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Exercise 3.1

In this exercise we wish to show the linear independence of two cubic B-splines on the knot vector $\hat{t} \coloneqq (0, 0, 1, 3, 4, 5)$ over the interval $[1, 3)$. Using the knot dependecies, these two B-splines are

$$\hat{B}_{1,3}(x \mid 0, 0, 1, 3, 4), \qquad \hat{B}_{2,3}(x \mid 0, 1, 3, 4, 5).$$

In order to show the linear independence, we wish to apply Theorem 3.9, restated here:

> **Theorem.** Suppose that t is a $p+1$-extended knot vector. Then the B-splines in $\mathbb{S}_{p,t}$ are linearly independent on the interval $[t_{p+1}, t_{n+1})$.

Note that the vector $\hat{t}$ is *not* $p+1$-extended, so we introduce the new knot vector $t \coloneqq (0, 0, 0, 1, 3, 4, 5, 5)$. We now have enough knots to define four cubic B-splines, and these are

$$\begin{aligned} &B_{1,3}(x \mid 0, 0, 0, 1, 3), && B_{2,3}(x \mid 0, 0, 1, 3, 4), \\ &B_{3,3}(x \mid 0, 1, 3, 4, 5), && B_{4,3}(x \mid 1, 3, 4, 5, 5). \end{aligned}$$

By the above theorem, we now know that the B-splines $B_{i,3,t}$ for $i = 1, \ldots, 4$ are linearly independent on the interval $[t_4, t_5)$. And by examining the knot dependencies, we see that $B_{2,3} = \hat{B}_{1,3}$ and $B_{3,3} = \hat{B}_{2,3}$. Hence, the two B-splines on the coarses knot vector are linearly independent on the interval $[1, 3)$.