

Introduction to Algorithms

Due March 27, 2017, 10 a.m.

Exercise 1

6 points

- (a) Show that an algorithm to find the median in linear time can be used to solve the general selection problem in linear time.
- (b) Suppose you are given a deterministic (i.e. not randomized) algorithm A of linear runtime for the selection problem (such algorithms exist). Explain how A can be used to design a deterministic variant of Quicksort of runtime $O(n \log n)$ in the worst case.

Exercise 2

8 points

Implement the randomized selection algorithm (“Quickselect”) and determine experimentally the constant in the $O(n)$ many comparisons, and in the $O(n)$ runtime on your computer (in seconds).

Exercise 3

6 points

- (a) Give the comparison tree for Mergesort (splitting into subsequences of sizes $\lfloor n/2 \rfloor$ and $\lceil n/2 \rceil$) and $n = 3$.
- (b) Show by a comparison tree argument that any comparison based algorithm for searching an array $A[1], \dots, A[n]$ needs at least $\lceil \log n \rceil$ comparisons in the worst case.