FYS-MEK1100 OBLIGATORISK OPPGAVE 1

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January 29, 2014

Moddeling a 100m race

(a)

See appendix 1.3 for free body diagram. Didn't know whether to include all the forces used in this assignment, so I just added vectors for the main driving force F, the friction R, and the air drag D.

(b)

Find the position, $x(t)$, of the sprinter as a function of time. I know that the acceleration is the double derivative of the position with respect to time. First, I must find the acceleration as a function of time by using Newton's second law of motion.

$$\begin{aligned}\sum F_x^{\rightarrow} &= ma_x^{\rightarrow} \Leftrightarrow a_x^{\rightarrow} = \frac{\sum F_x^{\rightarrow}}{m} \\ &= \frac{400N}{80kg} = 5m/s^2\end{aligned}$$

Finding an expression for the velocity with respect to time

$$\begin{aligned}a(t) = v'(t) \implies v(t) &= v_0 + \int_0^t a(t)\,dt \\ v(t) &= \int_0^t a(t)\,dt = \int_0^t 5\,dt \\ v(t) &= 5t\end{aligned}$$

Finding an expression for the position with respect to time

$$x(t) = x_0 + \int_0^t v(t)\,dt = \int_0^t 5t\,dt = 2.5t^2$$

(c)

Show that the sprinter uses $t = 6.3s$ to reach the 100m line

Here I'll just plug in $t = 6.3$ into my expression for the position. You could also set $x(t) = 100$ and solve for t.

$$x(6.3) = 2.5 * (6.3)^2 = 99.225 \approx 100m$$

(d)

Assuming that the runner is only affected by the constant driving force, F, and the air resistance force, D.

$$D = (1/2)\rho C_D A(v - w)^2$$

Again, I start by applying Newton's second law of motion. I also assume that there is no wind: $w = 0m/s$

$$\sum F_x = F - D = 400N - 0.5 * 1.293kg/m^3 * 1.2 * 0.45m^2 v^2 = ma_x$$

$$\Downarrow$$

$$a(t) = \left(\frac{-0.6 * 1.293 * 0.45}{80} v(t)^2 + 5 \right) m/s^2$$

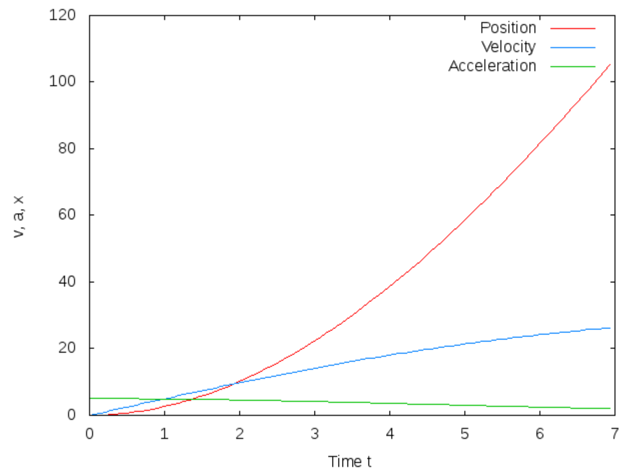
(e)

Find the velocity $v(t)$ and position $x(t)$ as a function of time for the runner. The runner starts from rest at the time $t = 0s$. This means that the initial values are $a(0) = 5m/s^2$. Using the Euler-Crome numerical method:

$$v(t_i + \Delta t) = v(t_i) + \Delta t * a(x(t_i), v(t_i), t_i)$$

$$x(t_i + \Delta t) = x(t_i) + \Delta t * v(t_i + \Delta t)$$

Figure 1: Position, velocity and acceleration as functions of time



See appendix 1.1 for source code.

(f)

Use the result to find the race time for the 100m race. I implemented an if test in the solver that prints out the time at which the runner passes 100 meters. According to this program the runner uses 6.79 seconds to finish the race.

(g)

Show that the (theoretical) maximum velocity of a runner driven by these forces is:

$$v_T = \sqrt{\frac{2F}{\rho C_D A}}$$

The maximum velocity is reached, given our conditions, when the air-drag force

F_D is equal in magnitude to the driving force F. We have:

$$\begin{aligned}
F &= D \\
F &= 1/2\rho C_D A v_T^2 \\
2F &= \rho C_D A v_T^2 \\
\frac{2F}{\rho C_D A} &= v_T^2 \\
v_T &= \sqrt{\frac{2F}{\rho C_D A}}
\end{aligned}$$

Which is what we wanted to show. Using the same program as before but integrating over a bigger timespan, I get that the runner reaches his terminal velocity after 33.0s, and his velocity at that time is 33.8 m/s. Which is not very realistic at all.

(h)

What is the runners maximum velocity, given that the runner is subject only to F and F_V? Ignoring the drag term, D.

For this task I have to replace the formula for $a(t)$ in the program. Newton's second law gives:

$$a(t) = \frac{400N - 25.8v(t)}{80kg}$$

Using the numerical method, this gives a maximum (terminal) velocity of 15,5 m/s at the time t=31,0s.

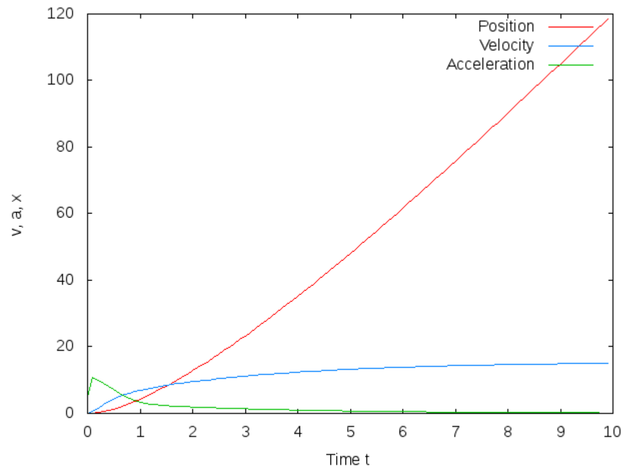
(i)

Running the simulation again with the new forces.

$$\begin{aligned}
F_V &= -f_V v \\
F_C &= f_c exp(-(t/t_c)^2) \\
F_D &= F + f_c exp(-(t/t_c)^2) - f_v v \\
D &= \frac{1}{2}A(1 - 0.25exp(-(t/t_c)^2))\rho C_D (v-w)^2 \\
F_{net} &= F + F_C - F_V - D \\
&= F + f_c exp(-(t/t_c)^2) - f_v v - D
\end{aligned}$$

See appendix 1.2 for source code.

Figure 2: Position, velocity and acceleration with a more realistic force model



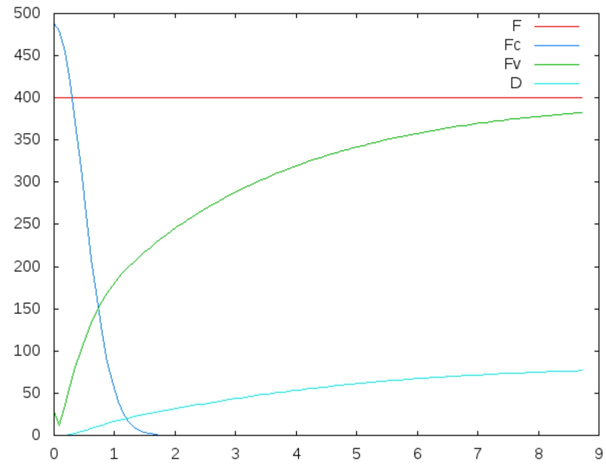
(j)

With the new force model he runs 100 meters in 8.7 seconds.

(k)

Extending the program to also plot the forces F, F_C, F_V and D versus time. It is

Figure 3: The forces F, F_C, F_V and D with respect to time



important to note that here I have plotted the absolute value of the forces, their magnitude. The forces D and F_v are working against the runner. As you can see from the graph, the initial drive force F_c contributes to the initial acceleration, but as the force diminishes the acceleration starts to decline, as you can see in figure 2. We can also see that the force F_D is the major contributor to the deacceleration of the runner, which makes sense. The physical limitations of the humans should be a bigger factor than the air-drag force F_D, so I completely agree with the graph and how it corresponds to Figure 2.

(l)

The difference in speed, both with head- and tail-wind is, according to my program, so small that in can be seen as negligible. Only with winds up towards 10 m/s did I see a significant difference in the velocity of the runner. Since I didn't see any significant changes with a wind of -1m/s or 1m/s I feel I can't comment.

```
# APPENDIX 1.1
# Obligatorisk oppgave 1: Euler-Cromer method
# Emne: FYS-MEK1110
# Navn: IVAR STANGEBY
# Dato: 28. JANUAR 2014

from scitools.std import linspace, zeros, plot, savefig

# Variables and initialization of arrays
n = 100
dt = 7. / n

t = zeros(n)
x = zeros(n)
v = zeros(n)
a = zeros(n)

# Initial values

t[0] = 0
x[0] = 0
v[0] = 0
a[0] = 5

won = False # Boolean variable used to mark the
            #time at which the runner passes a 100 meters.

# Simulation of the run
for i in range(n-1):
     a[i+1] = (-0.6*1.293*0.45)/80 * v[i]**2 + 5
     v[i+1] = v[i] + dt*a[i]
     x[i+1] = x[i] + dt*v[i+1]
     t[i+1] = t[i] + dt

     # An if loop that prints the current time as the runner passes 100 meters.
     if x[i+1] > 100 and not won:
         print "Won at time: ", t[i+1]
         won = True

# Plot of the position, velocity and acceleration with respect to time

plot(t, x, t, v, t, a,
legend=("Position", "Velocity", "Acceleration"),
xlabel="Time t", ylabel="v, a, x")
savefig("Graph.png")
```

```
# APPENDIX 1.2
# Obligatorisk oppgave 1: Euler-Cromer method
# Emne: FYS-MEK1110
# Navn: IVAR STANGEBY
# Dato: 28. JANUAR 2014

from scitools.std import linspace, zeros, plot, savefig, exp

# Variables and initialization of arrays
n = 100
dt = 10. / n

t = zeros(n)
x = zeros(n)
v = zeros(n)
a = zeros(n)

# Initial values

t[0] = 0
x[0] = 0
v[0] = 0
a[0] = 5

won = False # Boolean variable used to mark the time at
            #which the runner passes a 100 meters.
terminalVel = False # Boolean variable used to mark the
                    #time at which the runner reaches terminal velocity.
eps = 1e-4  # A small epsilon-value for the if-test further down

# Simulation of the run
for i in range(n-1):

    t[i+1] = t[i] + dt
    v[i+1] = v[i] + dt*a[i]
    a[i+1] = (400+488*exp(-(t[i+1]/0.67)**2)-25.8*v[i+1]-0.5*0.45*\
    (1-0.25*exp(-(t[i+1]/0.67)**2)*1.293*1.2*v[i+1]**2))/80

    x[i+1] = x[i] + dt*v[i+1]


    # An if loop that prints the current time as the runner reaches
    #terminal velocity as well as the velocity at that time.
    if -eps < a[i+1] < eps  and not terminalVel:
        print "Terminal velocity at time: ", t[i+1],"s
Terminal velocity: ", v[i+1]
        terminalVel = True
    if x[i+1] > 100 and not won:
        print "Runner passes 100m at time: ", t[i+1], "s
with a velocity of: ", v[i+1]
        won = True
```



```
# Plot of the position, velocity and
# acceleration with respect to time, and saving the graph as a png.

plot(t, x, t, v, t, a,
legend=("Position", "Velocity", "Acceleration"),
xlabel="Time t", ylabel="v, a, x")
savefig("GraphTerminalVelPhysiologicalCrossSection.png")
```

Figure 4: Appendix 1.3: Free body diagram