Linear Differential Equations:-

A first order ODE of the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

is called linear D.E because it is linear in 'y' & $\frac{dy}{dx}$.

To solve this D.E find integrating factor

$$I.F = \mu(x) = e^{\int P(x)dx}$$

then Multiply D.E by I.F

$$\mu(x)\frac{dy}{dx} + \mu(x)P(x)y = \mu(x)\,Q(x).$$

$$\frac{d}{dx}\left(\mu(x)y\right) = \mu(x)Q(x)$$

$$d\left(\mu(x)y\right) = \mu(x)Q(x)dx$$

on applying Integration

$$\boxed{\mu(x)y = \int \mu(x)Q(x)dx + c}$$

Similarly,
A diff. eq of the form

$$\frac{dx}{dy} + P(y)x = Q(y)$$

is linear in x & dx/dy.

Ex # 9.6

Q1:- $$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x} \quad \text{——} \; (1)$$

Sol..

It is linear in y

$$I.F = \exp\left(\int P(x)dx\right) = \exp\left(\int \frac{2x+1}{x} dx\right)$$

$$= \exp\left(\int \left(2 + \tfrac{1}{x}\right)dx\right) = e^{2x + \ln x} = e^{2x} \cdot e^{\ln x}$$

$$I \cdot F = x e^{2x}.$$

Multiplying eq (1) by I.F

$$xe^{2x} \cdot \frac{dy}{dx} + xe^{2x}\left(\frac{2x+1}{x}\right)y = xe^{2x} \cdot e^{-2x}$$

$$\frac{d}{dx}\left(xe^{2x}y\right) = x$$

$$d\left(xe^{2x}y\right) = x\,dx$$

On Integrating

$$\int d\left(xe^{2x}y\right) = \int x\,dx$$

$$xe^{2x}y = \frac{x^2}{2} + C$$

Q5:- $\cos^3 x \frac{dy}{dx} + y\cos x = \sin x$

Sol:

$$\frac{dy}{dx} + \frac{y\cos x}{\cos^3 x} = \frac{\sin x}{\cos^3 x}$$

$$\frac{dy}{dx} + \sec^2 x\, y = \sec^2 x \tan x \quad \text{--- (1)}$$

It is linear in 'y'

$$I\cdot F = \exp\left(\int P(x)\,dx\right) = \exp\left(\int \sec^2 x\,dx\right)$$

$$I\cdot F = e^{\tan x}$$

multiplying eq (1) by I·F

$$e^{\tan x}\frac{dy}{dx} + e^{\tan x}\sec^2 x\, y = e^{\tan x}\sec^2 x \tan x$$

$$\frac{d}{dx}\left(e^{\tan x} y\right) = e^{\tan x}\sec^2 x \tan x$$

$$d\left(e^{\tan x}\, y\right) = e^{\tan x} \sec^2 x \tan x \, dx$$

on Integrating

$$e^{\tan x}\, y = \int e^{\tan x} \sec^2 x \tan x \, dx$$

Let $\tan x = z$

$\sec^2 x \, dx = dz$

Then

$$e^{z} y = \int z e^{z} \, dz$$

$$= z e^{z} - \int (1) e^{z} \, dz$$

$$e^{z} y = z e^{z} - e^{z} + C \qquad \therefore z = \tan x$$

$$e^{\tan x}\, y = e^{\tan x}(\tan x - 1) + C$$

$$y = \text{[illegible]} + C e^{-\tan x}.$$

is required solution.

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(11) $$\frac{dy}{dx} = \frac{1}{e^y - x}$$

$$\frac{dx}{dy} = e^y - x$$ by reciprocal

$$\frac{dx}{dy} + x = e^y \text{ --- (1)}$$ (LDE in x)

$$I.F = e^{\int P(y)\,dy} = e^{\int 1\,dy} = e^y$$

Multiplying by eq (1)

$$e^y \frac{dx}{dy} + xe^y = e^y \cdot e^y$$

$$\frac{d}{dy}(e^y \cdot x) = e^{2y}$$

$$d(e^y \cdot x) = e^{2y}\,dy$$

on Integrating

$$e^y x = \int e^{2y} dy$$

$$e^y x = \frac{e^{2y}}{2} + C$$

$$\Rightarrow x = \frac{e^y}{2} + c e^{-y}$$

is required solution.