HW5

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1. (a) $H(0)=0$, $H(1)=1$, $H(2)=1$, $H(3)=1+1-0=2$, $H(4)=2+1-1=2$
$H(5)=2+2-1=3$, $H(6)=3+2-2=3$, $H(7)=3+3-2=4$, $H(8)=4+3-3=4$
$H(9)=4+4-3=5$, $H(10)=5+4-4=5$
$\therefore H(1)=1, H(2)=1, H(3)=2, H(4)=2, H(5)=3, H(6)=3, H(7)=4, H(8)=4, H(9)=5, H(10)=5$

(b) $H(0)=0$ $H(2)=1$ $H(4)=2$ $H(6)=3$ --- $H(98)=49$ $\boxed{H(100)=50}$

2. (a) Let Number of edges in $K_{3,n} \Rightarrow N_n$
$N_n = N_{n-1} + |V_1| = N_{n-1} + 3$

(b) Let Number of edges in $K_{n,n} \Rightarrow N_n$
$N_n = N_{n-1} + (|V_2| + |V_1| - 1) = N_{n-1} + 2n - 1$

3. (i) $C(0) = 0 = \frac{3^{0+1}-2\times 0-3}{4} = \frac{3-3}{4} = 0$
(ii) if $C(K-1) = \frac{3^{(K-1)+1}-2(K-1)-3}{4} = \frac{3^{K}-2K-1}{4}$
$C(K) = K + 3\cdot C(K-1) = K + \frac{3^{K+1}-6K-3}{4} = \frac{3^{K+1}-6K-3+4K}{4}$
$= \frac{3^{K+1}-2K-3}{4}$
$\therefore$ by Inductive Hypothesis,
$C(n) = \frac{3^{n+1}-2n-3}{4}$ for all $n \geq 0$

4 (a) $G(0)=1$, $G(1)=1+2-1=2$, $G(2)=2+4-1=5$, $G(3)=5+6-1=10$
$G(4)=10+8-1=17$, $G(5)=17+10-1=26$

(b)
1 2 5 10 17 26
1 3 5 7 9
2 2 2 2

The second sequence of differences is constant.

$G(n) = An^2 + Bn + C$
$G(0) = C = 1$ $G(1) = A+B+C = A+B+1 = 2$
$G(2) = 4A+2B+C = 4A+2B+1 = 5$
$\Rightarrow A+B=1$ $2A+B=2$ $\Rightarrow \boxed{\therefore G(n) = n^2+1}$
$A=1$ $B=0$ $C=1$

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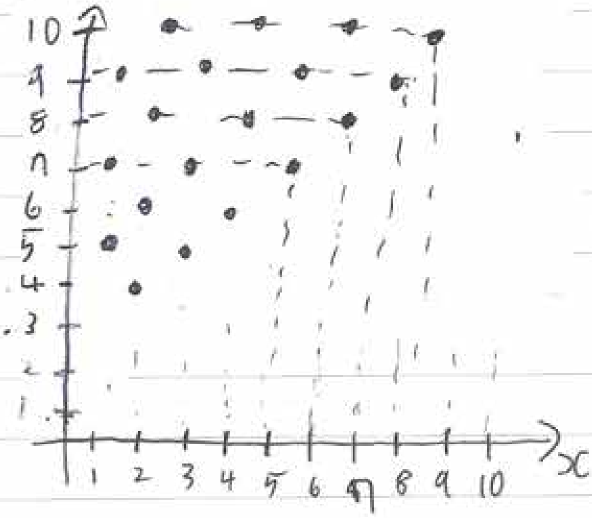
(c) (i) for $G(0) = 1 = 0^2+1$

(ii) if $G(k-1) = (k-1)^2+1$

$G(k) = G(k-1)+2k-1 = (k-1)^2+1+2k-1 = k^2-2k+1+1+2k-1 = k^2+1$

by inductive hypothesis, $\boxed{G(n) = n^2+1}$

5. (a)




6 (a) $\phi, \{\phi\}, \{\phi, \{\phi\}\}, \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$

(b) Let the components of $S$ be named $X_n$ and $X_0 = \phi$

$X_1 = \{\phi\}$ $X_2 = \{\phi, \{\phi\}\}$, ......

There is allways $X_{k+1} = \{X_k, \{X_{k+1}\}\}$

and $X_{k+1} \neq X_k$ and $k$ has no limit,

$S$ has infinitely many elements

7. (a) (i) line map without a line has 1 regions $\geq 0+1=1$

(ii) suppose a line map with $k$ distinct lines has at least $k+1$ regions,

when a line is added to be $k+1$ distinct lines, at least one region

is cut into two, the number of regions $\geq k+2$

$\therefore$ by ind. hyp. linemap with $n$ distinct lines has at least $n+1$ regions.

(b) (i) line map with a line has 2 regions $\leq 2^1=2$

(ii) suppose a line map with $k$ distinct lines has most $2^k$ regions,

when a extra line cut every $2^k$ regions, new line map with $k+1$

lines has $2^k \times 2 = 2^{k+1}$ regions as the most,

$\therefore$ by ind. hyp. linemap with $n$ distinct line has at

most $2^n$ regions.

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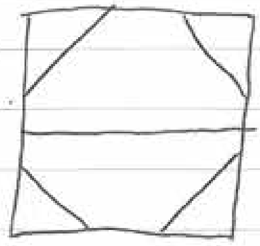
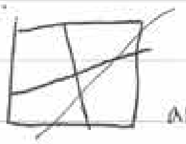
(c)



(d)



when 2 lines make the most numbers of regions, those has to cross each other, and getting into map with 3 lines, the new line cannot cut every four region. because not every region has external tangent(접선) with each other. In maximum there can be at most 3 region getting crossed, then the maximum region number is $3\times2+1 = 7 \neq 8$

8. (i) for $n=1$
$H(2) = H(1) = 1 = 1$

(ii) if for any $1 \le i \le K$ $H(2i) = H(2i-1) = i$,

$H(2(K+1)) = H(2K+2) = H(2K+1) + H(2K) - H(2K-1)$

$= H(2K+1) + (\underbrace{H(2K) - H(2K-1)}_{0}) = H(2K+1) = H(2(K+1)-1)$

$H(2K+1) = H(2K) + H(2K-1) - H(2K-2) = K + K - (K-1) = K+1$

$\Rightarrow H(2(K+1)) = H(2(K+1)-1) = K+1$ //

$\therefore$ by strong induction, $H(2n) = H(2n-1) = n$

9. (i) for $n=1, 2$.

$\left(\alpha = \frac{1+\sqrt5}{2}, \beta = \frac{1-\sqrt5}{2}\right)$

$\frac{1+\sqrt5}{2} + \frac{1-\sqrt5}{2} = 1 = L(1)$

$\left(\frac{1+\sqrt5}{2}\right)^2 + \left(\frac{1-\sqrt5}{2}\right)^2 = \frac{12}{4} = 3 = L(2)$ $\therefore$ true

(ii) for $1 \le i \le K$
suppose $L(i) = \alpha^i + \beta^i$

$L(K+1) = L(K) + L(K-1) = \alpha^K + \beta^K + \alpha^{K-1} + \beta^{K-1}$

$= \alpha^{K-1}(1+\alpha) + \beta^{K-1}(1+\beta)$

$= \alpha^{K-1}\left(\frac{3+\sqrt5}{2}\right) + \beta^{K-1}\left(\frac{3-\sqrt5}{2}\right)$

because $\alpha^2 = \frac{3+\sqrt5}{2}$, $\beta^2 = \frac{3-\sqrt5}{2}$,

$L(K+1) = \alpha^{K+1} + \beta^{K+1}$ //

$\therefore$ by strong induction, $L(n) = \alpha^n + \beta^n$

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10. Let if

The sum of the numbers in any Q-sequence is 4

B. $\langle x, 4-x \rangle \Rightarrow$ sum of $\langle x, 4-x \rangle = x+4-x = 4$ //

R. If sum of $\langle x_1, x_2, \cdots x_{j-1}, x_j \rangle = x_1 + x_2 + \cdots + x_j = 4$

and sum of $\langle y_1, y_2, \cdots, y_k \rangle = y_1 + y_2 + \cdots + y_k = 4$

sum of $\langle x_1 - 1, x_2, \cdots, x_{j-1}, x_j, y_1, y_2, \cdots y_{k-1}, y_k - 3 \rangle$

$= -1 + (x_1 + \cdots + x_j) + (y_1 + \cdots + y_k) - 3 = -4 + 4 + 4 = 4$ //

$\therefore$ The sum of the numbers in any Q-sequence is 4