

Simulation of SNI for Bias Estimation

We cannot compute $\hat{\mathcal{N}}_{st}^{-1}$ exactly, and using Monte Carlo to directly estimate $\hat{\mathcal{N}}_{st}^{-1}\mathcal{N}_{st}$ introduces statistical errors due to the finite size of samples M , which may obscure the information about the bias. Therefore, we use a small trick in the SNI.py: we simulate the relative error between $\hat{\mathcal{N}}_{st}$ and the true \mathcal{N}_{st} , as shown below:

$$\begin{aligned}
\hat{\mathcal{N}}_{st}^{-1}\mathcal{N}_{st} &= \frac{1}{(1-\hat{P})\mathcal{I} + \hat{P}\mathcal{E}} \{(1-P)\mathcal{I} + P\mathcal{E}\} \\
&= \frac{1}{(1-\hat{P})\mathcal{I} + \hat{P}\mathcal{E}} \{(1-\hat{P})\mathcal{I} + \hat{P}\mathcal{E} + (P-\hat{P})(\mathcal{E}-\mathcal{I})\} \\
&= \mathcal{I} + \frac{1}{(1-\hat{P})\mathcal{I} + \hat{P}\mathcal{E}} \{(P-\hat{P})(\mathcal{E}-\mathcal{I})\} \\
&= \mathcal{I} + \sum_{k=0}^{\infty} \frac{(-1)^k \hat{P}^k}{(1-\hat{P})^{k+1}} \mathcal{E}^k \{(P-\hat{P})(\mathcal{E}-\mathcal{I})\} \\
&= \mathcal{I} + (P-\hat{P}) \sum_{k=0}^{\infty} \left\{ \frac{(-1)^k \hat{P}^k}{(1-\hat{P})^{k+1}} \mathcal{E}^{k+1} - \frac{(-1)^k \hat{P}^k}{(1-\hat{P})^{k+1}} \mathcal{E}^k \right\} \\
&= \left(1 + \frac{P-\hat{P}}{1-\hat{P}}\right) \mathcal{I} + (P-\hat{P}) \sum_{k=1}^{\infty} \frac{(-1)^{k-1} \hat{P}^{k-1}}{(1-\hat{P})^{k+1}} \mathcal{E}^k
\end{aligned} \tag{1}$$

We can use Monte Carlo simulation to estimate the second term, $\sum_{k=1}^{\infty} \frac{(-1)^{k-1} \hat{P}^{k-1}}{(1-\hat{P})^{k+1}} \mathcal{E}^k$, obtain $\langle O \rangle_{circuit}$, and ultimately estimate the absolute value of the bias:

$$|Bias| = \left| \frac{P-\hat{P}}{1-\hat{P}} \langle O \rangle_I + (P-\hat{P}) \langle O \rangle_{circuit} \right| \tag{2}$$