MATRIX - [TYPE 1]

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Q. (18) $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$

$\Delta = 4$

BY CH. Eq, $|A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & 6 & 6 \\ 1 & 3-\lambda & 2 \\ -1 & -5 & -2-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 5\lambda^2 + \left[\begin{vmatrix} 3 & 2 \\ -5 & -2 \end{vmatrix} + \begin{vmatrix} 4 & 6 \\ -1 & -2 \end{vmatrix} + \begin{vmatrix} 4 & 6 \\ 1 & 3 \end{vmatrix}\right]\lambda - 4 = 0$$

$$\lambda^3 - 5\lambda^2 + [4 - 2 + 6]\lambda - 4 = 0$$

$$\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$$

$$\boxed{\lambda = 1, 2, 2}$$

$\lambda = 2$, $|A - \lambda I| X = 0$

$$\begin{bmatrix} 2 & 6 & 6 \\ 1 & 1 & 2 \\ -1 & -5 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$2x_1 + 6x_2 + 6x_3 = 0$$
$$1x_1 + 1x_2 + 2x_3 = 0$$
$$-1x_1 - 5x_2 - 4x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 6 & 6 \\ 1 & 2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 2 & 6 \\ 1 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 2 & 6 \\ 1 & 1 \end{vmatrix}}$$

$$\frac{x_1}{6} = \frac{-x_2}{+2} = \frac{x_3}{-4}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} \quad \text{— (1)}$$

$\lambda = 1$, $|A - \lambda I| Y = 0$

$$\begin{bmatrix} 3 & 6 & 6 \\ 1 & 2 & 2 \\ -1 & -5 & -3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3 \cdot R_2 - 1 \cdot R_1$
$3 R_3 + 1 \cdot R_1$

$$\begin{bmatrix} 3 & 6 & 6 \\ 0 & 0 & 0 \\ 0 & -9 & -3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$3 \cdot y_1 + 6 \cdot y_2 + 6 \cdot y_3 = 0$$
$$0 \cdot y_1 + 0 \cdot y_2 + 0 \cdot y_3 = 0$$
$$0 \cdot y_1 - 9 \cdot y_2 - 3 \cdot y_3 = 0$$

$R = 2$
$N = 3$
$U = 1$

LET $y_2 = t$

$\therefore$
$$-9t - 3 \cdot y_3 = 0$$
$$-9t = 3y_3$$
$$-3t = y_3$$

$$3y_1 + 6t + 6 \cdot -3t = 0$$
$$3y_1 + 6t - 18t = 0$$
$$3y_1 - 12t = 0$$
$$y_1 = 4t$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4t \\ t \\ -3t \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ -3 \end{bmatrix} \quad \text{— (11)}$$

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Q. $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

14

$\Delta = 12$

BY CH, EQ $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & 10 & 5 \\ -2 & -3-\lambda & 4 \\ 3 & 5 & 7-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 7\lambda^2 + \left[\begin{vmatrix} -3 & -4 \\ 5 & 7 \end{vmatrix} + \begin{vmatrix} 3 & 5 \\ 3 & 7 \end{vmatrix} + \begin{vmatrix} 3 & 10 \\ -2 & -3 \end{vmatrix}\right]$$

$$\lambda - 12 = 0$$

$$\lambda^3 - 7\lambda^2 + [-1 + 6 + 11]\lambda - 12 = 0$$

$$\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$$

$$\boxed{\lambda = 3, 2, 2}$$

$\lambda = 3$, $|A - \lambda I| X = 0$

$$\begin{bmatrix} 3-\lambda & 10 & 5 \\ -2 & -3-+3 & -4 \\ 3 & 5 & 7-3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\begin{bmatrix} 0 & 10 & 5 \\ -2 & -6 & -4 \\ 3 & 5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0.x_1 + 10x_2 + 5x_3 = 0$$
$$-2x_1 - 6x_2 - 4x_3 = 0$$
$$3x_1 + 5x_2 + 4x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 10 & 5 \\ -6 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 5 \\ -2 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 10 \\ -2 & -6 \end{vmatrix}}$$

$$\frac{x_1}{-10} = \frac{-x_2}{10} = \frac{x_3}{20}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \quad \text{—} \quad (1)$$

$\lambda = 2$, $|A - \lambda I| Y = 0$

$$\begin{bmatrix} 3-2 & 10 & 5 \\ -2 & -3-2 & -4 \\ 3 & 5 & 7-2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 10 & 5 \\ -2 & -5 & -4 \\ 3 & 5 & 5 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$+1\,y_1 \quad +10\,y_2 \quad +5\,y_3 = 0$$
$$-2\,y_1 \quad -5\,y_2 \quad -4\,y_3 = 0$$
$$+3\,y_1 \quad +5\,y_2 \quad +5\,y_3 = 0$$

$$\frac{y_1}{\begin{vmatrix} 10 & 5 \\ -5 & -4 \end{vmatrix}} = \frac{-y_2}{\begin{vmatrix} 1 & 5 \\ -2 & -4 \end{vmatrix}} = \frac{y_3}{\begin{vmatrix} 1 & 10 \\ -2 & -5 \end{vmatrix}}$$

$$\frac{y_1}{-15} = \frac{-y_2}{6} = \frac{y_3}{15}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -5 \end{bmatrix} \quad \text{— (11)}$$

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Q. $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

22

$\Delta = 45$

BY CH. Eq, $|A - \lambda I| = 0$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{vmatrix} = 0$$

$$\lambda^3 + 1\lambda^2 + \left[\begin{vmatrix} 1 & -6 \\ -2 & 0 \end{vmatrix} + \begin{vmatrix} -2 & -3 \\ -1 & 0 \end{vmatrix} + \begin{vmatrix} -2 & 2 \\ 2 & 1 \end{vmatrix}\right] \lambda - 45 = 0$$

$$\lambda^3 + \lambda^2 + [-12 - 3 - 6]\lambda - 45 = 0$$

$$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$$

$$\boxed{\lambda = 5, -3, -3}$$

$\lambda = 5$, $|A - \lambda I| X = 0$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$-7x_1 + 2x_2 - 3x_3 = 0$$
$$2x_1 - 4x_2 - 6x_3 = 0$$
$$-1x_1 - 2x_2 - 5x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 2 & -3 \\ -4 & -6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -7 & -3 \\ 2 & -6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -7 & 2 \\ 2 & -4 \end{vmatrix}}$$

$$\frac{x_1}{-24} = \frac{-x_2}{48} = \frac{x_3}{24}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \longrightarrow (1)$$

$\lambda = -3$, $|A - \lambda I| Y = 0$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & +3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$1y_1 + 2y_2 - 3y_3 = 0$$
$$2y_1 + 4y_2 - 6y_3 = 0$$
$$-1y_1 - 2y_2 + 3y_3 = 0$$

$$\frac{y_1}{\begin{vmatrix} 2 & -3 \\ 4 & -6 \end{vmatrix}} = \frac{-y_2}{\begin{vmatrix} 1 & -3 \\ 2 & -6 \end{vmatrix}} = \frac{y_3}{\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}}$$

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$$\frac{y_1}{0} = \frac{-y_2}{0} = \frac{y_3}{0}$$

[CRAMER RULE FAILED]

$1 \cdot R_2 - 2 \cdot R_1$
$1 \cdot R_3 + 1 \cdot R_1$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

HERE,
$R = 1$
$N = 3$
$U = N - R = 3 - 1 = 2$

LET $y_2 = s$, $y_3 = t$

$y_1 + 2s - 3t = 0$
$y_1 = -2s + 3t$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -2s+3t \\ s \\ t \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -2s \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} 3t \\ 0 \\ t \end{bmatrix}$$

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$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} \quad \text{--- (11)}$$

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Q. $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

(5)

$\Delta = 36$

BY CH Eq. $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 11\lambda^2 + \left[\begin{vmatrix} 5 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ -1 & 5 \end{vmatrix}\right]\lambda - 36 = 0$$

$$\lambda^3 - 11\lambda^2 + [14 + 8 + 14]\lambda - 36 = 0$$

$$\lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0$$

$$\boxed{\lambda = 6, 3, 2}$$

$\lambda = 6$, $|A - \lambda I| X = 0$

$$\begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$-3x_1 - x_2 + x_3 = 0$$
$$-x_1 - x_2 - 1x_3 = 0$$
$$x_1 - x_2 - 3x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} -1 & 1 \\ -1 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -3 & 1 \\ -1 & -1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -3 & -1 \\ -1 & -1 \end{vmatrix}}$$

$$\frac{x_1}{2} = \frac{-x_2}{4} = \frac{x_3}{2}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \quad \text{—} \quad (1)$$

$\lambda = 3$, $|A - \lambda I| Y = 0$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0y_1 - 1y_2 + 1y_3 = 0$$
$$-1y_1 + 2y_2 - 1y_3 = 0$$
$$1y_1 - 1y_2 + 0y_3 = 0$$

$$\frac{y_1}{\begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix}} = \frac{-y_2}{\begin{vmatrix} 0 & 1 \\ -1 & -1 \end{vmatrix}} = \frac{y_3}{\begin{vmatrix} 0 & -1 \\ -1 & 2 \end{vmatrix}}$$

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$$\frac{y_1}{-1} = \frac{-y_2}{1} = \frac{y_3}{-1}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ +1 \\ 1 \end{bmatrix} \quad \text{— (II)}$$

$\lambda = 2, \ |A - \lambda I| Z = 0$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$1 Z_1 - 1 Z_2 + 1 Z_3 = 0$
$-1 Z_1 + 3 Z_2 - 1 Z_3 = 0$
$1 Z_1 - 1 Z_2 + 1 Z_3 = 0$

$$\frac{Z_1}{\begin{vmatrix} -1 & 1 \\ 3 & -1 \end{vmatrix}} = \frac{-Z_2}{\begin{vmatrix} 1 & +1 \\ -1 & -1 \end{vmatrix}} = \frac{Z_3}{\begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix}}$$

$$\frac{Z_1}{-2} = \frac{-Z_2}{0} = \frac{Z_3}{2}$$

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \text{— (III)}$$

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Q. $A = \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$

$\Delta = 216$

BY CH Eq. $|A - \lambda I| = 0$

$$\begin{vmatrix} -2-\lambda & 5 & 4 \\ 5 & 7-\lambda & 5 \\ 4 & 5 & -2-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 3\lambda^2 + \left[\begin{vmatrix} 7 & 5 \\ 5 & -2 \end{vmatrix} + \begin{vmatrix} -2 & 4 \\ 4 & -2 \end{vmatrix} + \begin{vmatrix} -2 & 5 \\ 5 & 7 \end{vmatrix} \right] \lambda - 216 = 0$$

$$\lambda^3 - 3\lambda^2 + [-39 - 12 - 39]\lambda - 216 = 0$$

$$\lambda^3 - 3\lambda^2 - 90\lambda - 216 = 0$$

$$\boxed{\lambda = 12, -3, -6}$$

$\lambda = 12$, $|A - \lambda I| X = 0$

$$\begin{bmatrix} -14 & 5 & 4 \\ 5 & -5 & 5 \\ 4 & 5 & -14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$-14x_1 + 5x_2 + 4x_3 = 0$$
$$5x_1 - 5x_2 + 5x_3 = 0$$
$$4x_1 + 5x_3 - 14x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 5 & 4 \\ -5 & 5 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -14 & 4 \\ 5 & 5 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -14 & 5 \\ 5 & -5 \end{vmatrix}}$$

$$\frac{x_1}{45} = \frac{+x_2}{+90} = \frac{x_3}{45}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \longrightarrow (1)$$

$$\lambda = -3, \quad |A - \lambda I| Y = 0$$

$$\begin{bmatrix} 1 & 5 & 4 \\ 5 & 10 & 5 \\ 4 & 5 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$y_1 + 5y_2 + 4y_3 = 0$$
$$5y_1 + 10y_2 + 5y_3 = 0$$
$$4y_1 + 5y_2 + y_3 = 0$$

$$\frac{y_3}{\begin{vmatrix} 1 & 5 \\ 5 & 10 \end{vmatrix}} = \frac{-y_2}{\begin{vmatrix} 1 & 4 \\ 5 & 5 \end{vmatrix}} = \frac{y_1}{\begin{vmatrix} 5 & 4 \\ 10 & 5 \end{vmatrix}}$$

$$\frac{y_3}{-15} = \frac{+y_2}{+15} = \frac{y_1}{-15}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad \text{—} \quad \text{(II)}$$

$\lambda = -6, \ |A - \lambda I| Z = 0$

$$\begin{bmatrix} 4 & 5 & 4 \\ 5 & 13 & 5 \\ 4 & 5 & 4 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$4Z_1 + 5Z_2 + 4Z_3 = 0$$
$$5Z_1 + 13Z_2 + 5Z_3 = 0$$
$$4Z_1 + 5Z_2 + 4Z_3 = 0$$

$$\frac{Z_{111}}{\begin{vmatrix} 4 & 5 \\ 5 & 13 \end{vmatrix}} = \frac{-Z_2}{\begin{vmatrix} 4 & 4 \\ 5 & 5 \end{vmatrix}} = \frac{Z_{21}}{\begin{vmatrix} 5 & 4 \\ 13 & 5 \end{vmatrix}}$$

$$\frac{Z_3}{27} = \frac{-Z_2}{0} = \frac{Z_1}{-27}$$

$$\begin{bmatrix} Z_1 \\ Z_{11} \\ Z_{111} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \text{—} \quad \text{(III)}$$