

Introduction to Algorithms**Due:** May 1, 2017, 10 a.m.

Exercise 1

8 points

In the main loop of Floyd-Warshall's algorithm there is the instruction

$$d_{i,j}^k := \min(d_{i,j}^{k-1}, d_{i,k}^{k-1} + d_{k,j}^{k-1}),$$

where the costs of the edges are from the set $\mathbb{R}_{\geq 0}$.

Give application examples if the operations \min and $+$ and the cost set $\mathbb{R}_{\geq 0}$ are replaced by

- (a) \min, \max , and \mathbb{R}
- (b) \max, \min , and \mathbb{R}
- (c) $\max, *$ (multiplication), and $[0, 1]$
- (d) $+, *$, and $\{0, 1\}$
where all pairs of vertices (u, v) are assigned 0 or 1: 1 if (u, v) is an edge and 0 otherwise.

In each case also indicate, how the values $d_{i,j}^0$ are set in the initialization phase of the algorithm.

Exercise 2

7 points

- (a) Just as a warmup: In how many ways can you change 200 Won in 10-, 50-, and 100-Won coins?
- (b) Generalize Problem (a): Suppose some currency, call it Penny, has coins of values c_1, \dots, c_k . Find an algorithm which determines in how many ways an amount of n Pennies can be changed into coins in that currency.

Hint: Show that, if $w_{i,m}$ is the number of ways to change m Pennies into coins only from c_1, \dots, c_i then

$$w_{i,0} = 1, w_{0,m} = 0 \text{ for all } i \in \{0, \dots, k\}, m \in \mathbb{N} \text{ and}$$

$$w_{i,m} = \begin{cases} w_{i,m-c_i} + w_{i-1,m} & \text{if } m \geq c_i \\ w_{i-1,m} & \text{if } m < c_i \end{cases}$$

for $i \geq 1$.

This formula can be used to compute $w_{k,n}$ by dynamic programming.

What is the runtime of the resulting algorithm assuming that k is a constant?

- (c) Implement the algorithm of part (b) and solve part (a) again. In how many ways can you change 2 Euros? In that currency there are coins of 1, 2, 5, 10, 20, and 50 Cents and 1 Euro (= 100 Cents) and 2 Euros.

Exercise 3*5 points*

Suppose that all edge weights in an graph are integers from 1 to the number n of vertices. How fast can you make Prim's algorithm run? What if the edge weights are integers in the range from 1 to k for some constant k ?