8th Homework

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## Introduction to Algorithms

**Due:** May 8, 2017, 10 a.m.

**Exercise 1** UNION-FIND *10 points*

(a) Give the state of the UNION-FIND data structure after the following sequence of operations, starting from singleton sets $\{1\}, ..., \{8\}$. Use path compression. In case of ties, always make the lower numbered root point to the higher numbered one.
$union(1,2); union(3,4); union(5,6); union(7,8); union(2,4); union(6,8);$
$union(4,8); find(1).$

(b) Suppose that starting from the singleton partititon $\{x_1\}, ..., \{x_n\}$ of some set $S = \{x_1, ..., x_n\}$ first some UNION- and then some FIND-Operations are carried out, altogether $m$ operations. Suppose that union-by-height and path-compression is used. Show that the amortized runtime per operation is $\Theta(1)$, i.e., the total runtime of these operations is $\Theta(m)$.

*Hint:* Let $k$ be the number of times the parent pointer of some vertex is accessed if it is pointing to the root of some tree. Find an upper bound on $k$? How often is a parent pointer accessed otherwise before it is pointing to the root again.

**Exercise 2** greedy *10 points*

(a) Suppose that a long straight stretch of a highway has to be equipped with mobile phone base stations. The company has investigated that possible positions for stations are at kilometers $x_1, ..., x_n$ from the beginning of the highway stretch. No point of the highway should be more than 5 km away from the nearest station. Give an algorithm to find the minimum number of stations to achieve that goal, if possible.

(b) A server has $n$ customers waiting to be served. The service time required by each customer is known in advance: it is $t_i$ minutes for customer $i$. So if, for example, the customers are served in order of increasing $i$, then the ith customer has to wait $\sum_{j=1}^{i} t_j$ minutes. We wish to minimize the total waiting time
$\sum_{i=1}^{n}$(time spent waiting by customer $i$).
Give an efficient algorithm for computing the optimal order in which to process the customers.