Let $(x_1, \dots, x_n)$ be a multiple of $(y_1, \dots, y_n)$. We can then write the matrix

$$\begin{bmatrix} x_1 & \dots & x_n \\ y_1 & \dots & y_n \end{bmatrix} = \begin{bmatrix} x_1 & \dots & x_n \\ \lambda x_1 & \dots & \lambda x_n \end{bmatrix}$$

for some scalar λ. However, we can now subtract a multiple λ of the first row from the second row and achieve

$$\begin{bmatrix} x_1 & \dots & x_n \\ 0 & \dots & 0 \end{bmatrix}.$$

If we now let x_i be the smallest non-zero component for $1 \leq i \leq n$ and divide the first row by x_i we achieve the following matrix

$$\begin{bmatrix} 0 & \dots & 1 & \frac{x_i+1}{x_i} & \dots \\ \frac{x_n}{x_i} & & & & \\ 0 & \dots & 0 & 0 & \dots \\ 0 & & & & \end{bmatrix}.$$

This matrix has exactly one pivot-column if there is at least one i for which $x_i \neq 0$ and exactly zero pivot columns if $x_i = 0$ for all i. Consequently, the rank of this matrix is either 1 or 0, and more specifically, less than or equal to 1.