

The University of Hong Kong
FACULTY OF ENGINEERING
DEPARTMENT OF COMPUTER SCIENCE

COMP 7802 Introduction to financial computing

Date: December 19, 2015

Time: 2:30pm–4:30pm

Time allowed : 2 hours

Student I.D. : _____

Only approved calculators as announced by the Examinations Secretary can be used in this examination. It is candidates' responsibility to ensure that their calculator operates satisfactorily, and candidates must record the name and type of the calculator used on the front page of the examination script.

**Brand and type
of calculator** : _____

Candidates are permitted to bring to the examination one piece of A4-sized paper with printed or written notes on both sides.

Answer ALL questions.

1. [20 points]

On 16 October 2014

GBP LIBOR 3 months 1%

GBP FRA 3x6 months 1.1%

Assume that year basis is 365, days to spot is 2.

| September 2014 | | | | | | | October 2014 | | | | | | | November 2014 | | | | | | | December 2014 | | | | | | |
|----------------|----|----|----|----|----|----|--------------|----|----|----|----|----|----|---------------|----|----|----|----|----|----|---------------|----|----|----|----|----|----|
| Su | Mo | Tu | We | Th | Fr | Sa | Su | Mo | Tu | We | Th | Fr | Sa | Su | Mo | Tu | We | Th | Fr | Sa | Su | Mo | Tu | We | Th | Fr | Sa |
| | 1 | 2 | 3 | 4 | 5 | 6 | | | | 1 | 2 | 3 | 4 | | | | | | 1 | | | 1 | 2 | 3 | 4 | 5 | 6 |
| 7 | 8 | 9 | 10 | 11 | 12 | 13 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 14 | 15 | 16 | 17 | 18 | 19 | 20 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
| 28 | 29 | 30 | | | | | 26 | 27 | 28 | 29 | 30 | 31 | | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 28 | 29 | 30 | 31 | | | |
| | | | | | | | | | | | | | | 30 | | | | | | | | | | | | | |

| January 2015 | | | | | | | February 2015 | | | | | | | March 2015 | | | | | | | April 2015 | | | | | | |
|--------------|----|----|----|----|----|----|---------------|----|----|----|----|----|----|------------|----|----|----|----|----|----|------------|----|----|----|----|----|----|
| Su | Mo | Tu | We | Th | Fr | Sa | Su | Mo | Tu | We | Th | Fr | Sa | Su | Mo | Tu | We | Th | Fr | Sa | Su | Mo | Tu | We | Th | Fr | Sa |
| | | | | 1 | 2 | 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | | | | 1 | 2 | 3 | 4 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 18 | 19 | 20 | 21 | 22 | 23 | 24 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| 25 | 26 | 27 | 28 | 29 | 30 | 31 | | | | | | | | 29 | 30 | 31 | | | | | 26 | 27 | 28 | 29 | 30 | | |

- a. [3 points] What are the start date and maturity date of the 3-month LIBOR? What is the settlement date of the FRA? Note that besides weekends, only the 25th and 26th of December and 1st of January are holidays.
- b. [3 points] Suppose Company A would like to transact a spot fixed rate borrowing of GBP 1 Million for 6 months using the given LIBOR and FRA. What transaction(s) should Company A execute(s) on 16 October 2014? Note that details of transactions should be included.
- c. [10 points] Suppose that 2 business days before the settlement date of the FRA, the GBP LIBOR for the FRA period is 2%. Show the detail cashflows throughout the transactions' period. For each cashflow, show the date and the amount (+ve for receiving; -ve for paying), indicate the transaction from which the cashflow is generated. If on a particular date, there are +ve and -ve cashflows, do not just show the net, show all the cashflows. If the amount requires calculation, show the expression.
- d. [2 points] What is the fixed rate Company A able to achieve?
- e. [2 points] Suppose that 2 business days before the settlement date of the FRA, the GBP LIBOR for the FRA period is 1% instead. Will Company A still achieve the same fixed rate as in part (d)? Explain.

2. [10 points] Given the following Certificate of Deposit

CHINA CITIC BANK INTERNATIONAL LTD - KWHKFC14004

| | | |
|-----------------------------|-----------------|--|
| | | |
| Currency | CNY | |
| Notional | 50,000 | |
| Issue Date | 2/18/2014 | |
| Maturity Date | 2/18/2015 | |
| Coupon Rate | 2.85% | |
| Day Count Fraction | ACT/365 (fixed) | |
| | | |
| Today | 3/13/2014 | |
| Settlement Date | 3/18/2014 | |
| Indicative Buy Price | 100.3750 | |

- [3 points] Calculate the *Accrued Interest*. *Accrued Interest* is the interest that has accumulated, but not yet paid, up to the settlement date.
 - [3 points] Calculate the *Maturity Proceed*.
 - [4 points] Calculate the (simple) *Yield*. Note that the *Indicative Buy Price* of 100.3750 means for every \$100 worth of the Certificate of Deposit at maturity, you need to pay \$100.3750. You are also required to pay the *Accrued Interest* on settlement.
3. [10 points] Today is 18th December 2015. I borrow \$100,000 from the bank, which I receive the money in my account today. That is, settlement date is today. The terms of the loan are such that I will repay the bank \$52,000 on 18th January 2016 and again \$52,000 on 18th February 2016. Suppose interest calculation is based on ACT/365. \$52,000 includes interest payment on the remaining loan balance plus part of the principal. After the two repayments, the remaining loan balance is equal to 0. What is the (annualized simple) rate the bank is charging for my loan? (Please answer part *a* through part *d* and show the percentage in 4 decimal points accuracy)

Let r be the (annualized simple) rate.

- [3 points] In terms of r , show the remaining balance after the 1st repayment.
- [2 points] In terms of r , show the interest payment in the 2nd repayment.
- [3 points] Show the equation for solving r .
- [2 points] Solve the equation for r .

4. [10 points] Arbitrage

Market prices:

2 v 5 FRA 7.25% / 7.27%

3 month Futures 92.69 / 92.71

Futures delivery date is in 2 months' time

Is there any must win strategy? If yes, what's the strategy? Assuming interest in 2 months' time is $r\%$, show an expression for the (i) FRA profit, (ii) Futures profit, and (iii) the total profit. It is sufficient to express profit in terms of the interest differential. (E.g. Sold FRA at 7.25%, interest in 2 months' time is 7.27%, profit is -0.02%)

5. [18 points] The following market data for HKD dollar is available from the broker screen for the rates (in %) of respective financial instruments.

| | | | | | |
|-----------------------|------|-------------|------|---------------------|------|
| Date 18 November 2015 | | | | | |
| Cash | | 3 month FRA | | Interest Rate Swaps | |
| 1W | 0.09 | 1x4 | 0.42 | 1Y | 0.56 |
| 1M | 0.24 | 2x5 | 0.52 | 2Y | 0.68 |
| 2M | 0.32 | 3x6 | 0.55 | 3Y | 0.92 |
| 3M | 0.38 | 6x9 | 0.58 | 4Y | 1.22 |
| | | | | 5Y | 1.24 |
| | | | | 7Y | 1.55 |
| | | | | 10Y | 1.87 |

Banker A is working on the financial model to construct the market yield curve. The following assumptions are made:

- day / year convention is ACT / 365
- holiday effect is ignored
- day to Spot is 2

In his worksheet model, the discount factors calculation schedule for Money Market is shown below:

| MM Discount Factor | | | | | | |
|--------------------|----------|----------|--------|---------------------------|----------|----------|
| Tenor | From | To | Period | ^t (Act/365) | Rate (r) | Dt |
| Spot | 20/11/15 | 20/11/15 | 0 | 0.000000 | | 1.000000 |
| 1Wk | 20/11/15 | 27/11/15 | 7 | 0.019178 | 0.09% | 0.999983 |
| 1M | 20/11/15 | 20/12/15 | 30 | 0.082192 | 0.24% | ??? |
| 2M | 20/11/15 | 20/01/16 | 61 | 0.167123 | 0.32% | ??? |
| 3M | 20/11/15 | 20/02/16 | 92 | 0.252055 | 0.38% | 0.999043 |

- a) [2 points] As shown above, the schedule cannot work out the discount factor for 1M and 2M. He would like to seek your help to calculate these 2 discount factors. Please show your expression for calculation. (the calculated results are rounded to nearest 6 decimal points)
- b) [4 points] He would like to seek your help to calculate the discount factor on 20/08/2016 as well. Please demonstrate to him the relevant steps and calculations. (the calculated results are rounded to the nearest 6 decimal points)

- c) Customer B would like to do a 9 months borrowing, however, he cannot see the rate quotation for 9 months borrowing from the broker screen above. Customer B is asking Banker A to offer the 9 months borrowing rate valued on Spot. The proposed termsheet listed out below:

Trade Date : 18 Nov 2015
Value Date : 20 Nov 2015
Maturity Date: 20 Aug 2015
Currency: HKD
Day/Year Convention : ACT/365

Banker A is planning to construct the 9 months borrowing rate based on 3M cash rate, FRA 3x6 and FRA 6x9 rate as of 18 Nov 2015 in question 5). He would like to seek your help on this and ask the following questions:

- i) [4 points] How to construct the synthetic 9 months borrowing from 3M cash instrument, FRA 3x6 and FRA 6x9?
- ii) [3 points] Based on your feedback in 5c i) above, Banker A creates the table below for the period coverage of each financial instrument. Please fill in the blank in below table.

| Instruments | From | To | No. of Days | Rate (%) |
|--------------|------|----|-------------|----------|
| 3M Cash rate | | | | |
| FRA 3x6 | | | | |
| FRA 6x9 | | | | |

- iii) [5 points] Please calculate the 9 months borrowing rate from 3M cash instrument, FRA 3x6 and FRA 6x9. Please show your calculation expression (the calculated results are rounded to the nearest 6 decimal points)

6. [32 points] Mr. B tries to build the spreadsheet model to estimate the historical volatility of Hang Seng Index (HSI) for last year. His spreadsheet model shows part of the data results.

| Month | Observed Price (S_t) | X_t | $(x_t - \bar{x})^2$ |
|-------|--------------------------|-----------|---------------------|
| 0 | 23936 | | |
| 1 | 23452 | -0.020428 | 0.000381 |
| 2 | 21561 | -0.084070 | 0.006917 |
| 3 | 22630 | 0.048390 | 0.002430 |
| 4 | 22292 | -0.015049 | 0.000200 |
| 5 | 22194 | -0.004406 | 0.000012 |
| 6 | 23327 | 0.049790 | 0.002570 |
| 7 | 23326 | -0.000043 | 0.000001 |
| 8 | 24594 | 0.052934 | 0.002898 |
| 9 | 24703 | 0.004422 | 0.000028 |
| 10 | 22691 | -0.084956 | 0.007065 |
| 11 | 24133 | 0.061612 | 0.003908 |
| 12 | 23678 | -0.019034 | 0.000329 |
| Total | | | 0.026739 |

where:

S_t = observed HSI at time period t

$X_t = \text{LN}(S_t / S_{t-1})$

$\bar{x} = \text{Average}(X_t) = (\sum X_t) / n$

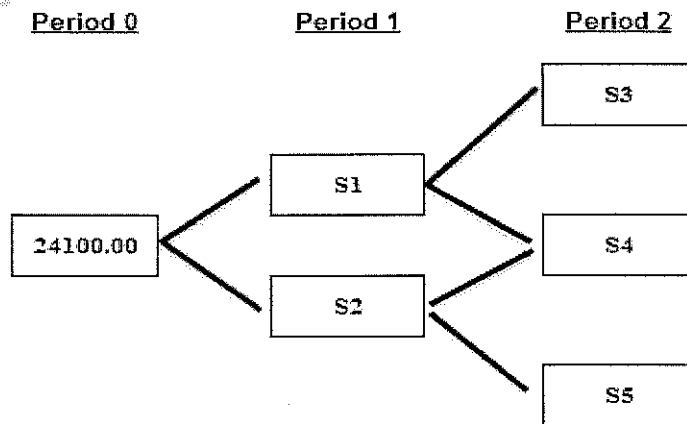
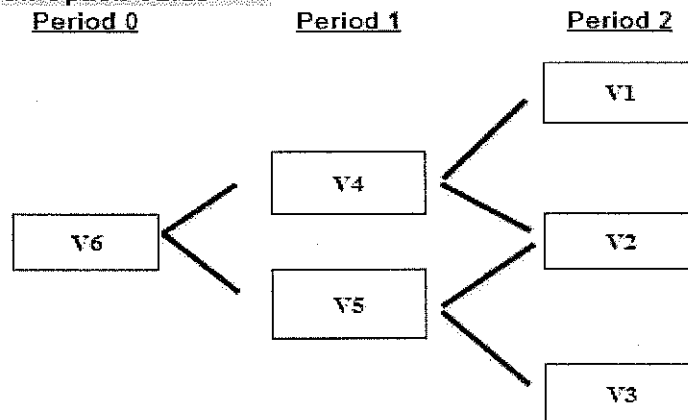
n = no. of month over the year

a) [4 points] Use the above data, please help him to complete the annual volatility calculation. Please show your calculation expression (round the annual volatility in percentage to the nearest 2 decimal points)

b) In addition to 6 a), Mr. B has done additional research and the obtained the following current market data:

- HSI level = 24100
- continuous compound risk-free interest rate = 1.5% per annum

He would like to use 2 steps Binomial Model (Cox, Ross & Rubinstein) to calculate the theoretical price of European Call Option for HSI at strike 22000 with 180 days time to maturity. The day / year convention is ACT / 360. In his spreadsheet model, the following table are shown:

Index Price**European Call Option Value**

i) [3 points] What is the moneyness of the above Call Option and calculate its intrinsic value? Please explain with reason for your answer and show your calculation expression as appropriate.

ii) [5 points] He asked you to help calculate below intermediary parameters:

- Δt = period interval in each binomial nodes
- DF = discount factor in each binomial nodes
- u = up jump size
- d = down jump size
- p = risk neutral probability of up jump size

Please show the expression of your calculation of each intermediary parameters (round the calculated results to the nearest 6 decimal points)

iii) [17 points] Please can you help him to finish the remaining computation work of European Option Pricing spreadsheet model in 6b). Please show all your calculation expression and result of each items from S1 to S5, V1 to V6. (For all calculated results, please round to the nearest 2 decimal points)

- iv) [1 point] After completion of all computation work in 6b iii), please advise Mr. B what is the theoretical price of European Call Option for HSI at strike 22000 with 180 days time to maturity based on the 2 steps Binomial Model (Cox, Ross & Rubinstein)?
- v) [2 points] Based on European Call Option theoretical price result in 6b iv) above, please calculate the corresponding theoretical price of European Put Option at a strike of 22000 based on Put-Call Parity theory. Please show your calculation expression (round the calculated results to the nearest 2 decimal points)

END OF PAPER