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PHIL 379 - Assignment 1

Definitions[1]:

A function *f* from set A to set B is an assignment for some or all elements a of A of an associated element *f(a)* of B. If *f(a)* is defined for every element a of A, then the function *f* is total. If every element b of B is assigned to some element a of A, then the function *f* is onto. If no element b of B is assignment to more than one element a of A, then function *f* is one-to-one. If a function is both onto and one-to-one, it is bijective. An inverse function f^{-1} from B to A is defined by letting $f^{-1}(b)$ be the one and only a such that *f(a)* = b.

Let *f* be a function from A to B and *g* be a function from B to C. Then the composite function *h* = *gf* from A to C is defined by *h(a) = g(f(a))*.

A correspondence between sets A and B is a one-to-one total function from A onto B. Two sets A and B are said to be equinumerous if and only if there is a correspondence between A and B.

With the above definitions, we can now proceed to answer question 1.

1. Let X, Y and Z be some sets.

A. Using identity function $1_X : X \to X$ where $1_X(x) = x$ for all $x \in X$, we see that X is equinumerous with itself (since 1_X is bijective)

B. Suppose X is equinumerous with Y. Then there is a bijective *f: X -> Y.* Since *f* is bijective, its inverse f^{-1} exists and is also bijective. Hence, f^{-1}*: Y -> X* is a bijective function from Y to X, so Y is also equinumerous with X.

C. Suppose that X is equinumerous with Y via the bijective function *f: X -> Y* and that Y is equinumerous with Z via bijective function *g: Y -> Z*. Then the composition of *g o f: X -> Z* is bijective and X is thus equinumerous with Z.

Having shown the above 3 properties, equinumerousity is an equivalence relation[2].

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[1] Boolos, G. S., Burgess, J. P., & Jeffrey, R. C. (2010). *Computability and logic*. Cambridge: Cambridge Univ. Press.

[2] Zach, Richard (2016). *Sets, Logic, Computation* University of Calgary: Univ. Press.

2. Let $A = \{A_1, A_2, A_3, \ldots\}$ be an enumerable family of sets. Then we have for each $A_i = \{a_{i1}, a_{i2}, a_{i3}, \ldots\}$. $\cup A$ is defined as the union of the family A, the set whose elements are precisely the elements of the elements of A.

$\cup A$ would then be the set containing all $a_{ij,}$ for all i and j. This can be enumerated as ordered pairs (i, j). First comes the pairs whose sum is 2, then the pairs whose sums are 3, then the pairs whose sums are 4, and so on. The following are some entries in the list

$$(1,1), (1,2), (2,1), (1,3), (2,2), (3,1), \ldots$$

And so we can apply this to define an enumeration

$$\cup A = \{a_{11,}\ a_{12,}\ a_{21,}\ a_{13,}\ a_{22,} \ldots\}$$

3. Given some denumerable alphabet $A_1, A_2, A_3, \ldots$ we can take as a code number for any finite string (or word)

$$A_{s1}A_{s2}A_{s3}\ldots A_{sk}$$

as the finite sequence of positive integers

$$(s_1, s_2, s_3, \ldots s_k),$$

often via Euclid's prime decomposition method, the code number being $2^{s1} * 3^{s2} * 5^{s3} * 7^{s4} * 11^{s5} \ldots$

(i) For the set of all two-letter words, we have $A_{s1}A_{s2}$, with s_1, s_2 being any positive integers. Via prime decomposition, we would have unique code numbers $2^{s1} * 3^{s2}$ for each finite 2 letter string/word and so an enumeration is shown.

(ii) For the set of all n-letter words, we have $A_{s1}A_{s2}A_{s3\ldots}A_{sn}$, with $s_1, s_2, s_3, \ldots s_n$ being any positive integers. Via prime decomposition, we would have unique code numbers $2^{s1} * 3^{s2} * 5^{s3} * \ldots n^{sn}$ until the nth prime, for each finite string of length n. Thus, an enumeration is shown.

(iii) For the set of all words, let the denumerable alphabet $A_1, A_2, A_3, \ldots$ be denoted as Σ. The set of all words would then be Σ*, which contains within it elements of the set of all zero letter words, the elements of the set of all one letter words, the elements of the set of all two letter words and so on. Since it contains within it countable elements, the set then is itself countably infinite by definition. A symbolization for the enumeration is also given;

$$\Sigma^* = \{\ \varepsilon, \quad \Sigma_0^1, \quad \Sigma_1^1, \Sigma_0^2, \quad \Sigma_2^1, \Sigma_1^2, \Sigma_0^3, \quad \ldots\ \}$$

4. We can consider the symbols '0' and '1' an alphabet of two symbols. The set of all finite strings composed of a two symbol alphabet is a subset of all finite strings composed of a denumerable symbol alphabet. Since the set of all finite strings composed of a denumerable symbol alphabet was shown to be enumerable in 3(iii), this subset is then also enumerable. This is because we can obtain a gappy listing of the elements of any subset B of A simply by erasing any entry in the list that does not belong to B, leaving a gap.

The set of all infinite strings of the two symbol alphabet '0' and '1' is not enumerable. We can prove this by contradiction. Assume first that it is enumerable. Consider the following table

	s_1	s_2	s_3	s_4
1	a_{11}	a_{21}	a_{31}	a_{41}
2	a_{12}	a_{22}	a_{32}	a_{42}
3	a_{13}	a_{23}	a_{33}	a_{43}
4	a_{14}	a_{24}	a_{34}	a_{44}

where s_i are infinite strings in the top row and a_{ij} are the entries of the list s_i in the jth index.

Consider the diagonal entries $a_{11}, a_{22}, a_{33}, a_{44} \dots$ which themselves form an infinite string. As such, that infinite string must belong to some column in the top row, given that the first row is a set containing all infinite strings.

Consider now an anti diagonal entries $a_{11} + 1$, $a_{22} + 1$, $a_{33} + 1$, $a_{44} + 1 \dots$ which also form an infinite string. And so this must also lie in the top row at some column. However, this presents a contradiction because at some column k, we have $s_{kk} = 1 + s_{kk}$.

So the entries of the anti diagonal forming an infinite string cannot lie in the top row, that being the set containing all infinite strings which then is a contradiction. And so the set of all infinite strings of the two symbol alphabet is not enumerable.