12th Homework

Prof. Helmut Alt

Introduction to Algorithms

This homework need not be submitted, is not counted, but working on it is recommended.

Exercise 1 NP *0 points*

Show that the following problems are in NP. Of which can you show that they are in P?

(a) Given a graph, does it contain a simple cycle of length 4 ?

(b) Set cover (SC):
Given sets (of integers) $S_1, ..., S_n$ and $k \in \mathbb{N}$. Are there k of the given sets whose union equals the union of all sets?

(c) Is a given natural number a the product of 2 prime numbers?

Exercise 2 reductions *0 points*

Show the following polynomial time reductions:

(a) subset sum $\leq_p$ PARTITION
Hint: $(a_1, ..., a_n, b) \mapsto (a_1, ..., a_n, |\sum_{i=1}^{n} a_i - 2b|)$

(b) Let SGI be the following problem: Given two graphs G_1, G_2. Is G_2 isomorphic to a subgraph of G_1. Show that A $\leq_p$ SGI where A is one of the NP-problems given in class.

(c) SAT $\leq_p$ 3SAT
Hint: replace any SAT-clause $(y_1 \vee ... \vee y_k), k > 3$ by 3SAT-clauses
$(y_1 \vee y_2 \vee z_1) \wedge (\overline{z_1} \vee y_3 \vee z_2) \wedge ... \wedge (\overline{z_{k-3}} \vee y_{k-1} \vee y_k)$
where $z_1, ..., z_{k-3}$ are new additional variables.

Exercise 3 SAT-solver *0 points*

A Boolean formula $\phi(x_1, ..., x_n)$ in CNF is obviously satisfiable if $\phi|_{x_1=1}$ or if $\phi|_{x_1=0}$ are satisfiable by some truth assignment to $x_2, ..., x_n$. Here, $\phi|_{x_1=1}$ means the formula ϕ simplified by omitting clauses containing the literal x_1 and taking out the literal $\overline{x_1}$ from all clauses where it occurs. $\phi|_{x_1=0}$ is defined analogously.

Once some clause contains no more literals the formula is not satisfiable, if the formula is empty (no more clauses) it is satisfiable.

Use these properties to implement a recursive *SAT-solver*, i.e., a program, that decides whether the input formula is satisfiable and, if so, produces a satisfying assignment.