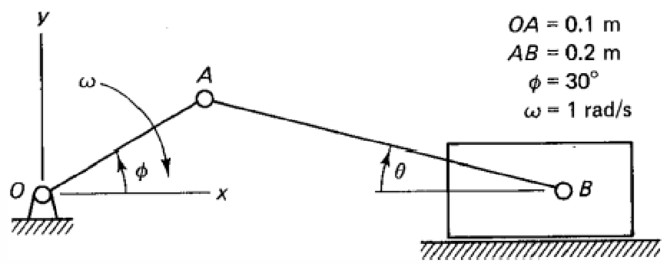
Using Newton-Raphson's Method on Crank Mechanism

Introduction

The task was about creating a program in MATLAB, that plots displacement, velocity and acceleration of a connecting rod and a piston of a Slider Crank Mechanism.

$$a\cos\phi + b\cos\theta - d = 0$$
$$a\sin\phi - b\sin\theta = 0$$

1 Solution for constant ϕ



Source: P. E. Nikravesh, *Computer-aided analysis....*

$$\boldsymbol{x} = \begin{bmatrix}\theta \\ d\end{bmatrix}, \quad \dot{\boldsymbol{x}} = \begin{bmatrix}\dot\theta \\ \dot d\end{bmatrix}, \quad \ddot{\boldsymbol{x}} = \begin{bmatrix}\ddot\theta \\ \ddot d\end{bmatrix}, \quad \phi = 30° = \frac{\pi}{6}.$$

$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} a\cos\phi + b\cos\theta - d \\ a\sin\phi - b\sin\theta\end{bmatrix} = 0,$ constrain equations.

Solution for $\boldsymbol{x} = \begin{bmatrix}0.2527 \\ 0.2803\end{bmatrix}$,

$\boldsymbol{J} = \begin{bmatrix}-b\sin\theta & -1 \\ -b\cos\theta & 0\end{bmatrix}$ Jacobian.

$\dot{\boldsymbol{f}}(\boldsymbol{x}) = \begin{bmatrix}-a\,\dot\phi\sin\phi - b\,\dot\theta\sin\theta - \dot d \\ a\,\dot\phi\cos\phi - b\,\dot\theta\cos\theta\end{bmatrix} = 0.$ First time derivative (velocity), where $\dot\phi = \omega = 1\,\frac{rad}{s}$ and is constant.

This can be also represented in a form $\boldsymbol{J}\,\dot{\boldsymbol{x}} = -\boldsymbol{f}_{\boldsymbol{\phi}}$, where the terms associated with the known crank velocity are moved to the right side:

$\begin{bmatrix}-b\sin\theta & -1 \\ -b\cos\theta & 0\end{bmatrix}\begin{bmatrix}\dot\theta \\ \dot d\end{bmatrix} = \begin{bmatrix}a\,\dot\phi\sin\phi \\ -a\,\dot\phi\cos\phi\end{bmatrix}$. Solution for $\dot{\boldsymbol{x}} = \begin{bmatrix}0.4472 \\ -0.0724\end{bmatrix}$.

$\ddot{\boldsymbol{f}}(\boldsymbol{x}) = \begin{bmatrix}-a\,\dot\phi^2\cos\phi - b\,\ddot\theta\sin\theta - b\,\dot\theta^2\cos\theta - \ddot d \\ -a\,\dot\phi^2\sin\phi - b\,\ddot\theta\cos\theta + b\,\dot\theta^2\sin\theta\end{bmatrix} = 0.$ Second time derivative (acceleration).

This can be also represented in a form $\boldsymbol{J}\,\ddot{\boldsymbol{x}} = \boldsymbol{G}$, where the terms associated with the known crank acceleration and the quadratic velocity are moved to the right side:

$\begin{bmatrix}-b\sin\theta & -1 \\ -b\cos\theta & 0\end{bmatrix}\begin{bmatrix}\ddot\theta \\ \ddot d\end{bmatrix} = \begin{bmatrix}a\,\dot\phi^2\cos\phi + b\,\dot\theta^2\cos\theta \\ a\,\dot\phi^2\sin\phi - b\,\dot\theta^2\sin\theta\end{bmatrix}$. Solution for $\ddot{\boldsymbol{x}} = \begin{bmatrix}-0.2066 \\ -0.1150\end{bmatrix}$.

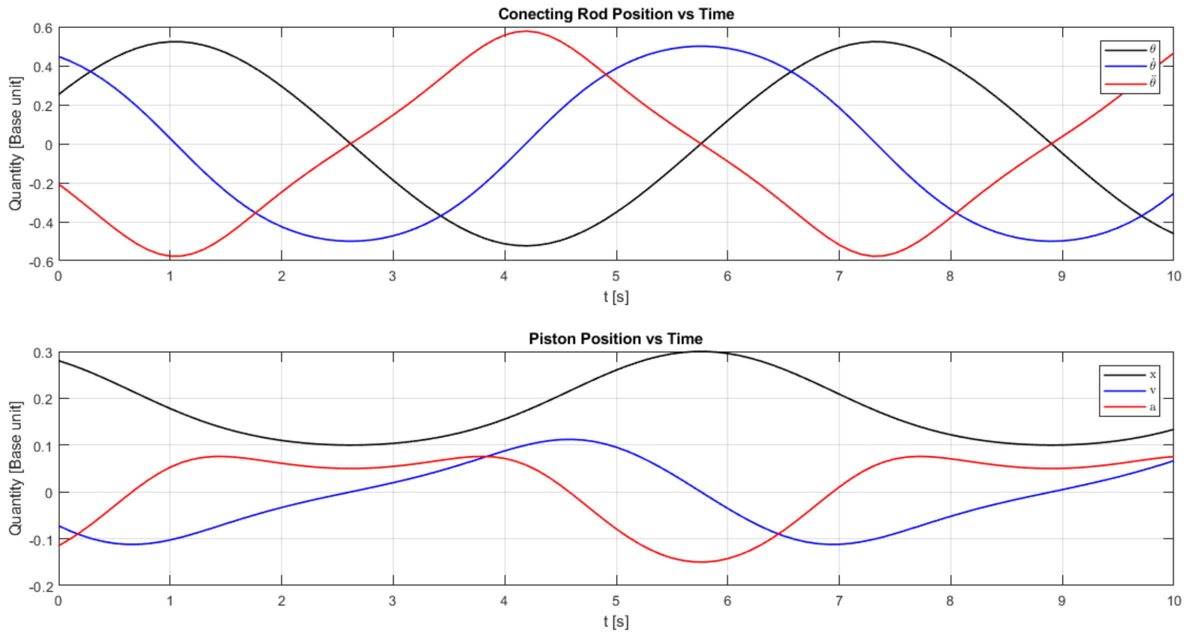
2 Program

A program Crank_Mechanism.m was created, which calls function NR_method.m. In the beginning of the program, parameters and time vector is defined. Next, initial guess and tolerance for the solution of the equations is set. The vectors of displacement *x*, velocity *v* and acceleration *aa* are initialized with zeros to match size of the time vector.

After that, function *for* is used to call one by one each element of the vector *phi*, which corresponds to the angle of rotation of the crankshaft and save it to corresponding row of each wanted vectors *x*, *v* and *aa*. These vectors are computed using function NR_method.m, which uses Newton-Raphson's method to solve systems of nonlinear algebraic equations. Above mentioned equations are used in its input. Finally, these vectors are saved to extra variables and then plotted.

3 Results

The results for constant angular velocity $\omega = 1\,\frac{rad}{s}$, time interval (0, 10 s), $\mathrm{phi} = \frac{\pi}{6} + \omega\, t$, tolerance 10^{-4} and initial guess $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ looks like this:



Conclusions

All sources were pushed to GitHub. Link to the repository is: https://github.com/q2493/HW_4_Jakub_Trusina

The correct results very depend on initial guess. For example, with initial guess $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ solution diverge: