

Introduction to Algorithms

Due April 17, 2017, 10 a.m.

Exercise 1

6 points

Design a graph-theoretic model for the search of train connections in a railway system, like, e.g., <http://www.hyperdia.com/> (I couldn't find one for Korea). Consider possibilities for transfer, the time needed for transferring, and departure times. Which means, indicate the meaning of vertices and edges of the graph, as well as the edge costs. What algorithm should be used when a customer types in departure and destination station and desired earliest departure time. Explain why your model solves the problem and demonstrate it with a few examples.

Exercise 2

7 points

- (a) Modify Dijkstra's algorithm so that it not only gives the lengths of the shortest paths but also a structure for finding the shortest paths themselves.
- (b) What is the space requirement of the algorithm of Floyd-Warshall for a graph with n vertices if the values $d_{i,j}^k$, $k = 0, \dots, n$; $i, j = 1, \dots, n$ are just stored in a three-dimensional array? How can it be improved?
- (c) Modify Floyd-Warshall's algorithm so that it not only gives the lengths of the shortest paths but also a structure for finding the shortest paths themselves.

Hint: Compute each predecessor vertex $\pi_{i,j}^k$ of vertex j in some shortest path from i to j with intermediate vertices in $\{1, \dots, k\}$.

Exercise 3

7 points

Implement Floyd-Warshall's algorithm. As input for a directed graph with n vertices and edge costs use an $n \times n$ -matrix where the (i, j) th entry is $c(i, j)$ if (i, j) is an edge of the graph and ∞ otherwise.