Oblig 9 - FYSMEK1110

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a)

Before the collision we have:

$$\begin{aligned} L_0 &= I_{0,r}\omega_0 = I_{0,b}\frac{v_0}{L} \\ &= mL^2\frac{v_0}{L} = mLv_0 \end{aligned}$$

b)

After the collision the angular momentum must necessary be conserved, and thus the same as in a).

$$L_0 = mLv_0$$

c)

Using superposition, we find that the moment of inertia is:

$$\begin{aligned} I &= \frac{M}{L}\int_0^L y^2 \, \mathrm{d}y + mL^2 \\ &= \frac{M}{3}L^2 + mL^2 \\ &= \left(\frac{M}{3} + m\right) L^2 \end{aligned}$$

d)

$$\begin{aligned} L = I\omega &\Rightarrow \omega_1 = \frac{L_1}{I_1} \\ \omega_1 &= \frac{mLv_0}{\left(\frac{M}{3} + m\right) L^2} = \frac{mv_0}{\left(\frac{M}{3} + m\right) L} \end{aligned}$$

e)

The linear momentum before the collision is $p = mv_0$, only the bullet is involved. However, after the collision both the bullet and the rod is moving. When we want to calculate the linear momentum we have to take into account that it is the speed of the center of mass of the rod we want, and not the speed at the end of the rod where the bullet hit. The velocity of the rod at the center of mass is proportional to the length of the position vector of the position of mass and the speed at the end of the rod.

f)

Using Newton's second law we get:

$$\alpha(\theta) = \frac{\tau}{I} = \frac{\vec{r} \times \vec{F}}{I} = -\frac{g}{\left(\frac{M}{3} + m\right) L} \sin\theta$$

g)

Using energy conservation

$$\begin{aligned} \frac{1}{2} m v_0^2 = (M+m)gh = (M+m)\frac{\cos\theta_{max}}{L} \\ \cos\theta_{max} = \frac{m v_0^2 L}{2g(m+M)} \end{aligned}$$

h)

We have

$$\sin\theta = \frac{\Delta x}{\left(\frac{h}{2}\right)} \Rightarrow \Delta x = \sin\theta \left(\frac{h}{2}\right)$$

From a first degree taylor approximation to $\sin\theta$ we get

$$\sin\theta \approx \theta$$

thus

$$\Delta x \approx \left(\frac{h}{2}\right) \theta$$

i)

Net toeque around O is given by

$$\begin{aligned} \tau_{net} &= \vec{r} \times \vec{F} - \tau_b \\ &= (L\cos\theta, L\sin\theta) \times \vec{F} - \tau_b \end{aligned}$$