum_{k=0}^{8} \sum_{i=0}^{8-k} \sum_{j=0}^{k} \binom{8}{k} \binom{8-k}{i} \binom{k}{j} (2w)^{8-k-i} (-x)^i y^{k-j} (-2z)^j$$

$$= \sum_{k=0}^{8} \sum_{i=0}^{8-k} \sum_{j=0}^{k} \binom{8}{k} \binom{8-k}{i} \binom{k}{j} (-1)^{i+j} 2^{8-k-i+j} w^{8-k-i} x^i y^{k-j} z^j$$

Then by condition

$8-k-i=1$

$i=3$

$k-j=2$

$j=2$

Then $\boxed{j=2, \quad i=3, \quad k=4}$

Thus coefficient is

$$\binom{8}{4}\binom{8-4}{3}\binom{4}{2}(-1)^{3+2} 2^{8-4-3+2} = \binom{8}{4}\binom{4}{3}\binom{4}{2}(-1)^5(2^3) = -2^3\left(\frac{8!}{4!4!}\right)\left(\frac{4!}{3!1!}\right)\left(\frac{4!}{2!2!}\right)$$

$$= -8 \frac{4 \cdot 5 \cdot 6 \cdot 7 \cdot 8}{2 \cdot 2} = \boxed{-13440}$$

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9. $\sum_{k=0}^{8} \binom{8}{k} = \sum_{k=0}^{8} \binom{8}{k} \cdot 1^{8-k} \cdot 1^{k} = (1+1)^8 = 2^8 = \boxed{256}$

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10. We have 2 scoops, 30 toppings and 3 cones. That means $30 \cdot 30 \cdot 3 = 2700$ different icecream combinations.
Now pick 12 but do not let any one of them be same.
This latter one is thus a combination in which order does not matter. So 2700 choose 12.

If assume order of flavors matters and can be the same

~~24x~~ $\binom{2700}{12} = \boxed{\cancel{3.0 \times 10^{32}}}$ (I use calculator)

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11. Recall $(a+b)^n$ has $n+1$ terms. So

$$(x+y+2)^{10} = (x+(y+2))^{10} = \sum_{k=0}^{10} \binom{10}{k} x^{10-k} (y+2)^k$$

Then the number of terms in expansion of $(x+y+2)^{10}$ is

How about use the method of combination with repetitions? You used a more elementary method.

$$\sum_{k=0}^{10} (k+1) = 1+2 \ldots 10+11 = \frac{1+11}{2} \cdot 11 = 6 \cdot 11 = \boxed{66}$$

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. The number of integer solutions of $x_1+x_2 \ldots x_5 = 32$ where $x_i \geq 3$ for $1 \leq i \leq 5$ is equal to number of integer solutions of

$$y_1+y_2+y_3+y_4+y_5 = 12 \text{ where } y_i \geq 0, \quad 1 \leq i \leq 5$$

because $4+4+4+4+4 = 20$ and $32-20 = 12$ (second eqn obtained from first eqn substitution $x_i = y_i + 4$, $1 \leq i \leq 5$).

Then number of integer solutions of last equation is

$$\binom{12+5-1}{12} = \binom{16}{12} = \boxed{1820} \checkmark \text{ (I use calculator)}$$

Hence, number of integer solutions of $x_1 + x_2 \dots x_5 = 32$, $x_i \geq 3$, $1 \leq i \leq 5$ is also $\boxed{1820}$

17. We write prime decomposition

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$$4312440 = 2 \cdot 2156220 = 2^2\, 1078110 = 2^3\, 539055 = 2^3\, 3\, 179685$$
$$= 2^3 3^2\, 59895 = 2^3 3^3\, 19965 = 2^3 3^4\, 6655 = 2^3 3^4 5^1\, 1331$$
$$2^3 3^4 5^1 11^1\, 121 = \boxed{2^3 3^4 5^1 11^3}$$

Hence, number of divisors of integer 4312440 are

$$(3+1)(4+1)(1+1)(3+1) = 4 \cdot 5 \cdot 2 \cdot 4 = \boxed{160} \checkmark$$