MATRICES - [TYPE II]

Q. (22)

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$\Delta = 45$

BY CH EQ,

$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$

(1) → $\lambda = 5, -3, -3$

AM OF 5 ⇒ 1
AM OF −3 ⇒ 2

FOR $\lambda = -3$, $|A - \lambda I| X = 0$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 - 2R_1$
$R_3 + R_1$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{AM = GM}$

$$x_1 + 2x_2 - 3x_3 = 0$$

LET $x_2 = s, \quad x_3 = t$

$$x_1 + 2s - 3t = 0$$

$$x_1 = -2s + 3t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2s + 3t \\ s \\ t \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ +1 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} - \text{(I)}$$

FOR $\lambda = 5, \quad |A - \lambda I| Y = 0$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{y_1}{-24} = \frac{-y_2}{48} = \frac{y_3}{24}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} +1 \\ +2 \\ -1 \end{bmatrix} - \text{(II)}$$

$\rightarrow$ $D = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ $M = \begin{bmatrix} -2 & 3 & +1 \\ +1 & 0 & +2 \\ 0 & 1 & -1 \end{bmatrix}$

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Q. $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$

2

$\Delta = 10$

BY CH. EQ. $|A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 8\lambda^2 + \left[\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} + \begin{vmatrix} 4 & -2 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} 4 & 2 \\ -5 & 3 \end{vmatrix} \right] \lambda - 10 = 0$$

$$\lambda^3 - 8\lambda^2 + [-5 + 0 + 22]\lambda - 10 = 0$$

$$\lambda^3 - 8\lambda^2 + 17\lambda - 10 = 0$$

$$\boxed{\lambda = 5, 2, 1}$$

EIGN VALUES ARE DISTINCT
IT IS DIAGNALIZABLE

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FOR $\lambda = 5$, $|A - \lambda I| X = 0$

$$\begin{bmatrix} -1 & 2 & -2 \\ -5 & -2 & 2 \\ -2 & 4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-1\,x_1 + 2\,x_2 - 2\,x_3 = 0$$
$$-5\,x_1 - 2\,x_2 + 2\,x_3 = 0$$
$$-2\,x_1 + 4\,x_2 - 4\,x_3 = 0$$

$$\frac{x_3}{\begin{vmatrix} -1 & 2 \\ -5 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -1 & -2 \\ -5 & 2 \end{vmatrix}} = \frac{x_1}{\begin{vmatrix} 2 & -2 \\ -2 & 2 \end{vmatrix}}$$

$$\frac{x_3}{12} = \frac{+x_2}{+12} = \frac{x_1}{0}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

FOR $\lambda = 2$, $|A - \lambda I| Y = 0$

$$\begin{bmatrix} 2 & 2 & -2 \\ -5 & 1 & 2 \\ -2 & 4 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$2y_1 + 2y_2 - 2y_3 = 0$$
$$-5y_1 + 1y_2 + 2y_3 = 0$$
$$-2y_1 + 4y_2 - 1y_3 = 0$$

$$\frac{y_1}{\begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix}} = \frac{-y_2}{\begin{vmatrix} 2 & -2 \\ -5 & 2 \end{vmatrix}} = \frac{y_3}{\begin{vmatrix} 2 & 2 \\ -5 & 1 \end{vmatrix}}$$

$$\frac{y_1}{6} = \frac{+y_2}{+6} = \frac{y_3}{12}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \quad \text{— (11)}$$

FOR $\lambda = 1$, $|A - \lambda I|\, Z = 0$

$$\begin{bmatrix} 3 & 2 & -2 \\ -5 & 2 & 2 \\ -2 & 4 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3z_1 + 2z_2 - 2z_3 = 0$$
$$-5z_1 + 2z_2 + 2z_3 = 0$$
$$-2z_1 + 4z_2 + 0.z_3 = 0$$

$$\frac{z_1}{\begin{vmatrix} 2 & -2 \\ 2 & 2 \end{vmatrix}} = \frac{-z_2}{\begin{vmatrix} 3 & -2 \\ -5 & 2 \end{vmatrix}} = \frac{z_3}{\begin{vmatrix} 3 & 2 \\ -5 & 2 \end{vmatrix}}$$

$$\frac{z_1}{8} = \frac{+z_2}{+4} = \frac{z_3}{16}$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} \quad \text{— (III)}$$

→ THE MATRIX $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$

IS DIAGONALIZABLE ~~BY~~ TO

$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

BY TRANSFORM $M^{-1}AM = D$

WHERE $M = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

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Q. (4)

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

$\Delta = 0$

BY CH EQ,

$|A - \lambda I| = 0$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 18\lambda^2 + \left[\begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}\right]\lambda - 0 = 0$$

$$\lambda^3 - 18\lambda^2 + [5 + 20\ 20]\lambda - 0 = 0$$

$$\lambda^3 - 18\lambda^2 + 45\lambda - 0 = 0$$

$$\boxed{\lambda = 15, 3, 0}$$

EIGN VALUES ARE DISTINCT
THUS IT IS DIAGONALIZABLE

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FOR $\lambda = 15$, $|A - \lambda I| X = 0$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{40} = -\frac{x_2}{40} = \frac{x_3}{20}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \quad - \text{I}$$

FOR $\lambda = 3$, $|A - \lambda I| Y = 0$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{y_1}{16} = \frac{-y_2}{+8} = \frac{y_3}{-16}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} \quad - \text{II}$$

FOR $\lambda = 0$, $|A - \lambda I| \, Z = 0$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{z_1}{10} = \frac{+z_2}{+20} = \frac{z_3}{20}$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \quad - \quad (11)$$

→ THE MATRIX $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

IS DIAGONALIZABLE TO

$$D = \begin{bmatrix} 15 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

BY THE TRANSFORM

$M^{-1}AM = D$ WHERE $M = \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

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Q. 10 $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$

$\Delta = 3$

BY CH. EQ, $|A - \lambda I| = 0$

$$\begin{vmatrix} -9-\lambda & 4 & 4 \\ -8 & 3-\lambda & 4 \\ -16 & 8 & 7-\lambda \end{vmatrix} = 0$$

$\lambda^3 - \lambda^2 + [-11 + 1 + 5]\lambda - 3 = 0$

$\lambda^3 - \lambda^2 - 5\lambda - 3 = 0$

$\boxed{\lambda = 3, -1, -1}$

AM OF 3 $\Rightarrow$ 1

AM OF -1 $\Rightarrow$ 2

FOR $\lambda = -1$, $|A - \lambda I| X = 0$

$$\begin{bmatrix} -8 & 4 & 4 \\ -8 & 4 & 4 \\ -16 & 8 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$8R_2 - 8R_1$
$8R_3 - 16R_1$

$$\begin{bmatrix} -8 & 4 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R = 1$
$N = 3$
$U = GM = R - N = 3 - 1 = 2$

$\boxed{Am = Gm}$ IT IS DIAGONALIZABLE

$-8x_1 + 4x_2 + 4x_3 = 0$
LET $x_2 = s, \; x_3 = t$

$$-8x_1 + 4s + 4t = 0$$
$$-8x_1 = -4s - 4t$$
$$8x_1 = 4s + 4t$$
$$2x_1 = s + t$$
$$x_1 = \frac{s+t}{2}$$
$$x_1 = \frac{s}{2} + \frac{t}{2}$$

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$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} s/2 + t/2 \\ s \\ t \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} s/2 \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} t/2 \\ 0 \\ t \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} - \text{(I)}$$

FOR $\lambda = 3$, $|A - \lambda I|\, Y = 0$

$$\begin{bmatrix} -12 & 4 & 4 \\ -8 & 0 & 4 \\ -16 & 8 & 4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{y_1}{16} = \frac{-y_2}{-16} = \frac{y_3}{32}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} - \text{(II)}$$