Name: ______________________

ID #: ______________________

PHIL 379 Lec 01
Logic II
Winter 2018

Assignment 4
DUE IN CLASS AT 11:00 AM ON MARCH 20, 2018

1. An interpretation M of a sentence S is a finite model of S if and only if S is true in M and |M| is finite. Show that the following sentence is satisfiable, but does not have a finite model. *(2 marks)*

$$(\forall x\exists yRxy \ \& \ \forall x\forall y\forall z\ ((Rxy \ \& \ Ryz) \rightarrow Rxz) \ \& \ -\exists xRxx)$$

2. A sentence is satisfiable if and only if there is an interpretation of it in which it is true, a sentence A implies a sentence B if and only if there is no interpretation of A and B in which A is true and B is false, and a sentence is valid if and only if it is true in all of its interpretations. Show that the decision problem for satisfiability is solvable if and only if the decision problem for implication is solvable, and the decision problem for implication is solvable if and only if the decision problem for validity is solvable. *(2 marks)*

3. Write the members of Δ and the sentence H for the following Turing machine and input 2. Explain informally why Δ implies a description of time 3. *(3+1marks)*

$q_1S_1Lq_2$, $q_2S_0Lq_3$, $q_3S_0S_1q_3$, $q_3S_1Rq_4$, $q_4S_0S_1q_4$, $q_4S_1Lq_5$

4. A two-place relation R on a set S is reflexive if and only if for any member x of S, Rxx; R is symmetric if and only if for any members x and y of S, (Rxy→Ryx); and R is transitive if and only if for any members x, y, z of S, ((Rxy & Ryz) → Rxz). R is an equivalence relation on S if and only if R is reflexive, symmetric, and transitive.
   If R is an equivalence relation on a non-empty set S, then for any member x of S, the equivalence class of x under R, with respect to S, $[x]_{R,}$ is the set of all members y of S such that Rxy.
   Show that $[x\}_R = [y]_R$ if and only if Rxy.
   (2 marks)