20140174 Hyungrok Kim

CS 204 | Discrete Mathematics | Fall 2016

Homework 3(85 pts)

Sungwon Kang | Due date Oct 6, 2016

1. (10 pts) Consider the following definition of the "$\lhd$" symbol.

Definition. Let x and y be integers. Write $x \lhd y$ if $3x + 5y = 7k$ for some integer k.

(a) Show that $1 \lhd 5$, $3 \lhd 1$, and $0 \lhd 7$.

i) $x=1, y=5$ $7K=3+25=28$ $k=4$

ii) $x=3$ $y=1$ $K=\frac{9+5}{7}=2$

iii) $x=0$. $y=7$ $K=\frac{35}{7}=5$

(b) Find a counterexample to the following statement:

If $a \lhd b$ and $c \lhd d$, then $a \cdot c \lhd b \cdot d$.

$3a+5b=7K_1$

$3c+5d=7K_2$

$3ac+5bd=7K_3$

if) $a=2$ $b=3$ $c=5$ $d=4$

$a \lhd b$, $c \lhd b$ but $ac \not\lhd bd$

$ac=10$

$bd=12$

$10\times3+5\times12=90\neq 7K$

2. (10 pts) Let the following statements be given.

Definition. A triangle is *scalene* if all of its sides have different lengths.

$P(x)$ $Q(x)$

Theorem. A triangle is scalene if it is a right triangle that is not isosceles.

$P(x)$ $R(x)$ ~~$R(x)$~~

$P(x)$: A triangle is scalene

$Q(x)$: All sides of a triangle have different lengths

$R(x)$: A triangle is a right triangle

Suppose ΔABC is a scalene triangle. Which of the following conclusions are valid? Why or why not?

$P(x)$

(a) All of the sides of ΔABC have different lengths.

(b) ΔABC is a right triangle that is not isosceles.

given statements are

$P(x) \leftrightarrow Q(x)$

$(R(x) \land Q(x)) \rightarrow P(x)$

(a) : $P(x) \rightarrow Q(x)$

(a) is valid

(b) : $P(x) \rightarrow (R(x) \land Q(x))$

(b) is not valid

it's not given in the statements.

3. (12 pts) Let P(n,x,y,z) be the predicate "$x^n + y^n = z^n$."

(a) Write the following statement in predicate logic, using positive integers as the domain.

For every positive integer n, there exist positive integers x, y, and z such that $x^n + y^n = z^n$.

$(\forall n)(\exists x)(\exists y)(\exists z)P(n,x,y,z)$

(b) Formally negate your predicate logic statement from part (a). Simplify so that no quantifier lies within the scope of a negation.

$(\exists n)(\forall x)(\forall y)(\forall z)P(n,x,y,z)$

(c) In order to produce a counterexample to the statement in part (a), what, specially, would you have to find among x, y, z and n?

Find a counterexample to the negated statement which is (b)

if $n\geq 3$

n, x, y. positive integer $x^n+y^n=z$ has no solution (proven by Andrew Wiles)

4. (9 pts) Consider the following theorem.

Theorem. Let x be a wamel. If x has been schlumpfed, then x is a borfin.

Answer the following questions.
(a) Give the converse of this theorem. Let x be a wamel, If x hasn't been schlumpfed, then x is not a borfin.
(b) Give the contrapositive of this theorem.
(c) Which statement, (a) or (b), is logically equivalent to the Theorem?

(b) Let x be a wamel, If x is not a borfin, then x has not been schlumpfed.

(C) (b) is logically equivalent to the Theorem.

5. (8 pts) In four-point geometry discussed in class, do triangles exist? In other words, is it possible to have three distinct points, not on the same line, such that a line passes through each pair of points? Why or why not?

let's say there're distinct point x, y, z,
and x, y on l_1, y, z on l_2, x, z on l_3 (l_n is a line).
by axiom 4, z is not on l_1, x is not on l_2, y is not on l_3 $\therefore$ x, y, z & l_1, l_2, l_3 make a triangle

6. (6 pts) Give a direct proof.

Let a, b, and c be integers. If a | b and b | c, then a | c.

k_1, k_2: integers, k_3 $a | b \Rightarrow b = k_1 a$
$b | c \Rightarrow c = k_2 b = k_1 k_2 a$ (by Def). $\because k_1 \cdot k_2$ is an integer k_3 (by axioms)
$c = k_3 a \Rightarrow \boxed{a | c}$

7. (8 pts) Consider the following definition.

Definition. An integer n is sane if $3 | (n^2 + 2n)$.

(a) Give a counterexample to the following: All odd integers are sane. if $n = 5$, $n^2 + 2n = 35$ which is not $3k$ (k: integer)
(b) Give a direct proof of the following: If $3 | n$, then n is sane.

$3 | n \Rightarrow n = 3k$ (k: integer)
$n^2 + 2n = 9k^2 + 6k = 3(3k^2 + 2k) = 3k'$ ($k' = 3k^2 + 2k$: integer) so n is sane.

8. (6 pts) Prove that the rational numbers are closed under addition. That is, prove that, if a and b are rational numbers, then a + b is a rational number.

by saying l is a rational number $\Rightarrow l = \frac{k_1}{k_2}$ (k_1, k_2: integer)
if a, b is rational $a = \frac{k_1}{k_2}$, $b = \frac{k_3}{k_4}$ (k_n: integers)
$a + b = \frac{k_1 k_4 + k_2 k_3}{k_2 k_4} = \frac{k_5}{k_6}$ ($k_5 = k_1 k_4 + k_2 k_3$, $k_6 = k_2 k_4$ integers)
so $a + b$ is a rational number.

9. (8 pts) Recall the Badda-Bing axiomatic system discussed in class. Prove:

If q and r are distinct bings, both of which are hit by baddas x and y, then x = y.

if $x \neq y$, when x, y is hit by q, by axiom 3 there's no more bings that hit both x & y.
This conflict the supposition so $x = y$

10. (8 pts) In the axiomatic system for four-point geometry, prove the following assertion using a proof by contradiction.

Suppose that a and b are distinct points on line u. Let v be a line such that u ≠ v. Then a is not on v or b is not on v.

$u \neq V$, if a and b are on V,
by axiom 1, there's only one line having a and b on it, this means $u = V$
so a and b cannot be on V at once,
a is not on v or b is not on V.