83/100

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MATH 309 Assignment 3

8/8

1\. a) $A \times B = \{(a,1),(a,2)\}$ ✓ b) $P(A \times B) = \{\emptyset, \{(a,1)\}, \{(a,2)\}, \{(a,1),(a,2)\}\}$ ✓

c) $P(A) = \{\emptyset, \{a\}\}$, $P(B) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$ ✓

d) $P(A) \times P(B) = \{\emptyset, \{a\} \times \{1\}, \{a\} \times 2, \{a\} \times \{1,2\}\}$ ✓
$= \{\emptyset, \{(a,1)\}, \{(a,2)\}, \{(a,1),(a,2)\}\}$

2\. a) R is a function if ① For all $x \in \mathbb{Z}^+$, there is $y \in \mathbb{Z}^+$ such that xRy
② For all $x,y,z \in \mathbb{Z}^+$, if xRy and xRz then $y = z$

12/12

In our case, second condition is not true. Counterexample $x=2$, $y=2$, $z=4$ then xRy and xRz but $2 \neq 4$, $y \neq z$. ✓

b) $A = \{m \in \mathbb{Z}^+ \mid m \mid 52\} = \{1,2,4,13,26,52\}$, $|A| = 6$ ✓

c) $B = \{n \in \mathbb{Z}^+ \mid 52 \mid n\} = \{52, 104, 156, \ldots\} = \{52k \mid k \in \mathbb{Z}^+\}$. ✓

d) $\{1,2,4,13,26,52\} \cap \{52k \mid k \in \mathbb{Z}^+\} = \{52\} \neq \emptyset$. ✓ No, $A \cap B \neq \emptyset$

3\. a) i $P_3 = (5\ 1\ 4\ 2\ 3) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 4 & 2 & 3 \end{pmatrix}$

10/10

$P_3^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 4 & 2 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} 5 & 1 & 4 & 2 & 3 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 3 & 1 \end{pmatrix} = \boxed{(2\ 4\ 5\ 3\ 1)}$ ✓

ii $P_3 \circ P_1 = (5\ 1\ 4\ 2\ 3) \circ (3\ 4\ 1\ 2\ 5) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 4 & 2 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 1 & 2 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 1 & 3 \end{pmatrix} = (4\ 2\ 5\ 1\ 3)$ ✓

$(P_3 \circ P_1)^{-1} = \begin{pmatrix}1&2&3&4&5\\4&2&5&1&3\end{pmatrix}^{-1} = \begin{pmatrix}4&2&5&1&3\\1&2&3&4&5\end{pmatrix} = \begin{pmatrix}1&2&3&4&5\\4&2&5&1&3\end{pmatrix} = \boxed{(4\,2\,5\,1\,3)}$ ✓

iii $P_1 = (3\,4\,1\,2\,5) = \begin{pmatrix}1&2&3&4&5\\3&4&1&2&5\end{pmatrix}$, $P_1^{-1} = \begin{pmatrix}1&2&3&4&5\\3&4&1&2&5\end{pmatrix} = \begin{pmatrix}3&4&1&2&5\\1&2&3&4&5\end{pmatrix} = \begin{pmatrix}1&2&3&4&5\\3&4&1&2&5\end{pmatrix}$

$P_1^{-1} \circ P_2 = \begin{pmatrix}1&2&3&4&5\\3&4&1&2&5\end{pmatrix} \circ \begin{pmatrix}1&2&3&4&5\\3&5&1&2&4\end{pmatrix}$, $\begin{matrix}1&2&3&4&5\\1&5&3&4&2\end{matrix} = \boxed{(1\,5\,3\,4\,2)}$ ✓

b) $P_1 \circ P_2 = \begin{pmatrix}1&2&3&4&5\\3&4&1&2&5\end{pmatrix} \circ \begin{pmatrix}1&2&3&4&5\\3&5&1&2&4\end{pmatrix} = \begin{pmatrix}1&2&3&4&5\\1&5&3&4&2\end{pmatrix} = (1\,5\,3\,4\,2)$ ✓

$P_2 \circ P_1 = \begin{pmatrix}1&2&3&4&5\\3&5&1&2&4\end{pmatrix} \circ \begin{pmatrix}1&2&3&4&5\\3&4&1&2&5\end{pmatrix} = \begin{pmatrix}1&2&3&4&5\\1&2&3&5&4\end{pmatrix} = (1\,2\,3\,5\,4)$

So $\boxed{15342 \neq (12354) \text{ thus } P_1 \circ P_2 \neq P_2 \circ P_1}$

4 By definition, $[3][4] = [3 \cdot 4] = [12]$ but $[12] = [2]$ in Z_5 because 2 is remainder of division of $\cancel{12}$ by 5.

6/6

5. $A^2 = \{(a,a),(a,b),(a,c),(a,d),(b,a),(b,b),(b,c),(b,d),(c,a),(c,b),(c,c),(c,d),(d,a),(d,b),(d,c),(d,d)\}$.

A relation R on A^2 which is reflexive, symmetric and not transitive is

5/9 $R = \{(a,a),(a,b),(a,d),(b,a),(b,b),(c,c),(d,a),(d,d)\}$

Need to give examples rather than say things in general(copy definitions).
Also, it is actually transitive, so this is not correct.

① Relation is reflexive because each element is related to itself. −3, −1

② Relation is symmetric because when one element is related to second element the second element is related to first.

Relation is not transitive because bRa and aRd but not bRd.

MATH 309 Assignment 3

6. Partitioning of A into subsets results in relation

6/6

$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (1,3), (3,1), (1,5), (5,1), (3,5), (5,3), (4,6), (6,4)\}$

We can see by inspection

$a, b \in A,\ aRb \iff a, b \in A_1$ or $a, b \in A_2$ or $a, b \in A_3$

7. a) $A \cap B = \{00, 11, 000, 111, 0000, 1111\}$ b) $A \cap C = \{1, 0, 00, 11\}$

13/16

c) $B \cup C = \Sigma^*$ d) ~~$A \oplus B = \Sigma^1 \cup \{w \in \Sigma^* \mid 2 \le ||w|| \le 4$ and w contains 0s and 1s$\}$~~

Missing terms and also have terms should not be included. -3

d) $A \oplus B = \Sigma^1 \cup \{w \in \Sigma^* \mid \underbrace{2 \le ||w|| \le 4 \text{ and } w \text{ contains 0s and 1s}}_{X}\} \cup \{w \in \Sigma^* \mid 5 \le ||w||\}$

8 a) Language $\{00\}\{0\}^*\{10\}$ consists of all words in Σ^* that begin with 00, end with 10 and have any number of zeros (including no zeros) between ~~the~~ these two segments. Form of $00(0)^n 10$ where $n \geq 0$ and x^n means a string of n x's. Then for $n=1$, we have word 00010 belongs to $\{00\}\{0\}^*\{10\}$.

9/9

b) In second language $\{000\}^*\{1\}^*\{0\}$, we have form $(000)^n 1^m 0$ where $n, m \geq 0$. Then for $n, m = 1$ we have word 00010 belong to $\{000\}^*\{1\}^*\{0\}$.

c) In last language $\{00\}^*\{10\}^*$ we have form $(00)^n(10)^m$ where $n, m \geq 0$. Then there are no m, n such that 00010 belong to $\{00\}^*\{10\}^*$

9. Any derivation from S starts with 0X or 1Y. Suppose we start with 0X then

4/10

0X → 01Y → 010X → 0101Y → ....

↓ ↓ ↓ ↓

01Z 010Z 0101Z 01010Z → ....

↓ ↓ ↓ ↓

01 or 010 010 or 0100 0101 or 01010 01010 or 010100

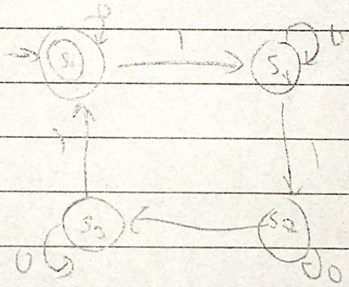
−6

The bottom row contains exactly words that begin with 01 alternate 0s and 1s and then optionally add final 0 if alternating strings ends in 0.

It may begin with 10. Also need to explain clearly. the format should be like example 6.11 on P307 of the study guide.

10. Exercise 12.2 16 (From book)

9/9

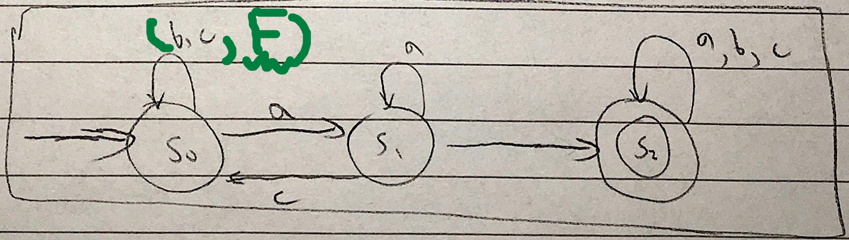


This automaton accepts set of strings of 0's and 1's for which number of 1's is divisible by 4 ✓

Regular expression: $\boxed{0^*1(0^*10^*10^*10^*10^*)^*}$ ✓

11. Diagram:

4/8



: state ⟶ : transition ⟶ © ↳ transition condition

S_0 : starting state

−4

Missing outputs. Notations not correct (missing brackets).