# The University of Hong Kong

## FACULTY OF ENGINEERING
## DEPARTMENT OF COMPUTER SCIENCE

### COMP 7802 Introduction to financial computing

Date: December 22, 2018                    Time: 2:30pm–4:30pm

Time allowed     :        2 hours
Student I.D.     :        _____

*Only approved calculators as announced by the Examinations Secretary can be used in this examination. It is candidates' responsibility to ensure that their calculator operates satisfactorily, and candidates must record the name and type of the calculator used on the front page of the examination script.*

Brand and type
of calculator            :        _____

*Candidates are permitted to bring to the examination one piece of A4-sized paper with printed or written notes on both sides.*

## Answer ALL questions.

1.  [20 points] Loan

An amortization schedule is a complete table of periodic loan payments showing the amount of principal and the amount of interest that comprise each payment until the loan is paid off at the end of its term.

| | | | | | |
|---|---|---|---|---|---|
| | Loan Amount | $ 400,000 | | | |
| | Annual Interest Rate | 3.00% | | | |
| | Term of Loan in Years | 1 | | | |
| | Start Date of Loan | 30/11/2017 | Thursday | | |
| | | | | | |
| | Due Date | Monthly payment due date (Modified Following convention) | | | |
| | Interest | Interest payment for the month (ACT/365) | | | |
| | | | | | |
| | Holidays | | | | |
| | 12/25/2017 | 16/2/2018 | 1/5/2018 | 25/9/2018 | |
| | 12/26/2017 | 30/3/2018 | 22/5/2018 | 1/10/2018 | |
| | 1/1/2018 | 2/4/2018 | 18/6/2018 | 17/10/2018 | |
| | 15/2/2018 | 5/4/2018 | 2/7/2018 | | |
| | | | | | |

| Amortization Schedule | | | | | |
|---|---|---|---|---|---|
| No. | Due Date | Monthly Payment | Interest | Repaid Principal | Remaining Balance |
| | | | | | $400,000.00 |
| 1 | 29/12/2017 | 33,875.27 | 953.42 | 32,921.85 | 367,078.15 |
| 2 | 31/1/2018 | 33,875.27 | 995.64 | 32,879.63 | 334,198.52 |
| 3 | 28/2/2018 | 33,875.27 | 769.11 | 33,106.16 | 301,092.36 |
| 4 | (a) | 33,875.27 | (b) | (c) | (d) |
| 5 | 30/4/2018 | 33,875.27 | 704.71 | 33,170.56 | 234,764.20 |
| 6 | 31/5/2018 | 33,875.27 | 598.17 | 33,277.10 | 201,487.10 |
| 7 | 29/6/2018 | 33,875.27 | 480.26 | 33,395.01 | 168,092.09 |
| 8 | 31/7/2018 | 33,875.27 | 442.11 | 33,433.16 | 134,658.93 |
| 9 | 31/8/2018 | 33,875.27 | 343.10 | 33,532.17 | 101,126.76 |
| 10 | 28/9/2018 | 33,875.27 | 232.73 | 33,642.54 | 67,484.22 |
| 11 | 31/10/2018 | 33,875.27 | 183.04 | 33,692.23 | 33,791.99 |
| 12 | 30/11/2018 | 33,875.27 | 83.32 | 33,791.95 | 0.04 |

a.  [3 points] Figure out the Due Date of the 4th Monthly Payment.
b.  [3 points] Calculate the Interest that comprises the 4th Monthly Payment.
c.  [2 points] Calculate the Repaid Principal that comprises the 4th Monthly Payment.
d.  [2 points] Calculate the Remaining Balance after the 4th Monthly Payment.
e.  [3 points] Show how to use the Excel PMT function to approximate the Monthly Payment of the above Amortization Schedule. (See a description of PMT function on the next page)

Below is a description of Microsoft Excel PMT function:

**Description**

Calculates the payment for a loan based on constant payments and a constant interest rate.

**Syntax**

*PMT(rate, nper, pv, [fv], [type])*

The PMT function syntax has the following arguments:

- *Rate  Required. The interest rate for the loan.*
- *Nper  Required. The total number of payments for the loan.*
- *Pv  Required. The present value, or the total amount that a series of future payments is worth now; also known as the principal.*

**Remarks:**
*Make sure that you are consistent about the units you use for specifying rate and nper. If you make monthly payments on a four-year loan at an annual interest rate of 12 percent, use 12%/12 for rate and 4\*12 for nper. If you make annual payments on the same loan, use 12 percent for rate and 4 for nper.*

f.　[7 points] Referring to the Excel PMT function, express (i.e., an equation) *pv* in terms of *pmt* (payment), *rate* and *nper*. Full marks will be given for an answer with a closed form (i.e., no summation). Show your work.

2. [15 points] Arbitrage
   Given the following market rates:

   | 3 months Rates | 5.05% - 5.20% |
   | 6 months Rates | 5.10% - 5.30% |

   For simplicity, assume the days to spot is 0 and the date count conventions are $\frac{1}{4}$ for 3 months and $\frac{1}{2}$ for 6 months.

   a. [5 points] Suppose a trader can buy an FRA 3v6 at a rate of $r_b$. With the given market rates, what is the value of $r_b$ (percentage with 2 decimal points) below which the trader can execute an arbitrage?

   b. [8 points] Suppose the trader buy a 10mio FRA 3v6 at a rate of $(r_b - 5bp)$, that is 5 bp below the value you calculated in part a, draw a cash flow diagram that clearly indicate the cash flows (amounts and dates) of each transaction the trader has to execute in order to profit today (that is, +ve net cash flow today and no net cash flow at 3M and 6M). Note that you should still show the +ve and -ve cash flows at 3M and 6M despite each net is 0. Assume that the 3-month spot rate in 3-month time is exactly $(r_b - 5bp)$.

   c. [2 points] Assume the 3-month spot rate in 3-month time is $(r_b - 2bp)$, calculate the profit as in part b.

3. [15 points] Today is 1 May 2006. Given the following market data of an IRS:

|                      |                                  |
|----------------------|----------------------------------|
| Notional amount:     | 10 million                       |
| Start of swap:       | 3 May 2006                       |
| Maturity of swap:    | 3 May 2007                       |
| Receive (Fixed leg): | ? % (semi-annual ACT/365)        |
| Pay (Floating leg):  | LIBOR (semi-annual ACT/365)      |

Zero-coupon discount factors:

3 May 2006:  0.999602858

Rates:

$$3\ May\ 2006 - 3\ Nov\ 2006 \quad 7.4966\%$$
$$3\ Nov\ 2006 - 3\ May\ 2007 \quad 7.0439\%$$

a. [1 point] Suppose a company would like to enter the above IRS deal with a counterparty. What is the "ideal" NPV of the IRS on 1 May 2006?

b. [4 points] Estimate the discount factors on 3 Nov 2006 and 3 May 2007.

c. [4 points] Show the equation for calculating the swap rate, that is, rate of the fixed leg.

d. [2 points] Solve the equation in part (c).

e. [2 points] Show the cashflows of the floating leg.

f. [2 points] Calculate the NPV of the fixed leg.

4. **[20 marks]** Below information is the current market data available for stock and stock option of Company XYZ.

    Current Stock price = $65 per share
    Option Strike = $70
    Option time to expiry = 90 days
    Day / year convention = ACT/360
    Annualized Volatility = 13.49%
    Continuous compounding risk-free interest rate = 1.25% per annum

The Investor would like to use 2 periods binomial option pricing model (Cox, Ross & Rubinstein) to calculate the American Put option theoretical price of the above option. In his spreadsheet model, the following tree diagram is shown.
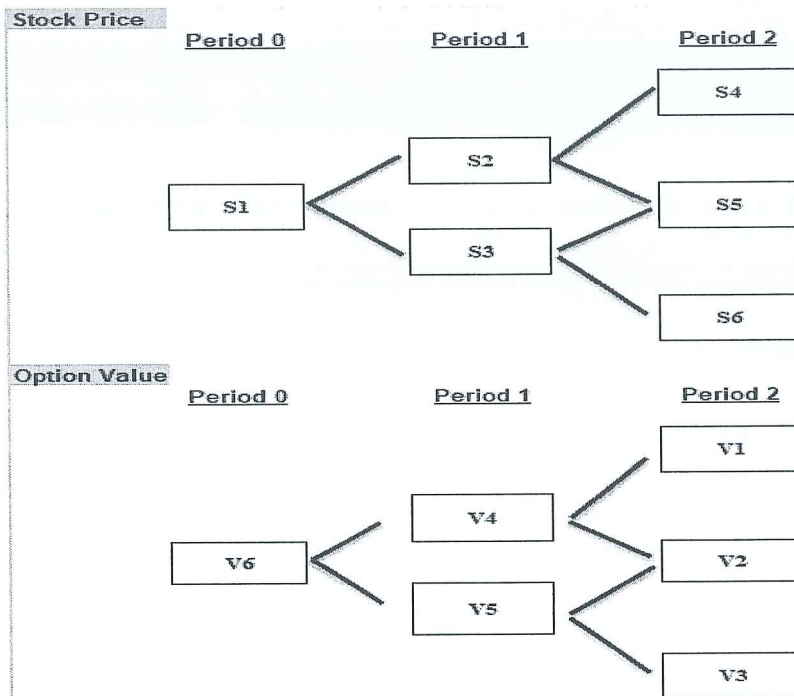


a) **[5 points]** Please calculate below intermediary parameters of binomial option pricing model.

- $\Delta t$ = period interval in each binomial nodes
- DF = discount factor in each binomial nodes
- u = up jump size
- d = down jump size
- p = risk neutral probability of up jump size

Please show the calculation expression of each intermediary parameters *(round the calculated results to 6 decimal points accuracy)*

b)   **[6 points]** Please compute the stock value projection of S1 to S6.

Show the calculation expression *( round to 2 decimal points accuracy)*

c)   **[7 points]** Please compute the option value of V1 to V6, then indicate the theoretical price of American put option clearly.

Show the calculation expression *(round to 6 decimal points accuracy)*

d)   **[2 points]** For European put option of the same underlying stock, strike and time to maturity, you expect the theoretical option price will be more expensive or cheaper? Explain why.

5.  [20 marks] Below are the current market data observation for Company ABC and its stock option on a given day T.

| Today = T | | | | |
|---|---|---|---|---|
| **Current Stock price on T ($)**   68.95 | | | | |
| Stock Option Style = European | | | | |
| Stock option conversion ratio: 1 option = 1 share | | | | |
| **Option Reference** | **Call/Put** | **Time to Expiry** | **Strike** | **Price** |
| 1 | Call | 30 days | 70 | 0.74 |
| 2 | Call | 60 days | 70 | 1.26 |
| 3 | Call | 90 days | 70 | 1.70 |

Investor A and B have positive outlook to the future stock price movement of Company ABC in coming 2 months but with different option strategy listed out below. Both plan to exit the position after 2 months regardless of profit or loss.

- Investor A: Covered Call strategy that he buy 1 stock at current stock price and short 1 call option.
- Investor B: Long Call strategy that he buy 1 call option

a)   **[4 points]** State the pros and cons of Investor A and B's strategy.

(1 pros and 1 cons for each strategy.)

b)   **[2 points]** Based on the above options available in the market, which Option (Option Reference 1 to 3) is the best option to select for Investor A and B's option strategy? Please justify.

c)   **[8 points]** If 2 months later at Option expiry, the stock price moved up to $70.5, please advise who can exit the position with a profit at expiry?

Show all the calculation expression indicating the exit value of stock and option. Please also indicate the option exercise status at expiry.

d)   **[6 points]** Investor A anticipated that his strategy will be in profit zone at a lower stock price level than Investor B's strategy at Option expiry.
Based on breakeven analysis to prove that Investor A's anticipation is correct and indicate the stock price range that Investor A position are breakeven and/or making profit before Investor B does.

6. **[10 marks]** Below are the current market data observation for Company ABC stock option on a given day T.

| Stock Option Style = European | | | | | |
|---|---|---|---|---|---|
| Stock option conversion ratio: 1 option = 1 share | | | | | |
| Option Reference | Call/Put | Time to Expiry | Strike | Price | Delta |
| 1 | Call | 30 days | 67.5 | 2.30 | 0.7652 |
| 2 | Call | 30 days | 70.0 | 0.77 | 0.5000 |
| 3 | Call | 30 days | 72.5 | 0.21 | 0.1353 |

a)   **[3 points ]** Based on the above market data, please advise which Option is in-the-money and out-of-money with justification.

b)   **[7 points]** Assuming that the current stock price of Company ABC is 70, what is the theoretical new Option price of the 3 Options when the stock price move up by $1 to $71?

(Show all your calculation expression)

—END OF PAPER—