CS 204	Discrete Mathematics	Fall 2016

Homework 3(85 pts)

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1. (10 pts) Consider the following definition of the "$\lhd$" symbol.

Definition. Let x and y be integers. Write $x \lhd y$ if $3x + 5y = 7k$ for some integer k.

(a) Show that $1 \lhd 5$, $3 \lhd 1$, and $0 \lhd 7$.

(b) Find a counterexample to the following statement:

If $a \lhd b$ and $c \lhd d$, then $a \cdot c \lhd b \cdot d$.

2. (10 pts) Let the following statements be given.

Definition. A triangle is *scalene* if all of its sides have different lengths.

Theorem. A triangle is scalene if it is a right triangle that is not isosceles.

Suppose ΔABC is a scalene triangle. Which of the following conclusions are valid? Why or why not?

(a) All of the sides of ΔABC have different lengths.

(b) ΔABC is a right triangle that is not isosceles.

3. (12 pts) Let P(n,x,y,z) be the predicate "$x^n + y^n = z^n$."

(a) Write the following statement in predicate logic, using positive integers as the domain.

For every positive integer n, there exist positive integers x, y, and z such that $x^n + y^n = z^n$.

(b) Formally negate your predicate logic statement from part (a). Simplify so that no quantifier lies within the scope of a negation.

(c) In order to produce a counterexample to the statement in part (a), what, specially, would you have to find among x, y, z and n?

4. (9 pts) Consider the following theorem.

Theorem. Let x be a wamel. If x has been schlumpfed, then x is a borfin.

Answer the following questions.

(a) Give the converse of this theorem.
(b) Give the contrapositive of this theorem.
(c) Which statement, (a) or (b), is logically equivalent to the Theorem?

5. (8 pts) In four-point geometry discussed in class, do triangles exist? In other words, is it possible to have three distinct points, not on the same line, such that a line passes through each pair of points? Why or why not?

6. (6 pts) Give a direct proof.

Let a, b, and c be integers. If $a \mid b$ and $b \mid c$, then $a \mid c$.

7. (8 pts) Consider the following definition.

Definition. An integer n is sane if $3 \mid (n^2 + 2n)$.

(a) Give a counterexample to the following: All odd integers are sane.
(b) Give a direct proof of the following: If $3 \mid n$, then n is sane.

8. (6 pts) Prove that the rational numbers are closed under addition. That is, prove that, if a and b are rational numbers, then a + b is a rational number.

9. (8 pts) Recall the Badda-Bing axiomatic system discussed in class. Prove:

If q and r are distinct bings, both of which are hit by baddas x and y, then x = y.

10. (8 pts) In the axiomatic system for four-point geometry, prove the following assertion using a proof by contradiction.

Suppose that a and b are distinct points on line u. Let v be a line such that u ≠ v. Then a is not on v or b is not on v.