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Study of Predicting Combined Chaotic Time Series Using Neural Networks*

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Abstract: The combined chaotic time series prediction by using the standard feed-forward neural networks. Henon and Lozi systems are used to generate the combined chaotic time series. From the forecasting results obtained, it can be concluded that the NN, which is trained by improved back-propagation (BP) algorithms, can be well applicable for combined chaotic time series prediction.

Key words: combined chaotic time series; back-propagation (BP) algorithms; feed-forward neural networks; time series prediction
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基于神经网络的联合混沌时间序列的预测研究

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摘要：提出了利用前馈神经网络预测联合混沌序列，通过引用著名的 Henon 和 Lozi 混沌系统作为仿真实验产生联合混沌信号序列。预测结果证明，用改进的 BP 算法训练的 NN 可以完全预测联合混沌信号序列。

关键词：联合混沌时间序列；BP 算法；前馈神经网络；时间序列预测

1 Introduction

Neural networks (NN) are capable of learning complex nonliniear relationship. It had been theoretically proved that multilayer feed-forward networks with as few as one hidden layer is indeed capable of universal approximation in a very precise and satisfactory sense[1]. Such NN had been applied successfully to solve the problem of time series prediction and other problems of forecasing[2~4]. A hybrid linear-neural model for time series forecasting was mentioned in Ref. [2], and that NN obtained good performance for time series forecasting. The authors[3] applied NN to the problem of forecasting the flow of the river Nile and concluded that the NN produced fairly accurate forecasts. The application of NN to data mining problems, which require proper explanations for prediction, was mentioned in Ref. [4]. Experimental results illustrated the advantages in scalability and learning speed and had shown that the NN has a high potential in solving data mining problems. It was well know that chaotic time series is difficult to be predicted because it is extremely sensitive to the ini-

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tial conditions and similar to noise. Recently, many publications have proposed feed-forward or recurrent NN for chaotic time series prediction[5~8].

The prediction of simple chaotic processes was carried out in Ref. [5]. Two-layer feed-forward NN was used to forecast chaotic time series with very promising results, especially for the Lorenz system. Other authors[6] used NN to learn chaotic dynamics and reasonable prediction was achievable. A recurrent NN was used to predict chaotic time series in Ref. [7]. The authors addressed the use of dynamical recurrent NN for chaotic time series prediction and illustrated by several experiments on well-known chaotic processes that the dynamical recurrent NN be well applicable for chaotic time series prediction. The paper [8] described two basic structures for identifying chaotic systems based on the Wiener and Hammerstein cascade models, in which three-layer feed-forward artificial NN was employed as the nonlinear static subsystem and a simple linear plant was used as the dynamic subsystem. Through training the NN and choosing an appropriate linear subsystem, various chaotic systems can be well identified by these two basic structures. However, these processes were at the expenses of structural and computational complexity.

In this paper, the authors have concentrated their attentions on predicting combined chaotic time series[9] using standard feed-forward NN. Henon and Lozi systems[10] are employed to generate combined chaotic time series as an illustrative example to demonstrate the effectiveness of the proposed approach. The idea of chaotic masking[11] is employed in this letter. The chaotic time series used in this paper is generated by directly adding Lozi time series in the Henon chaotic time series. In this work the number of neurons in hidden layer and in input layers is selected to be different integers to fully exercise the networks. The selections range from 5 neurons through 20 neurons in the hidden layers and 3 neurons through 9 neurons in the input layers. The architecture provides a comparable degree of accuracy to the prediction when selecting appropriate structures. The learning and generalization abilities of the architecture have been demonstrated by its application to function approximation. The authors also draw a conclusion that it is possible to predict the combined chaotic time series to any desired degree of accuracy when selecting the appropriate structure of the standard feed-forward networks.

2 Model description

The NN architecture used in this application has M neurons in input layer. N neurons in the hidden layer, and a single neuron in the output layer. Such a NN is denoted as $(M : N : 1)$. The NN model used is defined as follows

$$[Y(t)] = F_{NN}\{[Y(t-\tau)][W][B]\} \tag{1}$$

Where F_{NN} is an approximation of the nonlinear function f, $[W]$ represents the weight matrices of the hidden and output layers and $[B]$ represents the Bias value of neurons of the hidden layer. Here, the activation function of hidden layer is sigmoid function. And the activation function of output layer is linear function.

The scalar time series is denoted by $y(i+m)$, and the problem of prediction using neural network model will be defined by following equation[3]

$$y(i+m) = F_{NN}\{y(i), y(i+1), \cdots, y(i+m-1)\} + \varepsilon(i+m) \tag{2}$$

where $\varepsilon(i)$ is the prediction errors. The main difficulty in implementing prediction model is that function f is actually unknown. The only information available is the set of observables: $y(1), y(2), y(3), \cdots, y(n)$, where n is the total length of the time series. It is the goal of prediction scheme to approximate this function. NN are known to be the good function approximators[1~6]. Their real attraction lies in their ability to learn by examples. However, in order to obtain a network that produces a desirable output, the network weights must typically be trained upon the available examples many times.

During training, a NN is presented with several input/output pairs, and is expected to learn the functional relationship between inputs and outputs of simulation model. Therefore, the trained neural networks can predict the output for inputs other than the ones presented during training.

This architecture can be readily implemented on the MATLAB NN toolbox and trained using improved back-propagation(BP) algorithms[5]. For comparison the number of neurons in hidden layer changes from 5 nodes to 20 nodes when use 3 input in the NN structure. And for further analysis, the number of inputs changes form 3 to 9 when using 14 nodes in hidden layer.

3 Simulation results

Combined chaotic time series used in this work are generated by Henon and Lozi chaotic system[10]. The systems are defined by Eq. (3) and (4) below. In this work we also have taken $a=1.4$, $b=0.3$[7], $p=1.8$ and $q=0.4$ as we did before[9].

$$x_L(t+2) = 1 - p \mid x_L(t+1) \mid + q x_L(t) \quad (3)$$

$$x_H(t+2) = 1 - a x_H^2(t+1) + b x_H(t) \quad (4)$$

The combined chaotic time series data was obtained by the Eq. (5). The first 500 points of this series using an initial condition of $x_H(0) = -0.005$, $x_H(1)=0.001$ and $x_L(0)=-0.002$, $x_L(1)=0.008$.

$$y(t) = x_H(t) + x_L(t) \quad (5)$$

The prediction of future values of this series can be formulated as given values $y(t-m)$, $y(t-m+1)$, $y(t-m+1)$, $\cdots$, $y(t-1)$; determine $y(t)$ or $y(t-m+n)$, where m and n are fixed positive integers and t is the series index.

The results presented in Tab. 1 and Tab. 2 illustrate the different degree of prediction accuracy in terms of the prediction error when selecting different number of neurons in hidden layer and input layer.

A summary of the implementation results obtained are presented in Tab. 1. All the simulations use 500 points from the series as training data and use a further 300 points as test data. And the network is trained for an unlimited time period until the error metric reach a steady-state value.

The results are presented in terms of the accuracy of the prediction using the root-mean-square error ($RMSE$) metric. The values for $RMSE_{\text{training}}$ are obtained using the training data and similarly the values of $RMSE_{\text{testing}}$ are obtained using the test data.

Tab. 1 Implementation results using 3 input neurons for prediction

Number of nodes of hidden layer	$RMSE_{\text{training}}$	$RMSE_{\text{testing}}$
5	0.034	0.030
11	0.023	0.021
14	0.016	0.013
17	0.026	0.022
20	0.028	0.025

Tab. 2 Implementation results using 14 neurons in hidden layer

Number of neurons of input layer	$RMSE_{\text{training}}$	$RMSE_{\text{testing}}$
3	0.016	0.013
5	0.013	0.011
7	0.017	0.014
9	0.023	0.020

Tab. 1 illustrate that the prediction accuracy is improved using 14 neurons in hidden layer rather than 5 or 11 nodes in hidden layer. This is an expected results as the combined chaotic time series prediction is more difficult than simple chaotic time series prediction. The result we get, which is not presented here, indicated that the only 9 neurons of hidden layers are enough to predict the chaotic time series generated by Henon system. Another notable result of the implementations is that the prediction accuracy degrades when moving from 14 nodes of hidden layer to 20 nodes of hidden layer. This can be explained by the larger size of the network with higher number of hidden layer which introduced extra parameters and hence increases the training difficulties.

Further analysis which is presented in Tab. 2 using 14 nodes in hidden layer to predict the next step indicates that implementations using less than 5 previous values of the series provided a very poor prediction of the series. Similarly, using more than 7 previous values of the series provides inaccurate estimates and indeed introduced excessive training times for the networks to obtain reasonable prediction results.

The accuracy of the predictions is illustrated in Fig. 1(a). The simulation results, which is obtained by using 5 inputs and 14 neurons in hidden layer, demonstrates that the predicted and actual points of the series are almost heavy to match. The difference between the prediction and actual results is also illustrated in Fig. 1(b) which is the section of the Fig. 1(a). We zoom the Fig. 1(a) at the end of points 700 to 800 and indeed observe that the predicted and actual points of the series are largely indistinguishable. The results establish that a single hidden layer feed-forward network can approximate the combined chaotic time series to the satisfied degree of accuracy.

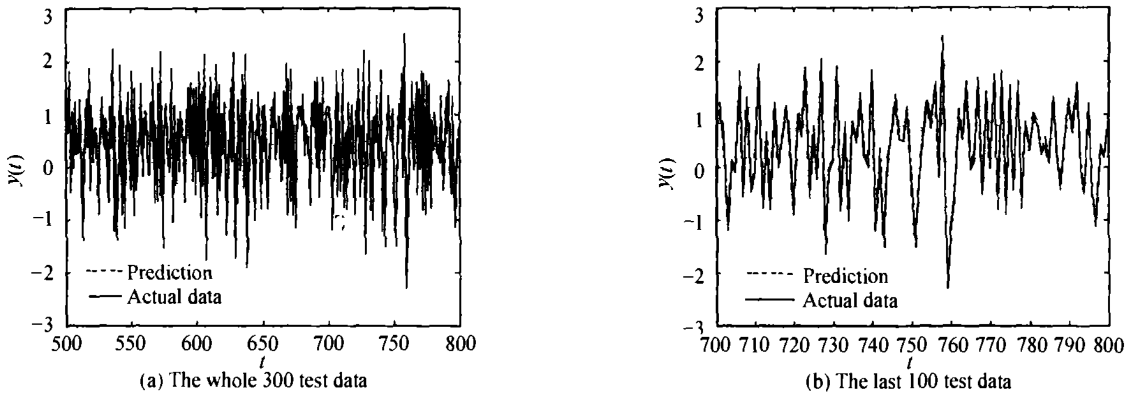

Fig. 1 The prediction of the combined chaotic series as compared with the real data

4 Conclusions

A three-layer feed-forward NN has been used for predicting combined chaotic nonlinear time series, which is more complicated than the simple chaotic time series that is very difficult to forecast using traditional methods. The simulation results demonstrate that the NN is capable of predicting the combined chaotic time series to any desired degree of accuracy. In particular, the prediction results illustrate that the numbers of hidden layers and input layers will produce the direct influence on the accuracy of the prediction. This approach not only solves the particular problem of predicting combined chaotic nonlinear time series, but also promotes the development of the NN with applications to other nonlinear problems and the chaos synchronization problems which have been employed to secure communication.

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