CS 204	Discrete Mathematics	Fall 2016

Homework 7(88 pts)

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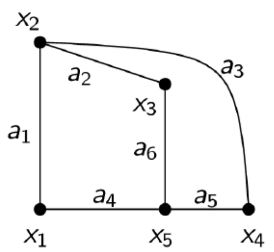
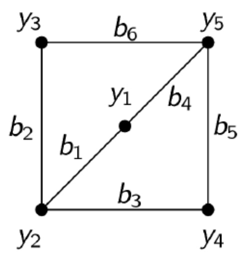
1. (10 pts) For the graphs G1 and G2 below, α and β are defined as follows to show that G1 ≅ G2.

G1:



G2:



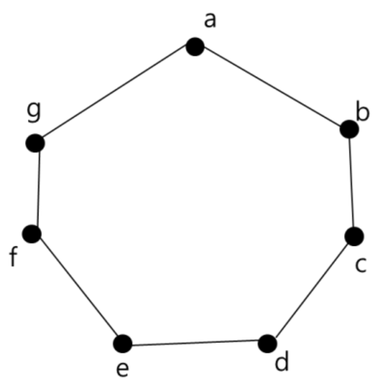
α(x1) = y4 α(x2) = y2 α(x3) = y1 α(x4) = y3 α(x5) = y5	β(a1) = b3 β(a2) = b1 β(a3) = b2	β(a4) = b5 β(a5) = b6 β(a6) = b4

Fill out the following table to check that G1 ≅ G2.

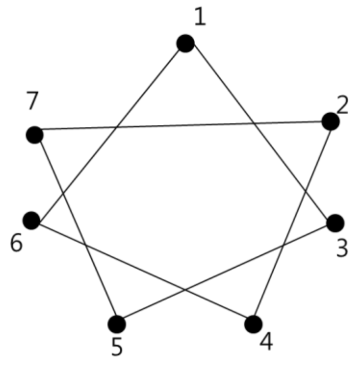
G1			α(n1)	α(n2)	Edge of G2 joining α(n1) and α(n2)	β(e)	Edge of G2 joining α(n1) and α(n2) = β(e)? (Answer with Yes/No)
n1	e	n2					
x1	a1	x2					
x2	a2	x3					
x2	a3	x4					
x1	a4	x5					
x4	a5	x5					
x3	a6	x5					

2. (10 pts) Prove that the graph G1 and G2 are isomorphic.

G1:



G2:



3. (8 pts)
   (a) Is it possible for a group of 11 teams to each play six games against different opponents in the group? How many games will be played?

   (b) Is it possible for a group of 11 teams to each play five games against different opponents in the group? If impossible, explain why it is in terms of the Euler's theorem on degrees discussed in class.

4. (10 pts) Prove that a tree with n vertices has n-1 edges.

5. (10 pts) Consider a numerical function Sum(L), where L is an SList discussed in class, as follows:

B. If L = n, then Sum(L) = n.

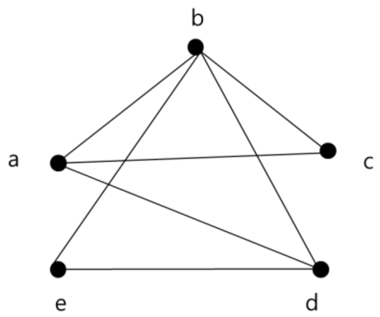
R. If L = (X, Y), then Sum(L) = Sum(X) + Sum(Y).

Prove that Sum(L) works. In other words, prove that, for any $p \geq 0$, that if $a_1, a_2, \ldots, a_{2^p}$, are the elements of L (so, there are 2^P elements altogether), then Sum(L) = $a_1 + a_2 + \ldots + a_{2^p}$.

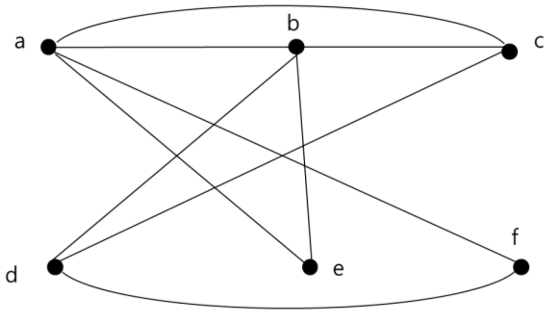
(Hint: Use induction on p.)

6. (12 pts) A graph is called *planar* if it can be drawn in the plane without its edges crossing. If a connected, planar graph is drawn in the plane, the plane is divided into contiguous regions called *faces*. A face is characterized by the cycle that forms its boundary.

G1:



G2:



(a) Show that the above graphs are planar by redrawing it so that no edges cross.

(b) How many faces do G1 and G2 have after redrawing? Describe the cycles for the faces of G1 and G2 as sequences of vertices.

7. (6 pts) If a graph is a connected planar graph with e edges, v vertices and f faces, then the following equation holds

$$f = e - v + 2$$

A connected, planar graph has nine vertices having degrees 2, 2, 2, 3, 3, 3, 4, 4 and 5. How many edges are there? How many faces are there?

8. (12 pts)

(a) Draw all non-isomorphic rooted trees (i.e. trees with their unique roots) having three vertices.

(b) Draw all non-isomorphic binary trees having four vertices.

9. (10 pts) Recall the definition of *full binary tree* discussed in class. Prove that if T is a full binary tree with i internal vertices (i.e. non-terminal nodes), then T has i+1 terminal vertices (i.e. leaf nodes) and 2i+1 total vertices.