CS 204 Discrete Mathematics Fall 2016

Homework 6(85 pts)

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1. (8 pts) Give a recursive definition for the set X of all binary strings with an even number of 0's.

2. (5 pts) Give a recursive definition for the set X of even integers (including both positive and negative even integers).

3. (6 pts) We gave a recursive definition of a binary tree in class. Suppose we modify this definition by deleting part B1, so that an empty tree is not a binary tree. A tree satisfying this revised definition is called a *full binary tree*.

(a) Give an example of a full binary tree with five nodes.

(b) Give an example of a binary tree with five nodes that is not a fully binary tree.

4. (12 pts) Consider the following recursively defined function

$$f(m,n) = \begin{cases} n + 1 & \text{if } m = 0 \\ f(m-1, 1) & \text{if } m > 0 \text{ and } n = 0 \\ f(m-1, f(m, n-1)) & \text{if } m > 0 \text{ and } n > 0 \end{cases}$$

a) What is the value of f(3,4)?

b) Prove that $f(3,n) = 2^{n+3} - 3$.

5. (10 pts) Let

$$f(m, n) = \begin{cases} 5 & \text{if } m = n = 1 \\ f(m-1,n) + 2 & \text{if } n = 1 \text{ and } m > 1 \\ f(m, n-1) + 2 & \text{if } n > 1 \end{cases}$$

Prove that

$$f(m,n) = 2(m+n) + 1 \text{ for all, } m, n \in \mathbb{N}^{+}.$$

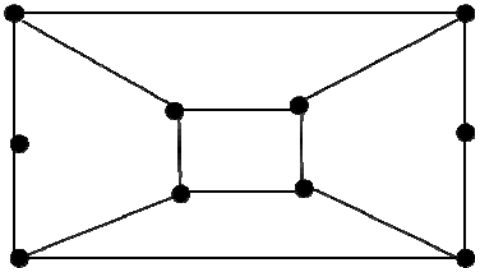
(Hint: First, define (x1,y1) <= (x2,y2) iff x1 < x2 or (x1 = x2 and y1 < y2). Then use (m,n) = (1,1) as the basis case.)

6. (6 pts) An undirected graph is called *complete* if every vertex shares an edge with every other vertex. Draw a complete graph on five vertices. How many edges does it have?

7. (12 pts) Think of the Internet as one big graph, where each web page is a vertex and each link is an edge.

(a) Is this a directed graph? Why or why not?
(b) Is this graph connected? Why or why not?
(c) Is this graph complete? Why or why not?
(d) Is this graph simple? Why or why not?
(e) For a given web page p, what does the outdegree of p represent?
(e) For a given web page p, what does the indegree of p represent?

8. (8 pts) Color the vertices of the following graph so that no vertices of the same color share an edge. Use as few colors as possible. Explain why the graph cannot be colored using fewer colors. Be specific.



9. (9 pts) A round-robin tournament among four teams - Canadiens, Canucks, Flames, and Oilers - has the following results: Canucks defeat Canadiens; Canucks defeat Flames; Canucks defeat Oilers; Canadiens defeat Oilers; Flames defeat Canadiens; Oilers defeat Flames.

(a) Model these results with a directed graph, where ach vertex represents a team and each edge represents a game, pointing from the winner to the loser.

(b) Find a circuit in this graph.

(c) Explain why the existence of a circuit in such a graph makes it hard to rank the teams from best to worst.

10. (9 pts) Consider the following list of numbers.

123, 684, 121, 511, 602, 50, 43

(a) Place the numbers, in the order given, into a binary search tree.

(b) The height of a binary search tree is the maximum number of edges you have to go through to reach the bottom of the tree, starting at the root. What is the height of the tree in part (a)?

(c) Reorder the numbers so that when they are put into a binary search tree, the height of the resulting tree is less than the height of the tree in part (a). Give both your new list and the search tree it produces.