PHIL 379
Assignment 4

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1. In order to show that the sentence is satisfiable, all conjuncts must be true. To start, we assume there is at least one element a_0 in the domain of M. Then there needs to be some element a_1 such that Ra_0a_1. This is due to conjuncts ∀x∃yRxy and due to -∃xRxx, the elements must be different. There then needs to be an additional element a_2 such that Ra_1a_2. This a_2 cannot be a_1 because of -∃xRxx. It also cannot be a_0 because then we would have Ra_1a_0 and Ra_0a_1 and by transitivity we would have Ra_0a_0 which contradicts -∃xRxx. Due to the way the sentence is constructed, each additional element added must require a new element which cannot be any of the previous elements. Thus, the only way for the sentence to be satisfiable is to have an infinite domain. Finally, in an infinite sized domain, the way to make the conjunction true would be to let Rxy be the relation ‘x is greater than y’.

We can also show by contradiction that the sentence does not have a finite model. Suppose first that it does have a finite model, M, such that the domain is $\{a_1, a_2, \ldots, a_n\}$. Suppose $x \in M$. Then we can use induction to find $x_0 = x$ and x_{n+1} = some arbitrary $y \in M$ where Rxy. Since R is a transitive relation, Rx_ix_j if $i < j$. Since M is finite, there must be $i \neq j$ such that $x_i = x_j$. Then $x = x_i$ such that Rxx. This presents a contradiction with one of the conjuncts in the original sentence in a finite domain and so the domain must be infinite.

2. Suppose the decision problem for satisfiability is solvable. Now let S be a set of sentences: $S = \{T \cup A\}$ under some model M. Consider the set S satisfiable. Then, by definition, we have S is satisfiable if and only if -A is false. Then we have $A \vDash B$ if and only if A —> B by definition. So if we can solve the decision problem for satisfiability, we can solve the decision problem for implication.

Suppose the decision problem for implication is solvable. Then A —> B under some model M. Then, by definition of implication, there is no interpretation of A and B in which A is true and B is false. So if we can solve the decision problem for implication, we can solve the decision problem for satisfaction. Thus, that concludes the first biconditional.

Suppose the decision problem for implication is not solvable and show contradiction. Then there exists a model/interpretation in which -A is valid. This contradicts with the definition of validity because -A must be invalid. Since validity is much stronger than implication, there can’t exist a model in which implication is not solvable, because validity demands all models to be solvable. As such, if the decision problem for implication is solvable, then the decision problem for validity must be solvable.

Suppose the decision problem for validity is solvable. Then by definition, for every model, a sentence is true ie true in all of its interpretations. As such, every model implies and so because validity is stronger than implication, the decision problem for implication must be solvable or else it would contradict with the definition of validity. Thus, if the decision problem for validity is solvable, the decision problem for implication is solvable. Since we have shown both sides of the second biconditional, that concludes our proof.

3. The members of Δ:

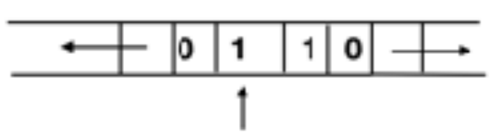
1) $\forall x \forall y \forall z((Sxy \wedge Sxz) \rightarrow y = z)$
2) $\forall x \forall y \forall z((Syx \wedge Szx) \rightarrow y = z)$
3) $\forall x \forall y(Sxy \rightarrow x < y)$
4) $\forall x \forall y \forall z((x < y \wedge y < z) \rightarrow x < z)$
5) $\forall x(x \not< x)$
6) D(0): $[Q_1 0 \wedge @00 \wedge M00 \wedge M01 \wedge \forall x((x \neq 0 \wedge x \neq 1) \rightarrow -M0x)]$
7) $\forall x \forall y[(Q_1 x \wedge @xy \wedge Mxy) \rightarrow \exists u \exists w(Sxu \wedge Swy \wedge Q_2 u \wedge \forall z((z \neq y \wedge Mxz) \rightarrow Muz) \wedge \forall z((z \neq y \wedge -Mxz) \rightarrow -Muz))]$
8) $\forall x \forall y[(Q_2 x \wedge @xy \wedge -Mxy) \rightarrow \exists u \exists w(Sxu \wedge Swy \wedge Q_3 u \wedge \forall z((z \neq y \wedge Mxz) \rightarrow Muz) \wedge \forall z((z \neq y \wedge -Mxz) \rightarrow -Muz))]$
9) $\forall x \forall y[(Q_3 x \wedge @xy \wedge -Mxy) \rightarrow \exists u(Sxu \wedge @uy \wedge Muy \wedge Q_3 u \wedge \forall z((z \neq y \wedge Mxz) \rightarrow Muz) \wedge \forall z((z \neq y \wedge -Mxz) \rightarrow -Muz))]$
10) $\forall x \forall y[(Q_3 x \wedge @xy \wedge Mxy) \rightarrow \exists u \exists w(Sxu \wedge Syw \wedge Q_4 u \wedge \forall z((z \neq y \wedge Mxz) \rightarrow Muz) \wedge \forall z((z \neq y \wedge -Mxz) \rightarrow -Muz))]$
11) $\forall x \forall y[(Q_4 x \wedge @xy \wedge -Mxy) \rightarrow \exists u(Sxu \wedge @uy \wedge Muy \wedge Q_4 u \wedge \forall z((z \neq y \wedge Mxz) \rightarrow Muz) \wedge \forall z((z \neq y \wedge -Mxz) \rightarrow -Muz))]$
12) $\forall x \forall y[(Q_4 x \wedge @xy \wedge Mxy) \rightarrow \exists u \exists w(Sxu \wedge Swy \wedge Q_5 u \wedge \forall z((z \neq y \wedge Mxz) \rightarrow Muz) \wedge \forall z((z \neq y \wedge -Mxz) \rightarrow -Muz))]$

The sentence H: $[\exists x \exists y(Q_1 x \wedge @xy \wedge -Mxy) \vee \exists x \exists y(Q_2 x \wedge @xy \wedge Mxy)]$.

The reason why the above members of Δ imply a description of time 3 is because of the description of D(0) and the instructions themselves in Δ. If the machine has not yet halted at time 3, then the members in Δ along with specifically D(0) make it so that Δ implicates time 3.

Turing machine T:

Input of 2 at time 0, starting at state 1



4. Suppose first the equivalence class $[x]_R$ is equal to the equivalence class $[y]_R$ for some equivalence relation R, $[x]_R = [y]_R$. Then since $x \in [x]_R$, $x \in [y]_R$ also due to our supposition. Then $x \in [y]_R$ means that $(x,y) \in R$ or equivalently Rxy.

Conversely now suppose $(x,y) \in R$ or equivalently Rxy. Consider any $z \in [y]_R$. By definition, Ryz and since R is transitive, we have Rxz so $z \in [x]_R$ and so $[y]_R \subseteq [x]_R$. Similarly, consider any $z \in [x]_R$. By definition, Rxz and since R is transitive, we have Ryz so $z \in [y]_R$ and so $[x]_R \subseteq [y]_R$. By showing $[x]_R \subseteq [y]_R$ and $[y]_R \subseteq [x]_R$, we have shown $[x]_R = [y]_R$. Thus since both sides of the biconditional are shown, the proof is complete.