Name: ______________________

ID #: ______________________

PHIL 379 Lec 01
Logic II
Winter 2018

Assignment 3
DUE IN CLASS ON AT 11:00 AM ON MARCH 13, 2018

1. For each of the following sentences, specify a model in which the sentence is true and another model in which it is false. In all cases, the domain of the model must be {1, 2, 3}. *(3 marks)*
   (i) $\exists y \forall x(F(x) \leftrightarrow x=y)$
   (ii) $\forall x \forall y \forall z((R(x,y) \wedge R(x,z) \rightarrow y=z)$
   (iii) $\exists x \forall y(R(y,x) \vee R(f(y),x))$

2. Using the definition of truth in a model, show that (ii) implies (i). *(3 marks)*
   (i) $\forall x \exists y R(x,y)$
   (ii) $\exists y \forall x R(x,y)$

3. A universal closure of an open formula F is the result of attaching universal quantifiers at the beginning of F so that all occurrences of variables in the resulting formula are bound. Likewise, an existential closure of a formula F is the result of attaching existential quantifiers at the beginning of F so that all occurrences of variables in the resulting formula are bound. For example, '$\forall x \forall y \forall z((R(x,y) \wedge R(x,z) \rightarrow y=z)$' is a universal closure of '$((R(x,y) \wedge R(x,z) \rightarrow y=z)$' and '$\exists x \forall y(R(y,x) \vee R(f(y),x))$' is the existential closure of '$\forall y(R(y,x) \vee R(f(y),x))$'.

   Write an open formula whose universal closure is invalid, but whose existential closure is valid. *(1 marks)*

4. A model M^+ is an extension of a model M of a language L if and only if (i) $|M| = |M^+|$, and (ii) for any member e of L, $v_M(e) = v_{M+}(e)$. Show, by induction on complexity, that for any sentence S of L, S is true in M if and only if S is true in M^+. *(3 marks)*

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