Again let l, m, n be the multipliers

$$\frac{l\,dx + m\,dy + n\,dz}{l(mz - ny) + m(nx - lz) + n(ly - mx)}$$

$$\frac{l\,dx + m\,dy + n\,dz}{0}$$

$$l\,dx + m\,dy + n\,dz = 0$$

$$\Rightarrow lx + my + nz = c_2$$

Example..

$$x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$$

Sol.

AE-

$$\frac{dx}{x(z^2 - y^2)} = \frac{dy}{y(x^2 - z^2)} = \frac{dy}{z(y^2 - x^2)}$$

let x, y, z be the multipliers

$$\frac{x\,dx + y\,dy + z\,dz}{x^2(z^2 - y^2) + y^2(x^2 - z^2) + z^2(y^2 - x^2)}$$

$$\frac{x\,dx + y\,dy + z\,dz}{0}$$

$$\Rightarrow x\,dx + y\,dy + z\,dz = 0$$

Integrating,

$$x^2+y^2+z^2 = 2c_1$$

let $1/x, 1/y, 1/z$ be the multipliers

$$\frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{(z^2-y^2)+(x^2-z^2)+(y^2-x^2)}$$

$$\Rightarrow \frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$$

$$\Rightarrow \ln x + \ln y + \ln z = c_2$$

$$\Rightarrow \ln xyz = c_2'$$

Example.

Solve $(y+z)p - (x+z)q = x-y$

A.E.

$$\frac{dx}{y+z} = \frac{dy}{-(x+z)} = \frac{dz}{x-y}$$

let $1, 1, 1$ be the multipliers

$$\frac{dx+dy+dz}{y+z-(x+z)+x-y}$$

$$\Rightarrow dx+dy+dz = 0$$

Integrating,

$$x+y+z=c_1$$

Let $x, y, -z$ be the multipliers

$$\frac{x\,dx+y\,dy-z\,dz}{x(y+z)-y(x+z)-z(x-y)}$$

$$\Rightarrow x\,dx+y\,dy-z\,dz=0$$

$$x^2+y^2+z^2=c_2'$$

$$\phi\left(x+y+z,\ x^2+y^2+z^2\right)=0$$

Example..

$$z(x+y)p+z(x-y)q=x^2+y^2$$

Sol.

A.E

$$\frac{dx}{z(x+y)}=\frac{dy}{z(x-y)}=\frac{dz}{x^2+y^2} \quad \text{—— (1)}$$

Let $-x, y, z$ be the multipliers

$$\frac{-x\,dx+y\,dy+z\,dz}{-xz(x+y)+yz(x-y)+z(x^2+y^2)}$$

$$-xdx + ydy + zdz = 0$$

Integrating

$$-\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = c_1$$

let $y, x, -z$ be the multipliers of eq (1)

$$\frac{ydx + xdy - zdz}{yz(x+y) + xz(x-y) - z(x^2+y^2)}$$

$$\Rightarrow \quad ydx + xdy - zdz = 0$$

$$d(xy) - zdz = 0$$

Integrating

$$xy - z^2/2 = c_2$$

$$\phi\left(\frac{-x^2+y^2+z^2}{2},\ xy - z^2/2\right) = 0$$

Example..

$$(z^2-2yz-y^2)p+x(y+z)q = x(y-z)$$

Sol

A.E

$$\frac{dx}{z^2-2yz-y^2} = \frac{dy}{x(y+z)} = \frac{dz}{x(y-z)}$$

Let x, y, z be the multipliers

$$\frac{xdx+ydy+zdz}{x(z^2-2yz-y^2)+xy(y+z)+zx(y-z)}$$

$$\Rightarrow \quad xdx+ydy+zdz = 0$$

Integrating

$$\frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2} = c_1$$

Let

$0, -(y-z), y+z$ be the multipliers

$$\frac{0dx-(y-z)dy+(y+z)dz}{0-x(y+z)(y-z)+x(y-z)(y+z)}$$

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$$\Rightarrow \quad -(y-z)dy + (y+z)dz = 0$$

$$\Rightarrow \quad -ydy + zdy + ydz + zdz = 0$$

$$-ydy + zdz + d(zy) = 0$$

$$-\frac{y^2}{2} + \frac{z^2}{2} + zy = 0$$

$$\phi\left(\frac{x^2+y^2+z^2}{2},\ -\frac{y^2}{2} + \frac{z^2}{2} + zy\right) = 0$$