Game Concept for Quantum Hashi

The idea is to use Classical Hashi to understand the concept **of correcting errors** on **entangled states** using **syndrome measurements**.

This will be done through a **two part game** (for each level).

Description:

- Each island corespond to a Qubit
- The number on each qubit (island) corresponds to the number of qubits that are entangled with the island.
- The bridges correspond in the first part to the entanglement between qubits and the parity measurement in the second part. I will discuss this more in details.

Part 1

Part 1 plays like a classical Hashi with a possible change in one of rules. This part will play the role of the encoding of the state.

Rules

- The number of bridges connected to an island (the number of entanglement link) must match the number written on the island.

- This is the only rule that has to change compared to Classical Hashi. **There must be at most a single bridge between two qubits.** If anybody can give an explanation of what having two entanglement links between same qubits could mean, that rule could stay also. And because of that, in Quantum Hashi, you can have at max 4 bridges for a single qubit, instead of 8 for an island in the classical case.
- The bridges cannot cross each other at a place where there is no qubit. This could mean we cannot copy quantum unknown quantum states du to the No Cloning Theorem.
- Qubits can only be linked perpendicularly : this could be a constraint on the hardware in which we want to encode our qubits.
- **The goal** is to connect all the qubits in the islands in a single group of entangled qubits, while respecting the above rules. This could be for example finding a map that allows us to encode a GHZ state :

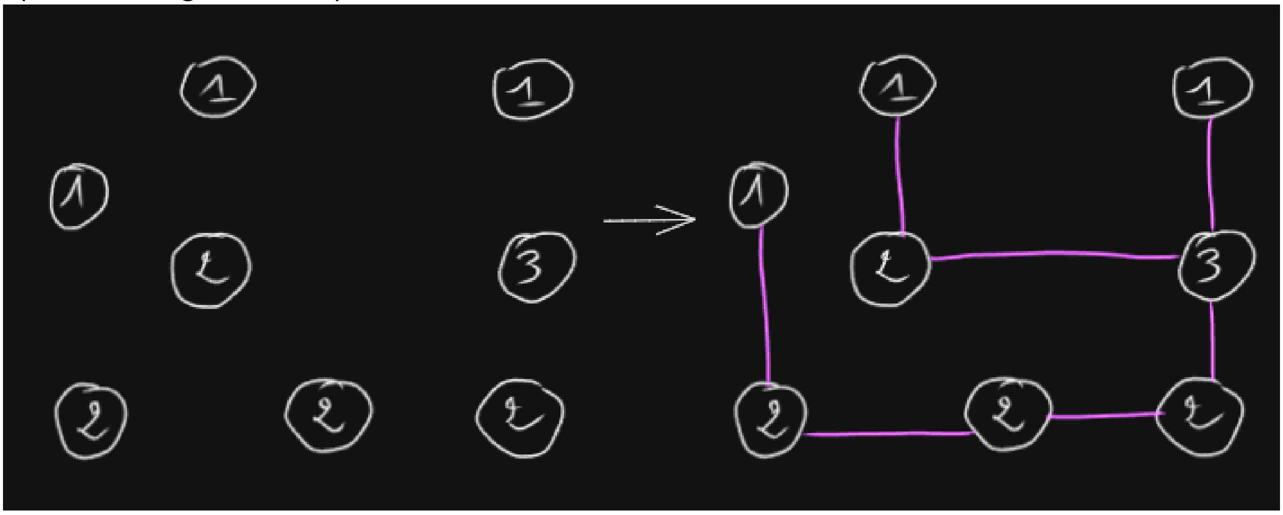
$$|GHZ\rangle = \frac{1}{\sqrt{2}}\Big(|00\cdots 0\rangle + |11\cdots 1\rangle\Big).$$

This encoding will help us teach concept of syndrome measurement for Quantum Error Correction in the next part.

Finding the most optimal entanglement map the **fastest possible** wins you the first part of the level $\implies$ playing original Hashi with one rule change.

Example

We start with qubits and the number of qubits they are entangled with and we try to find the most optimal entanglement map.



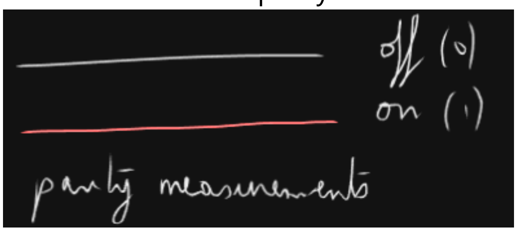
How do we increase difficulty in this part :

More qubits + larger grids. The example is for a grid of size 5x5.

Part 2

Once you find the entanglement map, let's say your **GHZ** enconding we keep the same configuration, but **the bridges are now considered syndrome measurements**.
Syndrome measurement are any measurements that will give us information about the errors in our qubits without disturbing their states. In this particular case of GHZ encoding, I will consider **parity measurement** between pairs of qubits as a syndrome measurement. The concept of syndrome measurement is very important here since measurement directly the qubits to infer some errors will collapse the whole state of the system.
Parity measurement between two qubits will tell us if they are in the same state or not. The measurement will yield 0 if both qubits are in the same state and 1 if not.
In the particular case of these GHZ states, pairs of qubits are always in the same state $|0\rangle$ or $|1\rangle$, therefore the measurement will always yield the value 0.
If there is any bitflip in one of the pair of qubits, for example, the first qubit, the state of the entangled pairs become $|10\rangle + |10\rangle$, which will yield a value of 1 when we make a parity measurement.

So the bridges will be the parity measurement between the two linked qubits. We can use different color to show if the parity measurement is 0 or 1.



If we measure 0, there is no error and if we measure 1 (on), there is an bit flip error in one of the qubit. The interesting part of the game is that observations, based solely on the parity measurement of a pair of qubits, will not tell us which of the two qubits of the pairs is the faulty one. We need to take into account the other part of neighbouring parity measurements also.

This simple example of GHZ encoding and parity measurements is just to bring more intuition into the game. But the idea works for different types of encodings, different types of syndrome measurements and different types of errors.

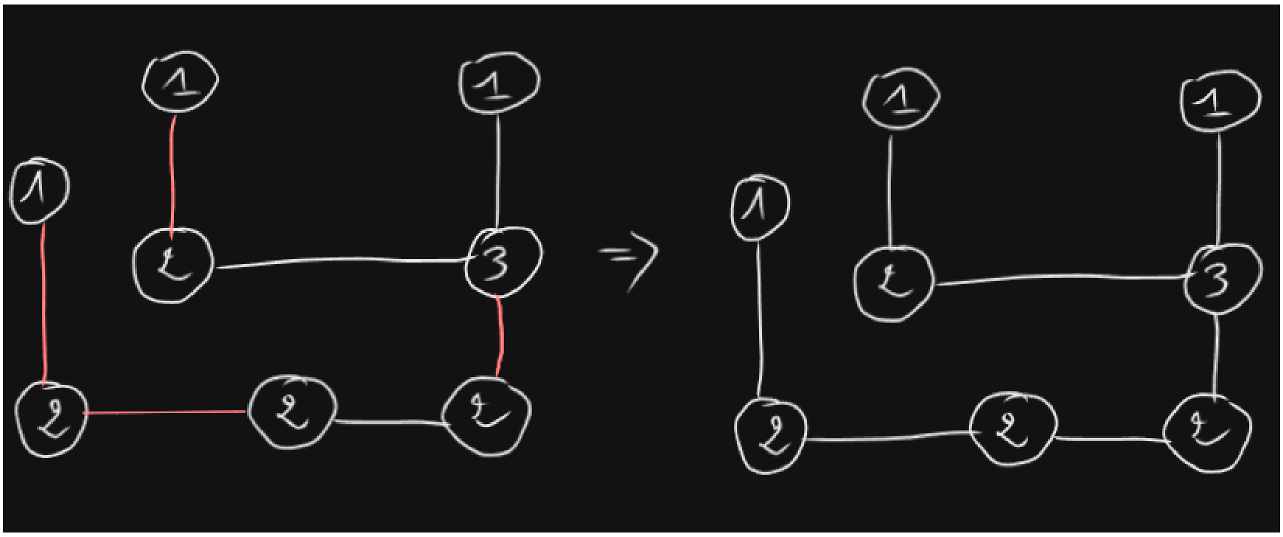
Rules :

- We start with a full map where some parity measurements have been flipped to 1, indicating the presence of a bit flip error for example in some of our qubits or the parity measurement itself.
- **The goal** is to find the faulty qubits or parity measurement and correct them (by using some interactive clics) to ensure all the parity measurements are back to 0. And that's what will be considered a win.

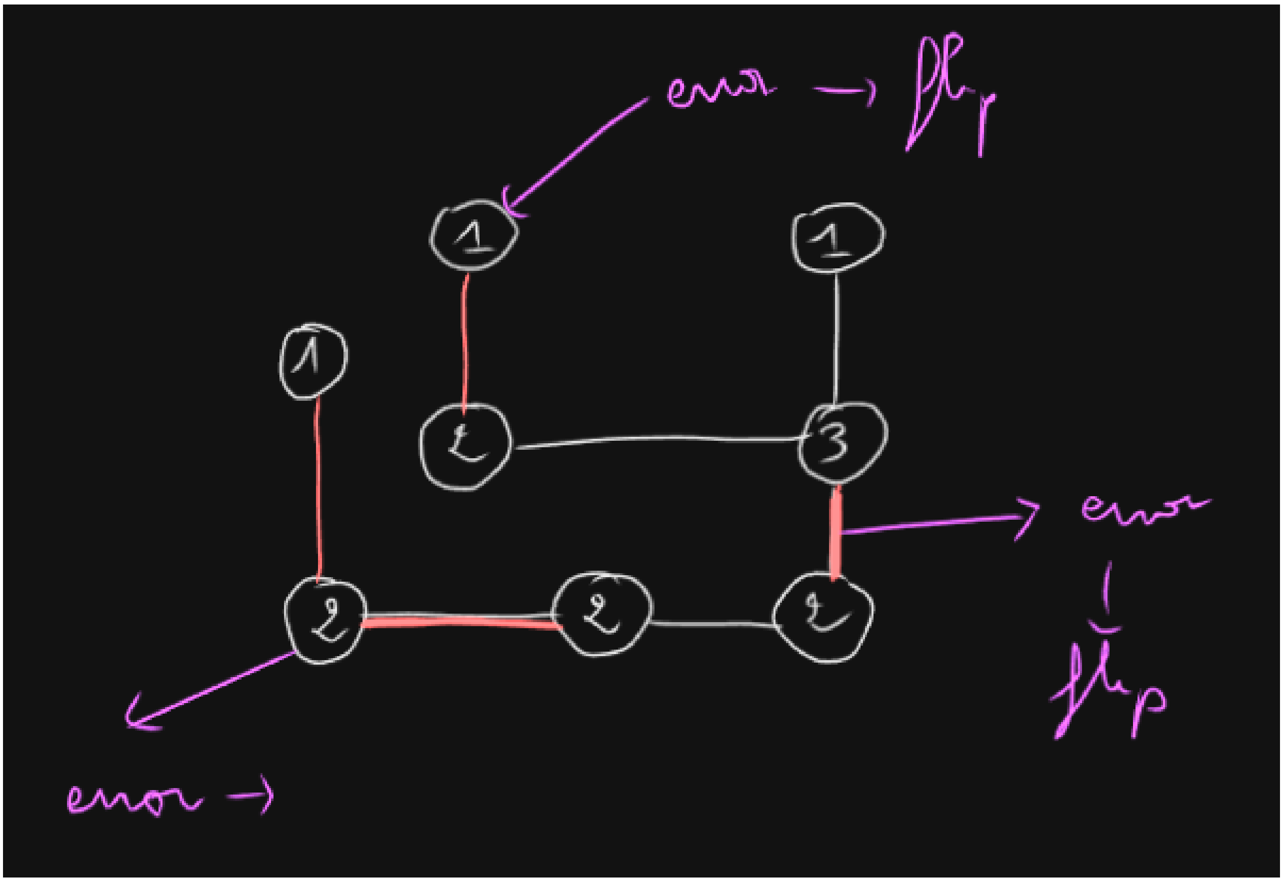
Perhaps for the simple case, let's stick to only faulty qubits.

Example

So we start with some parity measurements flipped, and the goal to figure out the errors and correct them.

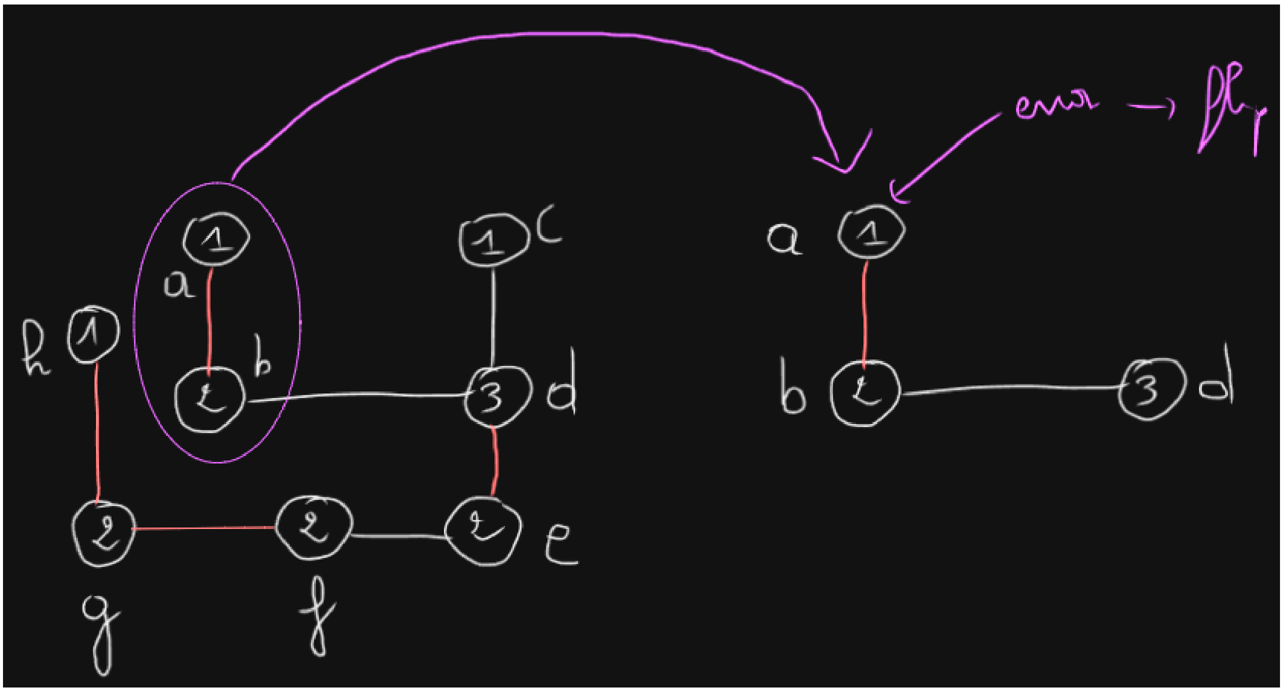


How do we figure out what the errors are?

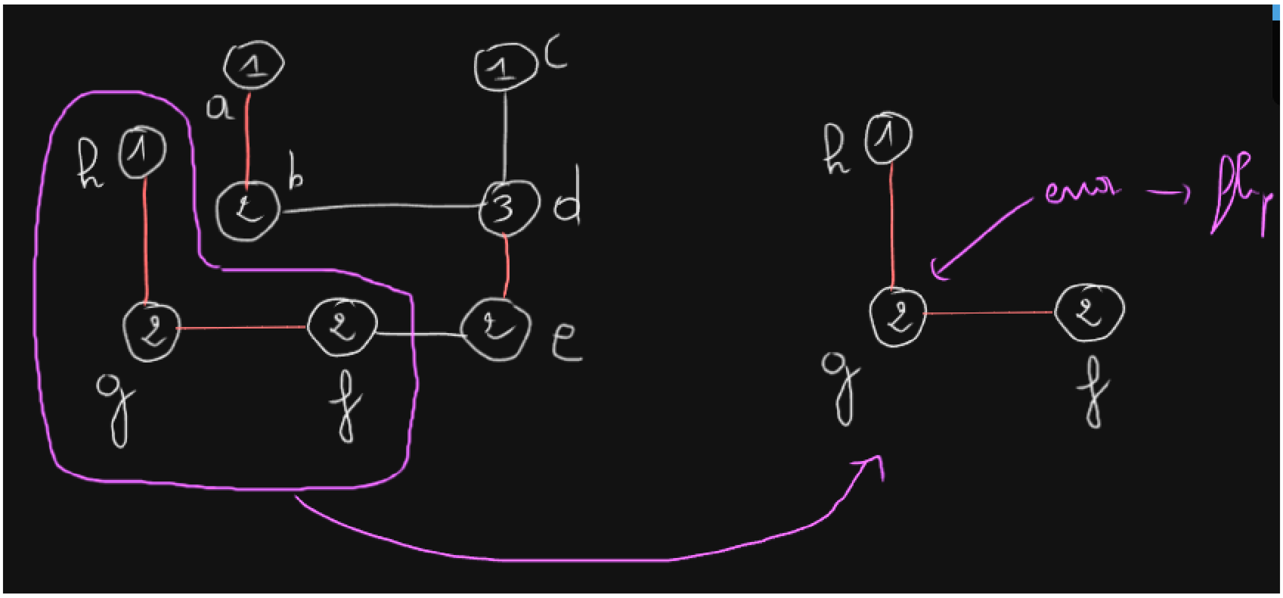


We need to observe the flipped parity measurement but also the neighboring parity measurements too.

Let's look case by case. Let's label the qubits a to h for the sake of explanation.



The parity measurement between qubit a and qubit b is on (value = 1). So how do we figure which one between the qubit a and qubit b has been flipped. For that, let's also look at the parity measurement between qubit b and qubit d. That parity measurement is off, none of the qubit b and d has been flipped. I can conclude from that the error is necessary on the qubit a and all I have to do is click on qubit a to correct the error and therefore flip the parity measurement between a and b to off.

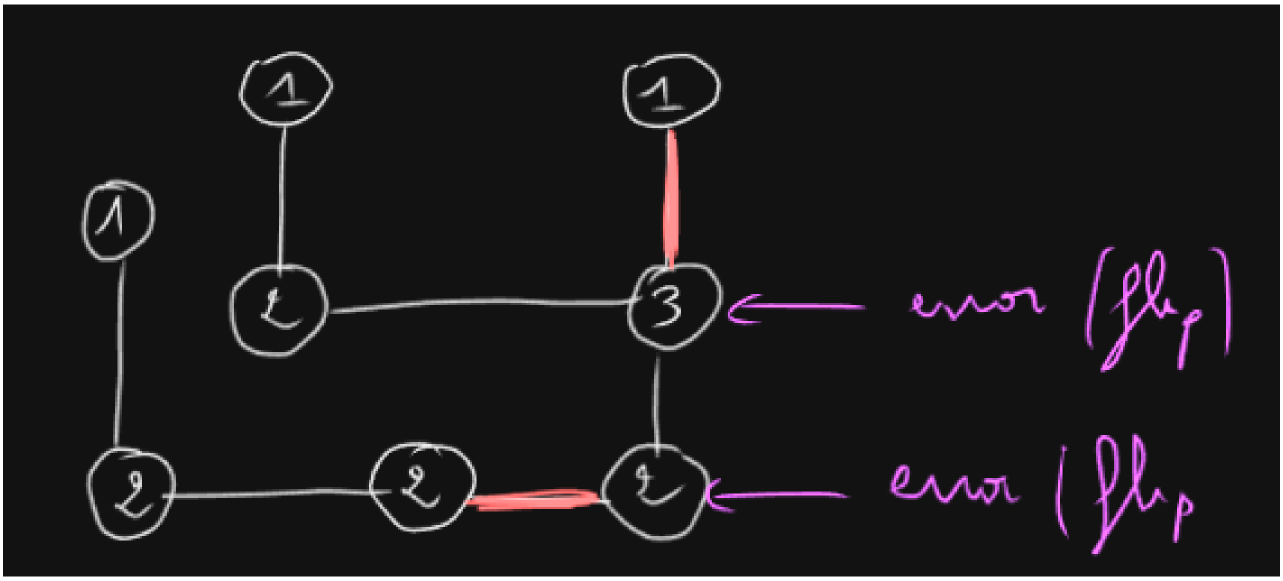


Here similarly, we have a flip of the parity measurement between qubits h and g meaning one of them has an error. Same for the parity measurement between qubits g and f. The common denominator being qubit g here, it's the one that has suffered a bit flip, and we need to correct it.

We can use the same logic to figure why the parity measurement between qubits d and e is the one who suffered an error also. Feel free to ask if you can't figure it out.

Again, the win happens once all the parity measurement are back to 0.

We can also have much more complex types of error that's are not obvious to deduce. Feel free to try also.

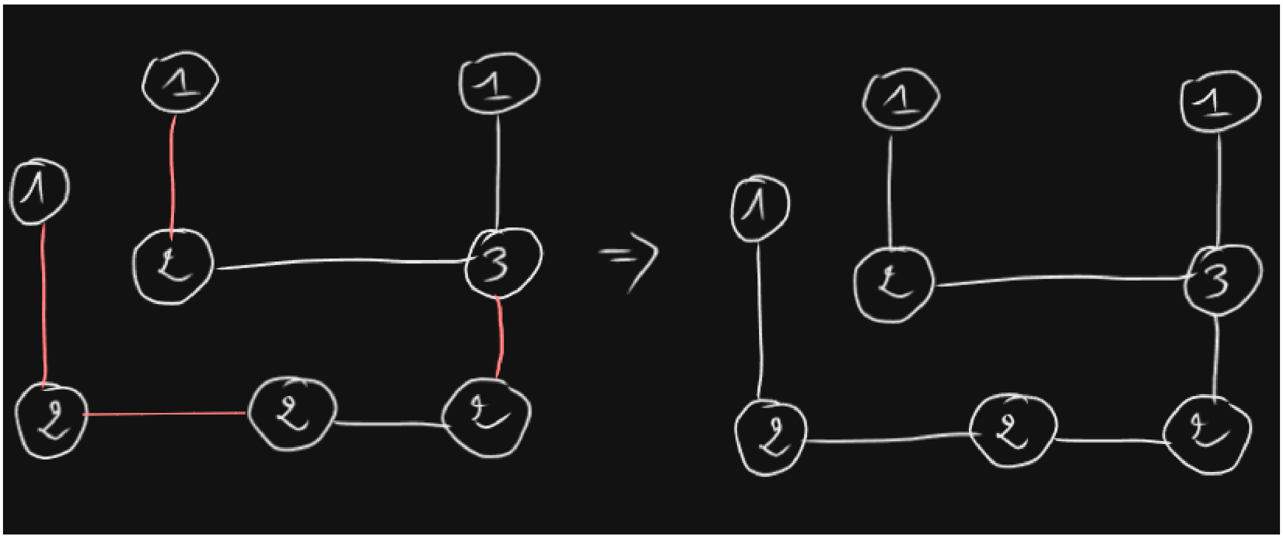


How do we increase difficulty in this part :

More qubits + larger grids.
Harder to deduce types of error and larger number of errors in the map.

Scoring

This is not priority yet but as Abdullah suggested, a type of scoring for this part of the game could be through fildelity measurement between these two mappings. And everytime, an error is corrected such that a parity measurement is flipped back to 0, the fildelity should increase until reaching 1 when all errors have been corrected.



Conclusion

Through this two part game (basically each level is a two part game : finding the entanglement mapping and correct all the errors), the players should be able to learn intuitively (with or without knowledge of quantum computing or quantum error correction):

- How to encode quantum information using entangled qubits
- How to detect errors using syndrome measurementrs
- How to correct errors using feedback operations