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Group Assignment for Computational Methods in Mechanics

1 Introduction

In the first part of this assignment, the mechanical system, shown in Figure 1, is analyzed. The analysis consists of a kinematic analysis in which the position, velocity as well as acceleration of each part of the system in calculated. Furthermore, a dynamic analysis is performed in order to obtain the behavior of the system resulting from gravity and external applied forces. In the second part, the kinematic as well as dynamic analysis is applied to another system, which is illustrated in Figure 2.

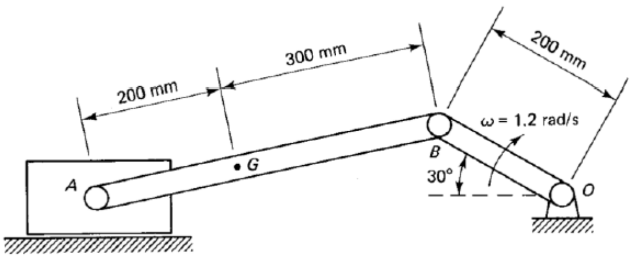


Figure 1: Crank Mechanism

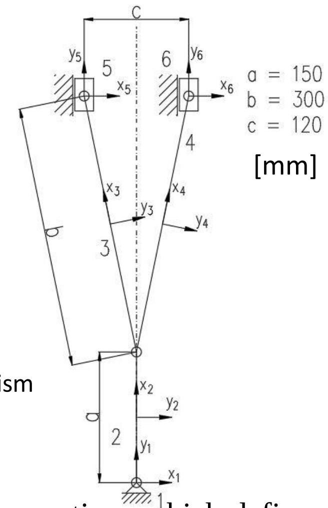


Figure 2 Double Crank Mechanism

2 Methods

The kinematics of the mechanical systems shown above can be described by constraint equations which define the possible movements of the different parts of the system. Within the scope of this task, three different constraints are taken into account, namely constraints for revolute joints, simple and driving constraints. The constraint equations for revolute joints read

$$\mathbf{r}_\mathrm{i} + \mathbf{A}_\mathrm{i}\bar{\mathbf{s}}_\mathrm{i}^\mathrm{p} - \mathbf{r}_j - \mathbf{A}_{\boldsymbol{j}}\bar{\mathbf{s}}_{\boldsymbol{j}}^\mathrm{p} = 0 \qquad (1)$$

where the vectors with index i describe the position of the joint in the body i with respect to the global coordinate system and the same holds for the body j. Simple constraints can be described by a constant value that is assigned to the coordinate of a body, so that

$$\begin{bmatrix} \mathrm{x_i} \\ \mathrm{y_i} \\ \varphi_i \end{bmatrix} = \begin{bmatrix} \mathrm{c}_1 \\ \mathrm{c}_2 \\ c_3 \end{bmatrix} \qquad (2)$$

holds for a body i. Unlike these two types of constraints, driving constraints are dependent on time. They can be defined similarly to simple constraints as

$$\begin{bmatrix} \mathrm{x_i} \\ \mathrm{y_i} \\ \varphi_i \end{bmatrix} = \begin{bmatrix} \mathrm{d}_1(t) \\ \mathrm{d}_2(t) \\ d_3(t) \end{bmatrix} \qquad (3)$$

whereas in the two systems that are analyzed in this assignment, there is only a driving constraints for the angular coordinate which is dependent on time and the angular velocity ω. It follows that

$$d_3(t) = \varphi_i^0 + \omega t. \qquad (4)$$

The constraint equation can be summarized in a matrix $\boldsymbol{C}$ and the resulting system of equations for the kinematic analysis reads

$$\boldsymbol{C} = \boldsymbol{0} \qquad (5)$$

Link to Code on Github: https://github.com/q2493/TEAM-WORK

$$\begin{bmatrix} \boldsymbol{C_q} \\ \boldsymbol{C_q^d} \end{bmatrix} \dot{\boldsymbol{q}} = \begin{bmatrix} 0 \\ -\boldsymbol{C_t^d} \end{bmatrix} \tag{6}$$

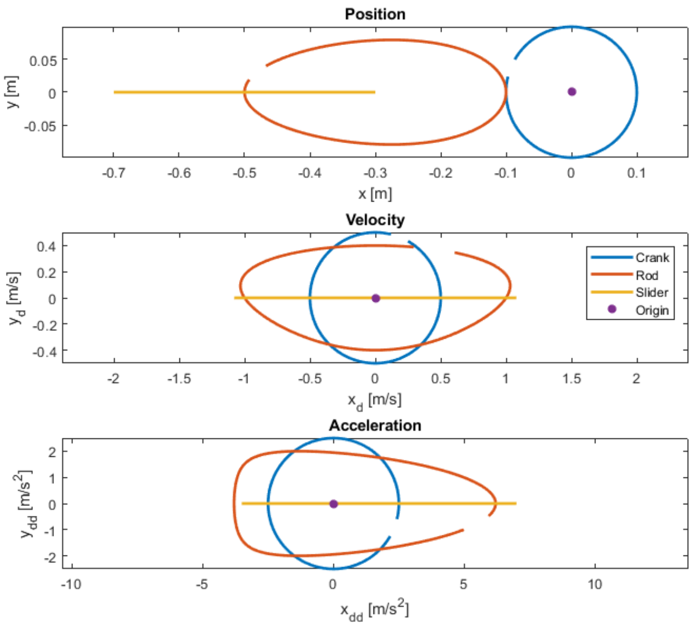
$$\begin{bmatrix} \boldsymbol{C_q} \\ \boldsymbol{C_q^d} \end{bmatrix} \ddot{\boldsymbol{q}} = \begin{bmatrix} -(\boldsymbol{C_q}\dot{\boldsymbol{q}})_{\boldsymbol{q}}\dot{\boldsymbol{q}} \\ -(\boldsymbol{C_q^d}\dot{\boldsymbol{q}})_{\boldsymbol{q}}\dot{\boldsymbol{q}} - 2\boldsymbol{C_{qt}^d}\dot{\boldsymbol{q}} - \boldsymbol{C_{tt}^d} \end{bmatrix}. \tag{7}$$

For the dynamic analysis, the masses of the bodies are taken into account in a mass matrix and the applied forces such as gravity are stored in a force vector. Together with the constraints from the previous task and the previously unknown Lagrange multipliers $\boldsymbol{\lambda}$, the system of equations for the dynamic analysis in matrix form reads

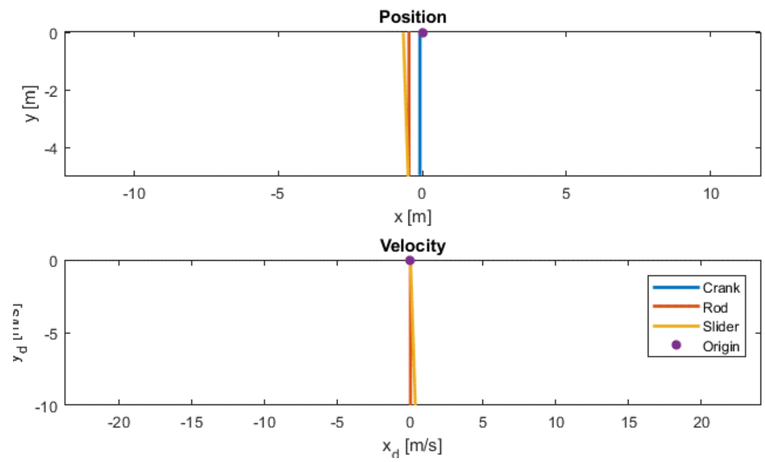
$$\begin{bmatrix} \boldsymbol{M} & \boldsymbol{C_q^T} \\ \boldsymbol{C_q} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f} \\ \hat{\boldsymbol{g}} \end{bmatrix} \tag{8}$$

where $\hat{\boldsymbol{g}} = \boldsymbol{g} - 2\alpha\dot{\boldsymbol{C}} - \beta^2\boldsymbol{C}$ is calculated with the Baumgartle method with constants $\alpha$and β.

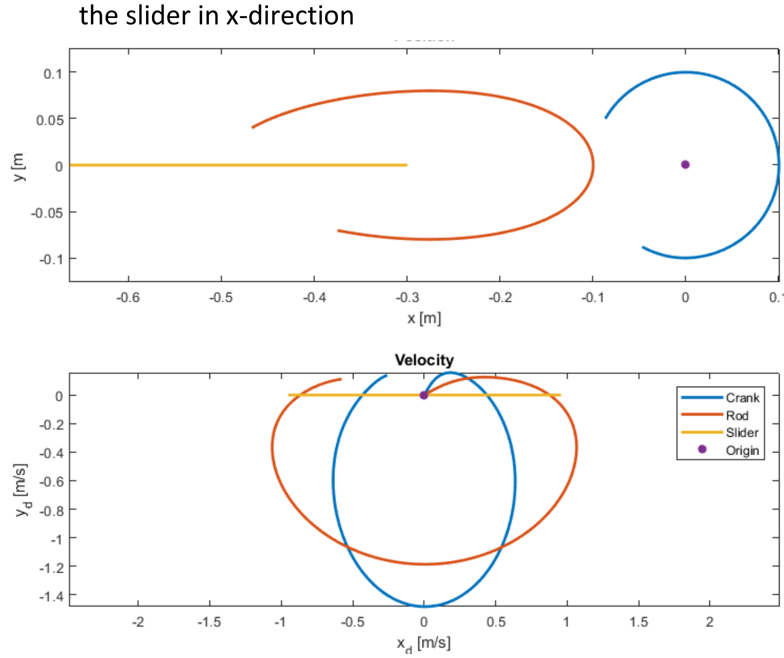
3 Results for 1 second



Kinematics of Crank Mechanism with angular velocity of the crank -6 rad/s



Dynamics of non-constrained Crank Mechanism with gravity in minus y-direction and force 1 N acting on the slider in x-direction



Dynamics of constrained Crank Mechanism with force 10 N acting on the slider in x-direction with no gravity

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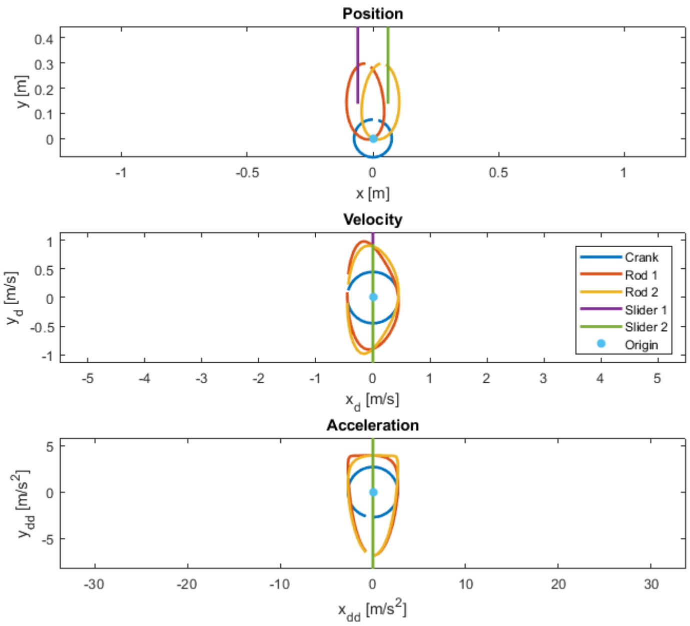
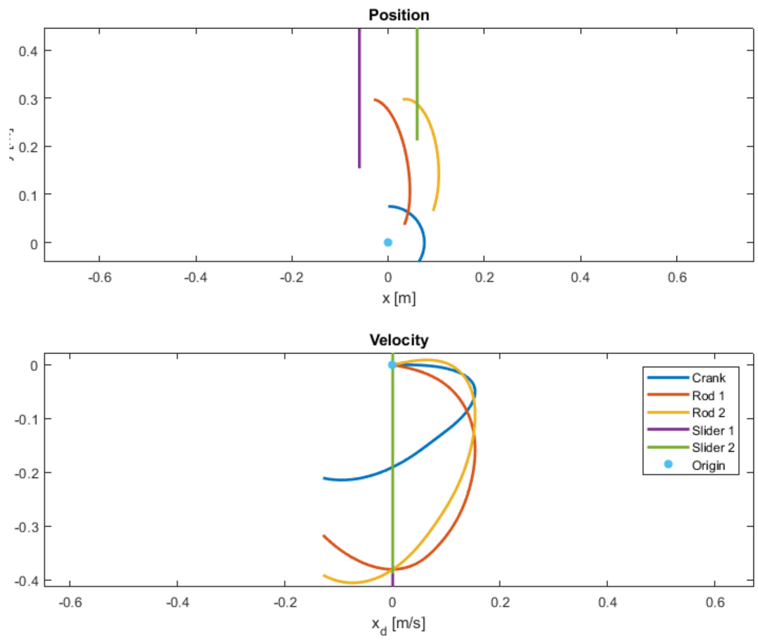


Kinematics of Double Crank Mechanism with angular velocity of the crank 6 rad/s



Dynamics of constrained Double Crank Mechanism with force 10 N acting on the crank in x-direction with no gravity

Link to Code on Github: https://github.com/q2493/TEAM-WORK