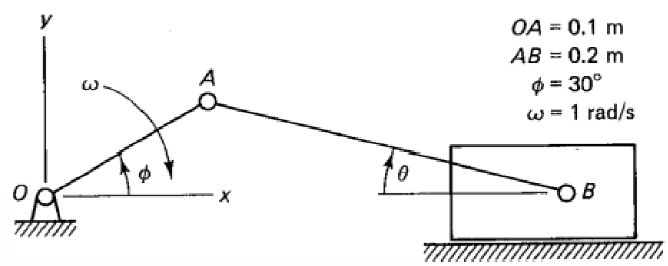
# Using Newton-Raphson's Method on Crank Mechanism

## Introduction

The task was about creating a program that plots displacement, velocity and acceleration of a connecting rod and a piston of a Short Crank Mechanism.

## 1 First and Second Time Derivatives

$$a\cos\phi + b\cos\theta - d = 0$$
$$a\sin\phi - b\sin\theta = 0$$




Source: P. E. Nikravesh, *Computer-aided analysis....*

$$\boldsymbol{x} = \begin{bmatrix}\theta\\ d\end{bmatrix}$$

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix}a\cos\phi + b\cos\theta - d\\ a\sin\phi - b\sin\theta\end{bmatrix}$$ constrain equations

$$\boldsymbol{J} = \begin{bmatrix}-b\sin\theta & -1\\ -b\cos\theta & 0\end{bmatrix}$$ Jacobian

$$\dot{\boldsymbol{f}}(\boldsymbol{x}) = \begin{bmatrix}-a\,\dot\phi\sin\phi - b\,\dot\theta\sin\theta - \dot d\\ a\,\dot\phi\cos\phi - b\,\dot\theta\cos\theta\end{bmatrix}$$ First time derivative (velocity), where $\dot\phi = \omega$ and is constant

This can be also represented in a form $\boldsymbol{J}\,\dot{\boldsymbol{x}} = -\boldsymbol{f}_{\boldsymbol{\phi}}$ ,where the terms associated with the known crank velocity are moved to the right side:

$$\begin{bmatrix}-b\sin\theta & -1\\ -b\cos\theta & 0\end{bmatrix}\begin{bmatrix}\dot\theta\\ \dot d\end{bmatrix} = \begin{bmatrix}a\,\dot\phi\sin\phi\\ -a\,\dot\phi\cos\phi\end{bmatrix}$$

$$\ddot{\boldsymbol{f}}(\boldsymbol{x}) = \begin{bmatrix}-a\,\dot\phi^2\cos\phi - b\,\ddot\theta\sin\theta - b\,\dot\theta^2\cos\theta - \ddot d\\ -a\,\dot\phi^2\sin\phi - b\,\ddot\theta\cos\theta + b\,\dot\theta^2\sin\theta\end{bmatrix}$$ Second time derivative (acceleration)

This can be also represented in a form $\boldsymbol{J}\,\ddot{\boldsymbol{x}} = \boldsymbol{G}$ , where the terms associated with the known crank acceleration and the quadratic velocity are moved to the right side:

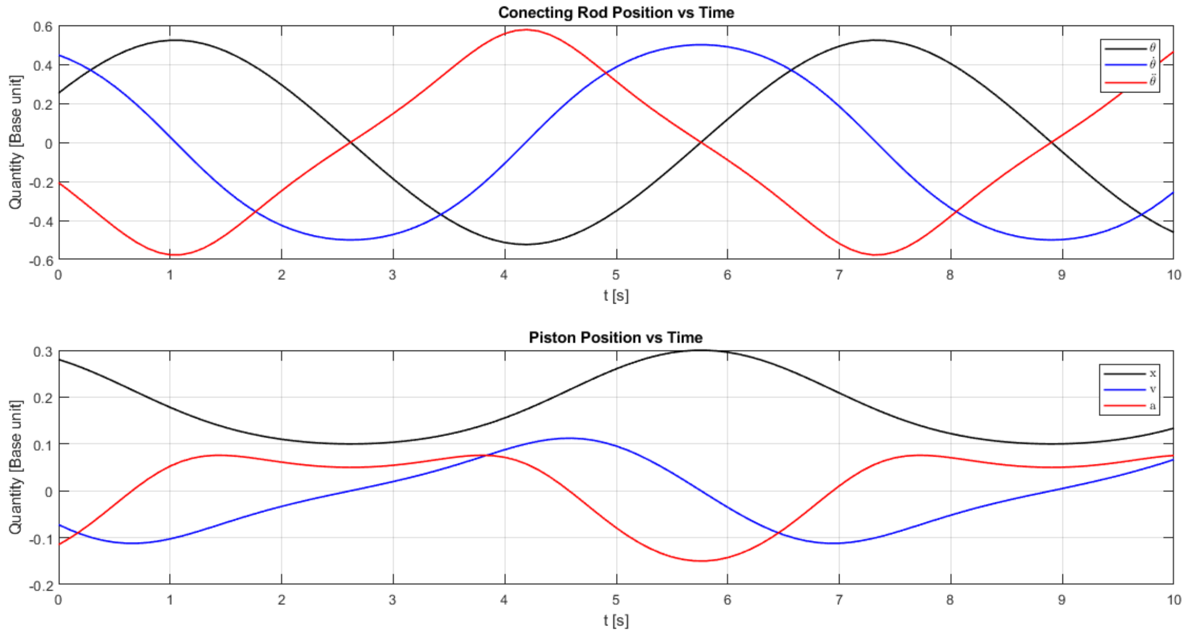
$$\begin{bmatrix}-b\sin\theta & -1\\ -b\cos\theta & 0\end{bmatrix}\begin{bmatrix}\ddot\theta\\ \ddot d\end{bmatrix} = \begin{bmatrix}a\,\dot\phi^2\cos\phi + b\,\dot\theta^2\cos\theta\\ a\,\dot\phi^2\sin\phi - b\,\dot\theta^2\sin\theta\end{bmatrix}$$

## 2 Program

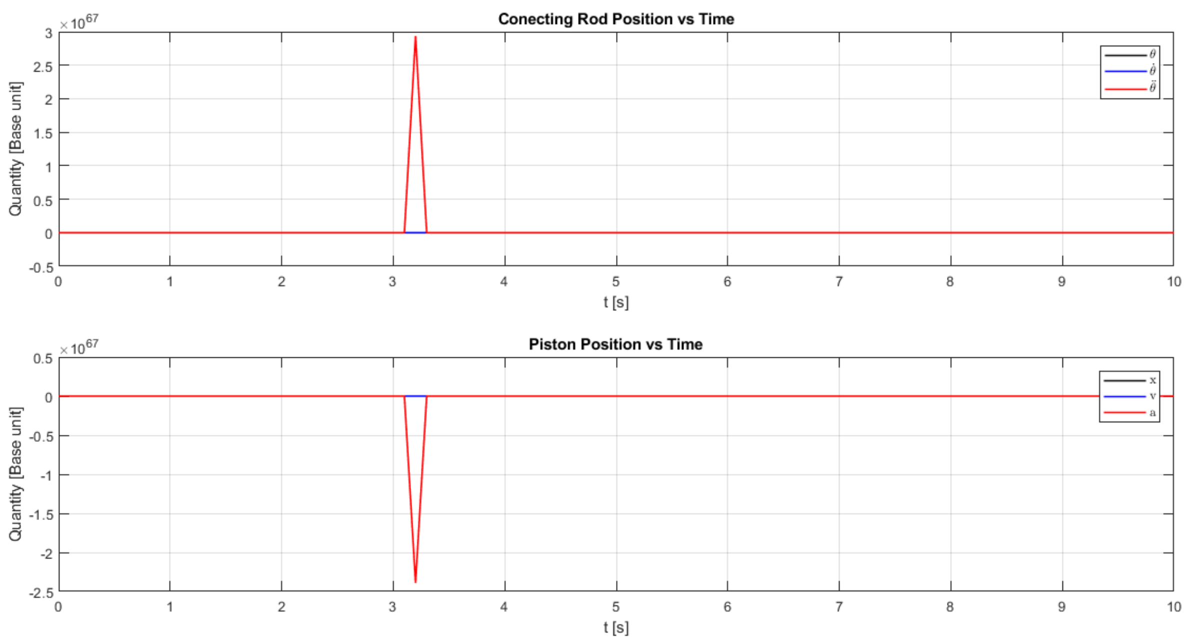
A program Crank_Mechanism.m was created which calls function NR_method.m. In the beginning of the program, parameters and time vector is defined. Next, initial guess for the solution of the equations is defined and a tolerance. The vectors of displacement *x*, velocity *v* and acceleration *aa* are initialized with zeros to match size of the time vector. After that, function *for* is used to call one by one each element of the vector *phi*, which corresponds to angle of rotation of the crankshaft, and save it to corresponding row of each wanted vectors *x*, *v* and *aa*. These vectors are computed using function NR_method.m, which uses Newton-Raphson's method to solve systems of nonlinear algebraic equations. Finally, these vectors are saved to extra variables and then plotted.

## 3 Results

The results for constant angular velocity $\omega = 1\,\frac{rad}{s}$, time interval (0, 10 s), $\mathrm{phi} = \frac{\pi}{6} + \omega\,t$, tolerance $10^{-4}$ and initial guess $\begin{bmatrix}0\\0\end{bmatrix}$ looks like this:




The correct results very depend on initial guess. For example, with initial guess $\begin{bmatrix}1\\1\end{bmatrix}$ the solution diverge:




## Conclusions

All sources were pushed to GitHub. Link to the repository is: https://github.com/q2493/HW_3_Trusina.git