Weather Balloon - FYS-MEK1110

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(a)

I define positive directions as in the diagram.

(b)

What is the acceleration of the balloon?
Using Newton's second law of motion. I decompose the forces in x, y and z direction, to find the respective accelerations. I see that the only forces acting upon the weather balloon are doing so in the z direction. So I can already tell that the x and y components of $\vec{a}$ are 0.

$$\sum \vec{F} = m\vec{a} \Rightarrow \vec{a} = \frac{\sum \vec{F}}{m}$$
$$a_z = \frac{B - G}{m} = \frac{B}{m} - g$$
$$\vec{a} = \left(\frac{B}{m} - g\right)\hat{k}$$

(c)

Find the position and velocity of the balloon as a function of time.
I will assume that the balloon is let go from ground level, and that it has no initial velocity. This gives: $v_0 = 0m/s$ and $x_0 = 0m$.

$$\vec{a}(t) = a_x(t)\hat{i} + a_y(t)\hat{j} + a_z(t)\hat{k} = \left(\frac{B}{m} - g\right)\hat{k}$$
$$\vec{v}(t) = \int_0^t \vec{a}(t)\,dt = \int_0^t \left(\frac{B}{m} - g\right)\hat{k}\,dt = t\left(\frac{B}{m} - g\right)\hat{k}$$
$$\vec{r}(t) = \int_0^t \vec{v}(t)\,dt = \int_0^t t\left(\frac{B}{m} - g\right)\hat{k}\,dt = \frac{t^2}{2}\left(\frac{B}{m} - g\right)\hat{k}$$

Which seems reasonable, seing as there are no forces acting upon the weather balloon in the x and y direction it follows that there is no acceleration, velocity or displacement in those directions.

(d)

Introducing air resistance: $\vec{F}_D = -Dv\vec{v}$.
I again use Newton's second law. Seeing as the air resistance is a force that acts in the opposite direction of the velocity I know that a_x and a_y are going to remain 0. The absolute value in the formula is to ensure that the air resistance always works in the opposite direction of the velocity.

$$a_z = \frac{\sum F_z}{m} = \frac{B + F_{Dz} - G}{m} = \frac{B}{m} - g - \frac{D}{m}|v_z|v_z$$

(e)

(f)

Find the asymptotic (terminal) velocity of the balloon.
The balloon is going to reach it's terminal velocity when $B + F_{Dz} - G = 0 \Leftrightarrow B = G - F_{Dz}$. This implies that:

$$a_z = \frac{B}{m} - g - \frac{D}{m}|v_z|v_z = 0$$

Solving for v_z:

$$\frac{B}{m} - g - \frac{D}{m}|v_z|v_z = 0$$
$$\frac{D}{m}|v_z|v_z = \frac{B}{m} - g$$
$$D|v_z|v_z = B - G$$
$$|v_z|v_z = \frac{B - G}{D}$$

Because of the absolute value of v_z in F_{Dz} we have two different cases, one for $v_z < 0$ and one for $v_z \geq 0$.

$v_z < 0$ gives:

$$-v_z^2 = \frac{B-G}{D}$$
$$v_z = \sqrt{\frac{G-B}{D}}$$

$v_z \geq 0$ gives:

$$v_z^2 = \frac{B-G}{D}$$
$$v_z = \sqrt{\frac{B-G}{D}}$$

Because we will only be working with positive velocities for now, we can neglect the negative solution $v_z = \sqrt{\frac{G-B}{D}}$. Where therefore have an expression for the terminal velocity of the balloon in the z-direction:

$$v_{Tz} = \sqrt{\frac{B-G}{D}}$$

(g)

We now have to take into account a wind blowing with a velocity $\vec{w} = w\hat{i} = [w, 0, 0]$ along the horizontal x-axis. How does this wind modify the air resistance force $\vec{F_D}$ on the balloon?

The air resistance force $\vec{F}_D$ is now going to have two non-zero components $F_D x$ and $F_D z$

(h)

(i)

Find an expression for the acceleration $\vec{a}$ of the balloon. What are the initial conditions? Because of the new air resistance, we have to again apply Newton's second law.

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{(B-G)\hat{k} + \vec{F}_D}{m} = \frac{1}{m}\left([0,0,B-G] - D|v|\vec{v}\right)$$
$$= \left[\frac{-D|v_x|v_x}{m}, 0, \frac{B-D|v_z|v_z}{m} - g\right]$$

The initial conditions is going to be $\vec{v}_0 = [3, 0, 0], \vec{r}_0 = [0, 0, 0]$ and $\vec{a}_0 = [0, 0, \frac{B}{m} - g]$

(j)

The motion in the z- and x-direction is called coupled because they are dependent on eachother. They both occur in each others expressions. I can't see a way for us to find the motion of the ball analytically, unless we have really simple values to work with.

(k)

```
for i in range(1, len(a)-1):
    # Using numpys matrix and linalg library,
    # where v, a and r are matrices.
    v[i+1] = v[i] + dt*a[i]
    FD = -D*np.linalg.norm(v[i+1])*v[i+1]
    a[i+1] = [0, 0, float(B-G)/m] + FD
    r[i+1] = r[i] + dt*v[i+1]
```

(l)

(m)

The ball reaches its terminal velocity when $|\vec{a}| = 0$. This can be tested for by a simple if test in our python program:

```
eps = 0.001
termVel = True

for i in range(1, len(a)-1):
    # Using numpys matrix and linalg library,
    # where v, a and r are matrices.
    v[i+1] = v[i] + dt*a[i]
    FD = -D*np.linalg.norm(v[i+1])*v[i+1]
    a[i+1] = [0, 0, float(B-G)/m] + FD
    r[i+1] = r[i] + dt*v[i+1]

    if np.linalg.norm(a[i+1] < eps)\
    and not termVel:
        print "Terminal velocity: ", v[i+1]
        termVel = False
```

(p)

When the fire is extinguished, the boyancy force B is going to be reduced to 0. This can easily be implemented in our model by an if test inside our solution-loop that sets $B = 0$ when our conditions are met.