HW 1

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1. p = "There is water in the cylinders."
   q = "The head gasket is blown."
   r = "The car will start."
   (a) $(q \wedge p) \rightarrow \neg r$
   (b) If the car doesn't start then it is not true that there is water in the cylinders nor that the head gasket is blown.

2. p = "You are in Seoul."
   q = "You are in Kwangju."
   r = "You are in South Korea."
   (a) $\neg r \rightarrow \neg (p \vee q)$
   (b) If you are in Kwangju then you are in south Korea or you are not in Seoul.

3. Show $(a \vee b) \wedge (\neg(a \wedge b))$ is equivalent to $a \leftrightarrow \neg b$

a	b	$\neg b$	$a \vee b$	$a \wedge b$	$\neg(a \wedge b)$	$(a \vee b) \wedge (\neg(a \wedge b))$	$a \leftrightarrow \neg b$
T	T	F	T	T	F	F	F
T	F	T	T	F	T	T	T
F	T	F	T	F	T	T	T
F	F	T	F	F	T	F	F

$(a \vee b) \wedge (\neg(a \wedge b)) \Leftrightarrow a \leftrightarrow \neg b$

4. Statement P : x is a multiple of 4.
   Statement Q : $x/2$ is an even integer.
   By these conditions saying $P \rightarrow Q$,
   it is sufficient.

5.

p	q	$p \uparrow q$	$p \wedge q$	$\neg(p \wedge q)$
T	T	F	T	F
T	F	T	F	T
F	T	T	F	T
F	F	T	F	T

$p \uparrow q \Leftrightarrow \neg(p \wedge q)$

6.

p	q	$p \uparrow q$	$\neg(p \wedge p)$	$\neg(q \wedge q)$	$p \uparrow p$	$q \uparrow q$
T	T	F	F	F	F	F
T	F	T	F	T	F	T
F	T	T	T	F	T	F
F	F	T	T	T	T	T

addition to the table above.

(a)

$(p \uparrow q) \uparrow (p \uparrow q)$	$p \wedge q$
T	T
F	F
F	F
F	F

By this truth table, $(p \uparrow q) \uparrow (p \uparrow q)$ is logically equivalent to $p \wedge q$

(b)

$(p \uparrow p) \uparrow (q \uparrow q)$	$p \vee q$
T	T
T	T
T	T
F	F

$(p \uparrow p) \uparrow (q \uparrow q) \Leftrightarrow p \vee q$

(c)

$p \uparrow (q \uparrow q)$	$p \to q$
T	T
F	F
T	T
T	T

$p \uparrow (q \uparrow q) \Leftrightarrow p \to q$

$(\neg a) \vee (\neg a)$

7.

Statement	Reasons
1. $p \wedge (q \vee r)$	given
2. $\neg(p \wedge q)$	given
3. $\neg p \vee \neg q$	De Morgan's laws, 2
4. $\neg q \vee \neg p$	commutativity, 3
5. $q \rightarrow \neg p$	implication, 4
6. p	simplification, 1
7. $\neg(\neg p)$	double negation, 6
8. $\neg q$	modus tollens, 6, 5
9. $(q \vee r) \wedge p$	commatativity, 1
10. $q \vee r$	simplification, 9
11. $r \vee q$	commutativity, 10
12. $\neg(\neg r) \vee q$	double negation, 11
13. $\neg r \rightarrow q$	implication, 12
14. $\neg(\neg r)$	modus tollens, 8, 13
15. r	double negation, 14
16. $p \wedge r$	conjunction, 6, 16

8.

a	$\neg a$	$a \rightarrow \neg a$
T	F	F
F	T	T

It is not a contradiction,
if a is true $a \rightarrow \neg a$ is false,
if a is false $a \rightarrow \neg a$ is True,
$a \rightarrow \neg a$ is a statement that is a declarative sentence that is either true or false, but not both. And It doesn't contradict the assumption.