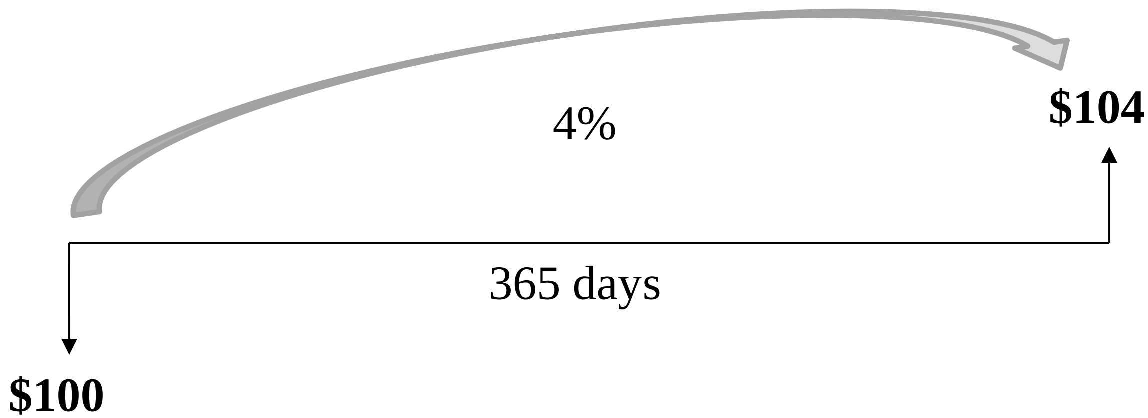
Time Value of Money

Basic Financial Arithmetic

Simple Interest

$$\text{Total Proceed} = \text{Principal} \times \left(1 + \text{interest rate} \times \frac{\text{days}}{\text{year}}\right)$$

$$\$104 \quad = \quad \$100 \quad \times \left(1 + \quad 4\% \quad \times \frac{365}{365}\right)$$

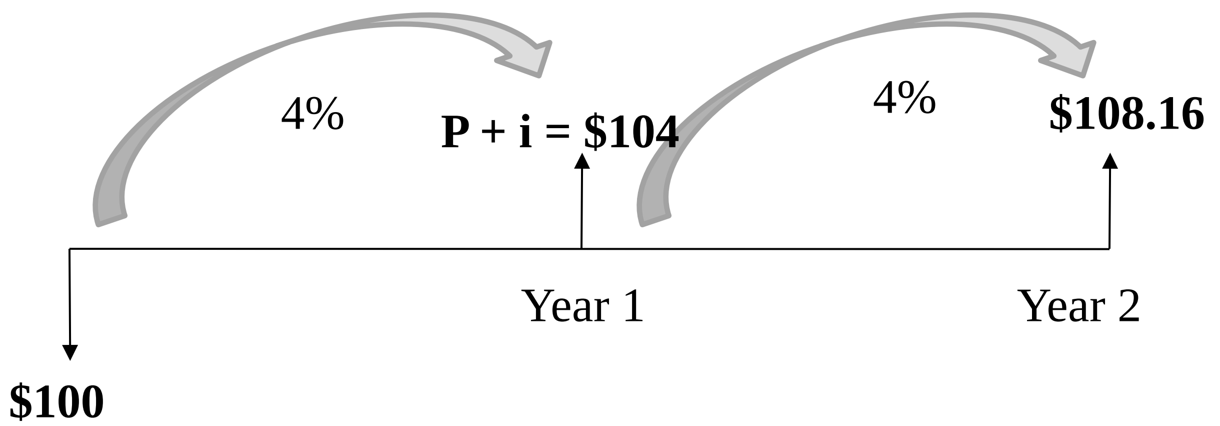


invest at 4% for 1 year (i.e. simple interest at 4%)

Compound Interest

$$\text{Total Proceed} = \text{Principal} \times \left(1 + \text{interest rate} \times \frac{\text{days}}{\text{year}}\right)^{N}$$

$$\$108.16 \quad = \quad \$100 \quad \times \left(1 + \quad 4\% \quad \times \frac{365}{365}\right)^{2}$$



2 years – assume reinvest at 4% after 1 year

i.e. compounding yearly at 4% for 2 years

Nominal and Effective Rates

- Consider 4% per annum and *quarterly* interest payments
- Assume reinvest at 4%

$$\text{Total Return} = \text{Principal} \times \left(1 + \frac{\text{interest rate}}{\text{n}}\right)^{\text{n}}$$

$$\$104.06 = \$100 \times \left(1 + \frac{4\%}{4}\right)^{4}$$

What are the nominal and effective interest rate?

Nominal and Effective Rates

- 4% is the nominal rates (annualised)
- 4.06% is the effective rates (annualised)

$$1 + \text{effective rate} = \left(1 + \frac{\text{nominal rate}}{\text{n}}\right)^{\text{n}}$$

$$\text{effective rate} = \left[\left(1 + \frac{\text{nominal rate}}{\text{n}}\right)^{\text{n}} - 1\right]$$

$$\text{nominal rate} = \left[(1 + \text{effective rate})^{\frac{1}{\text{n}}} - 1\right] \times \text{n}$$

where n = no. of compounding periods in a year

Example

5% is the nominal interest rate quoted for a 1-year deposit when the interest is paid all at maturity. What is the quarterly equivalent?

$$\left[(1.05)^{\frac{1}{4}}-1\right]\times 4 = 4.91\%$$

Interest Rate

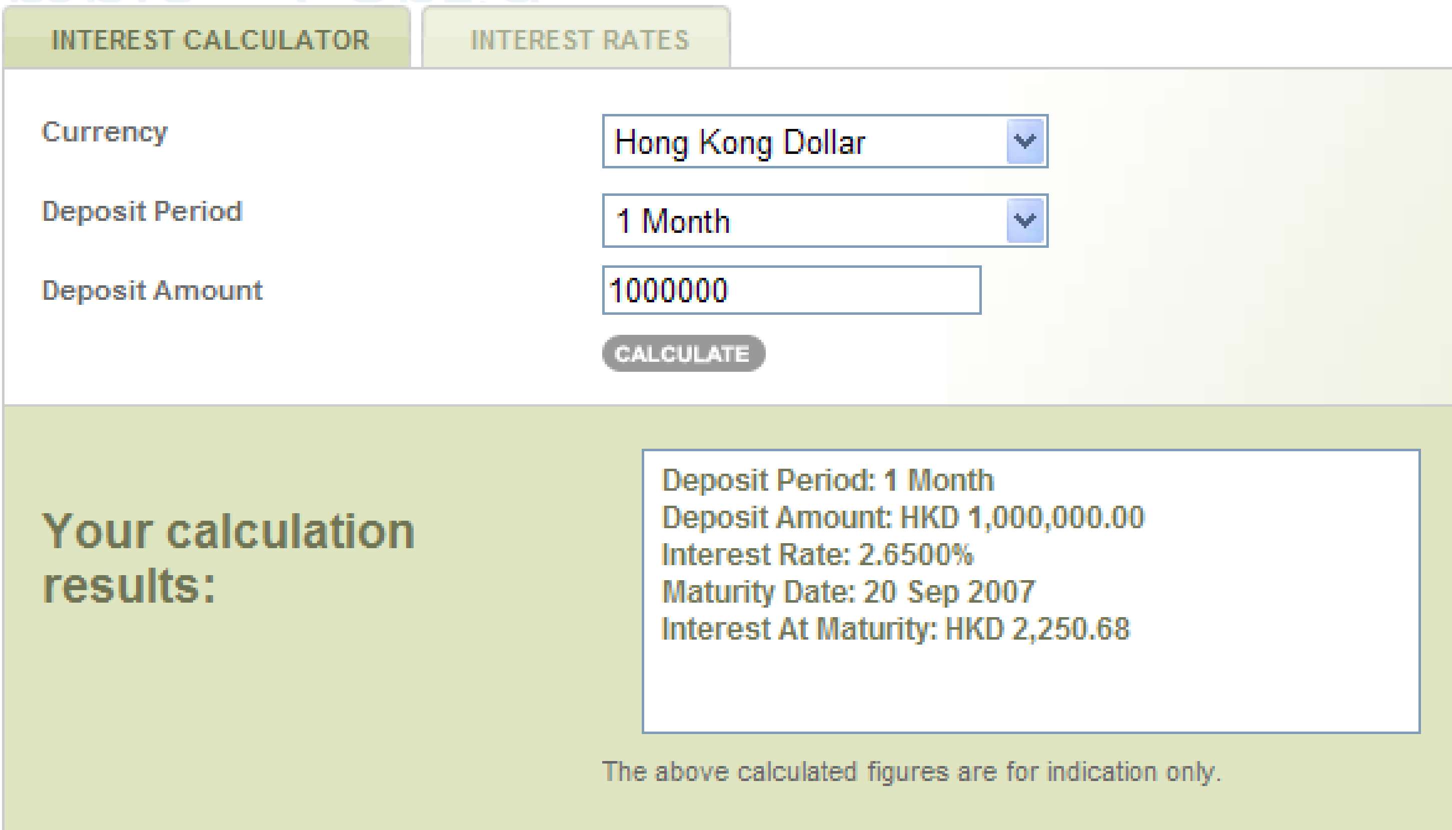
- The period for which the investment/loan will last
- The absolute period to which the quoted interest rate applies
  - 10% for 6-month?
- The frequency with which interest is paid

Example

Deposit period	HK $10,000 to HK $99,999	HK $100,000 to HK $499,999	HK $500,000 to HK $999,999	HK $1,000,000 or above
1 day				2.5000%
1 week	2.5000%	2.5000%	2.5000%	2.5000%
2 weeks	2.5000%	2.5000%	2.5000%	2.5000%
1 month	2.5000%	2.5500%	2.6000%	2.6500%
2 months	2.5000%	2.5500%	2.6000%	2.6500%
3 months	2.5500%	2.6000%	2.6500%	2.7000%
6 months	2.6000%	2.6500%	2.7000%	2.7500%
9 months	2.6500%	2.7000%	2.7500%	2.8500%
12 months	2.7500%	2.8500%	2.9500%	3.0500%

Interest is calculated on the following year basis:
Hong Kong Dollar - 365 days or 366 days (in leap years), Pound Sterling, Singapore Dollar, Thai Baht - 365 days and other currencies - 360 days.

Example – Cont'd



$$\text{Interest At Maturity} = \$1000000 \times \frac{2.65}{100} \times \frac{31}{365}$$

Annually **Compound** Rates

The interest rate for a 5-month (153-day) investment is 10.2%. What is the annually compounded rates?

$$\text{Annually Compound Rate} = \left(1 + 0.102 \times \frac{153}{365}\right)^{\frac{365}{153}} - 1 = 10.5038\%$$

$$\text{Annually Compound Rate} = \left(1 + \text{nominal rate} \times \frac{\text{days}}{\text{year}}\right)^{\frac{365}{\text{days}}} - 1$$

$$\text{Interest} = (1 + 10.5038\%)^{\frac{153}{365}} - 1 = 0.042756 \quad or$$

$$\text{Interest} = \left(1 + 0.102 \times \frac{153}{365}\right) - 1 = 0.042756$$

Compound Yield

The interest rate for a 5-month (153-day) investment is 10.2%. What is the compound yield?

$$\text{Compound Yield} = \left(1 + 0.102 \times \frac{153}{365}\right)^{\frac{365}{153}} - 1 = 10.50\%$$

$$\text{Compound Yield} = \left(1 + \text{nominal rate} \times \frac{\text{days}}{\text{year}}\right)^{\frac{365}{\text{days}}} - 1$$

Daily Compounding

Daily equivalent rate

Equivalent rate with daily compounding for an annual rate of 9.3%

$$\left[\left(1+9.3\%\right)^{\frac{1}{365}}-1\right]\times 365=8.894\%$$

$$\left(1+\frac{8.894\%}{365}\right)^{365}=?$$

Continuous Compounding

Equivalent rate with continuous compounding for an annual rate of 9.3%

$$\lim_{n\to\infty}\left(1+\frac{r_c}{n}\right)^n = 1+9.3\%$$

$$e^{r_c} = 1+9.3\%$$

$$r_c = \ln(1+9.3\%) = 8.8926\%$$

Continuous Compounding

Continuously compounded rate

$$r = \ln(1 + \mathrm{i})$$

where i is the nominal rate for a year

Or,

$$\mathrm{i} = \left(e^{r} - 1\right)$$

Time Value of Money

You have 2 payment options:

- Receive $100 now ?

Or

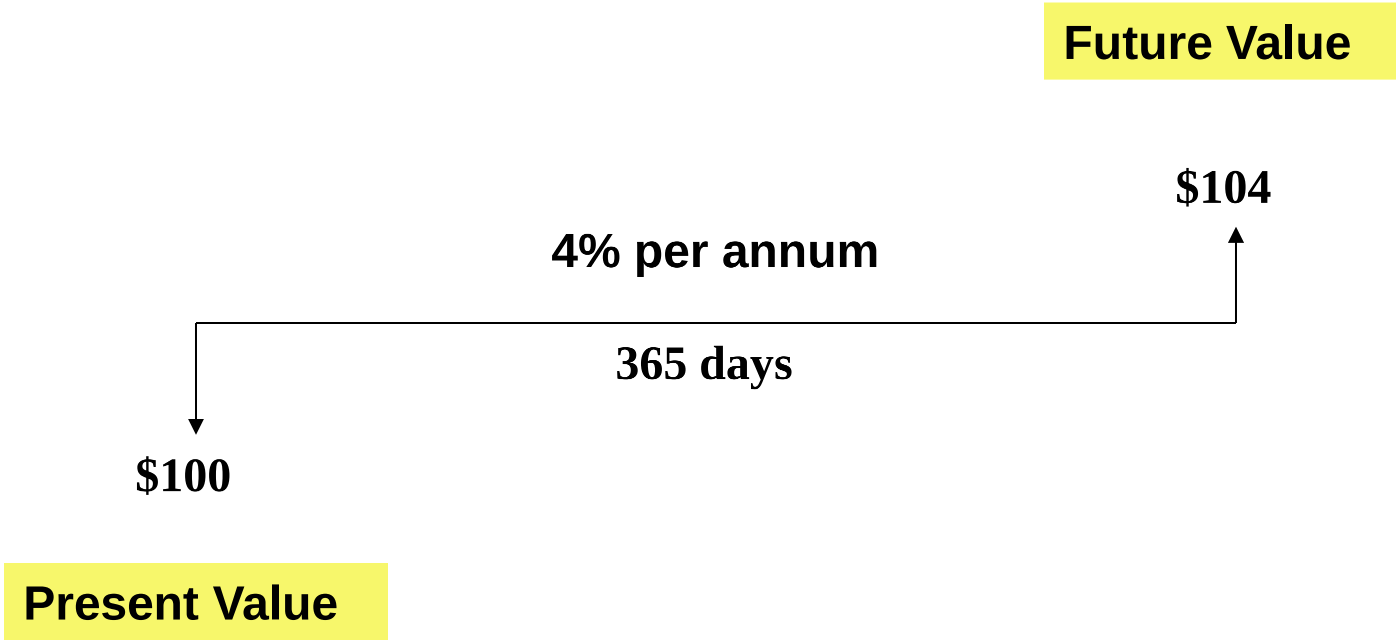
- Receive $100 after 1 years ?

Time value of money? What are the key factors you consider ?

Time Value of Money

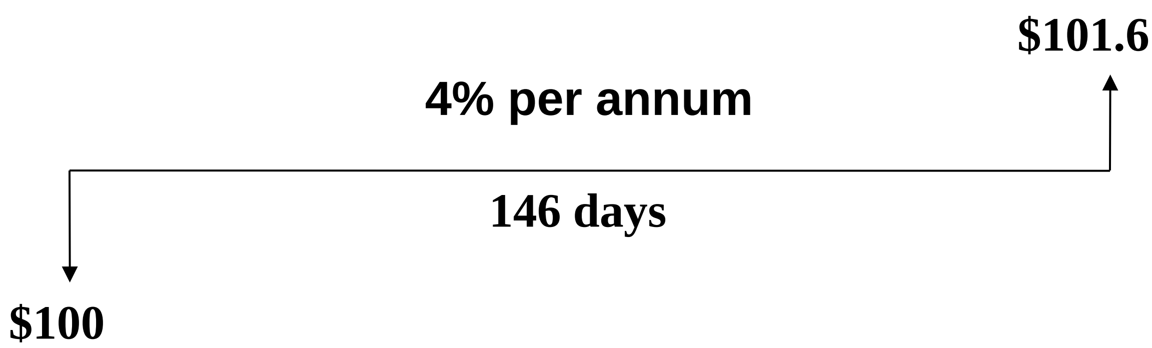
- **Future Value (FV**) the amount of money received in the future, including interest by investing a given amount of money now
- **Present Value (PV)** the amount of money that needs to be invested now to received a given amount in the future when interest is added

Future Value/Present Value



Future Value/Present Value

Future Value



Present Value

$$100 = \frac{101.6}{\left(1 + 0.04 \times \frac{146}{365}\right)}$$

Time Value of Money

- \$100 received today is worth more than \$100 received at some time in the future, because over time you can earn more interest on your money
- These concepts based on a relationship between **Present Value (PV)** and **Future Value (FV)**

For Simple Interest

$$FV = PV\left(1 + i \times \frac{days}{year}\right)$$

Time Value of Money

- **Present Value (PV)** and **Future Value (FV)**

For Annually Compound Interest

$$FV = PV \times (1 + i_c)^{\frac{days}{year}}$$

For Continuous Compound Interest

$$FV = PV \times e^{i_{cc} \times \frac{days}{year}}$$

Future Value / Present Value Yield/Rate of Return

For short-term investments

$$\text{Simple Yield} = \left(\frac{\text{FV}}{\text{PV}} - 1\right) \times \frac{\text{year}}{\text{days}}$$

$$\text{Compound Yield} = \left(1 + \text{simple yield} \times \frac{\text{days}}{\text{year}}\right)^{\frac{\text{year}}{\text{days}}} - 1$$

$$\text{Compound Yield} = \left(\frac{\text{FV}}{\text{PV}}\right)^{\frac{\text{year}}{\text{days}}} - 1$$

Future Value / Present Value Long-Term Investment

For long-term investments,

$$FV = PV \times (1 + i_c)^{\frac{days}{year}}$$

$$PV = \frac{FV}{(1 + i_c)^{\frac{days}{year}}}$$

$$Compound\,Yield = \left(\frac{FV}{PV}\right)^{\frac{year}{days}} - 1$$

Example

I invest \$138 now. After 64 days I receive back a total (principal + interest) of \$139.58. What is my (simple) yield on this investment?

$$\text{simple yield} = \left(\frac{139.58}{138.00} - 1\right) \times \frac{365}{64} = 0.0653 = 6.53\%$$

Discount Factors

- Discounting is the process to bring the future cashflow to the present value cashflow

$$\text{PV} = \text{FV} \times \text{Discount Factor}$$

For simple interest

$$Discount\ Factor = \frac{1}{1 + i \times \dfrac{days}{year}}$$

Discount Factors

$$\text{PV} = \text{FV} \times \text{Discount Factor}$$

For Compound Interest

$$Discount\ Factor = \frac{1}{(1 + i_c)^{\frac{days}{year}}}$$

Discount Factors

$$\mathrm{PV} = \mathrm{FV} \times \mathrm{Discount\ Factor}$$

For Continuous Compound Interest

$$Discount\ Factor = e^{-i_{cc} \times \frac{days}{year}}$$

Examples

What is the 3-year discount factor based on a 3-year interest rate of 8.5% compounded annually?

$$\text{discount factor} = \frac{1}{(1+0.085)^3} = 0.7829$$

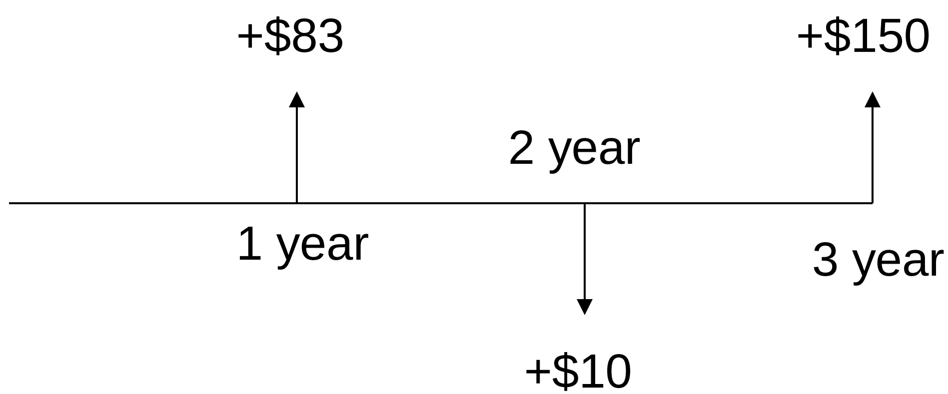
What is the present value of $100 in 3 years time?

$$\$100 \times 0.7829 = \$78.29$$

Net Present Value

$$\text{NPV} = \text{sum of all the present values}$$

Cashflow

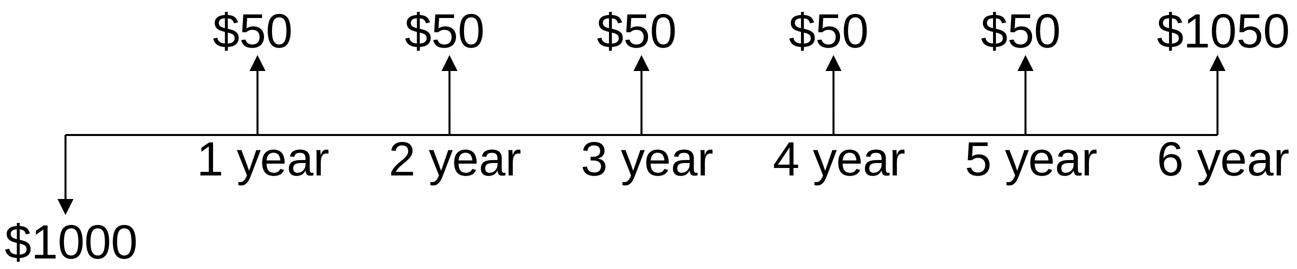


Discounting at rate of 7.5%

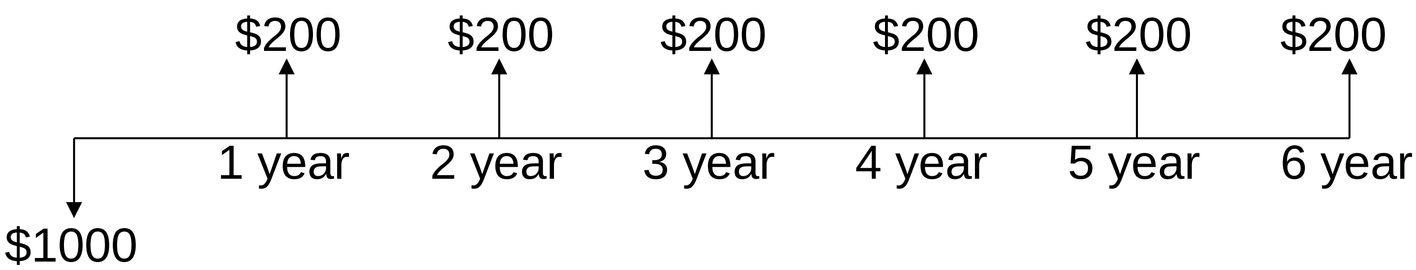
$$\text{NPV} = \frac{83}{(1.075)} - \frac{10}{(1.075)^2} + \frac{150}{(1.075)^3}$$

Which is better?

Investment 1:



Investment 2:



Internal Rate of Return

- the one single interest rate used when discounting a series of future value to achieve a given net present value

Investment 1:

$$\text{IRR} = 5.0000\%$$

$$1000 = \frac{50}{(1+\text{IRR})} + \frac{50}{(1+\text{IRR})^2} + \frac{50}{(1+\text{IRR})^3} + \frac{50}{(1+\text{IRR})^4} + \frac{50}{(1+\text{IRR})^5} + \frac{1050}{(1+\text{IRR})^6}$$

Investment 2:

$$\text{IRR} = 5.4718\%$$

$$1000 = \frac{200}{(1+\text{IRR})} + \frac{200}{(1+\text{IRR})^2} + \frac{200}{(1+\text{IRR})^3} + \frac{200}{(1+\text{IRR})^4} + \frac{200}{(1+\text{IRR})^5} + \frac{200}{(1+\text{IRR})^6}$$