Q14:- $\dfrac{dy}{dx} = e^{2x} + y - 1$

$dy = (e^{2x} + y - 1)dx$

$(e^{2x} + y - 1)dx - dy = 0$ ----(i)

$M = e^{2x} + y - 1$, $N = -1$

$M_y = 1$ $\quad N_x = 0$

$M_y \neq N_x$

① is not an Exact Diff eq.

To find Integrating factor.

$\dfrac{M_y - N_x}{N} = \dfrac{1-0}{-1} = -1 = -x^0 = f(x)$

Now $I.F = e^{\int -1 dx} = e^{-x}$

multiply both sides of eq (i) by e^{-x}.

$e^{-x}(e^{2x} + y - 1)dx - e^{-x}dy = 0$

$(e^{x} + e^{-x}y - e^{-x})dx - e^{-x}dy = 0$

$M = e^{x} + e^{-x}y - e^{-x}$ $\quad N = -e^{-x}$

$M_y = e^{-x}$ $\quad N_x = e^{-x}$

$M_y = N_x$

② is an Exact Diff. eq.

So

$$\int M dx + \int \bar{N} dy = C$$

$$\int (e^x + e^{-y} y - e^{-x}) dx = C$$

$$e^x - e^x y + e^{-x} = C$$

Q15.-

$$(y^2 + xy)dx - x^2 dy = 0 \quad \text{--- (1)}$$

$M = y^2 + xy$ $\qquad$ $N = -x^2$

$M_y = 2y + x$ $\qquad$ $N_x = -2x$

$$M_y \neq N_x$$

(1) is Non-Exact diff. eq.

$$\frac{M_y - N_x}{N} = \frac{2y + x + 2x}{-x^2} \neq f(x)$$

$$\frac{N_x - M_y}{M} = \frac{-2x - 2y - x}{y^2 + xy} \neq f(y)$$

(1) is Homogenous diff. eq. of degree 2

$$xM + yN \neq 0$$

$$x(y^2 + xy) + y(-x^2) \neq 0$$

$$xy^2 + x^2 y - x^2 y \neq 0$$

$$xy^2 \neq 0$$

$$I.F = \frac{1}{xM+yN} = \frac{1}{xy^2}$$

Multiply both sides of ① by I.F

$$\frac{1}{xy^2}(y^2+xy)dx - \frac{1}{xy^2}x^2dy = 0$$

$$\left(\frac{1}{x}+\frac{1}{y}\right)dx - \frac{x}{y^2}dy = 0 \quad \text{---} ②$$

$$M = \frac{1}{x}+\frac{1}{y} \qquad N = -\frac{x}{y^2}$$

$$M_y = \frac{-1}{y^2} \qquad N_x = -\frac{1}{y^2}$$

$$M_y = N_x$$

② is Exact diff. eq.

$$\int M dx + \int \bar{N} dy = C$$

$$\int \left(\frac{1}{x}+\frac{1}{y}\right)dx + 0 = C$$

$$\ln x + x/y = C$$

is required solution

Q15..

$(y^2+xy)dx - x^2dy = 0$ Alternatively

$$\frac{dy}{dx} = \frac{y^2+xy}{x^2} \quad \text{--- (1)}$$

It is an Homogeneous diff. eq.

put $y = vx$ in (1)

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{v^2x^2 + xvx}{x^2}$$

$$v + x\frac{dv}{dx} = \frac{x^2(v^2+v)}{x^2}$$

$$x\frac{dv}{dx} = v^2+v-v$$

$$\frac{dv}{v^2} = \frac{dx}{x}$$ Now It is Separable eq.

$$\int \frac{dv}{v^2} = \int \frac{dx}{x}$$

$$-\frac{1}{v} = \ln x + c$$

$$\ln x + \frac{1}{v} = c$$

$$\ln x + \frac{x}{y} = c \qquad \because v = y/x$$

is required Sol.

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(20) $(x+2)\sin y\,dx + x\cos y\,dy = 0$ —— (1)

$M = (x+2)\sin y \qquad N = x\cos y$

$M_y = (x+2)\cos y \qquad N_x = \cos y$

$M_y \neq N_x$

(1) is Non - Exact

To find I.F

$$\frac{M_y - N_x}{N} = \frac{(x+2)\cos y - \cos y}{x\cos y}$$

$$= \frac{(x+2-1)\cos y}{x\cos y} = \frac{x+1}{x} = f(x)$$

$$I.F = e^{\int (1+\frac{1}{x})dx} = e^{x+\ln x} = e^x e^{\ln x}$$

$$I.F = xe^x$$

Multiply eq (1) by I.F

$$\underbrace{xe^x(x+2)\sin y}_{M}\,dx + \underbrace{xe^x x\cos y}_{N}\,dy = 0 \quad \text{—— (2)}$$

$N = x^2e^x\cos y$

$M_y = (x^2e^x + 2xe^x)\cos y \qquad N_x = (x^2e^x + 2xe^x)\cos y$

$M_y = N_x$

(2) is Exact diff. eq.

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$$\int M\,dx + \int \bar{N}\,dy = c$$

$$\int (x^2 e^x \sin y + 2x e^x \sin y)\,dx + 0 = c$$

$$x^2 e^x \sin y - \int 2x e^x \sin y\,dx + \int 2x e^x \sin y\,dx = c$$

$$x^2 e^x \sin y = c$$

is required solution.