HW 9

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Date No.

1. $\frac{3\times 99P_2}{100P_3} = \frac{3\times 99\times 98}{100\times 99\times 98} = \frac{3}{100}$

2. (a) $1-\left(\frac{5}{6}\right)^4 \simeq 0.518$

(b) $1-\left(1-\frac{1}{6^2}\right)^{24} \simeq 0.49 < \frac{1}{2}$

(c) a six in roll of a die four times is more likely to come out.

3. (a) $-2-3-$ or $-3-2-$
$-N-N-N-$
(when one of {1,4} goes in)
$\frac{3\times 4}{4!} = \frac{1}{2}$

(b) $1-\frac{1}{2} = \frac{1}{2}$

(c) $-4-1-2-$ or $-4-2-1-$ $\quad \frac{8}{24} = \frac{1}{3}$

(d) $4, \{1,2,3\}$ $\quad \frac{3!}{4!} = \frac{1}{4}$

(e) $\left(\frac{3\times 4}{4!}\right)^2 = \frac{1}{4}$

4. (a) $\left(\frac{1}{2}\right)^5 = 0.03125$

(b) $(1-0.1)^5 \simeq 0.59$

(c) ~~$(0.50)(0.49)(0.48)(0.47)(0.46) \simeq$~~
$(0.50)(0.51)(0.52)(0.53)(0.54) \simeq 0.038$

5. $50\times 2\times 0.9 + 25\times 4\times 0.8 = 170$

6. $E(X) = \frac{1+2+\cdots+6}{6} = \frac{7}{2}$

$E(Y) = \frac{1}{36}(2\times1 + 3\times2 + 4\times3 + 5\times4 + 6\times5 + 7\times6 + 8\times5 + 9\times4 + 10\times3 + 11\times2 + 12)$

$= 7$

$E(XY) = \frac{1}{36}(1\times(2+3+4+5+6+7) + 2\times(3+4+5+6+7+8) + 3\times(4+5+6+7+8+9)$

$+ 4\times(5+6+7+8+9+10) + 5\times(6+7+8+9+10+11) + 6\times(7+8+9+10+11+12))$

$= \frac{987}{36} \simeq 27.42$

$E(X)\times E(Y) = \frac{49}{2} = 24.5$

$E(X)E(Y) \neq E(XY)$

7. $P(\text{both are boys} \mid \text{at least one is a boy}) = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \boxed{\frac{1}{3}}$

8. $P(\overset{\text{Knows the}}{\text{answer}}) = p \quad P(\text{Guess}) = 1-p$

$P(\text{correct} \mid \text{Guess}) = \frac{1}{m}$

$\frac{p}{p+(1-p)\frac{1}{m}} = \frac{mp}{mp+1-p} = \boxed{\frac{mp}{(m-1)p+1}}$

9 Y: $\overset{\text{Tested}}{\text{positive}}$ D: people having disease H: people healthy

$P(D) = 0.005 \quad P(Y|D) = 0.95 \quad P(H) = 0.995$

$P(Y|H) = 0.01$

$P(D|Y) = \frac{P(D)P(Y|D)}{P(D)P(Y|D) + P(H)P(Y|H)}$

$= \frac{0.005\times0.95}{0.005\times0.95 + 0.995\times0.01} = \frac{95}{294} = \boxed{0.323}$

10. $\frac{4(2\times2C_2)}{2^4} = \frac{4\times3}{16} = \frac{3}{8} = \boxed{0.375}$