# HW4

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1 (a) $(A\cup B)' = \{1,7,8,9\}$

(b) $(A\cap B)\times A = \{(3,2),(3,3),(3,4),(4,2),(4,3),(4,4)\}$

(c) $P(B\setminus A) = \{\emptyset, \{5\}, \{6\}, \{5,6\}\}$

2. ~~(a)~~ $|F\cap(M')| > |S\cap C|$

(b) All freshmen who are math majors, are also cs majors.

3. Need to show that

finite sets A and B disjointed $(A\cap B=\emptyset)$ $\longleftrightarrow$ $|A|+|B|=|A\cup B|$

(i) for $\rightarrow$

for finite sets A and B

$|A\cup B| = |A|+|B|-|A\cap B|$

because $A\cap B=\emptyset \Rightarrow |A\cap B|=0$

$|A\cup B| = |A|+|B|-0$

$\therefore |A|+|B| = |A\cup B|$

(ii) for $\leftarrow$

if $|A|+|B|=|A\cup B|$

because $|A\cup B| = |A|+|B|-|A\cap B|$

$|A\cup B| = |A|+|B| = |A|+|B|-|A\cap B|$

$|A\cap B|=0 \Rightarrow A\cap B=\emptyset$

finite sets A, B are disjoint

4. (i) about P2,

P2 are sets of any two elements of X, this means $|P2| = {}_nC_2$ $(n=|X|)$

$|P2| = \frac{n(n+1)}{2}$

(ii) about P1,

P1 has difference that it can even have sets that are made of two same elements.

(continuing 4)

That means $|P1| = n^2$

$\therefore |P1| = n^2$, $|P2| = \frac{n(n+1)}{2} = \frac{1}{2}n^2+\frac{1}{2}n$

$|P1|-|P2| = n^2-(\frac{1}{2}n^2+\frac{1}{2}n) = \frac{1}{2}(n^2-n)$

$= \frac{1}{2}n(n-1)$

because $|X| = n > 1$

$|P1|-|P2| > 0 \Rightarrow |P1| > |P2|$

$\therefore$ P1 has more elements.

5. to define function $f: P\rightarrow Q$

setting $f(p)$ equal to p's occupation,

every domain p's $f(p)$ has to have a single assigned element of Q.

for that,

$\therefore$ Every person has to have one occupation.

6. (a) NO, as a counterexample

$\{0,1\}\in P(S)^*$ and $\{1\}\in P(S)^*$

Then $m(\{0,1\}) = m(\{1\}) = 1$

but $\{0,1\}\neq\{1\}$

(b) Yes.

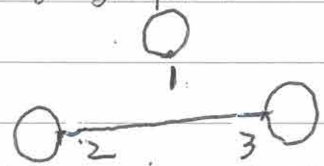
$\{\{0\},\{1\},\{2\},\{3\},\{4\},\{5\}\}\subseteq P(S)^*$

for each $\{N\}$, $(N=0,1,2,3,4,5)$ $m(\{N\}) = N$

This means for every element N in $S=\{0,1,2,3,4,5\}$

has at least one domain element $\{N\}$

7. Drawing graph associated with $(\{1,2,3\}, R)$




equivalence classes are

$\{1\}$, $\{2,3\}$

8. $R_1=\{1\}$ $R_2=\{2,3,4\}$ $R_3=\{3,2\}$ $R_4=\{4,2\}$

as a counter example,

$3R_2$ and $2R_4$ but $3 \not R 4$

It's not transitive

$\therefore$ R does not define an equivalence relation on the set $\{1,2,3,4\}$

9. (i) ① if $X \neq \emptyset$, then,

when $A \neq \emptyset$,

$A \wedge A = A \neq \emptyset$

$A \not R A \Rightarrow$ It's not reflexive

② if $X = \emptyset$,

$P(x) = \{\emptyset\}$

$A \wedge A = A = \emptyset$ It's reflexive

(ii) if $ARB \Rightarrow A \wedge B = \emptyset$

$\Rightarrow B \wedge A = \emptyset \Rightarrow BRA$

$\therefore ARB \Rightarrow BRA$

It's symmetric

(iii) if $C \in R(X)$ and $ARB$, $BRC$

$A \cap B = \emptyset$, $B \cap C = \emptyset$

about $A \cap C = ?$, as a counter example

if $A=\{1,3\}$, $B=\{2\}$, $C=\{1,4\}$

$A \wedge B = B \cap C = \emptyset$

but $A \wedge C = \{1\} \neq \emptyset$

so $A \not R C$

$\Rightarrow$ It's not transitive

10. $P(X)=\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$R_\emptyset = \{\emptyset\}$, $R_{\{1\}} = \{\{1\}, \{2\}, \{3\}\}$

$R_{\{1,2\}} = \{\{1,2\}, \{1,3\}, \{2,3\}\}$, $R_{\{1,2,3\}} = \{\{1,2,3\}\}$

Continuing 10) Date . . No.

$\therefore$ equivalence classes are

$\{\emptyset\}$, $\{\{1\}, \{2\}, \{3\}\}$, $\{\{1,2\}, \{2,3\}, \{1,3\}\}$, $\{\{1,2,3\}\}$

11. (a)

(i) for integer $a$

$aRa$ because $a+a=2a$ is even

$\therefore$ Reflexive

(ii) for integers $a, b$ ($k$ is an integer)

if $aRb \Rightarrow a+b$ is even $\Rightarrow a+b=2k$

to prove $bRa$, we need to show $b+a$ is even

$b+a = a+b = 2k$ is even

$aRb \Rightarrow bRa$

$\therefore$ Symmetric

(iii) for integers $a, b, c$

if $aRb$, $bRc$

$a+b=2k_1$ $b+c=2k_2$ ($k_1, k_2$ are integers)

$(a+b)+(b+c) = 2k_1+2k_2$

$a+2b+c = 2(k_1+k_2)$

$a+c = 2(k_1+k_2-b)$ ($k_1+k_2-b$ is an integer)

$a+c$ is even

$\Rightarrow aRc$ $\therefore$ Transitive

By these three prooves,

$\therefore$ R is an equivalence relation.

(b) for any even number $2k$, ($k$ is an integer)

if $(2k)Rb \Rightarrow 2k+b = 2k'$ ($k'$ is an integer)

$b = 2(k'-k)$ is even

for any odd number $2k+1$,

if $(2k+1)R_b \Rightarrow (2k+1)+b = 2k'$

$b = 2(k'-k)-1$ is odd

for $aR_b$ if $a$ is even, $b$ is even

if $a$ is odd, $b$ is odd,

and because $k$ can be any integer,

equivalence classes are

one set of all of even numbers

and one set of all of odd numbers.

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