Example ..

$$p + 3q = 5z + \tan(y - 3x)$$

Sol/ $P = 1$, $Q = 3$, $R = 5z + \tan(y - 3x)$

A.E ..

$$\frac{dx}{1} = \frac{dy}{3} = \frac{dz}{5z + \tan(y - 3x)}$$

① $\frac{dx}{1} = \frac{dy}{3}$

$$3x = y + c_1$$

$$3x - y = c_1$$

$$\frac{dy}{3} = \frac{dz}{5z + \tan(y - 3x)}$$

from ①

$$3x - y = c_1$$

or

$$y - 3x = -c_1$$

$$\Rightarrow \frac{dy}{3} = \frac{dz}{5z + \tan(-c_1)}$$

$$\Rightarrow \frac{dy}{3} = \frac{dz}{5z - \tan c_1}$$

$$\frac{1}{3}y = \frac{1}{5}\ln(5z - \tan c_1) + c_2$$

$$\frac{1}{3}y - \frac{1}{5}\ln(5z - \tan(3x - y)) = c_2$$

$$\phi\left(3x - y, \ \frac{1}{3}y - \frac{1}{5}\ln(5z - \tan(3x - y))\right) = 0$$

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Example..

$$p + q = x + y + z$$

$$P = 1 \quad , \quad Q = 1 \quad , \quad R = x + y + z$$

A.E

$$\frac{dx}{1} = \frac{dy}{1} = \frac{dz}{x+y+z}$$

$$dx = dy$$

$$x = y + c_1$$

$$x - y = c_1$$

$$dy = \frac{dz}{x+y+z}$$

$$dy = \frac{dz}{c_1 + y + y + z}$$

$$dy = \frac{dz}{c_1 + 2y + z}$$

$$\Rightarrow \frac{dz}{dy} = c_1 + 2y + z$$

$$\Rightarrow \frac{dz}{dy} - z = 2y + c_1$$

It is linear in z

$$IF = e^{\int -1 dy} = e^{-y}$$

$$\Rightarrow e^{-y}\frac{dz}{dy} - ze^{-y} = (2y + c_1)e^{-y}$$

$$\frac{d}{dy}(e^{-y}z) = 2ye^{-y} + c_1e^{-y}$$

$$d(e^{-y}z) = (2ye^{-y} + c_1e^{-y})dy$$

$$e^{-y}z = 2y\frac{e^{-y}}{-1} - \int\left(2\frac{e^{-y}}{-1}\right)dy + c_1\frac{e^{-y}}{-1}$$

$$e^{-y}z = -2ye^{-y} - 2e^{-y} - c_1e^{-y} + c_2$$

$$e^{-y}z + 2ye^{-y} + 2e^{-y} + (x-y)e^{-y} = c_2$$

$$e^{-y}z + ye^{-y} + xe^{-y} + 2e^{-y} = c_2$$

$$\phi\left(x-y,\ e^{-y}z + ye^{-y} + xe^{-y} + 2e^{-y}\right) = 0$$

Method of Multipliers :-

For PDE

$$P_p + Q_q = R$$

the auxiliary lagrange eq is

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \quad \text{--- (1)}$$

let P_1, Q_1, R_1 are the multipliers of eq (1) and each & every fraction in (1) will be equal to

$$\frac{P_1dx + Q_1dy + R_1dz}{P_1P + Q_1Q + R_1R}$$

If $P_1P + Q_1Q + R_1R = 0$

then $P_1dx + Q_1dy + R_1dz = 0$ --- (2)

Now Integrate eq ② we get

$$u(x, y, z) = c_1$$

Repeat the process for another set of multipliers say P_2, Q_2, R_2 to get

$$v(x, y, z) = c_2$$

Example..

$$(mz - ny)p + (nx - lz)q = ly - mx$$

Sol,

$$P = mz - ny \quad , Q = nx - lz, \quad R = ly - mx$$

A.E..

$$\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx}$$

let x, y, z be the multipliers

then

$$\frac{x\,dx + y\,dy + z\,dz}{x(mz - ny) + y(nx - lz) + z(ly - mx)}$$

$$\frac{x\,dx + y\,dz + z\,dz}{0}$$

$$\Rightarrow \quad x\,dx + y\,dz + z\,dz = 0$$

Integrating

$$x^2 + y^2 + z^2 = 2c_1$$

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Again let l, m, n be the multipliers

$$\frac{l\,dx + m\,dy + n\,dz}{l(mz - ny) + m(nx - lz) + n(ly - mx)}$$

$$\frac{l\,dx + m\,dy + n\,dz}{0}$$

$$l\,dx + m\,dy + n\,dz = 0$$

$$\Rightarrow lx + my + nz = c_2$$