

Deterministic Near-Linear Time Minimum Cut for Weighted Graphs

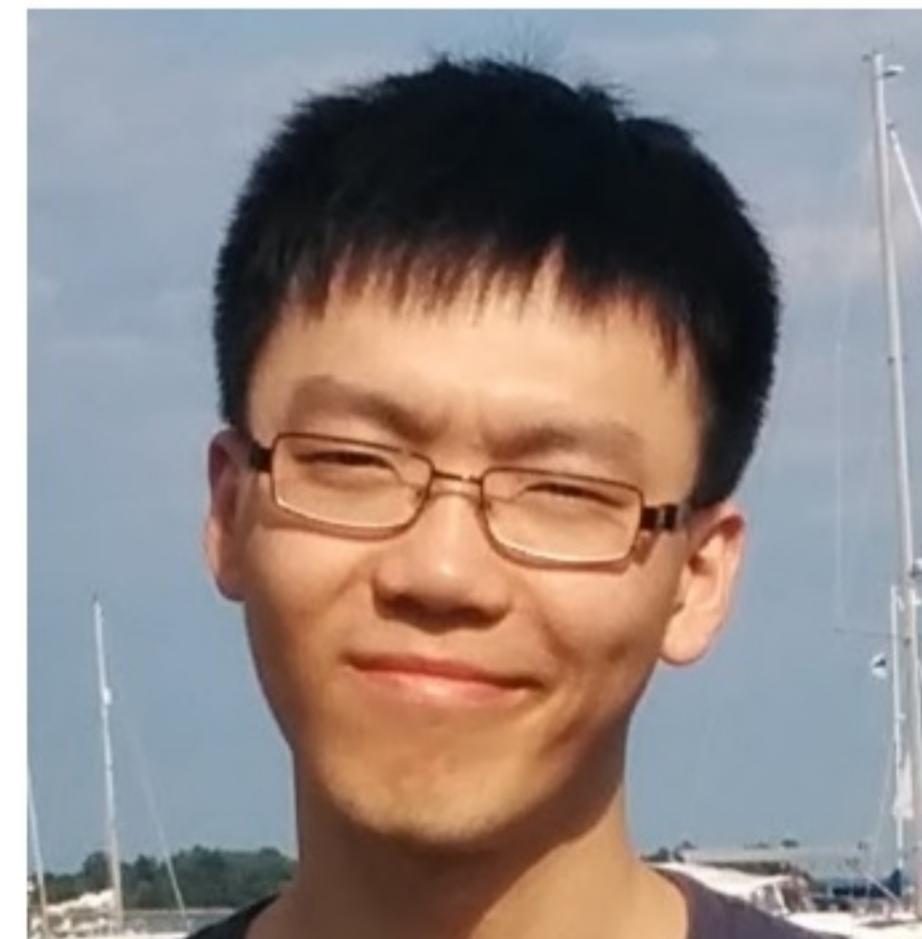
Jason Li (Berkeley→CMU)



with Monika Henzinger



Satisf Rao



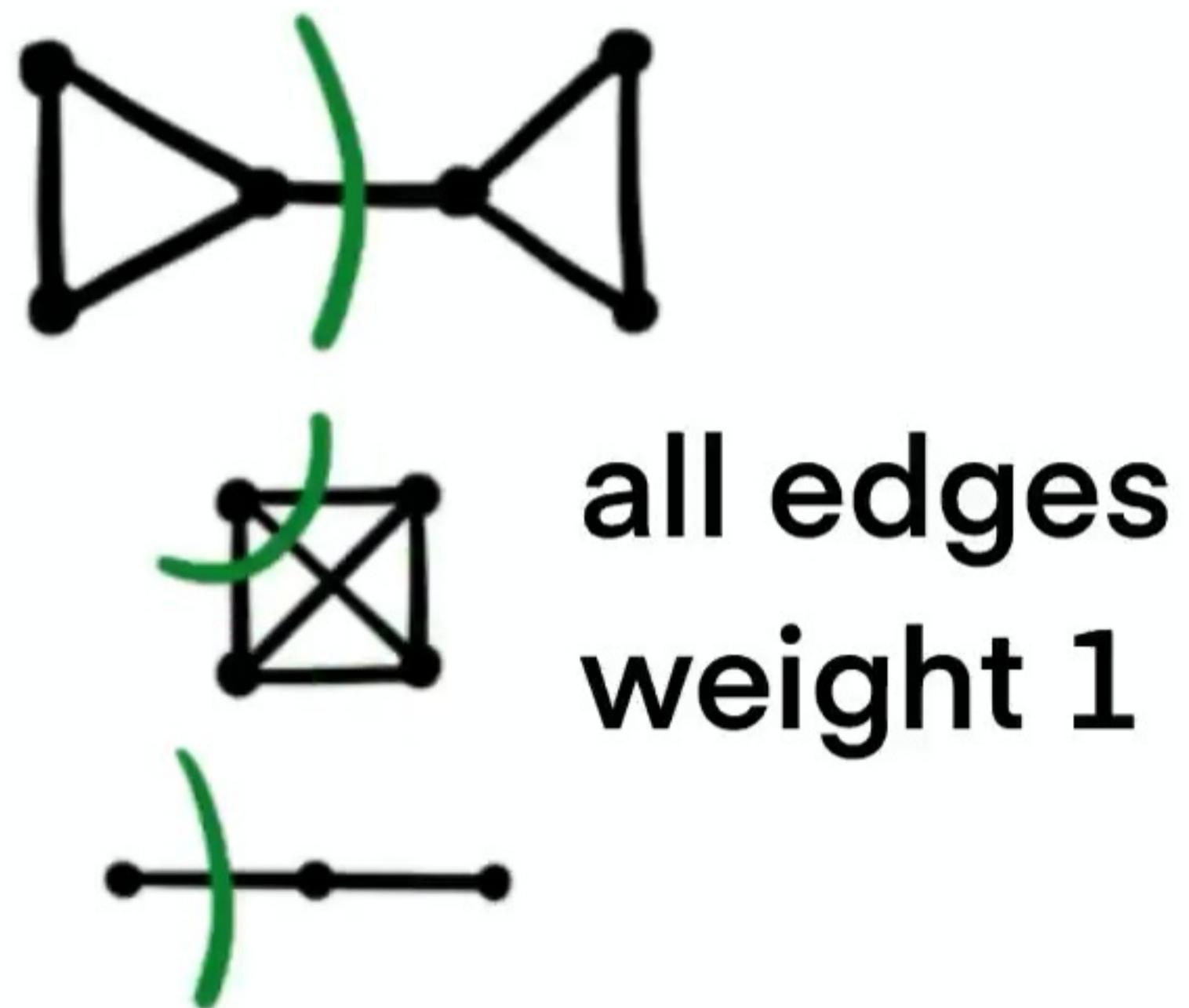
Di Wang

Introduction

- Global mincut problem:
given a weighted undirected graph,
delete edges of minimum weight
to disconnect the graph

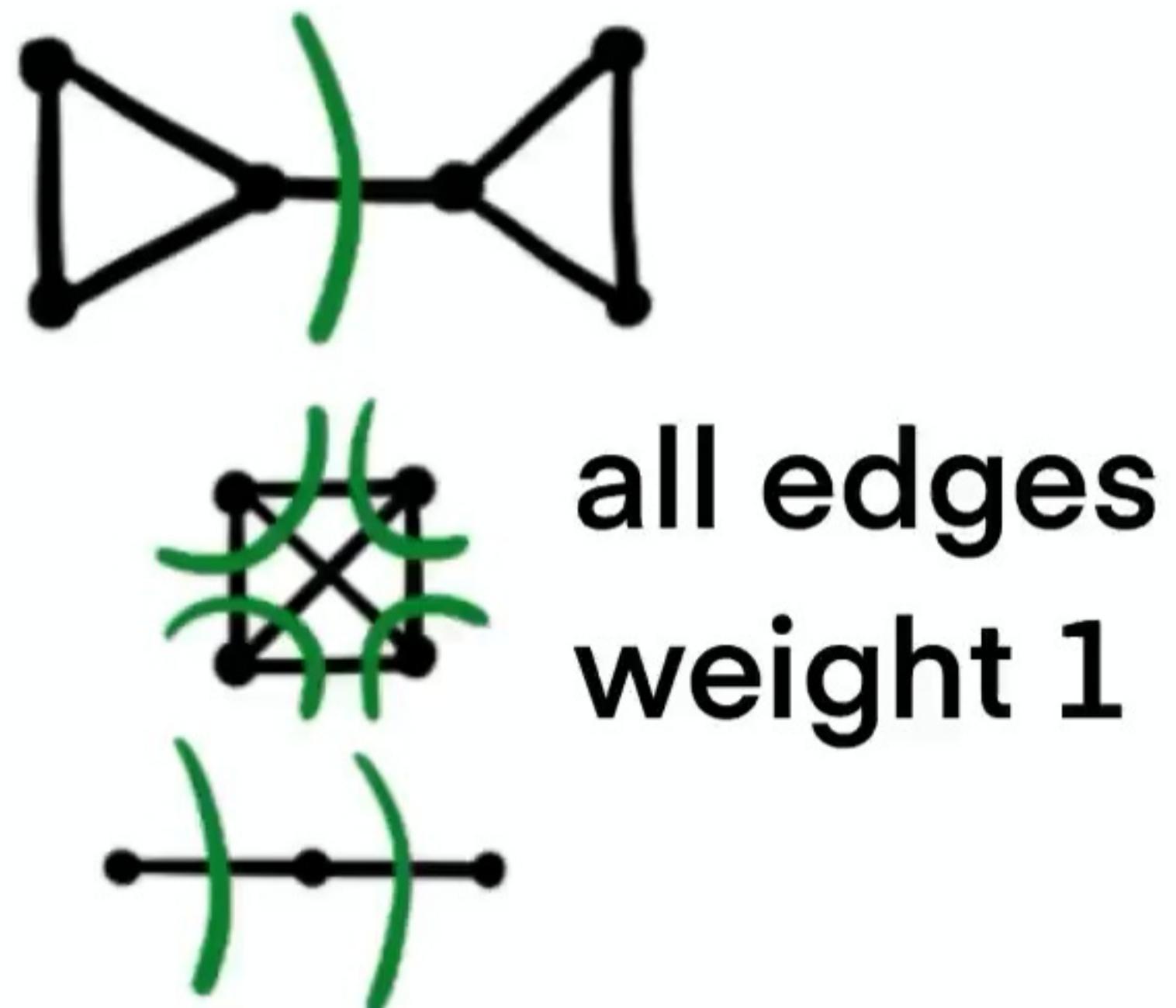
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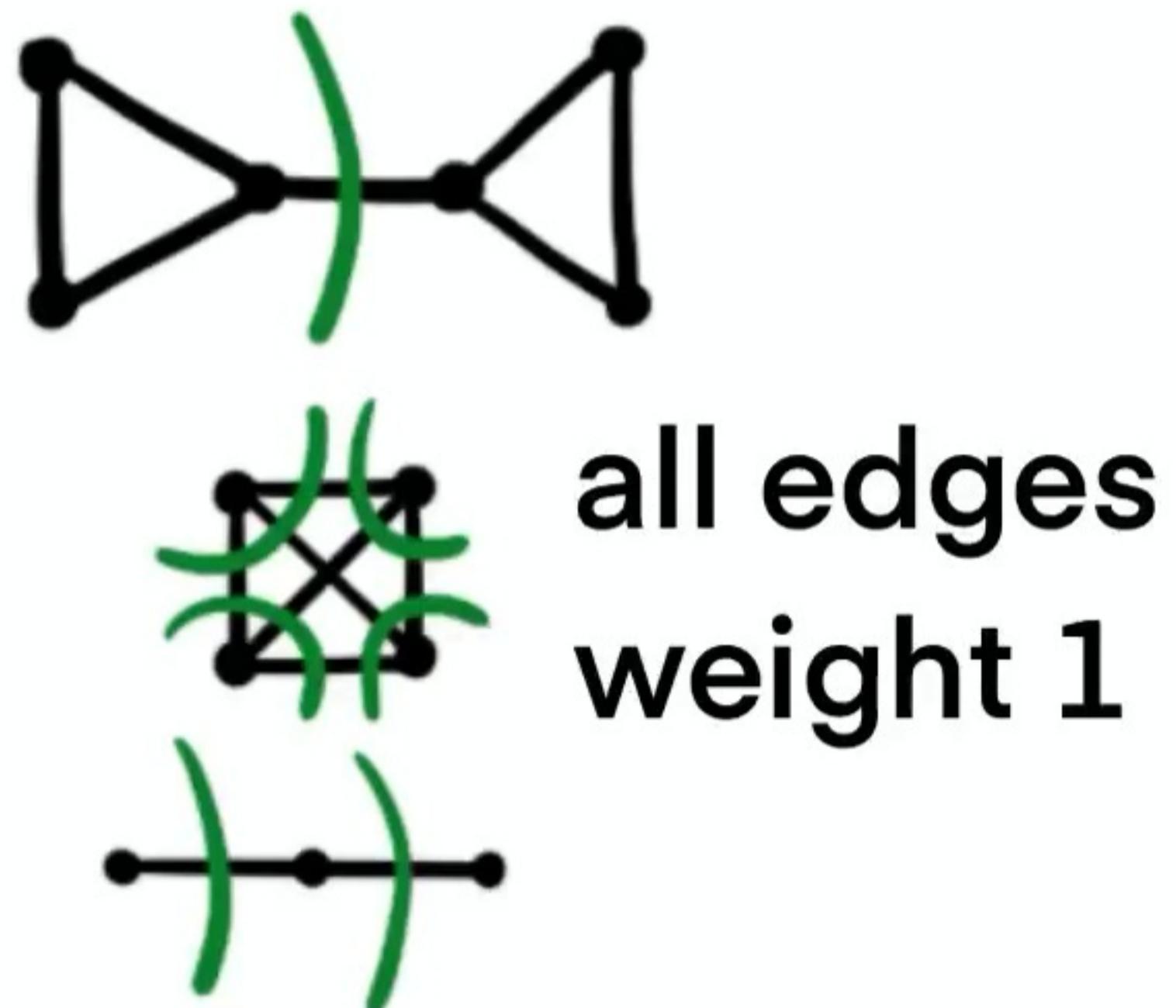
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- [Karger'96] randomized algorithm in $O(m \log^3 n)$ time
“Is there a **deterministic** near-linear time algorithm?”

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"almost-linear time"
- [This work] $\tilde{O}(m)$ time, answering Karger's question

Outline

- Local win/win approach to global mincut

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- New randomized $(1+\epsilon)$ -approximate global mincut
(main conceptual contribution)

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- Local win/win approach to global mincut
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- New randomized $(1+\epsilon)$ -approximate global mincut
(main conceptual contribution)
- Derandomize and obtain exact mincut
(technical: 40+ pages)

Local Method

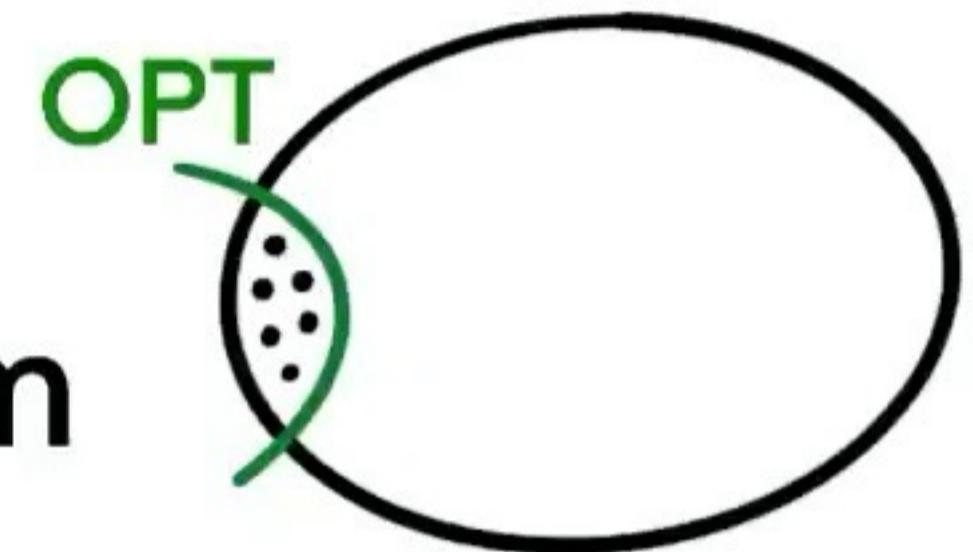
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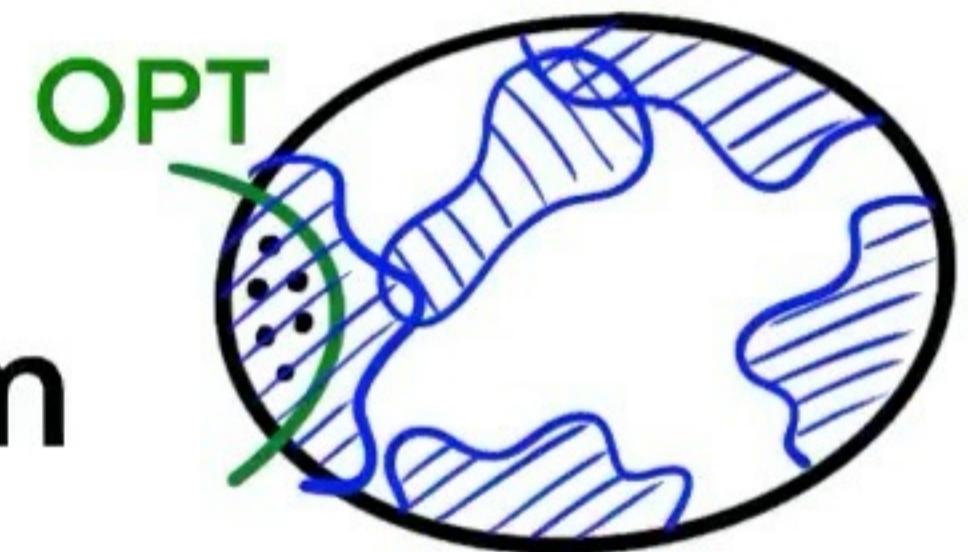
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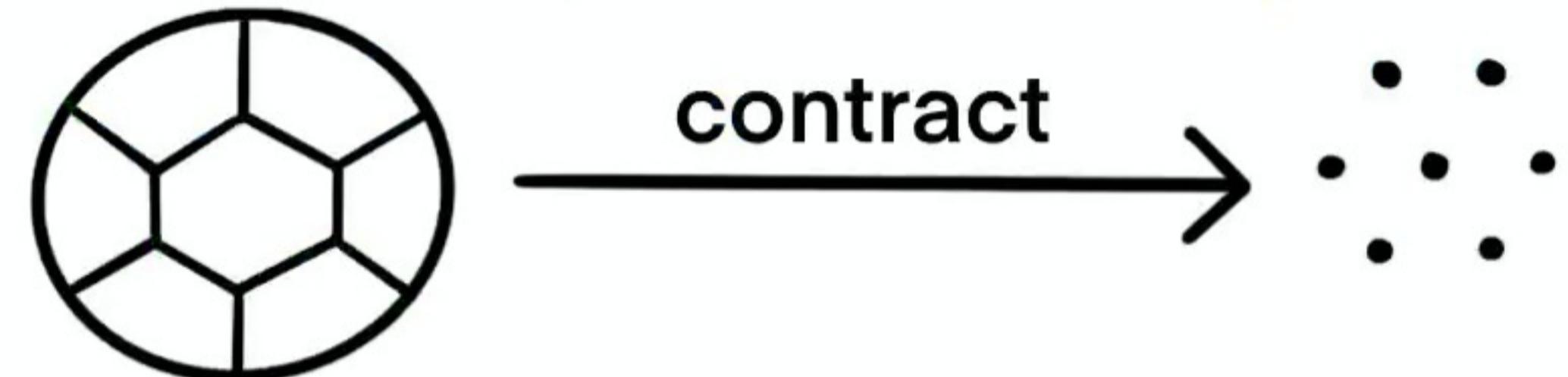


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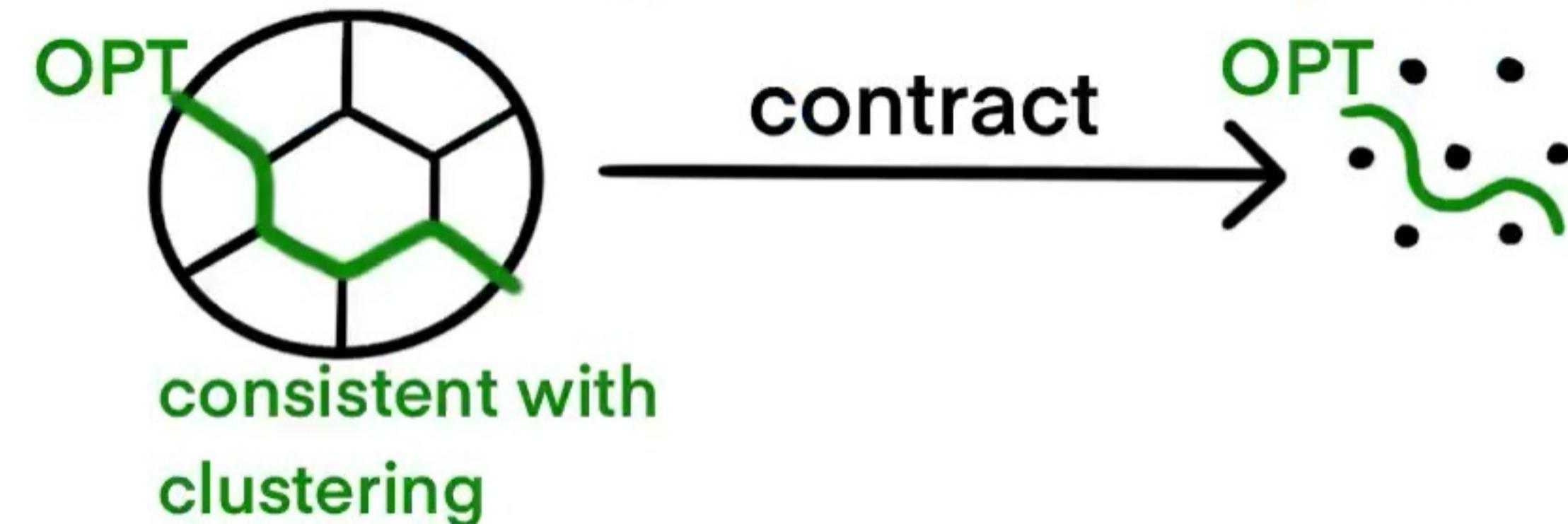


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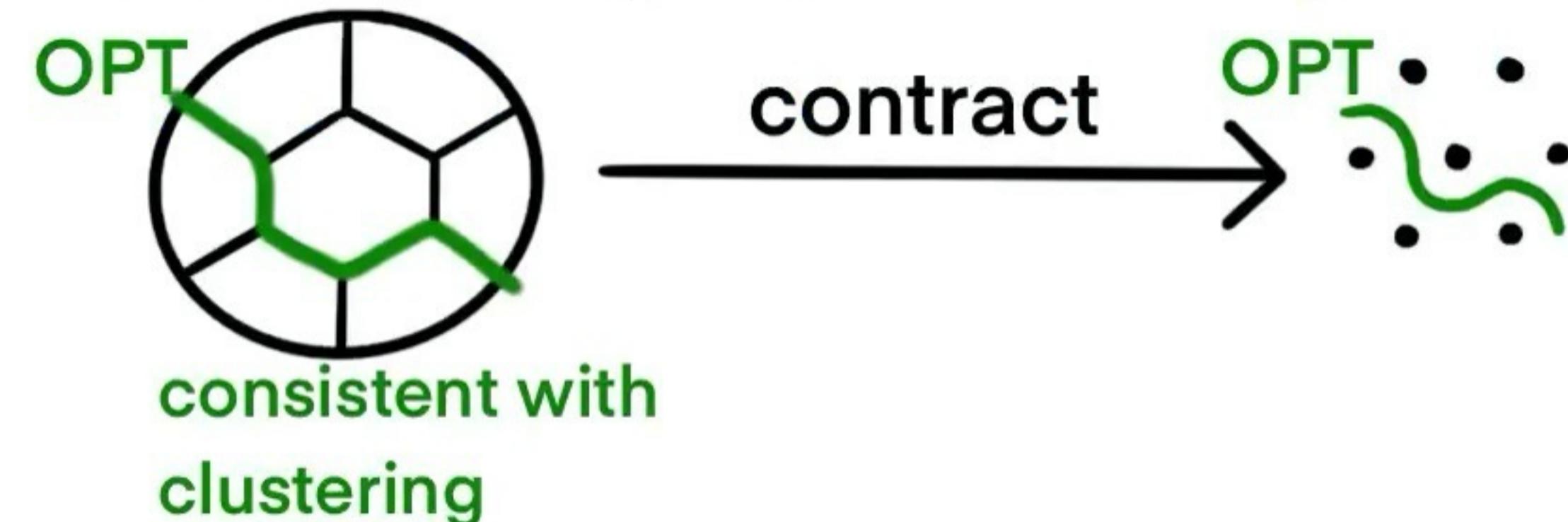


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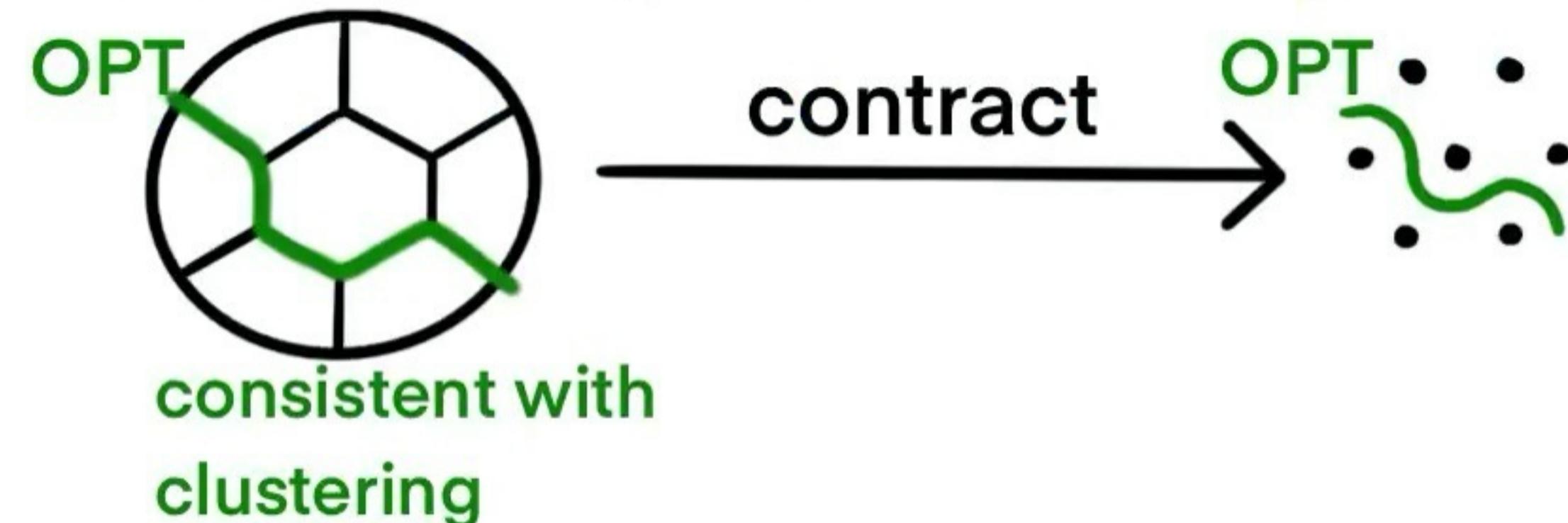
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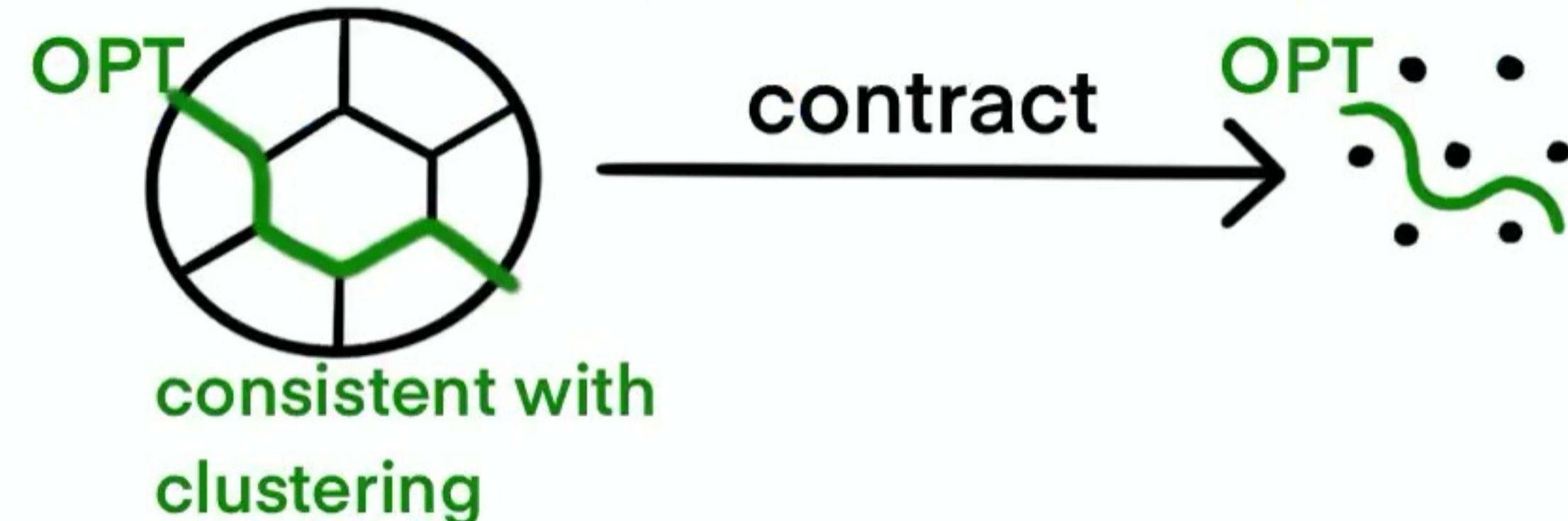
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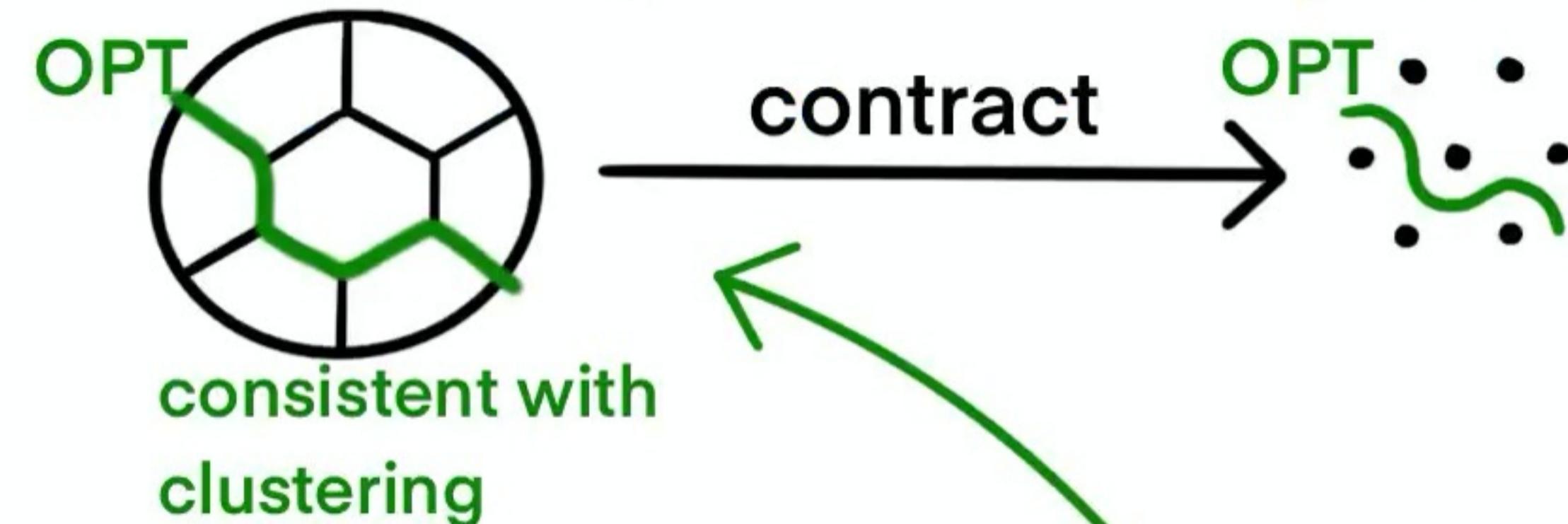
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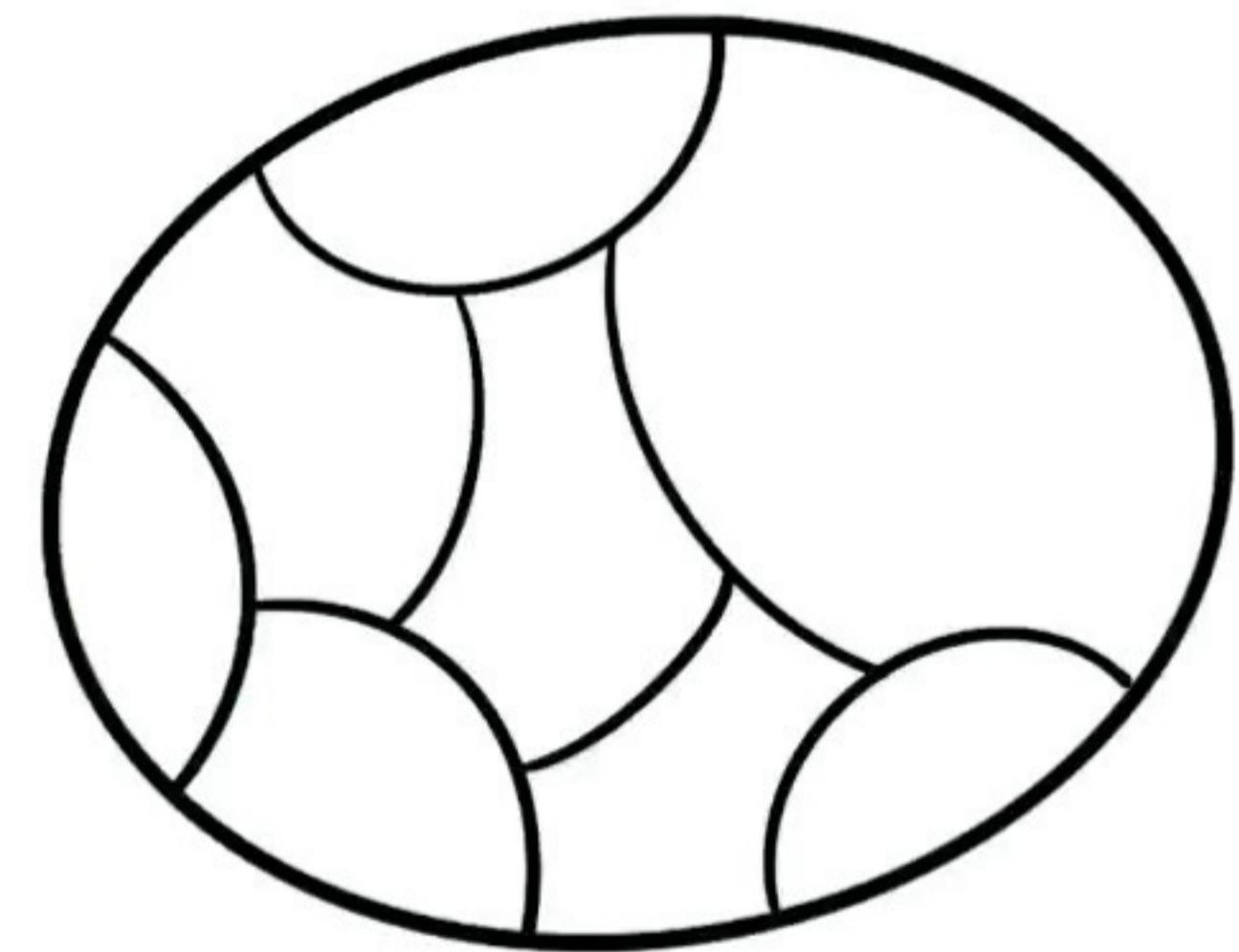
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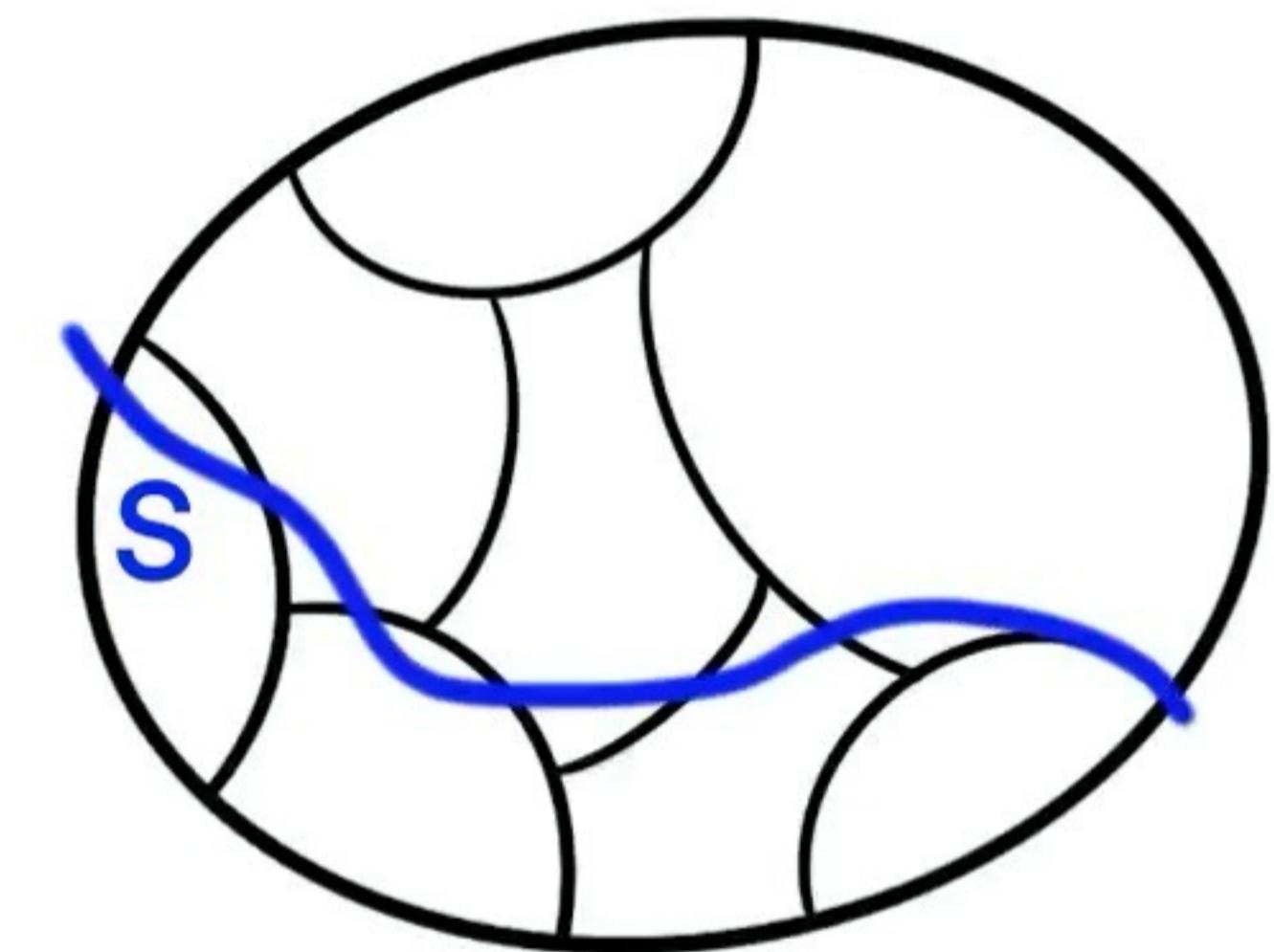
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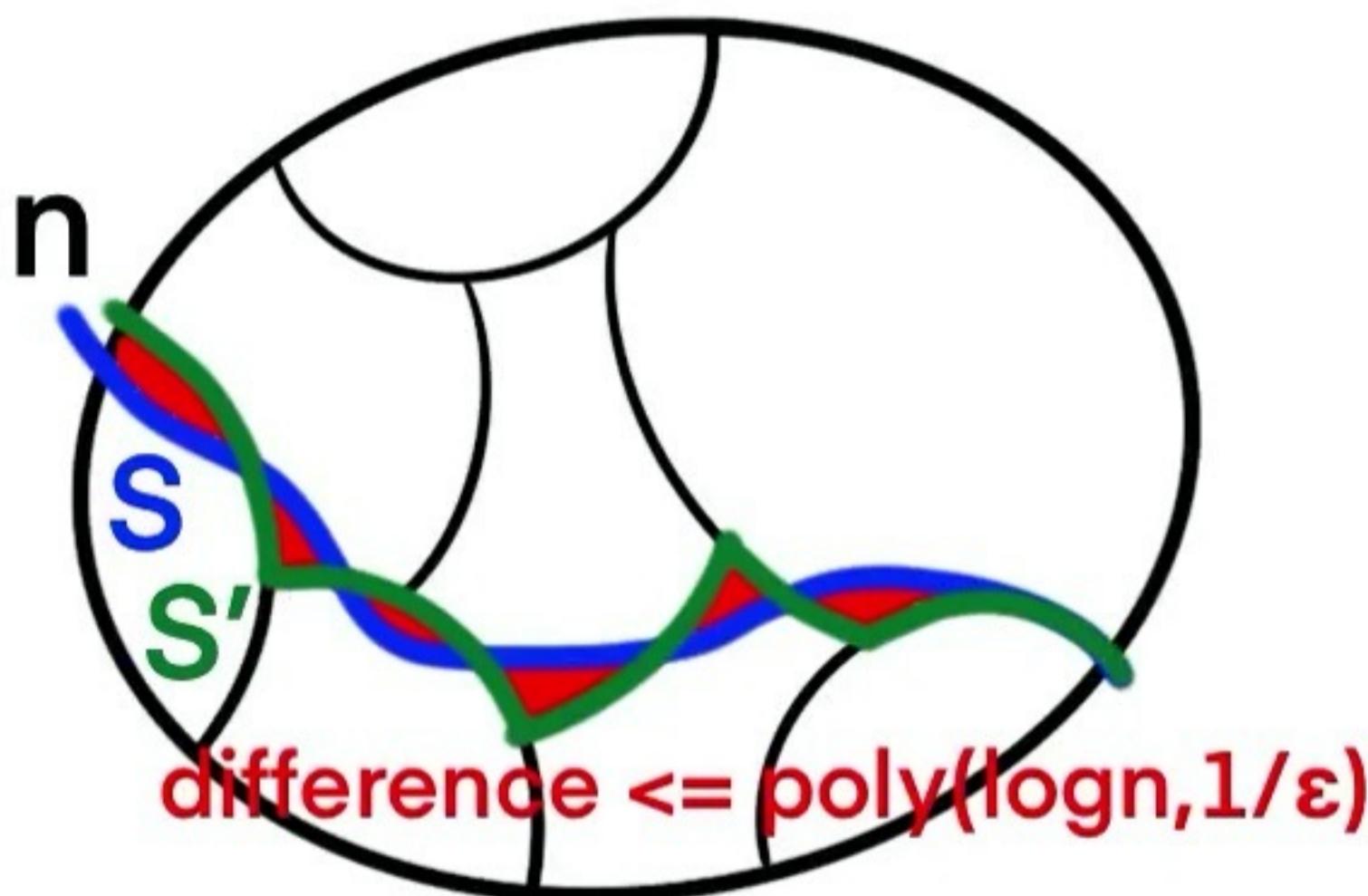
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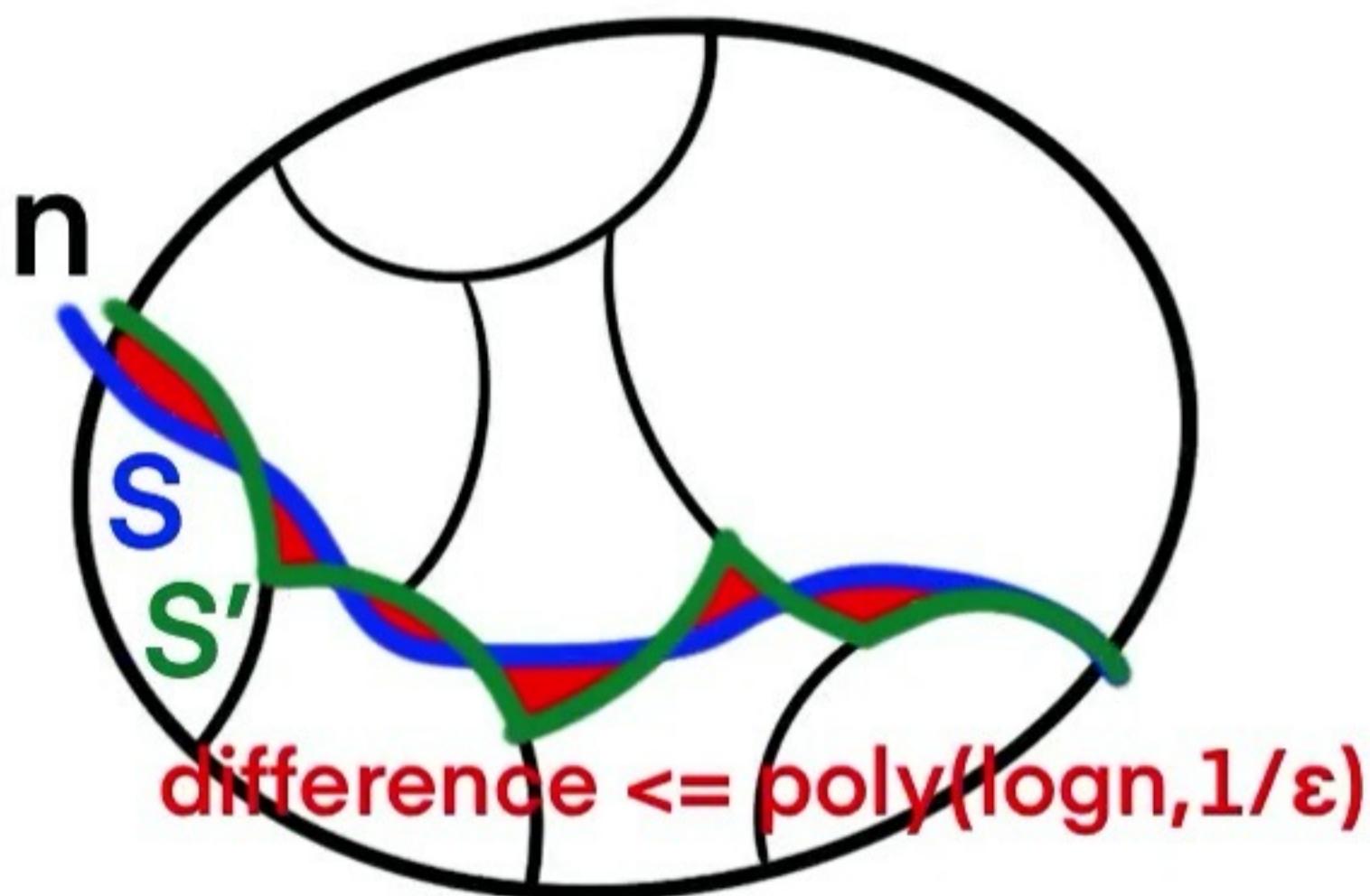
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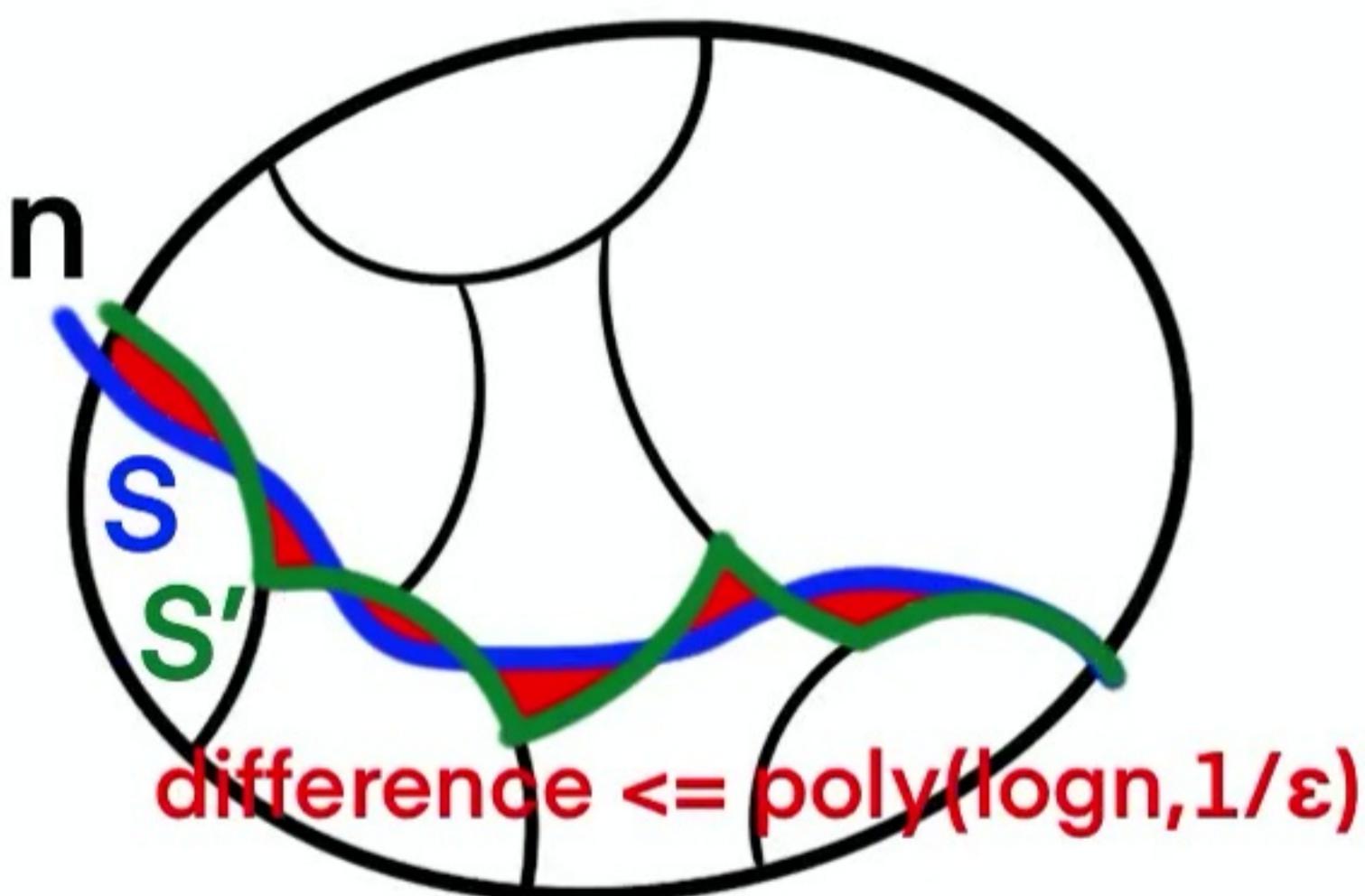
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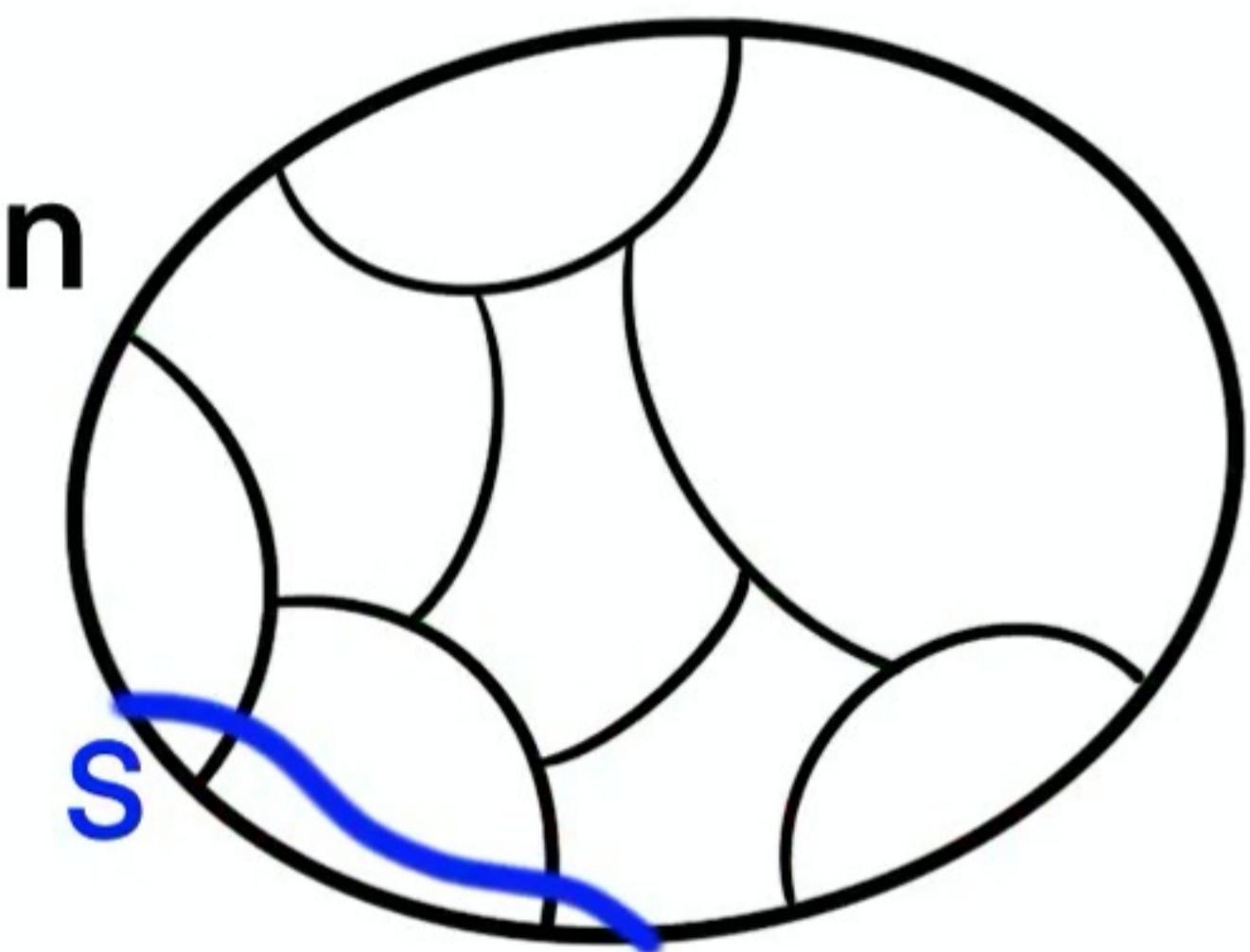
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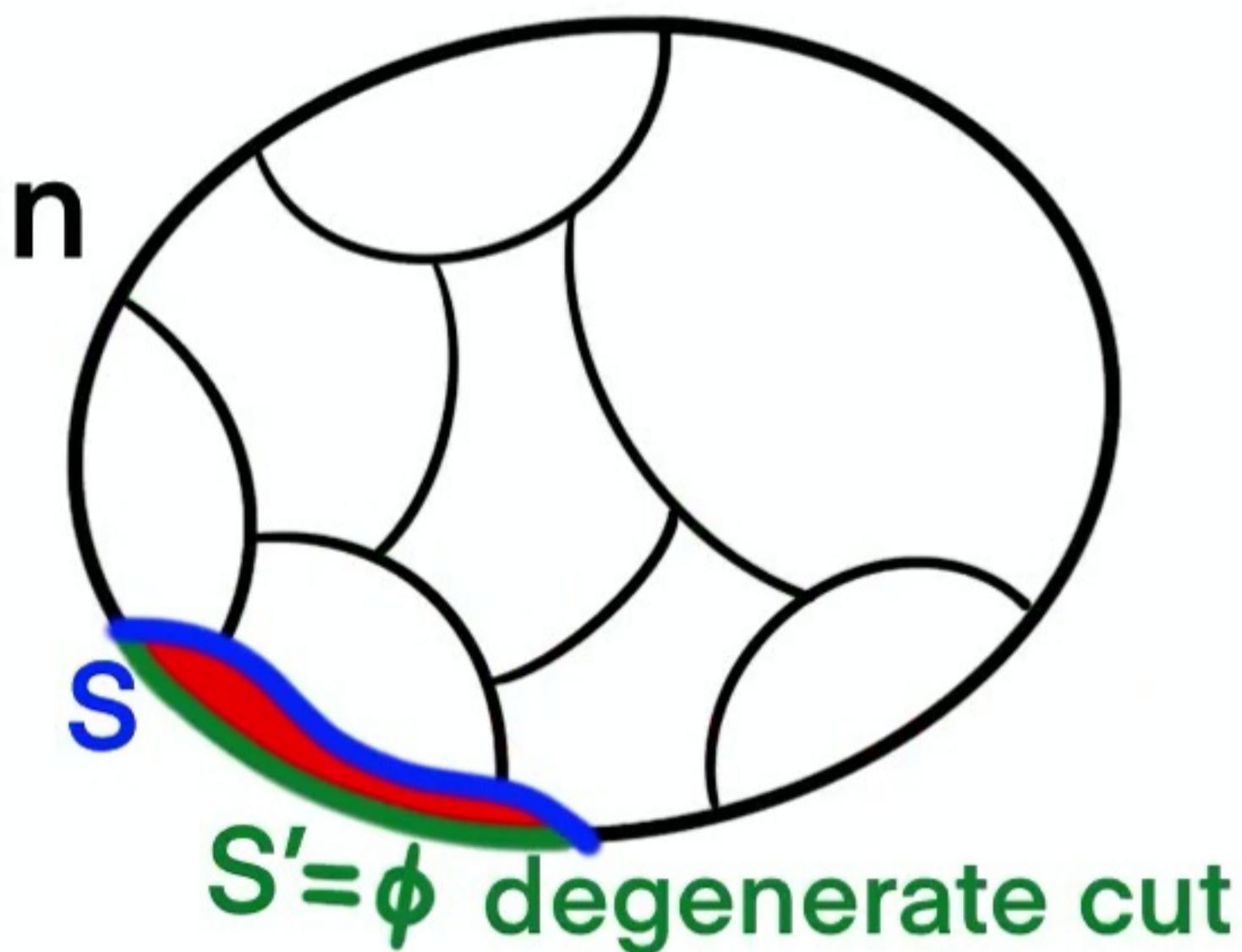
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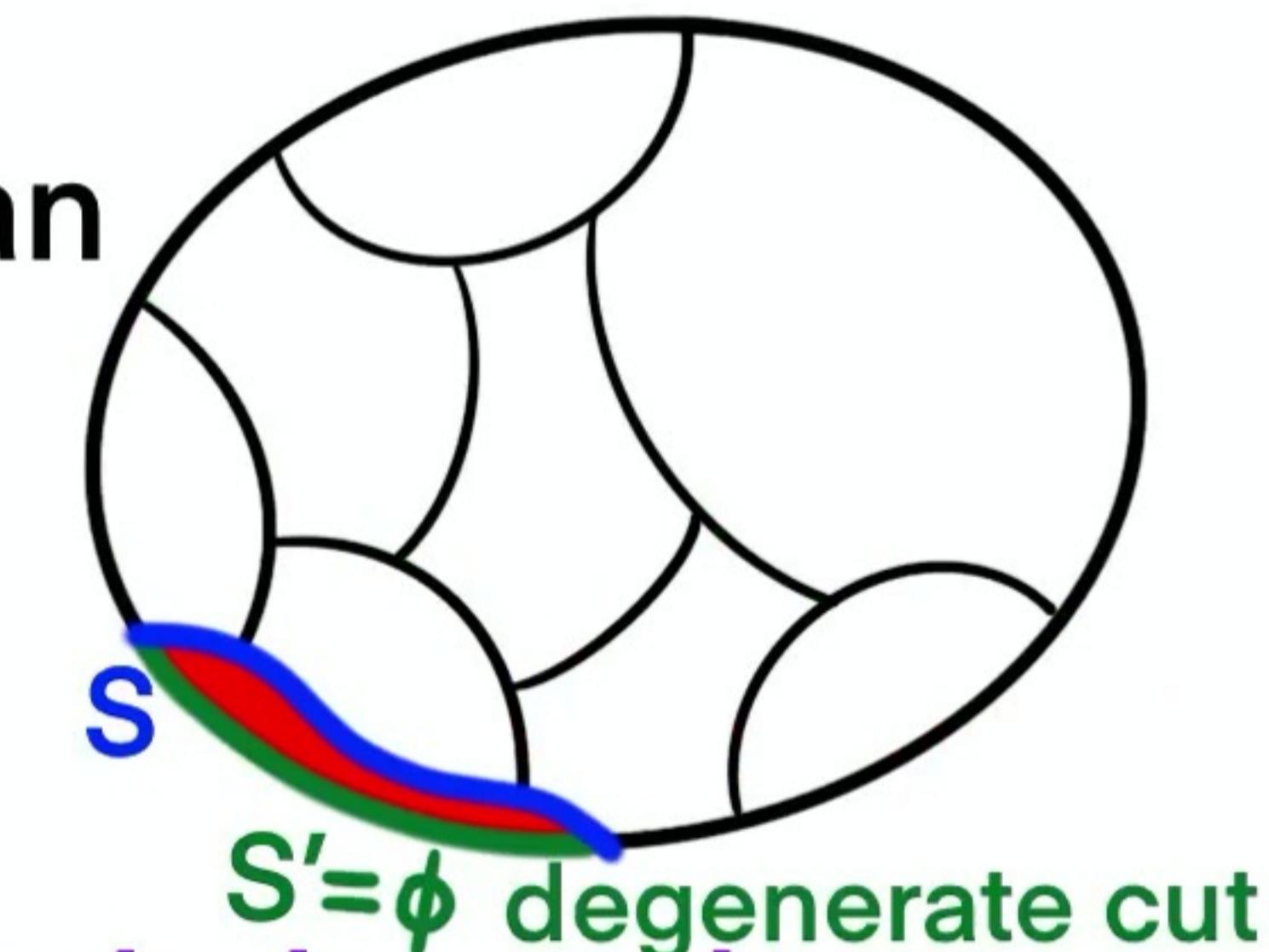
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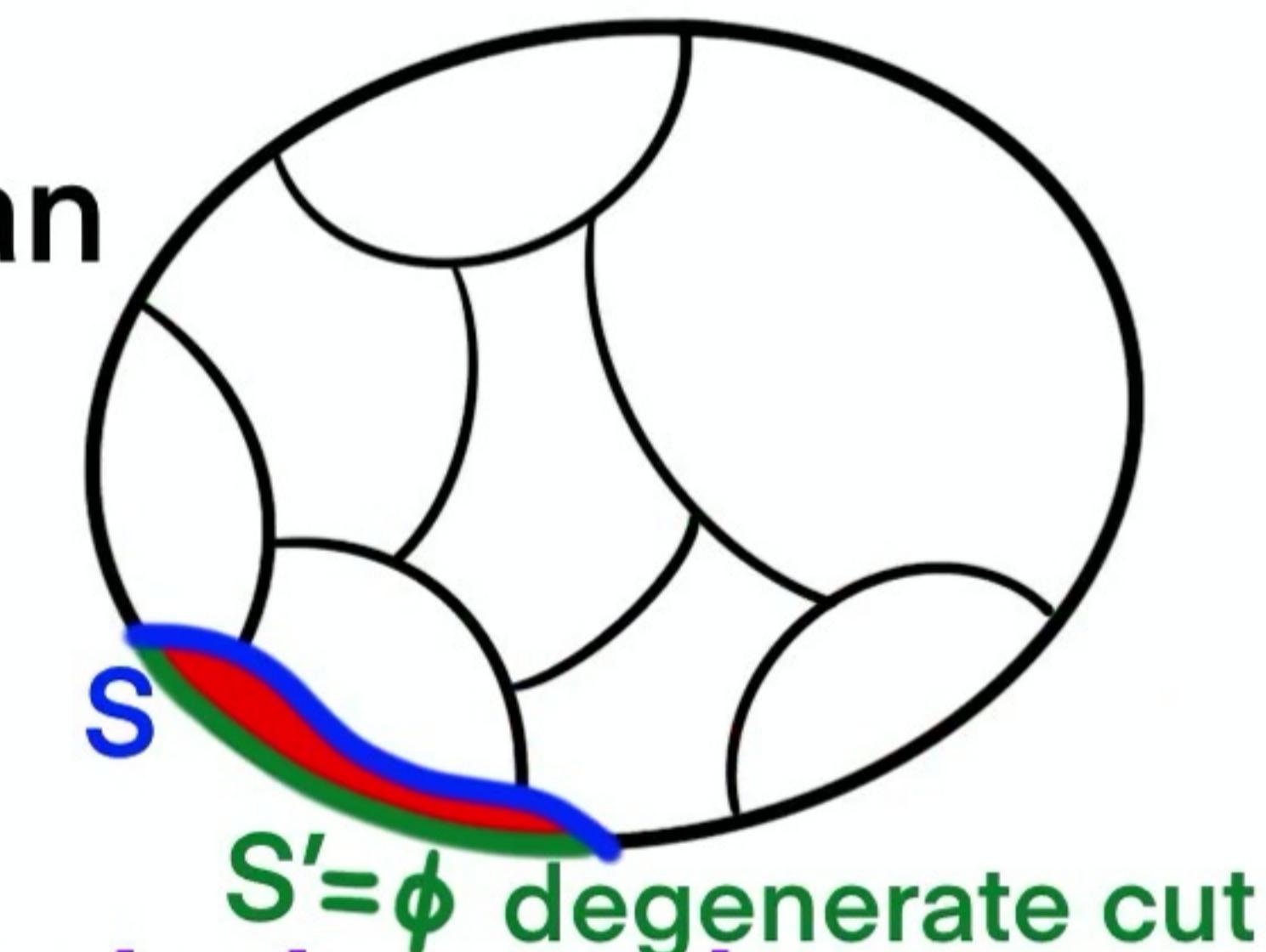
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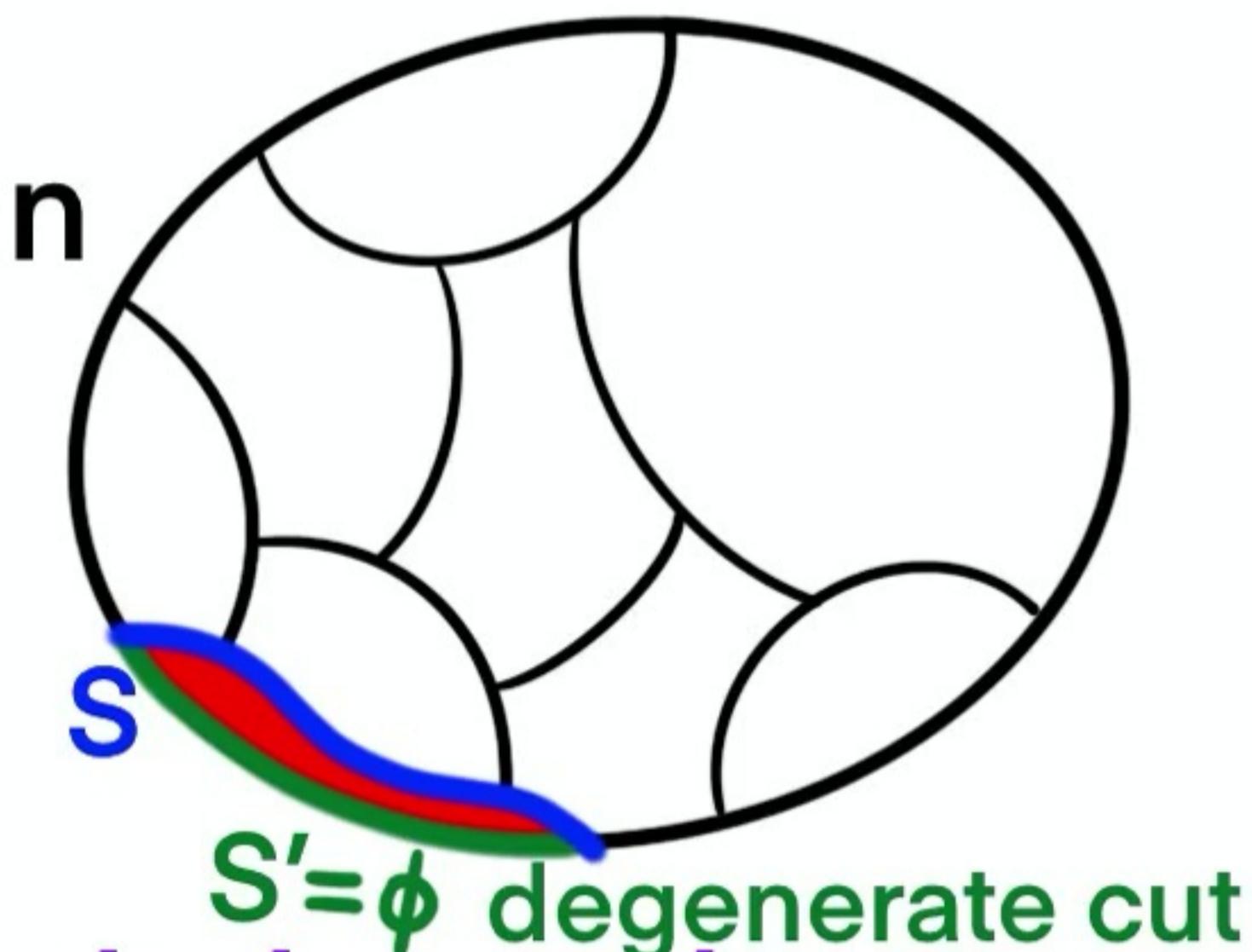
Example 1: path graph



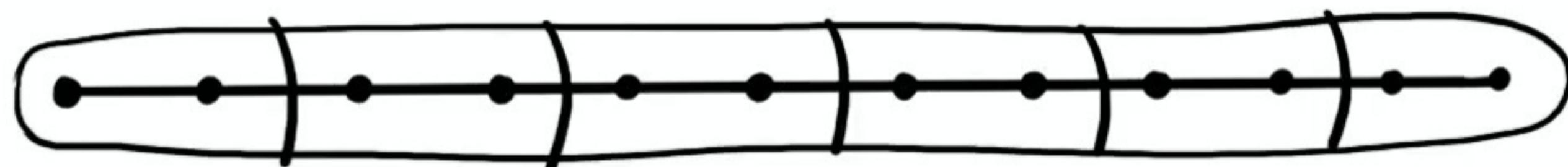
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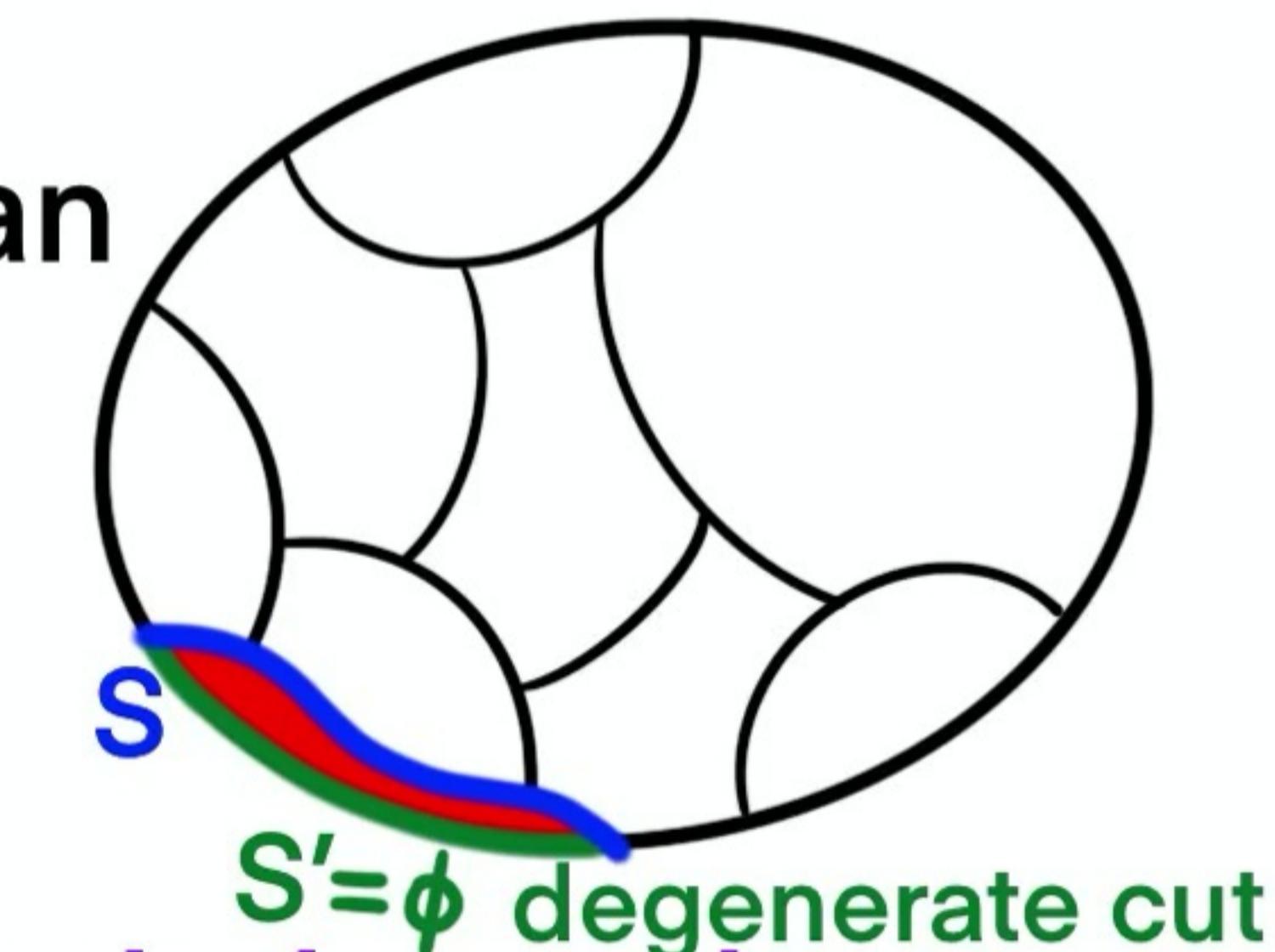
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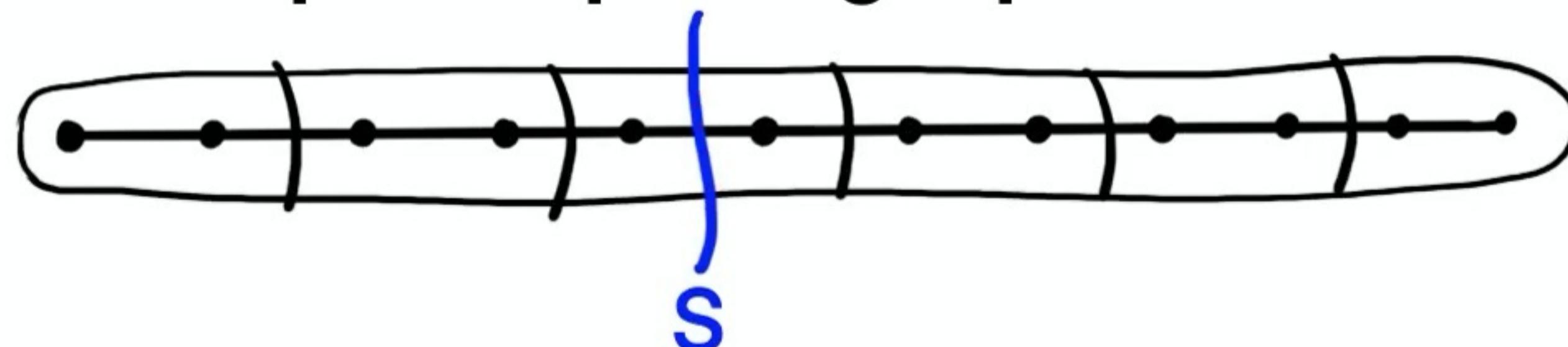
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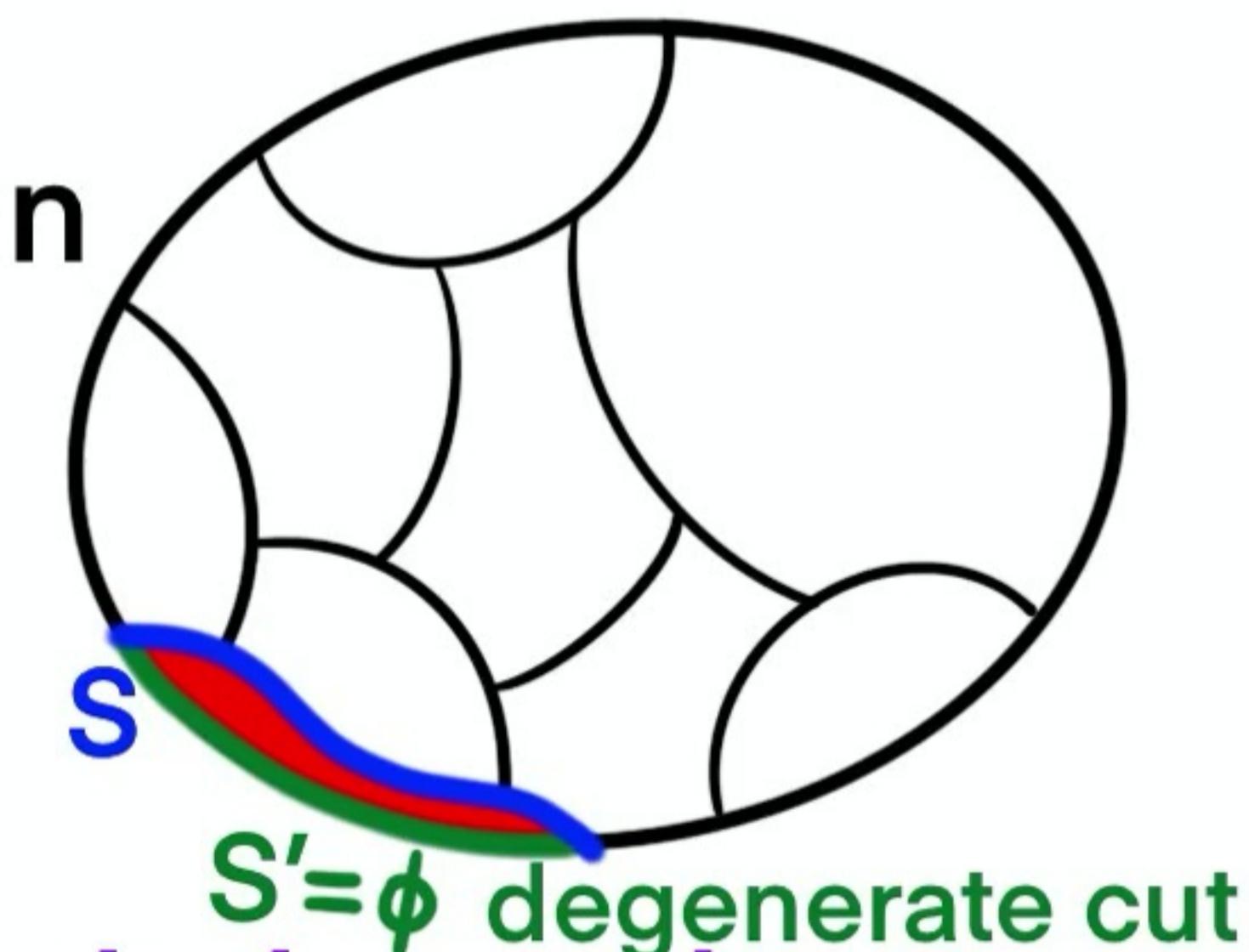
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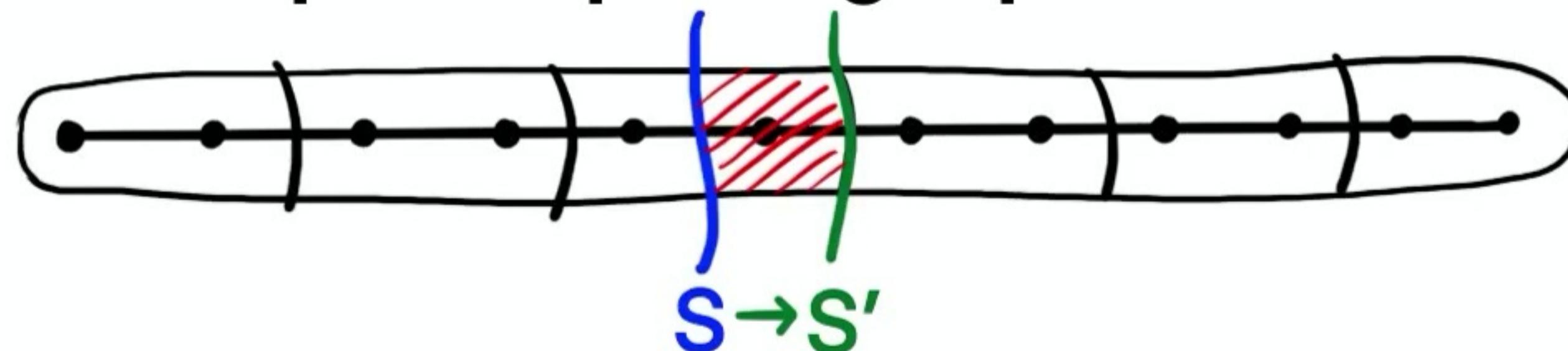
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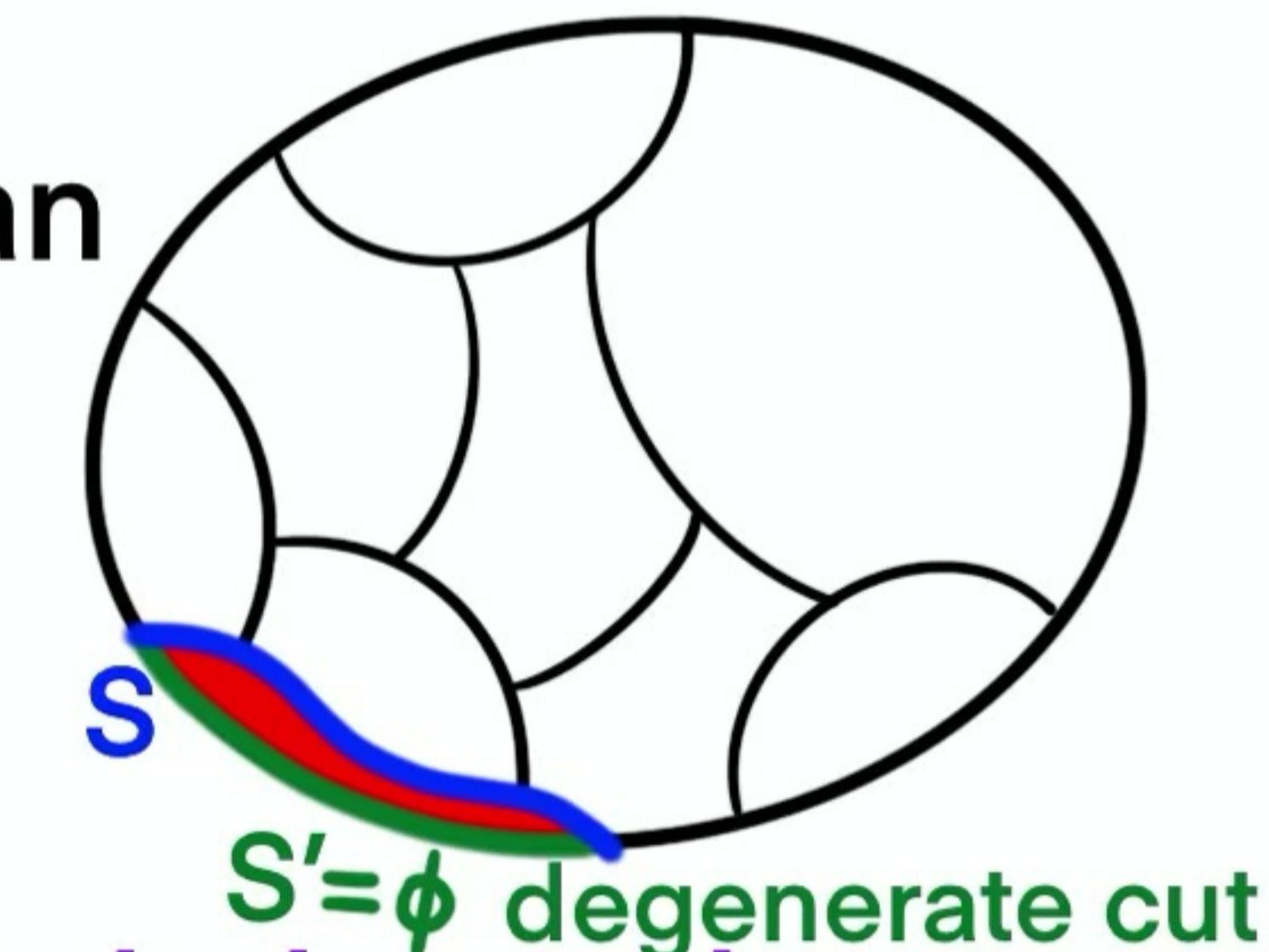
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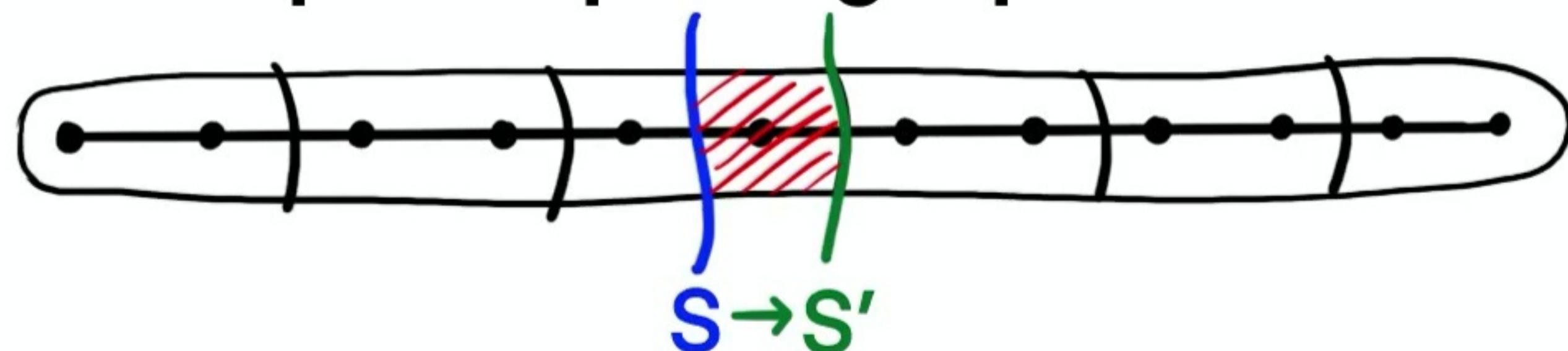
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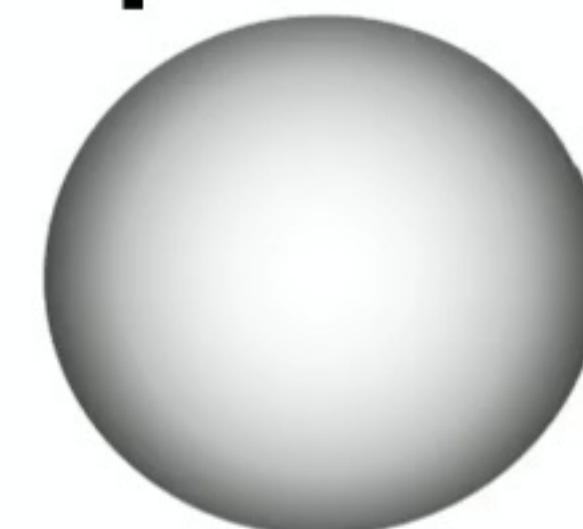
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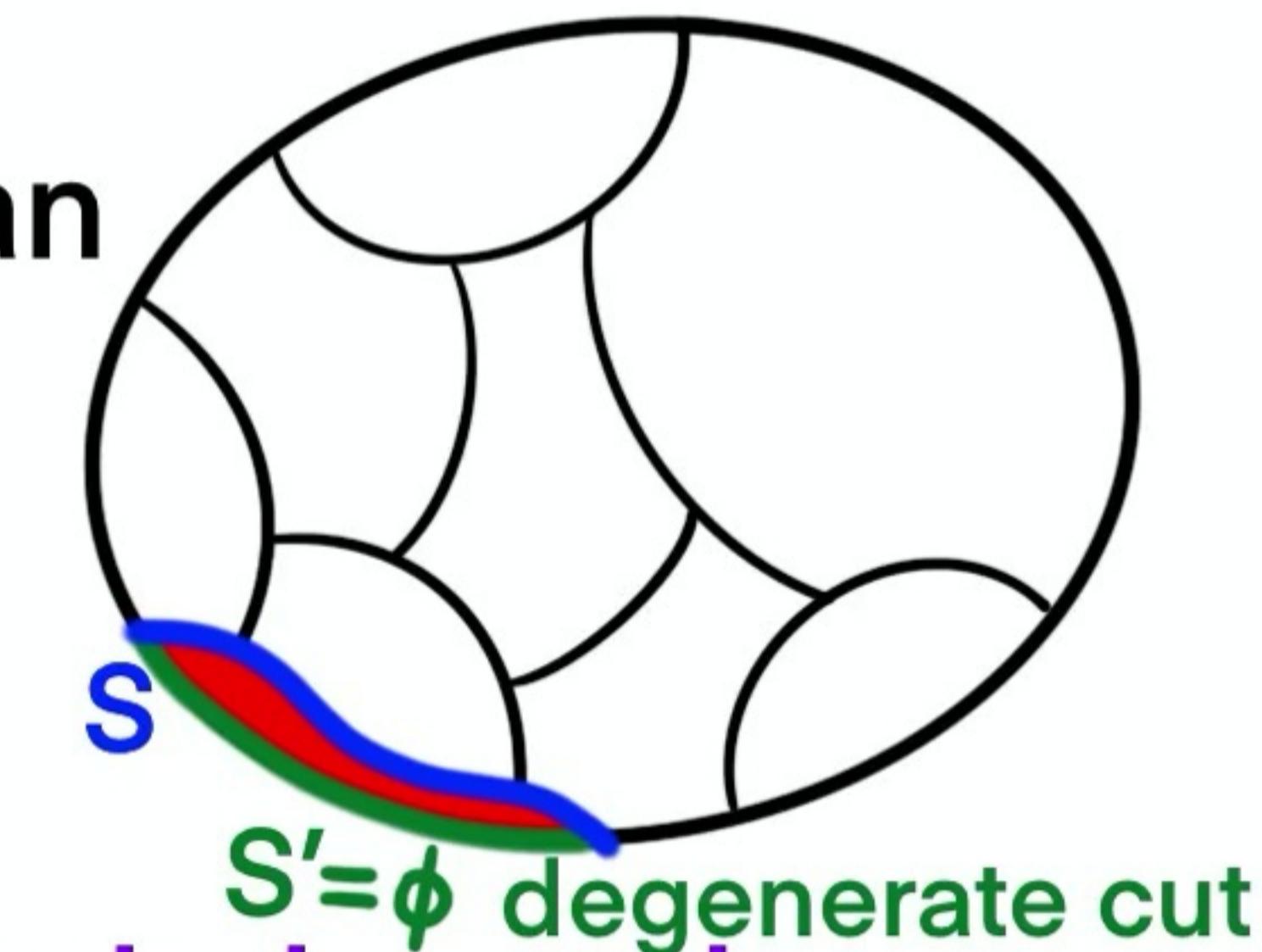
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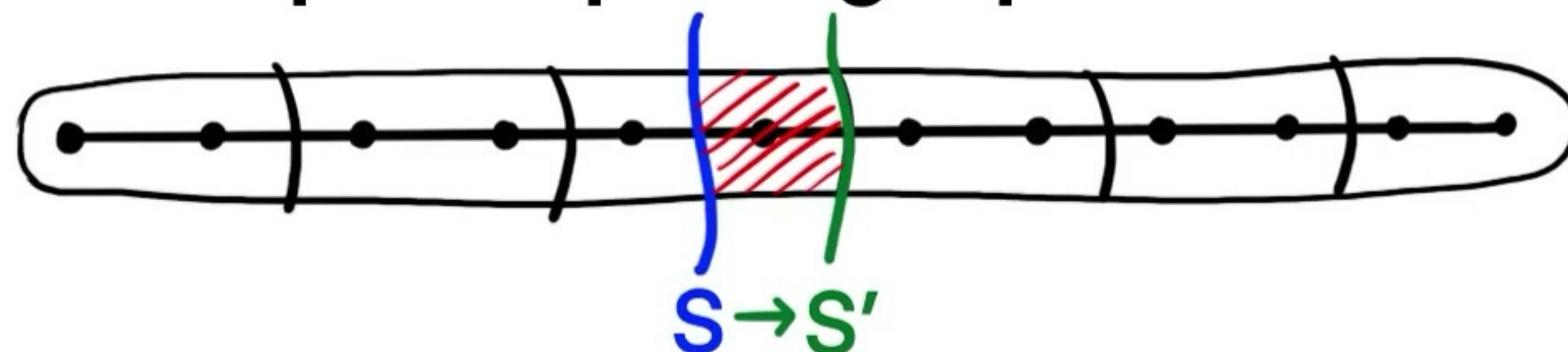
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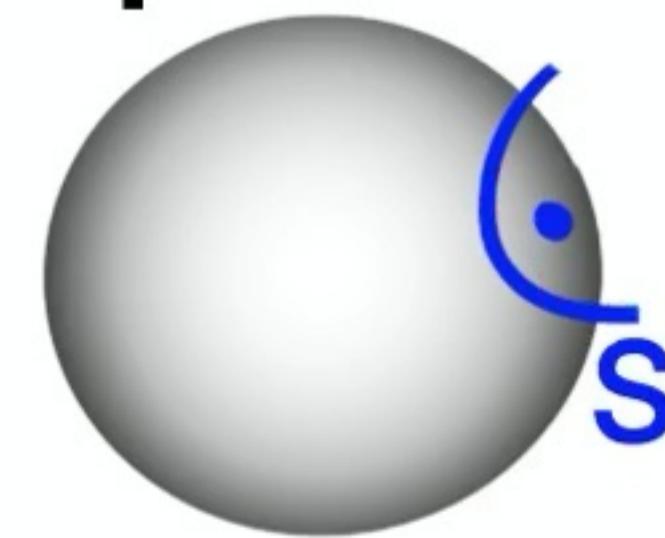
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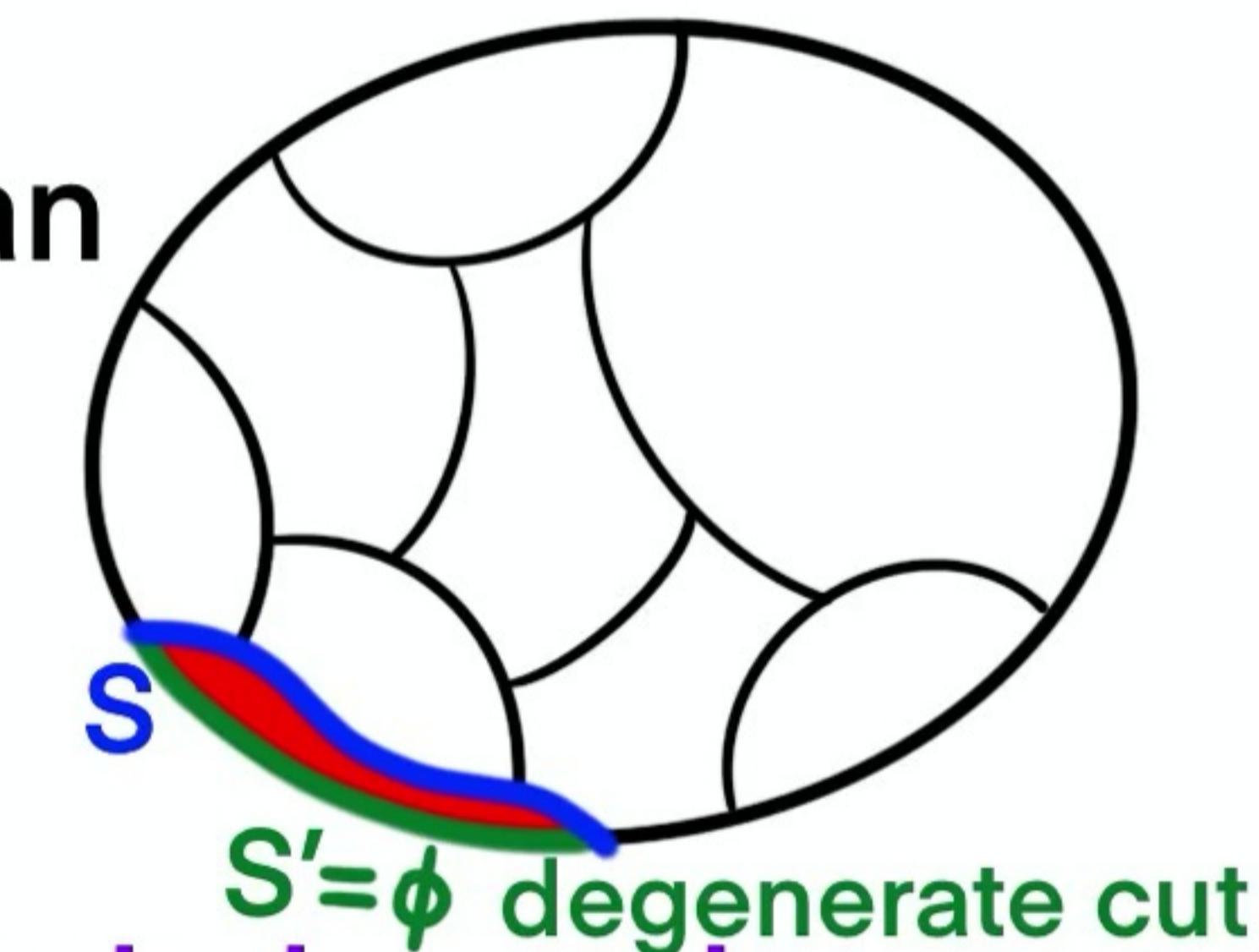
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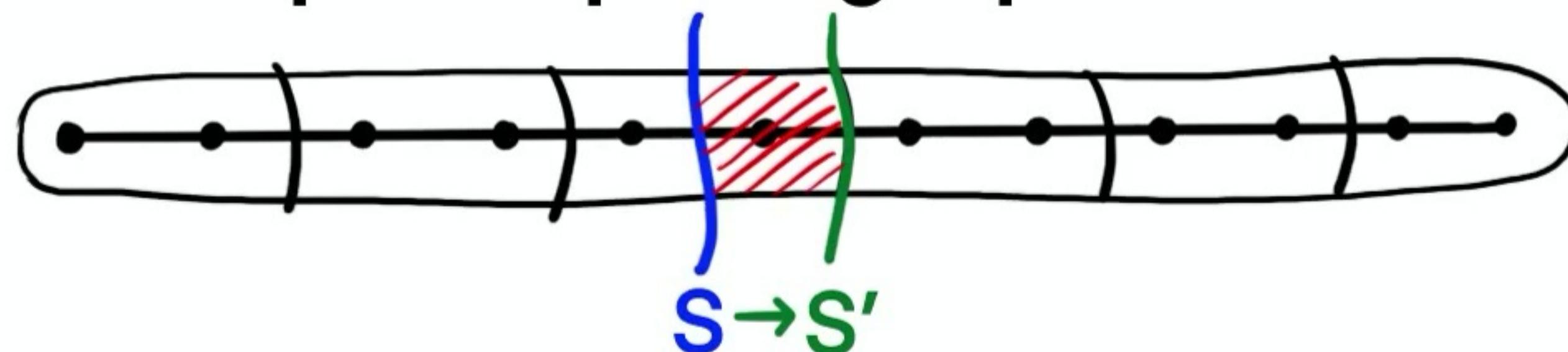
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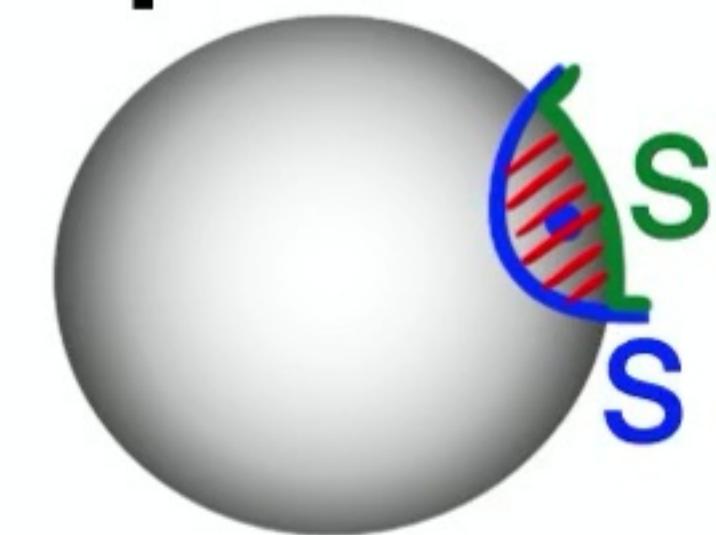
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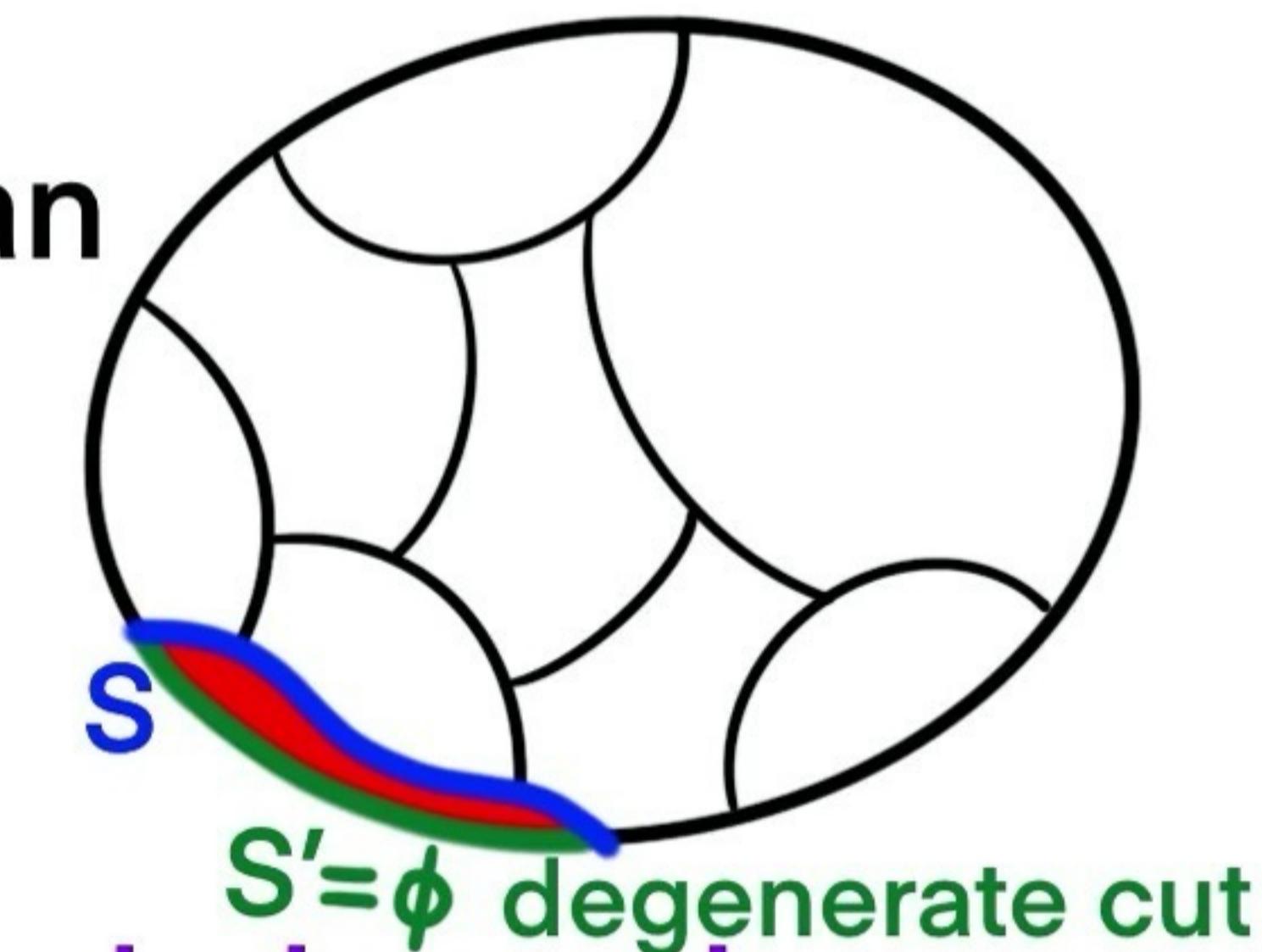
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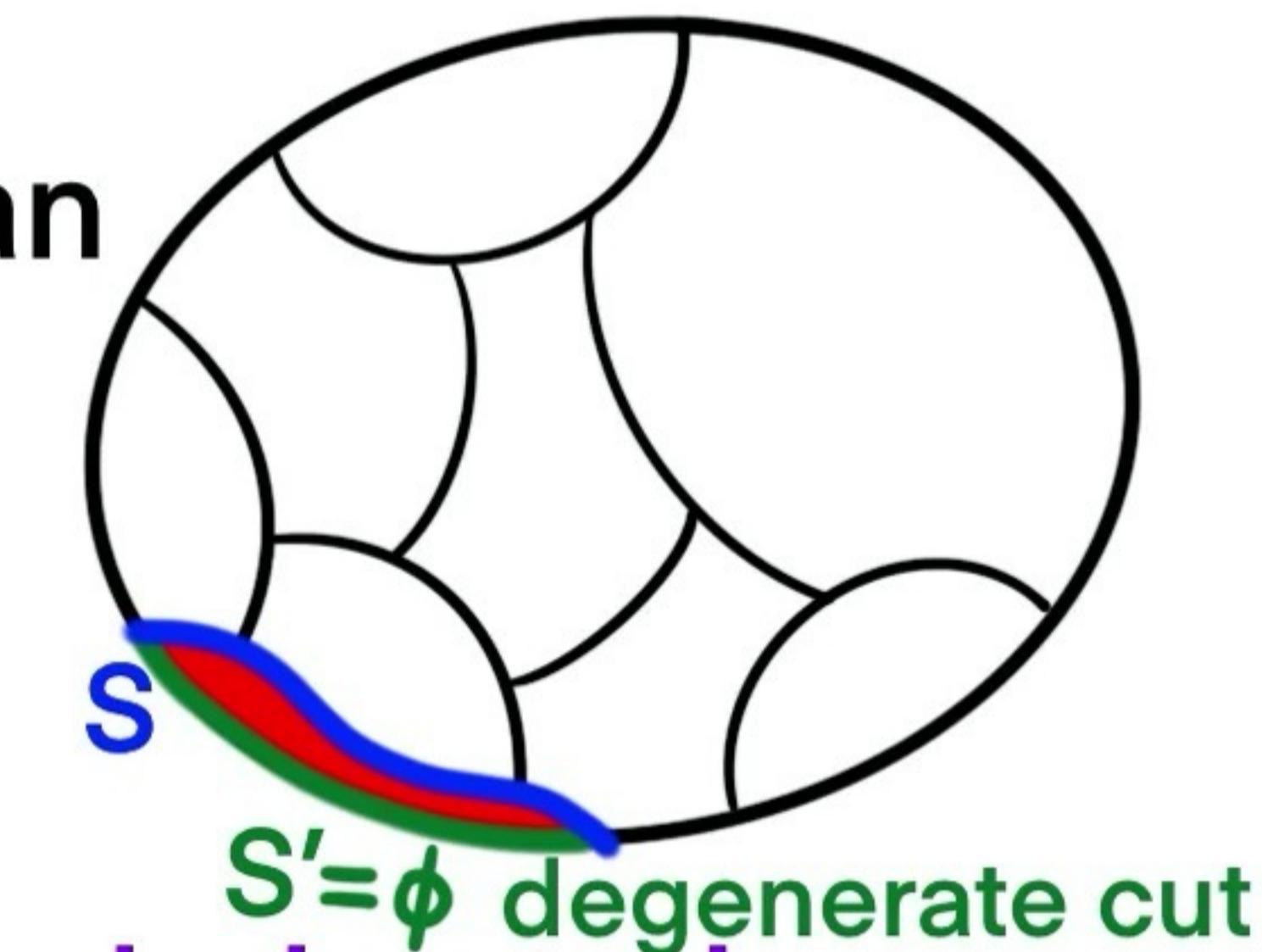
local algorithm [Kawarabayashi-Thorup]:

- if mincut is **trivial** (single vertex on one side): find min degree
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Algorithm **unbalanced $\leq \text{poly}(\log n, 1/\epsilon)$ vertices**

run local algorithm

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up to $(1+\epsilon)$ factor

$(1+\varepsilon)$ -approximate mincut

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Repeat $\log_2 n$ times: run local algorithm, then cluster+contract
 $\rightarrow (1+\varepsilon)^{\log n}$ approximation. Set $\varepsilon \ll 1/\log n \rightarrow (1+o(1))$ -approx

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“Local Karger contraction” [Nalam-Saranurak’23]:

$m k^2 \text{polylog}(n)$ randomized time if k vertices on one side

$k = \text{poly}(\log n, 1/\varepsilon) = \text{polylog}(n)$

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$m k^2 \text{polylog}(n)$ randomized time if k vertices on one side

$$k = \text{poly}(\log n, 1/\epsilon) = \text{polylog}(n)$$

Overall: $m \text{polylog}(n)$ randomized for $(1+o(1))\text{-approx}$ mincut

Structure Theorem Proof Outline

Theorem: for any weighted undirected graph, can group the vertices into $\leq n/2$ clusters s.t.

- for any 1.01-approx mincut S , can modify few vertices $\rightarrow S'$ s.t.
(1) $\text{cost}(S') \leq (1+\varepsilon) \text{cost}(S)$, (2) S' is consistent with clustering

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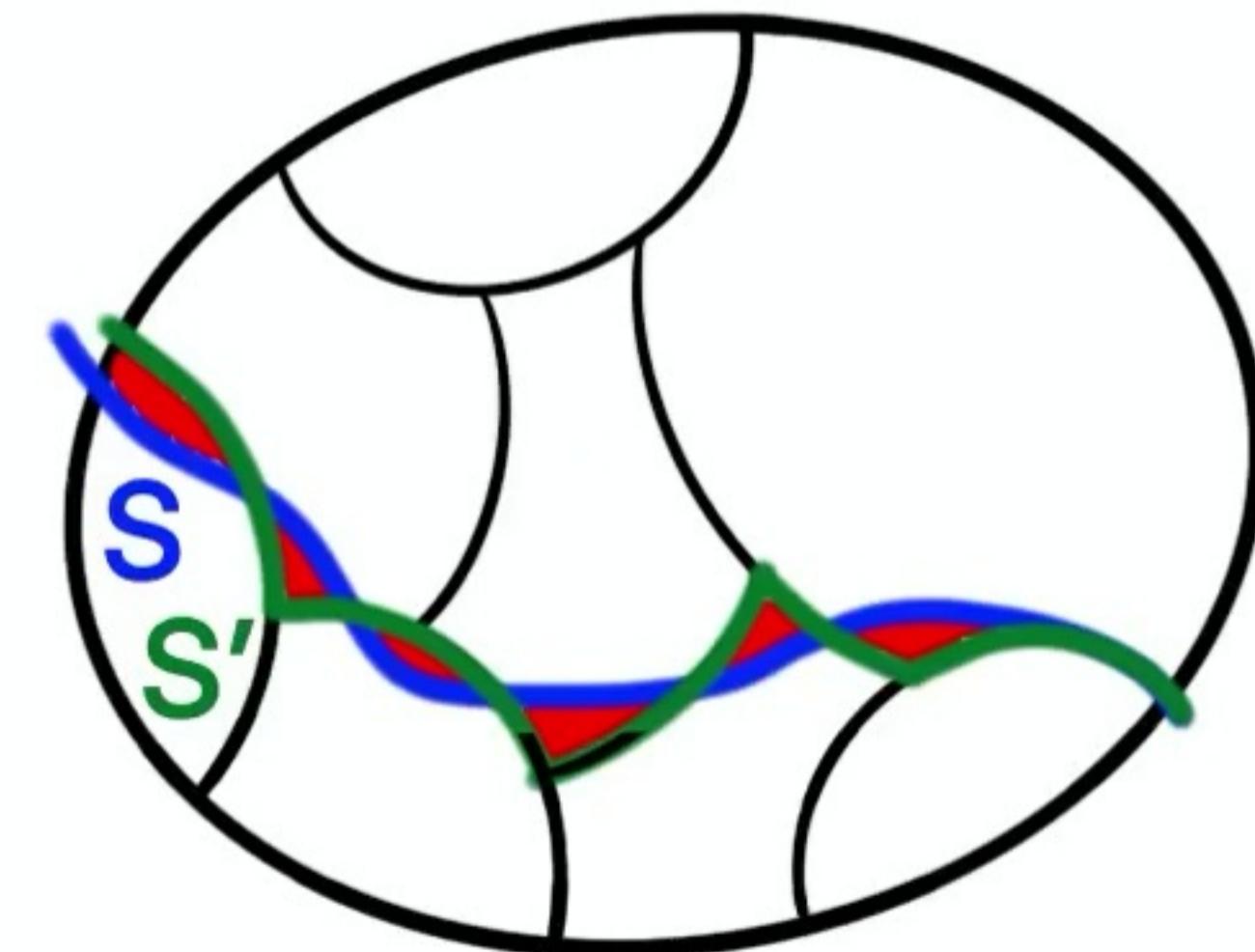
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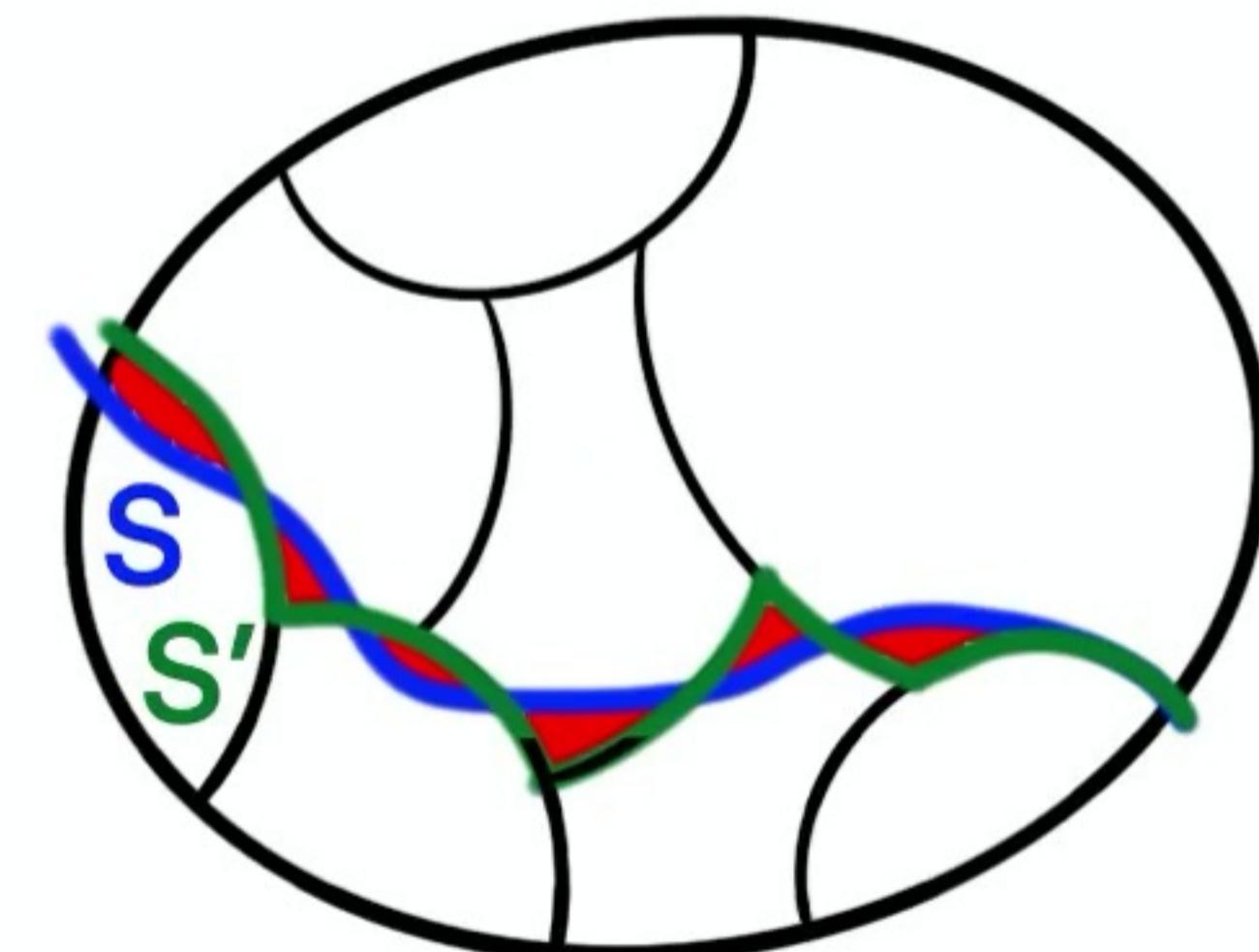


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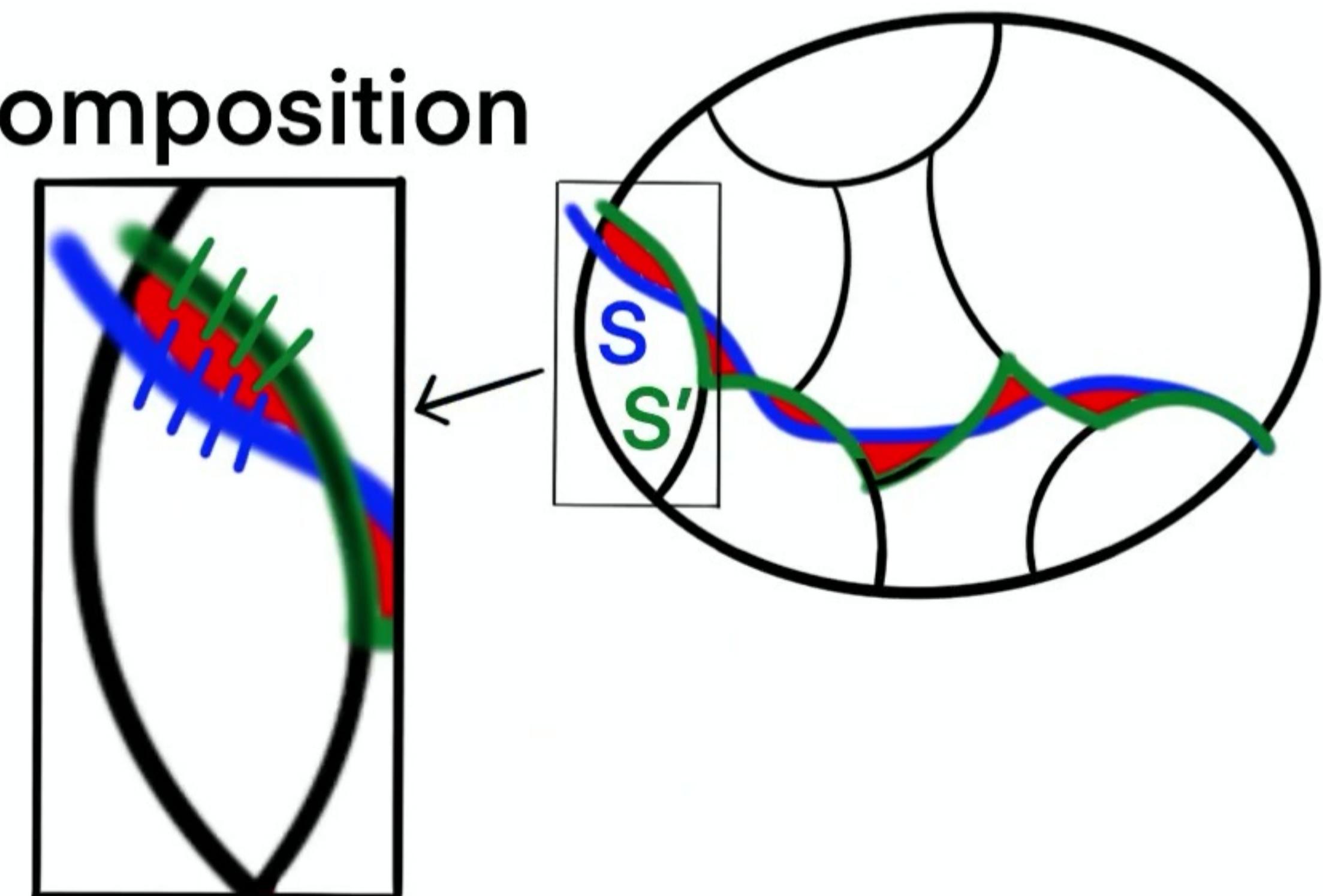


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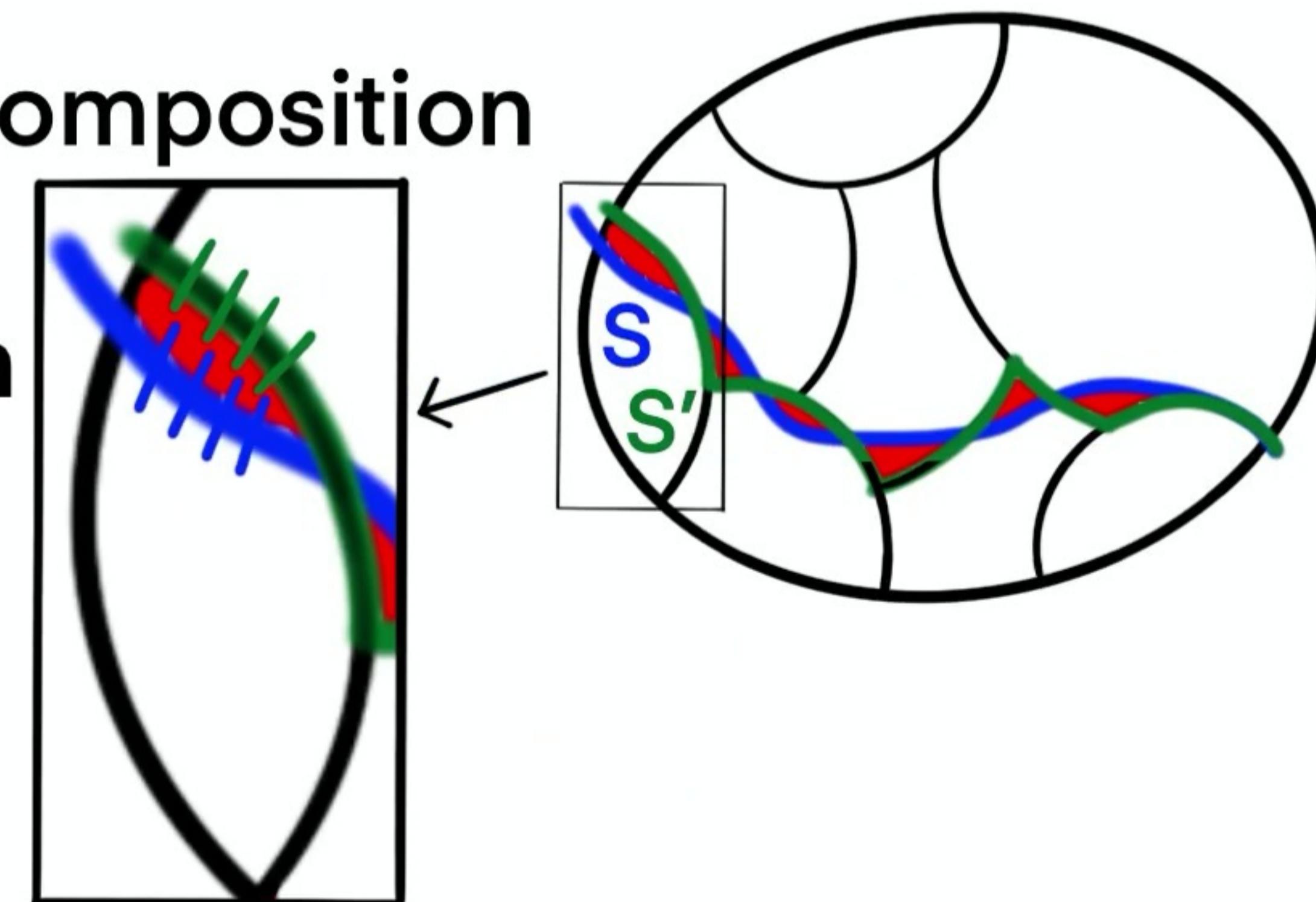
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Step 1: compute expander decomposition

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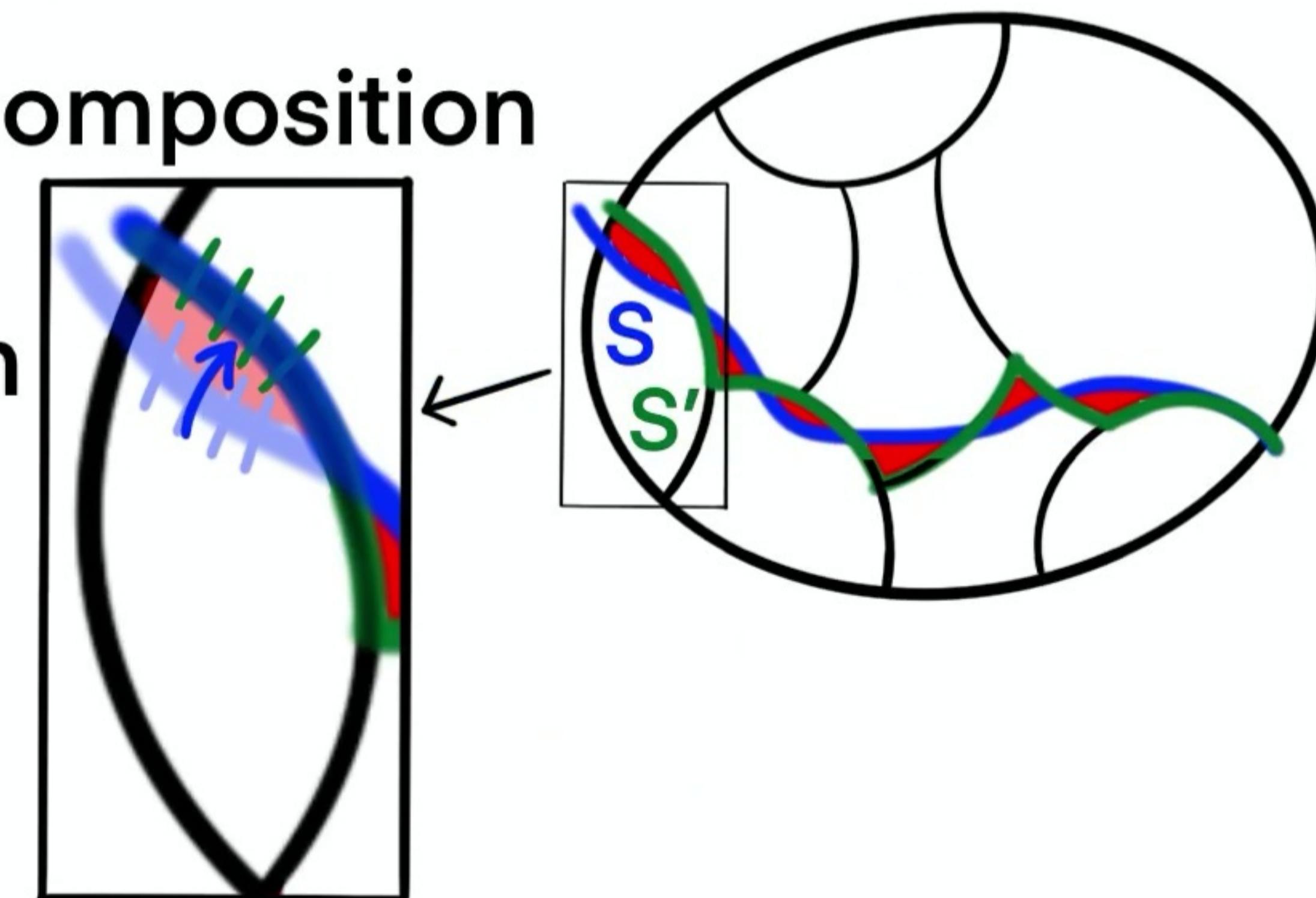
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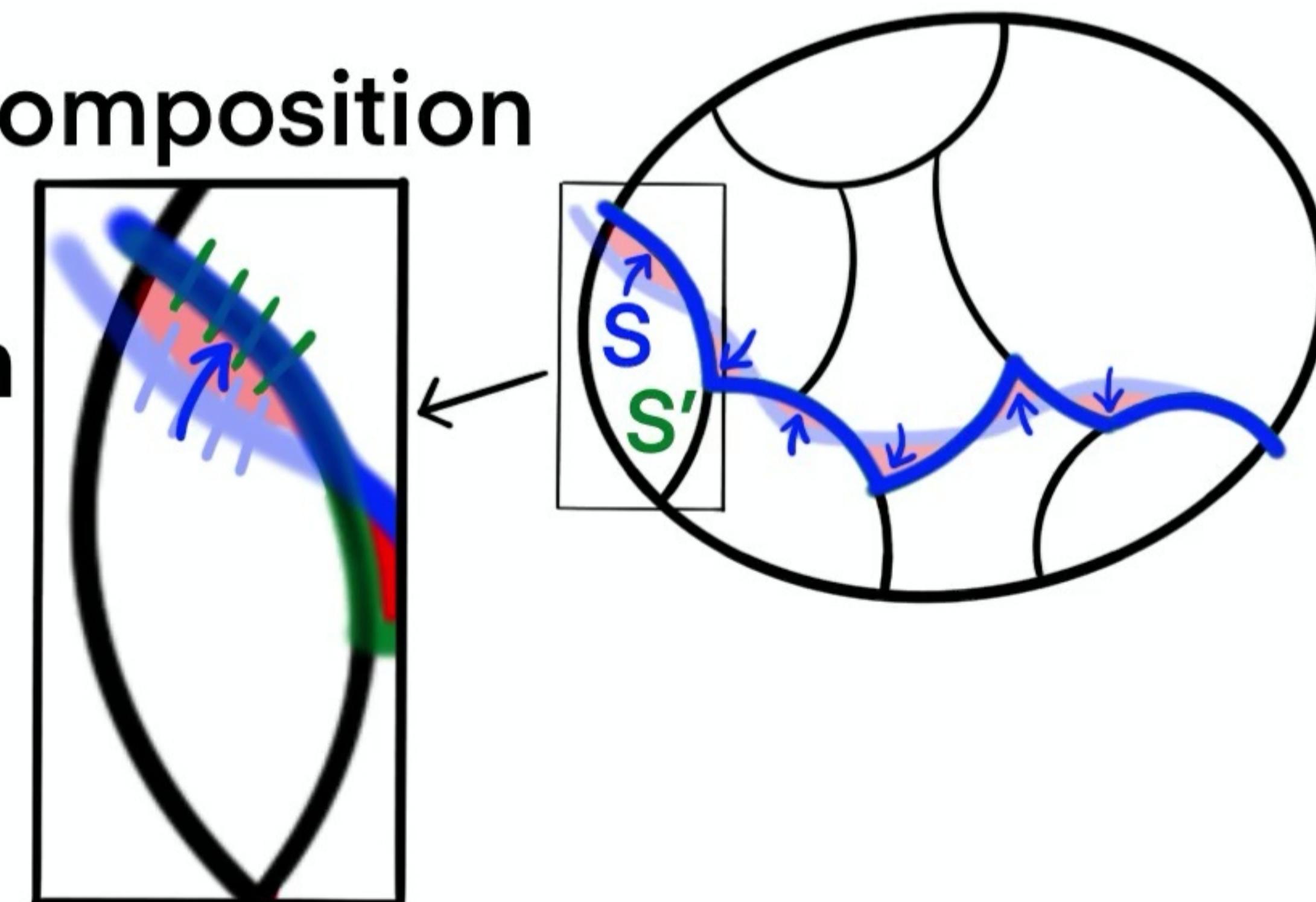
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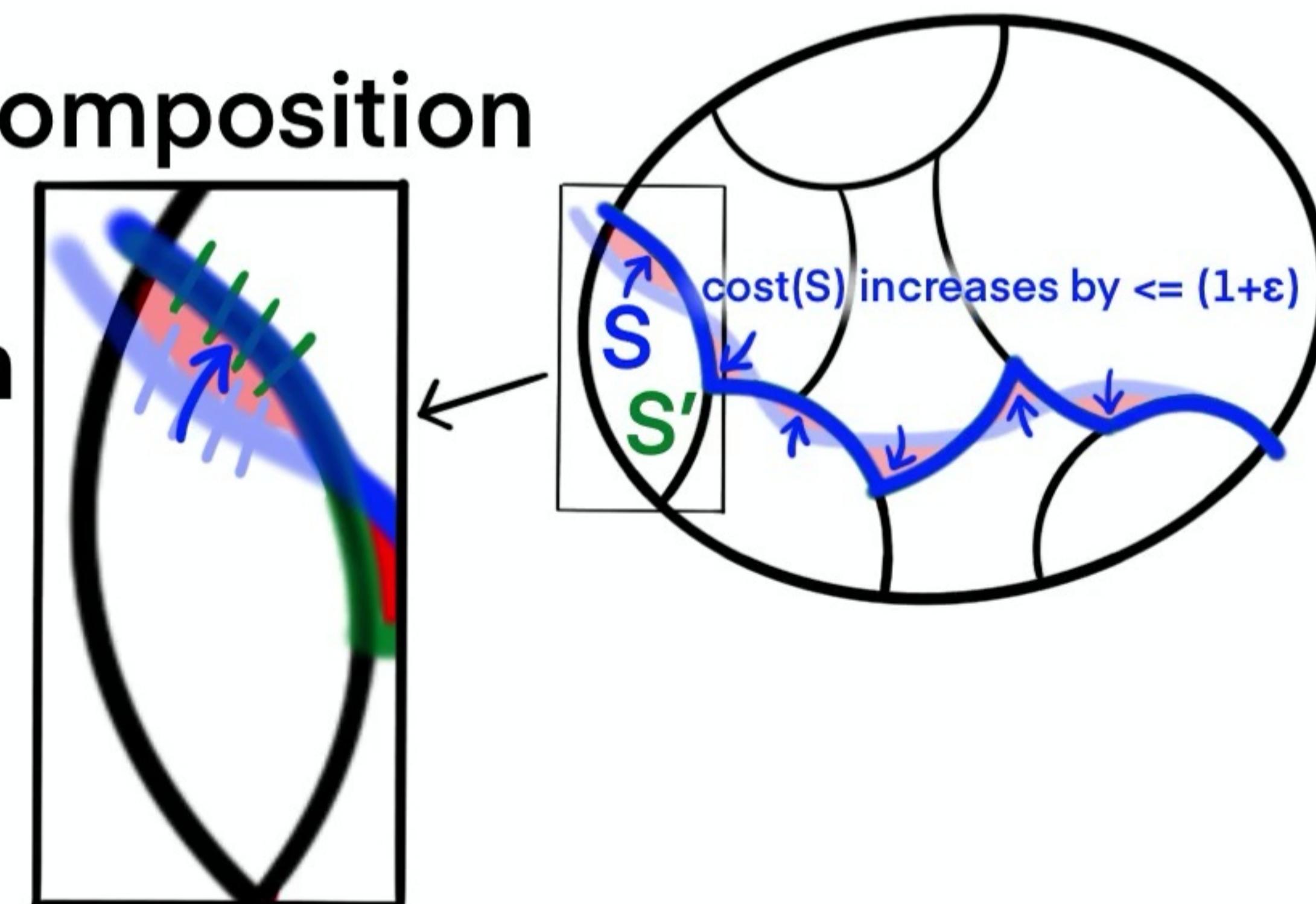
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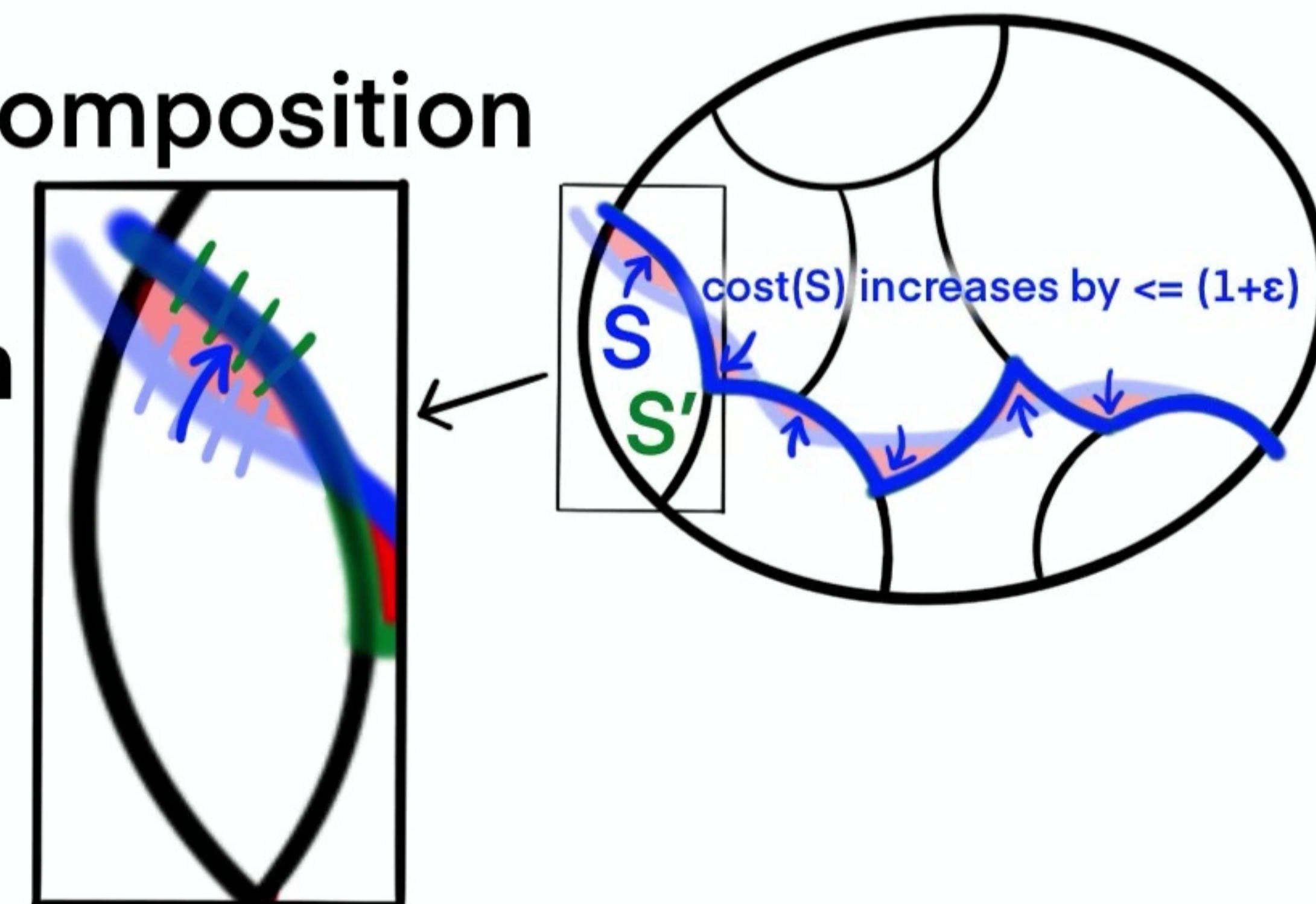
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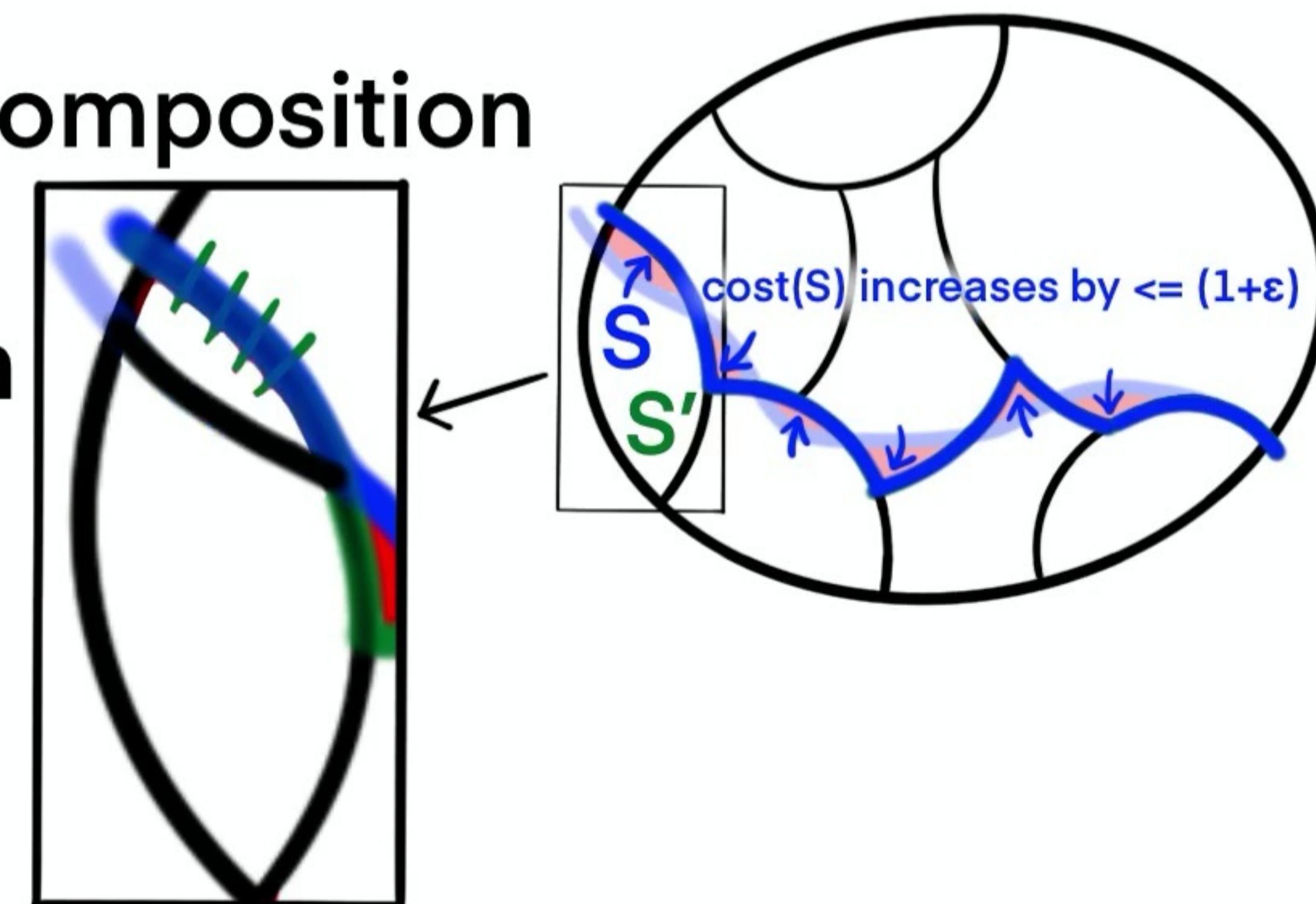
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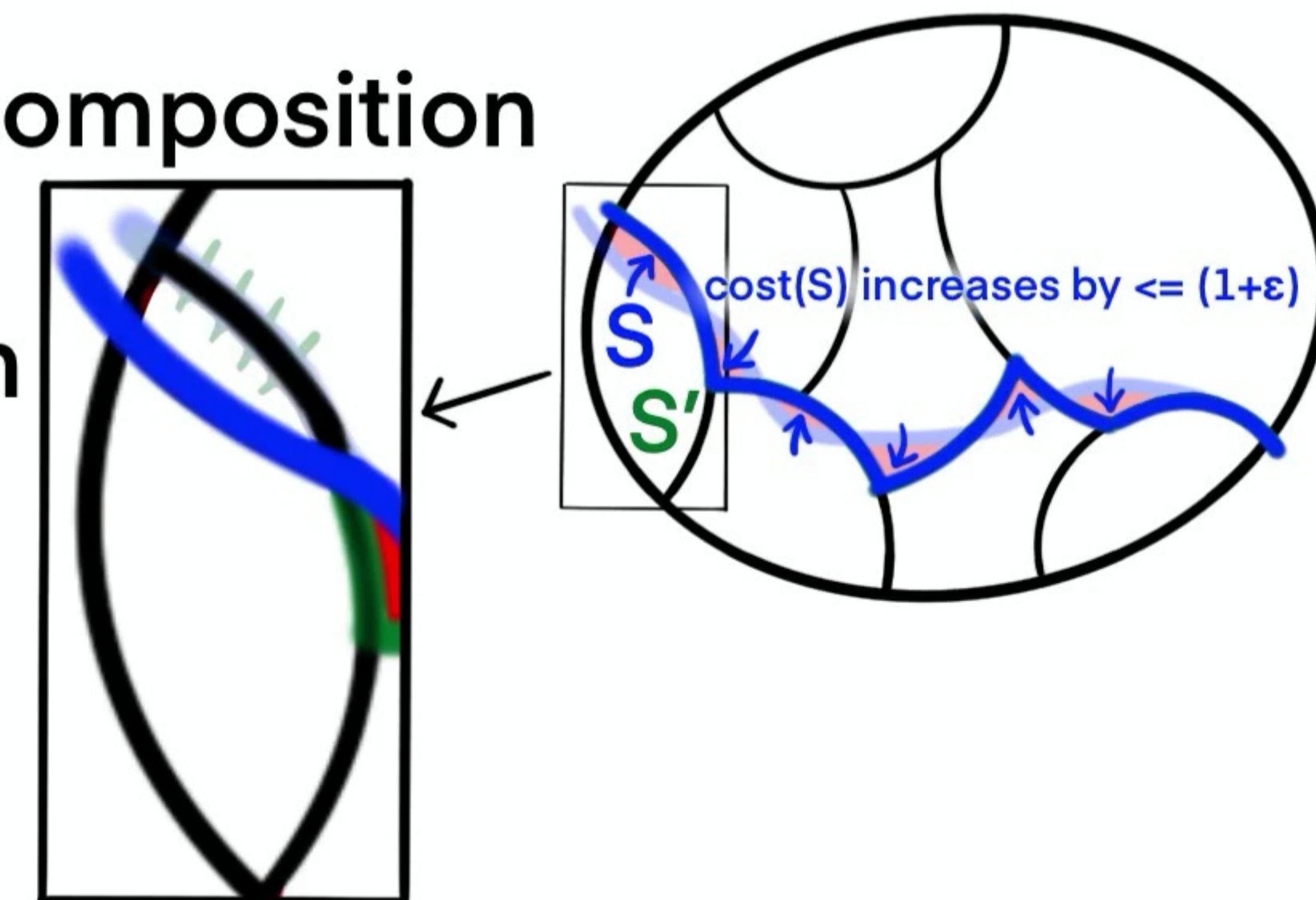
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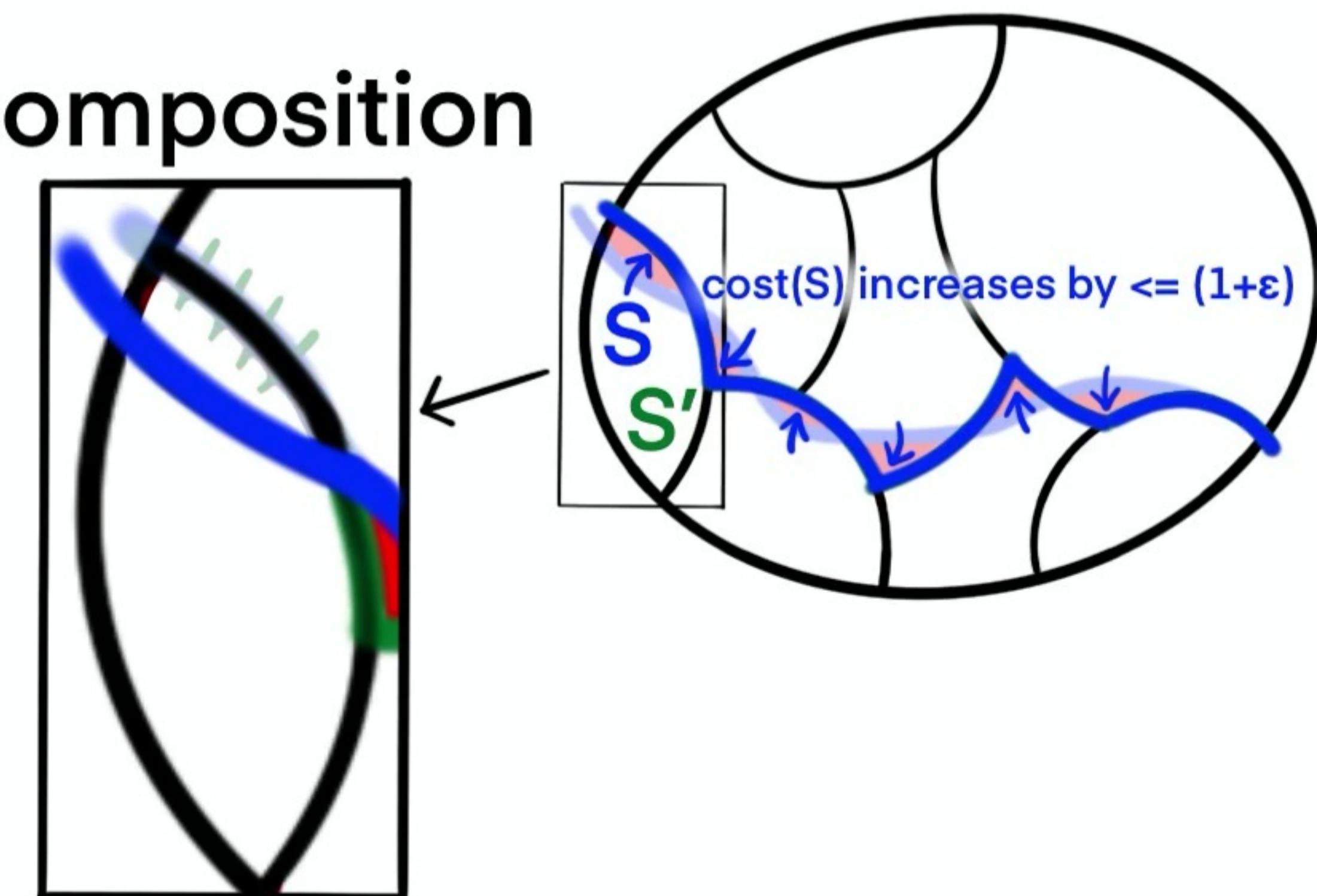
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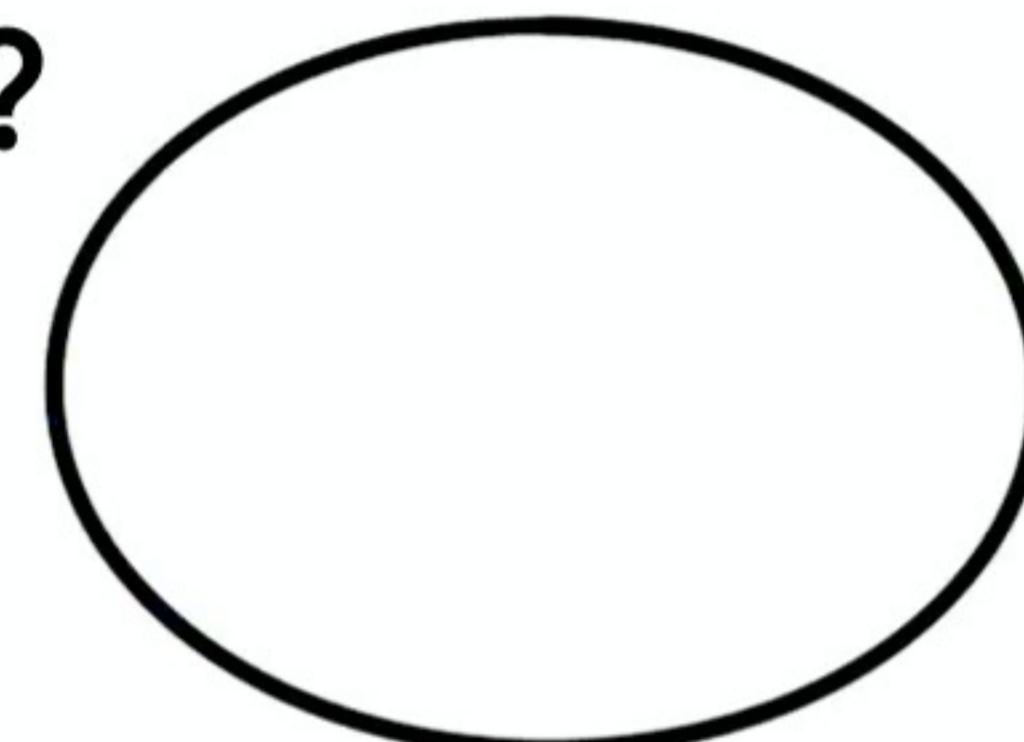
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- Should be useful for global mincut in other settings (dynamic/streaming/distributed)