CS 204	Discrete Mathematics	Fall 2016
	Homework 2(75 pts)	
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1. (8 pts) In the domain of all penguins, let D(x) be the predicate "x is dangerous." Translate the following quantified statements into simple, everyday English..

(a) $(\forall x)\ D(x)$

(b) $(\exists x)\ D(x)$

(c) $\neg(\exists x)\ D(x)$

(d) $(\exists x)\ \neg D(x)$

2. (4 pts) Let the following predicates be given. The domain is all animals.

L(x) = "x is a lion."

F(x) = "x is fuzzy."

Translate the following statements into predicate logic.

(a) All lions are fuzzy.

(b) Some lions are fuzzy.

3. (6 pts) The domain of the following predicates is the set of all plants.

P(x) = "x is poisonous."

Q(x) = "Jeff has eaten x."

Translate the following statements into predicate logic.

(a) Some plants are poisonous.

(b) Jeff has never eaten a poisonous plant.

(c) There are some nonpoisonous plants that Jeff has never eaten.

4. (12 pts) In the domain of integers, consider the following predicates: Let N(x) be the statement "$x \neq 0$". Let P(x,y) be the statement that "xy = 1."

(a) Translate the following statement into the symbols of predicate logic.

For all integers x, there is some integer y such that if $x \neq 0$, then xy =1.

(b) Write the negation of your answer to part (a) in the symbols of predicate logic, Simplify your answer so that it uses the $\wedge$ connective.

(c) Translate your answer from part (b) into an English sentence.

(d) Which statement, (a) or (b), is true in the domain of integers? Explain.

5. (9 pts) The domain of the following predicates is the set of all traders who work at the Tokyo Stock Exchange.

P(x,y) = "x makes more money than y."

Q(x,y) = "$x \neq y$."

Translate the following predicate logic statements into ordinary, everyday English. (Don't simply give a word-for-word translation; try to write sentences that make sense.)

(a) $(\forall x)\ (\exists y)\ P(x,y)$

(b) $(\exists x)\ (\forall y)\ (Q(x,y) \rightarrow P(x,y))$

(c) Which statement is impossible in this context? Why?

6. (6 pts) Write the following statement in predicate logic, and negate it. Say what your predicates are, along with the domains.

Let x and y be real numbers. If x is rational and y is irrational, then x + y is irrational.

7. (10 pts) Let the following predicates be given in the domain of triangles.

$R(x)$ = "x is a right triangle."

$B(x)$ = "x has an obtuse angle."

Consider the following statements.

$$S1 = \neg(\exists x)\ (R(x) \wedge B(x))$$

$$S2 = (\forall x)\ (R(x) \rightarrow \neg B(x))$$

(a) Write a proof sequence to show that $S1 \Leftrightarrow S2$.

(b) Write S1 in ordinary English.

(c) Write S2 in ordinary English.

8. (10 pts)

(a) Give an example interpretation of a pair of predicates $P(x)$ and $Q(x)$ in some domain (in the manner of Problems 2, 3, 5 and 7) to show that the $\exists$ quantifier does not distribute over the $\wedge$ connective. That is, give an example to show that the statements

$$(\exists x)(P(x) \wedge Q(x)) \quad \text{and} \quad (\exists x)P(x) \wedge (\exists x)Q(x)$$

are not logically equivalent.

(b) It is true, however, that $\exists$ distributes over $\vee$. That is,

$$(\exists x)(P(x) \vee Q(x)) \quad \Leftrightarrow \quad (\exists x)P(x) \vee (\exists x)Q(x)$$

is an equivalence rule for predicate logic. Verify that your example from part (a) satisfies this equivalence.

9. (10 pts) Show each of the following, using only the 12 inference rules of the natural deduction discussed in class:

(a) $\exists x\ T(x), \forall x\ (T(x) \rightarrow P(x)) \vdash \exists y(T(y) \wedge P(y))$

(b) $\forall x\ (P(x) \wedge Q(x)) \vdash \forall x\ P(X) \wedge \forall y\ Q(y)$