Finite State machine

Finite State machine (FSM)

Defination - FSM consists of finite set of states(S) which alter on receiving the I/P set (I) to produce the output set (O)

FSm divides the two functions

State function STF - $S \times I \rightarrow S$

machine function MAF - $S \times I \rightarrow O$

✓(P) Design a FSM to check whether the given decimal number is divisible by 3

Solution Step I Theory

(defination of FSm)

Step II Logic

$I = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$O = \{y, n\}$ $y \rightarrow f$ $n \rightarrow nf$

$S = \{q_s, q_0, q_1, q_2\}$

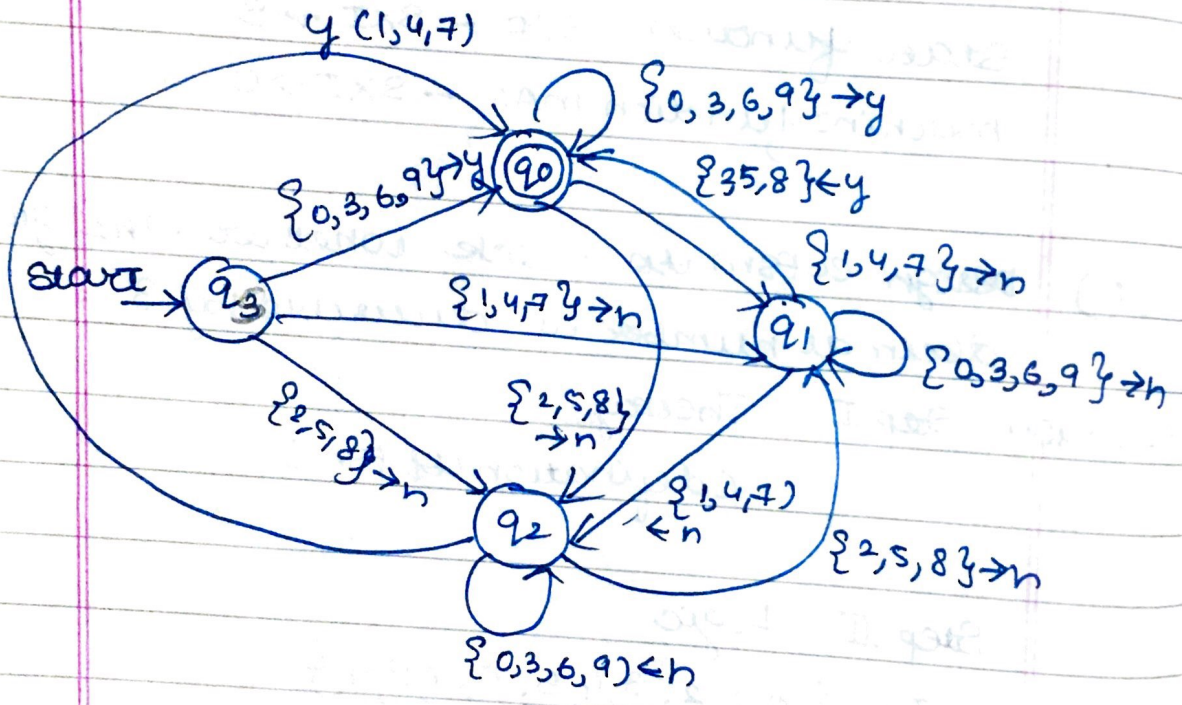
Step III Implementation

S \ I	{0,3,6,9}	{1,4,7}	{2,5,8}
→ q_s	q_0	q_1	q_2
q_0*	q_0	q_1	q_2
q_1	q_1	q_2	q_0
q_2	q_2	q_0	q_1

STF - $S \times I \rightarrow S$

S \ I	{0,3,6,9}	{1,4,7}	{2,5,8}
→qs	y	n	n
q0*	y	n	n
q1	n	n	y
q2	n	y	n

MAF - SXI → O



Step 4

⊢ → transition symbol

① (qs, 5248)
⊢ (q2, 248)
⊢ (q1, 48)
⊢ (q2, 8)
⊢ (q1) → n

② (qs, 312)
⊢ (q0, 12)
⊢ (q1, 2)
⊢ (q0) → y

(P) Design a FSM to check whether the given decimal number is divisible by 4

Solution $I = \{0,1,2,3,4,5,6,7,8,9\}$

$O = \{y, n\}$ $y \rightarrow$ yes $n \rightarrow$ no

$S = \{q_3, q_0, q_1, q_2, q_3\}$

S \ I	{0,4,8}	{1,5,9}	{2,6}	{3,7}
→ q_3	q_0	q_1	q_2	q_3
{0,4,8} q_0*	q_0	q_1	q_2	q_3
{1,5,9} q_1	q_2	q_3	q_0	q_1
{2,6} q_2	q_0	q_1	q_2	q_3
{3,7} q_3	q_2	q_3	q_0	q_1

S \ I	{0,4,8}	{1,5,9}	{2,6}	{3,7}
→ q_3	y	n	n	n
q_0*	y	n	n	n
q_1	n	n	y	n
q_2	y	n	n	n
q_3	n	n	y	n

$(q_3, 7328)$

$\vdash (q_3, 328)$

$\vdash (q_1, 28)$

$\vdash (q_0, 8)$

$\vdash (q_0) \rightarrow y$

③ Design FSM to check whether the given binary number is divisible by 4

Solution

$I = \{0, 1\}$

$O = \{y, n\}$

$S = \{q_s, q_0, q_1, q_2, q_3\}$

↑ $(2R+0) \bmod 4$

S \ I	0	1 → $(2R+1) \bmod 4$		
R →	→ q_s	q_0	q_1	0/4	1/4
0	* q_0	q_0	q_1	1000/4	1001/4
1	q_1	q_2	q_3	1010/4	1011/4
2	q_2	q_0	q_1	1100/4	1101/4
3	q_3	q_2	q_3		

$(q_s, 10110000)$

$\vdash (q_1, 0110000)$

$\vdash (q_2, 110000)$

$\vdash (q_1, 10000)$

$\vdash (q_3, 0000)$

$\vdash (q_2, 000)$

$\vdash (q_0, 00)$

$\vdash (q_0, 0)$

$q_0 \rightarrow y$

(7) Design FSM to check whether the given ternary number is divisible by 5

$I = \{0, 1, 2\}$

$O = \{y, n\}$

$S = \{q_s, q_0, q_1, q_2, q_3, q_4\}$

	S \ I	0 $(3R+0)\%5$	1 $(3R+1)\%5$	2 $(3R+2)\%5$
R →	q_s	q_0	q_1	q_2
0 *	q_0	q_0	q_1	q_2
1	q_1	q_3	q_4	q_0
2	q_2	q_1	q_2	q_3
3	q_3	q_4	q_0	q_1
4	q_4	q_2	q_3	q_4

$(q_s, 2102)$

$\vdash (q_2, 102)$

$\vdash (q_2, 02)$

$\vdash (q_1, 2)$

$\vdash (q_0) \rightarrow y$

(8) Design FSM in which the I/P is valid if it ends in "100" over $\Sigma = \{0, 1\}$ (alphabet)

$I = \{0, 1\}$

$y \rightarrow y$

$n \rightarrow n$

$O = \{y, n\}$

$S = \{q_s, q_0, q_1, q_2, q_3\}$

DATE : / /

S \ I	0	1
ends in → q_s	q_0	q_1
0 q_0	q_0	q_1
1 q_1	q_2	q_1
10 q_2	q_3	q_1
100 q_3*	q_0	q_1

0 1
00 01
10 11
100 101
1000 1001

$(q_s, 10100)$
$\vdash q_1 (0100)$
$\vdash q_2 (100)$
$\vdash q_3 (00)$
$\vdash q_2 (0)$
$\vdash (q_3) \rightarrow y$

⑥ if it ends in babb over $\Sigma = \{a, b\}$

solution

$I = \{a, b\}$

$O = \{n, y\}$

$S = \{q_s, q_1, q_2, q_3, q_4\}$

babb

S \ I	a	b
→ ends in q_s	q_0	q_1
a q_0	q_0	q_1
b q_1	q_2	q_1
ba q_2	q_0	q_3
bab q_3	q_2	q_4
babb* q_4	q_2	q_1

aa ab
ba bb
baa bab
baba
babb

(7) if it ends either in "101" or "110" over $\Sigma = \{0,1\}$

output?

S \ I	0	1
→q5	q0	q1
0 q0	q0	q1
1 q1	q2	q4
10 q2	q0	q3
101 q3*	q2	q4
11 q4	q5	q4
110 q5*	q0	q3

(8) if the second last symbol is 'a' over

$\Sigma = \{a,b\}$

$I = \{a,b\}$

$O = \{n,y\}$

$S = \{$

S \ I	a	b
→q5	q0	q1
a q0	q2	q3
b q1	q0	q1
aa q2*	q2	q3
ab q3*	q0	q1

"a a" OR "a b"

(9) if it contains "1011" over $\Sigma = \{0, 1\}$

Solution

	S\I	0	1
→	q_s	q_0	q_1
0	q_0	q_0	q_1
1	q_1	q_2	q_1
10	q_2	q_0	q_3
101	q_3	q_2	q_4
1011	q_4*	q_4	q_4

(10) if it contains atleast one occurrence of substring "bba" over $\Sigma = \{a, b\}$

	S\I	a	b
→	q_s	q_0	q_1
a	q_0	q_0	q_1
b	q_1	q_0	q_2
bb	q_2	q_3	q_2
* bba	q_3	q_3	q_3

= trap state

(Reverse logic)

(11) if it does not contain any occurence of three consecutive b's over $\Sigma = \{a, b\}$

S \ I	a	b
→ q_0*	q_0	q_1
a q_0*	q_0	q_1
b q_1*	q_0	q_2
bb q_2*	q_0	q_3
bbb	q_3	q_3	q_3	= dead state

Points to remember

(1) There is always one start state

(2) There can be one/many final states

(3) Start state can also be final
(in such cases Blank I/P is valid)

(4) Dead state is a trap state which is non final

(5) Every dead state is a trap state but a

(6) trap state may/may not be dead
start state ⟺ initial state
final state ⟺ accepting state
non final state ⟺ rejecting state

(7) Dead state is always a rejecting state

(12) if it starts with three consecutive a's over $\Sigma = \{a, b\}$

Solution

S \ I	a	b
A E → q_5	q_0	q_1 (q_3)
a aa q_0	q_1	q_3
aa a q_1	q_2	q_3
aaa − q_2^*	q_2	q_2
deadstate q_3	q_3	q_3

(13) if it starts either with "011" OR "100" over $\Sigma = \{0, 1\}$

Solution

S \ I	0	1
A E → q_5	q_0	q_3
0 11 q_0	q_6	q_1
01 1 q_1	q_6	q_2
011 − q_2^*	q_2	q_2
1 00 q_3	q_4	q_6
10 0 q_4	q_5	q_6
100 − q_5^*	q_5	q_5
dead state q_6	q_6	q_6

(14) if it contains odd number of b's over $\Sigma = \{a, b\}$

Solution

S \ I	a	b
$n_b(x) \rightarrow q_s$	q_0	q_1
even q_0	q_0	q_1
odd q_1	q_1	q_0

final state?

(15) if it contains even number of a's and odd number of b's over $\Sigma = \{a, b\}$

		S \ I	a	b
$n_a(x)$	$n_b(x)$	$\rightarrow q_s$	q_2	q_1
E	E	q_0	q_2	q_1
E	O	q_1*	q_3	q_0
O	E	q_2	q_0	q_3
O	O	q_3	q_1	q_2

(16) if it contains

(1) atleast 3a's

q_3 & q_4

(2) exactly 3a's

q_3

(3) ~~atmost~~ ~~atleast~~ 3a's

q_3, q_0, q_1, q_2 & q_3

over $\Sigma \{a, b\}$

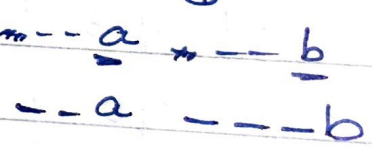
S / I	a	b
$n_a(x) \to q_5$	q_1	q_0
0 q_0	q_1	q_0
1 q_1	q_2	q_1
2 q_2	q_3	q_2
3 q_3	q_4	q_3
more than 3 q_4	q_4	q_4

(17) if it contains 'b' at every even position over $\Sigma = \{a, b\}$

8 states are made final?

Solution

S / I	a	b
$\|x\| \to q_5$*	q_1	q_1
even q_0*	q_1	q_1
odd q_1*	q_2	q_0
dead state q_2	q_2	q_2



(18) if it contains 'a' at every even position and 'b' at every odd position over $\Sigma = \{a, b\}$

	S / I	a	b
	q_5	q_2	q_1
*	even q_0	q_2	q_1
*	odd q_1	q_0	q_2
	dead q_2	q_2	q_2