

## Problem B. Stacking Paper Boxes

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We are given a collection of paper boxes  $B_1, B_2, \dots, B_n$ . Each box  $B_i$  has the dimension  $(\ell_i, \omega_i, d_i)$  where  $\ell_i, \omega_i, d_i$  are positive integers and  $d_i \leq \omega_i \leq \ell_i$ . We say  $B_j$  is smaller than  $B_i$ , written as  $B_j \propto B_i$ , if

$$(1) \ell_j < \ell_i, (2) \omega_j < \omega_i, \text{ and } (3) d_j < d_i.$$

Here are the rules to stack the paper boxes. Initially, all paper boxes are vacant. If  $B_i$  is vacant and  $B_j \propto B_i$ , then we can place  $B_j$  inside  $B_i$  and set  $B_i$  occupied. This forbids us to **directly** place more than one boxes into another. However, if  $B_k \propto B_j \propto B_i$ , it is allowed to place  $B_k$  into  $B_j$ , set  $B_j$  occupied, place  $B_j$  (with  $B_k$  inside) into  $B_i$ , and set  $B_i$  occupied. This gives a way to **indirectly** place more than one boxes into another. The task is to stack the paper boxes in the fewest number of piles. In other words, stack the paper boxes so as to minimize the number of boxes that are not placed inside another.

### Hint

This problem can be reduced to finding the minimum path cover on a DAG (See Problem 26-2 in I2A). In what follows, we will see how to solve this problem by reducing it to a maximum flow problem.

Construct a directed graph  $G = (\{s, t\} \cup U \cup V, E)$ . Initially,  $U = V = E = \emptyset$ . For each box  $B_i$ , add a new node  $u_i$  to  $U$  and add a new node  $v_i$  to  $V$ . If  $B_j \propto B_i$ , then add a directed edge  $(u_j, v_i)$  to  $E$  with capacity 1. Lastly, add a directed edge  $(s, u_i)$  with capacity 1 for all  $u_i \in U$  and add a directed edge  $(v_i, t)$  with capacity 1 for all  $v_i \in V$ . It is known that:

$G$  has  $\ell$  units of flow from  $s$  to  $t$  iff the given boxes can be stacked in  $n - \ell$  piles.

### Input

The first line contains  $n$ , an integer in  $[1, 100]$ . Each of the subsequent  $n$  lines has three integers  $\ell_i, \omega_i, d_i$ , the dimension of box  $B_i$ , where  $1 \leq d_i \leq \omega_i \leq \ell_i \leq 100$ .

### Output

The minimum number of boxes that are not placed inside another.

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### Sample Input

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3
3 2 1
4 3 2
5 3 2
```

### Sample Output

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2
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