CS470 Introduction to Artificial Intelligence
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Problem 1. Select all of the propositions from the following list:

(a) Write your names on the note.

(b) Where do you come from?

(c) Look out for that horse!

(d) Jeju Island is the largest island off the coast of the Korean Peninsula.

(e) Jupiter is the closest planet to the Sun.

(f) if x=0 then $x^2 = 0$.

(g) x+9=0

answer: (d), (e), (f)

Problem 2. Construct truth tables for each of the following propositions. Keep in mind that it may be useful to record the truth values of intermediate expressions as well (e.g., when constructing a truth table for $(q \wedge r) \Longrightarrow \neg j$, it may be helpful to first write down the truth values for $q \vee r$).

(a) $(p \vee q) \Longrightarrow (p \wedge q)$

by implication elimination,

$(p \vee q) \Longrightarrow (p \wedge q) \equiv \neg(p \vee q) \vee (p \wedge q)$, T: true, F: false

p	q	$p \vee q$	$p \wedge q$	$\neg(p \vee q)$	$\neg(p \vee q) \vee (p \wedge q)$	$(p \vee q) \Longrightarrow (p \wedge q)$
F	F	F	F	T	T	T
F	T	T	F	F	F	F
T	F	T	F	F	F	F
T	T	T	T	F	T	T

(b) $(\neg p \Leftrightarrow \neg q) \Leftrightarrow (p \Leftrightarrow q)$

by biconditional elimination,

$$(\neg p \Leftrightarrow \neg q) \Leftrightarrow (p \Leftrightarrow q) \equiv ((\neg p \Rightarrow \neg q) \wedge (\neg q \Rightarrow \neg p)) \Leftrightarrow ((p \Rightarrow q) \wedge (q \Rightarrow p))$$

by contraposition

$$((\neg p \Rightarrow \neg q) \wedge (\neg q \Rightarrow \neg p)) \Leftrightarrow ((p \Rightarrow q) \wedge (q \Rightarrow p)) \equiv ((q \Rightarrow p) \wedge (p \Rightarrow q)) \Leftrightarrow ((p \Rightarrow q) \wedge (q \Rightarrow p))$$

by commutativity of $\wedge$

$$((q \Rightarrow p) \wedge (p \Rightarrow q)) \Leftrightarrow ((p \Rightarrow q) \wedge (q \Rightarrow p)) \equiv ((p \Rightarrow q) \wedge (q \Rightarrow p)) \Leftrightarrow ((p \Rightarrow q) \wedge (q \Rightarrow p))$$

so,

$$(\neg p \Leftrightarrow \neg q) \Leftrightarrow (p \Leftrightarrow q) \equiv ((p \Rightarrow q) \wedge (q \Rightarrow p)) \Leftrightarrow ((p \Rightarrow q) \wedge (q \Rightarrow p))$$

let $R \equiv (p \Rightarrow q) \land (q \Rightarrow p)$, then,

$$(\neg p \Leftrightarrow \neg q) \Leftrightarrow (p \Leftrightarrow q) \equiv R \Leftrightarrow R$$

$R \Leftrightarrow R$ is valid(true in all models).

$(\neg p \Leftrightarrow \neg q) \Leftrightarrow (p \Leftrightarrow q)$ is valid.

So given $(\neg p \Leftrightarrow \neg q) \Leftrightarrow (p \Leftrightarrow q)$ is always true.

p	q	$\neg p$	$\neg q$	$\neg p \lor q$	$p \Rightarrow q$	$\neg q \lor p$	$q \Rightarrow p$	$(p \Rightarrow q) \land (q \Rightarrow p)$
F	F	T	T	T	T	T	T	T
F	T	T	F	T	T	F	F	F
T	F	F	T	F	F	T	T	F
T	T	F	F	T	T	T	T	T

$((p \Rightarrow q) \land (q \Rightarrow p)) \Leftrightarrow ((p \Rightarrow q) \land (q \Rightarrow p))$	$(\neg p \Leftrightarrow \neg q) \Leftrightarrow (p \Leftrightarrow q)$
T	T
T	T
T	T
T	T

(c) $(p \Leftrightarrow q) \lor (\neg q \land \neg r)$

by biconditional elimination,

$$(p \Leftrightarrow q) \lor (\neg q \land \neg r) \equiv ((p \Rightarrow q) \land (q \Rightarrow p)) \lor (\neg q \land \neg r)$$

by implication elimination

$$((p \Rightarrow q) \land (q \Rightarrow p)) \lor (\neg q \land \neg r) \equiv ((\neg p \lor q) \land (\neg q \lor p)) \lor (\neg q \land \neg r)$$

so,

$$(p \Leftrightarrow q) \lor (\neg q \land \neg r) \equiv ((\neg p \lor q) \land (\neg q \lor p)) \lor (\neg q \land \neg r)$$

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg p \lor q$	$\neg q \lor p$	$(\neg p \lor q) \land (\neg q \lor p)$	$\neg q \land \neg r$
F	F	F	T	T	T	T	T	T	T
F	F	T	T	T	F	T	T	T	F
F	T	F	T	F	T	T	F	F	F
F	T	T	T	F	F	T	F	F	F
T	F	F	F	T	T	F	T	F	T
T	F	T	F	T	F	F	T	F	F
T	T	F	F	F	T	T	T	T	F
T	T	T	F	F	F	T	T	T	F

$((\neg p \vee q) \wedge (\neg q \vee p)) \vee (\neg q \wedge \neg r)$	$(p \Leftrightarrow q) \vee (\neg q \wedge \neg r)$
T	T
T	T
F	F
F	F
T	T
F	F
T	T
T	T

Problem 3. Each item below offers a pair of compound propositions. In each case, prove whether the two are logically equivalent (i.e., prove by deriving one to another by equivalence rules such as De Morgan: $\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$). If they are not, give a truth table that show the two compound propositions have different truth values.

(a) $r \rightarrow (p \vee \neg q),\ \neg(p \wedge \neg q \wedge r)$

by implication elimination,

$$r \rightarrow (p \vee \neg q) \equiv \neg r \vee (p \vee \neg q)$$

by double-negation,

$$\neg r \vee (p \vee \neg q) \equiv \neg r \vee (\neg(\neg p) \vee \neg q)$$

by De Morgan,

$$\neg r \vee (\neg(\neg p) \vee \neg q) \equiv \neg r \vee \neg(\neg p \wedge q) \equiv \neg(r \wedge \neg p \wedge q)$$

so,

$$r \rightarrow (p \vee \neg q) \equiv \neg(r \wedge \neg p \wedge q)$$

let's implement the truth table,

p	q	r	$\neg p$	$\neg q$	$r \wedge \neg p \wedge q$	$\neg(r \wedge \neg p \wedge q)$	$r \rightarrow (p \vee \neg q)$	$p \wedge \neg q \wedge r$	$\neg(p \wedge \neg q \wedge r)$
F	F	F	T	T	F	T	T	F	T
F	F	T	T	T	F	T	T	F	T
F	T	F	T	F	F	T	T	F	T
F	T	T	T	F	T	F	F	F	T
T	F	F	F	T	F	T	T	F	T
T	F	T	F	T	F	T	T	T	F
T	T	F	F	F	F	T	T	F	T
T	T	T	F	F	F	T	T	F	T

$r \to (p \vee \neg q)$ and $\neg(p \wedge \neg q \wedge r)$ are not logically equivalent by comparing them in this truth table.

(b) $(p \vee q) \to (\neg p \vee \neg q),\ p \to \neg q$

by implication elimination,

$$(p \vee q) \to (\neg p \vee \neg q) \equiv \neg(p \vee q) \vee (\neg p \vee \neg q)$$

by De Morgan,

$$\neg(p \vee q) \vee (\neg p \vee \neg q) \equiv \neg(p \vee q) \vee \neg(p \wedge q) \equiv \neg((p \vee q) \wedge (p \wedge q))$$

by associativity of $\wedge$,

$$\neg((p \vee q) \wedge (p \wedge q)) \equiv \neg(((p \vee q) \wedge p) \wedge q)$$

by distributivity of $\wedge$ over $\vee$,

$$\neg(((p \vee q) \wedge p) \wedge q) \equiv \neg(((p \wedge p) \vee (q \wedge p)) \wedge q) \equiv \neg((p \vee (q \wedge p)) \wedge q)$$

$$\neg((p \vee (q \wedge p)) \wedge q) \equiv \neg((p \wedge q) \vee ((q \wedge p) \wedge q))$$

by commutativity of $\wedge$,

$$\neg((p \wedge q) \vee ((q \wedge p) \wedge q)) \equiv \neg((p \wedge q) \vee (p \wedge q \wedge q)) \equiv \neg((p \wedge q) \vee (p \wedge q))$$

$$\neg\big((p \wedge q) \vee (p \wedge q)\big) \equiv \neg(p \wedge q)$$

by De Morgan,

$$\neg(p \wedge q) \equiv \neg p \vee \neg q \equiv \neg(p) \vee (\neg q)$$

by implication elimination,

$$\neg(p) \vee (\neg q) \equiv p \to \neg q$$

so,

$$(p \vee q) \to (\neg p \vee \neg q) \equiv p \to \neg q$$

given two propositions are logically equivalent

(c) $p \to (\neg q \to r),\ \neg r \to p$

by implication elimination

$$p \to (\neg q \to r) \equiv p \to (q \vee r) \equiv \neg p \vee (q \vee r) \equiv \neg p \vee q \vee r$$

$$\neg r \to p \equiv r \vee p$$

let's implement the truth table,

p	q	r	$\neg p$	$\neg p \lor q \lor r$	$p \to (\neg q \to r)$	$r \lor p$	$\neg r \to p$
F	F	F	T	T	T	F	F
F	F	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	F	F	F	F
T	F	T	F	T	T	T	T
T	T	F	F	T	T	T	T
T	T	T	F	T	T	T	T

$p \to (\neg q \to r)$ and $\neg r \to p$ are not logically equivalent by comparing them in this truth table.

Problem 4. Represent the following sentences in first-order logic using quantifiers. Remember to define a consistent vocabulary and write its semantics in English.

(a) Some AI topics are symbolic.

Vocabulary:

$Symbolic(x)$: True if x is symbolic, in otherwise, it's false.

$AItopic(x)$: True if x is an AI topic, in otherwise, it's false.

Answer: $\exists x\ AItopic(x) \land Symbolic(x)$

(b) Only one CS class is named "Introduction to Artificial Intelligence".

Vocabulary:

$CSclass(x)$: True if x is a CS class, in otherwise, it's false.

$CS470(x)$: True if x is a class named "Introduction to Artificial Intelligence", in otherwise, it's false

Answer: $\exists x\ \ CSclass(x) \land CS470(x) \land \forall y\ (CSclass(y) \land CS470(y) \Longrightarrow y = x)$

(c) Everybody who takes CS470 needs to take two exams.

Vocabulary:

TakeCourse(x, y): True if x takes y course, in otherwise, it's false.

CS470: Course named CS470.

ExamNum(x): Number of exams that x needs to take.

Answer: $\forall x\ \ TakeCourse(x, CS470) \Longrightarrow (ExamNum(x) = 2)$

Problem 5. Using resolution with refutation, construct the resolution tree to show that the given query can be inferred from the given knowledge base.

- Knowledge base:
  - $\forall x\ \ Elephant(x) \lor Giraffe(x) \lor Ostrich(x) \Longrightarrow Mammal(x)$
  - $\forall x, y\ \ Offspring(x, y) \land Elephant(y) \Longrightarrow Elephant(x)$
  - $Elephant(Rocky)$
  - $Parent(Rocky, Cooper)$
  - $\forall x, y\ \ Offspring(x, y) \Leftrightarrow Parent(y, x)$
  - $\forall x\ \ Mammal(x) \Longrightarrow \exists y Parent(y, x)$

- Query: $Elephant(Cooper)$

Let's start from converting few definite clauses to CNF form.

First, $\forall x, y\ \ Offspring(x, y) \land Elephant(y) \Longrightarrow Elephant(x)$,

by implication elimination,

$$\forall x, y\ \ Offspring(x, y) \land Elephant(y) \Longrightarrow Elephant(x)$$
$$\equiv \forall x, y\ \ \neg(Offspring(x, y) \land Elephant(y)) \lor Elephant(x)$$

by De Morgan,

$$\forall x, y\ \ \neg(Offspring(x, y) \land Elephant(y)) \lor Elephant(x)$$
$$\equiv \forall x, y\ \ \neg Offspring(x, y) \lor \neg Elephant(y) \lor Elephant(x)$$

Drop universal quantifiers,

$$\neg Offspring(x, y) \lor \neg Elephant(y) \lor Elephant(x)$$

Second, $\forall x, y\ \ Offspring(x, y) \Leftrightarrow Parent(y, x)$,

by biconditional elimination,

$$\forall x, y\ \ Offspring(x, y) \Leftrightarrow Parent(y, x)$$
$$\equiv \forall x, y\ \ (Offspring(x, y) \Rightarrow Parent(y, x)) \land (Parent(y, x) \Rightarrow Offspring(x, y))$$

by implication elimination,

$$\forall x, y\ \left(Offspring(x,y) \Rightarrow Parent(y,x)\right) \wedge \left(Parent(y,x) \Rightarrow Offspring(x,y)\right)$$

$$\equiv \forall x, y\ \left(\neg Offspring(x,y) \vee Parent(y,x)\right) \wedge \left(\neg Parent(y,x) \vee Offspring(x,y)\right)$$

Drop universal quantifiers,

$$\left(\neg Offspring(x,y) \vee Parent(y,x)\right) \wedge \left(\neg Parent(y,x) \vee Offspring(x,y)\right)$$

Sentences that we are going to use from CNF$(KB \wedge \neg Elephant(Cooper))$ are

$Parent(Rocky, Cooper)$

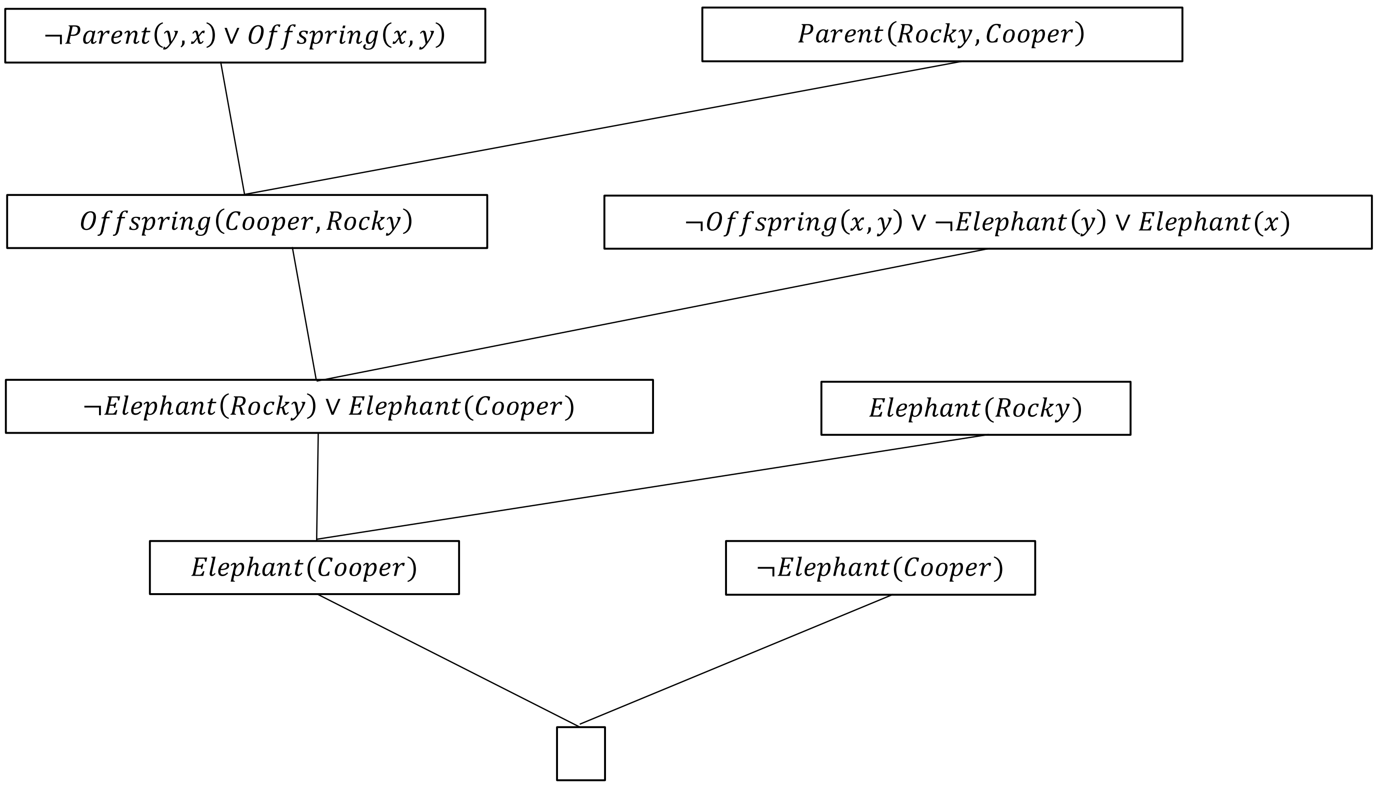
$Elephant(Rocky)$

$\neg Offspring(x,y) \vee \neg Elephant(y) \vee Elephant(x)$

$\neg Parent(y,x) \vee Offspring(x,y)$

$\neg Elephant(Cooper)$

The resolution tree is like below,



By this derived empty clause, $KB \wedge \neg Elephant(Cooper)$ is unsatisfiable,

which proves that $KB \vDash Elephant(Cooper)$.