Ball in a spring - FYS-MEK1110

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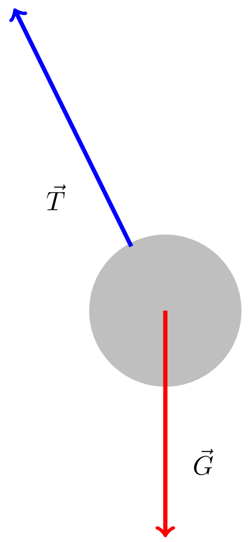

Figure 1: Free body diagram

(a)

Identify the forces and draw a free-body diagram. We have got the force of gravity $G = mg$ pointing i the downward y-direction, as well as the force from the rope $\vec{T}$.

(b)

Show that the net external force acting on the ball can be written as:

$$\sum \vec{F}_j = -mg\hat{j} - k(r - L_0)\frac{\vec{r}}{r}$$

Since we are assuming that the rope can be modelled as a spring, we will use the full spring model to find an expression for $\vec{T}$.

$$\vec{T} = -k(r - r_0)\frac{\vec{r}}{r},$$

where k is the spring constant, $\frac{\vec{r}}{r}$ is the unit vector pointing in the direction of the rope, r_0 is the length of the rope in equilibrium. In our case L_0 and r is the length of the rope.
We also have the long range force of gravity acting upon the ball, which gives us $G = -mg\hat{j}$. We therefore have the sum of all forces:

$$\begin{aligned}\sum_j \vec{F} &= \vec{F}_S - G\hat{j} \\ &= -k(r - L_0)\frac{\vec{r}}{r} - mgj \\ &= -mg\hat{j} - k(r - L_0)\frac{\vec{r}}{r}\end{aligned}$$

Just as we wanted to show.

(c)

Rewrite the expression of the exteral force on component form by writing the force components F_x and F_y as functions of the components x and y of the position vector $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$.

$$\vec{F}(t) = [F_x(t), F_y(t)] \qquad \vec{r}(t) = [x(t), y(t)]$$

$$\vec{F}_x(t) = -k\left(\sqrt{x(t)^2 + y(t)^2} - L_0\right)\frac{x(t)}{\sqrt{x(t)^2 + y(t)^2}}$$

$$\vec{F}_y(t) = -mg - k\left(\sqrt{x(t)^2 + y(t)^2} - L_0\right)\frac{y(t)}{\sqrt{x(t)^2 + y(t)^2}}$$

(d)

For a pendulum with constant rope length it is sufficient to use just the angle to describe the position of the ball. However, in our case the rope length is not constant, so we have to take that account whenever we want to find the current position of the ball.

(e)

If the ball is at rest at $\theta = 0$ with no velocity and no acceleration, what is the position of the ball? What happens if you increase the value of k for the rope?

We know that in this case $x(t) = 0$ because the ball is positioned directly beneath the origin $(0,0)$. This gives us $F_x = 0$. We also know, that since the ball is stationary with zero acceleration and that $x(t) = 0$, that

$$
\begin{aligned}
-k\left(\sqrt{y(t)^2} - L_0\right)\frac{y(t)}{\sqrt{y(t)^2}} &= mg \\
y(t) - L_0 &= -\frac{mg}{k} \\
y(t) &= L_0 - \frac{mg}{k}
\end{aligned}
$$

This gives us the position:

$$\vec{r} = \left[0, L_0 - \frac{mg}{k}\right]$$

If we increase the value of k, the length the rope stretches is smaller, because the rope is stiffer.

SPECIFIC PENDULUM

$$
\begin{aligned}
m &= 0.1 & v_0 &= 0 \\
L_0 &= 1 & \theta_0 &= 30 \\
k &= 200 & r &= 1
\end{aligned}
$$

(f)

Find an expression for the acceleration, $\vec{a}$, both on vector form and on component form. Using $g = 9.81 m/s^2$.

$$
\begin{aligned}
\vec{a} = \frac{\sum F}{m} &= -g\hat{j} - \frac{k(r - L_0)}{m}\frac{\vec{r}}{r} \\
&= -9.81\hat{j} - \frac{200(r-1)}{0.1}\frac{\vec{r}}{r} \\
&= -9.81\hat{j} - 2000(r-1)\frac{\vec{r}}{r}
\end{aligned}
$$

$$a_x(t) = -200\left(\sqrt{x(t)^2 + y(t)^2} - 1\right)\frac{x(t)}{0.1\sqrt{x(t)^2 + y(t)^2}}$$

$$a_y(t) = -9.81 - 200\left(\sqrt{x(t)^2 + y(t)^2} - 1\right)\frac{y(t)}{m\sqrt{x(t)^2 + y(t)^2}}$$

(g)

$$
\begin{aligned}
\vec{v} &= 0 + \int_0^t a\ dt = at \\
r &= \int_0^t at\ dt = r_0 + \frac{1}{2}at^2 \\
v' &= -\frac{v}{t}
\end{aligned}
$$

(h)

Using the Euler-Cromer method:

$$
\begin{aligned}
&\vec{v}(t_i + \Delta t) = \vec{v}(t_i) + \Delta t \cdot \vec{a}(x(t_i), v(t_i), t_i) \\
&\vec{r}(t_i + \Delta t) = \vec{r}(t_i) + \Delta t \cdot \vec{v}(t_i + \Delta t) \\
&\vec{a}(t_i + \Delta t) = -9.81\hat{j} - 2000(\vec{r}(t_i + \Delta t) - 1)\frac{\vec{r}(t_i + \Delta t)}{r(t_i + \Delta t)}
\end{aligned}
$$

Again out of time, and my program is not running as it's supposed to. I am therefore only going to comment on the following questions.

(i)

```
# INCOMPLETE CODE - DOES NOT RUN
from scitools.std import *
import numpy as np
T = 10
n = 1000
dt = float(T)/n
a_const = [0, -9.81]

a = zeros([n, 2])
v = zeros([n, 2])
r = zeros([n, 2])
t = linspace(0, T, n)
L_0 = 1
a[0] = [0, -9.81]
r[0] = [np.sin(30)*L_0, np.cos(30)*L_0]
for i in range(1, n-1):
    v[i+1] = v[i] + dt*a[i]
    r[i+1] = r[i] + dt*v[i+1]
    a[i+1] = a_const - 2000*(linalg.norm(r[i+1]) -
```

;

(j)

I have no plot.

(k)

I can't comment on what happens when you change Δt. However, I think that Euler's method is not capable of following the periodic oscilations that our ball has. This is going to lead to a plot that does not resemble anything physical at all. The energy of the ball is not going to decrease according to Eulers method.

(l)

Setting$k = 2000$ is going to stiffen the rope. This is going to lead to a higher velocity because of less energy being lost in the exstension of the rope. Whereas $k = 20$ will do the opposite.

(m)

This is easy to implement in our program. Move the calculation of the acceleration in the next timestep to a function, and make it test for if $r < L_0$. What this means is that whenever the length of the rope r is less than the length of the rope in equilibrium L_0, which means that the rope is compressed, and the rope tension is therefore zero.