Project 1: Radiative Transfer

PHY4905/5905: Computational Physics-Spring 2022

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Description of the code architecture

Python functions in the code:

- To generate new cartesian coordinates of photons after interaction with gas molecules using the old cartesian coordinates and randomly generated distance to travel with scattering angle φ and polar angle, θ , as well if it is 3-dimensional case.
    1. move2D(s, phi, x, y)
    2. move3D(s, phi, theta, x, y, z)
- To set the plotting environment: setplot(Ndim, maxD)
  If motion of photons is considered in 2-dimensional space then, a circle is drawn to show the boundary of maximum distance from the star. For motion in 3D space, it's a sphere.
- Function to perform actual simulation of photons and print their path:
  MC_RadiativeTransfer(Nphotons, Ndim, tau, albedo, maxD=10, steps=int(1e3), verbose=True, plot=True, quiet=True, tau_funcOFs=False)

Description of all the input parameters and outputs of the above-mentioned functions are mentioned in the code itself.

Algorithm steps of the code:

1. Initializing
    a. empty arrays to store:
        i. indices of absorbed photons
        ii. indices of escaped photons
        iii. No. Of scattering events of escaped photons
        iv. No. Of scattering events of absorbed photons
    b. Declare list of interaction events = ["scatter","absorb"]
    c. Counter for photons with zero scattering events
    d. Store the initial clock timing
    e. Set plotting environment

2. Start looping over photons (perform following steps for each photon one by one)
    a. Set the counter for scattering events to 0
    b. Define Initial coordinates and store them in the array ‘path’
    c. Randomly generate:
        i. Distance to travel
        ii. Scattering angle, φ
    d. Move the photons accordingly and generate new coordinates
        i. If it is 2D: $(x, y) = (s\cos(\varphi), s\sin(\varphi))$
        ii. If it is 3D:
            1. Generate new polar angle, θ
            2. $(x, y, z) = (s\cos(\varphi)\sin(\theta), s\sin(\varphi)\sin(\theta), s\cos(\theta))$
    e. Calculate the distance from the initial coordinates, here we are assuming the origin (0.0,0.0,0.0)
    f. Start another loop for interactions which will run till predefined maximum number of scattering events allowed per photon (here, it is 1000)
        i. Randomly choose what type of interaction event can occur. Two choices: ‘scatter’ with probability $'albedo'$ and ‘absorb’ with probability $'1 - albedo'$
        ii. If it is $'scatter'$:
            1. IF $radius \in [-\max D, \max D]$: generate new coordinates according to the dimensions of motions as described above in step 2.c and 2.d. *(Here, maxD is the predefined maximum distance from the star beyond which photon will be assumed to escape the cloud.)*

               Also increase the count of of #scattering events by 1.

            2. OTHERWISE declare that photon escaped the gas cloud and store its index and corresponding #scattering events.

           BREAK THE LOOP of the events and go for next photon! No more events for current photon!

        iii. If it is $'absorb'$, then store its #scattering events. Check if its #scattering events = 0 then increase the count of photons with zero scattering events by 1. BREAK THE LOOP and go for next photon!

           BREAK THE LOOP of the events and go for next photon! No more events for current photon!

    g. Print path of the photons as per the mentioned dimensions.
3. Calculate different fractions and averages as a part of the analysis.

Efficiency of the code:

1. We are using 2 ‘for loop’s, first one for the number of photons. The second loop is nested inside the first one to loop over interaction events for the ongoing photon until the loop termination occurs. In order to use less memory, coordinates of motion are saved for a single photon in each loop, and then plotted it.
2. I tested time required for various options for random number generators: like *np.random.random vs. Random.random, etc.*

Some useful functions from the random module:

random.random() -- gives a random float uniformly distributed in [0.0,1.0)

random.choices(list, k=n) -- selects a random sample of length n from a list of any type, with replacement

```
[25]: %timeit rand = np.random.random()
259 ns ± 5.61 ns per loop (mean ± std. dev. of 7 runs, 1000000 loops each)
[26]: %timeit rand = random.random()
44.7 ns ± 0.623 ns per loop (mean ± std. dev. of 7 runs, 10000000 loops each)
[27]: %timeit interaction = random.choices(events, weights=[0.95,0.05])
1.08 µs ± 5.62 ns per loop (mean ± std. dev. of 7 runs, 1000000 loops each)
[28]: %timeit interaction = np.random.choice(events, p=[0.95,0.05])
11.3 µs ± 56.9 ns per loop (mean ± std. dev. of 7 runs, 100000 loops each)
```

Considering time required, I chose to use *random.random()* over *np.random.random()*. And I used *np.random.choice() because of the ability to randomly choose the elements from a list with associated probability*.

The calculation is mainly dependent on following INPUT parameters:

1) Nphotons: Number of Photons
2) Ndim: Dimension of space of motion of photons
3) tau: Optical Depth (distance traveled before interaction)
4) albedo: Scattering vs. Absorption probability
5) maxD: Maximum distance from the star, beyond which it will be assumed to have "escaped" the gas cloud surrounding the star.
6) steps: maximum number of scattering events as a failsafe. (Here we used: 1000)

The calculation gives following OUTPUTS:

1) fraction of emitted photons that are absorbed
2) fraction of emitted photons that are escaped
3) fraction of emitted photons that have zero scattering events
4) average number of scattering events for the absorbed photons
5) average number of scattering events for the escaped photons
6) time required to run the calculation

Background Information

Optical depth (distance traveled before interaction)

A population of photons propagating through a medium will interact after traversing different distances s, distributed according to an exponential, **non-uniform PDF:**

$$p(s) = \tau e^{-\tau s}$$

Here, $q(z) = 1$ is a uniform distribution on [0,1) by definition, and we want to end up with $p(s) = \tau e^{-\tau s}$ defined from 0 to ∞.

$$|p(s)ds| = |q(z)dz|$$

$$\int_{-\infty}^{s} p(s')ds' = \int_{-\infty}^{z} q(z')dz'$$

To find s(z) that will yield an exponential distribution in x when z are drawn uniformly from 0 to 1.

$$\tau \int_{0}^{s} e^{-\tau s'} ds' = \int_{0}^{z} dz' = z$$

$$1 - e^{-\tau s} = z$$

$$s = -\frac{1}{\tau} \ln(1 - z)$$

So, to generate random numbers s according to an exponential distribution, we simply need to generate uniform random numbers z and then transform them according to the above equation.

Generate Scattering Angle (for 2D: Azimuthal Angle) (ϕ) using random number

Note that if we use random numbers x_i drawn from [0,1], we must transform them to the correct interval [a,b] using

$$x = x_i(b - a) + a$$

$$\phi = x_i(2\pi) + (-\pi), \quad \phi \in [-\pi, \pi]$$

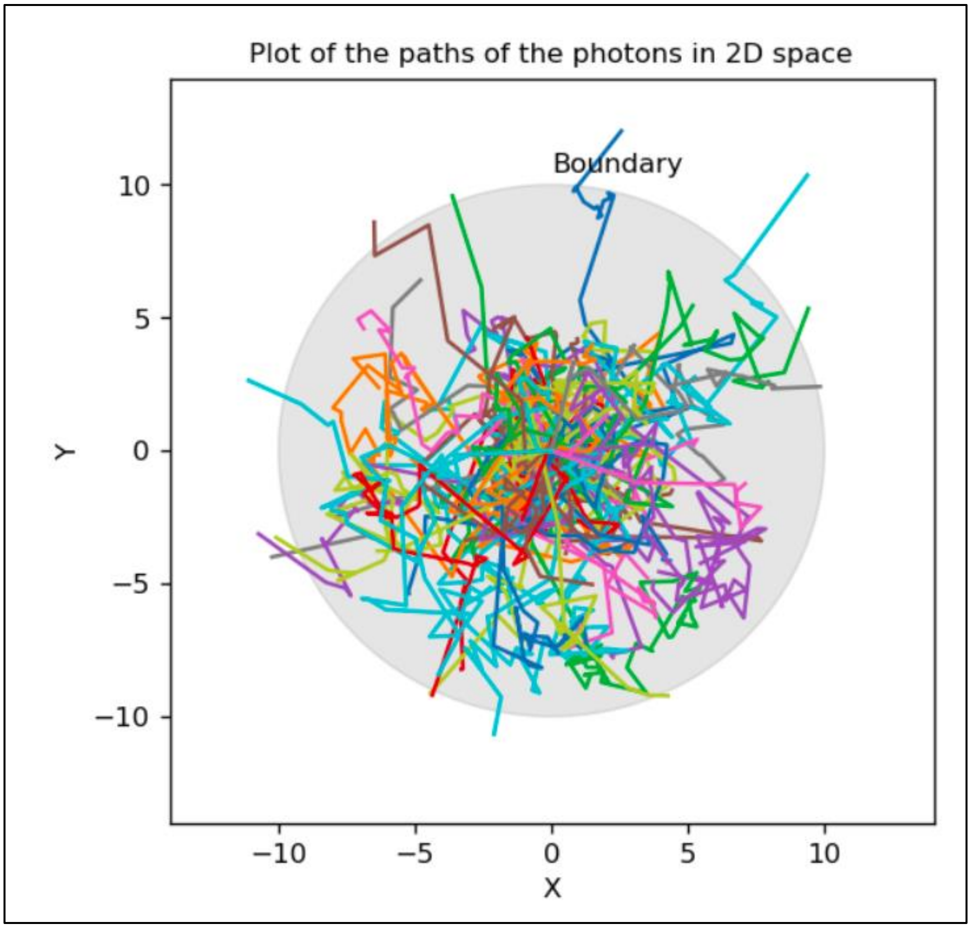
ANALYSIS

Output of calculation for motion of photons in 2D

The input parameters and the results of the 2D calculation are presented below:

```
Beginning MONTE CARLO RADIATIVE TRANSFER run with:

Nphotons=10000, Ndim=2
tau=1, albedo=0.95, maxD=10, steps=1000

Calculation took 3.48 s

Fraction of emitted photons that are ABSORBED = 0.852
Fraction of emitted photons that are ESCAPED = 0.148


Avg. number of scatterings for ABSORBED photons = 14.297769953051644
Avg. number of scatterings for ESCAPING photons = 26.120945945945945


Fraction of photons that experience zero scattering events = 0.05
```

The plot shows the simulation of photon in 2D space considering the star is at the center of the 2D space and the light-gray circle represents the boundary of the area of radius 10 distance units around the star beyond which photon is considered as escaping photon.



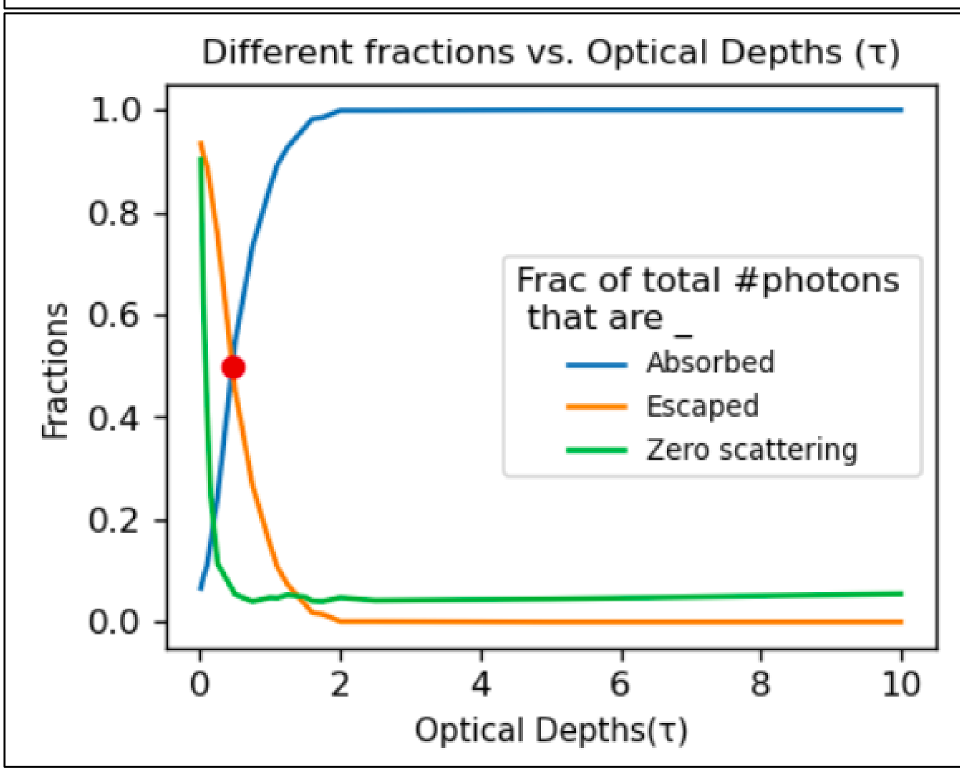
Optical depth, $\tau = 1$, means photons travel for a length of mean free path of photons before absorption.

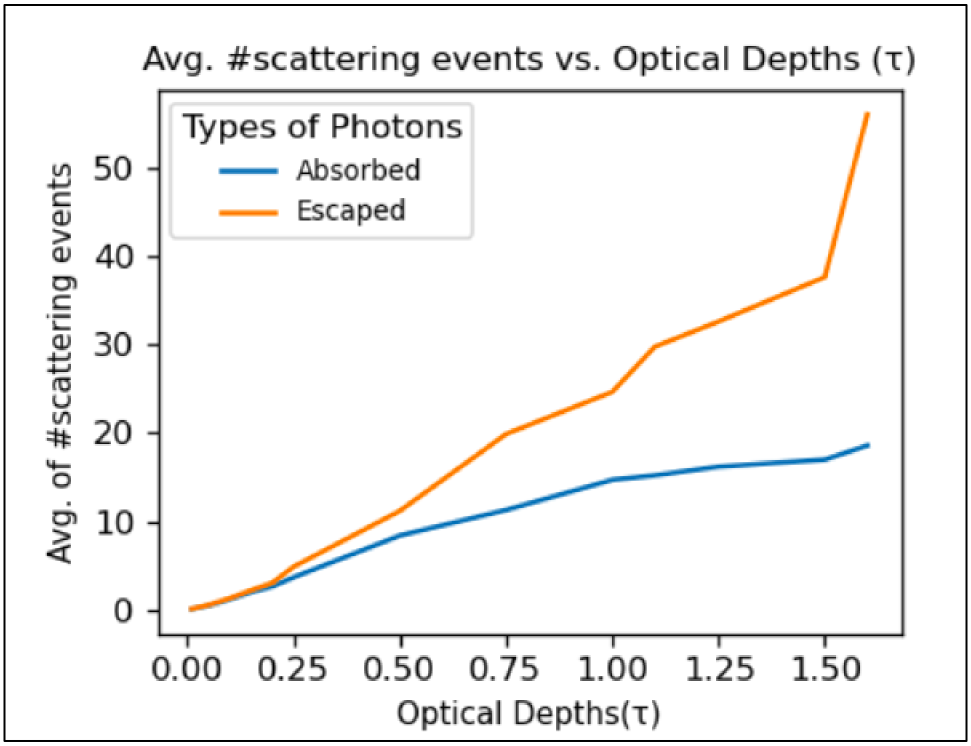
(i) Varying the optical depth

To check the results of varying optical depth, I gave following list of values of tau as an input and compared the resulted outputs by printing them in a tabular form and plotting them accordingly.

```
taus = [0.01,0.05,0.1,0.15, 0.2, 0.25, 0.5, 0.75, 1,1.1,1.25, 1.5,1.6,1.75, 2, 2.5, 5, 10,50,100]
```

```
Beginning MONTE CARLO RADIATIVE TRANSFER run with:

Nphotons=1000, Ndim=2
albedo=0.95, maxD=10, steps=1000


*********Results from VARYING THE OPTICAL DEPTH***************


tau      |       FRACTIONS of ___ Photons                |       Avg. #scatterings   | Time Required(s)

         |Absorbed        Escaped         0scattering    | Escaped         Absorbed   |
-----------------------------------------------------------------------------------------------------
0.01     |0.0660          0.934           0.9030         | 0.104925        0.106061 |  0.031009
0.05     |0.0890          0.911           0.6190         | 0.556531        0.471910 |  0.041743
0.1      |0.1110          0.889           0.4090         | 1.311586        1.189189 |  0.057817
0.15     |0.1540          0.846           0.2470         | 2.215130        1.993506 |  0.072219
0.2      |0.2010          0.799           0.1830         | 3.056320        2.641791 |  0.087790
0.25     |0.2440          0.756           0.1140         | 4.867725        3.672131 |  0.134108
0.5      |0.5480          0.452           0.0540         | 11.176991       8.395985 |  0.225633
0.75     |0.7340          0.266           0.0400         | 19.875940       11.282016 |  0.297431
1        |0.8500          0.15            0.0480         | 24.640000       14.694118 |  0.350389
1.1      |0.8920          0.108           0.0470         | 29.740741       15.193946 |  0.373967
1.25     |0.9270          0.073           0.0530         | 32.589041       16.181230 |  0.376888
1.5      |0.9660          0.034           0.0490         | 37.617647       16.986542 |  0.393364
1.6      |0.9820          0.018           0.0410         | 56.000000       18.578411 |  0.411457
1.75     |0.9850          0.015           0.0400         | 43.066667       17.290355 |  0.381135
2        |0.9990          0.001           0.0480         | 39.000000       18.446446 |  0.405757
2.5      |0.9990          0.001           0.0420         | 62.000000       19.855856 |  0.420032
5        |1.0000          0.0             0.0450         |  nan    18.241000 |  0.400446
10       |1.0000          0.0             0.0550         |  nan    18.962000 |  0.404080
50       |1.0000          0.0             0.0520         |  nan    18.945000 |  0.406243
100      |1.0000          0.0             0.0430         |  nan    19.712000 |  0.421935

The intersection point of the graph of both fractions (x,y) = (0.46,0.5).
```





As we see from the plot of different fractions vs. Optical depths, the fraction of the total number of photons that are absorbed increases whereas the fraction of the total number of photons that are escaped decreases, with the increasing optical depth. The last line of the printed results shows that both fractions get equal to 0.5 around $\tau = 0.46$. From the printed table of outputs, we can see that for , $\tau = [0.01,\ 0.05,\ 0.1,\ 0.15,\ 0.2,\ 0.25,]$ the fraction of emitted photons that are escaped is greater than that of absorbed and this trend reverses for the rest of the τ values. For $\tau > 1,\ i.e.,\ for\, \tau = [1.1, 1.25, 1.5, 1.6, 1.75]$, in the optically thick medium, the fraction of photons getting absorbed >> than that of getting escaped, means photons are getting quickly absorbed. Here, for $\tau > 1.75,\ i.e.,\ \tau \gg 1$ the fraction for escaping photons is almost equal to zero, whereas the average number of scattering events for escaping photons increases more rapidly than that of absorbed photons. For very large optical depths ($here,\ \tau = 5 - 100$), the fraction of absorbed photons.

These observations say that: if the gas is optically thick, then it is certain that a photon will interact many, many times with particles before it finally escapes from the cloud. Any photon entering the cloud will have its direction changed many times by collisions -- which means that its "output" direction has nothing to do with its "input" direction. In general terms, we can't see anything at all through the cloud: it is opaque. And if the gas is optically thin ($\tau \ll 1$), then the chances are small that a photon will interact with a single particle and really small that it will interact with more than one. We can safely ignore multiple scatterings or absorptions. In general terms, we can see right through the cloud.

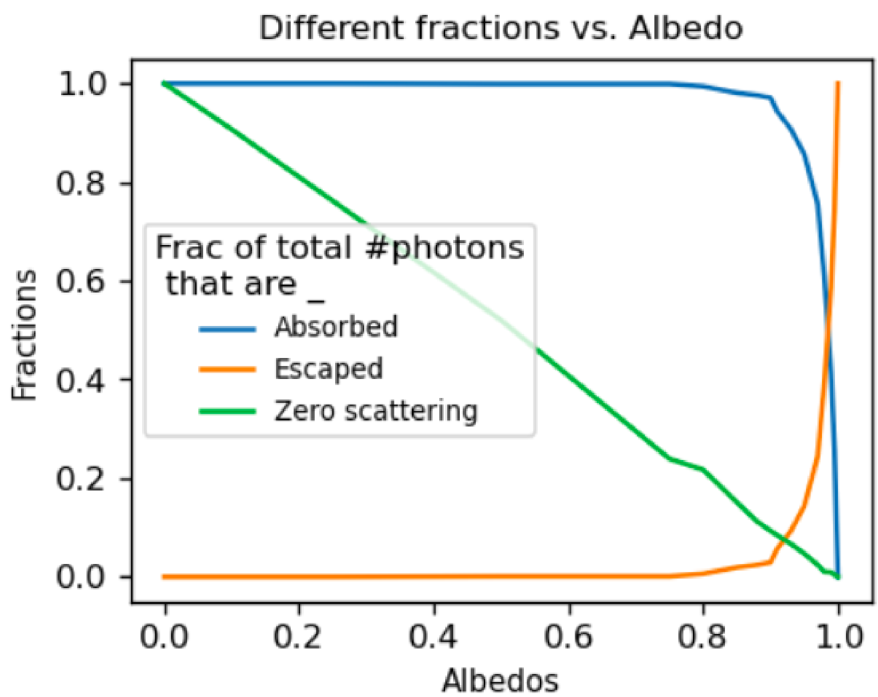
Ref: http://spiff.rit.edu/classes/phys440/lectures/optd/optd.html

(ii) Varying the albedo

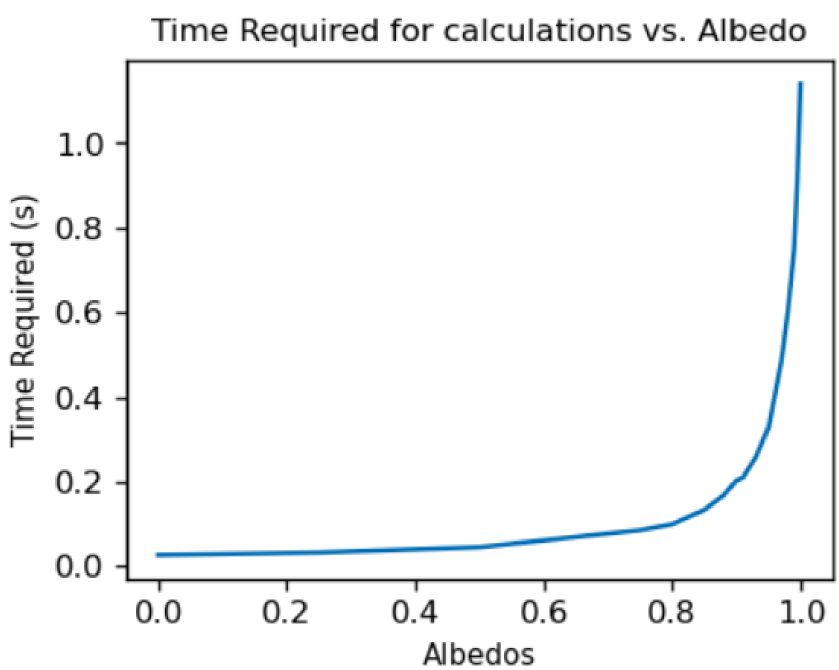
To check the results of varying albedos, I gave following list of values of albedos as an input and compared the resulted outputs by printing them in a tabular form and plotting them accordingly.

```
albedos = [0.0, 0.1, 0.25, 0.5, 0.75, 0.80, 0.85, 0.88, 0.90,0.91, 0.93, 0.95,0.97, 0.98, 0.99,0.995, 1]
```

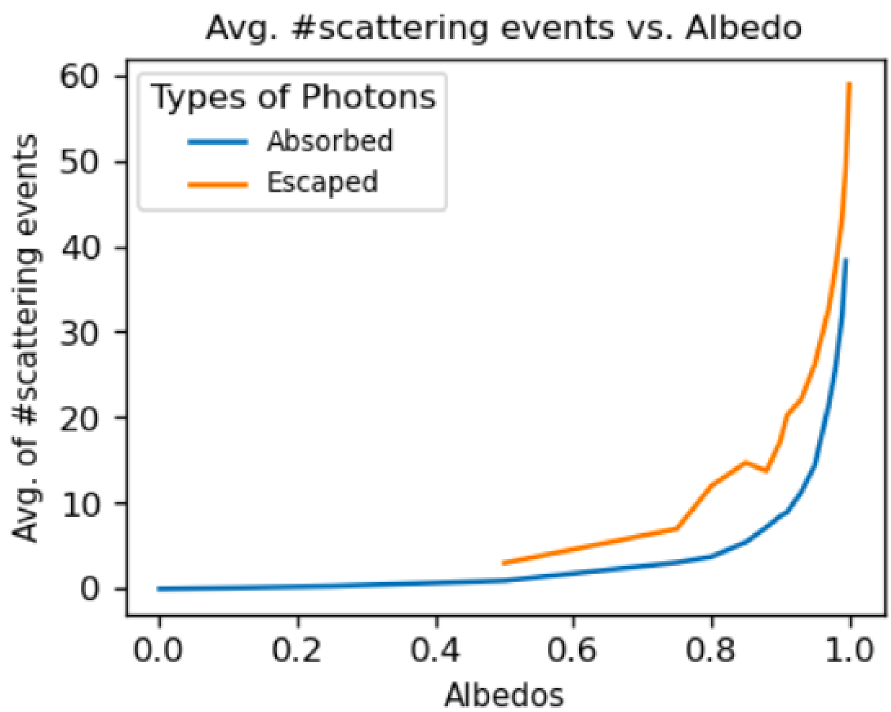
```
Beginning MONTE CARLO RADIATIVE TRANSFER run with:

Nphotons=1000, Ndim=2
tau=1, maxD=10, steps=1000


*********Results from VARYING THE ALBEDO****************


albedo  |       FRACTIONS of ___ Photons                |       Avg. #scatterings   | Time Required(s)

        |Absorbed        Escaped         0scattering    | Escaped        Absorbed   |
-------------------------------------------------------------------------------------------------------
0.0     |1.0000          0.0             1.0000         | nan            0.0000     |  0.0271
0.1     |1.0000          0.0             0.9070         | nan            0.1030     |  0.0292
0.25    |1.0000          0.0             0.7620         | nan            0.3070     |  0.0329
0.5     |0.9990          0.001           0.5200         | 3.0000                 0.9389     |  0.0455
0.75    |0.9990          0.001           0.2390         | 7.0000                 3.0541     |  0.0853
0.8     |0.9940          0.006           0.2170         | 12.0000                3.7304     |  0.0996
0.85    |0.9810          0.019           0.1510         | 14.7368                5.4404     |  0.1339
0.88    |0.9760          0.024           0.1120         | 13.7500                7.1834     |  0.1676
0.9     |0.9710          0.029           0.0930         | 17.2414                8.4902     |  0.2035
0.91    |0.9430          0.057           0.0840         | 20.2807                8.9512     |  0.2101
0.93    |0.9080          0.092           0.0670         | 22.0652                11.2390     |  0.2572
0.95    |0.8570          0.143           0.0470         | 26.1958                14.4049     |  0.3283
0.97    |0.7560          0.244           0.0240         | 32.6189                21.3571     |  0.4857
0.98    |0.6070          0.393           0.0100         | 37.4478                25.7381     |  0.5995
0.99    |0.4140          0.586           0.0090         | 43.4727                32.0531     |  0.7475
0.995   |0.2570          0.743           0.0040         | 49.1494                38.2763     |  0.9081
1       |0.0000          1.0             0.0000         | 58.9140                nan    |  1.1409
```



Different fractions vs. Albedo
Fractions
Albedos
Frac of total #photons
that are _
Absorbed
Escaped
Zero scattering
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0



Time Required for calculations vs. Albedo
Time Required (s)
Albedos
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0



If we compare the nature of the plots of different fractions for varying optical depth with that of varying albedo, we see that the latter one is the mirror image of the former one. The fraction of total photons that are getting absorbed drastically decreases as the albedo value goes beyond 0.93 and at albedo=1, the fraction equals 0, *i.e.* , no photon gets absorbed. Exactly opposite behavior can be observed for the fraction for escaped photons, which increases rapidly for albedo > 0.93 and equals 1 for albedo=1. This highlights the definition of albedo which is the measure of the reflectivity of the gas cloud. A perfectly reflective surface would get an albedo score of 1, while a completely dark object would have an albedo of 0. From our observations, a greater number of photons escape the cloud as albedo goes beyond 0.93, so light comes out of the cloud thus cloud reflects almost all the incident light.

Another significant observation: the fraction of photons with zero scattering events linearly decreases with increasing albedo from 0 to 1. The increasing fraction of photons that are escaping the cloud and increasing the average number of scattering events for the escaped photons with increasing albedo complements the decreasing linearity observed for the fraction for photons with zero scattering events. As we increase the albedo, the fraction of escaping photons as well as the average number of scattering events for the escaping photons increases rapidly. This increase in the scattering events affects the time required for the calculation that is why we obtained a plot for the time required for the calculations of similar nature as of the fraction of escaping photons vs. Varying Albedo.

Ref. http://irina.eas.gatech.edu/EAS8803_Fall2017/petty_13.pdf

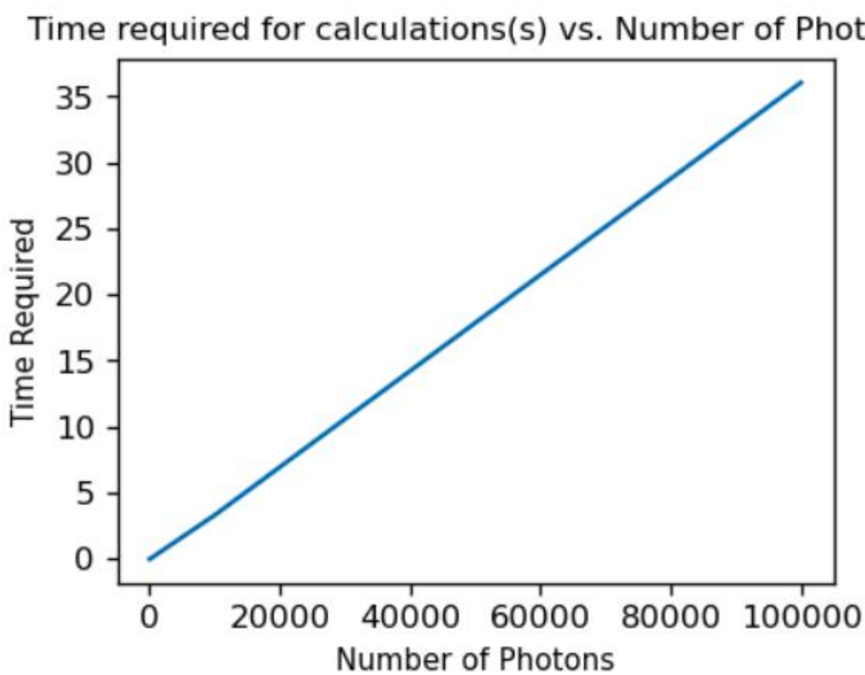
(iii) Varying the number of rays and examining the convergence of the results

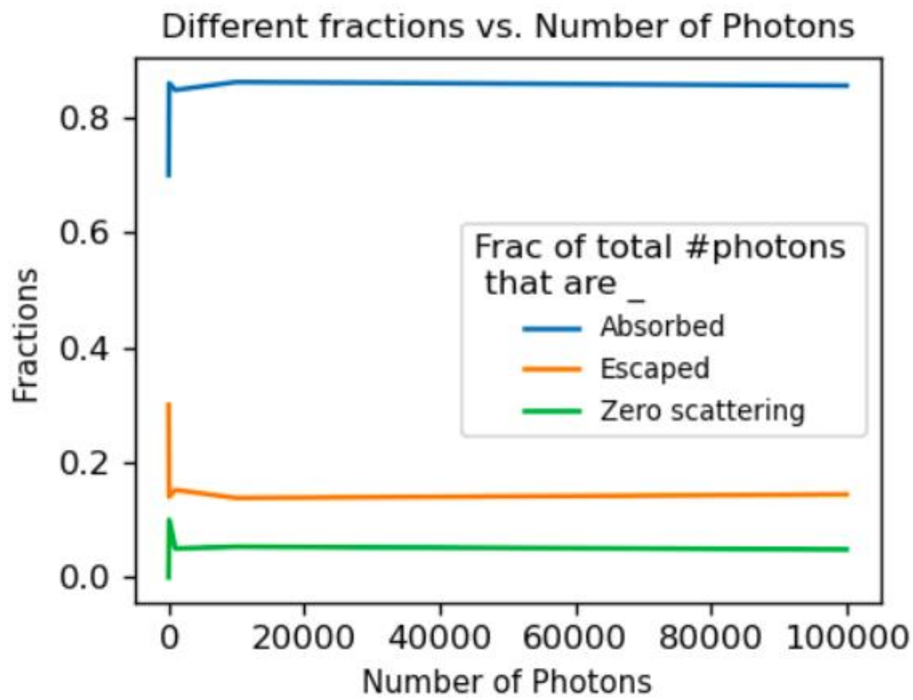
To check the results of varying the number of rays, I gave following list of values of number of photons as an input and compared the resulted outputs by printing them in a tabular form and plotting them accordingly.

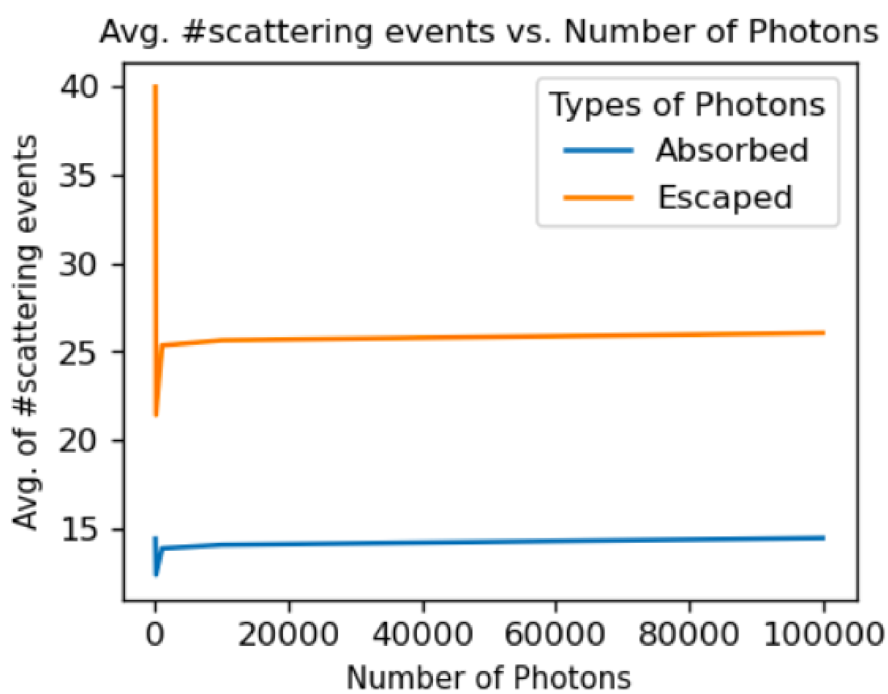
```python
Nphotons = [10**i for i in range(1,6,1)]
print(Nphotons)
```

```
[10, 100, 1000, 10000, 100000]
```

```
Beginning MONTE CARLO RADIATIVE TRANSFER run with:

Nphotons= varying, Ndim=2
tau=1,albedo=0.95, maxD=10, steps=1000


*********Results from VARYING THE #PHOTONS***************


#Photons |       FRACTIONS of ___ Photons                |       Avg. #scatterings   | Time Required(s)

         |Absorbed        Escaped         0scattering    | Escaped         Absorbed  |
---------------------------------------------------------------------------------------------------
10       |0.7000          0.3             0.0000         | 40.0000                 14.4286    |  0.0048
100      |0.8600          0.14            0.1000         | 21.4286                 12.3837    |  0.0293
1000     |0.8480          0.152           0.0500         | 25.3553                 13.8821    |  0.3285
10000    |0.8619          0.1381                  0.0532         | 25.6488                 14.0769    |  3.3308
100000   |0.8557          0.1443                  0.0485         | 26.0630                 14.4669    |  36.0424
```







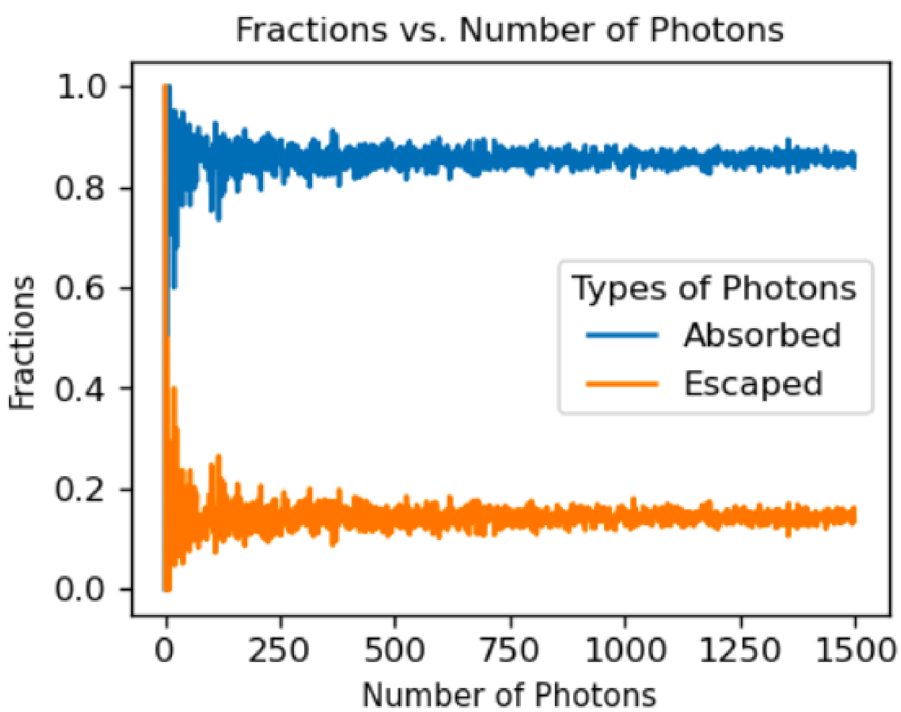
The noticeable observation from increasing the number of rays is that time required for the calculations increases by 10-fold if we increase the number of photons by 10 times. This shows that the time required for the calculations is linear with the increasing number of photons. It's obvious that it will take extra time to simulate the extra photon.

We are analyzing the results of various numbers of photons for albedo = 0.95 and τ =1, which signifies that the cloud surface is more reflective (almost all photons will escape with higher scattering events, almost negligible number of photons will be absorbed). As τ =1, the cloud is neither that thick nor that thin. So, the plot for all the fractions (for absorbed, escaped photons, and photons with zero scattering events) saturate very quickly, almost around 250 photons.

Calculations for varying number of photons from 1-1500 by 1

This part of the code is commented out in the file as it takes more time to run as it is doing whole calculations for first for 1 photon, then for 2 photons, so on up to 1500 photons. So, I ran the respective code and presenting the corresponding results here for reference. I commented out this part of the code after obtaining results so that if you run the whole file of the code in one click, it will skip this part and save some extra time required for the calculations.

I performed this analysis just to zoom out the initial portion of the above plot of fractions vs. the Number of photons. It shows more closely how the fractions of photons that are absorbed and escaped saturate easily for albedo = 0.95 and τ =1.



(iv) Adding additional Elements

a. Extension of the calculation to 3D

To go from 2D to 3D space, we need to consider the polar angle (θ), in addition to the distance (s) and the scattering angle ($i.e.,\ azimuthal\ angle,\ \varphi$). We randomly generate the polar angle first using the uniform distribution for the range [0,1) and then change the range to [0, π]. The description of the same is as follow:

Extending the calculation to 3D, assuming spherical symmetry around the star.

Because the differential surface element in spherical coordinates $d\Omega = sin(\theta)d\theta d\phi$ depends on the polar angle θ, its probability distribution is not uniform. In order to draw uniformly distributed points on a sphere, we define a random variable z that is uniform in $[0.0, 1.0)$ and we use the transformation method to find a function $\theta(z)$ that returns the correct probability distribution $p(\theta) = sin(\theta)/2$.

Here, $q(z) = 1$ is a uniform distribution on $[0, 1)$ by definition, and we want to end up with $p(\theta) = sin(\theta)/2$ defined from 0 to π.

To find $\theta(z)$ that will yield an sinusodial distribution in θ when z are drawn uniformly from 0 to 1:

$$\int_0^\theta \frac{sin(\theta')}{2} d\theta' = \int_0^z dz' = z$$

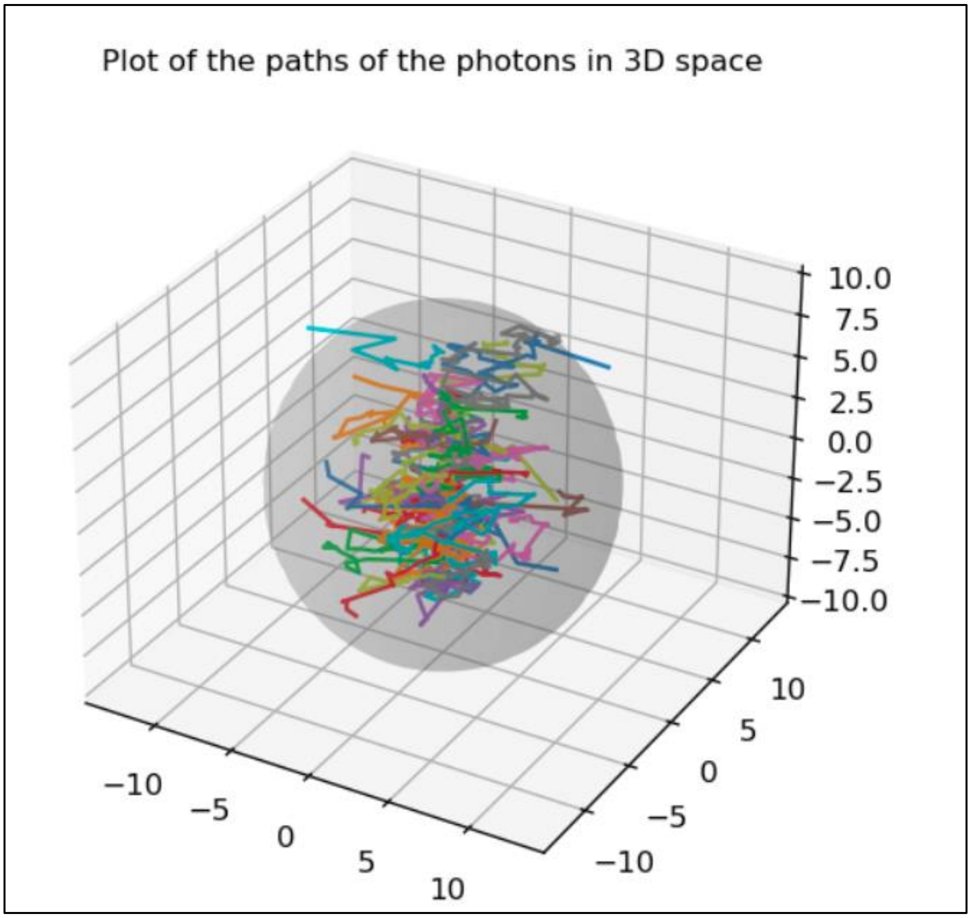
$$\frac{1}{2}[-cos(\theta') + C]_0^\theta = z$$

$$\frac{1 - cos(\theta)}{2} = z$$

$$\theta = cos^{-1}(1 - 2z)$$

The results of the Monte Carlo Radiative Transfer calculation in 3D space are presented below:

```
Beginning MONTE CARLO RADIATIVE TRANSFER run with:

Nphotons=10000, Ndim=3
tau=1, albedo=0.95, maxD=10, steps=1000

Calculation took 3.89 s

Fraction of emitted photons that are ABSORBED = 0.8709
Fraction of emitted photons that are ESCAPED = 0.1291


Avg. number of scatterings for ABSORBED photons = 14.616833161097714
Avg. number of scatterings for ESCAPING photons = 28.727343144848955


Fraction of photons that experience zero scattering events = 0.0483
```

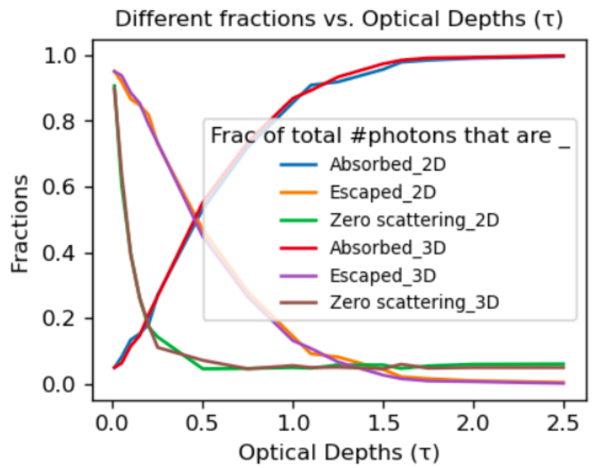


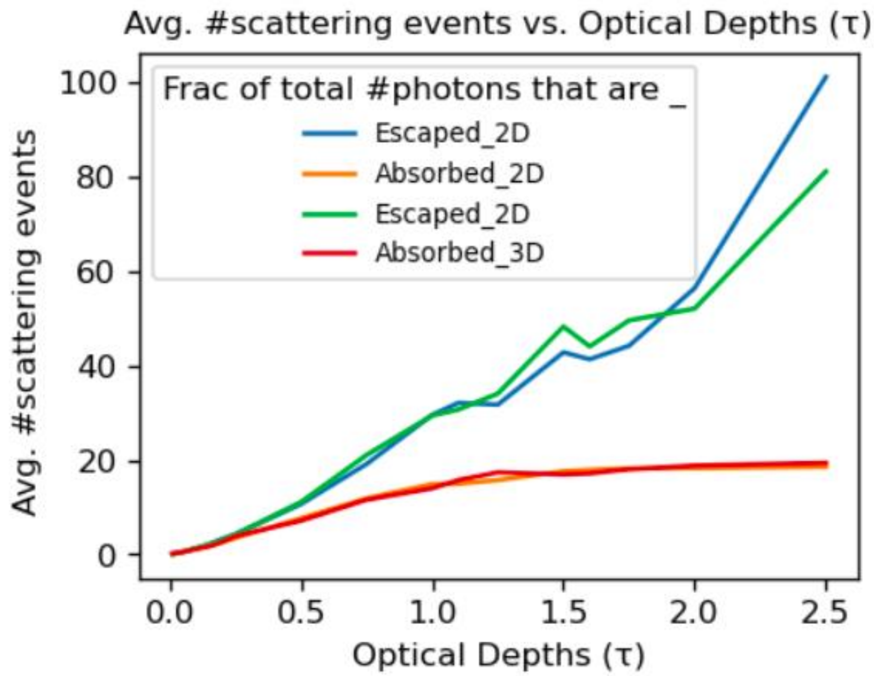
The nature of the motion of the photons is not supposed to change after adding an extra dimension for the motion of the photons. The consistency of the results from the 3D calculation is checked with the results from the 2D calculation. As expected, the corresponding plots almost overlap each other shows that we get the same trends for 3D as well.

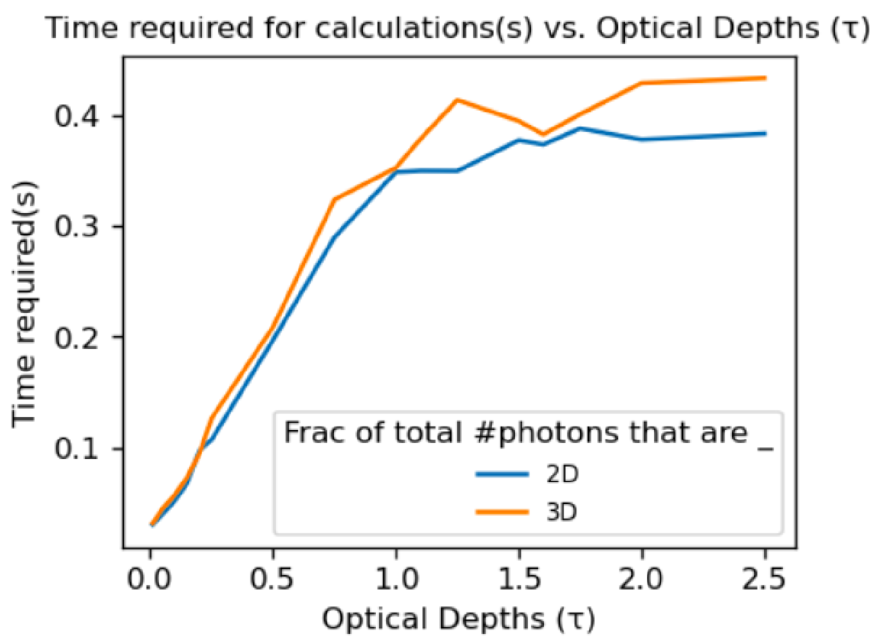
Comparison of results from 2D with 3D

(I) Varying optical depth

```
Beginning MONTE CARLO RADIATIVE TRANSFER run with:

Nphotons=1000, Ndim=2
albedo=0.95, maxD=10, steps=1000


*********Results from VARYING THE Optical Depths****************
```

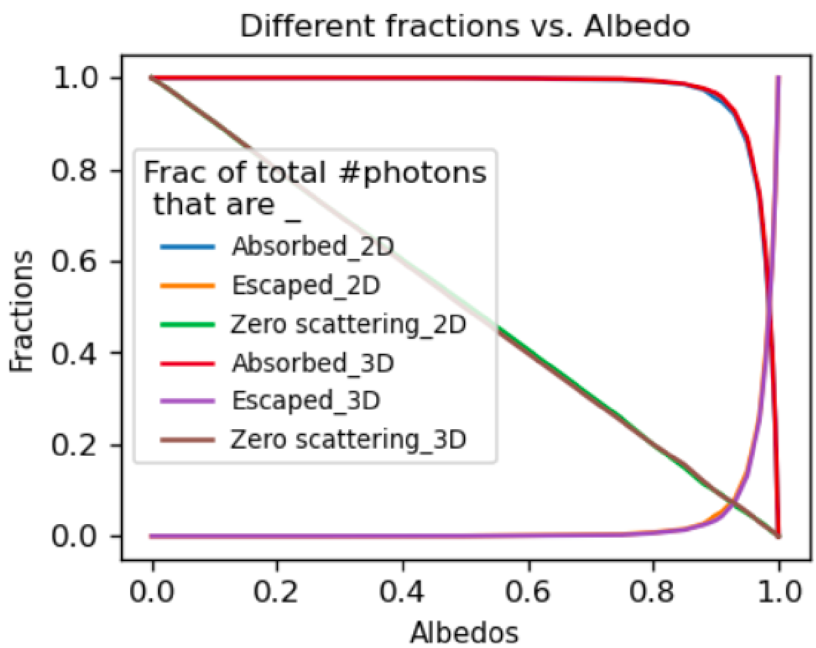


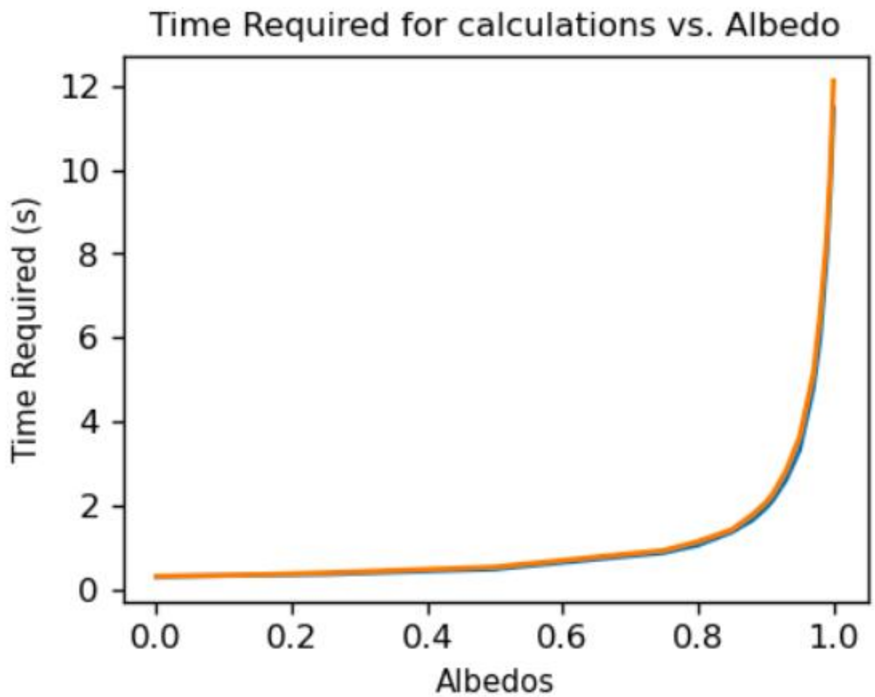

Avg. #scattering events vs. Optical Depths (τ)
Frac of total #photons that are _
Escaped_2D
Absorbed_2D
Escaped_2D
Absorbed_3D
Avg. #scattering events
Optical Depths (τ)

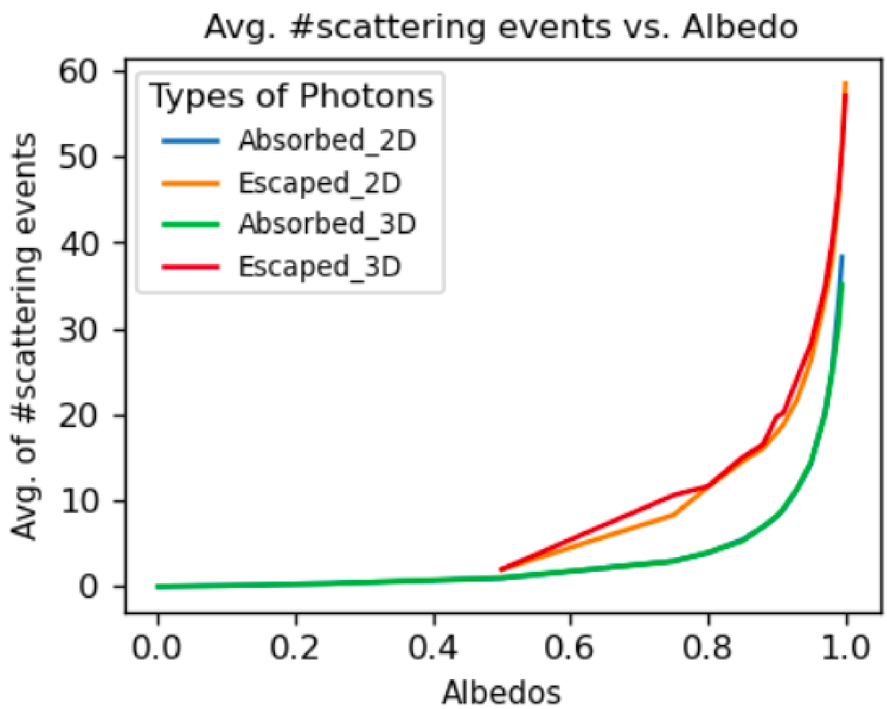

Time required for calculations(s) vs. Optical Depths (τ)
Frac of total #photons that are _
2D
3D
Time required(s)
Optical Depths (τ)

(II) Varying albedo

```
Beginning MONTE CARLO RADIATIVE TRANSFER run with:

Nphotons=10000, Ndim=2
tau=1, maxD=10, steps=1000


*********Results from VARYING ALBEDO****************
```




Time Required for calculations vs. Albedo
Time Required (s)
Albedos
0
2
4
6
8
10
12
0.0
0.2
0.4
0.6
0.8
1.0


Avg. #scattering events vs. Albedo
Avg. of #scattering events
Albedos
Types of Photons
Absorbed_2D
Escaped_2D
Absorbed_3D
Escaped_3D
0
10
20
30
40
50
60
0.0
0.2
0.4
0.6
0.8
1.0

(iv)Adding additional Elements

b. Optical depth a function of the distance from the star ($\tau(s)$)

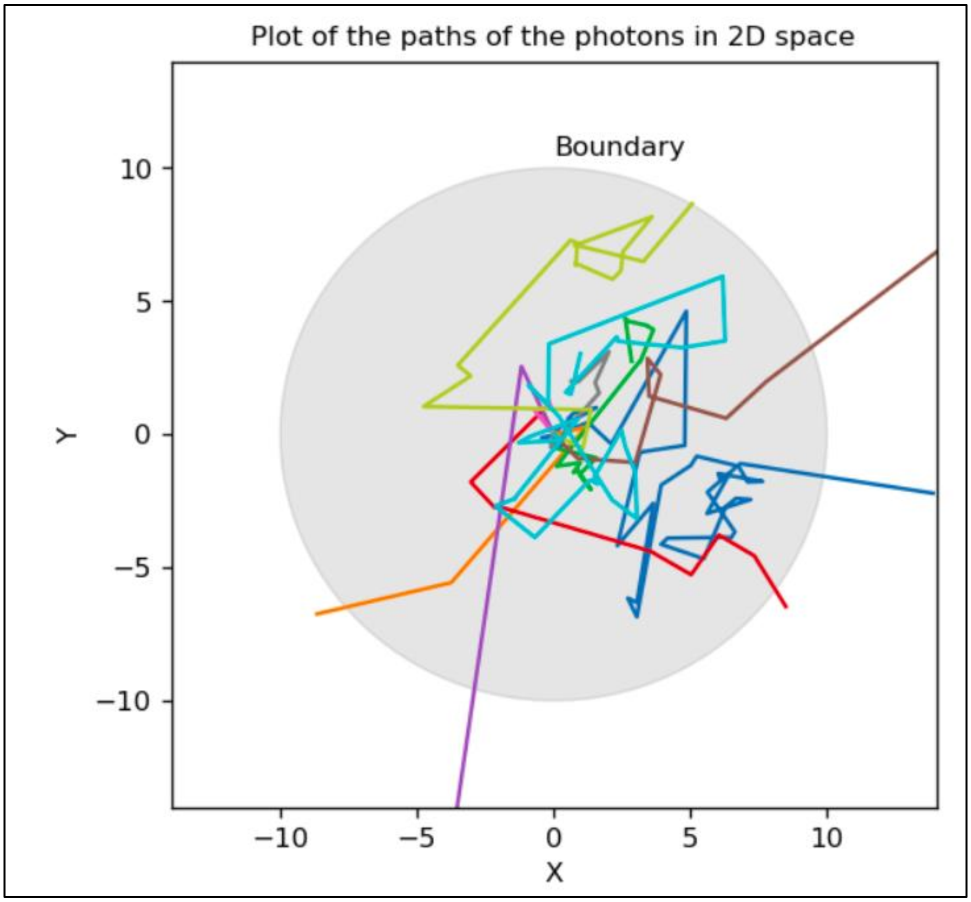
We are calculating the distance traveled by using the exponential PDF: $s = -\frac{1}{\tau}\ln(1-z)$, where z is the random number generated from uniform distribution on [0,1).

In order to generate a new tau which is dependent on 's', we will use old s and then generate new τ using: $\tau(s) = ce^{-cs}$, where c is some constant.

Implementation: If tau_funcOFs == True, for each photon, initial tau == input_tau from function call and it will exponentially decrease as per the above mentioned function with some constant c until the loop terminates for that particular photon. Again new loop will start for new photon with initial tau == input_tau and then it keeps on decreasing exponentially...repeat again....

Here, we considering const. **c = 1.0.**

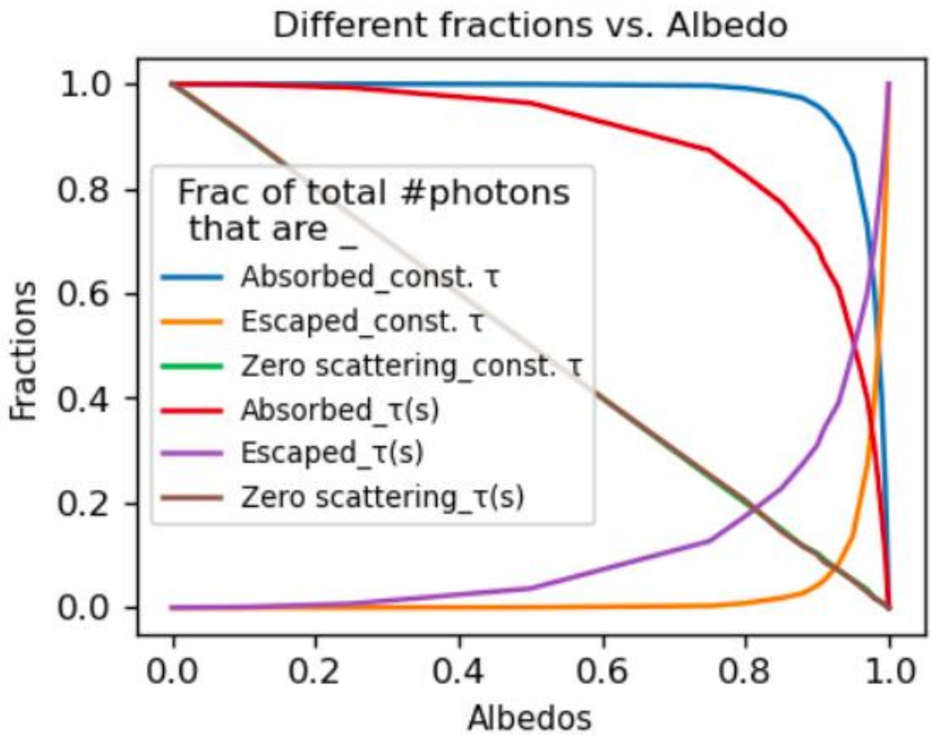
```
Beginning MONTE CARLO RADIATIVE TRANSFER run with:

Nphotons=1000, Ndim=2
tau=0.6999860823343379, albedo=0.95, maxD=10, steps=1000

Calculation took 0.23 s

Fraction of emitted photons that are ABSORBED = 0.518
Fraction of emitted photons that are ESCAPED = 0.482


Avg. number of scatterings for ABSORBED photons = 10.287644787644787
Avg. number of scatterings for ESCAPING photons = 8.506224066390041


Fraction of photons that experience zero scattering events = 0.05
```

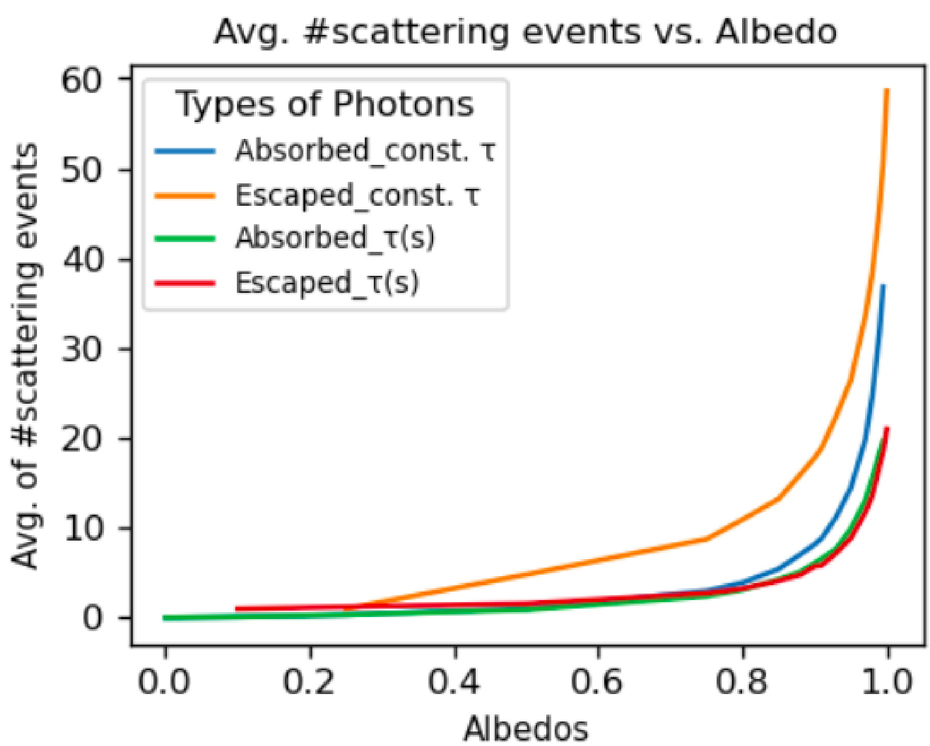


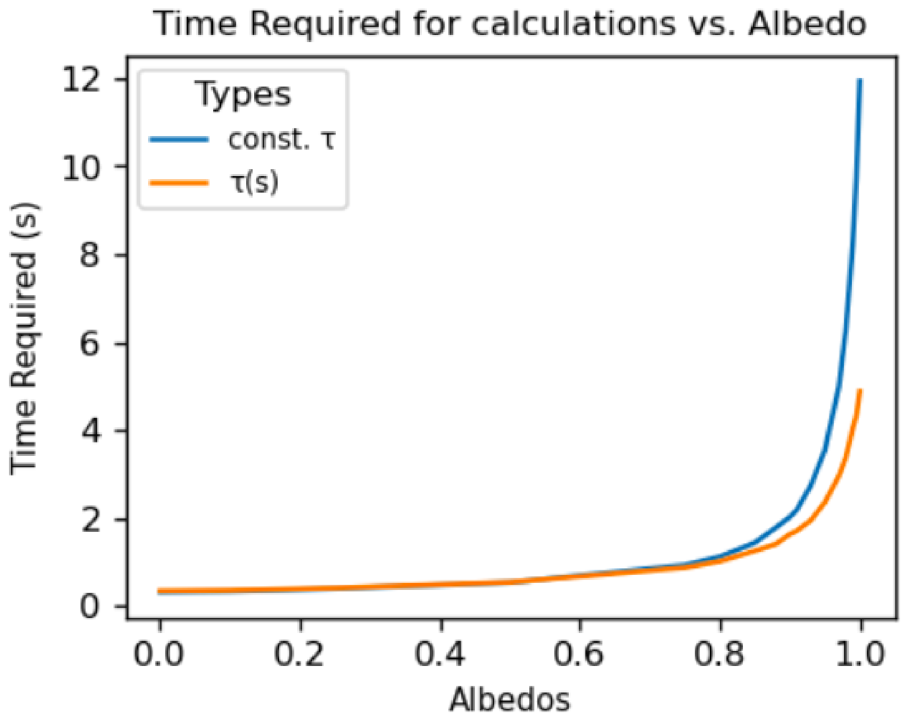
With optical depth as function of distance, $\tau(s)$, and with varying albedo

For the const. **c = 1.0,** we vary the optical depth as a function of the distance travelled. To check the effects of constant optical depth and distance dependent optical depth, we compare the results for both the calculations for varying albedo.

```
Beginning MONTE CARLO RADIATIVE TRANSFER run with:

Nphotons=10000, Ndim=2
maxD=10, steps=1000


*********Results with  Optical Depths (τ) and VARYING ALBEDO***************



In every section, 1st column: constant τ, 2nd column: τ = f(s)

Albedo |                        FRACTIONS of ___ Photons                        |       Avg. #scatterings                         | Time Required(s)

        |Absorbed                Escaped                 0scattering            | Escaped                        Absorbed  |
--------------------------------------------------------------------------------------------------------------------------------------------
0.0      |1.0000,  1.0000         0.0000,  0.0000         1.0000,  1.0000         | nan,  nan                0.0000,  0.0000   |  0.3258,  0.3484
0.1      |1.0000,  0.9989         0.0000,  0.0011         0.9031,  0.9052         | nan,  1.0000             0.1068,  0.1039   |  0.3397,  0.3609
0.25     |0.9999,  0.9927         0.0001,  0.0073         0.7502,  0.7494         | 1.0000,  1.2192          0.3309,  0.3192   |  0.3970,  0.4106
0.5      |0.9995,  0.9636         0.0005,  0.0364         0.5012,  0.4989         | 4.8000,  1.5549          0.9989,  0.8998   |  0.5366,  0.5430
0.75     |0.9966,  0.8731         0.0034,  0.1269         0.2492,  0.2528         | 8.7353,  2.7242          2.9794,  2.3879   |  0.9321,  0.8739
0.8      |0.9916,  0.8253         0.0084,  0.1747         0.2001,  0.2047         | 10.9167,  3.2341         3.8980,  3.1100   |  1.1229,  1.0176
0.85     |0.9818,  0.7729         0.0182,  0.2271         0.1500,  0.1456         | 13.2143,  4.1083         5.4243,  4.2659   |  1.4416,  1.2563
0.88     |0.9729,  0.7254         0.0271,  0.2746         0.1176,  0.1178         | 15.9262,  4.7651         7.0325,  5.1042   |  1.7839,  1.4091
0.9      |0.9581,  0.6905         0.0419,  0.3095         0.1040,  0.1014         | 17.8067,  5.7283         8.1547,  6.0776   |  2.0264,  1.6367
0.91     |0.9479,  0.6575         0.0521,  0.3425         0.0921,  0.0868         | 18.9347,  5.8496         8.8606,  6.5872   |  2.1908,  1.7129
0.93     |0.9166,  0.6093         0.0834,  0.3907         0.0714,  0.0716         | 22.4580,  7.2073         11.2345,  7.6819   |  2.7441,  1.9505
0.95     |0.8633,  0.5125         0.1367,  0.4875         0.0530,  0.0516         | 26.4221,  8.8361         14.4456,  9.8913   |  3.5426,  2.3581
0.97     |0.7296,  0.4006         0.2704,  0.5994         0.0342,  0.0299         | 33.4338,  11.6879                19.7378,  13.0147   |  4.9867,  2.9597
0.98     |0.6062,  0.3085         0.3938,  0.6915         0.0189,  0.0178         | 38.3459,  13.6110                24.7100,  15.5468   |  6.2905,  3.3805
0.99     |0.4072,  0.1809         0.5928,  0.8191         0.0097,  0.0098         | 45.5845,  16.9192                31.8819,  18.3975   |  8.2306,  4.0400
0.995    |0.2429,  0.1040         0.7571,  0.8960         0.0052,  0.0046         | 50.5561,  18.4356                36.8267,  19.6875   |  9.7406,  4.3297
1        |0.0000,  0.0000         1.0000,  1.0000         0.0002,  0.0000         | 58.6114,  20.9797                nan,  nan   |  11.9370,  4.8815
```


Different fractions vs. Albedo
Frac of total #photons
that are _
Absorbed_const. τ
Escaped_const. τ
Zero scattering_const. τ
Absorbed_τ(s)
Escaped_τ(s)
Zero scattering_τ(s)
Fractions
Albedos
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


Avg. #scattering events vs. Albedo
Types of Photons
Absorbed_const. τ
Escaped_const. τ
Absorbed_τ(s)
Escaped_τ(s)
Avg. of #scattering events
Albedos
60
50
40
30
20
10
0
0.0
0.2
0.4
0.6
0.8
1.0



From the above plot of fractions vs. Albedo, we see that the graphs for distance-dependent optical depths are curvier than that of the constant optical depth ones. This shows that for increasing albedos the fractions from results of the function dependent optical depth, change rapidly compared to that of the constant optical depths.