Linear Programming Problem

Weight Distribution of Types

Comp/IT/AI

Type	Name	Nov 2017	May 2018	Nov 2018	May 2019	Nov 2019	May 2022	Nov 2022	May 2023	Dec 2023	May 2024	Dec 2024
I	Basic Solution	---	05	05	05	05	---	---	05	06	05	06
II	Simplex	06	08	08	06	08	10	08	08	---	08	---
III	Big M	08	---	---	---	---	---	---	---	08	---	08
IV	Dual Simplex	---	08	08	08	08	05	08	08	08	08	08
V	Duality	05	---	---	---	---	02	05	---	---	---	---
Total Marks		**19**	**21**	**21**	**19**	**21**	**17**	**21**	**21**	**22**	**21**	**22**

Type I: Basic solutions of L.P.P.

1. Determine all basic solutions to the following problem. Which of them are basic feasible, degenerate, infeasible basic and optimal basic feasible solutions?

Maximise $z = x_1 - 2x_2 + 4x_3$

subject to $x_1 + 2x_2 + 3x_3 = 7$

$3x_1 + 4x_2 + 6x_3 = 15$

$x_1, x_2, x_3 \geq 0$

[D23/D24/CompITAI/6M]

Solution:

No	Non-basic var = 0	Basic var	Equations & solutions	Is the solution feasible?	Is the solution degenerate?	Value of z	Is the solution optimal?
1	$x_3 = 0$	x_1, x_2	$x_1 + 2x_2 = 7$ $3x_1 + 4x_2 = 15$ $x_1 = 1, x_2 = 3$	Yes	No	-5	No
2	$x_2 = 0$	x_1, x_3	$x_1 + 3x_3 = 7$ $3x_1 + 6x_3 = 15$ $x_1 = 1, x_3 = 2$	Yes	No	9	Yes
3	$x_1 = 0$	x_2, x_3	$2x_2 + 3x_3 = 7$ $4x_2 + 6x_3 = 15$ Unbounded solution	-	-	-	-

2. Determine all basic solutions to the following problem. Which of them are basic feasible, degenerate, infeasible basic and optimal basic feasible solutions?

Maximise $z = x_1 + 3x_2 + 3x_3$

subject to $x_1 + 2x_2 + 3x_3 = 4$

$2x_1 + 3x_2 + 5x_3 = 7$

$x_1, x_2, x_3 \geq 0$

[N15/M23/CompIT/5M][M18/N19/Comp/5M][N18/MechCivil/5M]
[M24/CompITAI/5M]

Solution:

No	Non-basic var = 0	Basic var	Equations & solutions	Is the solution feasible?	Is the solution degenerate?	Value of z	Is the solution optimal?
1	$x_3 = 0$	x_1, x_2	$x_1 + 2x_2 = 4$ $2x_1 + 3x_2 = 7$ $x_1 = 2, x_2 = 1$	Yes	No	5	Yes
2	$x_2 = 0$	x_1, x_3	$x_1 + 3x_3 = 4$ $2x_1 + 5x_3 = 7$ $x_1 = 1, x_3 = 1$	Yes	No	4	No
3	$x_1 = 0$	x_2, x_3	$2x_2 + 3x_3 = 4$ $3x_2 + 5x_3 = 7$ $x_2 = -1, x_3 = 2$	No	No	3	No

3. Determine all basic solutions to the following problem. Which of them are basic feasible, degenerate, infeasible basic and optimal basic feasible solutions?

Maximise $z = 2x_1 - 2x_2 + 4x_3 - 5x_4$

subject to $x_1 + 4x_2 - 2x_3 + 8x_4 \le 2$

$-x_1 + 2x_2 + 3x_3 + 4x_4 \le 1$

$x_1, x_2, x_3, x_4 \ge 0$

[N19/MechCivil/5M]

Solution:

No	Non-basic var = 0	Basic var	Equations & Solutions	Is the solution feasible?	Is the solution degenerate?	Value of z	Is the solution optimal?
1	$x_3 = 0$ $x_4 = 0$	x_1, x_2	$x_1 + 4x_2 = 2$ $-x_1 + 2x_2 = 1$ $x_1 = 0, x_2 = 0.5$	Yes	Yes	-1	No
2	$x_2 = 0$ $x_4 = 0$	x_1, x_3	$x_1 - 2x_3 = 2$ $-x_1 + 3x_3 = 1$ $x_1 = 8, x_3 = 3$	Yes	No	28	Yes
3	$x_1 = 0$ $x_4 = 0$	x_2, x_3	$4x_2 - 2x_3 = 2$ $2x_2 + 3x_3 = 1$ $x_2 = 0.5, x_3 = 0$	Yes	Yes	-1	No
4	$x_2 = 0$ $x_3 = 0$	x_1, x_4	$x_1 + 8x_4 = 2$ $-x_1 + 4x_4 = 1$ $x_1 = 0, x_4 = \frac{1}{4}$	Yes	Yes	$-\frac{5}{4}$	No
5	$x_1 = 0$ $x_3 = 0$	x_2, x_4	$4x_2 + 8x_4 = 2$ $2x_2 + 4x_4 = 1$ Unbounded soln	-	-	-	-
6	$x_1 = 0$ $x_2 = 0$	x_3, x_4	$-2x_3 + 8x_4 = 2$ $3x_3 + 4x_4 = 1$ $x_3 = 0, x_4 = \frac{1}{4}$	Yes	No	$-\frac{5}{4}$	No

4. Find all basic solutions to the following problem

Maximise $z = x_1 + x_2 + 3x_3$

subject to $x_1 + 2x_2 + 3x_3 = 9$

$3x_1 + 2x_2 + 2x_3 = 15$

$x_1, x_2, x_3 \geq 0$

[N18/Comp/5M]

Solution:

No	Non-basic var = 0	Basic var	Equations & solutions	Is the solution feasible?	Is the solution degenerate?	Value of z
1	$x_3 = 0$	x_1, x_2	$x_1 + 2x_2 = 9$ $3x_1 + 2x_2 = 15$ $x_1 = 3, x_2 = 3$	Yes	No	6
2	$x_2 = 0$	x_1, x_3	$x_1 + 3x_3 = 9$ $3x_1 + 2x_3 = 15$ $x_1 = \frac{27}{7}, x_3 = \frac{12}{7}$	Yes	No	9
3	$x_1 = 0$	x_2, x_3	$2x_2 + 3x_3 = 9$ $2x_2 + 2x_3 = 15$ $x_2 = \frac{27}{2}, x_3 = -6$	No	No	-9/2

5. Find all basic feasible solutions of the following system of equations

$x_1 + 2x_2 + 4x_3 + x_4 = 7$

$2x_1 - x_2 + 3x_3 - 2x_4 = 4$

Solution:

No	Non-basic var = 0	Basic var	Equations & solutions	Is the solution feasible?
1	$x_3 = 0$ $x_4 = 0$	x_1, x_2	$x_1 + 2x_2 = 7$ $2x_1 - x_2 = 4$ $x_1 = 3, x_2 = 2$	Yes
2	$x_2 = 0$ $x_4 = 0$	x_1, x_3	$x_1 + 4x_3 = 7$ $2x_1 + 3x_3 = 4$ $x_1 = -1, x_3 = 2$	No
3	$x_1 = 0$ $x_4 = 0$	x_2, x_3	$2x_2 + 4x_3 = 7$ $-x_2 + 3x_3 = 4$ $x_2 = \frac{1}{2}, x_3 = \frac{3}{2}$	Yes
4	$x_2 = 0$ $x_3 = 0$	x_1, x_4	$x_1 + x_4 = 7$ $2x_1 - 2x_4 = 4$ $x_1 = \frac{9}{2}, x_4 = \frac{5}{2}$	Yes
5	$x_1 = 0$ $x_3 = 0$	x_2, x_4	$2x_2 + x_4 = 7$ $-x_2 - 2x_4 = 4$ $x_2 = 6, x_4 = -5$	No
6	$x_1 = 0$ $x_2 = 0$	x_3, x_4	$4x_3 + x_4 = 7$ $3x_3 - 2x_4 = 4$ $x_3 = \frac{18}{11}, x_4 = \frac{5}{11}$	Yes

6. Find all the basic feasible solutions to the following system of equations.
$2x_1 + 6x_2 + 2x_3 + x_4 = 3$
$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$
[M19/Comp/5M]
Solution:

No	Non-basic var = 0	Basic var	Equations & Solutions	Is the solution feasible?
1	$x_3 = 0$ $x_4 = 0$	x_1, x_2	$2x_1 + 6x_2 = 3$ $6x_1 + 4x_2 = 2$ $x_1 = 0, x_2 = 0.5$	Yes
2	$x_2 = 0$ $x_4 = 0$	x_1, x_3	$2x_1 + 2x_3 = 3$ $6x_1 + 4x_3 = 2$ $x_1 = -2, x_3 = 3.5$	No
3	$x_1 = 0$ $x_4 = 0$	x_2, x_3	$6x_2 + 2x_3 = 3$ $4x_2 + 4x_3 = 2$ $x_2 = 0.5, x_3 = 0$	Yes
4	$x_2 = 0$ $x_3 = 0$	x_1, x_4	$2x_1 + x_4 = 3$ $6x_1 + 6x_4 = 2$ $x_1 = \frac{8}{3}, x_4 = -\frac{7}{3}$	No
5	$x_1 = 0$ $x_3 = 0$	x_2, x_4	$6x_2 + x_4 = 3$ $4x_2 + 6x_4 = 2$ $x_2 = 0.5, x_4 = 0$	Yes
6	$x_1 = 0$ $x_2 = 0$	x_3, x_4	$2x_3 + x_4 = 3$ $4x_3 + 6x_4 = 2$ $x_3 = 2, x_4 = -1$	No

Type II: Simplex Method

1. Solve by using Simplex method.

   Maximise $z = 3x_1 + 2x_2$

   subject to $x_1 + x_2 \leq 4$

   $x_1 - x_2 \leq 2$

   $x_1, x_2 \geq 0$

   [M17/CompIT/6M]

   Solution:

   The standard form,

   Max $z - 3x_1 - 2x_2 + 0s_1 + 0s_2 = 0$

   s.t. $x_1 + x_2 + s_1 + 0s_2 = 4$

   $x_1 - x_2 + 0s_1 + s_2 = 2$

   $x_1, x_2, s_1, s_2 \geq 0$

   Simplex table,

Iteration No.	Basic Var	Coefficient of				RHS	Ratio	Formula
		x_1	x_2	s_1	s_2			
0	z	**-3**	-2	0	0	0	-	$X + 3Y$
s_2 leaves x_1 enters	s_1	**1**	1	1	0	4	$\frac{4}{1} = 4$	$X - Y$
	s_2	**1**	**-1**	**0**	**1**	**2**	$\frac{2}{1} = 2$	-
1	z	0	**-5**	0	3	6	-	$X + \frac{5}{2}Y$
s_1 leaves x_2 enters	s_1	**0**	**2**	**1**	**-1**	**2**	$\frac{2}{2} = 1$	$\frac{Y}{2}$
	x_1	1	**-1**	0	1	2	-	$X + \frac{1}{2}Y$
2	z	0	0	$\frac{5}{2}$	$\frac{1}{2}$	11		
	x_2	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	1		
	x_1	1	0	$\frac{1}{2}$	$\frac{1}{2}$	3		

   Thus, the solution is

   $x_1 = 3, x_2 = 1, z_{max} = 11$

2. Solve by using Simplex method.

Maximise $z = x_1 + 4x_2$

subject to $2x_1 + x_2 \leq 3$

$3x_1 + 5x_2 \leq 9$

$x_1 + 3x_2 \leq 5$

$x_1, x_2 \geq 0$

[N14/CompIT/6M]

Solution:

The standard form,

Max $z - x_1 - 4x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $2x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 3$

$3x_1 + 5x_2 + 0s_1 + s_2 + 0s_3 = 9$

$x_1 + 3x_2 + 0s_1 + 0s_2 + s_3 = 5$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

Simplex table,

Iteration No.	Basic Var	Coefficient of					RHS	Ratio	Formula
		x_1	x_2	s_1	s_2	s_3			
0	z	-1	**-4**	0	0	0	0	-	$X + \frac{4}{3}Y$
s_3 leaves x_2 enters	s_1	2	**1**	1	0	0	3	$\frac{3}{1} = 3$	$X - \frac{1}{3}Y$
	s_2	3	**5**	0	1	0	9	$\frac{9}{5} = 1.8$	$X - \frac{5}{3}Y$
	s_3	**1**	$\boxed{\mathbf{3}}$	**0**	**0**	**1**	**5**	$\frac{5}{3} = 1.67$	$\frac{Y}{3}$
1	z	1/3	0	0	0	4/3	20/3		
	s_1	5/3	0	1	0	-1/3	4/3		
	s_2	4/3	0	0	1	-5/3	2/3		
	x_2	1/3	1	0	0	1/3	5/3		

Thus, the solution is

$x_1 = 0, x_2 = \frac{5}{3}, z_{max} = \frac{20}{3}$

3. Solve the following L.P.P. by simplex method

Maximise $z = 10x_1 + x_2 + x_3$

subject to $x_1 + x_2 - 3x_3 \le 10$

$4x_1 + x_2 + x_3 \le 20$

$x_1, x_2, x_3 \ge 0$

[N17/CompIT/6M]

Solution:

The standard form,

Max $z - 10x_1 - x_2 + x_3 + 0s_1 + 0s_2 = 0$

s.t. $x_1 + x_2 - 3x_3 + s_1 + 0s_2 = 10$

$4x_1 + x_2 + x_3 + 0s_1 + s_2 = 20$

$x_1, x_2, x_3, s_1, s_2 \ge 0$

Simplex table,

Iteration No.	Basic Var	Coefficient of					RHS	Ratio	Formula
		x_1	x_2	x_3	s_1	s_2			
0	z	**-10**	-1	1	0	0	0	-	$X + \frac{10}{4}Y$
s_2 leaves x_1 enters	s_1	**1**	1	-3	1	0	10	$\frac{10}{1} = 10$	$X - \frac{1}{4}Y$
	s_2	**4**	**1**	**1**	**0**	**1**	**20**	$\frac{20}{4} = 5$	$\frac{Y}{4}$
1	z	0	$\frac{3}{2}$	$\frac{7}{2}$	0	$\frac{5}{2}$	50		
	s_1	0	$\frac{3}{4}$	$-\frac{13}{4}$	1	$-\frac{1}{4}$	5		
	x_1	1	$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{4}$	5		

Thus, the solution is

$x_1 = 5, x_2 = 0, x_3 = 0, z_{max} = 50$

4. Solve the following L.P.P. by simplex method

Maximise $z = 15x_1 + 6x_2 + 9x_3 + 2x_4$

subject to $2x_1 + x_2 + 5x_3 + 6x_4 \leq 20$

$3x_1 + x_2 + 3x_3 + 25x_4 \leq 24$

$7x_1 + x_4 \leq 70$

$x_1, x_2, x_3, x_4 \geq 0$

[M14/CompIT/8M]

Solution:

Max $z - 15x_1 - 6x_2 - 9x_3 - 2x_4 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $2x_1 + x_2 + 5x_3 + 6x_4 + s_1 + 0s_2 + 0s_3 = 20$

$3x_1 + x_2 + 3x_3 + 25x_4 + 0s_1 + s_2 + 0s_3 = 24$

$7x_1 + 0x_2 + 0x_3 + x_4 + 0s_1 + 0s_2 + s_3 = 70$

$x_1, x_2, x_3, x_4, s_1, s_2, s_3 \geq 0$

Simplex table,

Iteration No.	Basic Var	Coefficient of							RHS	Ratio	Formula
		x_1	x_2	x_3	x_4	s_1	s_2	s_3			
0	z	**-15**	-6	-9	-2	0	0	0	0	-	$X + 5Y$
s_2 leaves x_1 enters	s_1	**2**	1	5	6	1	0	0	20	$\frac{20}{2} = 10$	$X - \frac{2}{3}Y$
	s_2	**3**	**1**	**3**	**25**	**0**	**1**	**0**	**24**	$\frac{24}{3} = 8$	$\frac{Y}{3}$
	s_3	**7**	0	0	1	0	0	1	70	$\frac{70}{7} = 10$	$X - \frac{7}{3}Y$
1	z	0	**-1**	6	123	0	5	0	120	-	$X + 3Y$
s_1 leaves x_2 enters	s_1	**0**	**1/3**	**3**	**-32/3**	**1**	**-2/3**	**0**	**4**	$\frac{4}{\frac{1}{3}} = 12$	$3Y$
	x_1	1	**1/3**	1	25/3	0	1/3	0	8	$\frac{8}{\frac{1}{3}} = 24$	$X - Y$
	s_3	0	**-7/3**	-7	-172/3	0	-7/3	1	14	-	$X + 7Y$
2	z	0	0	15	91	3	3	0	132		
	x_2	0	1	9	-32	3	-2	0	12		
	x_1	1	0	-2	19	-1	1	0	4		
	s_3	0	0	14	-132	7	-7	1	42		

Thus, the solution is

$x_1 = 4, x_2 = 12, x_3 = 0, x_4 = 0, z_{max} = 132$

5. Solve the following L.P.P. by simplex method

Minimise $z = x_1 - 3x_2 + 3x_3$

subject to $3x_1 - x_2 + 2x_3 \leq 7$

$2x_1 + 4x_2 \geq -12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$

Solution:

Maximise $z' = -z = -x_1 + 3x_2 - x_3$

s.t. $3x_1 - x_2 + 3x_3 \leq 7$

$-2x_1 + 4x_2 + 0x_3 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

Converting into standard form,

Max $z' + x_1 - 3x_2 + x_3 - 0s_1 - 0s_2 - 0s_3 = 0$

s.t. $3x_1 - x_2 + 3x_3 + s_1 = 7$

$-2x_1 + 4x_2 + 0x_3 + s_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Simplex Table

Iteration No.	Basic Var	Coefficient of						RHS	Ratio	Formula
		x_1	x_2	x_3	s_1	s_2	s_3			
0	z'	1	**-3**	1	0	0	0	0	-	$X - \frac{-3}{4}Y$
s_2 leaves x_2 enters	s_1	3	**-1**	3	1	0	0	7	-	$X - \frac{-1}{4}Y$
	s_2	**-2**	**4**	**0**	**0**	**1**	**0**	**12**	3	$Y \div 4$
	s_3	-4	**3**	8	0	0	1	10	3.333	$X - \frac{3}{4}Y$
1	z'	**-1/2**	0	1	0	3/4	0	9	-	$X - \frac{-1}{5}Y$
s_1 leaves x_1 enters	s_1	**5/2**	**0**	**3**	**1**	**1/4**	**0**	**10**	4	$\frac{2}{5}Y$
	x_2	**-1/2**	1	0	0	1/4	0	3	-	$X - \frac{-1}{5}Y$
	s_3	**-5/2**	0	8	0	-3/4	1	1	-	$X + Y$
2	z'	0	0	8/5	2/5	4/5	0	11		
	x_1	1	0	6/5	2/5	1/10	0	4		
	x_2	0	1	3/5	1/5	3/10	0	5		
	s_3	0	0	11	1	-1/2	1	11		

$z'_{max} = 11,$

$z_{min} = -z'_{max} = -11, x_1 = 4, x_2 = 5, x_3 = 0$

6. Solve by Simplex Method

Maximise $z = 7x_1 + 5x_2$

subject to $-x_1 - 2x_2 \geq -6$

$4x_1 + 3x_2 \leq 12$

$x_1, x_2 \geq 0$

[M22/CompITAI/5M]

Solution:

The standard form,

Max $z - 7x_1 - 5x_2 + 0s_1 + 0s_2 = 0$

s.t. $x_1 + 2x_2 + s_1 + 0s_2 = 6$

$4x_1 + 3x_2 + 0s_1 + s_2 = 12$

$x_1, x_2, s_1, s_2 \geq 0$

Simplex table,

Iteration No.	Basic Var	Coefficient of				RHS	Ratio	Formula
		x_1	x_2	s_1	s_2			
0	z	$\mathbf{-7}$	-5	0	0	0	-	$X + \frac{7}{4}Y$
$s_2$leaves x_1 enters	s_1	**1**	2	1	0	6	$\frac{6}{1} = 6$	$X - \frac{1}{4}Y$
	s_2	**4**	**3**	**0**	**1**	**12**	$\frac{12}{4} = 3$	$Y \div 4$
1	z	0	$\frac{1}{4}$	0	$\frac{7}{4}$	21		
	s_1	0	$\frac{5}{4}$	1	$-\frac{1}{4}$	3		
	x_1	1	$\frac{3}{4}$	0	$\frac{1}{4}$	3		

Thus, the solution is

$x_1 = 3, x_2 = 0, z_{max} = 21$

7. Solve by using Simplex method.

Maximise $z = 4x_1 + 10x_2$

subject to $2x_1 + x_2 \leq 10$

$2x_1 + 5x_2 \leq 20$

$2x_1 + 3x_2 \leq 18$

$x_1, x_2 \geq 0$

[N19/Comp/8M]

Solution:

Max $z - 4x_1 - 10x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $2x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 10$

$2x_1 + 5x_2 + 0s_1 + s_2 + 0s_3 = 20$

$2x_1 + 3x_2 + 0s_1 + 0s_2 + s_3 = 18$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

Simplex table,

Iteration No.	Basic Var	Coefficient of					RHS	Ratio	Formula
		x_1	x_2	s_1	s_2	s_3			
0	z	-4	**-10**	0	0	0	0	-	$X + 2Y$
s_2 leaves x_2 enters	s_1	2	**1**	1	0	0	10	$\frac{10}{1} = 10$	$X - \frac{1}{5}Y$
	s_2	**2**	**5**	**0**	**1**	**0**	**20**	$\frac{20}{5} = 4$	$\frac{Y}{5}$
	s_3	2	**3**	0	0	1	18	$\frac{18}{3} = 6$	$X - \frac{3}{5}Y$
1	z	0	0	0	2	0	40		
	s_1	$\frac{8}{5}$	0	1	$-\frac{1}{5}$	0	6		
	x_2	$\frac{2}{5}$	1	0	$\frac{1}{5}$	0	4		
	s_3	$\frac{4}{5}$	0	0	$-\frac{3}{5}$	1	6		

Thus, the solution is

$x_1 = 0, x_2 = 4, z_{max} = 40$

8. Solve by using Simplex method.

Maximise $z = 4x_1 + 10x_2$

subject to $2x_1 + x_2 \leq 50$

$2x_1 + 5x_2 \leq 100$

$2x_1 + 3x_2 \leq 90$

$x_1, x_2 \geq 0$

[N22/CompITAI/8M]

Solution:

Max $z - 4x_1 - 10x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $2x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 50$

$2x_1 + 5x_2 + 0s_1 + s_2 + 0s_3 = 100$

$2x_1 + 3x_2 + 0s_1 + 0s_2 + s_3 = 90$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

Simplex table,

Iteration No.	Basic Var	Coefficient of					RHS	Ratio	Formula
		x_1	x_2	s_1	s_2	s_3			
0	z	-4	**-10**	0	0	0	0	-	$X + 2Y$
s_2 leaves x_2 enters	s_1	2	**1**	1	0	0	50	$\frac{50}{1} = 50$	$X - \frac{1}{5}Y$
	s_2	**2**	**5**	**0**	**1**	**0**	**100**	$\frac{100}{5} = 20$	$\frac{Y}{5}$
	s_3	2	**3**	0	0	1	90	$\frac{90}{3} = 30$	$X - \frac{3}{5}Y$
1	z	0	0	0	2	0	200		
	s_1	$\frac{8}{5}$	0	1	$-\frac{1}{5}$	0	30		
	x_2	$\frac{2}{5}$	1	0	$\frac{1}{5}$	0	20		
	s_3	$\frac{4}{5}$	0	0	$-\frac{3}{5}$	1	30		

Thus, the solution is

$x_1 = 0, x_2 = 20, z_{max} = 200$

9. Solve the following L.P.P. by simplex method

Maximise $z = 3x_1 + 2x_2$

subject to $3x_1 + 2x_2 \leq 18$

$0 \leq x_1 \leq 4$

$0 \leq x_2 \leq 6$

$x_1, x_2 \geq 0$

[N16/CompIT/6M][N18/MechCivil/8M][M24/CompITAI/8M]

Solution:

Max $z - 3x_1 - 2x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $3x_1 + 2x_2 + s_1 + 0s_2 + 0s_3 = 18$

$x_1 + 0x_2 + 0s_1 + s_2 + 0s_3 = 4$

$0x_1 + x_2 + 0s_1 + 0s_2 + s_3 = 6$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

Simplex table,

Iteration No.	Basic Var	Coefficient of					RHS	Ratio	Formula
		x_1	x_2	s_1	s_2	s_3			
0	z	**-3**	-2	0	0	0	0	-	$X + 3Y$
s_2 leaves x_1 enters	s_1	**3**	2	1	0	0	18	$\frac{18}{3} = 6$	$X - 3Y$
	s_2	**1**	**0**	**0**	**1**	**0**	**4**	$\frac{4}{1} = 4$	-
	s_3	**0**	1	0	0	1	6	-	-
1	z	0	**-2**	0	3	0	12	-	$X + Y$
s_1 leaves x_2 enters	s_1	**0**	**2**	**1**	**-3**	**0**	**6**	$\frac{6}{2} = 3$	$\frac{Y}{2}$
	x_1	1	**0**	0	1	0	4	-	-
	s_3	0	**1**	0	0	1	6	$\frac{6}{1} = 6$	$X - \frac{1}{2}Y$
2	z	0	0	1	0	0	18		
	x_2	0	1	1/2	-3/2	0	3		
	x_1	1	0	0	1	0	4		
	s_3	0	0	-1/2	3/2	1	3		

Thus, the solution is

$x_1 = 4, x_2 = 3, z_{max} = 18$

10. Use Simplex method to

Maximize $z = 3x_1 + 5x_2$

Subject to $3x_1 + 2x_2 \leq 18$

$x_1 \leq 4$

$x_2 \leq 6$

$x_1, x_2 \geq 0$

[N18/Comp/8M][M19/Comp/6M]

Solution:

Max $z - 3x_1 - 5x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $3x_1 + 2x_2 + s_1 + 0s_2 + 0s_3 = 18$

$x_1 + 0x_2 + 0s_1 + s_2 + 0s_3 = 4$

$0x_1 + x_2 + 0s_1 + 0s_2 + s_3 = 6$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

Simplex table,

Iteration No.	Basic Var	Coefficient of					RHS	Ratio	Formula
		x_1	x_2	s_1	s_2	s_3			
0	z	-3	**-5**	0	0	0	0	-	$X + 5Y$
s_3 leaves x_2 enters	s_1	3	**2**	1	0	0	18	$\frac{18}{2} = 9$	$X - 2Y$
	s_2	1	**0**	0	1	0	4	-	-
	s_3	**0**	**1**	**0**	**0**	**1**	**6**	$\frac{6}{1} = 6$	-
1	z	**-3**	0	0	0	5	30	-	$X + Y$
s_1 leaves x_1 enters	s_1	**3**	**2**	**1**	**0**	**-2**	**6**	$\frac{6}{3} = 2$	$\frac{Y}{3}$
	s_2	**1**	0	0	1	0	4	$\frac{4}{1} = 4$	$X - \frac{1}{3}Y$
	x_2	**0**	1	0	0	1	6	–	–
2	z	0	2	1	0	3	36		
	x_1	1	$\frac{2}{3}$	$\frac{1}{3}$	0	$-\frac{2}{3}$	2		
	s_2	0	$-\frac{2}{3}$	$-\frac{1}{3}$	1	$\frac{2}{3}$	2		
	x_2	0	1	0	0	1	6		

Thus, the solution is

$x_1 = 2, x_2 = 6, z_{max} = 36$

11. Solve the following L.P.P. by simplex method

Maximise $z = 6x_1 - 2x_2 + 3x_3$

subject to $2x_1 - x_2 + 2x_3 \leq 2$

$x_1 + 4x_3 \leq 4$

$x_1, x_2, x_3 \geq 0$

[N15/CompIT/6M][M18/Comp/8M]

Solution:

The standard form,

Max $z - 6x_1 + 2x_2 - 3x_3 + 0s_1 + 0s_2 = 0$

s.t. $2x_1 - x_2 + 2x_3 + s_1 + 0s_2 = 2$

$x_1 + 0x_2 + 4x_3 + 0s_1 + s_2 = 4$

$x_1, x_2, x_3, s_1, s_2 \geq 0$

Simplex table,

Iteration No.	Basic Var	Coefficient of					RHS	Ratio	Formula
		x_1	x_2	x_3	s_1	s_2			
0	z	**-6**	2	-3	0	0	0	-	$X + 3Y$
s_1 leaves x_1 enters	s_1	**2**	**-1**	**2**	**1**	**0**	**2**	$\frac{2}{2} = 1$	$\frac{Y}{2}$
	s_2	**1**	0	4	0	1	4	$\frac{4}{1} = 4$	$X - \frac{1}{2}Y$
1	z	0	**-1**	3	3	0	6	-	$X + 2Y$
s_2 leaves x_2 enters	x_1	1	**-1/2**	1	1/2	0	1	-	$X + Y$
	s_2	**0**	**1/2**	**3**	**-1/2**	**1**	**3**	$\frac{3}{\frac{1}{2}} = 6$	$2Y$
2	z	0	0	9	2	2	12		
	x_1	1	0	4	0	1	4		
	x_2	0	1	6	-1	2	6		

Thus, the solution is

$x_1 = 4, x_2 = 6, x_3 = 0, z_{max} = 12$

12. Using Simplex Method

Maximise $z = 10x_1 + 6x_2 + 5x_3$

subject to $2x_1 + 2x_2 + 6x_3 \leq 300$

$10x_1 + 4x_2 + 5x_3 \leq 600$

$x_1 + x_2 + x_3 \leq 100$

$x_1, x_2, x_3 \geq 0$

[M22/CompITAI/5M]

Solution:

The standard form,

Max $z - 10x_1 - 6x_2 - 5x_3 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $2x_1 + 2x_2 + 6x_3 + s_1 + 0s_2 + 0s_3 = 300$

$10x_1 + 4x_2 + 5x_3 + 0s_1 + s_2 + 0s_3 = 600$

$x_1 + x_2 + x_3 + 0s_1 + 0s_2 + s_3 = 100$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Simplex table,

Iteration No.	Basic Var	Coefficient of						RHS	Ratio	Formula
		x_1	x_2	x_3	s_1	s_2	s_3			
0	z	**-10**	-6	-5	0	0	0	0	-	$X+Y$
s_2 leaves x_1 enters	s_1	**2**	2	6	1	0	0	300	150	$X-\frac{1}{5}Y$
	s_2	**10**	**4**	**5**	**0**	**1**	**0**	**600**	60	$Y \div 10$
	s_3	**1**	1	1	0	0	1	100	100	$X-\frac{1}{10}Y$
1	z	0	**-2**	0	0	1	0	600	-	$X+\frac{10}{3}Y$
s_3 leaves x_2 enters	s_1	0	**6/5**	5	1	-1/5	0	180	150	$X-2Y$
	x_1	1	**2/5**	1/2	0	1/10	0	60	150	$X-\frac{2}{3}Y$
	s_3	**0**	**3/5**	**1/2**	**0**	**-1/10**	**1**	**40**	66.66	$\frac{5}{3}Y$
2	z	0	0	5/3	0	2/3	10/3	2200/3		
	s_1	0	0	4	1	0	-2	100		
	x_1	1	0	1/6	0	1/6	-2/3	100/3		
	x_2	0	1	5/6	0	-1/6	5/3	200/3		

Thus, the solution is

$x_1 = \frac{100}{3}, x_2 = \frac{200}{3}, x_3 = 0, z_{max} = \frac{2200}{3}$

13. Solve the following L.P.P. by simplex method

Maximise $z = 4x_1 + 3x_2 + 6x_3$

subject to $2x_1 + 3x_2 + 2x_3 \leq 440$

$4x_1 + 3x_3 \leq 470$

$2x_1 + 5x_2 \leq 430$

$x_1, x_2, x_3 \geq 0$

[M15/CompIT/6M]

Solution:

Max $z - 4x_1 - 3x_2 - 6x_3 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $2x_1 + 3x_2 + 2x_3 + s_1 + 0s_2 + 0s_3 = 440$

$4x_1 + 0x_2 + 3x_3 + 0s_1 + s_2 + 0s_3 = 470$

$2x_1 + 5x_2 + 0x_3 + 0s_1 + 0s_2 + s_3 = 430$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Simplex table,

Iteration No.	Basic Var	Coefficient of						RHS	Ratio	Formula
		x_1	x_2	x_3	s_1	s_2	s_3			
0	z	-4	-3	**-6**	0	0	0	0	-	$X + 2Y$
s_2 leaves x_3 enters	s_1	2	3	**2**	1	0	0	440	$\frac{440}{2} = 220$	$X - \frac{2}{3}Y$
	s_2	**4**	**0**	**3**	**0**	**1**	**0**	**470**	$\frac{470}{3} = 156.67$	$\frac{Y}{3}$
	s_3	2	5	**0**	0	0	1	430	-	-
1	z	4	**-3**	0	0	2	0	940	-	$X + Y$
s_1 leaves x_2 enters	s_1	**-2/3**	**3**	**0**	**1**	**-2/3**	**0**	**380/3**	$\frac{380}{9} = 42.22$	$\frac{Y}{3}$
	x_3	4/3	**0**	1	0	1/3	0	470/3	-	-
	s_3	2	**5**	0	0	0	1	430	$\frac{430}{5} = 86$	$X - \frac{5}{3}Y$
2	z	10/3	0	0	1	4/3	0	3200/3		
	x_2	-2/9	1	0	1/3	-2/9	0	380/9		
	x_3	4/3	0	1	0	1/3	0	470/3		
	s_3	28/9	0	0	-5/3	10/9	1	1970/9		

Thus, the solution is

$x_1 = 0, x_2 = \frac{380}{9}, x_3 = \frac{470}{3}, z_{max} = \frac{3200}{3}$

14. Solve the following L.P.P. by simplex method

Maximise $z = 3x_1 + 2x_2 + 5x_3$

subject to $x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$

$x_1 + 4x_2 \le 420$

$x_1, x_2, x_3 \ge 0$

[M23/CompIT/8M]

Solution:

Max $z - 3x_1 - 2x_2 - 5x_3 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $x_1 + 2x_2 + x_3 + s_1 + 0s_2 + 0s_3 = 430$

$3x_1 + 0x_2 + 2x_3 + 0s_1 + s_2 + 0s_3 = 460$

$x_1 + 4x_2 + 0x_3 + 0s_1 + 0s_2 + s_3 = 420$

$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$

Simplex table,

Iteration No.	Basic Var	Coefficient of						RHS	Ratio	Formula
		x_1	x_2	x_3	s_1	s_2	s_3			
0	z	-3	-2	**-5**	0	0	0	0	-	$X + \frac{5}{2}Y$
s_2 leaves x_3 enters	s_1	1	2	**1**	1	0	0	430	430	$X - \frac{1}{2}Y$
	s_2	**3**	**0**	**2**	**0**	**1**	**0**	**460**	230	$Y \div 2$
	s_3	1	4	**0**	0	0	1	420	-	-
1	z	9/2	**-2**	0	0	5/2	0	1150	-	$X + Y$
s_1 leaves x_2 enters	s_1	**-1/2**	**2**	**0**	**1**	**-1/2**	**0**	**200**	100	$Y \div 2$
	x_3	3/2	**0**	1	0	1/2	0	230	-	-
	s_3	1	**4**	0	0	0	1	430	107.5	$X - 2Y$
2	z	4	0	0	1	2	0	1350		
	x_2	-1/4	1	0	1/2	-1/4	0	100		
	x_3	3/2	0	1	0	1/2	0	230		
	s_3	2	0	0	-2	1	1	30		

Thus, the solution is

$x_1 = 0, x_2 = 100, x_3 = 230, z_{max} = 1350$

15. Solve the following L.P.P. by simplex method

Maximise $z = 4x_1 + x_2 + 3x_3 + 5x_4$

subject to $-4x_1 + 6x_2 + 5x_3 + 4x_4 \leq 20$

$-3x_1 - 2x_2 + 4x_3 + x_4 \leq 10$

$-8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20$

$x_1, x_2, x_3, x_4 \geq 0$

[M16/CompIT/6M]

Solution:

The standard form,

Max $z - 4x_1 - x_2 - 3x_3 - 5x_4 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $-4x_1 + 6x_2 + 5x_3 + 4x_4 + s_1 + 0s_2 + 0s_3 = 20$

$-3x_1 - 2x_2 + 4x_3 + x_4 + 0s_1 + s_2 + 0s_3 = 10$

$-8x_1 - 3x_2 + 3x_3 + 2x_4 + 0s_1 + 0s_2 + s_3 = 20$

$x_1, x_2, x_3, x_4, s_1, s_2, s_3 \geq 0$

Simplex table,

Iteration No.	Basic Var	Coefficient of							RHS	Ratio	Formula
		x_1	x_2	x_3	x_4	s_1	s_2	s_3			
0	z	-4	-1	-3	**-5**	0	0	0	0	-	$X + \frac{5}{4}Y$
s_1 leaves x_4 enters	s_1	**-4**	**6**	**5**	**4**	**1**	**0**	**0**	**20**	$\frac{20}{4} = 5$	$\frac{Y}{4}$
	s_2	-3	-2	4	**1**	0	1	0	10	$\frac{10}{1} = 10$	$X - \frac{1}{4}Y$
	s_3	-8	-3	3	**2**	0	0	1	20	$\frac{20}{2} = 10$	$X - \frac{1}{2}Y$
1	z	**-9**	13/2	13/4	0	5/4	0	0	25	-	
	x_4	**-1**	3/2	5/4	1	1/4	0	0	5	-	
	s_2	**-2**	-7/2	11/4	0	-1/4	1	0	5	-	
	s_3	**-6**	-6	1/2	0	-1/2	0	1	10	-	

Since, there are no positive ratio obtained and the coefficient is still negative the solution is unbounded.

Type III: Big M Method

1. Using Penalty (Big-M or Charne's) method to solve the following L.P.P.

Minimise $z = 2x_1 + 3x_2$

subject to $x_1 + x_2 \geq 5$

$x_1 + 2x_2 \geq 6$

$x_1, x_2 \geq 0$

[N16/CompIT/8M]

Solution:

The standard form,

Max $z' = -z = -2x_1 - 3x_2 + 0s_1 + 0s_2 - MA_1 - MA_2$

Max $z' + 2x_1 + 3x_2 + 0s_1 + 0s_2 + MA_1 + MA_2 = 0$ ……(1)

s.t. $x_1 + x_2 - s_1 + 0s_2 + A_1 + 0A_2 = 5$ ……(2)

$x_1 + 2x_2 + 0s_1 - s_2 + 0A_1 + A_2 = 6$ ……(3)

Multiplying eqn (2) & (3) by M and subtracting both with eqn (1), we get

$z' + (2 - 2M)x_1 + (3 - 3M)x_2 + Ms_1 + Ms_2 + 0A_1 + 0A_2 = -11M$

Simplex table,

Iteration No.	Basic Var	Coefficient of						RHS	Ratio	Formula
		x_1	x_2	s_1	s_2	A_1	A_2			
0	z'	2-2M	**3-3M**	M	M	0	0	-11M	-	$X - \frac{(3-3M)}{2}Y$
A_2 leaves x_2 enters	A_1	1	**1**	-1	0	1	0	5	$\frac{5}{1} = 5$	$X - \frac{1}{2}Y$
	A_2	**1**	**2**	**0**	**-1**	**0**	**1**	**6**	$\frac{6}{2} = 3$	$\frac{Y}{2}$
1	z'	**1/2-M/2**	0	M	3/2 –M/2	0		-9-2M	-	$X - (1 - M)Y$
A_1 leaves x_1 enters	A_1	**1/2**	**0**	**-1**	**1/2**	**1**		**2**	$\frac{2}{\frac{1}{2}} = 4$	$2Y$
	x_2	**1/2**	1	0	-1/2	0		3	$\frac{3}{\frac{1}{2}} = 6$	$X - Y$
2	z'	0	0	1	1			-11		
	x_1	1	0	-2	1			4		
	x_2	0	1	1	-1			1		

Thus, the solution is

$x_1 = 4, x_2 = 1, z'_{max} = -11, \therefore\ z_{min} = 11$

2. Using Penalty (Big-M or Charne's) method to solve the following L.P.P.

Minimise $z = 10x_1 + 3x_2$

subject to $x_1 + 2x_2 \geq 3$

$x_1 + 4x_2 \geq 4$

$x_1, x_2 \geq 0$

[M18/MechCivil/6M]

Solution:

The standard form,

Max $z' = -z = -10x_1 - 3x_2 + 0s_1 + 0s_2 - MA_1 - MA_2$

Max $z' + 10x_1 + 3x_2 + 0s_1 + 0s_2 + MA_1 + MA_2 = 0$ ……(1)

s.t. $x_1 + 2x_2 - s_1 + 0s_2 + A_1 + 0A_2 = 3$ ……(2)

$x_1 + 4x_2 + 0s_1 - s_2 + 0A_1 + A_2 = 4$ ……(3)

Multiplying eqn (2) & (3) by M and subtracting both with eqn (1), we get

$z' + (10 - 2M)x_1 + (3 - 6M)x_2 + Ms_1 + Ms_2 + 0A_1 + 0A_2 = -7M$

Simplex table,

Iteration No.	Basic Var	Coefficient of x_1	x_2	s_1	s_2	A_1	A_2	RHS	Ratio	Formula
0	z'	10-2M	**3-6M**	M	M	0	0	-7M	-	$X - \frac{(3-6M)}{4}Y$
A_2 leaves x_2 enters	A_1	1	**2**	-1	0	1	0	3	$\frac{3}{2} = 1.5$	$X - \frac{1}{2}Y$
	A_2	**1**	**4**	**0**	**-1**	**0**	**1**	**4**	$\frac{4}{4} = 1$	$\frac{Y}{4}$
1	z'	37/4-M/2	0	M	**3/4 –M/2**	0		-M-3	-	$X - (3/2 - M)Y$
A_1 leaves s_2 enters	A_1	1/2	**0**	**-1**	**1/2**	**1**		**1**	$\frac{1}{\frac{1}{2}} = 2$	$2Y$
	x_2	1/4	1	0	**-1/4**	0		1	-	$X + \frac{1}{2}Y$
2	z'	17/2	0	3/2	0			-9/2	-	
	s_2	1	0	-2	1			2	-	
	x_2	1/2	1	-1/2	0			3/2	-	

Thus, the solution is

$x_1 = 0, x_2 = \frac{3}{2}, z'_{max} = -\frac{9}{2}, \therefore\ z_{min} = \frac{9}{2}$

3. Using Penalty (Big-M or Charne's) method to solve the following L.P.P.

Minimise $z = 2x_1 + x_2$

subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

[N17/CompIT/8M][M19/MechCivil/6M]

Solution:

The standard form,

Max $z' = -z = -2x_1 - x_2 + 0s_2 + 0s_3 - MA_1 - MA_2$

Max $z' + 2x_1 + x_2 + 0s_2 + 0s_3 + MA_1 + MA_2 = 0$ ……(1)

s.t. $3x_1 + x_2 + A_1 = 3$ ……(2)

$4x_1 + 3x_2 - s_2 + A_2 = 6$ ……(3)

$x_1 + 2x_2 + s_3 = 3$

Multiplying eqn (2) & (3) by M and subtracting both with eqn (1), we get

$z' + (2 - 7M)x_1 + (1 - 4M)x_2 + Ms_2 + 0s_3 + 0A_1 + 0A_2 = -9M$

Simplex table,

Iteration No.	Basic Var	Coefficient of						RHS	Ratio	Formula
		x_1	x_2	s_2	s_3	A_1	A_2			
0	z'	**2-7M**	1-4M	M	0	0	0	-9M	-	$X - \frac{(2-7M)}{3}Y$
A_1 leaves x_1 enters	A_1	**3**	**1**	**0**	**0**	**1**	**0**	**3**	$\frac{3}{3} = 1$	$\frac{Y}{3}$
	A_2	**4**	3	-1	0	0	1	6	$\frac{6}{4} = 1.5$	$X - \frac{4}{3}Y$
	s_3	**1**	2	0	1	0	0	3	$\frac{3}{1} = 3$	$X - \frac{1}{3}Y$
1	z'	0	$\frac{1-5M}{3}$	M	0		0	-2-2M	-	$X - \frac{1-5M}{5}Y$
A_2 leaves x_2 enters	x_1	1	$\frac{1}{3}$	0	0		0	1	3	$X - \frac{1}{5}Y$
	A_2	**0**	$\frac{5}{3}$	**-1**	**0**		**1**	**2**	1.2	$\frac{3}{5}Y$
	s_3	0	$\frac{5}{3}$	0	1		0	2	1.2	$X - Y$
2	z'	0	0	$\frac{1}{5}$	0			$-\frac{12}{5}$		
	x_1	1	0	$\frac{1}{5}$	0			$\frac{3}{5}$		
	x_2	0	1	$-\frac{3}{5}$	0			$\frac{6}{5}$		
	s_3	0	0	1	1			0		

Thus, the solution is

$x_1 = \frac{3}{5}, x_2 = \frac{6}{5}, z'_{max} = -\frac{12}{5}, \therefore\ z_{min} = \frac{12}{5}$

4. Using Penalty (Big-M or Charne's) method to solve the following L.P.P.

Minimise $z = 6x_1 + 4x_2$

subject to $2x_1 + 3x_2 \leq 30$

$3x_1 + 2x_2 \leq 24$

$x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$

[N18/MechCivil/8M]

Solution:

The standard form,

Max $z' = -z = -6x_1 - 4x_2 + 0s_1 + 0s_2 + 0s_3 - MA_3$

Max $z' + 6x_1 + 4x_2 + 0s_1 + 0s_2 + 0s_3 + MA_3 = 0$ ……(1)

s.t. $2x_1 + 3x_2 + s_1 = 30$

$3x_1 + 2x_2 + s_2 = 24$

$x_1 + x_2 - s_3 + A_3 = 3$ …… (2)

Multiplying eqn (2) by M and subtracting with eqn (1), we get

$z' + (6 - M)x_1 + (4 - M)x_2 + 0s_1 + 0s_2 + Ms_3 + 0A_3 = -3M$

Simplex table,

Iteration No.	Basic Var	Coefficient of						RHS	Ratio	Formula
		x_1	x_2	s_1	s_2	s_3	A_3			
0	z'	6-M	**4-M**	0	0	M	0	-3M	-	$X - (4 - M)Y$
A_3 leaves x_2 enters	s_1	2	**3**	1	0	0	0	30	$\frac{30}{3} = 10$	$X - 3Y$
	s_2	3	**2**	0	1	0	0	24	$\frac{24}{2} = 12$	$X - 2Y$
	A_3	**1**	**1**	**0**	**0**	**-1**	**1**	**3**	$\frac{3}{1} = 3$	-
1	z'	2	0	0	0	4		-12		
	s_1	-1	0	1	0	3		21		
	s_2	1	0	0	1	2		18		
	x_2	1	1	0	0	-1		3		

Thus, the solution is

$x_1 = 0, x_2 = 3, z'_{max} = -12, \therefore\ z_{min} = 12$

5. Using Penalty (Big-M or Charne's) method to solve the following L.P.P.

Maximise $z = 3x_1 - 2x_2$

subject to $2x_1 + x_2 \leq 2$

$x_1 + 3x_2 \geq 3$

$x_1, x_2 \geq 0$

[D23/CompITAI/8M]

Solution:

The standard form,

Max $z = 3x_1 - 2x_2 + 0s_1 + 0s_2 - MA_2$

Max $z - 3x_1 + 2x_2 + 0s_1 + 0s_2 + MA_2 = 0$ ……(1)

s.t. $2x_1 + x_2 + s_1 = 2$ ……(2)

$x_1 + 3x_2 - s_2 + A_2 = 3$ ……(3)

Multiplying eqn (3) by M and subtracting with eqn (1), we get

$z + (-3 - M)x_1 + (2 - 3M)x_2 + 0s_1 + Ms_2 + 0A_2 = -3M$

Simplex table,

Iteration No.	Basic Var	Coefficient of					RHS	Ratio	Formula
		x_1	x_2	s_1	s_2	A_2			
0	z	-3-M	**2-3M**	0	M	0	-3M	-	$X - \frac{(2-3M)}{3}Y$
A_2 leaves x_2 enters	s_1	2	**1**	1	0	0	2	2	$X - \frac{1}{3}Y$
	A_2	**1**	**3**	**0**	**-1**	**1**	**3**	1	$\frac{Y}{3}$
1	z	**-11/3**	0	0	2/3		-2	-	$X + \frac{11}{5}Y$
s_1 leaves x_1 enters	s_1	**5/3**	**0**	**1**	**1/3**		**1**	0.6	$\frac{3Y}{5}$
	x_2	**1/3**	1	0	-1/3		1	3	$X - \frac{1}{5}Y$
2	z	0	0	11/5	7/5		1/5		
	x_1	1	0	3/5	1/5		3/5		
	x_2	0	1	-1/5	-2/5		4/5		

Thus, the solution is

$x_1 = \frac{3}{5}, x_2 = \frac{4}{5}, z_{max} = \frac{1}{5}$

6. Using Penalty (Big-M or Charne's) method to solve the following L.P.P.

Maximise $z = x_1 + 2x_2 + 3x_3 - x_4$

subject to $x_1 + 2x_2 + 3x_3 = 15$

$2x_1 + x_2 + 5x_3 = 20$

$x_1 + 2x_2 + x_3 + x_4 = 10$

$x_1, x_2, x_3, x_4 \geq 0$

[D24/CompITAI/8M]

Solution:

The standard form,

Max $z = x_1 + 2x_2 + 3x_3 - x_4 - MA_1 - MA_2 - MA_3$

Max $z - x_1 - 2x_2 - 3x_3 + x_4 + MA_1 + MA_2 + MA_3 = 0$ ……(1)

s.t. $x_1 + 2x_2 + 3x_3 + A_1 = 15$ ……(2)

$2x_1 + x_2 + 5x_3 + A_2 = 20$ ……(3)

$x_1 + 2x_2 + x_3 + x_4 + A_3 = 10$ ……. (4)

Multiplying eqn (2), (3), (4) by M and subtracting all with eqn (1), we get

$z + (-1 - 4M)x_1 + (-2 - 5M)x_2 + (-3 - 9M) + (1 - M)x_4 + 0A_1 + 0A_2 + 0A_3 = -45M$

Simplex table,

Iteration No.	Basic Var	Coefficient of							RHS	Ratio	Formula
		x_1	x_2	x_3	x_4	A_1	A_2	A_3			
0	z	-1-4M	-2-5M	**-3-9M**	1-M	0	0	0	-45M	-	$X - \frac{-3-9M}{5}Y$
A_2 leaves x_3 enters	A_1	1	2	**3**	0	1	0	0	15	5	$X - \frac{3}{5}Y$
	A_2	**2**	**1**	**5**	**0**	**0**	**1**	**0**	**20**	4	$Y \div 5$
	A_3	1	2	**1**	1	0	0	1	10	10	$X - \frac{1}{5}Y$
1	z	1/5 -2M/5	**-7/5-16M/5**	0	1-M	0		0	12-9M	-	$X - \frac{-7-16M}{7}Y$
A_1 leaves x_2 enters	A_1	**-1/5**	**7/5**	**0**	**0**	**1**		**0**	**3**	15/7	$\frac{5}{7}Y$
	x_3	2/5	**1/5**	1	0	0		0	4	20	$X - \frac{1}{7}Y$
	A_3	3/5	**9/5**	0	1	0		1	6	10/3	$X - \frac{9}{7}Y$
2	z	**-6M/7**	0	0	1-M			0	15-15M/7	-	$X + MY$
A_3 leaves x_1 enters	x_2	**-1/7**	1	0	0			0	15/7	-	$X + \frac{1}{6}Y$
	x_3	**3/7**	0	1	0			0	25/7	25/3	$X - \frac{1}{2}Y$
	A_3	**6/7**	**0**	**0**	**1**			**1**	**15/7**	5/2	$\frac{7}{6}Y$
3	z	0	0	0	1				15		
	x_2	0	1	0	1/6				5/2		
	x_3	0	0	1	-1/2				5/2		
	x_1	1	0	0	7/6				5/2		

Thus, $x_1 = x_2 = x_3 = \frac{5}{2}, x_4 = 0, z_{max} = 15$

Type IV: Dual Simplex Method

1. Use Dual simplex method to solve the following LPP

Minimize $z = 6x_1 + x_2$

Subject to $2x_1 + x_2 \geq 3$

$x_1 - x_2 \geq 0$

$x_1, x_2 \geq 0$

[M17/CompIT/6M][M23/CompIT/8M]

Solution:

The standard form,

Min $z = 6x_1 + x_2$

$z - 6x_1 - x_2 + 0s_1 + 0s_2 = 0$

s.t. $-2x_1 - x_2 + s_1 = -3$

$-x_1 + x_2 + s_2 = 0$

Simplex table,

Iteration No.	Basic Var	Coefficient of				RHS	Formula
		x_1	x_2	s_1	s_2		
0	z	-6	**-1**	0	0	0	$X - Y$
s_1 leaves	s_1	**-2**	**-1**	**1**	**0**	**-3**	$-Y$
x_2 enters	s_2	-1	**1**	0	1	0	$X + Y$
Ratio		$\frac{-6}{-2} = 3$	$\frac{-1}{-1} = 1$	-	-	-	
1	z	**-4**	0	-1	0	3	$X - \frac{4}{3}Y$
s_2 leaves	x_2	**2**	1	-1	0	3	$X + \frac{2}{3}Y$
x_1 enters	s_2	**-3**	**0**	**1**	**1**	**-3**	$-\frac{Y}{3}$
Ratio		$\frac{-4}{-3} = 1.33$	-	-	-	-	
2	z	0	0	$-\frac{7}{3}$	$-\frac{4}{3}$	7	
	x_2	0	1	$-\frac{1}{3}$	$\frac{2}{3}$	1	
	x_1	1	0	$-\frac{1}{3}$	$-\frac{1}{3}$	1	

The solution is

$x_1 = 1, x_2 = 1, z_{min} = 7$

2. Use Dual simplex method to solve the following LPP

Minimize $z = 6x_1 - x_2$

Subject to $2x_1 + x_2 \geq 3$

$x_1 - x_2 \geq 0$

$x_1, x_2 \geq 0$

[N19/MechCivil/6M]

Solution:

The standard form,

Min $z = 6x_1 - x_2$

$z - 6x_1 + x_2 + 0s_1 + 0s_2 = 0$

s.t. $-2x_1 - x_2 + s_1 = -3$

$-x_1 + x_2 + s_2 = 0$

Simplex table,

Iteration No.	Basic Var	Coefficient of				RHS	Formula
		x_1	x_2	s_1	s_2		
0	z	**-6**	1	0	0	0	$X - 3Y$
s_1 leaves x_1 enters	s_1	**-2**	**-1**	**1**	**0**	**-3**	$\frac{Y}{-2}$
	s_2	**-1**	1	0	1	0	$X - \frac{1}{2}Y$
Ratio		$\frac{-6}{-2} = 3$	–	-	-	-	
1	z	0	4	-3	0	9	
	x_1	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{3}{2}$	
	s_2	0	$\frac{3}{2}$	$-\frac{1}{2}$	1	$\frac{3}{2}$	

The solution is

$x_1 = \frac{3}{2}, x_2 = 0, z_{min} = 9$

3. Use the dual simplex method to solve the following L.P.P.

Minimise $z = 6x_1 + 3x_2 + 4x_3$

subject to $x_1 + 6x_2 + x_3 = 10$

$2x_1 + 3x_2 + x_3 = 15$

$x_1, x_2, x_3 \geq 0$

[N18/Comp/8M][M22/CompITAI/5M]

Solution:

Min $z = 6x_1 + 3x_2 + 4x_3$

s.t. $x_1 + 6x_2 + x_3 \leq 10$

$x_1 + 6x_2 + x_3 \geq 10$ i.e. $-x_1 - 6x_2 - x_3 \leq -10$

$2x_1 + 3x_2 + x_3 \leq 15$

$2x_1 + 3x_2 + x_3 \geq 15$ i.e. $-2x_1 - 3x_2 - x_3 \leq -15$

Standard form:

Min $z = 6x_1 + 3x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4$

$z - 6x_1 - 3x_2 - 4x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = 0$

s.t. $x_1 + 6x_2 + x_3 + s_1 + 0s_2 + 0s_3 + 0s_4 = 10$

$-x_1 - 6x_2 - x_3 + 0s_1 + s_2 + 0s_3 + 0s_4 = -10$

$2x_1 + 3x_2 + x_3 + 0s_1 + 0s_2 + s_3 + 0s_4 = 15$

$-2x_1 - 3x_2 - x_3 + 0s_1 + 0s_2 + 0s_3 + s_4 = -15$

Simplex Table,

Iteration No.	Basic Var	Coefficient of							RHS	Formula
		x_1	x_2	x_3	s_1	s_2	s_3	s_4		
0	z	-6	**-3**	-4	0	0	0	0	0	$X - Y$
s_4 leaves x_2 enters	s_1	1	**6**	1	1	0	0	0	10	$X + 2Y$
	s_2	-1	**-6**	-1	0	1	0	0	-10	$X - 2Y$
	s_3	2	**3**	1	0	0	1	0	15	$X + Y$
	s_4	**-2**	**-3**	**-1**	**0**	**0**	**0**	**1**	**-15**	$\frac{Y}{-3}$
Ratio		$\frac{-6}{-2} = 3$	$\frac{-3}{-3} = 1$	$\frac{-4}{-1} = 4$	-	-	-	-	-	
1	z	**-4**	0	-3	0	0	0	-1	15	$X - \left(\frac{4}{3}\right)Y$
s_1 leaves x_1 enters	s_1	**-3**	**0**	**-1**	**1**	**0**	**0**	**2**	**-20**	$\frac{Y}{-3}$
	s_2	**3**	0	1	0	1	0	-2	20	$X + Y$
	s_3	**0**	0	0	0	0	1	1	0	-
	x_2	**2/3**	1	1/3	0	0	0	-1/3	5	$X + \frac{2}{9}Y$
Ratio		$\frac{-4}{-3} = 1.33$	-	$\frac{-3}{-1} = 3$	-	-	-	-	-	
2	z	0	0	-5/3	-4/3	0	0	-11/3	125/3	
	x_1	1	0	1/3	-1/3	0	0	-2/3	20/3	
	s_2	0	0	0	1	1	0	0	0	
	s_3	0	0	0	0	0	1	1	0	
	x_2	0	1	1/9	2/9	0	0	1/9	5/9	

Thus the solution is $x_1 = \frac{20}{3}, x_2 = \frac{5}{9}, x_3 = 0, z_{min} = \frac{125}{3}$

4. Use Dual simplex method to solve the following LPP

Minimize $z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0$

[N18/MechCivil/8M][M19/Comp/8M][N22/CompITAI/8M]

Solution:

The standard form,

Min $z = x_1 + x_2$

$z - x_1 - x_2 + 0s_1 + 0s_2 = 0$

s.t. $-2x_1 - x_2 + s_1 = -2$

$x_1 + x_2 + s_2 = -1$

Simplex table,

Iteration No.	Basic Var	Coefficient of				RHS	Formula
		x_1	x_2	s_1	s_2		
0	z	**-1**	-1	0	0	0	$X - \frac{1}{2}Y$
s_1 leaves x_1 enters	s_1	**-2**	**-1**	**1**	**0**	**-2**	$\frac{Y}{-2}$
	s_2	**1**	1	0	1	-1	$X + \frac{1}{2}Y$
Ratio		$\frac{-1}{-2} = \frac{1}{2}$	$\frac{-1}{-1} = 1$	-	-	-	
1	z	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	1	
s_2 leaves x_1 enters	x_2	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	
	s_2	**0**	$\mathbf{\frac{1}{2}}$	$\mathbf{\frac{1}{2}}$	**1**	**-2**	
Ratio		-	-	-	-	-	

Since, there are no positive ratios obtained, the problem has no solution

5. Use the dual simplex method to solve the following L.P.P.

Minimise $z = 2x_1 + x_2$

subject to $3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

[M18/N19/Comp/8M][M18/MechCivil/8M][M24/CompITAI/8M]

Solution:

The standard form,

Min $z - 2x_1 - x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $-3x_1 - x_2 + s_1 + 0s_2 + 0s_3 = -3$

$-4x_1 - 3x_2 + 0s_1 + s_2 + 0s_3 = -6$

$x_1 + 2x_2 + 0s_1 + 0s_2 + s_3 = 3$

Simplex table,

Iteration No.	Basic Var	Coefficient of					RHS	Formula
		x_1	x_2	s_1	s_2	s_3		
0	z	-2	**-1**	0	0	0	0	$X - \frac{1}{3}Y$
s_2 leaves x_2 enters	s_1	-3	**-1**	1	0	0	-3	$X - \frac{1}{3}Y$
	s_2	**-4**	**-3**	**0**	**1**	**0**	**-6**	$\frac{Y}{-3}$
	s_3	1	**2**	0	0	1	3	$X + \frac{2}{3}Y$
Ratio		$\frac{-2}{-4} = \frac{1}{2}$	$\frac{-1}{-3} = \frac{1}{3}$	-	-	-	-	-
1	z	**-2/3**	0	0	-1/3	0	2	$X - \frac{2}{5}Y$
s_1 leaves x_1 enters	s_1	**-5/3**	**0**	**1**	**-1/3**	**0**	**-1**	$-\frac{3}{5}Y$
	x_2	**4/3**	1	0	-1/3	0	2	$X + \frac{4}{5}Y$
	s_3	**-5/3**	0	0	2/3	1	-1	$X - Y$
Ratio		$\frac{-\frac{2}{3}}{-\frac{5}{3}} = \frac{2}{5}$	-	-	$\frac{-\frac{1}{3}}{-\frac{1}{3}} = 1$	-	-	-
2	z	0	0	-2/5	-1/5	0	12/5	
	x_2	1	0	-3/5	1/5	0	3/5	
	x_1	0	1	4/5	-3/5	0	6/5	
	s_3	0	0	-1	1	1	0	

Thus, the solution is

$x_1 = \frac{6}{5}, x_2 = \frac{3}{5}, z_{min} = \frac{12}{5}$

6. Use the dual simplex method to solve the following L.P.P.

Minimise $z = 2x_1 + 2x_2 + 4x_3$

subject to $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

[N15/CompIT/8M][D23/CompITAI/8M]

Solution:

The standard form,

Min $z - 2x_1 - 2x_2 - 4x_3 + 0s_1 + 0s_2 + 0s_3 = 0$

s.t. $-2x_1 - 3x_2 - 5x_3 + s_1 + 0s_2 + 0s_3 = -2$

$3x_1 + x_2 + 7x_3 + 0s_1 + s_2 + 0s_3 = 3$

$x_1 + 4x_2 + 6x_3 + 0s_1 + 0s_2 + s_3 = 5$

Simplex table,

Iteration No.	Basic Var	Coefficient of						RHS	Formula
		x_1	x_2	x_3	s_1	s_2	s_3		
0	z	-2	**-2**	-4	0	0	0	0	$X - \frac{2}{3}Y$
s_1 leaves x_2 enters	s_1	**-2**	**-3**	**-5**	**1**	**0**	**0**	**-2**	$\frac{Y}{-3}$
	s_2	3	**1**	7	0	1	0	3	$X + \frac{1}{3}Y$
	s_3	1	**4**	6	0	0	1	5	$X + \frac{4}{3}Y$
Ratio		$\frac{-2}{-2} = 1$	$\frac{-2}{-3} = 0.67$	$\frac{-4}{-5} = 0.8$	-	-	-	-	
1	z	-2/3	0	-2/3	-2/3	0	0	4/3	
	x_2	2/3	1	5/3	-1/3	0	0	2/3	
	s_2	7/3	0	16/3	1/3	1	0	7/3	
	s_3	-5/3	0	-2/3	4/3	0	1	7/3	

The solution is, $x_1 = 0, x_2 = \frac{2}{3}, x_3 = 0, z_{min} = \frac{4}{3}$

7. Use the dual simplex method to solve the following L.P.P.

Maximise $z = -3x_1 - 2x_2$

subject to $x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \leq 10$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

[M16/CompIT/8M]

Solution:

Min $z' = -z = 3x_1 + 2x_2$

$z' - 3x_1 - 2x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = 0$

s.t. $-x_1 - x_2 + s_1 + 0s_2 + 0s_3 + 0s_4 = -1$

$x_1 + x_2 + 0s_1 + s_2 + 0s_3 + 0s_4 = 7$

$-x_1 - 2x_2 + 0s_1 + 0s_2 + s_3 + 0s_4 = -10$

$0x_1 + x_2 + 0s_1 + 0s_2 + 0s_3 + s_4 = 3$

Iteration No.	Basic Var	Coefficient of x_1	x_2	s_1	s_2	s_3	s_4	RHS	Formula
0	z'	-3	**-2**	0	0	0	0	0	$X - Y$
s_3 leaves x_2 enters	s_1	-1	**-1**	1	0	0	0	-1	$X - \frac{1}{2}Y$
	s_2	1	**1**	0	1	0	0	7	$X + \frac{1}{2}Y$
	s_3	**-1**	**-2**	**0**	**0**	**1**	**0**	-10	$\frac{Y}{-2}$
	s_4	0	**1**	0	0	0	1	3	$X + \frac{1}{2}Y$
Ratio		$\frac{-3}{-1} = 3$	$\frac{-2}{-2} = 1$	-	-	-	-		
1	z	**-2**	0	0	0	-1	0	10	$X - 4Y$
s_4 leaves x_1 enters	s_1	**-1/2**	0	1	0	-1/2	0	4	$X - Y$
	s_2	**1/2**	0	0	1	1/2	0	2	$X + Y$
	x_2	**1/2**	1	0	0	-1/2	0	5	$X + Y$
	s_4	**-1/2**	**0**	**0**	**0**	**1/2**	**1**	**-2**	$-2Y$
Ratio		$\frac{-2}{-\frac{1}{2}} = 4$	-	-	-	-	-	-	
2	z'	0	0	0	0	-3	-4	18	
	s_1	0	0	1	0	-1	-1	6	
	s_2	0	0	0	1	1	1	0	
	x_2	0	1	0	0	0	1	3	
	x_1	1	0	0	0	-1	-2	4	

Thus the solution is $x_1 = 4, x_2 = 3, z'_{min} = 18, \therefore\ z_{max} = -18$

8. Use dual simplex method, solve

Maximise $z = -2x_1 - x_3$

Subject to $x_1 + x_2 - x_3 \geq 5$

$x_1 - 2x_2 + 4x_3 \geq 8$

$x_1, x_2, x_3 \geq 0$

[M14/CompIT/8M]

Solution:

The standard form,

Min $z' = -z = 2x_1 + x_3$

$z' - 2x_1 + 0x_2 - x_3 + 0s_1 + 0s_2 = 0$

s.t. $-x_1 - x_2 + x_3 + s_1 + 0s_2 = -5$

$-x_1 + 2x_2 - 4x_3 + 0s_1 + s_2 = -8$

Simplex table,

Iteration No.	Basic Var	Coefficient of					RHS	Formula
		x_1	x_2	x_3	s_1	s_2		
0	z'	-2	0	**-1**	0	0	0	$X - \frac{1}{4}Y$
s_2 leaves x_3 enters	s_1	-1	-1	**1**	1	0	-5	$X + \frac{1}{4}Y$
	s_2	**-1**	**2**	**-4**	**0**	**1**	**-8**	$\frac{Y}{-4}$
Ratio		$\frac{-2}{-1} = 2$	-	$\frac{-1}{-4} = 0.25$	-	-	-	
1	z'	-7/4	**-1/2**	0	0	-1/4	2	$X - Y$
s_1 leaves x_2 enters	s_1	**-5/4**	**-1/2**	**0**	**1**	**1/8**	**-7**	$-2Y$
	x_3	1/4	**-1/2**	1	0	-1/4	2	$X - Y$
Ratio		$\frac{7}{5}$	1	–	-	-	-	
2	z'	-1/2	0	0	-1	-3/8	9	
	x_2	5/2	1	0	-2	-1/4	14	
	x_3	3/2	0	1	-1	-3/8	0	

The solution is

$x_1 = 0, x_2 = 14, x_3 = 9, z'_{min} = 9, \therefore\ z_{max} = -9$

9. Use dual simplex method, solve
Minimise $z = 2x_1 - x_2 + 3x_3$
Subject to $3x_1 - x_2 + 3x_3 \leq 7$
$2x_1 - 4x_2 \geq 12$
$x_1, x_2, x_3 \geq 0$
[D24/CompITAI/8M]
Solution:
The standard form,
Min $z = 2x_1 - x_2 + 3x_3 + 0s_1 + 0s_2$
$z - 2x_1 + x_2 - 3x_3 + 0s_1 + 0s_2 = 0$
s.t. $3x_1 - x_2 + 3x_3 + s_1 + 0s_2 = 7$
$-2x_1 + 4x_2 + 0x_3 + 0s_1 + s_2 = -12$
Simplex table,

Iteration No.	Basic Var	Coefficient of					RHS	Formula
		x_1	x_2	x_3	s_1	s_2		
0	z	-2	**1**	-3	0	0	0	$X - \frac{1}{4}Y$
s_2 leaves x_2 enters	s_1	3	**-1**	3	1	0	7	$X + \frac{1}{4}Y$
	s_2	**-2**	**4**	**0**	**0**	**1**	**-12**	$Y \div 4$
Ratio		1	1/4	-	-	-	-	
1	z	**-3/2**	0	-3	0	-1/4	3	$X - 3Y$
s_1 leaves x_2 enters	s_1	**5/2**	0	3	1	1/4	4	$X + 5Y$
	x_2	**-1/2**	**1**	**0**	**0**	**1/4**	**-3**	$-2Y$
Ratio		3	-	-	-	-	-	
2	z	0	-3	-3	0	-1	12	
x_2 leaves x_1 enters	s_1	**0**	**5**	**3**	**1**	**3/2**	**-11**	
	x_1	1	-2	0	0	-1/2	6	
Ratio		-	-	-	-	-	-	

The solution is unbounded

Type V: Duality

1. Write the dual of the following L.P.P.

Maximise $z = 2x_1 - x_2 + 4x_3$

subject to $x_1 + 2x_2 - x_3 \leq 5$

$2x_1 - x_2 + x_3 \leq 6$

$x_1 + x_2 + 3x_3 \leq 10$

$4x_1 + x_3 \leq 12$

$x_1, x_2, x_3 \geq 0$

[N17/CompIT/5M]

Solution:

Primal,

Max $z = 2x_1 - x_2 + 4x_3$

s.t. $x_1 + 2x_2 - x_3 \leq 5$

$2x_1 - x_2 + x_3 \leq 6$

$x_1 + x_2 + 3x_3 \leq 10$

$4x_1 + 0x_2 + x_3 \leq 12$

$x_1, x_2, x_3 \geq 0$

Its dual,

Min $w = 5y_1 + 6y_2 + 10y_3 + 12y_4$

s.t. $y_1 + 2y_2 + y_3 + 4y_4 \geq 2$

$2y_1 - y_2 + y_3 + 0y_4 \geq -1$

$-y_1 + y_2 + 3y_3 + y_4 \geq 4$

$y_1, y_2, y_3 \geq 0$

2. Write the dual of the following L.P.P.

Maximise $z = 4x_1 + 9x_2 + 2x_3$

subject to $2x_1 + 3x_2 + 2x_3 \leq 7$

$3x_1 - 2x_2 + 4x_3 = 5$

$x_1, x_2, x_3 \geq 0$

[M19/MechCivil/5M]

Solution:

Primal,

Max $z = 4x_1 + 9x_2 + 2x_3$

s.t. $2x_1 + 3x_2 + 2x_3 \leq 7$

$3x_1 - 2x_2 + 4x_3 \leq 5$

$3x_1 - 2x_2 + 4x_3 \geq 5$ i.e. $-3x_1 + 2x_2 - 4x_3 \leq -5$

$x_1, x_2, x_3 \geq 0$

Its dual,

Min $w = 7y_1 + 5y_2' - 5y_2''$

s.t. $2y_1 + 3y_2' - 3y_2'' \geq 4$

$3y_1 - 2y_2' + 2y_2'' \geq 9$

$2y_1 + 4y_2' - 4y_2'' \geq 2$

$y_1, y_2', y_2'' \geq 0$

Its dual,

Min $w = 7y_1 + 5y_2$

s.t. $2y_1 + 3y_2 \geq 4$

$3y_1 - 2y_2 \geq 9$

$2y_1 + 4y_2 \geq 2$

$y_1 \geq 0, y_2$ unrestricted

3. Construct dual of the following LPP:

Maximise $z = 8x_1 + 3x_2$

Subject to $x_1 - 6x_2 \geq 2$

$5x_1 + 7x_2 = -4$

$x_1, x_2 \geq 0$

Solution:

Max $z = 8x_1 + 3x_2$

s.t. $-x_1 + 6x_2 \leq -2$

$5x_1 + 7x_2 \leq -4$

$5x_1 + 7x_2 \geq -4$

$x_1, x_2 \geq 0$

Primal,

Max $z = 8x_1 + 3x_2$

s.t. $-x_1 + 6x_2 \leq -2$

$5x_1 + 7x_2 \leq -4$

$-5x_1 - 7x_2 \leq 4$

$x_1, x_2 \geq 0$

Its dual,

Min $w = -2y_1 - 4y_2' + 4y_2''$

s.t. $-y_1 + 5y_2' - 5y_2'' \geq 8$

$6y_1 + 7y_2' - 7y_2'' \geq 3$

$y_1, y_2', y_2'' \geq 0$

Dual,

Min $w = -2y_1 - 4y_2$

s.t. $-y_1 + 5y_2 \geq 8$

$6y_1 + 7y_2 \geq 3$

$y_1 \geq 0, y_2$ unrestricted

4. Construct the dual of the following LPP:

Maximize $z = x_1 + 3x_2 - 2x_3 + 5x_4$

Subject to $3x_1 - x_2 + x_3 - 4x_4 = 6$

$5x_1 + 3x_2 - x_3 - 2x_4 = 4$

$x_1, x_2 \geq 0, x_3, x_4$ unrestricted

Solution:

Max $z = x_1 + 3x_2 - 2(x_3' - x_3'') + 5(x_4' - x_4'')$

s.t. $3x_1 - x_2 + (x_3' - x_3'') - 4(x_4' - x_4'') \leq 6$

$3x_1 - x_2 + (x_3' - x_3'') - 4(x_4' - x_4'') \geq 6$

$5x_1 + 3x_2 - (x_3' - x_3'') - 2(x_4' - x_4'') \leq 4$

$5x_1 + 3x_2 - (x_3' - x_3'') - 2(x_4' - x_4'') \geq 4$

$x_1, x_2, x_3', x_3'', x_4', x_4'' \geq 0$

Primal,

Max $z = x_1 + 3x_2 - 2x_3' + 2x_3'' + 5x_4' - 5{x_4}''$

s.t. $3x_1 - x_2 + x_3' - x_3'' - 4x_4' + 4x_4'' \leq 6$

$-3x_1 + x_2 - x_3' + x_3'' + 4x_4' - 4x_4'' \leq -6$

$5x_1 + 3x_2 - x_3' + x_3'' - 2x_4' + 2x_4'' \leq 4$

$-5x_1 - 3x_2 + x_3' - x_3'' + 2x_4' - 2x_4'' \leq -4$

$x_1, x_2, x_3', x_3'', x_4', x_4'' \geq 0$

Dual,

Min $w = 6y_1' - 6y_1'' + 4y_2' - 4{y_2}''$

s.t. $3y_1' - 3y_1'' + 5y_2' - 5y_2'' \geq 1$

$-y_1' + y_1'' + 3y_2' - 3y_2'' \geq 3$

$y_1' - y_1'' - y_2' + y_2'' \geq -2$

$-y_1' + y_1'' + y_2' - y_2'' \geq 2$

$-4y_1' + 4y_1'' - 2y_2' + 2y_2'' \geq 5$

$4y_1' - 4y_1'' + 2y_2' - 2y_2'' \geq -5$

$y_1', y_1'', y_2', y_2'' \geq 0$

Min $w = 6y_1 + 4y_2$

s.t. $3y_1 + 5y_2 \geq 1$

$-y_1 + 3y_2 \geq 3$

$y_1 - y_2 \geq -2$

$-y_1 + y_2 \geq 2$ i.e. $y_1 - y_2 \leq -2$

$-4y_1 - 2y_2 \geq 5$

$4y_1 + 2y_2 \geq -5$ i.e. $-4y_1 - 2y_2 \leq 5$

y_1, y_2 unrestricted.

Dual,

Min $w = 6y_1 + 4y_2$

s.t. $3y_1 + 5y_2 \geq 1$

$-y_1 + 3y_2 \geq 3$

$y_1 - y_2 = -2$

$-4y_1 - 2y_2 = 5$

y_1, y_2 unrestricted

5. Write the dual of the following L.P.P.

Maximise $z = 2x_1 - x_2 + 3x_3$

subject to $x_1 - 2x_2 + x_3 \geq 4$

$2x_1 + x_3 \leq 10$

$x_1 + x_2 + 3x_3 = 20$

$x_1, x_3 \geq 0, x_2$ unrestricted

[M15/CompIT/5M]

Solution:

Primal,

Max $z = 2x_1 - x_2' + x_2'' + 3x_3$

s.t. $x_1 - 2x_2' + 2x_2'' + x_3 \geq 4$

$2x_1 + 0x_2' + 0x_2'' + x_3 \leq 10$

$x_1 + x_2' - x_2'' + 3x_3 \geq 20$

$x_1 + x_2' - x_2'' + 3x_3 \leq 20$

$x_1, x_2', x_2'', x_3 \geq 0$

Primal,

Max $z = 2x_1 - x_2' + x_2'' + 3x_3$

s.t. $-x_1 + 2x_2' - 2x_2'' - x_3 \leq -4$

$2x_1 + 0x_2' + 0x_2'' + x_3 \leq 10$

$-x_1 - x_2' + x_2'' - 3x_3 \leq -20$

$x_1 + x_2' - x_2'' + 3x_3 \leq 20$

$x_1, x_2', x_2'', x_3 \geq 0$

Its dual,

Min $w = -4y_1 + 10y_2 - 20y_3' + 20y_3''$

s.t. $-y_1 + 2y_2 - y_3' + y_3'' \geq 2$

$2y_1 + 0y_2 - y_3' + y_3'' \geq -1$

$-2y_1 + 0y_2 + y_3' - y_3'' \geq 1$

$-y_1 + y_2 - 3y_3' + 3y_3'' \geq 3$

$y_1, y_2', y_2'', y_3 \geq 0$

Its dual,

Min $w = -4y_1 + 10y_2 - 20y_3$

s.t. $-y_1 + 2y_2 - y_3 \geq 2$

$2y_1 + 0y_2 - y_3 \geq -1$

$-2y_1 + 0y_2 + y_3 \geq 1$

$-y_1 + y_2 - 3y_3 \geq 3$

$y_1, y_2 \geq 0$, y_3 unretsricted

Its dual,

Min $\quad w = -4y_1 + 10y_2 - 20y_3$

s.t. $\quad -y_1 + 2y_2 - y_3 \geq 2$

$2y_1 + 0y_2 - y_3 = -1$

$-y_1 + y_2 - 3y_3 \geq 3$

$y_1, y_2 \geq 0$, y_3 unretsricted

6. Find dual of the following LP model

Maximise $z = 2x_1 + 3x_2 + 5x_3$

Subject to $x_1 + x_2 - x_3 \geq -5$

$x_1 + x_2 + 4x_3 = 10$

$-6x_1 + 7x_2 - 9x_3 \leq 4$

$x_1, x_2 \geq 0, x_3$ unrestricted

[M14/CompIT/5M]

Solution:

Primal,

Max $z = 2x_1 + 3x_2 + 5x_3' - 5x_3''$

s.t. $x_1 + x_2 - x_3' + x_3'' \geq -5$

$x_1 + x_2 + 4x_3' - 4x_3'' \leq 10$

$x_1 + x_2 + 4x_3' - 4x_3'' \geq 10$

$-6x_1 + 7x_2 - 9x_3' + 9x_3'' \leq 4$

$x_1, x_2, x_3', x_3'' \geq 0$

Primal,

Max $z = 2x_1 + 3x_2 + 5x_3' - 5x_3''$

s.t. $-x_1 - x_2 + x_3' - x_3'' \leq 5$

$x_1 + x_2 + 4x_3' - 4x_3'' \leq 10$

$-x_1 - x_2 - 4x_3' + 4x_3'' \leq -10$

$-6x_1 + 7x_2 - 9x_3' + 9x_3'' \leq 4$

Its dual,

Min $w = 5y_1 + 10y_2' - 10y_2'' + 4y_3$

s.t. $-y_1 + y_2' - y_2'' - 6y_3 \geq 2$

$-y_1 + y_2' - y_2'' + 7y_3 \geq 3$

$y_1 + 4y_2' - 4y_2'' - 9y_3 \geq 5$

$-y_1 - 4y_2' + 4y_2'' + 9y_3 \geq -5$

$y_1, y_2', y_2'', y_3 \geq 0$

Its dual,

Min $w = 5y_1 + 10y_2 + 4y_3$

s.t. $-y_1 + y_2 - 6y_3 \geq 2$

$-y_1 + y_2 + 7y_3 \geq 3$

$y_1 + 4y_2 - 9y_3 \geq 5$

$-y_1 - 4y_2 + 9y_3 \geq -5$

$y_1, y_3 \geq 0, y_2$ is unrestricted

Its dual,

Min $w = 5y_1 + 10y_2 + 4y_3$

s.t. $-y_1 + y_2 - 6y_3 \geq 2$

$-y_1 + y_2 + 7y_3 \geq 3$

$y_1 + 4y_2 - 9y_3 = 5$

$y_1, y_3 \geq 0, y_2$ is unrestricted

7. Construct the dual of the following LPP

Maximise $z = 3x_1 + 17x_2 + 9x_3$

Subject to $x_1 - x_2 + x_3 \geq 3$

$-3x_1 + 2x_3 \leq 1$

$2x_1 + x_2 - 5x_3 = 1$

$x_1, x_2 x_3 \geq 0$

[M17/CompIT/5M]

Solution:

Primal,

Max $z = 3x_1 + 17x_2 + 9x_3$

s.t. $-x_1 + x_2 - x_3 \leq -3$

$-3x_1 + 0x_2 + 2x_3 \leq 1$

$2x_1 + x_2 - 5x_3 \leq 1$

$-2x_1 - x_2 + 5x_3 \leq -1$

$x_1, x_2, x_3 \geq 0$

Its dual,

Min $w = -3y_1 + y_2 + y_3' - y_3''$

s.t. $-y_1 - 3y_2 + 2y_3' - 2y_3'' \geq 3$

$y_1 + 0y_2 + y_3' - y_3'' \geq 17$

$-y_1 + 2y_2 - 5y_3' + 5y_3'' \geq 9$

$y_1, y_2, y_3', y_3'' \geq 0$

Its dual,

Min $w = -3y_1 + y_2 + y_3$

s.t. $-y_1 - 3y_2 + 2y_3 \geq 3$

$y_1 + 0y_2 + y_3 \geq 17$

$-y_1 + 2y_2 - 5y_3 \geq 9$

$y_1, y_2 \geq 0$ and y_3 unrestricted

8. Write dual of the given LPP

Minimize $z = 2x_1 + 3x_2 + 4x_3$

Subject to $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 = 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_3 \geq 0$ and x_2 is unrestricted

[M18/MechCivil/5M]

Solution:

Primal,

Minimize $z = 2x_1 + 3(x_2' - x_2'') + 4x_3$

Subject to $2x_1 + 3(x_2' - x_2'') + 5x_3 \geq 2$

$3x_1 + (x_2' - x_2'') + 7x_3 \geq 3$

$3x_1 + (x_2' - x_2'') + 7x_3 \leq 3$

$x_1 + 4(x_2' - x_2'') + 6x_3 \leq 5$

$x_1, x_3, x_2', x_2'' \geq 0$

Primal,

Minimize $z = 2x_1 + 3x_2' - 3x_2'' + 4x_3$

Subject to $2x_1 + 3x_2' - 3x_2'' + 5x_3 \geq 2$

$3x_1 + x_2' - x_2'' + 7x_3 \geq 3$

$-3x_1 - x_2' + x_2'' - 7x_3 \geq -3$

$-x_1 - 4x_2' + 4x_2'' - 6x_3 \geq -5$

$x_1, x_3, x_2', x_2'' \geq 0$

Its dual,

Maximise $w = 2y_1 + 3y_2' - 3y_2'' - 5y_3$

Subject to $2y_1 + 3y_2' - 3y_2'' - y_3 \leq 2$

$3y_1 + y_2' - y_2'' - 4y_3 \leq 3$

$-3y_1 - y_2' + y_2'' + 4y_3 \leq -3$

$5y_1 + 7y_2' - 7y_2'' - 6y_3 \leq 4$

$y_1, y_2', y_2'', y_3 \geq 0$

Its dual,

Maximise $w = 2y_1 + 3y_2 - 5y_3$

Subject to $2y_1 + 3y_2 - y_3 \leq 2$

$3y_1 + y_2 - 4y_3 = 3$

$5y_1 + 7y_2 - 6y_3 \leq 4$

$y_1, y_3 \geq 0$ and y_2 is unrestricted

9. Write the dual of the following problem

Maximise $z = 3x_1 + 10x_2 + 2x_3$

subject to $2x_1 + 3x_2 + 2x_3 \leq 8$

$3x_1 - 2x_2 + 4x_3 = 4$

$x_1, x_2, x_3 \geq 0$

[N22/CompITAI/5M]

Solution:

Primal,

Maximise $z = 3x_1 + 10x_2 + 2x_3$

Subject to $2x_1 + 3x_2 + 2x_3 \leq 8$

$3x_1 - 2x_2 + 4x_3 \leq 4$

$3x_1 - 2x_2 + 4x_3 \geq 4$

$x_1, x_2, x_3 \geq 0$

Primal,

Maximise $z = 3x_1 + 10x_2 + 2x_3$

Subject to $2x_1 + 3x_2 + 2x_3 \leq 8$

$3x_1 - 2x_2 + 4x_3 \leq 4$

$-3x_1 + 2x_2 - 4x_3 \leq -4$

$x_1, x_2, x_3 \geq 0$

Its dual,

Minimise $w = 8y_1 + 4y_2' - 4y_2''$

Subject to $2y_1 + 3y_2' - 3y_2'' \geq 3$

$3y_1 - 2y_2' + 2y_2'' \geq 10$

$2y_1 + 4y_2' - 4y_2'' \geq 2$

$y_1, y_2', y_2'' \geq 0$

Its dual,

Minimise $w = 8y_1 + 4y_2$

Subject to $2y_1 + 3y_2 \geq 3$

$3y_1 - 2y_2 \geq 10$

$2y_1 + 4y_2 \geq 2$

$y_1 \geq 0$ and y_2 is unrestricted

10. Construct dual of the following LPP and solve its dual

Minimise $z = 0.7x_1 + 0.5x_2$

Subject to $x_1 \geq 4$

$x_2 \geq 6$

$x_1 + 2x_2 \geq 20$

$2x_1 + x_2 \geq 18$

$x_1, x_2 \geq 0$

[M19/MechCivil/8M]

Solution:

Its dual,

Max $z = 4y_1 + 6y_2 + 20y_3 + 18y_4$

s.t. $y_1 + 0y_2 + y_3 + 2y_4 \leq 0.7$

$0y_1 + y_2 + 2y_3 + y_4 \leq 0.5$

$y_1, y_2 \geq 0$

Standard form,

Max $z = 4y_1 + 6y_2 + 20y_3 + 18y_4 + 0s_1 + 0s_2$

$z - 4y_1 - 6y_2 - 20y_3 - 18y_4 + 0s_1 + 0s_2 = 0$

s.t. $y_1 + 0y_2 + y_3 + 2y_4 + s_1 + 0s_2 = 0.7$

$0y_1 + y_2 + 2y_3 + y_4 + 0s_1 + s_2 = 0.5$

Simplex table,

Iteration No.	Basic Var	Coefficient of						RHS	Ratio	Formula
		y_1	y_2	y_3	y_4	s_1	s_2			
0	z	-4	-6	**-20**	-18	0	0	0		$X + 10Y$
s_2 leaves y_3 enters	s_1	1	0	**1**	2	1	0	0.7	$\frac{0.7}{1} = 0.7$	$X - \frac{1}{2}Y$
	s_2	**0**	**1**	**2**	**1**	**0**	**1**	**0.5**	$\frac{0.5}{2} = 0.25$	$\frac{Y}{2}$
1	z	-4	4	0	**-8**	0	10	5		$X + \frac{8}{1.5}Y$
s_1 leaves y_4 enters	s_1	**1**	**-0.5**	**0**	**1.5**	**1**	**-0.5**	**0.45**	$\frac{0.45}{1.5} = 0.3$	$\frac{Y}{1.5}$
	y_3	0	0.5	1	**0.5**	0	0.5	0.25	$\frac{0.25}{0.5} = 0.5$	$X - \frac{0.5}{1.5}Y$
2	z	$\frac{4}{3}$	$\frac{4}{3}$	0	0	$\frac{16}{3}$	$\frac{22}{3}$	7.4		
	y_4	$\frac{2}{3}$	$-\frac{1}{3}$	0	1	$\frac{2}{3}$	$-\frac{1}{3}$	0.3		
	y_3	$-\frac{1}{3}$	$\frac{2}{3}$	1	0	$-\frac{1}{3}$	$\frac{2}{3}$	0.1		

Thus, the solution is

$x_1 = \frac{16}{3}, x_2 = \frac{22}{3}, z_{min} = 7.4$

11. Using Duality solve the following L.P.P.

Minimize $z = 4x_1 + 3x_2 + 6x_3$

subject to $x_1 + x_3 \geq 2$

$x_2 + x_3 \geq 5$

$x_1, x_2, x_3 \geq 0$

Solution:

Min $z = 4x_1 + 3x_2 + 6x_3$

s.t. $x_1 + 0x_2 + x_3 \geq 2$

$0x_1 + x_2 + x_3 \geq 5$

Its dual,

Max $w = 2y_1 + 5y_2$

s.t. $y_1 + 0y_2 \leq 4$

$0y_1 + y_2 \leq 3$

$y_1 + y_2 \leq 6$

$y_1, y_2 \geq 0$

Converting into standard form,

Max $w = 2y_1 + 5y_2 + 0s_1 + 0s_2 + 0s_3$

$w - 2y_1 - 5y_2 - 0s_1 - 0s_2 - 0s_3 = 0$

s.t. $y_1 + 0y_2 + s_1 = 4$

$0y_1 + y_2 + s_2 = 3$

$y_1 + y_2 + s_3 = 6$

Iteration No.	Basic Var	Coefficient of					RHS	Ratio	Formula
		y_1	y_2	s_1	s_2	s_3			
0	w	-2	**-5**	0	0	0	0	-	$X + 5Y$
s_2 leaves y_2 enters	s_1	1	**0**	1	0	0	4	-	-
	s_2	**0**	**1**	**0**	**1**	**0**	**3**	3	-
	s_3	1	**1**	0	0	1	6	6	$X - Y$
1	w	**-2**	0	0	5	0	15	-	$X + 2Y$
s_3 leaves y_1 enters	s_1	**1**	0	1	0	0	4	4	$X - Y$
	y_2	**0**	1	0	1	0	3	-	-
	s_3	**1**	**0**	**0**	**-1**	**1**	**3**	3	-
2	w	0	0	0	3	2	21		
	s_1	0	0	1	1	-1	1		
	y_2	0	1	0	1	0	3		
	y_1	1	0	0	-1	1	3		

$w_{max} = z_{min} = 21$
$x_1 = s_1 = 0, x_2 = s_2 = 3, x_3 = s_3 = 2$

12. Using Duality to solve

Minimise $z = 4x_1 + 14x_2 + 3x_3$
Subject to $x_1 - 3x_2 - x_3 \leq -3$
$2x_1 + 2x_2 - x_3 \geq 2$
$x_1, x_2, x_3 \geq 0$

Solution:

Min $z = 4x_1 + 14x_2 + 3x_3$
s.t. $-x_1 + 3x_2 + x_3 \geq 3$
$2x_1 + 2x_2 - x_3 \geq 2$

Its dual,

Max $w = 3y_1 + 2y_2$
s.t. $-y_1 + 2y_2 \leq 4$
$3y_1 + 2y_2 \leq 14$
$y_1 - y_2 \leq 3$
$y_1, y_2 \geq 0$

Converting into standard form,

Max $w = 3y_1 + 2y_2 + 0s_1 + 0s_2 + 0s_3$
$w - 3y_1 - 2y_2 - 0s_1 - 0s_2 - 0s_3 = 0$
s.t. $-y_1 + 2y_2 + s_1 = 4$
$3y_1 + 2y_2 + s_2 = 14$
$y_1 - y_2 + s_3 = 3$

Simplex Table:

Iteration No.	Basic Var	Coefficient of					RHS	Ratio	Formula
		y_1	y_2	s_1	s_2	s_3			
0	w	**-3**	-2	0	0	0	0	-	$X-(-3)Y$
	s_1	**-1**	2	1	0	0	4	-	$X-(-1)Y$
s_3 leaves y_1 enters	s_2	**3**	2	0	1	0	14	$\frac{14}{3}$	$X-3Y$
	s_3	**1**	**-1**	**0**	**0**	**1**	**3**	$\frac{3}{1}=3$	-
1	w	0	**-5**	0	0	3	9	-	$X-(-1)Y$
	s_1	0	**1**	1	0	1	7	$\frac{7}{1}=7$	$X-\frac{1}{5}Y$
s_2 leaves y_2 enters	s_2	**0**	**5**	**0**	**1**	**-3**	**5**	$\frac{5}{5}=1$	$\frac{Y}{5}$
	y_1	1	**-1**	0	0	1	3	-	$X-\frac{-1}{5}Y$
2	w	0	0	0	1	0	14		
	s_1	0	0	1	-1/5	8/5	6		
	y_2	0	1	0	1/5	-3/5	1		
	y_1	1	0	0	1/5	2/5	4		

$w_{max} = z_{min} = 14$

$x_1 = s_1 = 0, x_2 = s_2 = 1, x_3 = s_3 = 0$

13. Using Duality solve the following L.P.P.

Maximise $z = 5x_1 + 8x_2$

subject to $x_1 + x_2 \leq 2$

$x_1 + 2x_2 \geq 0$

$-x_1 + 4x_2 \leq 1$

$x_1, x_2 \geq 0$

Solution:

Max $z = 5x_1 + 8x_2$

s.t. $x_1 + x_2 \leq 2$

$-x_1 - 2x_2 \leq 0$

$-x_1 + 4x_2 \leq 1$

Its dual,

Min $w = 2y_1 + 0y_2 + y_3$
s.t. $y_1 - y_2 - y_3 \geq 5$
$y_1 - 2y_2 + 4y_3 \geq 8$
Min $w = 2y_1 + 0y_2 + y_3$
s.t. $-y_1 + y_2 + y_3 \leq -5$
$-y_1 + 2y_2 - 4y_3 \leq -8$
converting into standard form
Min $w = 2y_1 + 0y_2 + y_3 + 0s_1 + 0s_2$
$w - 2y_1 + 0y_2 - y_3 - 0s_1 - 0s_2 = 0$
s.t. $-y_1 + y_2 + y_3 + s_1 = -5$
$-y_1 + 2y_2 - 4y_3 + s_2 = -8$

Iteration No.	Basic Var	Coefficient of					RHS	Formula
		y_1	y_2	y_3	s_1	s_2		
0	w	-2	0	**-1**	0	0	0	$X - \frac{1}{4}Y$
s_2 leaves y_3 enters	s_1	-1	1	**1**	1	0	-5	$X - \frac{1}{-4}Y$
	s_2	**-1**	**2**	**-4**	**0**	**1**	-8	$\frac{Y}{-4}$
Ratio		2	-	$\frac{1}{4}$	-	-		
1	w	**-7/4**	-1/2	0	0	-1/4	2	$X - \frac{7}{5}Y$
s_1 leaves y_1 enters	s_1	**-5/4**	**3/2**	**0**	**1**	**1/4**	-7	$-\frac{4}{5}Y$
	y_3	**1/4**	1/2	1	0	-1/4	2	$X - \frac{1}{-5}Y$
Ratio		$\frac{7}{5}$	-	-	-	-		
2	w	0	-13/5	0	-7/5	-3/5	59/5	
	y_1	1	-6/5	0	-4/5	-1/5	28/5	
	y_3	0	4/5	1	1/5	-1/5	3/5	

$w_{min} = z_{max} = \frac{59}{5}$

$x_1 = -s_1 = -\left(-\frac{7}{5}\right) = \frac{7}{5}$ and $x_2 = -s_2 = -\left(-\frac{3}{5}\right) = \frac{3}{5}$

14. Using Duality to solve

Maximise $z = 3x_1 + 4x_2$

Subject to $2x_1 + x_2 \le 5$

$x_1 + x_2 \le 3$

$x_1, x_2 \ge 0$

[N19/MechCivil/8M]

Solution:

The standard form,

Max $z = 3x_1 + 4x_2$

s.t. $2x_1 + x_2 \le 5$

$x_1 + x_2 \le 3$

Its dual,

Min $w = 5y_1 + 3y_2$

s.t. $2y_1 + y_2 \ge 3$

$y_1 + y_2 \ge 4$

Max $w' = -w = -5y_1 - 3y_2$

$w' + 5y_1 + 3y_2 + 0s_1 + 0s_2 + MA_1 + MA_2 = 0$ ………(1)

s.t. $2y_1 + y_2 - s_1 + 0s_2 + A_1 + 0A_2 = 3$ …………(2)

$y_1 + y_2 + 0s_1 - s_2 + 0A_1 + A_2 = 4$ ……….(3)

Multiplying eqn (2) & (3) by M and subtracting from eqn (1),

$w' + (5 - 3M)y_1 + (3 - 2M)y_2 + Ms_1 + Ms_2 + 0A_1 + 0A_2 = -7M$

Iteration No.	Basic Var	Coefficient of						RHS	Ratio	Formula
		y_1	y_2	s_1	s_2	A_1	A_2			
0	w'	**5–3M**	3 – 2M	M	M	0	0	-7M	-	$X - \frac{5-3M}{2}Y$
A_1 leaves	A_1	**2**	**1**	**-1**	**0**	**1**	**0**	**3**	1.5	$\frac{Y}{2}$
y_1 enters	A_2	**1**	1	0	-1	0	1	4	4	$X - \frac{Y}{2}$
1	w'	0	$\frac{1}{2} - \frac{M}{2}$	$\frac{5}{2} - \frac{M}{2}$	M		0	$-\frac{15}{2} - \frac{5M}{2}$	-	$X - (1 - M)Y$
y_1 leaves	y_1	**1**	$\frac{1}{2}$	$-\frac{1}{2}$	**0**		**0**	$\frac{3}{2}$	3	$2Y$
y_2 enters	A_2	0	$\frac{1}{2}$	$\frac{1}{2}$	-1		1	$\frac{5}{2}$	5	$X - Y$
2	w'	-1+M	0	**3-M**	M		0	-9-M	-	$X - (3 - M)Y$
A_2 leaves	y_2	2	1	**-1**	0		0	3	-	$X + Y$
s_1 enters	A_2	**-1**	**0**	**1**	**-1**		**1**	**1**	1	-
3	w'	2	0	0	3			-12		
	y_2	1	1	0	-1			4		
	s_1	-1	0	1	-1			1		

The solution,

$s_1 = 0, s_2 = 3, w'_{max} = -12, w_{min} = 12$

$\therefore x_1 = 0, x_2 = 3, z_{max} = 12$

15. Using Duality solve the following L.P.P

Minimise $z = 430x_1 + 460x_2 + 420x_3$

Subject to $x_1 + 3x_2 + 4x_3 \geq 3$

$2x_1 + 4x_3 \geq 2$

$x_1 + 2x_2 \geq 5$

$x_1, x_2, x_3 \geq 0$

Solution:

Min $z = 430x_1 + 460x_2 + 420x_3$

s.t. $x_1 + 3x_2 + 4x_3 \geq 3$

$2x_1 + 0x_2 + 4x_3 \geq 2$

$x_1 + 2x_2 + 0x_3 \geq 5$

Its dual,

Max $w = 3y_1 + 2y_2 + 5y_3$

s.t. $y_1 + 2y_2 + y_3 \leq 430$

$3y_1 + 0y_2 + 2y_3 \leq 460$

$4y_1 + 4y_2 + 0y_3 \leq 420$

$y_1, y_2, y_3 \geq 0$

Converting into standard form,

Max $w = 3y_1 + 2y_2 + 5y_3 + 0s_1 + 0s_2 + 0s_3$

$w - 3y_1 - 2y_2 - 5y_3 - 0s_1 - 0s_2 - 0s_3 = 0$

s.t. $y_1 + 2y_2 + y_3 + s_1 = 430$

$3y_1 + 0y_2 + 2y_3 + s_2 = 460$

$4y_1 + 4y_2 + 0y_3 + s_3 = 420$

Simplex Table:

Iteration No.	Basic Var	Coefficient of						RHS	Ratio	Formula
		y_1	y_2	y_3	s_1	s_2	s_3			
0	w	-3	-2	**-5**	0	0	0	0	-	$X+\frac{5}{2}Y$
s_2 leaves y_3 enters	s_1	1	2	**1**	1	0	0	430	430	$X-\frac{1}{2}Y$
	s_2	**3**	**0**	**2**	**0**	**1**	**0**	**460**	230	$Y \div 2$
	s_3	4	4	**0**	0	0	1	420	-	-
1	w	9/2	**-2**	0	0	5/2	0	1150	-	$X+Y$
s_1 leaves y_2 enters	s_1	**-1/2**	**2**	**0**	**1**	**-1/2**	**0**	**200**	100	$Y \div 2$
	y_3	3/2	**0**	1	0	1/2	0	230	-	-
	s_3	4	**4**	0	0	0	1	420	105	$X-2Y$
2	w	4	0	0	1	2	0	1350		
	y_2	-1/4	1	0	1/2	-1/4	0	100		
	y_3	3/2	0	1	0	1/2	0	230		
	s_3	5	0	0	-2	1	1	20		

$w_{max} = z_{min} = 1350$

$x_1 = s_1 = 1, x_2 = s_2 = 2, x_3 = s_3 = 0$

16. Using Duality solve the following L.P.P.

Maximise $z = 2x_1 + x_2$

subject to $2x_1 - x_2 \le 2$

$x_1 + x_2 \le 4$

$x_1 \le 3$

$x_1, x_2 \ge 0$

[N14/CompIT/6M]

Solution:

The standard form,

Max $z = 2x_1 + x_2$

s.t. $2x_1 - x_2 \le 2$

$x_1 + x_2 \le 4$

$x_1 + 0x_2 \le 3$

Its dual,

Min $w = 2y_1 + 4y_2 + 3y_3$

s.t. $2y_1 + y_2 + y_3 \ge 2$

$-y_1 + y_2 + 0y_3 \ge 1$

Max $w' = -w = -2y_1 - 4y_2 - 3y_3$

$w' + 2y_1 + 4y_2 + 3y_3 + 0s_1 + 0s_2 + MA_1 + MA_2 = 0$ ....(1)

s.t. $2y_1 + y_2 + y_3 - s_1 + 0s_2 + A_1 + 0A_2 = 2$ ....(2)

$-y_1 + y_2 + 0y_3 + 0s_1 - s_2 + 0A_1 + A_2 = 1$ ....(3)

Multiplying eqn (2) & (3) by M and subtracting from eqn (1),

$w' + (2 - M)y_1 + (4 - 2M)y_2 + (3 - M)y_3 + Ms_1 + Ms_2 + 0A_1 + 0A_2 = -3M$

Simplex table,

Iteration No.	Basic Var	Coefficient of							RHS	Ratio	Formula
		y_1	y_2	y_3	s_1	s_2	A_1	A_2			
0	w'	2-M	**4-2M**	3-M	M	M	0	0	-3M	-	$X - (4 - 2M)Y$
A_2 leaves y_2 enters	A_1	2	**1**	1	-1	0	1	0	2	2	$X - Y$
	A_2	**-1**	**1**	**0**	**0**	**-1**	**0**	**1**	1	1	-
1	w'	**6-3M**	0	3-M	M	4-M	0		-4-M	-	$X - \frac{(6-3M)}{3}Y$
A_1 leaves y_1 enters	A_1	**3**	**0**	**1**	**-1**	**1**	**1**		1	0.33	$\frac{Y}{3}$
	y_2	**-1**	1	0	0	-1	0		1	-	$X + \frac{1}{3}Y$
2	w'	0	0	1	2	2			-6		
	y_1	1/3	0	1/3	-1/3	1/3			1/3		
	y_2	0	1	1/3	-1/3	-2/3			4/3		

the solution is

$s_1 = 2, s_2 = 2, w'_{max} = -6, w_{min} = 6$

$\therefore x_1 = 2, x_2 = 2, z_{max} = 6$

17. Using Duality solve the following L.P.P

Maximise $z = 5x_1 - 2x_2 + 3x_3$

Subject to $2x_1 + 2x_2 - x_3 \geq 2$

$3x_1 - 4x_2 \leq 3$

$x_1 + 3x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

[M15/CompIT/6M]

Solution:

The standard form,

Max $z = 5x_1 - 2x_2 + 3x_3$

s.t. $-2x_1 - 2x_2 + x_3 \leq -2$

$3x_1 - 4x_2 + 0x_3 \leq 3$

$x_1 + 0x_2 + 3x_3 \leq 5$

Its dual,

Min $w = -2y_1 + 3y_2 + 5y_3$

s.t. $-2y_1 + 3y_2 + y_3 \geq 5$

$-2y_1 - 4y_2 + 0y_3 \geq -2$ i.e. $2y_1 + 4y_2 + 0y_3 \leq 2$

$y_1 + 0y_2 + 3y_3 \geq 3$

Max $w' = -w = 2y_1 - 3y_2 - 5y_3$

$w' - 2y_1 + 3y_2 + 5y_3 + 0s_1 + 0s_2 + 0s_3 + MA_1 + MA_3 = 0$....(1)

s.t. $-2y_1 + 3y_2 + y_3 - s_1 + 0s_2 + 0s_3 + A_1 + 0A_3 = 5$ …………(2)

$2y_1 + 4y_2 + 0y_3 + 0s_1 + s_2 + 0s_3 + 0A_1 + 0A_3 = 2$ ……….(3)

$y_1 + 0y_2 + 3y_3 + 0s_1 + 0s_2 - s_3 + 0A_1 + A_3 = 3$……..(4)

Multiplying eqn (2) & (4) by M and subtracting from eqn (1),

$w' + (-2 + M)y_1 + (3 - 3M)y_2 + (5 - 4M)y_3 + Ms_1 + 0s_2 + Ms_3 + 0A_1 + 0A_3 = -8M$

Simplex table,

Iteration No.	Basic Var	Coefficient of								RHS	Ratio	Formula
		y_1	y_2	y_3	s_1	s_2	s_3	A_1	A_3			
0	w'	-2+M	3-3M	**5-4M**	M	0	M	0	0	-8M	-	$X - \frac{5-4M}{3}Y$
A_3 leaves y_3 enters	A_1	-2	3	**1**	-1	0	0	1	0	5	5	$X - \frac{1}{3}Y$
	s_2	2	4	**0**	0	1	0	0	0	2	-	-
	A_3	**1**	**0**	**3**	**0**	**0**	**-1**	**0**	**1**	**3**	1	$\frac{Y}{3}$
1	w'	$\frac{-11+7M}{3}$	**3-3M**	0	M	0	$\frac{5-M}{3}$	0		-5-4M	-	$X - \frac{3-3M}{4}Y$
s_2 leaves y_2 enters	A_1	-7/3	**3**	0	-1	0	1/3	1		4	1.33	$X - \frac{3}{4}Y$
	s_2	**2**	**4**	**0**	**0**	**1**	**0**	**0**		**2**	0.5	$\frac{Y}{4}$
	y_3	1/3	**0**	1	0	0	-1/3	0		1	-	-
2	w'	$\frac{-31+23M}{6}$	0	0	M	$\frac{-3+3M}{4}$	$\frac{5-M}{3}$	0		$\frac{-13-5M}{2}$	-	$X - (5 - M)Y$
A_1 leaves s_3 enters	A_1	**-23/6**	**0**	**0**	**-1**	**-3/4**	**1/3**	**1**		**5/2**	7.5	$3Y$
	s_2	1/2	1	0	0	1/4	**0**	0		1/2	-	-
	y_3	1/3	0	1	0	0	**-1/3**	0		1	-	$X + Y$

3	w'	14	0	0	5	3	0			-19	-	
	s_3	-23/2	0	0	-3	-9/4	1			15/2	-	
	s_2	1/2	1	0	0	1/4	0			1/2	1	
	y_3	-7/2	0	1	-1	-3/4	0			7/2	-	

The solution is

$s_1 = 5, s_2 = 3, s_3 = 0, w'_{max} = -19, w_{min} = 19$

$\therefore x_1 = 5, x_2 = 3, x_3 = 0, z_{max} = 19$

18. If the primal LPP has an unbounded solution then the dual has
[M22/CompITAI/2M] Ans. Infeasible solution