

3차 저역 통과필터 설계와 분석

24년 1학기

2분반, 컴퓨터공학과 32231861 박준성

I. Project title

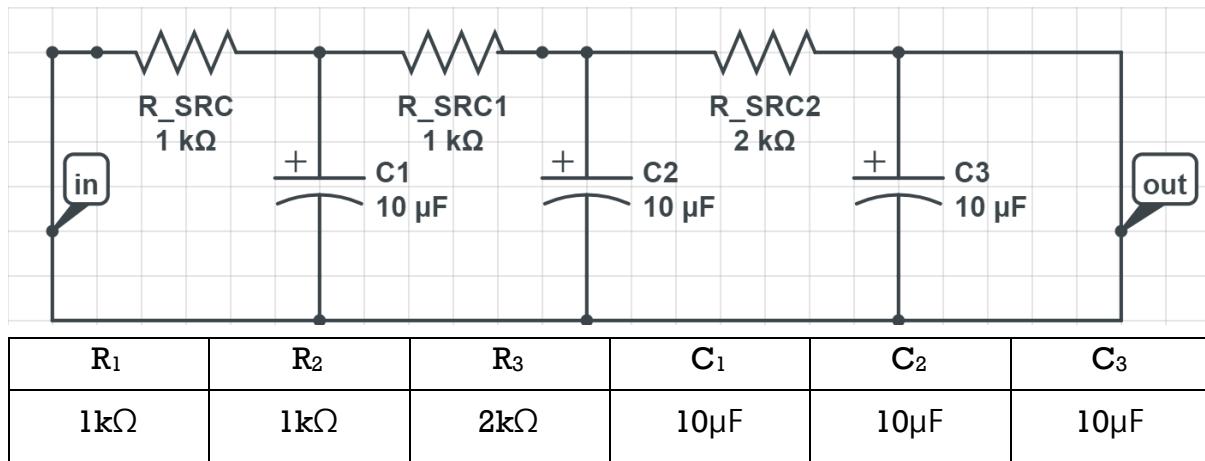
Design and construction of RC low-pass filter

II. Project objectives

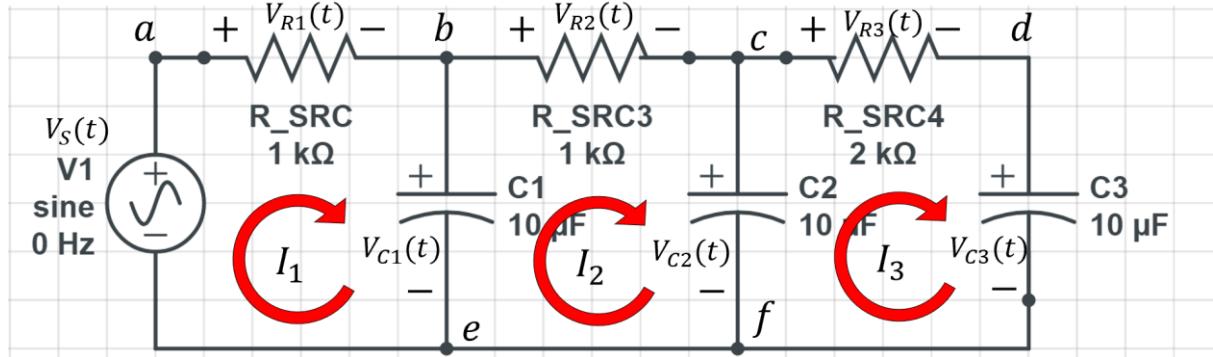
- ▶ Design of an n th order RC low-pass filter
- ▶ Calculation of the frequency response of the low-pass filter
- ▶ Calculation of the cut-off frequency
- ▶ Construction of the n th order RC low-pass filter
- ▶ Measurement of the frequency response of the low-pass filter
- ▶ Measurement of the cut-off frequency
- ▶ Comparison of measured cut-off frequency with theoretically calculated cut-off frequency
- ▶ Analysis of the difference between measured cut-off frequency and theoretically calculated cut-off frequency

III. Project

- ▶ An 3rd order RC low-pass filter consists of 3 pairs of resistor and capacitor in series. The figure below shows a 3th order RC low-pass filter



2. Calculation of frequency response of 3rd order RC low-pass filter



위 그림처럼 3개의 메시를 찾을 수 있다. 키르히호프의 전압법칙으로

$$(1) V_s(t) - V_{R1}(t) - V_{C1}(t) = 0$$

$$(2) V_{C1}(t) - V_{R2}(t) - V_{C2}(t) = 0$$

$$(3) V_{C2}(t) - V_{R3}(t) - V_{C3}(t) = 0,$$

커패시터의 전류와 전압의 관계를 통해

$$(4) I_1(t) = C_1 \frac{dV_{C1}(t)}{dt}, V_{R1}(t) = R_1 I_1(t) = R_1 C_1 \frac{dV_{C1}(t)}{dt}$$

$$(5) I_2(t) = C_2 \frac{dV_{C2}(t)}{dt}, V_{R2}(t) = R_2 I_2(t) = R_2 C_2 \frac{dV_{C2}(t)}{dt}$$

$$(6) I_3(t) = C_3 \frac{dV_{C3}(t)}{dt}, V_{R3}(t) = R_3 I_3(t) = R_3 C_3 \frac{dV_{C3}(t)}{dt}$$

(6)의 식을 (3)에 대입해

$$(7) V_{C2}(t) = R_3 C_3 \frac{dV_{C3}(t)}{dt} + V_{C3}(t)$$

V_{C2} 에서의 전류

$$(8) I_2(t) - I_3(t) = C_2 \frac{dV_{C2}(t)}{dt}, I_3(t) = I_2(t) - C_2 \frac{dV_{C2}(t)}{dt}$$

(3)에 (6)을 대입하고 미분한다.

$$V_{C2}(t) = R_3 C_3 \frac{dV_{C3}(t)}{dt} + V_{C3}(t)$$

$$(9) \frac{dV_{C2}(t)}{dt} = R_3 C_3 \frac{d^2 V_{C3}(t)}{dt^2} + \frac{dV_{C3}(t)}{dt}$$

(5)에 (9)를 대입

$$I_2(t) = C_2 \frac{dV_{C2}(t)}{dt} = C_2 R_3 C_3 \frac{d^2 V_{C3}(t)}{d^2 t} + C_2 \frac{dV_{C3}(t)}{dt}$$

(8)과 (6)에따라

$$I_2(t) = C_3 \frac{dV_{C3}(t)}{dt} + C_2 \frac{dV_{C2}(t)}{dt}$$

위 식에 (9)를 대입하여

$$\begin{aligned} I_2(t) &= C_3 \frac{dV_{C3}(t)}{dt} + C_2 \left(R_3 C_3 \frac{d^2 V_{C3}(t)}{d^2 t} + \frac{dV_{C3}(t)}{dt} \right) = \\ &C_2 R_3 C_3 \frac{d^2 V_{C3}(t)}{d^2 t} + (C_2 + C_3) \frac{dV_{C3}(t)}{dt} \end{aligned}$$

옴의 법칙으로

$$V_{R2}(t) = R_2 I_2(t) = R_2 (C_2 R_3 C_3 \frac{d^2 V_{C3}(t)}{d^2 t} + (C_2 + C_3) \frac{dV_{C3}(t)}{dt})$$

(2)에 대입해

$$V_{C1}(t) = \left(R_2 C_2 R_3 C_3 \frac{d^2 V_{C3}(t)}{d^2 t} + R_2 (C_2 + C_3) \frac{dV_{C3}(t)}{dt} \right) + R_3 C_3 \frac{dV_{C3}(t)}{dt} + V_{C3}(t)$$

(5)의 식을 (2)에 적용해

$$V_{C1}(t) = R_2 C_2 \frac{dV_{C2}(t)}{dt} + V_{C2}(t)$$

b(V_{C1})에서의 전류

$$(10) I_1(t) - I_2(t) = C_1 \frac{dV_{C1}(t)}{dt}, I_2(t) = I_1(t) - C_1 \frac{dV_{C1}(t)}{dt}$$

(2)에 (5)를 대입하고 미분한다.

$$V_{C1}(t) = R_2 C_2 \frac{dV_{C2}(t)}{dt} + V_{C2}(t)$$

$$(11) \frac{dV_{C1}(t)}{dt} = R_2 C_2 \frac{d^2 V_{C2}(t)}{d^2 t} + \frac{dV_{C2}(t)}{dt}$$

(4)에 (11)을 대입

$$I_1(t) = C_1 \frac{dV_{C_1}(t)}{dt} = C_1 R_2 C_2 \frac{d^2 V_{C_2}(t)}{d^2 t} + C_1 \frac{dV_{C_2}(t)}{dt}$$

(11)과 (5)에 따라

$$I_1(t) = C_2 \frac{dV_{C_2}(t)}{dt} + C_1 \frac{dV_{C_1}(t)}{dt}$$

위 식에 (11)을 대입해

$$\begin{aligned} I_1(t) &= C_2 \frac{dV_{C_2}(t)}{dt} + C_1 \left(R_2 C_2 \frac{d^2 V_{C_2}(t)}{d^2 t} + \frac{dV_{C_2}(t)}{dt} \right) = \\ &C_1 R_2 C_2 \frac{d^2 V_{C_2}(t)}{d^2 t} + (C_1 + C_2) \frac{dV_{C_2}(t)}{dt} \end{aligned}$$

옴의 법칙으로

$$V_{R1}(t) = R_1 I_1(t) = R_1 (C_1 R_2 C_2 \frac{d^2 V_{C_2}(t)}{d^2 t} + (C_1 + C_2) \frac{dV_{C_2}(t)}{dt})$$

(9)를 한 번 더 미분해

$$\frac{d^2 V_{C_2}(t)}{d^2 t} = R_3 C_3 \frac{d^3 V_{C_3}(t)}{d^3 t} + \frac{d^2 V_{C_3}(t)}{d^2 t}$$

$$\begin{aligned} V_{R1}(t) &= R_1 C_1 R_2 C_2 \left(R_3 C_3 \frac{d^3 V_{C_3}(t)}{d^3 t} + \frac{d^2 V_{C_3}(t)}{d^2 t} \right) + R_1 (C_1 + \\ &C_2) (R_3 C_3 \frac{d^2 V_{C_3}(t)}{d^2 t} + \frac{dV_{C_3}(t)}{dt}) \end{aligned}$$

(1)에 모아 정리하면

$$\begin{aligned} V_s(t) &= R_1 C_1 R_2 C_2 \left(R_3 C_3 \frac{d^3 V_{C_3}(t)}{d^3 t} + \frac{d^2 V_{C_3}(t)}{d^2 t} \right) + R_1 (C_1 + \\ &C_2) (R_3 C_3 \frac{d^2 V_{C_3}(t)}{d^2 t} + \frac{dV_{C_3}(t)}{dt}) + \left(R_2 C_2 R_3 C_3 \frac{d^2 V_{C_3}(t)}{d^2 t} + R_2 (C_2 + \\ &C_3) \frac{dV_{C_3}(t)}{dt} \right) + R_3 C_3 \frac{dV_{C_3}(t)}{dt} + V_{C_3}(t) \\ &= R_1 C_1 R_2 C_2 R_3 C_3 \frac{d^3 V_{C_3}(t)}{d^3 t} + (R_1 C_1 R_2 C_2 + R_1 C_1 R_3 C_3 + R_1 C_2 R_3 C_3 + \\ &R_2 C_2 R_3 C_3) \frac{d^2 V_{C_3}(t)}{d^2 t} + (R_1 C_1 + R_1 C_2 + R_2 C_2 + R_2 C_3 + R_3 C_3) \frac{dV_{C_3}(t)}{dt} + V_{C_3}(t) \end{aligned}$$

$F_{VS}(s) = L(V_s(t))$, $F_{VC2}(s) = L(V_{C3}(t))$ 라 두면

$$F_{VS}(s) = [R_1 C_1 R_2 C_2 R_3 C_3 s^3 + (R_1 C_1 R_2 C_2 + R_1 C_1 R_3 C_3 + R_1 C_2 R_3 C_3 + R_2 C_2 R_3 C_3)s^2 + (R_1 C_1 + R_1 C_2 + R_2 C_2 + R_2 C_3 + R_3 C_3)s + 1] F_{VC2}(s)$$

$$F_H(s) = \frac{1}{\alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0}$$

$$G_H(f) = F_H(i2\pi f) = \frac{1}{\alpha_0 - 4\alpha_2 \pi^2 f^2 + i(\alpha_1 2\pi f - 8\alpha_3 \pi^3 f^3)}$$

$$|G_H(f)| = \sqrt{\frac{1}{\alpha_0 - 4\alpha_2 \pi^2 f^2 + i(2\alpha_1 \pi f - 8\alpha_3 \pi^3 f^3)}} \sqrt{\frac{1}{\alpha_0 - 4\alpha_2 \pi^2 f^2 - i(2\alpha_1 \pi f - 8\alpha_3 \pi^3 f^3)}} = \\ \sqrt{\frac{1}{64\alpha_3^2 \pi^6 f^6 + (16\alpha_2^2 - 32\alpha_1 \alpha_3) \pi^4 f^4 + (4\alpha_1^2 - 8\alpha_2 \alpha_0) \pi^2 f^2 + \alpha_0}}$$

$$\alpha_3 = R_1 C_1 R_2 C_2 R_3 C_3$$

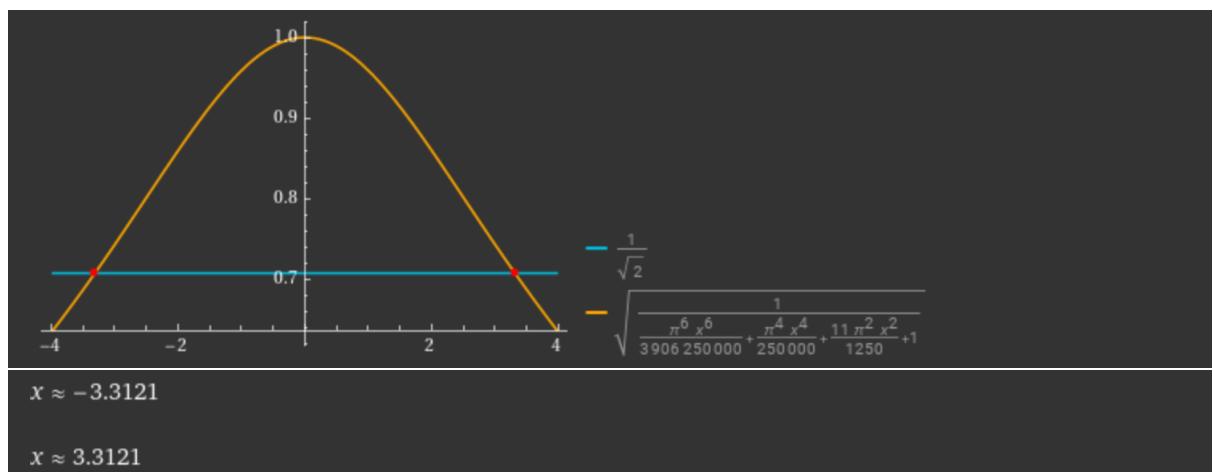
$$\alpha_2 = R_1 C_1 R_2 C_2 + R_1 C_1 R_3 C_3 + R_1 C_2 R_3 C_3 + R_2 C_2 R_3 C_3$$

$$\alpha_1 = R_1 C_1 + R_1 C_2 + R_2 C_2 + R_2 C_3 + R_3 C_3$$

$$\alpha_0 = 1$$

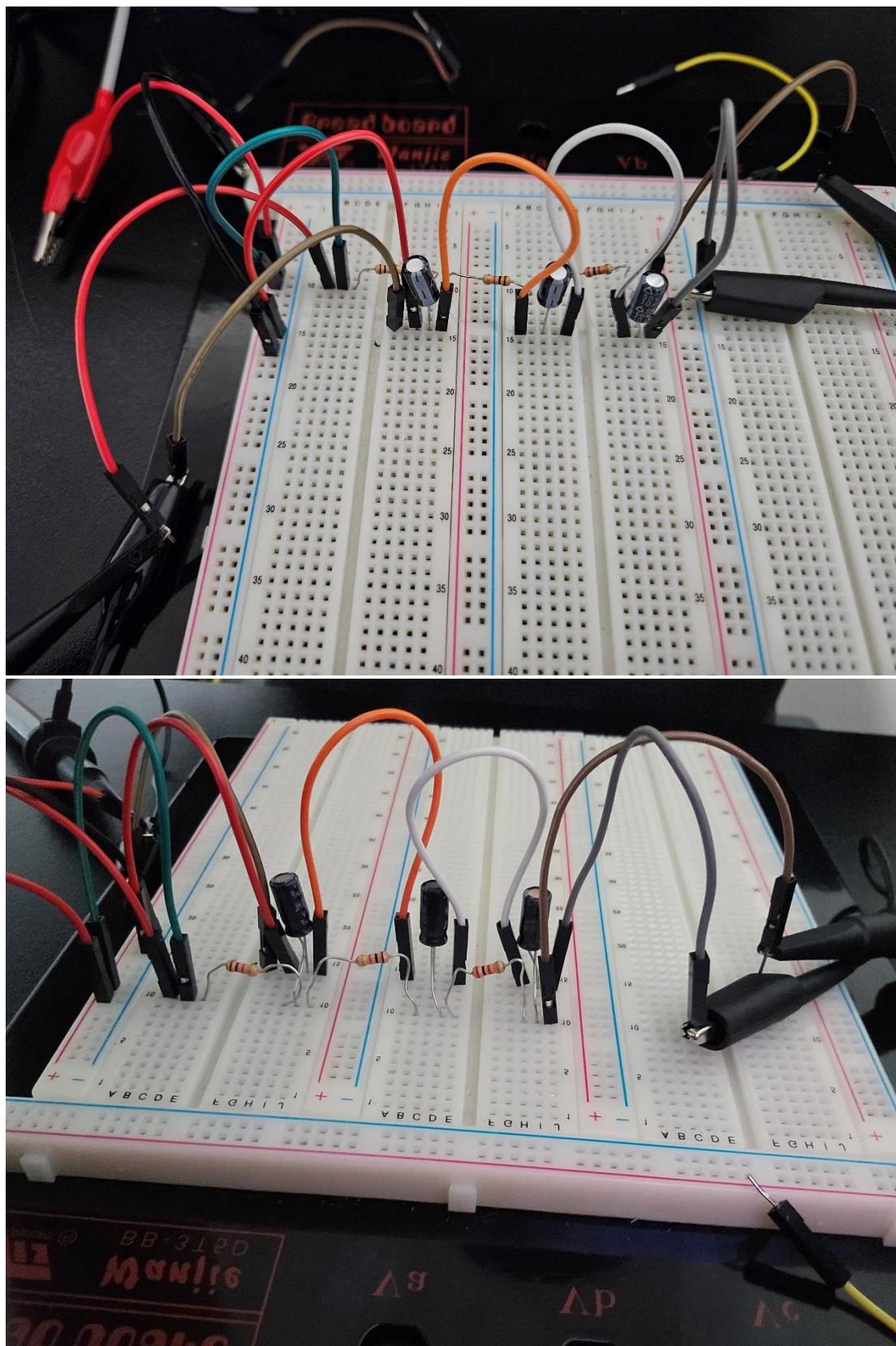
3. Calculation of cut-off frequency

$$\alpha_3 = 2 \times 10^{-6}, \alpha_2 = 700 \times 10^{-6}, \alpha_1 = 60 \times 10^{-3}, \alpha_0 = 1$$



컷오프 주파수	3.3121
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4. Construction of 3rd order RC low-pass filter



5. Measurement of frequency response of low-pass filter

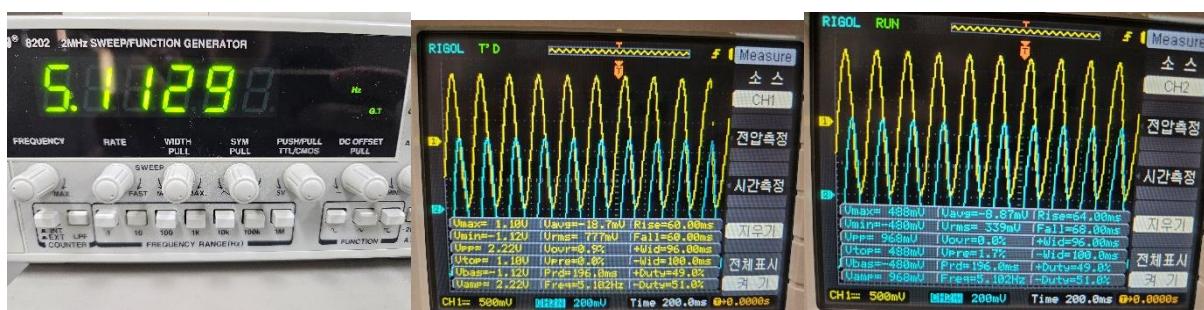
	전압을 관찰할 주파수의 수
m	4

	주파수(Hz)
f_1	1.1597
f_2	2.4735
f_3	4.1432
f_4	5.1129

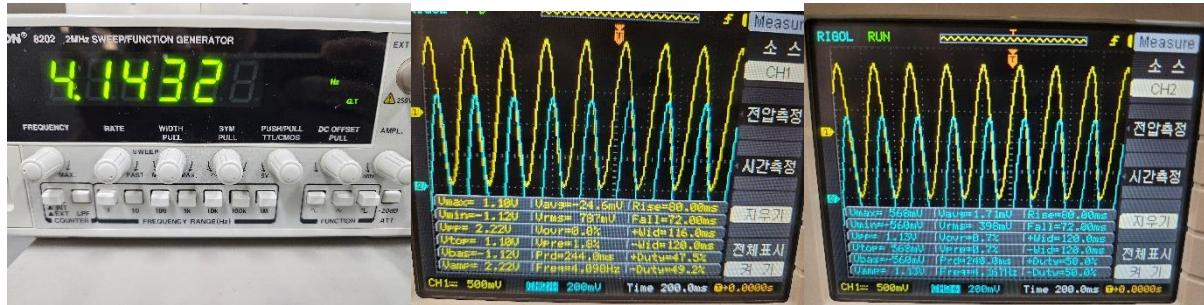
6. Measurement of cut-off frequency

	주파수(Hz)	input Vmax	output Vmax	$\frac{V_{out}(\omega)}{V_{in}(\omega)}$
f_1	5.1129	1.10V	488mV	0.443636
f_2	4.1432	1.10V	568mV	0.516363
f_c	3.3198	1.10V	656mV	0.596363
f_3	2.4735	1.10V	736mV	0.669090
f_4	1.1597	1.10V	696mV	0.632727

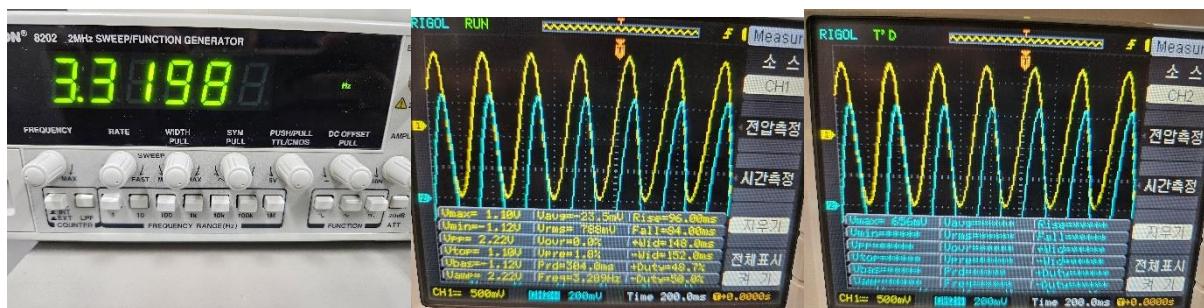
f_1



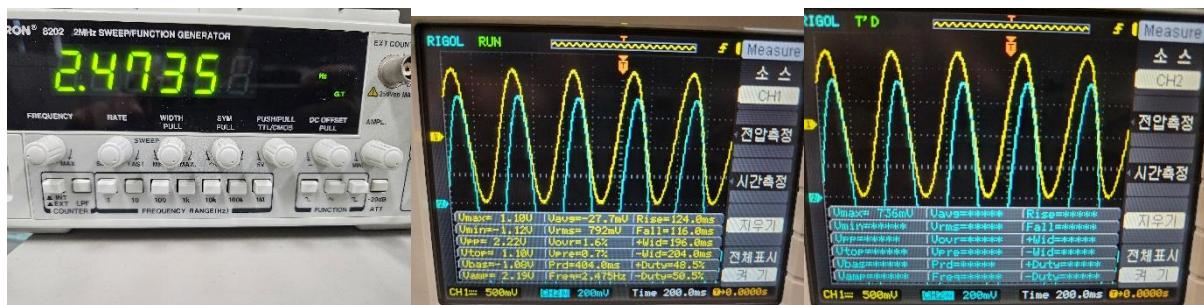
f_2



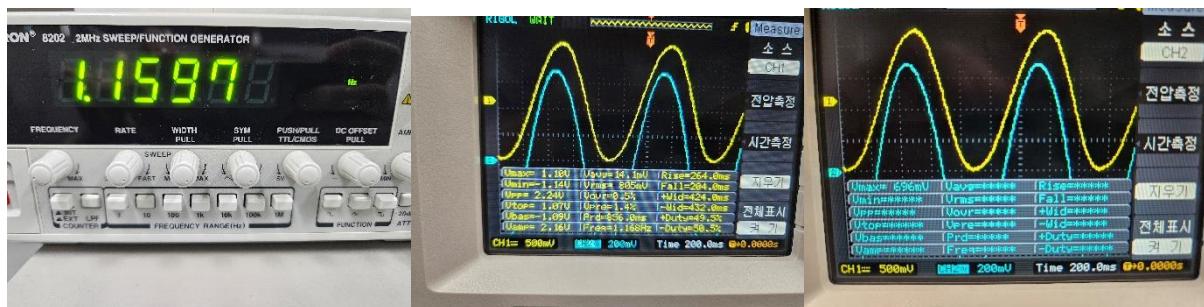
f_c



f_3

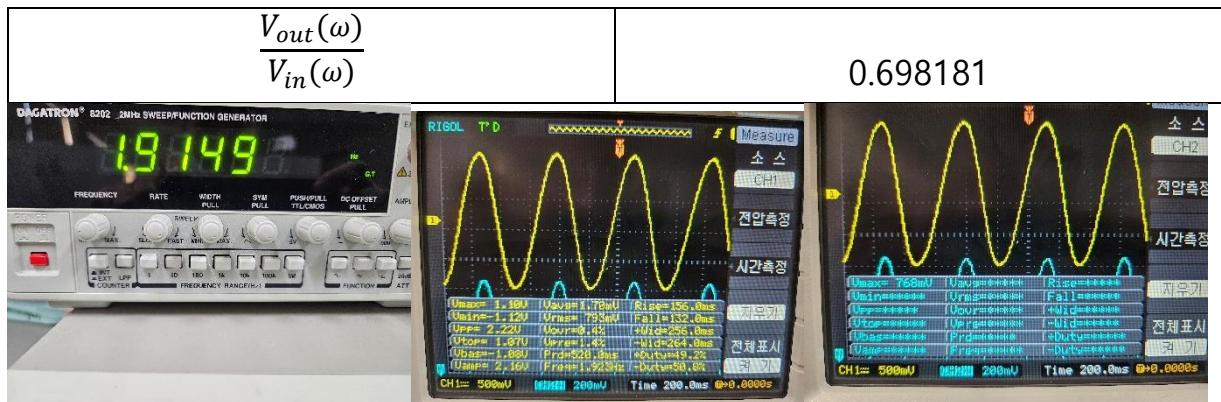


f_4



관측한 컷 오프 주파수

1.9149



8. Analysis of difference between theoretically calculated cut-off frequency and measured cut-off frequency

이론적인 분석에서는 이상적인 저항과 커패시터를 사용한다. 하지만 실제로 필터에 사용되는 부품은 주변 환경에도 영향을 받고 완벽하게 정확하지 않아 값의 차이가 발생한다. 마찬가지로 측정기와 전압 발생기 또한 현실에서 이상적인 출력을 내지 못하므로 차이가 발생한다.

또한 이상적인 필터는 사각 필스 신호 모양이다. 이의 임펄스 응답은 sinc함수로 정의역이 무한해 현실에서 구현하는 것이 불가능하다 따라서 유한 임펄스 응답 근사를 사용함에 있어 오차를 발생 할 수 있다.