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# [TAYLOR AND LAURENT SEIRIES]

— DEC 2022

Q. $f(z) = \frac{1}{z^2+4z+3}$

FOR ROC, PUT $z^2+4z+3=0$

$z=-1, \quad z=-3$

$|z|=1, \quad |z|=3$

ROCs ARE,

a) $|z|<1$
b) $1<|z|<3$
c) $|z|>3$

$\therefore$ LET, $f(z) = \frac{1}{(z+1)(z+3)}$

LET $f(z) = \frac{A}{z+1} + \frac{B}{z+3}$

$A = \left(\frac{1}{z+3}\right)_{z=1} = \frac{+1}{2}$

$B = \left(\frac{1}{z+1}\right)_{z=3} = \frac{-1}{2}$

$f(z) = \frac{+1/2}{z+1} + \frac{1/2}{z+3}$

a) $|z| < 1$

$$= \frac{+1/4}{+1+z} + \frac{1/4}{+3+z}$$

$$= \frac{+1/4}{+1(1+z)} + \frac{1/4}{+3\left(1+\frac{z}{+3}\right)}$$

$$= \frac{1}{4}(1+z)^{-1} + \frac{1}{12}\left(1+\frac{z}{+3}\right)^{-1}$$

$$= \frac{1}{2}\left[1 - z + z^2 - z^3 + \dots\right] +$$

$$\frac{1}{-6}\left[1 - \frac{z}{+3} + \left(\frac{z}{+3}\right)^2 - \left(\frac{z}{+3}\right)^3 + \dots\right]$$

b) $1 < |z| < 3$

$$= \frac{+1/4}{+1+z} + \frac{1/4}{+3+z}$$

$$= \frac{+1}{4z\left(1+\frac{1}{z}\right)} + \frac{1}{12\left(1+\frac{z}{+3}\right)}$$

$$= \frac{1}{4z}\left[1+\frac{1}{z}\right]^{-1} + \frac{1}{12}\left[1+\frac{z}{-3}\right]^{-1}$$

$$= \frac{1}{4z}\left[1 - \frac{1}{z} + \left(\frac{1}{z}\right)^2 - \left(\frac{1}{z}\right)^3 + \cdots\right] +$$

$$\frac{1}{12\cancel{-6}}\left[1 - \frac{2}{-3} + \left(\frac{2}{-3}\right)^2 - \left(\frac{2}{-3}\right)^3 + \cdots\right]$$

c) $|z| > 3$

$$= \frac{+1/4}{+1+z} + \frac{1/4}{+3+z}$$

$$= \frac{+1}{4z\left(1+\frac{1}{z}\right)} + \frac{1}{12z\left(1+\frac{3}{z}\right)}$$

$$= \frac{+1}{4z}\left(1+\frac{1}{z}\right)^{-1} + \frac{1}{12z}\left(1+\frac{3}{z}\right)^{-1}$$

$$= \frac{-1}{4z}\left[1 - \frac{1}{z} + \left(\frac{1}{z}\right)^2 - \left(\frac{1}{z}\right)^3 + \cdots\right] +$$

$$\frac{1}{12z}\left[1 - \frac{+3}{z} + \left(\frac{+3}{z}\right)^2 - \left(\frac{+3}{z}\right)^3 + \cdots\right]$$

— DEC 2023

Q. $f(z) = \frac{1}{(z-1)(z+2)}$

LET, $f(z) = \frac{A}{(z-1)} + \frac{B}{(z+2)}$

$A = \left[\frac{1}{z+2}\right]_{z=1} = \frac{1}{3}$

$B = \left[\frac{1}{z-1}\right]_{z=-2} = \frac{1}{-3}$

$f(z) = \frac{1}{3(z-1)} + \frac{-1}{3(z+2)}$

LET $z = u = 0$

$f(z) = \frac{1}{3(u-1)} - \frac{1}{3(u+2)}$ —— 1

FOR ROCs,

PUT $u-1=0$ , $u+2=0$

$u=1$ , $u=-2$

$|u|=1$ , $|u|=2$

ROCs ARE,

a) $|u|<1$

b) $1<|u|<2$

c) $|u|>2$

— a) $|u| < 1$

$$= \frac{1}{-3(1-u)} - \frac{1}{6\left(1+\frac{u}{2}\right)}$$

$$= \frac{1}{-3}(1-u)^{-1} - \frac{1}{6}\left(1+\frac{u}{2}\right)^{-1}$$

$$= \frac{1}{-3}\left[1-(-u)+(+u)^2-(-u)^3+\dots\right] - \frac{1}{6}\left[1-\left(\frac{u}{2}\right)+\left(\frac{u}{2}\right)^2-\left(\frac{u}{2}\right)^3+\dots\right]$$

— b) $1 < |u| < 2$

$$= \frac{1}{3u\left(1-\frac{1}{u}\right)} - \frac{1}{6\left(1+\frac{u}{2}\right)}$$

$$= \frac{1}{3u}\left(1-\frac{1}{u}\right)^{-1} - \frac{1}{6}\left(1+\frac{u}{2}\right)^{-1}$$

$$= \frac{1}{3u}\left[1-\left(\frac{-1}{u}\right)+\left(\frac{+1}{u}\right)^2-\left(\frac{-1}{u}\right)^3+\dots\right] - \frac{1}{6}\left[1-\frac{u}{2}+\left(\frac{u}{2}\right)^2-\left(\frac{u}{2}\right)^3+\dots\right]$$

— c) $|u| > 2$

$$= \frac{1}{3u\left(1-\frac{1}{u}\right)} - \frac{1}{3u\left(1+\frac{2}{u}\right)}$$

$$= \frac{1}{3u}\left(1-\frac{1}{u}\right)^{-1} - \frac{1}{3u}\left(1+\frac{2}{u}\right)^{-1}$$

$$= \frac{1}{3u}\left[1 + \left(\frac{1}{u}\right) + \left(\frac{1}{u}\right)^2 + \left(\frac{1}{u}\right)^3 + \cdots\right] - \frac{1}{3u}\left[1 - \frac{2}{u} + \left(\frac{2}{u}\right)^2 - \left(\frac{2}{u}\right)^3 + \cdots\right]$$

— PUT $u = z$

(a) $= \frac{1}{-3}\left[1 + z + (z)^2 + (z)^3 + \cdots\right] - \frac{1}{6}\left[1 - \frac{z}{2} + \left(\frac{z}{2}\right)^2 - \left(\frac{z}{2}\right)^3 + \cdots\right]$

(b) $= \frac{1}{3z}\left[1 + \frac{1}{z} + \left(\frac{1}{z}\right)^2 + \left(\frac{1}{z}\right)^3 + \cdots\right] - \frac{1}{6}\left[1 - \frac{z}{2} + \left(\frac{z}{2}\right)^2 - \left(\frac{z}{2}\right)^3 + \cdots\right]$

(c) $= \frac{1}{3z}\left[1 + \frac{1}{z} + \left(\frac{1}{z}\right)^2 + \left(\frac{1}{z}\right)^3 + \cdots\right] - \frac{1}{3z}\left[1 - \frac{2}{z} + \left(\frac{2}{z}\right)^2 - \left(\frac{2}{z}\right)^3 + \cdots\right]$

— MAY 2023

Q. $f(z) = \dfrac{z-1}{z^2-2z-3}$

PUT $z^2-2z-3=0$

$z=3$ , $z=-1$

$|z|=3$ , $|z|=1$

$\therefore$ $f(z) = \dfrac{z-1}{(z-3)(z+1)}$

ROCs ARE,

a) $|z|<1$

b) $1<|z|<3$

c) $|z|>3$

LET $f(z) = \dfrac{A}{z-3} + \dfrac{B}{z+1}$

$A = \left[\dfrac{z-1}{z+1}\right]_{z=3} = \dfrac{1}{2}$

$B = \left[\dfrac{z-1}{z-3}\right]_{z=-1} = \dfrac{1}{2}$

$\therefore$ $f(z) = \dfrac{1/2}{z-3} + \dfrac{1/2}{z+1}$

a) $|z| < 1$

$$= \frac{1/2}{-3+z} + \frac{1/2}{1+z}$$

$$= \frac{1}{-6}\left[1 - \frac{z}{3}\right]^{-1} + \frac{1}{2}\left[1+z\right]^{-1}$$

$$= \frac{1}{-6}\left[1 + \frac{z}{3} + \left(\frac{z}{3}\right)^2 + \left(\frac{z}{3}\right)^3 + \cdots\right]$$

$$+ \frac{1}{2}\left[1 - z + z^2 - z^3 + \cdots\right]$$

b) $1 < |z| < 3$

$$= \frac{1/2}{-3+z} + \frac{1/2}{z+1}$$

$$= \frac{1}{-6\left(1-\frac{z}{3}\right)} + \frac{1}{2z\left(1+\frac{1}{z}\right)}$$

$$= \frac{1}{-6}\left[1 + \frac{z}{3} + \left(\frac{z}{3}\right)^2 + \left(\frac{z}{3}\right)^3 + \cdots\right]$$

$$+ \frac{1}{2z}\left[1 - \frac{1}{z} + \left(\frac{1}{z}\right)^2 - \left(\frac{1}{z}\right)^3 + \cdots\right]$$

c) $|z| > 3$

$$= \frac{1/2}{z-3} + \frac{1/2}{z+1}$$

$$= \frac{1}{2z\left(1-\frac{3}{z}\right)} + \frac{1}{2z\left(1+\frac{1}{z}\right)}$$

$$= \frac{1}{2z}\left[1 + \frac{3}{z} + \left(\frac{3}{z}\right)^2 + \left(\frac{3}{z}\right)^3 + \cdots\right]$$
$$+ \frac{1}{2z}\left[1 - \frac{1}{z} + \left(\frac{1}{z}\right)^2 - \left(\frac{1}{z}\right)^3 + \cdots\right]$$

— MAY 2024

Q. $f(z) = \dfrac{4z+3}{(z-3)\,z\,(z+2)}$, $2 < |z| < 3$

LET $f(z) = \dfrac{4z+3}{(z-3)\,z\,(z+2)} = \dfrac{A}{z} + \dfrac{B}{z-3} + \dfrac{C}{z+2}$

$$4z+3 = A(z-3)(z+2) + B(z)(z+2) + C(z)(z-3)$$

$$4z+3 = A(z^2-3z+2z-6) + B(z^2+2z) + C(z^2-3z)$$

$$4z+3 = A(z^2-z-6) + B(z^2+2z) + C(z^2-3z)$$

ON COMPARING LHS AND RHS,

$$A+B+C=0$$
$$-A+2B-3C=4$$
$$-6A+0B+0C=3$$

$A = \dfrac{-1}{2}$, $B=1$, $C = \dfrac{-1}{2}$

$\therefore f(z) = \dfrac{-1/2}{z} + \dfrac{1}{z-3} + \dfrac{-1/2}{z+2}$

FOR $2 < |z| < 3$,

$$= \frac{-1}{2z} + \frac{1}{z-3} + \frac{-1}{2z+2}$$

$$= \frac{-1}{2z} + \frac{1}{-3+z} - \frac{1}{2z+2}$$

$$= \frac{-1}{2z} + \frac{1}{-3\left(1+\frac{z}{-3}\right)} - \frac{1}{2z\left(1+\frac{2}{z}\right)}$$

$$= \frac{-1}{2z} + \frac{1}{-3}\left[1+\frac{z}{-3}\right]^{-1} - \frac{1}{2z}\left[1+\frac{2}{z}\right]^{-1}$$

$$= \frac{-1}{2z} + \frac{1}{-3}\left[1 - \frac{z}{-3} + \left(\frac{z}{-3}\right)^2 - \left(\frac{z}{-3}\right)^3 + \dots\right]$$

$$-\frac{1}{2z}\left[1 - \frac{2}{z} + \left(\frac{2}{z}\right)^2 - \left(\frac{2}{z}\right)^3 + \dots\right]$$