Non Homogenous Higher Order Differential Equation

The D.E

$$F(D)y = F(x)$$

is non homogenous if $F(x)$ is not identically zero.

where $F(D) = a_0 D^n + a_1 D^{n-1} + \cdots a_{n-1} D + a_n$

Its Solution consist of two parts

(i) Complementary Function (C.F)

(ii) Particular Integral (P.I)

C.F is the solution of homogenous equation $f(D)y = 0$ discussed in previous exercise.

<u>To find P.I</u>

we write

$$y_p = \frac{1}{F(D)} F(x)$$

Its solution depends on $F(x)$ we will discuss using examples.

then general solution is

$$y = y_c + y_p$$

① If $F(x) = e^{ax}$

put $D = a$

$$\frac{1}{F(D)} F(x) = \frac{1}{F(D)} e^{ax} = \frac{1}{F(a)} e^{ax}$$

but if $F(a) = 0$

then

$$\frac{1}{F(D)} e^{ax} = \frac{x e^{ax}}{F'(D)} = \frac{x e^{ax}}{F'(a)}$$

if $F'(a) = 0$ then again differentiate

Q1..

$$(D^2 + 3D - 4)y = 15e^x$$

Sol. For y_c ..

Its characteristics eq is

$$D^2 + 3D - 4 = 0$$

$$D^2 + 4D - D - 4 = 0$$

$$D(D+4) - 1(D+4) = 0$$

$$(D+4)(D-1) = 0$$

$\Rightarrow$ $D = -4, 1$ real & distinct.

$$y_c = c_1 e^{-4x} + c_2 e^x$$

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For PI (y_p)

$$y_p = \frac{15e^x}{D^2+3D-4} \quad \text{put } D=1$$

on putting $D=1$ denominator is '0'

then

$$y_p = \frac{15xe^x}{2D+3} \quad \text{put } D=1$$

$$y_p = \frac{15xe^x}{2+3}$$

$$y_p = 3xe^x$$

So the G.S is

$$y = y_c + y_p$$

$$y = c_1e^{-4x} + c_2e^x + 3xe^x.$$

Q2:- $(D^2-3D+2)y = e^x + e^{-2x}$

Sol:-

For y_c:-

Its characteristics eq is

$$D^2-3D+2 = 0$$

$$(D-1)(D-2)=0$$

$$\Rightarrow D=1,\ 2$$

$$y_c = c_1 e^{x} + c_2 e^{2x}$$

For P.I :-

$$y_p = \frac{e^{x}+e^{-2x}}{D^2-3D+2}$$

$$y_p = \frac{e^{x}}{D^2-3D+2} + \frac{e^{-2x}}{D^2-3D+2}$$

$$= \frac{x e^{x}}{2D-3} + \frac{e^{-2x}}{(-2)^2-3(-2)+2}$$

$$= \frac{x e^{x}}{-1} + \frac{e^{-2x}}{4+6+2}$$

$$y_p = -x e^{x} + \frac{1}{12} e^{-2x}$$

So the G.S is

$$y = c_1 e^{x} + c_2 e^{2x} - x e^{x} + \frac{1}{12} e^{-2x}.$$

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(2) If $f(x) = \sin ax$ or $\cos ax$

put $D^2 = -a^2$

(3) $(D^2 - 2D - 3)y = 2e^x - 10\sin x$

Sol.

$(D^2 - 2D - 3)y = 0$

Its auxiliary eq is

$D^2 - 2D - 3 = 0$

$(D-3)(D+1) = 0$

$D = 3, -1$

So $y_c = c_1 e^{3x} + c_2 e^{-x}$

For P.I.:-

$$y_p = \frac{2e^x - 10\sin x}{D^2 - 2D - 3}$$

$$= \frac{2e^x}{D^2 - 2D - 3} - \frac{10\sin x}{D^2 - 2D - 3}$$

$$= \frac{2e^x}{1 - 2 - 3} - \frac{10\sin x}{-1^2 - 2D - 3}$$

$$= \frac{2e^x}{-4} - \frac{10\sin x}{-2D - 4}$$

$$= \frac{-e^x}{2} + \frac{10\sin x}{2D+4}$$

$$= \frac{-e^x}{2} + \frac{5\sin x}{D+2}$$

$$= \frac{-e^x}{2} + \frac{5(D-2)\sin x}{D^2-2^2}$$

$$= \frac{-e^x}{2} + \frac{5(D-2)\sin x}{-1^2-4}$$

$$= \frac{-e^x}{2} - \frac{5}{5}(D\sin x - 2\sin x)$$

$$y_p = \frac{-e^x}{2} - \cos x + 2\sin x$$

So the G.S is

$$y = c_1 e^{3x} + c_2 e^{-x} - \frac{e^x}{2} - \cos x + 2\sin x.$$

(7) $(D^2+4)y = 4\sin^2 x$

Sol:-

For y_c:-

Its auxiliary eq is

$$D^2 + 4 = 0$$

$$D^2 = -4$$

$$D = \pm 2i$$

$$y_c = (c_1 \cos 2x + c_2 \sin 2x) e^{0x}$$

$$y_p = \frac{4 \sin^2 x}{D^2+4}$$

$\therefore \cos 2x = 1 - 2\sin^2 x$

$\Rightarrow 2\sin^2 x = 1 - \cos 2x$

$$= \frac{2 \cdot 2\sin^2 x}{D^2+4}$$

$$= \frac{2(1-\cos 2x)}{D^2+4}$$

$$= \frac{2e^{0x}}{D^2+4} - \frac{2\cos 2x}{D^2+4}$$

$$= \frac{2}{4} - \frac{2x \cos 2x}{2D}$$

$$= \frac{1}{2} - x \frac{1}{D} \cos 2x$$

$$= \frac{1}{2} - x \frac{\sin 2x}{2}$$

The G.S is

$$y = y_c + y_p$$

$$y = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{2} - \frac{x \sin 2x}{2}$$