

HW6

20140174 김형록

Date

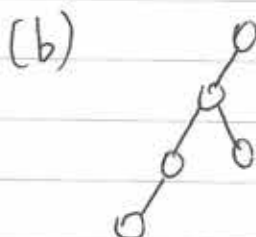
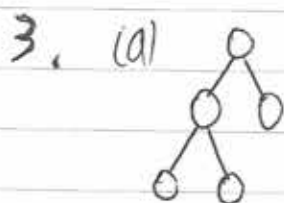
No.

1. B. empty string is in X

R. if $a \in X$, $00a$ and $0a0$ and $a00$ are also in X
if $a \in X$ and a is not an empty string,
 $1a$ and $a1$ are also in X

2. B. $0 \in X$

R. if $a \in X$, $(a-2)$ and $(a+2)$ are also in X .



4a $f(0, n) = n + 1$

$$f(1, 0) = 2, \quad f(1, 1) = f(0, f(1, 0)) = f(0, 2) = 3$$

i) if $f(1, k) = k + 2$,

$$f(1, k+1) = f(0, f(1, k)) = f(0, k+2) = k+3 = (k+1) + 2$$

and $f(1, 0) = 2 = 0 + 2$,
by ind. hyp.

$$f(1, n) = n + 2$$

ii) $f(2, 0) = f(1, 1) = 3$, $f(2, 1) = f(1, f(2, 0)) = f(1, 3) = 3 + 2 = 5$

if $f(2, k) = 2k + 3$,

$$f(2, k+1) = f(1, f(2, k)) = f(1, 2k+3) = (2k+3) + 2 = 2k+5 \\ = 2(k+1) + 3$$

and $f(2, 0) = 3 = 2 \times 0 + 3$

by ind. hyp.

$$f(2, n) = 2n + 3$$

$$f(3, 0) = f(2, 1) = 5, \quad f(3, 1) = f(2, f(3, 0)) = f(2, 5) = 13$$

$$f(3, 2) = f(2, f(3, 1)) = f(2, 13) = 29, \quad f(3, 3) = f(2, f(3, 2)) = f(2, 29) = 61$$

$$f(3, 4) = f(2, f(3, 3)) = f(2, 61) = 61 \times 2 + 3 = 125$$

$$\therefore f(3, 4) = 125$$

$$f(2, n) = 2n + 3$$

$$b \quad f(3, 0) = 5, \quad f(3, 1) = 13$$

$$\text{if } f(3, k) = 2^{k+3} - 3,$$

$$f(3, k+1) = f(2, f(3, k)) = f(2, 2^{k+3} - 3)$$

$$= 2 \times (2^{k+3} - 3) + 3 = 2^{k+4} - 3 = 2^{(k+1)+3} - 3 =$$

$$\text{and } f(3, 0) = 5 = 2^{0+3} - 3 //$$

$$\therefore \text{by ind. hyp. } f(3, n) = 2^{n+3} - 3 //$$

$$5 \quad (x_1, y_1) \leq (x_2, y_2) \text{ iff } x_1 < x_2 \text{ or } (x_1 = x_2 \text{ and } y_1 < y_2)$$

$$i) f(1, 1) = 5 = 2(1+1) + 1 //$$

$$ii) \text{ if } f(i, j) = 2(i+j) + 1 \quad \dots (1, 1) \leq (i, j) \leq (m, n)$$

$$f(i+1, j) = f(i, j) + 2 \quad \dots (\text{if } j=1)$$

$$= 2(i+j) + 3 = 2((i+1)+j) + 1 //$$

$$f(i, j+1) = f(i, j) + 2 \quad (\text{if } j \neq 1)$$

$$= 2(i+j) + 2 = 2(i+(j+1)) + 1 //$$

$$\text{by ind. hyp. } f(m, n) = 2(m+n) + 1 \text{ for all } m, n \in \mathbb{N}^+$$

6.



It has 10 edges.

7. (a) Yes, clicking a link lead to another page but there's not always a going back link.

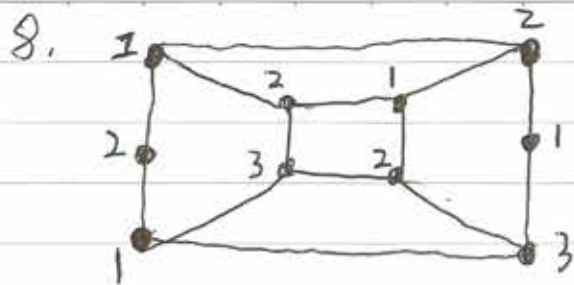
(b) No, personal private web sites can be isolated

(c) No, not all pages are linked directly.

(d) No, clicking home banner from the homepage is a loop.

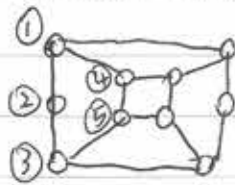
(e) number of links leading out from page P.

(f) number of links leading into page P.



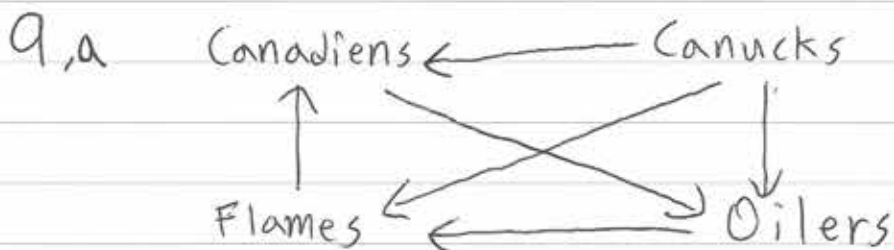
suppose that 1, 2, 3 represent each color, by putting those colors like this, there can be

using only 3 colors to correspond to the conditions in problem.



If trying to color it with two colors, if vertex ① has color a, ② and ④ will be colored with b, then ③ and ⑤

have to be colored with a, but they are directly linked, so it doesn't fit the conditions. Three colors are the minimum.



b Flames \rightarrow Canadiens \rightarrow Oilers \rightarrow Flames

c To make the rank, we need the statement saying one team is better than another. by the result, Flames won to Canadiens.

Flames \rightarrow Canadiens, This can be saying Flames is better, but the circuit says

~~to Flames~~ Canadiens \rightarrow Oilers \rightarrow Flames then

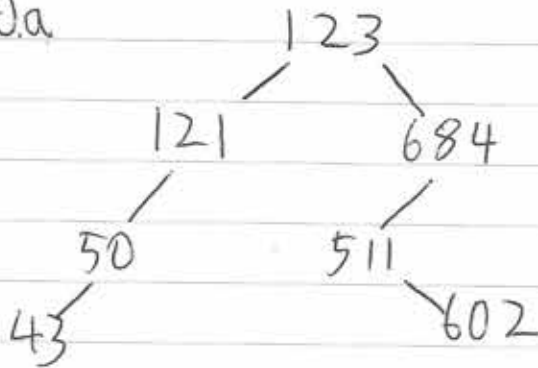
But this mean Canadiens is better than Flames by deduction. these two statements conflict each other. This is the reason that circuit makes it hard to rank.

123, 684, 121, 511, 602, 50, 43

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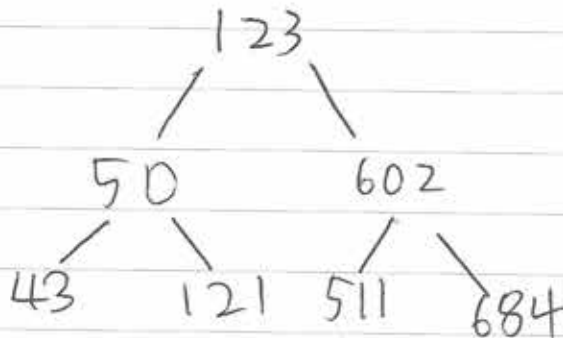
10a



b 3

(43, 50, 121, 123, 511, 602, 684)

c



height! 2

The new list is (123, 602, 684, 511, 50, 121, 43)