Q1.	**Choose the correct option for following questions. All the Questions are compulsory and carry equal marks**
1.	The Laplace transform of $\int_0^t \frac{1-e^{au}}{u}du$ is
Option A:	$\frac{1}{s}\log\left(\frac{s-a}{s}\right)$
Option B:	$\frac{1}{s}\log\left(\frac{s-a}{s}\right)$
Option C:	$\frac{1}{s}\log\left(\frac{s-a}{s}\right)$
Option D:	$\frac{1}{s}\log\left(\frac{s-a}{s}\right)$
2	If $f(x)=\sqrt{(1-\cos x)}, 0<x<2\pi$ then find a_0,
Option A:	$\frac{2\sqrt{2}}{\pi}$
Option B:	$\frac{\sqrt{2}}{\pi}$
Option C:	$\frac{\sqrt{2}}{3\pi}$
Option D:	$\frac{1}{\pi}$
3.	If $f(z)=u+iv$ is analytic then
Option A:	u is harmonic but v may or may not be harmonic.
Option B:	v is harmonic but u may or may not be harmonic.
Option C:	u and v both need not be harmonic.
Option D:	u and v both harmonic.
4.	If Var(X) = 4 then Var(3x+5) is
Option A:	12
Option B:	20
Option C:	26
Option D:	36
5.	If $f(x)$ is an even function in the interval $(-l,l)$ then in the Fourier series expansion of $f(x)$
Option A:	$a_n=0, b_n=0$.
Option B:	$a_n=0, a_0=0$.
Option C:	$b_n=0$.
Option D:	$a_0=0, b_n=0$.
6	If $b_{yx}=0.7764$, $b_{xy}=1.2321$ then coefficient of correlation
Option A:	0.9781
Option B:	0.6291
Option C:	1.2307
Option D:	0.0023
7	Find the constants a, b, c, d if $f(z)=x^2+2axy+2by^2+i(2cx^2+dxy+y^2)$
Option A:	$a=1, b=-\frac{1}{2}, c=-\frac{1}{2}, d=2$.
Option B:	$a=0, b=-\frac{1}{2}, c=-\frac{1}{2}, d=2$.
Option C:	$a=1, b=-2, c=-\frac{1}{2}, d=1$
Option D:	$a=3, b=-\frac{1}{2}, c=-\frac{1}{2}, d=2$.
8	If X_1 has mean 4 and variance 9 and If X_2 has mean -2 and variance 4 and they are independent then $Var(2X_1+X_2-3)$ is
Option A:	41
Option B:	40
Option C:	36
Option D:	37
9	Suppose two fair dice are thrown and sum of the numbers on dice is noted, what is the probability that the sum can be equal to 6, 7, 8 or 9.
Option A:	2/9
Option B:	5/9
Option C:	4/9
Option D:	7/9
10.	Let X denotes the demand in quintals and Y denotes the price in rupees per kg. Also if $\overline{X}=68$, $\overline{Y}=69$, $\sum(X-\overline{X})^2=36$, $\sum(Y-\overline{Y})^2=44$, $\sum(X-\overline{X})(Y-\overline{Y})=24$ then the Karl Pearson's coefficient (r) of correlation is
Option A:	0.4030
Option B:	0.5030
Option C:	0.7030
Option D:	0.6030

Q2	**Solve any Four out of Six 5 marks each**
A	If $L\{\sin\sqrt{t}\} = \frac{\sqrt{\pi}}{2s\sqrt{s}} \cdot e^{-1/(4s)}$, find $L\{\sin 2\sqrt{t}\}$
B	Find the inverse Laplace transform of $\frac{s+29}{(s+4)(s^2+9)}$
C	Find the Fourier series for $f(x)$ in $(0, 2\pi)$ where $f(x) = \begin{cases} x, & 0 < x \le \pi \\ 2\pi - x, & \pi \le x < 2\pi \end{cases}$
D	If $v = 3x^2y + 6xy - y^3$, show that v is harmonic function and find the corresponding analytic function.
E	Calculate the value of rank correlation coefficient from the following data regarding marks of 6 students in Statistics and Mathematics in a test: *Marks : Statistics* : 40, 42, 45, 35, 36, 39 *Marks : Mathematics* : 46, 43, 44, 39, 40, 43
F	Three factories A, B, C produces 30%, 50% and 20% of the total production of an item. Out of their production 80%, 50% and 10% are defective. An item is chosen at random and found to be defective. Find the probability that it was produced by the factory A.

A] → $L[\sin\sqrt{t}] = \frac{\sqrt{\pi}}{2s\sqrt{s}} e^{-1/4s}$ given

Now $L[\sin 2\sqrt{t}] = L[\sin\sqrt{4t}]$

$= \frac{1}{4} L[\sin\sqrt{t}]\Big|_{s \to s/4}$ — By the change of scale property

$= \frac{1}{4} \frac{\sqrt{\pi}}{2s\sqrt{s}} e^{-1/4s}\Big|_{s \to s/4}$

$= \frac{1}{4} \frac{\sqrt{\pi}}{2\frac{s}{4}\sqrt{s/4}} e^{-1/s}$

$= \frac{1}{2s} \frac{2\sqrt{\pi}}{\sqrt{s}} e^{-1/s}$

$= \frac{\sqrt{\pi}}{s\sqrt{s}} e^{-1/s}$

B] → $\frac{s+29}{(s+4)(s^2+9)} = \frac{s+4+25}{(s+4)(s^2+9)}$

$= \frac{1}{s^2+9} + \frac{25}{(s+4)(s^2+9)}$

$\therefore L^{-1}\left\{\frac{s+29}{(s+4)(s^2+9)}\right\} = L^{-1}\left\{\frac{1}{s^2+9}\right\} + 25L^{-1}\left\{\frac{1}{(s+4)(s^2+9)}\right\}$

$=$

$\frac{1}{(s+4)(s^2+9)} = \frac{A}{s+4} + \frac{Bs+C}{s^2+9}$

$\therefore 1 = A(s^2+9) + (Bs+C)(s^2+4)$

$= As^2 + 9A + Bs^3 + 4Bs + Cs^2 + 4C$

$1 = Bs^3 + (A+C)s^2 + 4Bs + (9A+4C)$

$\therefore B = 0,\ A + C = 0,\ 9A + 4C = 1$

$4A + 4C = 0$

$9A + 4C = 1$

$-5A = -1$

$\therefore \boxed{A = 1/5} \Rightarrow \boxed{C = -1/5}$

$\therefore L^{-1}\left\{\frac{s+29}{(s+4)(s^2+9)}\right\} = \frac{1}{3}\sin 3t + 25L^{-1}\left\{\frac{1}{5(s+4)} - \frac{1}{5(s^2+9)}\right\}$

$= \frac{1}{3}\sin 3t + 5\left[e^{-4t} - \frac{1}{3}\sin 3t\right]$

$= 5e^{-4t} - \frac{4}{3}\sin 3t$

c] $\rightarrow$ Let $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

here $a_0 = \frac{1}{2\pi}\left[\int_0^{\pi} x\,dx + \int_{\pi}^{2\pi}(2\pi - x)\,dx\right]$

$= \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_0^{\pi} + \frac{1}{2\pi}\left[2\pi x - \frac{x^2}{2}\right]_{\pi}^{2\pi}$

$= \frac{1}{2\pi}\cdot\frac{\pi^2}{2} + \frac{1}{2\pi}\left\{\left[4\pi^2 - \frac{4\pi^2}{2}\right] - \left[2\pi^2 - \frac{\pi^2}{2}\right]\right\}$

$= \frac{\pi}{4} + \frac{1}{2\pi}\left[2\pi^2 - \frac{3\pi^2}{2}\right] = \frac{\pi}{4} + \frac{1}{2\pi}\left[\frac{\pi^2}{2}\right]$

$= \frac{\pi}{4} + \frac{\pi}{4}$

$= \frac{\pi}{2}$

$a_n = \frac{1}{\pi}\int_0^{\pi} x\cos nx\,dx + \frac{1}{\pi}\int_{\pi}^{2\pi}(2\pi - x)\cos n\pi\,dx$

$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi} + \frac{1}{\pi}\left[(2\pi - x)\left(\frac{\sin nx}{n}\right) - (-1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{2\pi}$

$= \frac{-2}{\pi\cdot n^2}\left[1 - (-1)^n\right]$

$b_n = \frac{1}{\pi}\int_0^{\pi} x\sin nx\,dx + \frac{1}{\pi}\int_{\pi}^{2\pi}(2\pi - x)\sin n\pi\,dx$

$= \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi} + \frac{1}{\pi}\left[(2\pi - x)\left(-\frac{\cos nx}{n}\right) - (-1)\left(-\frac{\sin nx}{n^2}\right)\right]_{\pi}^{2\pi}$

$= 0$

$\therefore f(x) = \frac{\pi}{2} - \sum_{n=1}^{\infty}\frac{2}{\pi n^2}\left[1 - (-1)^n\right]\cos nx$

$\Rightarrow$ d] $v = 3x^2y + 6xy - y^3$

(I) We have

$$\frac{\partial v}{\partial x} = 6xy + 6y \qquad \therefore \frac{\partial^2 v}{\partial x^2} = 6y$$

$$\frac{\partial v}{\partial y} = 3x^2 + 6x - 3y^2 \qquad \therefore \frac{\partial^2 v}{\partial y^2} = -6xy$$

$$\therefore \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0 \qquad \therefore v \text{ is harmonic}$$

(II)

Now $\psi_1(x,y) = 6xy + 6y$ $\qquad$ $\psi_2(x,y) = 3x^2 + 6x - 3y^2$

$\therefore \psi_1(z,0) = 0$ $\qquad$ $\therefore \psi_2(z,0) = 3z^2 + 6z$

By Milne - Thompson method

$$f(z) = v_y + i v_x$$

$$= 3z^2 + 6z$$

$$\therefore f(z) = z^3 - 3z^2 + c$$

E]

X	Y	r_x	r_y	$d = r_x - r_y$	d^2
40	46	4	6	-2	4
42	43	5	3.5	1.5	2.25
45	44	6	5	1	1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Three factories A, B, C produces 30% , 50% and 20% of the total production of an item. Out of their production 80%, 50% and 10% are defective. An item is chosen at random and found to be defective. Find the probability that it was produced by the factory A.

$\Sigma = 6.50$

Series	repeated rank	no of times repeted $\frac{m(m^2-1)}{12}$	Correction factor
Y	3.5	2	$\frac{2(4-1)}{12} = \frac{1}{2}$

$$r_{xy} = 1 - 6\,\frac{\Sigma d^2 + \Sigma \text{ correction factor}}{n(n^2-1)}$$

$$= 1 - 6\left(\frac{6.50 + \frac{1}{2}}{6(6^2-1)}\right)$$

$$= 1 - \frac{7}{35} = \frac{4}{5} = 0.8$$

f]

A	B	C	
30%	50%	20%	production
80%	50%	10%	defective

Let D : product selected is defective

A : product produced by A

B : Product produced by B

C : Product produced by C

Given $P(A) = 0.3$, $P(B) = 0.5$, $P(C) = 0.2$

& $P(D/A) = 0.8$, $P(D/B) = 0.5$, $P(D/C) = 0.1$

$P(A/D) = ?$

$$\therefore P(D) = P(D/A)\cdot P(A) + P(D/B)\cdot P(B) + P(D/C)\,P(C)$$

$$= 0.8 \times 0.3 + 0.5 \times 0.5 + 0.1 \times 0.2$$

$$= 0.24 + 0.25 + 0.02$$

$$= 0.51$$

$\therefore$ By Bayes thm

$$P(A/D) = \frac{P(D/A)\cdot P(A)}{P(D)} = \frac{0.8 \times 0.3}{0.51}$$

$$= \frac{0.24}{0.51}$$

$$= 0.4705$$

Q3	Solve any Four out of Six	5 marks each
A	By using Laplace transform, prove that $\int_0^\infty e^{-t}\cdot\frac{\sin^2 t}{t}dt = \frac{1}{4}\log 5$	
B	Using convolution theorem, find the inverse Laplace transform of $\frac{1}{(s-2)^4(s+3)}$	
C	Obtain Fourier series for $f(x)=x+x^2;\ -1<x<1$	
D	Find an analytic function $f(z)=u+iv$, where $u+v=e^x(\cos y+\sin y)$	
E	State true or false with justification. "If two lines of regression are $x+3y-5=0$ and $4x+3y-8=0$, then the correlation coefficient is $+0.5$".	
F	If the mean of the following distribution is 16. Find m,n and variance. X : 8, 12, 16, 20, 24 $P(X)$: 1/8 m n 1/4 1/12	

A]

$$\int_0^\infty e^{-t}\frac{\sin^2 t}{t}dt = L\left[\frac{\sin^2 t}{t}\right]_{s=1}$$

Now $L[\sin^2 t] = L\left[\frac{1-\cos 2t}{2}\right] = \frac{1}{2s} - \frac{s}{2(s^2+4)}$

$$\therefore L\left[\frac{\sin^2 t}{t}\right] = \int_s^\infty \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4}\right]ds \quad \text{--- By } L\left[\frac{f(t)}{t}\right] = \int_s^\infty \phi(s)\,ds$$

$$= \frac{1}{2}\left[\log s - \frac{1}{2}\log(s^2+4)\right]$$

$$= \frac{1}{2}\log\frac{s}{(s^2+4)^{1/2}}$$

$$\therefore \int_0^\infty e^{-t}\frac{\sin^2 t}{t}dt = \frac{1}{2}\log\left(\frac{1}{(1^2+4)^{1/2}}\right)$$

$$= \frac{1}{2}\log\frac{1}{\sqrt{5}}$$

$$= -\frac{1}{4}\log 5$$

B] let $\phi_1(s) = \frac{1}{s+3}$, $\phi_2(s) = \frac{1}{(s-2)^4}$

$\therefore L^{-1}[\phi_1(s)] = e^{-3t} = f(t)$, $L^{-1}[\phi_2(s)] = e^{2t}L^{-1}\left[\frac{1}{s^4}\right] = e^{2t}\frac{t^3}{6} = g(t)$

$\therefore$ By the convolution thm

$$L^{-1}\left[\frac{1}{(s-2)^4(s+3)}\right] = \int_0^t f(u)\,g(t-u)\,du$$

$$= \int_0^t e^{-3u}\cdot e^{2(t-u)}\frac{(t-u)^3}{6}du$$

$$= \frac{e^{2t}}{6}\int_0^t e^{-5u}(t-u)^3du$$

$$= \frac{e^{2t}}{6}\left[(t-u)^3\left(\frac{e^{-5u}}{-5}\right) - \frac{3(t-u)^2}{-1}\left(\frac{e^{-5u}}{25}\right) + \frac{6(t-u)}{+1}\left(\frac{e^{-5u}}{-125}\right) - 6(-1)\left(\frac{e^{-5u}}{625}\right)\right]_0^t$$

$$= \frac{e^{2t}}{6}\left[\frac{e^{-5t}}{625} - \left\{-\frac{t^3}{30} + \frac{t^2}{50} - \frac{t}{125} + \frac{1}{625}\right\}\right]$$

c] $f(x) = f_1(x) + f_2(x)$

where $f_1(x) = x$ is an odd function

$f_2(x) = x^2$ is an even function

here $l = 1$

For $f_1(x)$, $a_n = 0$ and

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx = \frac{2}{1}\int_0^1 x\sin n\pi x\,dx$$

$$= 2\left[x\left(\frac{-\cos n\pi x}{n\pi}\right) - (1)\left(\frac{-\sin n\pi x}{n^2\pi^2}\right)\right]_0^1$$

$$= 2\left[-\frac{1}{n\pi}(-1)^n\right] = -\frac{2}{n\pi}(-1)^n$$

Further $f_2(x) = x^2$ which is even, $b_n = 0$

$$a_0 = \frac{1}{l}\int_0^l f(x)\,dx = \frac{1}{1}\int_0^1 x^2\,dx = \frac{1}{3}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx = \frac{2}{1}\int_0^1 x^2\cos n\pi x\,dx$$

$$= 2\left[x^2\left(\frac{\sin n\pi x}{n\pi}\right) - (2x)\left(\frac{-\cos n\pi x}{n^2\pi^2}\right) + (2)\left(\frac{-\sin n\pi x}{n^3\pi^3}\right)\right]_0^1$$

$$= 2\left\{\left[0 + 2\frac{(-1)^n}{n^2\pi^2} + 0\right] - \left[0 - 0 - 0\right]\right\}$$

$$= \frac{4(-1)^n}{n^2\pi^2}$$

$\therefore$ $f(x) = f_1(x) + f_2(x)$

$$= \frac{1}{3} + \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos n\pi x - \frac{2}{\pi}\sum\frac{(-1)^n}{n}\sin n\pi x$$

d) We have $f(z) = u + iv$ $\quad \therefore if(z) = iu - v$

$\therefore (1+i) f(z) = (u-v) + i(u+v)$

$= U + iV$ say

$\therefore \frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(u+v) = e^x(\cos y + \sin y) = \psi_1(x,y)$

$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(u-v) = e^x(-\sin y + \cos y) = \psi_2(x,y)$

By Milne-Thompson's method

$(1+i) f'(z) = U_x + iV_x$

$= V_y + iV_x$

$= \psi_2(z,0) + i\psi_1(z,0)$

$= e^z + i e^z$

$= (1+i) e^z$

$\therefore f'(z) = e^z$

$\therefore f(z) = \int e^z + c$

$\boxed{f(z) = e^z + c}$

e) Regression line

x on y	y on x
$x + 3y - 5 = 0$	$4x + 3y - 8 = 0$
$\therefore x = -3y + 5$	$y = \frac{8}{3} - \frac{4x}{3}$
$\therefore b_{xy} = -3$	$\therefore b_{yx} = -4/3$

$\therefore r^2 = b_{xy} \cdot b_{yx} = 4$

$\therefore r = -2$ $\quad \because$ both b_{xy} & b_{yx} are negative

not possible since $-1 \le r \le 1$

Hence let's have

Regression line

y on x	x on y
$4x + 3y - 8 = 0$	$x + 3y - 5 = 0$
$\therefore x = -\frac{3y}{4} + 2$	$y = -\frac{x}{3} + \frac{5}{3}$
$\therefore b_{xy} = -3/4$	$b_{yx} = -1/3$

$\therefore r^2 = b_{xy} \cdot b_{yx} = 1/4 = 0.25$

$\therefore \boxed{r = -0.5}$ $\quad \because$ b_{xy} & b_{yx} both are negative

F If the mean of the following distribution is 16. Find m, n and variance.

X	8,	12,	16,	20,	24
$P(X)$	1/8	m	n	1/4	1/12

$P(x)$ is pdf $\quad \therefore \sum P(x) = 1$

$\therefore \frac{1}{8} + m + n + \frac{1}{4} + \frac{1}{12} = 1$

$\therefore \frac{3+6+2}{24} + m + n = 1$

$\therefore m + n = 1 - \frac{11}{24} \Rightarrow \boxed{m + n = \frac{13}{24}}$

Also given mean 16

$\therefore \sum x P(x) = 16$

$\therefore 8 \cdot \frac{1}{8} + 12 \cdot m + 16 \cdot n + 20 \cdot \frac{1}{4} + 24 \cdot \frac{1}{12} = 16$

$\therefore 1 + 12m + 16n + 5 + 2 = 16$

$\therefore 12m + 16n = 8 \Rightarrow \boxed{3m + 4n = 2}$

$\therefore 4m + 4n = 13/6$

$-\ 3m \pm 4n = -2$

$\boxed{m = 1/6}$ & $n = \frac{13}{24} - \frac{1}{6} = \frac{9}{24}$ $\quad \therefore \boxed{n = \frac{3}{8}}$

Now

Variance $= E(x^2) - [E(x)]^2$

$E(x) = 16$ — given $\quad$ 32

$E(x^2) = \sum x^2 P(x)$

$= 64 \cdot \frac{1}{8} + 144 \cdot \frac{1}{6} + 256 \cdot \frac{3}{8} + 400 \cdot \frac{1}{4} + 576 \cdot \frac{1}{12}$

$= 8 + 24 + 64 + 100 + 48$

$= 244$

$\therefore$ Variance $= 244 - (16)^2$

$= 244 - 169$

$= 75$

Q4	Solve any Four out of Six 5 marks each
A	Find the Laplace transform of $e^{-4t}\int_0^t u\,\sin 3u\,du$
B	Find the inverse Laplace transform of $\tan^{-1}\left(\frac{a}{s}\right)$
C	Obtain half- range sine series for $f(x)$ where $f(x)=\begin{cases} x, & 0<x<(\pi/2) \\ \pi-x, & (\pi/2)<x<\pi \end{cases}$
D	Find the orthogonal trajectory of the family of curves given by $2x-x^3+3xy^2=a$
E	Fit a straight line to the following data. $(x,y)=(-1,-5),(1,1),(2,4),(3,7),(4,10)$ Estimate y when $x=7$
F	A random variable X has the following probability density function $f(x)=\begin{cases} ke^{-kx}, & x>0, k>0 \\ 0, & elsewhere \end{cases}$ Find the moment generating function and hence, the mean and variance.

B] $L^{-1}\left[\tan^{-1}\frac{a}{s}\right] = \frac{-1}{t} L^{-1}\left[\frac{d}{ds}\tan^{-1}\frac{a}{s}\right]$

$= \frac{-1}{t} L^{-1}\left[\frac{1}{1+(a/s)^2}\left(-\frac{a}{s^2}\right)\right]$

$= -\frac{1}{t} L^{-1}\left[\frac{-a}{s^2+a^2}\right]$

$= \frac{a}{t}\,\frac{1}{a}\sin at$

$= \frac{\sin at}{t}$

A] $L[\sin 3t] = \frac{3}{s^2+9}$

$\therefore L[t\sin 3t] = (-1)\frac{d}{ds}\frac{3}{s^2+9} = -3\left[\frac{-1}{(s^2+9)^2}(2s)\right]$

$= \frac{6s}{(s^2+9)^2}$

$\therefore L\left[\int_0^t u\sin 3u\,du\right] = \frac{1}{s}\frac{6s}{(s^2+9)^2} = \frac{6}{(s^2+9)^2}$

$\therefore L\left[e^{-4t}\int_0^t u\sin 3u\,du\right] = \frac{6}{((s+4)^2+9)^2}$

$= \frac{6}{s^2+8s+25}$ //

c] $f(x) = \begin{cases} x & 0<x<\pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$

Half range sin series of f(x) is

$$f(x) = \sum b_n \sin \frac{n\pi x}{l}$$

where $b_n = \frac{2}{l}\int_0^l f(x) \sin \frac{n\pi x}{l}\, dx$

here $\boxed{l = \pi}$

$$\therefore b_n = \frac{2}{\pi}\left[\int_0^{\pi/2} x \cdot \sin nx \, dx + \int_{\pi/2}^{\pi} (\pi - x) \sin nx \, dx\right]$$

$$= \frac{2}{\pi}\left[\left\{x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right\}_0^{\pi/2} + \left\{(\pi-x)\left(-\frac{\cos nx}{n}\right) - (-1)\left(-\frac{\sin nx}{n^2}\right)\right\}_{\pi/2}^{\pi}\right]$$

$$= \frac{2}{\pi}\left[-\frac{\pi}{2}\left(\frac{\cos n\pi/2}{n}\right) + \left(\frac{\sin n\pi/2}{n^2}\right) - (0+0) + \right.$$

$$\left. 0 - 0 - \left(-\frac{\pi}{2}\frac{\cos n\pi/2}{n} - \frac{\sin n\pi/2}{n^2}\right)\right]$$

$$= \frac{4}{\pi}\frac{\sin n\pi/2}{n^2}$$

$\therefore$ The half range sine series is

$$f(x) = \frac{4}{\pi}\sum \frac{\sin n\pi/2}{n^2} \sin nx$$

D] let $u = 2x - x^3 + 3xy^2$

$\phi_1 = u_x = 2 - 3x^2 + 3y^2$, $\phi_2 = u_y = 6xy$

$\therefore f'(z) = u_x + iv_x$

$= u_x - iu_y$

$= \phi_1(z,0) - i\phi_2(z,0)$ by milne thompson's method

$= (2 - 3z^2) - i(0)$

$= 2 - 3z^2$

$\therefore f(z) = 2z - z^3 + c$

$= 2(x+iy) - (x+iy)^3 + c$

$= 2x + i2y - (x^3 + 3ix^2y - 3xy^2 - iy^3) + c$

$= (2x - x^3 - 3xy^2) + i(2y + 3x^2y - y^3) + c$

$\therefore$ Orthogonal trajectory is $2y + 3x^2y - y^3 = 0$

E	Fit a straight line to the following data. $(x,y) = (-1,-5),(1,1),(2,4),(3,7),(4,10)$ Estimate y when $x = 7$

$\Rightarrow$ Let the equation of the line be $y = a + bx$

$\therefore$ Normal Equations are

$$\Sigma y = an + b\Sigma x$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

	x	y	x^2	xy
	-1	-5	1	5
	1	1	1	1
	2	4	4	8
	3	7	9	21
	4	10	16	40
Σ	9	17	31	75

$\therefore$ Normal equations are

$$17 = 5a + 9b$$

$$75 = 9a + 31b$$

$$\Rightarrow a = -2, \; b = 3$$

$\therefore$ The regression equation is

$$y = -2 + 3x$$

at $x = 7$, $\boxed{y = 19}$

F	A random variable X has the following probability density function $f(x) = \begin{cases} ke^{-kx}, & x > 0, k > 0 \\ 0, & elsewhere \end{cases}$ Find the moment generating function and hence, the mean and variance.

$\Rightarrow M_X(t) = E(e^{tx})$

$$= \int e^{tx} \cdot f(x)\, dx$$

$$= \int e^{tx} \cdot ke^{-kx}\, dx$$

$$= k \int e^{(t-k)x}\, dx$$

$$= k \frac{e^{(t-k)x}}{(t-k)} = \frac{k}{(t-k)} e^{tx} \cdot e^{-kx}$$

$$\text{mean} = \mu_1' = \frac{d}{dt} M_X(t) \Big|_{t=0}$$

$$\text{Variance} = \mu_2 = \mu_2' - (\mu_1')^2$$

$E(e^{tx})$

$\int e^{tx} \cdot ke^{-kx}\, dx$

$k\, e^{(t-k)x}$

$$\frac{k\, e^{(t-k)x}}{(t-k)}$$