Important Distributions

Weight Distribution of Types

MechCivil

Type	Name	May 2017	Nov 2017	May 2018	Nov 2018	May 2019	Nov 2019	May 2022	Nov 2022	May 2023	Dec 2023	May 2024	Dec 2024
I	Poisson	06	06	06	---	---	06	12	05	---	06	06	---
II	Normal	06	06	---	06	06	06	07	08	08	08	08	08
Total Marks		**12**	**12**	**06**	**06**	**06**	**12**	**19**	**13**	**08**	**14**	**14**	**08**

Comp/IT/AI

Type	Name	May 2017	Nov 2017	May 2018	Nov 2018	May 2019	Nov 2019	May 2022	Nov 2022	May 2023	Dec 2023	May 2024	Dec 2024
I	Poisson	---	---	---	---	---	04	04	---	06	05	---	05
II	Normal	08	08	06	06	06	06	11	06	06	06	---	06
III	Theory	06	---	---	04	04	---	---	---	---	---	08	---
Total Marks		**14**	**08**	**06**	**10**	**10**	**10**	**15**	**06**	**12**	**11**	**08**	**11**

Extc

Type	Name	May 2018	Nov 2018	May 2019	Nov 2019	May 2022	Nov 2022	May 2023	Dec 2023	May 2024	Dec 2024
I	Poisson	06	06	---	---	05	06	06	06	06	06
II	Normal	08	08	08	---	02	06	06	06	06	06
Total Marks		**14**	**14**	**08**	**00**	**07**	**12**	**12**	**12**	**12**	**12**

Elect

Type	Name	May 2018	Nov 2018	May 2019	Nov 2019	May 2022	Nov 2022	May 2023	Dec 2023	May 2024	Dec 2024
I	Poisson	08	06	---	06	---	06	---	---	---	---
II	Normal	06	08	06	06	07	06	08	08	08	---
III	Theory	---	---	---	06	---	---	---	---	---	---
Total Marks		**14**	**14**	**06**	**18**	**07**	**12**	**08**	**08**	**08**	**00**

Type I: Poisson distribution

1. A random variable X follows Poisson distribution with variance 3 calculate $P(X=2)$ and $P(X \geq 4)$
[N14/MechCivil/6M]
Solution:
$variance = m = 3$
By Poisson distribution,
$P(X=r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-3}.3^r}{r!}$
$P(X=2) = \frac{e^{-3}.3^2}{2!} = 0.2240$
$P(X \geq 4) = 1 - P(X<4)$
$= 1 - \{P(0) + P(1) + P(2) + P(3)\}$
$= 1 - \left\{\frac{e^{-3}.3^0}{0!} + \frac{e^{-3}.3^1}{1!} + \frac{e^{-3}.3^2}{2!} + \frac{e^{-3}.3^3}{3!}\right\}$
$= 1 - \{0.0498 + 0.1494 + 0.2240 + 0.2240\}$
$= 0.3528$

2. If a random variable X follows Poisson distribution such that $P(X=1) = 2P(X=2)$, find the mean, variance of the distribution. Also find $P(X=3)$.
[M16/N16/CompIT/6M][D24/Extc/6M]
Solution:
By Poisson distribution,
$P(X=r) = \frac{e^{-m}.m^r}{r!}$
It is given that,
$P(X=1) = 2P(X=2)$
$\frac{e^{-m}.m^1}{1!} = 2.\frac{e^{-m}.m^2}{2!}$
$m = m^2$
$m = 1$
$P(X=3) = \frac{e^{-1}.1^3}{3!} = 0.0613$

3. For a Poisson distribution $P(x=2)=9P(x=4)+90P(x=6)$. Find mean and variance **[M19/IT/6M][N22/MTRX/5M]**

Solution:

$P(X=x)=\frac{e^{-m}m^x}{x!}$

Given that,

$P(x=2)=9P(x=4)+90P(x=6)$

$\frac{e^{-m}.m^2}{2!}=9\frac{e^{-m}.m^4}{4!}+90\frac{e^{-m}.m^6}{6!}$

$\frac{m^2}{2}=\frac{9m^4}{24}+\frac{90m^6}{720}$

$\frac{1}{2}=\frac{3m^2}{8}+\frac{m^4}{8}$

$4=3m^2+m^4$

$\therefore m^4+3m^2-4=0$

$\therefore m^2=1$

$\therefore m=1$

4. If x is a Poisson variable such that $P(x=1)=P(x=2)$ find $E(x^2)$ **[N17/MechCivil/6M][M19/MTRX/5M][M22/CompITAI/2M]**

Solution:

$P(X=x)=\frac{e^{-m}m^x}{x!}$

Given that,

$P(x=1)=P(x=2)$

$\frac{e^{-m}m^1}{1!}=\frac{e^{-m}m^2}{2!}$

$m=\frac{m^2}{2}$

$m=2$

$\therefore mean=E(x)=2$

$\therefore var=V(x)=2$

Now,

$V(x)=E(x^2)-[E(x)]^2$

$2=E(x^2)-[2]^2$

$E(x^2)=6$

5. A random variable X follows Poisson distribution with variance 3 then the $P(X \geq 4)$ is
[M22/MTRX/2M]
Solution:
$variance = m = 3$
By Poisson distribution,
$$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-3}.3^r}{r!}$$
$$\begin{aligned} P(X \geq 4) &= 1 - P(X < 4) \\ &= 1 - \{P(0) + P(1) + P(2) + P(3)\} \\ &= 1 - \left\{\frac{e^{-3}.3^0}{0!} + \frac{e^{-3}.3^1}{1!} + \frac{e^{-3}.3^2}{2!} + \frac{e^{-3}.3^3}{3!}\right\} \\ &= 1 - \{0.0498 + 0.1494 + 0.2240 + 0.2240\} \\ &= 0.3528 \end{aligned}$$

6. A random variable X follows Poisson distribution with variance 3 calculate $P(X = 2)$ and $P(X \geq 2)$
[N18/MTRX/5M]
Solution:
$variance = m = 3$
By Poisson distribution,
$$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-3}.3^r}{r!}$$
$$P(X = 2) = \frac{e^{-3}.3^2}{2!} = 0.2240$$
$$\begin{aligned} P(X \geq 2) &= 1 - P(X < 2) \\ &= 1 - \{P(0) + P(1)\} \\ &= 1 - \left\{\frac{e^{-3}.3^0}{0!} + \frac{e^{-3}.3^1}{1!}\right\} \\ &= 1 - \{0.0498 + 0.1494\} \\ &= 0.8008 \end{aligned}$$

7. If a random variable X follows Poisson distribution such that $P(X=1)=2P(X=2)$, find the mean, variance of the distribution.
[D24/CompIT/5M]
Solution:
By Poisson distribution,
$P(X=r)=\frac{e^{-m}.m^r}{r!}$
It is given that,
$P(X=1)=2P(X=2)$
$\frac{e^{-m}.m^1}{1!}=2.\frac{e^{-m}.m^2}{2!}$
$m=m^2$
$m=1$

8. In a Poisson distribution if $P(X=2)=P(X=3)$ then $P(X=5)$ is
[M22/MechCivil/2M]
Solution:
$P(X=x)=\frac{e^{-m}m^x}{x!}$
Given that,
$P(x=2)=P(x=3)$
$\frac{e^{-m}m^2}{2!}=\frac{e^{-m}m^3}{3!}$
$\frac{m^2}{2}=\frac{m^3}{6}$
$m=3$
Thus, $P(x=5)=\frac{e^{-3}3^5}{5!}=0.1008$

9. If a random variable follows Poisson distribution such that $P(X=0)=6P(X=3)$, find the mean and variance of the distribution
[M22/CompITAI/2M]
Solution:
By Poisson distribution,
$P(X=r)=\frac{e^{-m}.m^r}{r!}$
It is given that,
$P(X=0)=6P(X=3)$
$\frac{e^{-m}.m^0}{0!}=6.\frac{e^{-m}.m^3}{3!}$
$1=m^3$
$m=1$

10. If x is a Poisson variate and $p(x=0)=6p(x=3)$ find $p(x=2)$
[N19/IT/6M][M22/MTRX/5M]
Solution:
$P(X=x)=\frac{e^{-m}m^x}{x!}$
Given that,
$P(x=0)=6P(x=3)$
$\frac{e^{-m}m^0}{0!}=6.\frac{e^{-m}m^3}{3!}$
$1=m^3$
$m=1$
$\therefore P(X=r)=\frac{e^{-m}.m^r}{r!}=\frac{e^{-1}.1^r}{r!}$
$\therefore P(X=2)=0.1839$

11. In a Poisson distribution, $P(x=3)$ is $\frac{2}{3}$ of $P(x=4)$. Find the mean and the standard deviation
[N19/MTRX/5M]
Solution:
$P(X=x)=\frac{e^{-m}m^x}{x!}$
Given that,
$P(x=3)=\frac{2}{3}P(x=4)$
$\frac{e^{-m}m^3}{3!}=\frac{2}{3}.\frac{e^{-m}m^4}{4!}$
$\frac{1}{6}=\frac{m}{36}$
$m=6$
Standard deviation $=\sqrt{m}=\sqrt{6}$

12. The number of accidents in a year attributed to taxi drivers in a city follows Poisson distribution with mean 3. Out of 1000 taxi drivers, find approximately the number of drivers with (i) no accidents in a year (ii) more than 3 accidents in a year. (Given: $e^{-3} = 0.0498$)
[M18/MechCivil/6M][N18/Extc/6M]
Solution:
$m = 3$
$N = 1000$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-3}.3^r}{r!}$
(i) $P(no\ accidents) = P(X = 0) = 0.0498$
Expected number of drivers $= N \times P = 1000 \times 0.0498 = 49.8 \approx 50$
(ii) $P(more\ than\ 3\ accidents) = P(X > 3)$
$= 1 - P(X \le 3)$
$= 1 - [P(0) + P(1) + P(2) + P(3)]$
$= 1 - [0.0498 + 0.1494 + 0.224 + 0.224]$
$= 0.3528$
Expected number of drivers $= N \times P = 1000 \times 0.3528 = 352.8 \approx 353$

13. An insurance company found that only $0.01\ \%$ of the population is involved in a certain type of accident every year. If its 1000 policy holders are randomly selected from the population, what is the probability that not more than two of its clients are involved in such accident next year?
[N18/Elect/6M][M23/Extc/6M]
Solution:
$p = 0.01\% = \frac{0.01}{100} = 0.0001$
$n = 1000$
$m = np = 0.1$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-0.1}.0.1^r}{r!}$
$P(not\ more\ than\ 2) = P(X \ngtr 2)$
$= P(X \le 2)$
$= P(0) + P(1) + P(2)$
$= 0.9048 + 0.0905 + 0.0045$
$= 0.9998$

14. The proofs of a 500-page book contain 500 misprints. Find the probability that there are at least 4 misprints in a randomly chosen page.

[N13/N18/Biot/5M]

Solution:

$m = \frac{500}{500} = 1$

By Poisson distribution,

$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{(e^{-1}.1^r)}{r!}$

$P(atleast\ 4) = P(X \geq 4)$

$= 1 - P(X < 4)$

$= 1 - \{P(0) + P(1) + P(2) + P(3)\}$

$= 1 - \left\{\frac{e^{-1}}{0!} + \frac{e^{-1}}{1!} + \frac{e^{-1}}{2!} + \frac{e^{-1}}{3!}\right\}$

$= 1 - \frac{8}{3}e^{-1} = 0.019$

15. After correcting 50 pages of the proof of a book, the reader finds that there are on an average 2 errors per 5 pages. How many pages would one expect to find 0, 1, 2 and 3 errors in 1000 pages of first print of the book?

[N22/Elex/8M]

Solution:

$m = \frac{2}{5} = 0.4$

$P(X = x) = \frac{e^{-m}.m^x}{x!} = \frac{e^{-0.4}.0.4^x}{x!}$

$P(0\ error) = P(X = 0) = \frac{e^{-0.4}(0.4)^0}{0!} = 0.6703$

Expected no of Pages = $NP = 1000 \times 0.6703 = 670.3 \approx 670$

$P(1\ error) = P(X = 1) = \frac{e^{-0.4}(0.4)^1}{1!} = 0.2681$

Expected no of pages $= NP = 1000 \times 0.2681 = 268.1 \approx 268$

$P(2\ errors) = 0.0536$

Expected no of pages = $NP = 1000 \times 0.0536 = 53.6 \approx 54$

$P(3\ errors) = 0.00715$

Expected no of pages $= NP = 1000 \times 0.00715 = 7.15 \approx 7$

16. A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as Poisson variate with mean 1.5. calculate the proportion of days on which (i) neither car is used (ii) some demand is refused.
[M14/MechCivil/6M][D23/Extc/6M]
Solution:
$m = 1..5$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-1.5}.1.5^r}{r!}$
(i) $P(neither\ car\ is\ used) = P(X = 0)$
$= \frac{e^{-1.5}.1.5^0}{0!}$
$= 0.2231$
(ii) $P(some\ demand\ is\ refused) = P(X > 2)$
$= 1 - P(X \le 2)$
$= 1 - \{P(0) + P(1) + P(2)\}$
$= 1 - \left\{\frac{e^{-1.5}.1.5^0}{0!} + \frac{e^{-1.5}.1.5^1}{1!} + \frac{e^{-1.5}.1.5^2}{2!}\right\}$
$= 1 - \{0.2231 + 0.3347 + 0.2510\}$
$= 0.1912$

17. In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would you expect to contain (i) 3 defectives (ii) less than 3 defectives
[N22/Elect/6M]
Solution:
$m = 2$
$N = 1000$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!}$
$P(X = 3) = \frac{e^{-2}.2^3}{3!} = 0.1804$
Expected number of defectives $= N \times P$
$= 1000 \times 0.1804 = 180.4 \approx 180$
$P(X < 3) = P(X = 0,1,2) = \frac{e^{-2}.2^0}{0!} + \frac{e^{-2}.2^1}{1!} + \frac{e^{-2}.2^2}{2!} = 0.6767$
Expected number of defectives $= N \times P$
$= 1000 \times 0.6767 = 676.7 \approx 677$

18. In a certain manufacturing process 5 % of the tools produced turnout to be defective. Find the probability that in a sample of 40 tools at most 2 will be defective
[N19/Comp/4M]
Solution:
$n = 40$
$p = 5\ \% = \frac{5}{100}$
$m = np = 40 \times \frac{5}{100} = 2$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{(e^{-2}.2^r)}{r!}$
$P(atmost\ 2) = P(X \leq 2)$
$= P(0) + P(1) + P(2)$
$= \frac{e^{-2}.2^0}{0!} + \frac{e^{-2}.2^1}{1!} + \frac{e^{-2}.2^2}{2!}$
$= e^{-2}[1 + 2 + 2] = 5e^{-2} = 0.6767$

19. Find the probability that at most 4 defective bulbs will be found in a box of 200 bulbs, if it is known that 2% of the bulbs are defective.
[M14/ChemBiot/6M][N22/MechCivil/5M][M22/Extc/5M][N22/Extc/6M]
[D23/MechCivil/6M]
Solution:
$n = 200$
$p = 2\% = \frac{2}{100}$
$m = np = 200 \times \frac{2}{100} = 4$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{(e^{-4}.4^r)}{r!}$
$P(atmost\ 4) = P(X \leq 4)$
$= P(0) + P(1) + P(2) + P(3) + P(4)$
$= \frac{e^{-4}.4^0}{0!} + \frac{e^{-4}.4^1}{1!} + \frac{e^{-4}.4^2}{2!} + \frac{e^{-4}.4^3}{3!} + \frac{e^{-4}.4^4}{4!}$
$= e^{-4}\left[1 + 4 + 8 + \frac{32}{2} + \frac{32}{3}\right] = \frac{103}{3}e^{-4} = 0.6288$

20. Find the probability that at most 5 defective diodes will be found in a pack of 600 diodes, if previous data shows that 3% of each diodes are defective.
[M24/Extc/6M]
Solution:
$n = 600$
$p = 3\% = \frac{3}{100}$
$m = np = 600 \times \frac{3}{100} = 18$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-18}.18^r}{r!}$
$P(atmost\ 5) = P(X \leq 5)$
$= P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$
$= \frac{e^{-18}.18^0}{0!} + \frac{e^{-18}.18^1}{1!} + \frac{e^{-18}.18^2}{2!} + \frac{e^{-18}.18^3}{3!} + \frac{e^{-18}.18^4}{4!} + \frac{e^{-18}.18^5}{5!}$
$= e^{-18}\left[1 + 18 + 162 + 972 + 4374 + \frac{78732}{5}\right]$
$= \frac{106367}{5}e^{-18}$
$= 0.0003$

21. If the probability that an individual suffers a bad reaction from a particular injection is 0.001. Determine the probability that out of 2000 individuals (i) exactly 3 (ii) more than 2 will suffer a bad reaction.
[N17/M18/N18/Chem/6M]
Solution:
$n = 2000$
$p = 0.001$
$m = np = 2000 \times 0.001 = 2$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{(e^{-2}.2^r)}{r!}$
$P(exactly\ 3) = P(X = 3) = 0.1804$
$P(more\ than\ 2) = P(X > 2)$
$= 1 - P(X \leq 2)$
$= 1 - [P(0) + P(1) + P(2)]$
$= 1 - \left\{\frac{e^{-2}.2^0}{0!} + \frac{e^{-2}.2^1}{1!} + \frac{e^{-2}.2^2}{2!}\right\}$
$= 1 - e^{-2}[1 + 2 + 2] = 1 - 5e^{-2} = 0.0.3233$

22. A hospital switch board receives an average of 4 emergency calls in a 10 minutes interval. What is the probability that (i) there are at least 2 emergency calls (ii) there are exactly 3 emergency call in an interval of 10 minutes?
[M15/ChemBiot/6M]
Solution:
$m = 4$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-4}.4^r}{r!}$
(i) $P(atleast\ 2) = P(X \geq 2)$
$= 1 - P(X < 2)$
$= 1 - \{P(0) + P(1)\}$
$= 1 - \{0.0183 + 0.0733\} = 0.9084$
(ii) $P(exactly\ 3) = P(X = 3) = 0.1954$

23. Between 2 pm and 4 pm, the average number of phone calls per minute coming into the switch board of a company is 2.5. Find the probability that during a particular minute there will be (i) no phone calls (ii) at least 5 calls.
[M18/M19/Inst/6M][M18/N18/M19/Biom/6M]
Solution:
$m = 2.5$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-2.5}.2.5^r}{r!}$
(i) $P(no\ phone\ calls) = P(X = 0) = 0.0821$
(ii) $P(atleast\ 5) = P(X \geq 5)$
$= 1 - P(X < 5)$
$= 1 - \{P(0) + P(1) + P(2) + P(3) + P(4)\}$
$= 1 - \{0.0821 + 0.2052 + 0.2565 + 0.2138 + 0.1336\}$
$= 0.1088$

24. Between 2 pm and 4 pm, the average number of phone calls per minute coming into the switch board of a company is 1.5. Find the probability that during a particular minute there will be (i) no phone calls (ii) at least 2 calls.
[N19/Elect/6M]
Solution:
$m = 1.5$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-1.5}.1.5^r}{r!}$
(i) $P(no\ phone\ calls) = P(X = 0) = 0.2231$
(ii) $P(atleast\ 2) = P(X \geq 2)$
$= 1 - P(X < 2)$
$= 1 - \{P(0) + P(1)\}$
$= 1 - \{0.2231 + 0.3347\}$
$= 0.4422$

25. It is known that the probability of an item produced by a certain machine will be defective is 0.05. if the produced items are sent to the market in the packets of 20, find the number of packets containing (i) at least (ii) exactly (iii) at most 2 defective items in a consignment of 1000 packets using Poisson distribution approximation to the Binomial distribution.
[N16/M17/MechCivil/6M][N18/MTRX/6M]
Solution:
$p = 0.05$
$n = 20$
$m = np = 1$
$N = 1000$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-1}.1^r}{r!}$
(i) $P(at\ least\ 2) = P(X \geq 2)$
$= 1 - P(X < 2)$
$= 1 - \{P(0) + P(1)\}$
$= 1 - \{0.3679 + 0.3679\}$
$= 0.2642$
Expected number of packets $= N \times P = 1000 \times 0.2642 = 264.2 \approx 264$
(ii) $P(exactly\ 2) = P(X = 2)$
$= \frac{e^{-1}.1^2}{2!} = 0.1839$
Expected number of packets $= N \times P = 1000 \times 0.1839 = 183.9 \approx 184$
(iii) $P(atmost\ 2) = P(X \leq 2)$
$= P(0) + P(1) + P(2)$
$= 0.3679 + 0.3679 + 0.1839 = 0.9197$
Expected number of packets $= N \times P = 1000 \times 0.9197 = 919.7 \approx 920$

26. The probability of an item produced by a certain machine will be defective is 0.05. if the produced items are sent to the market in the packets of 20, find the number of packets containing at least 2 defective items in a consignment of 1000 packets
[M24/MechCivil/6M]
Solution:
$p = 0.05$
$n = 20$
$m = np = 1$
$N = 1000$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-1}.1^r}{r!}$
$P(at\ least\ 2) = P(X \geq 2)$
$= 1 - P(X < 2)$
$= 1 - \{P(0) + P(1)\}$
$= 1 - \{0.3679 + 0.3679\}$
$= 0.2642$
Expected number of packets $= N \times P = 1000 \times 0.2642 = 264.2 \approx 264$

27. Assume that the probability of an individual coal miner being injured in a mine accident during a year is $\frac{1}{2400}$. Calculate the probability that in a mine employing 200 miners there will be atleast one fatal accident in a year
[N18/IT/4M]
Solution:
$p = \frac{1}{2400}$
$n = 200$
$m = np = \frac{1}{12}$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-\frac{1}{12}}.\left(\frac{1}{12}\right)^r}{r!}$
$P(at\ least\ 1) = P(X \geq 1)$
$= 1 - P(X = 0)$
$= 1 - 0.92 = 0.08$

28. In a study of the effectiveness of an insecticide against a certain insect, a large area of land was sprayed. Later the area was examined for live insects by randomly selecting squares and counting the number of live insects per square. Past experience has shown the average number of live insects per square after spraying to be 0.5. If the number of live insects per square follows á Poisson distribution, find the probability that a selected square will contain: (a) One or more live insects (b) Two live insects
[M22/MechCivil/5M]
Solution:
$m = 0.5$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-0.5}.(0.5)^r}{r!}$
$P(at\ least\ 1) = P(X \geq 1)$
$= 1 - P(X = 0)$
$= 1 - 0.6065 = 0.3935$
$P(two) = P(X = 2) = 0.0758$

29. On an average 20% of population in an area, suffer from T.B. What is the probability that out of 6 persons chosen at random from this area (a) at least 2, (b) none suffer from T.B.?
[M22/MechCivil/5M]
Solution:
$p = 20\% = \frac{20}{100} = 0.2$
$n = 6$
$m = np = 1.2$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-1.2}.(1.2)^r}{r!}$
$P(at\ least\ 2) = P(X \geq 2)$
$= 1 - P(X < 2)$
$= 1 - \{P(X = 0) + P(X = 1)\}$
$= 1 - \{0.3012 + 0.3614\}$
$= 1 - 0.6626 = 0.3374$
$P(none) = P(X = 0) = 0.3012$

30. A transmission channel has a per digit error probability $p = 0.01$. calculate the probability of more than 1 error in 10 received digit using Poisson distribution.
[D23/CompIT/5M]
Solution:
$p = 0.01, q = 1 - p = 0.99$
$n = 10$
$m = np = 10 \times 0.01 = 0.1$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-0.1}.(0.1)^r}{r!}$
$P(more\ than\ error) = P(X > 1)$
$= 1 - P(X \leq 1)$
$= 1 - [P(0) + P(1)]$
$= 1 - [0.9048 + 0.0905]$
$= 0.0047$

31. Fit a Poisson distribution to the following data:

X	0	1	2	3	4	5	6	7	8
F	56	156	132	92	37	22	4	0	1

[N15/CompIT/6M]
Solution:
$N = \sum F = 56 + 156 + 132 + 92 + 37 + 22 + 4 + 0 + 1 = 500$
Mean $= \frac{\sum F.x}{\sum F} = \frac{56(0)+156(1)+132(2)+92(3)+37(4)+22(5)+4(6)+0(7)+1(8)}{500} = 1.972$
$m = 1.972$
By Poisson distribution,
$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-1.972}.1.972^r}{r!}$

X	$P(X)$	$F(X) = NP = 500P$
0	0.1391	$69.589 \approx 70$
1	0.2745	$137.23 \approx 137$
2	0.2706	$135.31 \approx 135$
3	0.1779	$88.94 \approx 89$
4	0.0877	$43.85 \approx 44$
5	0.0346	$17.29 \approx 17$
6	0.0114	$5.68 \approx 6$
7	0.0032	$1.60 \approx 2$
8	0.0008	0

32. Fit a Poisson distribution to the following data:

No of deaths	0	1	2	3	4
Frequencies	123	59	14	3	1

[M22Elex/5M][M23/CompIT/6M]

Solution:

$N = \sum F = 123 + 59 + 14 + 3 + 1 = 200$

Mean $= \frac{\sum F.x}{\sum F} = \frac{123(0)+59(1)+14(2)+3(3)+1(4)}{200} = 0.5$

$m = 0.5$

By Poisson distribution,

$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-0.5}.(0.5)^r}{r!}$

X	$P(X)$	$F(X) = NP = 200P$
0	0.6065	$121.31 \approx 121$
1	0.3033	$60.65 \approx 61$
2	0.0758	$15.16 \approx 15$
3	0.0126	$2.53 \approx 3$
4	0.0016	$0.32 \approx 0$

33. Fit a Poisson distribution to the following data:

x	0	1	2	3	4
y	123	69	14	3	1

[M22/Chem/5M][N22/Chem/6M]

Solution:

$N = \sum F = 123 + 69 + 14 + 3 + 1 = 210$

Mean $= \frac{\sum F.x}{\sum F} = \frac{123(0)+69(1)+14(2)+3(3)+1(4)}{210} = 0.52$

$m = 0.52$

By Poisson distribution,

$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-0.52}.(0.52)^r}{r!}$

X	$P(X)$	$F(X) = NP = 210P$
0	0.5945	$124.85 \approx 125$
1	0.3091	$64.92 \approx 65$
2	0.0803	$16.88 \approx 17$
3	0.0139	$2.93 \approx 3$
4	0.0018	$0.38 \approx 0$

34. Fit a Poisson distribution to the following data:

X	0	1	2	3	4
F	122	60	15	2	1

[N19/Inst/8M]

Solution:

$N = \sum F = 122 + 60 + 15 + 2 + 1 = 200$

Mean $= \frac{\sum F.x}{\sum F} = \frac{122(0)+60(1)+15(2)+2(3)+1(4)}{200} = 0.5$

$m = 0.5$

By Poisson distribution,

$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-0.5}.(0.5)^r}{r!}$

X	$P(X)$	$F(X) = NP = 200P$
0	0.6065	$121.31 \approx 121$
1	0.3033	$60.65 \approx 61$
2	0.0758	$15.16 \approx 15$
3	0.0126	$2.53 \approx 3$
4	0.0016	$0.32 \approx 0$

35. Four dice were thrown 250 times and the number of appearance of 6 each time was noted

No. of success (x):	0	1	2	3	4
Frequency(f):	133	69	34	11	3

Fit a Poisson distribution and find the expected frequencies

[N19/MechCivil/6M]

Solution:

$N = \sum F = 133 + 69 + 34 + 11 + 3 = 250$

Mean $= \frac{\sum F.x}{\sum F} = \frac{133(0)+69(1)+34(2)+11(3)+3(4)}{250} = 0.728$

$m = 0.728$

By Poisson distribution,

$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-0.728}.(0.728)^r}{r!}$

X	$P(X)$	$F(X) = NP = 250P$
0	0.4829	$120.72 \approx 121$
1	0.3515	$87.88 \approx 88$
2	0.1280	$31.99 \approx 32$
3	0.0311	$7.76 \approx 8$
4	0.0057	$1.41 \approx 1$

36. A skilled typist on routine work kept a record of mistakes made per day during 300 working days. Fit a Poisson distribution to the following data and hence calculate the theoretical frequencies

Mistakes per day	0	1	2	3	4	5	6
No of days	143	90	42	12	9	3	1

[M18/Elect/8M]

Solution:

$N = \sum F = 143 + 90 + 42 + 12 + 9 + 3 + 1 = 300$

Mean $= \frac{\sum F.x}{\sum F} = \frac{143(0)+90(1)+42(2)+12(3)+9(4)+3(5)+1(6)}{300} = 0.89$

$m = 0.89$

By Poisson distribution,

$P(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-0.89}.0.89^r}{r!}$

X	$P(X)$	$F(X) = NP = 300P$
0	0.4107	$123.1967 \approx 123$
1	0.3655	$109.6451 \approx 110$
2	0.1626	$48.7921 \approx 49$
3	0.0482	$14.4750 \approx 14$
4	0.0107	$3.2207 \approx 3$
5	0.0019	$0.5733 \approx 1$
6	0.0003	0

Type II: Normal Distribution

1. If X is a normal variable with mean 10 & standard deviation 4. Find
   (i) $P(5 \leq X \leq 18)$ (ii) $P(X \leq 12)$ (iii) $P(|X-14| < 1)$
   [N16/N17/CompIT/8M][M18/M19/Elex/6M]
   Solution:
   $\mu = 10$
   $\sigma = 4$
   $z = \frac{x-\mu}{\sigma} = \frac{x-10}{4}$
   (i) $P(5 \leq x \leq 18) = P\left(\frac{5-10}{4} \leq z \leq \frac{18-10}{4}\right)$
   $= P(-1.25 \leq z \leq 2)$
   $= A(2) - A(-1.25)$
   $= A(2) + A(1.25)$
   $= 0.4772 + 0.3944$
   $= 0.8716$
   (ii) $P(x \leq 12) = P\left(z \leq \frac{12-10}{4}\right)$
   $= P(z \leq 0.5)$
   $= A(0.5) - A(-\infty)$
   $= A(\infty) + A(0.5)$
   $= 0.5 + 0.1915$
   $= 0.6915$
   (iii) $P(|x-14| < 1) = P(-1 < (x-14) < 1)$
   $= P(-1+14 < x < 1+14)$
   $= P(13 < x < 15)$
   $= P\left(\frac{13-10}{4} < z < \frac{15-10}{4}\right)$
   $= P(0.75 < z < 1.25)$
   $= A(1.25) - A(0.75)$
   $= 0.3944 - 0.2734$
   $= 0.1210$

2. A manufacturer knows from his past experience that the resistance of resistors he produces is normal with mean 100 ohms and standard deviation 2 ohms. What percentage of resistors will have resistance (i) between 98 ohms and 102 ohms (ii) less than 96 ohms (iii) more than 104 ohms?

Solution:

Here, $\mu = 100, \sigma = 2$

$z = \frac{x-\mu}{\sigma} = \frac{x-100}{2}$

$$\begin{aligned} P(between\ 98\ \&\ 102) &= P(98 < x < 102) \\ &= P\left(\frac{98-100}{2} < z < \frac{102-100}{2}\right) \\ &= P(-1 < z < 1) \\ &= A(1) - A(-1) \\ &= A(1) + A(1) \\ &= 2A(1) \\ &= 2(0.3413) \\ &= 0.6826 = 68.26\% \end{aligned}$$

$$\begin{aligned} P(less\ than\ 96) &= P(x < 96) \\ &= P\left(z < \frac{96-100}{2}\right) \\ &= P(z < -2) \\ &= P(-\infty < z < -2) \\ &= A(-2) - A(-\infty) \\ &= -A(2) + A(\infty) \\ &= A(\infty) - A(2) \\ &= 0.5000 - 0.4772 \\ &= 0.0228 = 2.28\% \end{aligned}$$

$$\begin{aligned} P(more\ than\ 104) &= P(x > 104) \\ &= P\left(z > \frac{104-100}{2}\right) \\ &= P(z > 2) \\ &= P(2 < z < \infty) \\ &= A(\infty) - A(2) \\ &= 0.0228 = 2.28\% \end{aligned}$$

3. If the heights of 500 students is normally distributed with mean 68 inches and standard deviation 4 inches. Find the expected number of students having heights: (i) greater than 72 inches (ii) less than 62 inches.

Solution:

Here, $N = 500, \mu = 68, \sigma = 4$

$z = \frac{x-\mu}{\sigma} = \frac{x-68}{4}$

$$
\begin{aligned}
P(greater\ than\ 72) &= P(x > 72) \\
&= P\left(z > \frac{72-68}{4}\right) \\
&= P(z > 1) \\
&= P(1 < z < \infty) \\
&= A(\infty) - A(1) \\
&= 0.5000 - 0.3413 \\
&= 0.1587
\end{aligned}
$$

Expected no of students $= NP = 500 \times 0.1587 = 79.35 \approx 79$

$$
\begin{aligned}
P(less\ than\ 62) &= P(x < 62) \\
&= P\left(z < \frac{62-68}{4}\right) \\
&= P(z < -1.5) \\
&= P(-\infty < z < -1.5) \\
&= A(-1.5) - A(-\infty) \\
&= -A(1.5) + A(\infty) \\
&= A(\infty) - A(1.5) \\
&= 0.5000 - 0.4332 \\
&= 0.0668
\end{aligned}
$$

Expected no of students $= NP = 500 \times 0.0668 = 33.4 \approx 33$

4. The mean inside diameter of a sample of 200 washers produced by a machine is 0.502 inches and the standard deviation is 0.005 inches. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 0.496 to 0.508 inches, otherwise the washers are considered defective. Determine the percentage of defective washers produced by the machinery assuming the diameters are normally distributed.
[N14/CompIT/8M]
Solution:
$N = 200$
$\mu = 0.502$
$\sigma = 0.005$
$z = \frac{x-\mu}{\sigma} = \frac{x-0.502}{0.005}$
$P(non-defective\ washers) = P(0.496 < X < 0.508)$
$= P\left(\frac{0.496-0.502}{0.005} < z < \frac{0.508-0.502}{0.005}\right)$
$= P(-1.2 < z < 1.2)$
$= A(1.2) - A(-1.2)$
$= A(1.2) + A(1.2)$
$= 2\,A(1.2)$
$= 2 \times 0.3849$
$= 0.7698$
$= 76.98\%$
Thus, $P(defective\ washers) = 1 - 0.7698 = 0.2302 = 23.02\%$

5. Which of the following is not true for a normal distribution?
[M22/MechCivil/2M] Ans. mean is always zero
6. In normal distribution
[M22/CompITAI/2M] Ans. mean=median=mode

7. If X and Y are two normal variables with mean 40 and 50 and standard deviation 4 and 3 respectively, what is the distribution of $X + Y$
[M22/Extc/2M]
Solution:
$E(X) = 40, E(Y) = 50$
$V(X) = 4^2, V(Y) = 3^2$
$E(X+Y) = E(X) + E(Y) = 40 + 50 = 90$
$V(X+Y) = V(X) + V(Y) = 16 + 9 = 25$
$\sigma(X+Y) = \sqrt{25} = 5$
Normal distribution : $N(90, 5)$

8. If X is normally distributed with mean 10 and standard deviation 2, find $P(8 \leq X \leq 10)$
[M22/Elect/2M]
Solution:
$\mu = 10, \sigma = 2$
$z = \frac{x-\mu}{\sigma} = \frac{x-10}{2}$
$$\begin{aligned} P(8 \leq x \leq 10) &= P\left(\frac{8-10}{2} \leq z \leq \frac{10-10}{2}\right) \\ &= P(-1 \leq z \leq 0) \\ &= A(0) - A(-1) \\ &= A(0) + A(1) \\ &= 0 + 0.3413 \\ &= 0.3413 \end{aligned}$$

9. For a normal variate with mean 2.5 and Sd 3.5, then which statement is true for standard normal variable
[M22/MTRX/2M]
Solution:
$P(2 \leq X \leq 4.5) = P(-0.14 \leq z \leq 0.57)$

10. If X is a normal variable with mean 10 & standard deviation 4. Find
(i) $P(5 \leq X \leq 18)$ (ii) $P(X \leq 12)$
[D24/CompIT/6M]
Solution:
$\mu = 10, \sigma = 4$
$z = \frac{x-\mu}{\sigma} = \frac{x-10}{4}$
(i) $$\begin{aligned} P(5 \leq x \leq 18) &= P\left(\frac{5-10}{4} \leq z \leq \frac{18-10}{4}\right) \\ &= P(-1.25 \leq z \leq 2) \\ &= A(2) - A(-1.25) \\ &= A(2) + A(1.25) \\ &= 0.4772 + 0.3944 \\ &= 0.8716 \end{aligned}$$
(ii) $$\begin{aligned} P(x \leq 12) &= P\left(z \leq \frac{12-10}{4}\right) \\ &= P(z \leq 0.5) \\ &= A(0.5) - A(-\infty) \\ &= A(\infty) + A(0.5) \\ &= 0.5 + 0.1915 \\ &= 0.6915 \end{aligned}$$

11. For a normal variate with mean 2.5 and Sd 3.5, find the probability that
(i) $2 \le x \le 4.5$ (ii) $-1.5 \le x \le 5.5$
[N15/MechCivil/6M][N18/Inst/8M][N19/Elex/6M][M22/Elex/5M] [M22/N22/Chem/6M]
Solution:
$\mu = 2.5, \sigma = 3.5$
$z = \frac{x-\mu}{\sigma} = \frac{x-2.5}{3.5}$
(i) $P(2 \le x \le 4.5) = P\left(\frac{2-2.5}{3.5} \le z \le \frac{4.5-2.5}{3.5}\right)$
$= P(-0.14 \le z \le 0.57)$
$= A(0.57) - A(-0.14)$
$= A(0.14) + A(0.57)$
$= 0.0557 + 0.2157 = 0.2714$
(ii) $P(-1.5 \le x \le 5.5) = P\left(\frac{-1.5-2.5}{3.5} \le z \le \frac{5.5-2.5}{3.5}\right)$
$= P(-1.14 \le z \le 0.86)$
$= A(0.86) - A(-1.14)$
$= A(1.14) + A(0.86)$
$= 0.3729 + 0.3051$
$= 0.6780$

12. A random variable X follows a normal distribution with mean 14 and standard deviation 2.5 find (1) $P(X < 8)$ (2) $P(X > 18)$ (3) $P(12 < X < 15)$
Given:
Area between $z = 0$ and $z = 2.4$ is 0.4918; Area between $z = 0$ and $z = 1.6$ is 0.4452; Area between $z = 0$ and $z = 0.8$ is 0.2882; Area between $z = 0$ and $z = 0.4$ is 0.1554
[N22/MechCivil/8M]
Solution:
$\mu = 14, \sigma = 2.5$
$z = \frac{x-\mu}{\sigma} = \frac{x-14}{2.5}$
(i) $P(x < 8) = P\left(z < \frac{8-14}{2.5}\right)$
$= P(z < -2.4)$
$= P(-\infty < z < -2.4)$
$= A(-2.4) - A(-\infty)$
$= -A(2.4) + A(\infty)$
$= -0.4918 + 0.5$
$= 0.0082$

(ii) $P(x > 18) = P\left(z > \frac{18-14}{2.5}\right)$
$= P(z > 1.6)$
$= P(1.6 < z < \infty)$
$= A(\infty) - A(1.6)$
$= 0.5 - 0.4452$
$= 0.0548$

(iii) $P(12 < x < 15) = P\left(\frac{12-14}{2.5} < z < \frac{15-14}{2.5}\right)$
$= P(-0.8 < z < 0.4)$
$= A(0.4) - A(-0.8)$
$= A(0.4) + A(0.8)$
$= 0.1554 + 0.2882$
$= 0.4436$

13. A manufacturer knows from his past experience that the resistance of resistors he produces is normal with mean 100 ohms and standard deviation 2 ohms. What percentage of resistors will have resistance between 98 ohms and 102 ohms?
[M17/CompIT/8M][N19/Chem/6M]
Solution:
$\mu = 100$
$\sigma = 2$
$z = \frac{x-\mu}{\sigma} = \frac{x-100}{2}$
$P(between\ 98\ and\ 102) = P(98 < X < 102)$
$= P\left(\frac{98-100}{2} < z < \frac{102-100}{2}\right)$
$= P(-1 < z < 1)$
$= A(1) - A(-1)$
$= A(1) + A(1)$
$= 2 \times 0.3413$
$= 0.6826$
$= 68.26\%$

14. In an intelligence test administered to 1000 students the average score was 42 and standard deviation 24. Find the number of students (i) exceeding the score 50 (ii) between 30 and 54

[N16/M17/MechCivil/6M]

Solution:

$N = 1000$

$\mu = 42$

$\sigma = 24$

$z = \frac{x-\mu}{\sigma} = \frac{x-42}{24}$

(i) $P(exceeding\ 50) = P(X > 50)$

$= P\left(z > \frac{50-42}{24}\right)$

$= P(z > 0.33)$

$= A(\infty) - A(0.33)$

$= 0.5 - 0.1293$

$= 0.3707$

Expected number of students $= N \times P = 1000 \times 0.3707 = 370.7 \approx 371$

(ii) $P(between\ 30\ and\ 54) = P(30 < X < 54)$

$= P\left(\frac{30-42}{24} < z < \frac{54-42}{24}\right)$

$= P(-0.5 < z < 0.5)$

$= A(0.5) - A(-0.5)$

$= A(0.5) + A(0.5)$

$= 2 \times 0.1915$

$= 0.383$

Expected number of students $= N \times P = 1000 \times 0.383 = 383$

15. In an intelligence test administered to 1000 students the average score was 42 and standard deviation 24. Find the number of students (i) exceeding the score 50 (ii) between 30 and 54 (iii) less than 30

[D24/MechCivil/8M]

Solution:

$N = 1000$

$\mu = 42$

$\sigma = 24$

$z = \frac{x-\mu}{\sigma} = \frac{x-42}{24}$

(i) $P(exceeding\ 50) = P(X > 50)$

$$= P\left(z > \frac{50-42}{24}\right)$$
$$= P(z > 0.33)$$
$$= A(\infty) - A(0.33)$$
$$= 0.5 - 0.1293$$
$$= 0.3707$$

Expected number of students $= N \times P = 1000 \times 0.3707 = 370.7 \approx 371$

(ii) $P(between\ 30\ and\ 54) = P(30 < X < 54)$

$$= P\left(\frac{30-42}{24} < z < \frac{54-42}{24}\right)$$
$$= P(-0.5 < z < 0.5)$$
$$= A(0.5) - A(-0.5)$$
$$= A(0.5) + A(0.5)$$
$$= 2 \times 0.1915$$
$$= 0.383$$

Expected number of students $= N \times P = 1000 \times 0.383 = 383$

(iii) $P(less\ \ than\ 30) = P(X < 30)$

$$= P\left(z < \frac{30-42}{24}\right)$$
$$= P(z < -0.5)$$
$$= P(-\infty < z < -0.5)$$
$$= A(-0.5) - A(-\infty)$$
$$= -A(0.5) + A(\infty)$$
$$= -0.1915 + 0.5$$
$$= 0.3085$$

Expected number of students $= N \times P = 1000 \times 0.3085 = 308.5 \approx 309$

16. If the heights of 500 students is normally distributed with mean 68 inches and standard deviation 4 inches. Find the expected number of students having heights: (i) less than 62 inches (ii) between 65 and 71 inches.
[N15/CompIT/8M][N19/Comp/6M]
Solution:
$N = 500$
$\mu = 68$
$\sigma = 4$
$z = \frac{x-\mu}{\sigma} = \frac{x-68}{4}$
(i) $P(less\ than\ 62) = P(X < 62)$
$$= P\left(z < \frac{62-68}{4}\right)$$
$$= P(z < -1.5)$$
$$= A(-1.5) - A(-\infty)$$
$$= -A(1.5) + A(\infty)$$
$$= A(\infty) - A(1.5)$$
$$= 0.5 - 0.4332$$
$$= 0.0668$$
Expected number of students $= N \times P = 500 \times 0.0668 = 33.4 \approx 33$
(ii) $P(between\ 65\ and\ 71) = P(65 < X < 71)$
$$= P\left(\frac{65-68}{4} < z < \frac{71-68}{4}\right)$$
$$= P(-0.75 < z < 0.75)$$
$$= A(0.75) - A(-0.75)$$
$$= A(0.75) + A(0.75)$$
$$= 2 \times 0.2734$$
$$= 0.5468$$
Expected number of students $= N \times P = 500 \times 0.5468 = 273.4 \approx 273$

17. If the heights of 500 students is normally distributed with mean 68 inches and standard deviation 4 inches. Find the expected number of students having heights: (i) greater than 72 inches (ii) less than 62 inches (iii) between 65 and 71 inches.
[M22/CompITAI/5M][N22/CompITAI/6M][D23/MechCivil/8M]
Solution:
$N = 500$
$\mu = 68$
$\sigma = 4$
$z = \frac{x-\mu}{\sigma} = \frac{x-68}{4}$
(i) $P(greater\ than\ 72) = P(X > 72)$
$= P\left(z > \frac{72-68}{4}\right)$
$= P(z > 1)$
$= P(1 < z < \infty)$
$= A(\infty) - A(1)$
$= 0.5 - 0.3413$
$= 0.1587$
Expected number of students $= N \times P = 500 \times 0.1587 = 79.35 \approx 79$
(ii) $P(less\ than\ 62) = P(X < 62)$
$= P\left(z < \frac{62-68}{4}\right)$
$= P(z < -1.5)$
$= A(-1.5) - A(-\infty)$
$= -A(1.5) + A(\infty)$
$= A(\infty) - A(1.5)$
$= 0.5 - 0.4332$
$= 0.0668$
Expected number of students $= N \times P = 500 \times 0.0668 = 33.4 \approx 33$
(ii) $P(between\ 65\ and\ 71) = P(65 < X < 71)$
$= P\left(\frac{65-68}{4} < z < \frac{71-68}{4}\right)$
$= P(-0.75 < z < 0.75)$
$= A(0.75) - A(-0.75)$
$= A(0.75) + A(0.75)$
$= 2 \times 0.2734$
$= 0.5468$
Expected number of students $= N \times P = 500 \times 0.5468 = 273.4 \approx 273$

18. The marks of 1000 students of a university are normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be (i) between 60 and 75 (ii) more than 75
[M14/MechCivil/6M][M16/CompIT/8M][M18/Extc/8M][M18/Biot/5M][M19/Elect/6M][M19/Inst/6M][M19/Biom/6M][M23/ElectElex/8M][N22/MTRX/8M]
(iii) less than 68.
[N18/MTRX/8M][M24/ElectECS/8M]
Solution:
$\mu = 70, \sigma = 5$
$z = \frac{x-\mu}{\sigma} = \frac{x-70}{5}$
$N = 1000$
(i) $P(between\ 60\ and\ 75) = P(60 < X < 75)$
$= P\left(\frac{60-70}{5} < z < \frac{75-70}{5}\right)$
$= P(-2 < z < 1)$
$= A(1) - A(-2)$
$= A(1) + A(2)$
$= A(2) + A(1)$
$= 0.4772 + 0.3413$
$= 0.8185$
Expected number of students $= N \times P = 1000 \times 0.8185 = 818.5 \approx 819$
(ii) $P(more\ than\ 75) = P(X > 75)$
$= P\left(z > \frac{75-70}{5}\right)$
$= P(z > 1)$
$= A(\infty) - A(1)$
$= 0.5 - 0.3413$
$= 0.1587$
Expected number of students $= N \times P = 1000 \times 0.1587 = 158.7 \approx 159$
(iii) $P(less\ than\ 68) = P(X < 68)$
$= P\left(z < \frac{68-70}{5}\right)$
$= P(z < -0.4)$
$= A(-0.4) - A(-\infty)$
$= -A(0.4) + A(\infty)$
$= A(\infty) - A(0.4)$
$= 0.5 - 0.1554$
$= 0.3446$
Expected number of students $= N \times P = 1000 \times 0.3446 = 344.6 \approx 345$

19. The weekly wages of 1000 workmen are normally distributed around a mean of Rs. 70 and standard deviation of Rs. 5. Estimate the number of workers whose weekly wages will be (i) between 69 and 72 (ii) more than 75

[D23/Extc/6M]

Solution:

$\mu = 70, \sigma = 5$

$z = \frac{x-\mu}{\sigma} = \frac{x-70}{5}$

$N = 1000$

(i) $P(between\ 69\ and\ 72) = P(69 < X < 72)$

$= P\left(\frac{69-70}{5} < z < \frac{72-70}{5}\right)$

$= P(-0.2 < z < 0.4)$

$= A(0.4) - A(-0.2)$

$= A(0.2) + A(0.4)$

$= 0.0793 + 0.1554 = 0.2347$

Expected number of workers $= N \times P = 1000 \times 0.2347 = 234.7 \approx 235$

(ii) $P(more\ than\ 75) = P(X > 75)$

$= P\left(z > \frac{75-70}{5}\right)$

$= P(z > 1)$

$= A(\infty) - A(1)$

$= 0.5 - 0.3413 = 0.1587$

Expected number of workers $= N \times P = 1000 \times 0.1587 = 158.7 \approx 159$

20. In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. assuming the distribution to be normal, find (i) how many students score between 12 & 15 (ii) how many score above 18?
[M19/MechCivil/6M]
Solution:
$\mu = 14$
$\sigma = 2.5$
$z = \frac{x-\mu}{\sigma} = \frac{x-14}{2.5}$
$N = 1000$
(i) $P(between\ 12\ and\ 15) = P(12 < X < 15)$

$$= P\left(\frac{12-14}{2.5} < z < \frac{15-14}{2.5}\right)$$
$$= P(-0.8 < z < 0.4)$$
$$= A(0.4) - A(-0.8)$$
$$= A(0.8) + A(0.4)$$
$$= 0.2881 + 0.1554$$
$$= 0.4435$$

Expected number of students $= N \times P = 1000 \times 0.4435 = 443.5 \approx 444$
(ii) $P(more\ than\ 18) = P(X > 18)$

$$= P\left(z > \frac{18-14}{2.5}\right)$$
$$= P(z > 1.6)$$
$$= A(\infty) - A(1.6)$$
$$= 0.5 - 0.4452$$
$$= 0.0548$$

Expected number of students $= N \times P = 1000 \times 0.0548 = 54.8 \approx 55$

21. The life of Army shoes normally distributed with mean 8 months and sd 2 months. If 5000 pairs are issued how many pairs would be expected to need replacement after 12 months? **[N13/Biot/6M]**

Solution:

$\mu = 8$

$\sigma = 2$

$z = \frac{x-\mu}{\sigma} = \frac{x-8}{2}$

$N = 2000$

$$
\begin{aligned}
P(atleast\ 12\ months) &= P(X \geq 12) \\
&= P\left(z \geq \frac{12-8}{2}\right) \\
&= P(z \geq 2) \\
&= A(\infty) - A(2) \\
&= 0.5 - 0.4772 \\
&= 0.0228
\end{aligned}
$$

Expected number of pairs $= N \times P = 5000 \times 0.0228 = 114$

22. The weights of 4000 students are found to be normally distributed with mean 50 kgs and standard deviation 5 kgs. Find the probability that the students selected at random will have weight (i) less than 45 (ii) between 45 and 60.
[N14/MechCivil/6M]
Solution:
$N = 4000$
$\mu = 50$
$\sigma = 5$
$z = \frac{x-\mu}{\sigma} = \frac{x-50}{5}$
(i) $P(less\ than\ 45) = P(X < 45)$
$= P\left(z < \frac{45-50}{5}\right)$
$= P(z < -1)$
$= A(-1) - A(-\infty)$
$= -A(1) + A(\infty)$
$= A(\infty) - A(1)$
$= 0.5 - 0.3413$
$= 0.1587$
(ii) $P(between\ 45\ and\ 60) = P(45 < X < 60)$
$= P\left(\frac{45-50}{5} < z < \frac{60-50}{5}\right)$
$= P(-1 < z < 2)$
$= A(2) - A(-1)$
$= A(1) + A(2)$
$= 0.3413 + 0.4772$
$= 0.8185$

23. It was found that the burning life of electric bulbs of a particular brand was normally distributed with the mean 1200 hrs. and the sd of 90 hours, estimate the number of bulbs in a lot of 2500 bulbs having the burning life: (i) more than 1300 hours (ii) between 1050 and 1400 hours

[M18/IT/6M]

Solution:

$\mu = 1200$

$\sigma = 90$

$z = \frac{x-\mu}{\sigma} = \frac{x-1200}{90}$

$N = 2500$

(i) $P(more\ than\ 1300) = P(X > 1300)$

$$= P\left(z > \frac{1300-1200}{90}\right)$$
$$= P(z > 1.11)$$
$$= A(\infty) - A(1.11)$$
$$= 0.5 - 0.3665$$
$$= 0.1335$$

Expected number of bulbs $= N \times P = 2500 \times 0.1335 = 333.75 \approx 334$

(ii) $P(between\ 1050\ and\ 1400) = P(1050 < X < 1400)$

$$= P\left(\frac{1050-1200}{90} < z < \frac{1400-1200}{90}\right)$$
$$= P(-1.67 < z < 2.22)$$
$$= A(2.22) - A(-1.67)$$
$$= A(1.67) + A(2.22)$$
$$= 0.4525 + 0.4868 = 0.9393$$

Expected number of bulbs $= N \times P = 2500 \times 0.9393 = 2348.25 \approx 2348$

24. If sizes of 10000 items are normally distributed with mean 20 cm & standard deviation of 4 cm. find the probability that an item selected at random will have size: (i) between 18 cm and 23 cm (ii) above 26 cm

[M19/Extc/8M][M24/MechCivil/8M]

Solution:

$\mu = 20$

$\sigma = 4$

$z = \frac{x-\mu}{\sigma} = \frac{x-20}{4}$

$N = 10000$

(i) $P(between\ 18\ and\ 23) = P(18 < X < 23)$

$= P\left(\frac{18-20}{4} < z < \frac{23-20}{4}\right)$

$= P(-0.5 < z < 0.75)$

$= A(0.75) - A(-0.5)$

$= A(0.5) + A(0.75)$

$= 0.1915 + 0.2734$

$= 0.4649$

(ii) $P(more\ than\ 26) = P(X > 26)$

$= P\left(z > \frac{26-20}{4}\right)$

$= P(z > 1.5)$

$= A(\infty) - A(1.5)$

$= 0.5 - 0.4332$

$= 0.0668$

25. The local authorities in a certain city install 10000 electric lamps in the streets of the city. If these lamps have an average life of 1000 burning hours with a standard deviation of 200 hours, how many lamps might be expected to fail (i) in the first 800 burning hours (ii) between 800 and 1200 burning hours

[N18/IT/6M]

Solution:

$\mu = 1000$

$\sigma = 200$

$z = \frac{x-\mu}{\sigma} = \frac{x-1000}{200}$

$N = 10000$

(i) $P(less\ than\ 800) = P(X < 800)$

$$= P\left(z < \frac{800-1000}{200}\right)$$
$$= P(z < -1)$$
$$= A(-1) - A(-\infty)$$
$$= -A(1) + A(\infty)$$
$$= A(\infty) - A(1)$$
$$= 0.5 - 0.3413$$
$$= 0.1587$$

Expected number of bulbs $= N \times P = 10000 \times 0.1587 = 1587$

(ii) $P(between\ 800\ and\ 1200) = P(800 < X < 1200)$

$$= P\left(\frac{800-1000}{200} < z < \frac{1200-1000}{200}\right)$$
$$= P(-1 < z < 1)$$
$$= A(1) - A(-1)$$
$$= A(1) + A(1)$$
$$= 0.3413 + 0.3413$$
$$= 0.6826$$

Expected number of bulbs $= N \times P = 10000 \times 0.6826 = 6826$

26. The income group of 10,000 people were found to be normally distributed with mean Rs. 520 and standard deviation of Rs. 60. Find the number of people having income (i) more than Rs. 600 (ii) between Rs. 400 and 550 (iii) less than Rs. 450
[D23/ElectElexECS/8M]
Solution:
$\mu = 520$
$\sigma = 60$
$z = \frac{x-\mu}{\sigma} = \frac{x-520}{60}$
$N = 10000$
(i) $P(more\ than\ 600) = P(X > 600)$
$= P\left(z > \frac{600-520}{60}\right)$
$= P(z > 1.33)$
$= A(\infty) - A(1.33)$
$= 0.5 - 0.4082$
$= 0.0918$
Expected number of people $= N \times P = 10000 \times 0.0918 = 918$
(ii) $P(between\ 400\ and\ 550) = P(400 < X < 550)$
$= P\left(\frac{400-520}{60} < z < \frac{550-520}{60}\right)$
$= P(-2 < z < 0.5)$
$= A(0.5) - A(-2)$
$= A(0.5) + A(2)$
$= 0.1915 + 0.4772$
$= 0.6687$
Expected number of people $= N \times P = 10000 \times 0.6687 = 6687$
(iii) $P(less\ than\ 450) = P(X < 450)$
$= P\left(z < \frac{450-520}{60}\right)$
$= P(z < -1.17)$
$= A(-1.17) - A(-\infty)$
$= -A(1.17) + A(\infty)$
$= A(\infty) - A(1.17)$
$= 0.5 - 0.3790$
$= 0.1210$
Expected number of people $= N \times P = 10000 \times 0.1210 = 1210$

27. The mean yield for one acre plot is 662 kilos with a standard deviation 32 kilos. Assuming normal distribution, how many one-acre plots in a batch of 1000 plots would you expect to have yield (i) over 700 kilos (ii) below 650 kilos (iii) what is the lowest yield of the best 100 plots?

Solution:

Here, $\mu = 662, \sigma = 32, N = 1000$

$z = \frac{x-662}{32}$

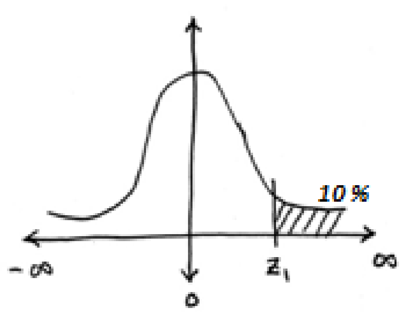
$$\begin{aligned}
P(over\ 700) &= P(x > 700) \\
&= P\left(z > \frac{700-662}{32}\right) \\
&= P(z > 1.19) \\
&= P(1.19 < z < \infty) \\
&= A(\infty) - A(1.19) \\
&= 0.5000 - 0.3830 \\
&= 0.1170
\end{aligned}$$

Expected no of plots $= NP = 1000 \times 0.1170 = 117$

$$\begin{aligned}
P(below\ 650) &= P(x < 650) \\
&= P\left(z < \frac{650-662}{32}\right) \\
&= P(z < -0.38) \\
&= P(-\infty < z < -0.38) \\
&= A(-0.38) - A(-\infty) \\
&= -A(0.38) + A(\infty) \\
&= A(\infty) - A(0.38) \\
&= 0.5000 - 0.1480 \\
&= 0.3520
\end{aligned}$$

Expected no of plots $= NP = 1000 \times 0.3520 = 352$

The best 100 plots out of the total 1000 plots belongs to the top 10% category.



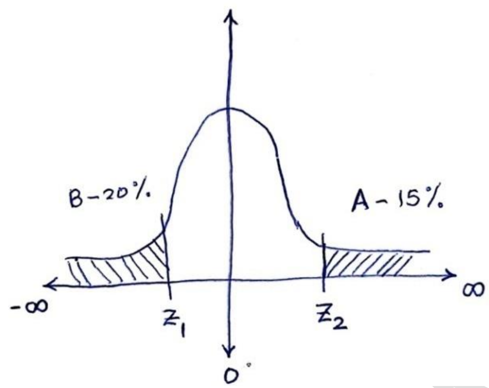
$Area(0\ to\ z_1) = 40\% = 0.40$

From table, $z_1 = 1.28$

$$z_1 = \frac{x_1-\mu}{\sigma}$$

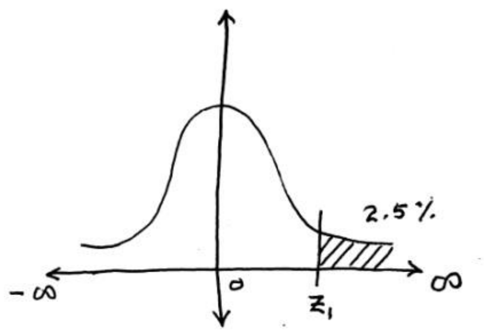
$1.28 = \frac{x_1 - 662}{32}$
$1.28 \times 32 = x_1 - 662$
$x_1 = 702.96 \approx 703$

28. In a competitive examination, the top 15% of the students appeared will get grade 'A', while the bottom 20% will be declared fail. If the grades are normally distributed with mean % of marks 75 and SD 10, determine the lowest % of marks to receive grade A and lowest % of marks that passes.
[M14/CompIT/8M]
Solution:
$\mu = 75, \sigma = 10$
$z = \frac{x-\mu}{\sigma} = \frac{x-75}{10}$



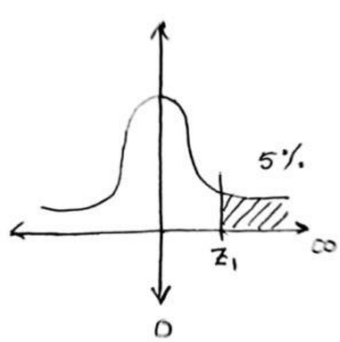
$A(0\ to\ z_1) = 30\% = 0.3$
From table, $z_1 = -0.84$
Thus, $z_1 = \frac{x_1-\mu}{\sigma}$
$-0.84 = \frac{x_1-75}{10}$
$-8.4 = x_1 - 75$
$x_1 = 66.6$
Thus, the highest of grade B is 66.6 i.e. the lowest percentage of marks that passes is $66.6 \approx 67$
$A(0\ to\ z_2) = 35\% = 0.35$
From table, $z_2 = 1.04$
Thus, $z_2 = \frac{x_2-\mu}{\sigma}$
$1.04 = \frac{x_2-75}{10}$
$10.4 = x_2 - 75$
$x_2 = 85.4$
Thus, the lowest percentage of marks of grade A is $85.4 \approx 85$

29. The incomes of a group of 10,000 persons were found to be normally distributed with mean Rs. 750 and s.d. Rs. 50. What is the lowest income of the richest 250?
[N18/M19/Comp/6M]
Solution:
$N = 10000, \mu = 750, \sigma = 50$
The richest 250 out of the total 10000 belongs to the top 2.5%



We see that, $A(0\ to\ z_1) = 0.4750$
From the table, we get $z_1 = 1.96$
$z_1 = \frac{x_1 - \mu}{\sigma}$
$1.96 = \frac{x_1 - 750}{50}$
$\therefore x_1 = 848$
Thus, the lowest income of the richest 250 is Rs. 848

30. The income of a group of 10,000 persons were found to be normally distributed with mean Rs. 520 and Sd Rs. 60. What is the lowest income of the richest 500?
[M23/CompIT/6M]
Solution:
$\mu = 520,\ \sigma = 60$
$z = \frac{x - \mu}{\sigma} = \frac{x - 520}{60}$
The richest 500 out of the total 10000 belongs to the top 5%



$A(0\ to\ z_1) = 45\% = 0.45$

From table, $z_1 = \frac{1.64+1.65}{2} = 1.645$

$z_1 = \frac{x_1 - 520}{60}$

$1.645 = \frac{x_1 - 520}{60}$

$\therefore x_1 = 618.7 \approx 619$

Thus, the minimum salary of workers should be Rs. 619/- so that they belong to the top 5% workers.

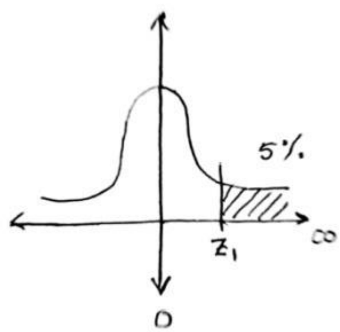
31. Monthly salary in a big organization is normally distributed with mean Rs. 3000 & s.d Rs. 250. What should be the minimum salary of workers in this organization so that the probability that he belongs to top 5% workers?

[M15/N17/N19/MechCivil/6M][M24/D24/Extc/6M]

Solution:

$\mu = 3000,\ \sigma = 250$

$z = \frac{x-\mu}{\sigma} = \frac{x-3000}{250}$



$A(0\ to\ z_1) = 45\% = 0.45$

From table, $z_1 = \frac{1.64+1.65}{2} = 1.645$

$z_1 = \frac{x_1 - 3000}{250}$

$1.645 = \frac{x_1 - 3000}{250}$

$\therefore x_1 = 3411.25 \approx 3411$

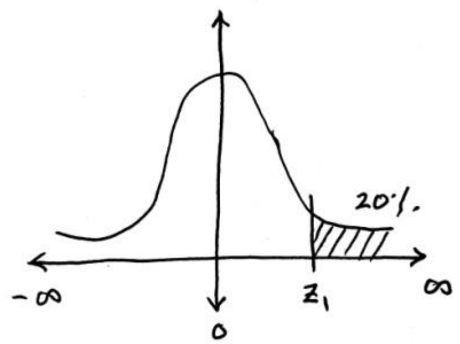
Thus, the minimum salary of workers should be Rs. 3411/- so that they belong to the top 5% workers.

32. The IQ's of army volunteers in a given year are normally distributed with mean 110 and standard deviation 10. The army wants to give advanced training to 20% of those recruits with the highest scores. What is the lowest IQ score acceptable for advanced training **[N18/Elex/6M]**

Solution:

$\mu = 110$

$\sigma = 10$



We see that, $A(0 \ to \ z_1) = 0.3$

From the table, we get $z_1 = 0.84$

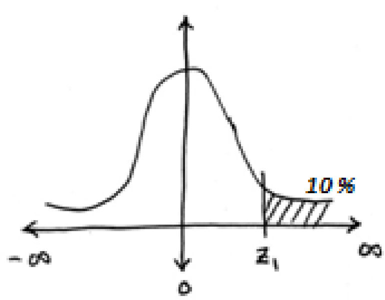
$z_1 = \frac{x_1 - \mu}{\sigma}$

$0.84 = \frac{x_1 - 110}{10}$

$x_1 = 118.4 \approx 118$

Thus, the lowest IQ score is 118

33. The weekly wages 1000 workmen are normally distributed around a mean Rs. 70 and s.d. Rs. 5. Estimate the number of workers whose weekly wages will be (i) between 69 and 72 (ii) less than 63. Also estimate the lowest wage of the 100 highest paid workers.
[M18/Biom/8M]
Solution:
$\mu = 70, \sigma = 5, N = 1000$ and $z = \frac{x-\mu}{\sigma} = \frac{x-70}{5}$
(i) $P(between\ 69\ and\ 72) = P(69 < X < 72)$
$= P\left(\frac{69-70}{5} < z < \frac{72-70}{5}\right)$
$= P(-0.2 < z < 0.4)$
$= A(0.4) - A(-0.2)$
$= A(0.2) + A(0.4)$
$= 0.0793 + 0.1554 = 0.2347$
Expected number of workers $= N \times P = 1000 \times 0.2347 = 234.7 \approx 235$
(ii) $P(less\ than\ 63) = P(X < 63)$
$= P\left(z < \frac{63-70}{5}\right)$
$= P(z < -1.4)$
$= A(-1.4) - A(-\infty)$
$-A(1.4) + A(\infty)$
$= A(\infty) - A(1.4)$
$= 0.5 - 0.4192 = 0.0808$
Expected number of workers $= N \times P = 1000 \times 0.0808 = 80.8 \approx 81$
Now, 100 highest paid workers out of the total 1000 workmen belongs to the top 10 % category



We see that, $A(0\ to\ z_1) = 0.4$
From the table, we get $z_1 = 1.28$
$z_1 = \frac{x_1-\mu}{\sigma}$
$1.28 = \frac{x_1-70}{5}$
$x_1 = 76.4 \approx 76$
Thus, the lowest wage of the 100 highest paid workers is 76

34. The weekly wages 1000 workmen are normally distributed around a mean Rs. 70 and s.d. Rs. 5. Estimate the number of workers whose weekly wages will be (i) between 65 and 75 (ii) more than 80. Also estimate the lowest wage of the 100 highest paid workers. **[N19/Elect/6M]**

Solution:

$\mu = 70, \sigma = 5, N = 1000$ and $z = \frac{x-\mu}{\sigma} = \frac{x-70}{5}$

(i) $P(between\ 65\ and\ 75) = P(65 < X < 75)$

$$= P\left(\frac{65-70}{5} < z < \frac{75-70}{5}\right)$$
$$= P(-1 < z < 1)$$
$$= A(1) - A(-1)$$
$$= A(1) + A(1)$$
$$= 0.3413 + 0.3413 = 0.6826$$

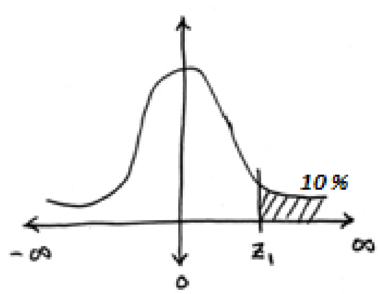
Expected number of workers $= N \times P = 1000 \times 0.6826 = 682.6 \approx 683$

(ii) $P(more\ than\ 80) = P(X > 80)$

$$= P\left(z > \frac{80-70}{5}\right)$$
$$= P(z > 2)$$
$$= A(\infty) - A(2)$$
$$= 0.5 - 0.4772 = 0.0228$$

Expected number of workers $= N \times P = 1000 \times 0.0228 = 22.8 \approx 23$

Now, 100 highest paid workers out of the total 1000 workmen belongs to the top 10 % category



We see that, $A(0\ to\ z_1) = 0.4$

From the table, we get $z_1 = 1.28$

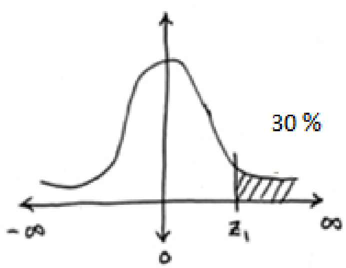
$z_1 = \frac{x_1-\mu}{\sigma}$

$1.28 = \frac{x_1-70}{5}$

$x_1 = 76.4 \approx 76$

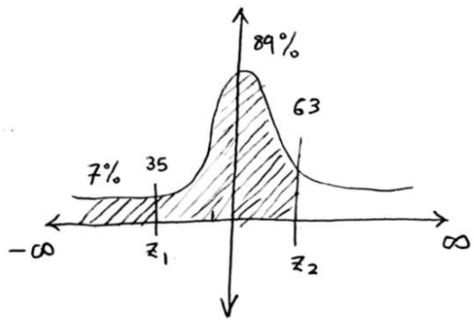
Thus, the lowest wage of the 100 highest paid workers is 76

35. The IQs of individuals admitted to a state school for the mentally retarded are approximately normally distributed with a mean of 60 and a standard deviation of 10. (a) What is the probability that an individual picked at random will have an IQ between 55 and 75? (b) What is the lowest IQ of top 30% individuals?
[M22/MechCivil/5M]
Solution:
$\mu = 60, \sigma = 10$
$z = \frac{x-\mu}{\sigma} = \frac{x-60}{10}$
$P(between\ 55\ and\ 75) = P(55 < X < 75)$
$= P\left(\frac{55-60}{10} < z < \frac{75-60}{10}\right)$
$= P(-0.5 < z < 1.5)$
$= A(1.5) - A(-0.5)$
$= A(1.5) + A(0.5)$
$= 0.4332 + 0.1915 = 0.6247$



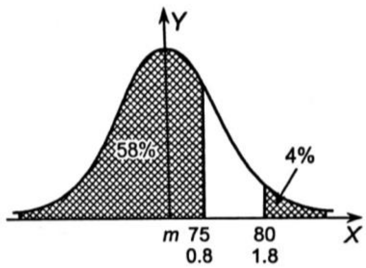
We see that, $A(0\ to\ z_1) = 20\% = 0.20$
From the table, we get $z_1 = 0.52$
$z_1 = \frac{x_1-\mu}{\sigma}$
$0.52 = \frac{x_1-60}{10}$
$x_1 = 65.2$
Thus, the lowest IQ of top 30% individuals is 65.2

36. In a normal distribution 7 % items are under 35 & 89% are under 63. Find mean & s.d.
[M19/MTRX/6M][N22/Extc/6M]
Solution:



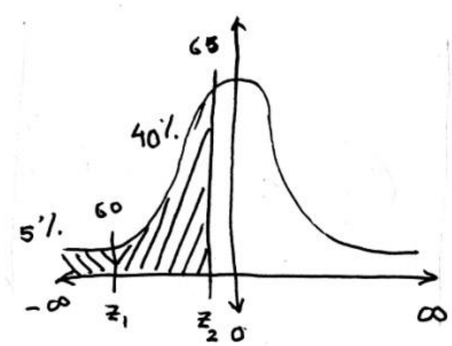
$A(0\ to\ z_1) = 43\% = 0.43$
From table, $z_1 = -1.48$
$z_1 = \frac{x_1-\mu}{\sigma}$
$-1.48 = \frac{35-\mu}{\sigma}$
$-1.48\sigma + \mu = 35$ ……………….(1)
$A(0\ to\ z_2) = 39\% = 0.39$
From table, $z_2 = 1.23$
$z_2 = \frac{x_2-\mu}{\sigma}$
$1.23 = \frac{63-\mu}{\sigma}$
$1.23\sigma + \mu = 63$ ………………….(2)
Solving (1) & (2), we get
$\sigma = 10.33, \mu = 50.29$

37. Find the mean & s.d of a normal distribution of marks in an exam where 58% of the candidates obtained marks below 75, 4% got above 80 & the rest between 75 & 80.
[N17/M18/N18/Chem/8M]
Solution:



$A(0\ to\ z_1) = 8\% = 0.08$
From table, $z_1 = 0.2$
$z_1 = \frac{x_1-\mu}{\sigma}$
$0.2 = \frac{75-\mu}{\sigma}$
$0.2\sigma + \mu = 75$ ……………….(1)
$A(0\ to\ z_2) = 46\% = 0.46$
From table, $z_2 = 1.75$
$z_2 = \frac{x_2-\mu}{\sigma}$
$1.75 = \frac{80-\mu}{\sigma}$
$1.75\sigma + \mu = 80$ ………………….(2)
Solving (1) & (2), we get
$\sigma = 3.23, \mu = 74.35$

38. Of a large group of men 5% are under 60" of height & 40% are between 60" & 65" assuming Normal distribution find the mean and s.d.
[M18/Inst/8M][N18/Biom/8M][N18/Elect/8M]
Solution:



we see that,

$A(0\ to\ z_1) = 0.45$ $\qquad$ $A(0\ to\ z_2) = 0.05$

From the table, we get

$z_1 = -1.645$ $\qquad$ $z_2 = -0.13$

$z_1 = \frac{x_1 - \mu}{\sigma}$ $\qquad$ $z_2 = \frac{x_2 - \mu}{\sigma}$

$-1.645 = \frac{60 - \mu}{\sigma}$ $\qquad$ $-0.13 = \frac{65 - \mu}{\sigma}$

$-1.645\sigma + \mu = 60$ ……………….. (1)

$-0.13\sigma + \mu = 65$ …………………. (2)

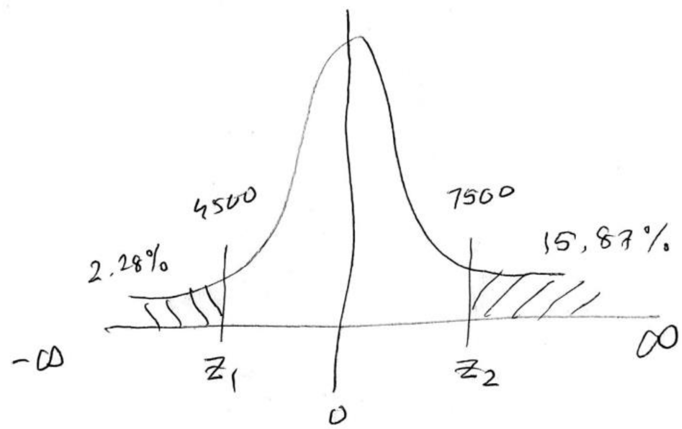
Solving eqn (1) & (2), we get

$\sigma = 3.3, \mu = 65.43$

39. In a large institution 2.28% employees receive income below Rs. 4500 and 15.87% employees receive income above Rs. 7500. Assuming the income to be normally distributed, find the mean and standard deviation.

[N19/MTRX/6M]

Solution:



$A(0\ to\ z_1) = 47.72\% = 0.4772$

From table, $z_1 = -02$

$z_1 = \frac{x_1 - \mu}{\sigma}$

$-2 = \frac{4500 - \mu}{\sigma}$

$-2\sigma + \mu = 4500$ ……………….(1)

$A(0\ to\ z_2) = 34.13\% = 0.3413$

From table, $z_2 = 1$

$z_2 = \frac{x_2 - \mu}{\sigma}$

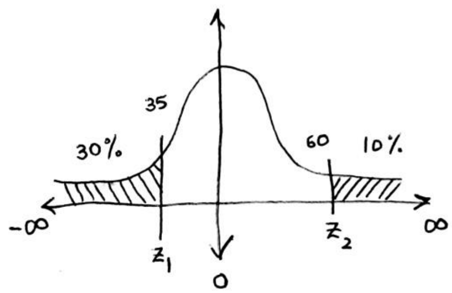
$1 = \frac{7500 - \mu}{\sigma}$

$\sigma + \mu = 7500$ ………………….(2)

Solving (1) & (2), we get

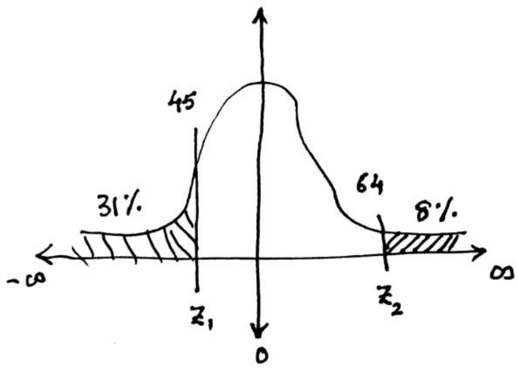
$\sigma = 1000, \mu = 6500$

40. Find the mean & s.d of a normal distribution of marks in an exam where 30% of the candidates obtained marks below 35, 10% got above 60.
[M16/MechCivil/6M][M19/IT/6M][N22/Elect/6M][M24/CompIT/8M]
Solution:



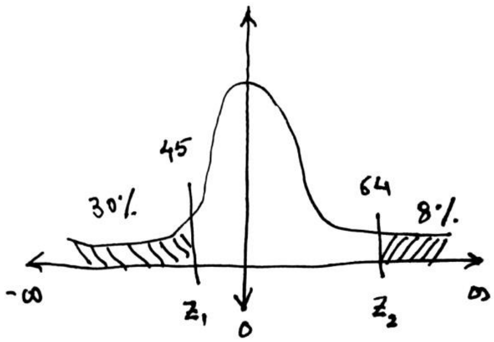
$A(0\ to\ z_1) = 20\% = 0.20$
From table, $z_1 = -0.52$
$z_1 = \frac{x_1-\mu}{\sigma}$
$-0.52 = \frac{35-\mu}{\sigma}$
$-0.52\sigma + \mu = 35$ ……………….(1)
$A(0\ to\ z_2) = 40\% = 0.40$
From table, $z_2 = 1.28$
$z_2 = \frac{x_2-\mu}{\sigma}$
$1.28 = \frac{60-\mu}{\sigma}$
$1.28\sigma + \mu = 60$ ………………….(2)
Solving (1) & (2), we get
$\sigma = 13.88, \mu = 42.22$

41. In a normal distribution 31 % items are under 45 & 8% are over 64. Find mean & s.d.
[M18/Elect/6M][N19/Biot/5M]
Solution:



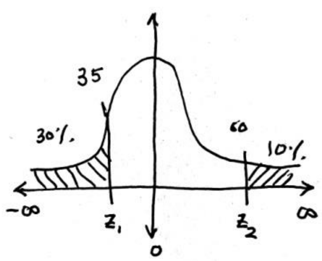
$A(0\ to\ z_1) = 19\% = 0.19$
From table, $z_1 = -0.5$
$z_1 = \frac{x_1-\mu}{\sigma}$
$-0.5 = \frac{45-\mu}{\sigma}$
$-0.5\sigma + \mu = 45$ ……………….(1)
$A(0\ to\ z_2) = 42\% = 0.42$
From table, $z_2 = 1.41$
$z_2 = \frac{x_2-\mu}{\sigma}$
$1.41 = \frac{64-\mu}{\sigma}$
$1.41\sigma + \mu = 64$ ………………….(2)
Solving (1) & (2), we get
$\sigma = 9.9476, \mu = 49.9738$

42. In a normal distribution 30 % items are under 45 & 8% are over 64. Find mean & s.d.
[M18/Comp/6M][M22/Elect/5M]
Solution:



$A(0\ to\ z_1) = 20\% = 0.2$
From table, $z_1 = -0.52$
$z_1 = \frac{x_1 - \mu}{\sigma}$
$-0.52 = \frac{45 - \mu}{\sigma}$
$-0.52\sigma + \mu = 45$ ……………….(1)
$A(0\ to\ z_2) = 42\% = 0.42$
From table, $z_2 = 1.41$
$z_2 = \frac{x_2 - \mu}{\sigma}$
$1.41 = \frac{64 - \mu}{\sigma}$
$1.41\sigma + \mu = 64$ …………………(2)
Solving (1) & (2), we get $\sigma = 9.8446, \mu = 50.1192$

43. Marks obtained by students in the examination follows normal distribution. If 30% of students got below 35 marks and 10% got above 60 marks. Find mean and standard deviation
[N18/MechCivil/6M][M23/MechCivil/8M]
Solution:



we see that,

$A(0\ to\ z_1) = 0.2$ $\qquad$ $A(0\ to\ z_2) = 0.4$

From the table, we get

$z_1 = -0.52$ $\qquad$ $z_2 = 1.28$

$z_1 = \frac{x_1-\mu}{\sigma}$ $\qquad$ $z_2 = \frac{x_2-\mu}{\sigma}$

$-0.52 = \frac{35-\mu}{\sigma}$ $\qquad$ $1.28 = \frac{60-\mu}{\sigma}$

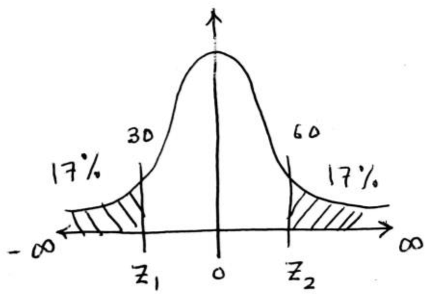
$-0.52\sigma + \mu = 35$ ……………….. (1)

$1.28\sigma + \mu = 60$ ………………… (2)

Solving eqn (1) & (2), we get

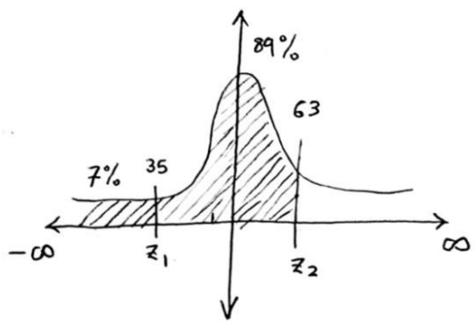
$\sigma = 13.89, \mu = 42.22$

44. In a normal distribution 17% of the items are below 30 and 17% of the items are above 60. Find the mean and standard deviation
[M23/Extc/6M]
Solution:



we see that,
$A(0\ to\ z_1) = 0.33$ $\qquad$ $A(0\ to\ z_2) = 0.33$
From the table, we get
$z_1 = -0.95$ $\qquad$ $z_2 = 0.95$
$z_1 = \frac{x_1-\mu}{\sigma}$ $\qquad$ $z_2 = \frac{x_2-\mu}{\sigma}$
$-0.95 = \frac{30-\mu}{\sigma}$ $\qquad$ $0.95 = \frac{60-\mu}{\sigma}$
$-0.95\sigma + \mu = 30$ ……………….. (1)
$0.95\sigma + \mu = 60$ …………………. (2)
Solving eqn (1) & (2), we get
$\sigma = 15.7895, \mu = 45$

45. In a normal distribution 7 % items are under 35 & 89% are under 63. Find probability of items lies between 45 and 56.
[M15/CompIT/8M]
Solution:



$A(0\ to\ z_1) = 43\% = 0.43$
From table, $z_1 = -1.48$
$z_1 = \frac{x_1-\mu}{\sigma}$
$-1.48 = \frac{35-\mu}{\sigma}$
$-1.48\sigma + \mu = 35$ ……………….(1)
$A(0\ to\ z_2) = 39\% = 0.39$
From table, $z_2 = 1.23$
$z_2 = \frac{x_2-\mu}{\sigma}$
$1.23 = \frac{63-\mu}{\sigma}$
$1.23\sigma + \mu = 63$ ………………….(2)
Solving (1) & (2), we get
$\sigma = 10.33, \mu = 50.29$
$\therefore z = \frac{x-\mu}{\sigma} = \frac{x-50.29}{10.33}$

$$
\begin{aligned}
P(between\ 45\ and\ 56) &= P(45 < x < 56)\\
&= P\left(\frac{45-50.29}{10.33} < z < \frac{56-50.29}{10.33}\right)\\
&= P(-0.51 < z < 0.55)\\
&= A(0.55) - A(-0.51)\\
&= A(0.51) + A(0.55)\\
&= 0.1950 + 0.2088\\
&= 0.4038
\end{aligned}
$$

46. If X_1 and X_2 are two independent random variables with means 30 and 25 and variances 16 and 12 and if $Y = 3X_1 - 2X_2$, find $P(60 \leq Y \leq 80)$

Solution:

$E(X_1) = 30, V(X_1) = 16$
$E(X_2) = 25, V(X_2) = 12$

We have,

$Y = 3X_1 - 2X_2$
$E(Y) = 3E(X_1) - 2E(X_2) = 40$
$\therefore \mu = 40$
$V(Y) = 3^2V(X_1) + (-2)^2V(X_2) = 192$
$\therefore \sigma^2 = 192$
$\therefore \sigma = 13.86$

Thus, $z = \frac{Y-\mu}{\sigma} = \frac{Y-40}{13.86}$

Now,

$P(60 \leq Y \leq 80)$
$= P\left(\frac{60-40}{13.86} \leq z \leq \frac{80-40}{13.86}\right)$
$= P(1.44 \leq z \leq 2.89)$
$= A(2.89) - A(1.44)$
$= 0.4981 - 0.4251$
$= 0.0730$

47. In an examination marks obtained by students in Mathematics, Physics and Chemistry are normally distributed with means 51, 53 and 46 with standard deviations 15, 12 and 16 respectively. Find the probability of securing total marks (i) 180 or above (ii) 90 or below. **[N18/Extc/8M]**

Solution:

Let the marks in Mathematics be x_1, physics be x_2, chemistry be x_3 and the total marks be $y = x_1 + x_2 + x_3$

$E(y) = E(x_1) + E(x_2) + E(x_3)$

$E(y) = 51 + 53 + 46$

$\therefore \mu = 150$

$V(y) = V(x_1) + V(x_2) + V(x_3)$

$V(y) = 15^2 + 12^2 + 16^2$

$\sigma^2 = 625$

$\therefore \sigma = 25$

Now, $z = \frac{y-\mu}{\sigma} = \frac{y-150}{25}$

(i) $P(180\ or\ above) = P(y \geq 180)$

$= P\left(z \geq \frac{180-150}{25}\right)$

$= P(z \geq 1.2)$

$= A(z = 1.2\ to\ z = \infty)$

$= A(\infty) - A(1.2)$

$= 0.5 - 0.3849$

$= 0.1151$

(ii) $P(90\ or\ below) = P(y \leq 90)$

$= P\left(z \leq \frac{90-150}{25}\right)$

$= P(z \leq -2.4)$

$= A(z = -\infty\ to\ z = -2.4)$

$= A(-2.4) - A(-\infty)$

$= -A(2.4) + A(\infty)$

$= A(\infty) - A(2.4)$

$= 0.5 - 0.4918$

$= 0.0082$

48. Using Normal distribution, find the probability of getting 55 heads in the toss of 100 fair coins.

[D23/CompIT/6M]

Solution:

$n = 100, p =$ probability of getting a head $= \frac{1}{2}, q = \frac{1}{2}$

$\mu = np = 100 \times \frac{1}{2} = 50$

$\sigma = \sqrt{npq} = \sqrt{100 \times \frac{1}{2} \times \frac{1}{2}} = 5$

$z = \frac{x-\mu}{\sigma} = \frac{x-50}{5}$

$$\begin{aligned} P(55\ heads) &= P(x = 55) \\ &= P(54.5 < x < 55.5) \\ &= P\left(\frac{54.5-50}{5} < z < \frac{55.5-50}{5}\right) \\ &= P(0.9 < z < 1.1) \\ &= A(1.1) - A(0.9) \\ &= 0.3643 - 0.3159 \\ &= 0.0484 \end{aligned}$$