HW2

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1. (a) $(\forall x) D(x)$
   All penguins are dangerous.

   (b) $(\exists x) D(x)$
   some penguins are dangerous.

   (c) $\neg(\exists x) D(x)$
   It's not true that, some penguins are dangerous.

   (d) $(\exists x) \neg D(x)$
   some penguins are not dangerous.

2. (a) $(\forall x)(L(x) \rightarrow F(x))$
   (b) $(\exists x)(L(x) \rightarrow F(x))$

3. (a) $(\exists x)(P(x))$
   (b) $(\forall x)(P(x) \rightarrow \neg Q(x))$
   (c) $(\exists x)(\neg P(x) \wedge \neg Q(x))$

4. (a) $(\forall x)(\exists y)(N(x) \rightarrow P(x,y))$
   (b) $\neg((\forall x)(\exists y)(N(x) \rightarrow P(x,y)))$
   $\Rightarrow (\exists x)(\forall y)(\neg(N(x) \rightarrow P(x,y)))$
   $\Rightarrow (\exists x)(\forall y)(\neg(\neg N(x) \vee P(x,y)))$
   $\Rightarrow (\exists x)(\forall y)(N(x) \wedge \neg P(x,y))$

   (c) For all y as an integer, and for some integer x, if $x \neq 0$, $xy \neq 1$.

   (d) (b) is true because.
   if $x \neq 0$, y as $\frac{1}{x}$ only makes $xy = 1$, but y has to be an integer.
   so if $x > 1$, $xy \neq 1$.

5. (a)

All traders in Tokyo stock Exchange makes more money than some traders in Tokyo stock Exchange.

(b) For all traders in Tokyo stock Exchange, there are some other traders in Tokyo stock Exchange making more money than them.

(c) (a) is impossible because is $x=y$, x cannot make more money than y.

6. domain is real number

$P(x)$: x is rational

$(\forall x)(\forall y)((P(x) \wedge \neg P(y)) \rightarrow \neg P(x+y))$

negate $\Rightarrow$ $(\exists x)(\exists y)((P(x) \wedge \neg P(y)) \wedge P(x+y))$

$\Rightarrow$ $(\exists x)(\exists y)(P(x) \wedge \neg P(y) \wedge P(x+y))$

7. (a)

	Statement	Reasons
1.	$\neg(\exists x)(R(x) \wedge B(x))$	given
2.	$(\forall x) \neg(R(x) \wedge B(x))$	existential negation
3.	$(\forall x)(\neg R(x) \vee \neg B(x))$	de morgan's law
4.	$(\forall x)(\neg(\neg R(x)) \rightarrow \neg B(x))$	implication
5.	$(\forall x)(R(x) \rightarrow \neg B(x))$	Double negation

(b) It's not true that some right triangle has an obtuse angle.

(c) All right triangles don't have an obtuse angle.

8. (a) domain : x is a human

$P(x)$: x has blue eyes

$Q(x)$: x has black skin.

$(\exists x)(P(x) \wedge Q(x))$ $\Rightarrow$ some human has blue eyes and black skin

$(\exists x)P(x) \wedge (\exists x)Q(x)$ $\Rightarrow$ some human has blue eyes and some human has black skin.

(b) $(\exists x)(P(x) \wedge Q(x))$ $\Rightarrow$ some human has blue eyes or black skin

Same $\leftarrow$ $(\exists x)(P(x)) \vee (\exists x)Q(x)$ $\Rightarrow$ some human has blue eyes or some has black skin

9. (a)

	Statement	reason
1.	$\exists x\, T(x)$	given
2.	$\forall x\,(T(x) \to P(x))$	given
3.	$T(x) \to P(x)$	$\forall$-elim 2
4.	$T(x)$	premise
5.	$P(x)$	$\to$elim 3, 4
6.	$T(x) \wedge P(x)$	$\wedge$-intro 4, 5
7.	$T(x) \wedge P(x)$	$\exists$-elim 4, 6
8.	$\exists x\, T(x) \wedge P(x)$	$\exists$-intro 7.

(b)

1.	$\forall x\,(P(x) \wedge Q(x))$	given
2.	$P(x) \wedge Q(x)$	$\forall$-elim 1
3.	$P(x)$	$\wedge$-elim 2
4.	$Q(x)$	$\wedge$-elim 2
5.	$\forall x\, P(x)$	$\forall$-intro 3
6.	$\forall x\, Q(x)$	$\forall$-intro 4
7.	$\forall x\, P(x) \wedge \forall x Q(x)$	$\wedge$-intro 5, 6