CS 204	Discrete Mathematics	Fall 2016

Homework 4(78 pts)

Sungwon Kang — Due date Oct 20, 2016

1. (6 pts) Let A = {2, 3, 4}, B = {3, 4, 5, 6}, and suppose the universal set if U = {1, 2, 3, 4, 5, 6, 7, 8, 9}. List all the elements in the following sets.

(a) $(A \cup B)'$

(b) $(A \cap B) \times A$

(c) $\mathcal{P}(B \setminus A)$

2. (8 pts) Let the following sets be given. The universal set for this problem is the set of all students at some university.

F = the set of all freshmen.

S= the set of all seniors.

M = the set of all math majors.

C = the set of all CS majors.

(a) Using only the symbols F, S, M, C, $|\quad|$, $\cap$, $\cup$, $'$, and $>$, translate the following statement into the language of set theory.

There are more freshmen who aren't math majors than
there are senior CS majors.

(b) Translate the following statement in set theory into everyday English.

$$(F \cap M) \subseteq C$$

3. (10 pts) Two sets are called *disjoint* if they have no elements in common, i.e., if their intersection is the empty set. Prove that finite sets A and B are disjoint if and only if $|A| + |B| = |A \cup B|$. Use the definition of $\varnothing$ and the inclusion-exclusion principle discussed in class in your proof.

4. (7 pts) Let X be a finite set with $|x| > 1$. What is the difference between $P1 = X \times X$ and $P2 = \{S \in \mathcal{P}(X) \mid |S| = 2\}$? Which set, P1 or P2, has more elements?

5. (5 pts) Let P be a set of people, and let Q be a set of occupations. Define a function $f: P \to Q$ by setting f(p) equal to p's occupation. What must be true about the people in P for f to be a total function?

6. (8 pts) Let S = {0, 1, 2, 3, 4, 5}, and let $\mathbb{P}(S)^*$ be the set of all nonempty subsets of S. Define a function $m: \mathcal{P}(S)^* \to S$ by

$$m(H) = \text{the largest elements in } H$$

for any nonempty subset $H \subseteq S$.

(a) Is *m* one-to-one? Why or why not?
(b) Does *m* map $\mathcal{P}(S)^*$ onto S? Why or why not?

7. (5 pts) The following set R defines an equivalence relation on the set {1, 2, 3}, where aRb means that $(a,b) \in R$.

$$R = \{(1,1), (2,2), (3,3), (2,3), (3,2)\}$$

What are the equivalence classes?

8. (5 pts) Give a specific reason why the following set R does not define an equivalence relation on the set {1, 2, 3, 4}.

$$R = \{(1,1), (2,2), (3,3), (4,4), (2,3), (3,2), (2,4), (4,2)\}$$

9. (6 pts) Let X be a set. Define a relation *R* on $\mathcal{P}(X)$ by

$$A \mathrel{R} B \Leftrightarrow A \cap B = \varnothing$$

for $A, B \in \mathcal{P}(X)$. Determine whether this relation is reflexive, symmetric, and/or transitive.

10. (8 pts) Let X be a finite set. For subsets, $A, B \in \mathcal{P}(X)$, let $A \mathrel{R} B$ if $|A| = |B|$. This is an equivalence relation on $\mathcal{P}(X)$. If X = {1, 2, 3}, list the equivalence classes.

11. (10 pts) Define a relation *R* on $\mathbb{Z}$ by $x \mathrel{R} y$ if $x + y$ is even.

(a) Show that *R* is an equivalence relation.

(b) Describe the equivalence classes formed by this relation.