

Introduction to Algorithms

Due March 13, 2017, 10 a.m.

Exercise 1

7 points

Suppose that you have three sorting algorithms where the first one makes $n\lceil\log n\rceil$, the second one $n(n-1)/2$, and the third one $n!$ comparisons on an input sequence of length n .

- (a) Determine the number of comparisons of the three algorithms for $n = 10, 20, 30, 40, 50$.
- (b) Problems of which size can be solved in 1 second and in 1 hour by the different algorithms on a computer where one comparison takes 1 nanosecond (ignoring the time for operations other than comparisons)?
- (c) Problems of which size can be solved in 1 second and in 1 hour on a ten times faster computer?

Exercise 2

7 points

- (a) Implement Algorithm 1 from the lecture (*Bogosort*) and determine experimentally its runtime for different input sizes on your computer. Describe how you generate all permutations systematically.
- (b) Consider a randomized variant of *Bogosort*, which consists of randomly generating a permutation, applying it to the input sequence, and then testing whether it is sorted. If not the experiment is repeated. Implement this method and determine experimentally its runtime for different input sizes on your computer. Describe how you randomly generate a permutation so that all are equally likely.

Exercise 3

6 points

Suppose that you have a collection of n nuts and n bolts, where each bolt fits into exactly one nut. Visually, you cannot find any difference, only by testing a bolt with a nut you can determine whether the bolt is too thick, too thin, or it fits.

Design an efficient algorithm to assign each bolt to the corresponding nut. Using an analysis from class (Th) determine the (worst case and expected) number of tests your algorithm makes.