HW 7

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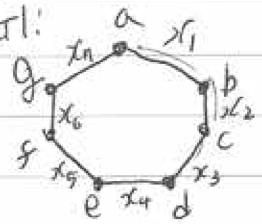
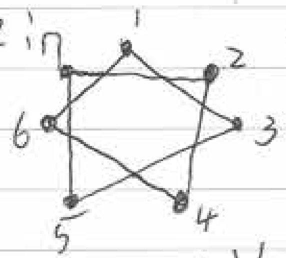
1.

G1			$\alpha(n1)$	$\alpha(n2)$	Edge of G2 Joining $\alpha(n1)$ and $\alpha(n2)$	$\beta(e)$	Edge of G2 Joining $\alpha(n1)$ and $\alpha(n2) = \beta(e)$? (Answer with Yes/No)
n1	e	n2					
x_1	a_1	x_2	y_4	y_2	b_3	b_3	Yes
x_2	a_2	x_3	y_2	y_1	b_1	b_1	Yes
x_2	a_3	x_4	y_2	y_3	b_2	b_2	Yes
x_1	a_4	x_5	y_4	y_5	b_5	b_5	Yes
x_4	a_5	x_5	y_3	y_5	b_6	b_6	Yes
x_3	a_6	x_5	y_1	y_5	b_4	b_4	Yes

2. G1:



G2 in



Edge made of 1&3 : y_1
" 3&5 : y_2
5&7 : y_3
7&2 : y_4
2&4 = y_5
4&6 : y_6
6&1 : y_7

Define α, β (one-to one) $\alpha : V_{G_1} \to V_{G_2}$ $\beta : E_{G1} \to E_{G2}$

$\alpha(a) = 1$ $\beta(x_1) = y_1$
$\alpha(b) = 3$ $\beta(x_2) = y_2$
$\alpha(c) = 5$ $\beta(x_3) = y_3$
$\alpha(d) = 7$ $\beta(x_4) = y_4$
$\alpha(e) = 2$ $\beta(x_5) = y_5$
$\alpha(f) = 4$ $\beta(x_6) = y_6$
$\alpha(g) = 6$ $\beta(x_7) = y_7$

then for any edge $e \in E_{G1}$, e joins vertex v to vertex w if and only if $\beta(e)$ joins vertex $\alpha(v)$ to vertex $\alpha(w)$ //

$\therefore$ Therefore $G1 \cong G2$

3. a For each of 11 teams to play 6 games each against opponents in the group, need $(6 \times 11)/2 = 33$ games
That's, 66 playing teams (=degree) and 33 games (= edges)
It's possible //

b For each of 11 teams to play 5 games against different opponents in the group, The sum of degree (total playing teams) is $5 \times 11 = 55$ and by Euler's theorem, the number of edges (total number of games) is $\frac{55}{2}$ is not a natural number, so It is not possible //

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4. (i) B: when there's only 1 vertex, there's 0 edge.

(ii) Suppose that if there are k vertices making a tree, it has $k-1$ edges. when a vertex is added, the new vertex gets connected with only one vertex, That makes one more edge, so there will be $k+1$ vertices and k edges.

by ind. hyp. The statement is true.

It's general to say that new vertex is added below the existing tree, then it's connected with only the higher one vertex

5. (i) when $p=0$, $2^p=1 \Rightarrow sum(L) = sum(a1) = a_1$

(ii) for $0 \le i \le p-1$, if $(a_1, a_2, \cdots, a_{2^i})$ are the elements of L,

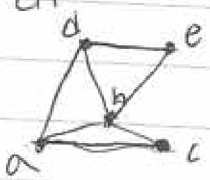
if $sum(L) = a_1 + a_2 + \cdots + a_{2^i}$ is true,

for $p \Rightarrow a_1, \cdots, a_{2^p}$ are the elements of L,

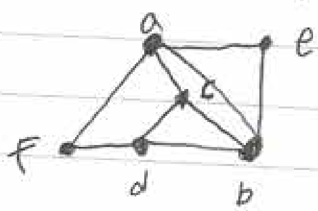
$$sum(L) = sum(a_1, a_2, \cdots, a_{2^p})$$
$$= sum(a_1, \cdots, a_{2^{p-1}}) + sum(a_{(2^{p-1}+1)}, \cdots, a_{2^p})$$
$$= (a_1 + \cdots + a_{2^{p-1}}) + (a_{(2^{p-1}+1)} + \cdots + a_{2^p})$$
$$= a_1 + a_2 + \cdots + a_{2^p}$$

$\therefore$ by ind. hyp, for any $p \ge 0$, if $a_1, a_2, \cdots, a_{2^p}$ are the elements of L, $sum(L) = a_1 + a_2 + \cdots + a_{2^p}$.

6. a G1



G2



b G1 has 3 faces (dba), (ebd), (acb)

G2 has 4 faces (afdc), (cdb), (abc), (aeb)

7. $e = (2\times3 + 3\times3 + 4\times2 + 5)/2 = 14$ (Euler's theorem)

$v = 9$ (given)

$f = 14 - 9 + 2 = 7$

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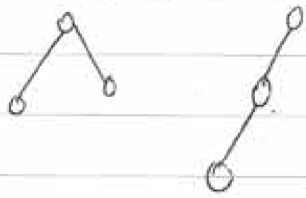
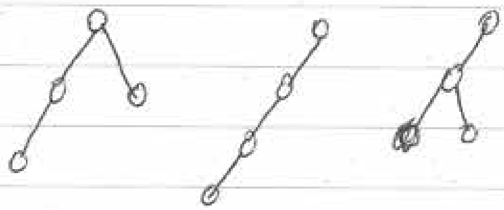
8.a

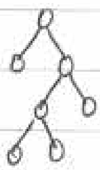


b

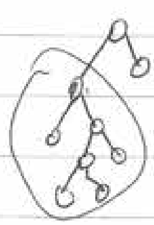


9. when $i=0$, there is 0 internal node and 1 terminal vertex.
Suppose that when $i=k$, T has k internal vertices and $k+1$ terminal vertices. ⇒ $2k+1$ in total

Think about adding one more internal vertex to T,



(i) because all internal vertices have each 2 terminal vertices and every terminal vertex has no more vertices connected below them, adding 2 vertices to an existing terminal vertex makes that terminal vertex into an internal vertex and makes 2 extra terminal vertices. (adding just one vertex is not possible because it makes an internal vertex with only one termial vertex connected to it.)
then new T has $k+1$ internal vertices and $(k+1)-1+2 = k+2$ terminal vertices ⇒ $2(k+1)+1$ total



(ii) we can add an internal vertex on the top of T, and to make it full an terminal vertex to it.
then new T has $k+1$ internal vertices and $k+1+1=k+2$ terminal vertices. ⇒ $2(k+1)+1$ total

by ind. hyp. if T has i internal vertices, T has $i+1$ in terminal and $2i+1$ in total.