II

X. I. E. [MATHS - IV] (A)

Mahim, Mumbai

[242466]

Page No. :

Date :

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

(1) $A^3 - 3A^{-1}$

(2) $\text{adj}\, A - 3A$

$|A| = 6$

CH. EQ = $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 0 & -1 \\ 0 & 2-\lambda & 0 \\ -1 & 0 & 2-\lambda \end{vmatrix} = 0$$

WE KNOW THAT,

$$\lambda^3 - 6\lambda^2 + \left[\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}\right]\lambda - 6 = 0$$

$$\lambda^3 - 6\lambda^2 + [4 + 3 + 4]\lambda - 6 = 0$$

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$\boxed{\lambda = 1, 3, 2}$$

FOR $\lambda = 1$, $[A - \lambda I]X = 0$

$1 \cdot x_1 + 0x_2 - 1x_3 = 0$

$0 \cdot x_1 + 1x_2 + 0x_3 = 0$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}}$$

$+1 \qquad 0 \qquad 1$

$1 \cdot x_1 + 0 \cdot x_2 - 1 \cdot x_3 = 0$

$0 \cdot x_1 + 1 \cdot x_2 + 0 \cdot 3x = 0$

BY CRAMER'S RULE,

$$\frac{x_1}{\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}}$$

$$\frac{x_1}{1} = \frac{-x_2}{0} = \frac{x_3}{1}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \text{ — } (1)$$

FOR $\lambda = 3$, $[A - \lambda I] Y = 0$

$$\begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-1y_1 + 0y_2 - 1y_3$
$0.y_1 - 1y_2 + 0y$

$-1.y_1 + 0.y_2 - 1.y_3 = 0$
$0.y_1 + (-1.y_2) + 0.y_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix}} \quad \frac{-x_2}{\begin{vmatrix} -1 & -1 \\ 0 & 0 \end{vmatrix}}$$

BY CRAMER'S RULE,

$$\frac{y_1}{\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix}} = \frac{-y_2}{\begin{vmatrix} -1 & -1 \\ 0 & 0 \end{vmatrix}} = \frac{y_3}{\begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}}$$

$$\frac{y_1}{-1} = \frac{-y_2}{0} = \frac{y_3}{1}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \text{ —}$$

FOR $\lambda = 2$, $[A - \lambda I]$

$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} =$$

HERE,
$r = 2$
$n = 3$
$u = n - r = 3 - 2 = 1$

$-z_3 = 0$
$-z_1 = 0$
LET $z_2 = s$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ s \\ 0 \end{bmatrix} =$$

(1) $A^3 - 3A^{-1}$

$\lambda = 1 \Rightarrow 1^3 - 3\,1^{-1}$
$\lambda = 3 \Rightarrow 3^3 - 3\,3^{-1}$
$\lambda = 2 \Rightarrow 2^3 - 3\,2^{-1}$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \text{—} \quad (2)$$

FOR $\lambda = 2$, $[A - \lambda I] Z = 0$

$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0 \cdot x_1 + 0 \cdot x_2 \quad -1 x_3$

$0 x_1 \quad 0 x_2 \quad 0 x_3$

x_1	$-x_2$	x_3
0 -1	0 -1	0 0
0 0	0 0	0 0

HERE,

$r = 2$

$n = 3$

$u = n - r = 3 - 2 = 1$

$-z_3 = 0$

$-z_1 = 0$

LET $z_2 = s$

$$\therefore \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ s \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \text{—} \quad (3)$$

(1) $A^3 - 3A^{-1}$

$\therefore \lambda = 1 \Rightarrow 1^3 - 3\,1^{-1} = -2$

$\therefore \lambda = 3 \Rightarrow 3^3 - 3\,3^{-1} = 26$

$\therefore \lambda = 2 \Rightarrow 2^3 - 3\,2^{-1} = \frac{65}{10}$

② adj A − 3A

$\therefore \lambda = 1 \Rightarrow 6/1 - 3 \cdot 1 = 3$

$\therefore \lambda = 3 \Rightarrow 6/3 - 3 \cdot 3 = -7$

$\therefore \lambda = 2 \Rightarrow 6/2 - 3 \cdot 2 = -3$

1] $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

① adj A
② $A^3 - 3I$

Soln $|A| = 5$

CH. EQ. $= |A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 2 \\ 2 & 1-\lambda & 2 \\ 2 & 2 & 1-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 3\lambda^2 + \left[\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} \right] \lambda - 5 = 0$$

$$\lambda^3 - 3\lambda^2 + (-3 - 3 - 3)\lambda - 5 = 0$$

$$\lambda^3 - 3\lambda^2 - 9\lambda - 5 = 0$$

$$\boxed{\lambda = 5, -1, -1}$$

$[A - \lambda I] X = 0$

FOR $\lambda = 5$,

$$\begin{bmatrix} -4 & 2 & 2 \\ 2 & -4 & 2 \\ 2 & 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-4x_1 + 2x_2 + 2x_3 = 0$

$2x_1 - 4x_2 + 2x_3 = 0$

BY CRAMER'S RULE

$$\frac{x_1}{\begin{vmatrix} 2 & 2 \\ -4 & 2 \end{vmatrix}} = \frac{-x}{\begin{vmatrix} -4 \\ 2 \end{vmatrix}}$$

$$\frac{x_1}{12} =$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

FOR $\lambda = -1$, $|A$

$$\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

$2R_2 - 2R_1$

$2R_3 - 2R_1$

$$\begin{bmatrix} 2 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$-4x_1 + 2x_2 + 2x_3 = 0$
$2x_1 - 4x_2 + 2x_3 = 0$

BY CRAMER'S RULE,

$$\frac{x_1}{\begin{vmatrix} 2 & 2 \\ -4 & 2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -4 & 2 \\ 2 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -4 & 2 \\ 2 & -4 \end{vmatrix}}$$

$$\frac{x_1}{12} = \frac{+x_2}{+12} = \frac{x_3}{12}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ --- } (1)$$

FOR $\lambda = -1$, $|A - \lambda I| Y = 0$

$$\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2R_2 - 2R_1$
$2R_3 - 2R_1$

$$\begin{bmatrix} 2 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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HERE,

$R = 1$

$N = 3$

$U = N - R = 3 - 1 = 2$

$2y_1 + 2y_2 + 2y_3 = 0$

$2y_1 = -2y_2 - 2y_3$

$y_1 = -y_2 - y_3$

Let $y_2 = s$, $y_3 = t$

$$\therefore \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -s-t \\ s \\ t \end{bmatrix}$$

$$= \begin{bmatrix} -s \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} -t \\ 0 \\ t \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad \text{---} \quad (2)$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \text{---} \quad (3)$$

① $adj\,A = |A| / \lambda$

$\lambda = 5 \Rightarrow 5/5 = 1$
$\lambda = -1 \Rightarrow 5/-1 = -5$
$\lambda = -1 \Rightarrow 5/-1 = -5$

② $A^3 - 3I = \lambda^3 - 3.1$

$\lambda = 5 \Rightarrow 125 - 3.1 = 122$
$\lambda = -1 \Rightarrow -4$
$\lambda = -1 \Rightarrow -4$

4) $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$, AM = ? GM = ?

$|A| = 4$

CH. EQ. $= |A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & 6 & 6 \\ 1 & 3-\lambda & 2 \\ -1 & -5 & -2-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 5\lambda^2 + \left[\begin{vmatrix} 3 & 2 \\ -5 & -2 \end{vmatrix} + \begin{vmatrix} 4 & 6 \\ -1 & -2 \end{vmatrix} + \begin{vmatrix} 4 & 6 \\ 1 & 3 \end{vmatrix} \right] \lambda - 4 = 0$$

$$\lambda^3 - 5\lambda^2 + [4 + (-2) + 6]\,\lambda - 4 = 0$$

$$\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$$

$\boxed{\lambda = 1, 2, 2}$ — CONDITION FOR E.VAL

ALGEBRIC MULTIPLICITY $\Longrightarrow$

FOR $\lambda = 1$ IS 1

FOR $\lambda = 2$ IS 2

FOR $\lambda = 1$, $[A - \lambda I] X = 0$

$$\begin{bmatrix} 3 & 6 & 6 \\ 1 & 2 & 2 \\ -1 & -5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

FOR $\lambda = 2$, $[A - \lambda I] Y = 0$

$$\begin{bmatrix} 2 & 6 & 6 \\ 1 & 1 & 2 \\ -1 & -5 & -4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

1 $\Longrightarrow$ $R = 3$

$N = 3$

$GM = N - R = 3 - 3 = 0$

2 $\Longrightarrow$ $R = 3$

$N = 3$

$GM = N - R = 3 - 3 = 0$

$\therefore$ $AM \neq GM$

$\therefore$ IT IS NOT DIAGONALISABLE

$A = \begin{bmatrix} -3 & 2+2i \\ 2-2i & 4 \end{bmatrix}$, IS

λ

WE KNOW THAT,

SUM OF E.VAL =

$\lambda_1 + \lambda_2$ =

$\lambda_1 + \lambda_2$ =

PROD. OF E.VAL

$\lambda_1 . \lambda_2$ =

$\lambda_1 . \lambda_2$ =

PUT $[\lambda_1 = 1 - \lambda_2$

$\lambda_2 (-\lambda_2 + 1)$

$-\lambda_2^2 + \lambda_2$ =

$\lambda_2^2 - \lambda_2$ -

$\boxed{\lambda = -4, 5}$

SINCE E.VAL.

DIAGNALIZABLE

FOR $\lambda = -4$, $[A$

$$\begin{bmatrix} -3+4 & 2+2i \\ 2-2i & 4+4 \end{bmatrix}$$

$A = \begin{bmatrix} -3 & 2+2i \\ 2-2i & 4 \end{bmatrix}$, IS IT DIAGNAUSABLE?
$\lambda_1, \lambda_2 = ?$

WE KNOW THAT,

SUM OF E.VAL = SUM OF DIAG.

$\lambda_1 + \lambda_2 = -3+4$

$\lambda_1 + \lambda_2 = +1$ — (1)

PROD. OF E.VAL = $|A|$

$\lambda_1 \cdot \lambda_2 = 3.(-4)$

$\lambda_1 \cdot \lambda_2 = -20$ — (2)

PUT $[\lambda_1 = 1 - \lambda_2]$ IN (2)

$\lambda_2(-\lambda_2 + 1) = -20$

$-\lambda_2^2 + \lambda_2 = -20$

$\lambda_2^2 - \lambda_2 - 20 = 0$

$\boxed{\lambda = -4, 5}$

∴ SINCE E.VAL. ARE DISTINCT, IT IS DIAGNALIZABLE

FOR $\lambda = -4$, $|A - \lambda I|X = 0$

$\begin{bmatrix} -3+4 & 2+2i \\ 2-2i & 4+4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Date :

$$\begin{bmatrix}1 & 2+2i\\ 2-2i & 8\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

$x_1 + (2+2i)x_2 = 0$ — (1)

$(2-2i)x_1 + 8x_2 = 0$ — (2)

(1) $\Longrightarrow$ $x_1 = -(2+2i)x_2$ — (3)

PUT (3) IN (2)

$(2-2i)(-(2+2i)x_2) + 8x_2 = 0$

$(2-2i)(-(2+2i)x_2) = -8x_2$

$-8x\cancel{x}_2 = -8x_2$

$-8x_2 + 8x_2 = 0$

$(2-2i)(2$

$4+4i-4$

$\therefore$ $x_2 = 1$

$\therefore$ FROM (3),

$\longrightarrow \begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}-2-2i\\ 1\end{bmatrix}$

$x_1 = -(2+2i)x_2$ — FROM (3)

$x_1 = -2-2i\ x_2$

$\frac{x_1}{-2-2i} = \frac{x_2}{1}$

FOR $\lambda = 5$, $|A - \lambda I|Y$

$$\begin{bmatrix}-8 & 2+2i\\ 2-2i & -1\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix}$$

$-8y_1 + (2+2i)y_2 = 0$ —

$(2-2i)y_1 - y_2 = 0$ —

(4) $\Longrightarrow$

$8y_1 = (2+2i)$

PUT (6) IN (5

$\left[\frac{(2+2i)}{8}y_2\right](2-2i)$

$\left[\frac{1+i}{4}y_2\right](2-2i)$

$\left[\frac{4}{4}y_2\right] - y_2$

$x_2 = 1$

$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1+i\\ 1\end{bmatrix}$

$8y_1 = (2+2i)y_2$

$\cancel{4y_1} = \cancel{(1+i)y_2}$

$\cancel{y_1}$

FOR $\lambda = 5$, $|A - \lambda I| Y = 0$

$$\begin{bmatrix} -8 & 2+2i \\ 2-2i & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-8y_1 + (2+2i)y_2 = 0$ — (4)
$(2-2i)y_1 - y_2 = 0$ — (5)

(4) $\Rightarrow$

$8y_1 = (2+2i)y_2$ — (6)

PUT (6) IN (5)

$$\left[\frac{(2+2i)}{8} y_2\right](2-2i) - y_2 = 0$$

$$\left[\frac{1+i}{4} y_2\right](2-2i) - y_2 = 0$$

$$\left[\frac{4}{4} y_2\right] - y_2 = 0$$

$(1+i)(2-2i)$
$= 2 - 2i + 2i + 2i^2$
$= 2 + 2$
$= 4$

$\therefore x_2 = 1$

$$\Rightarrow \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1+i/4 \\ 1 \end{bmatrix}$$

$8y_1 = (2+2i)y_2$ — FROM (6)

$\cancel{4y_1} = \cancel{(1+i)y_2}$

$\cancel{y_1}$

$y_1 = \frac{(1+2i)y_2}{4}$

$y_1 = \frac{1+i}{4}$

HERE,

R = 2
N = 2

ALGEBRIC MULTIPLICITY:
-4 = 1
5 = 1

GEOMETRIC MULTIPLICITY:
R - N = 2 - 2 = 0

∴ SINCE NO REPEATING EIGN VALUES THE MATRIX IS DIAGNALIZABLE.

→ THUS, THE MATRIX "A" = $\begin{bmatrix} -3 & 2+2i \\ 2-2i & 4 \end{bmatrix}$

IS DIAGNOLIZABLE TO "D" = $\begin{bmatrix} -4 & 0 \\ 0 & 5 \end{bmatrix}$

BY THE TRANSFORM $M^{-1} A.M = D$ WHER
$M = \begin{bmatrix} -2-2i & 1+i/4 \\ 1 & 1 \end{bmatrix}$

NXT Pg →

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$|A| = 3$

CHARACTERISTIC EQ. $= |A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ -1 & 3-\lambda & 1 \\ 1 & 0 & 2-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 6\lambda^2 + \left[\overset{6}{\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}} + \overset{-1}{\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix}} + \overset{4}{\begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix}} \right] \lambda - 3 = 0$$

$$\lambda^3 - 6\lambda^2 + [6 + (-1) + 4]\lambda - 3 = 0$$

$$\lambda^3 - 6\lambda^2 + 9\lambda - 3 = 0 \quad \text{—} \quad (1)$$

$\lambda = 3.879, 1.652, 0.467$

BY CAYLEY-HAMILTON THM.,

$$\lambda^3 - 6\lambda^2 + 9\lambda - 3 = 0$$

$$A^3 - 6A^2 + 9A - 3 = 0$$

NXT Pg $\rightarrow$

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$$\begin{bmatrix} 5 & 28 & 36 \\ -8 & 15 & 2 \\ 8 & 12 & 25 \end{bmatrix} - 6\begin{bmatrix} 2 & 8 & 11 \\ -3 & 7 & 2 \\ 3 & 2 & 7 \end{bmatrix} +$$

$$9\begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix} - 3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ LHS = RHS

$\therefore$ VERIFIED CAHILTON THM.

$\cancel{A^4} = A^3 - (6A^2) + 9A - 3I = 0$

$A^4 - 6A^3 + 9A^2 - 3A = 0$ — [MULTIPL

$$A^4 = 6\begin{bmatrix} 5 & 28 & 36 \\ -8 & 15 & 2 \\ 8 & 12 & 25 \end{bmatrix} - 9\begin{bmatrix} 2 & 8 & 11 \\ -3 & 7 & 2 \\ 3 & 2 & 7 \end{bmatrix}$$

$$3\begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 13 & 94 & 115 \\ -21 & 24 & -5 \\ 21 & 52 & 86 \end{bmatrix} \text{ — } (2)$$

$$A^3 - 6A^2 + 9A - 3I = 0$$

$$A^2 - 6A + 9I - 3A^{-1} = 0 - \boxed{\text{MULTIPLY BY } A^{-1}}$$

$$-3A^{-1} = -A^2 + 6A - 9I$$

$$3A^{-1} = A^2 - 6A + 9I$$

$$A^{-1} = \frac{1}{3}\left[A^2 - 6A + 9I\right]$$

$$= \frac{1}{3}\left[\begin{bmatrix} 2 & 8 & 11 \\ -3 & 7 & 2 \\ 3 & 2 & 7 \end{bmatrix} - 6\begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]$$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1.333 & -2.333 \\ 1 & -0.333 & -1.333 \\ -1 & 0.666 & 1.666 \end{bmatrix} - (3)$$

OR

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -4/3 & -7/3 \\ 1 & -1/3 & -4/3 \\ -1 & 2/3 & 5/3 \end{bmatrix}$$

NXT Pg →

$A^9 - 6A^8 + 10A^7 - 3A^6 + A + I = ?$

IF $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

$|A| = 3$

CH. EQ $= |A - \lambda I| = 0$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ -1 & 3-\lambda & 1 \\ 1 & 0 & 2-\lambda \end{bmatrix} = 0$$

$$\lambda^3 - 6\lambda^2 + \left[\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} \right] \lambda - 3 = 0$$

$$\lambda^3 - 6\lambda^2 + 10\lambda - 3 = 0 \quad \text{— } (1)$$

WE KNOW THAT,

$= A^9 - 6A^8 + 10A^7 - 3A^6 + A + I$

$= (A^9 - 6A^8 + 10A^7 - 3A^6) + A + I$

$= A^6(A^3 - 6A^2 + 10A - 3A) + A + I$

FROM (1), $A^6(A^3 - 6A^2 + 10A - 3A) = 0$

$\therefore = A + I$ — [NEW MATRIX]

$\therefore A = \begin{bmatrix} 2 & 2 & 3 \\ -1 & 4 & 1 \\ 1 & 0 & 3 \end{bmatrix}, \; |A| = 20$

CH. Eq. $= |A - \lambda I| = 0$

$$\begin{bmatrix} 2-\lambda & 2 & 3 \\ -1 & 4-\lambda & 1 \\ 1 & 0 & 3-\lambda \end{bmatrix} = 0$$

$$\lambda^3 - 9\lambda^2 + \left[\begin{vmatrix} 4 & 1 \\ 0 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ -1 & 4 \end{vmatrix} \right] \lambda - 20 = 0$$

$$\lambda^3 - 9\lambda^2 + [12 + 3 + 10] \lambda - 20 = 0$$

$$\lambda^3 - 9\lambda^2 + 25\lambda - 20 = 0$$

$\lambda = 1.381, 4, 3.618$

1:3

~~(1) A^{-1}~~ VIA CAYLEY HAMILTON :

$\Rightarrow \lambda^3 - 9\lambda^2 + 25\lambda - 20 = 0$ — [TH

$\lambda^3 - 9\lambda^2 + 25\lambda - 20 = 0$

LHS $= \lambda^3 - 9\lambda^2 + 25\lambda - 20 = 0$

$= A^3 - 9A^2 + 25A - 20 = 0$

$$= \begin{bmatrix} 15 & 58 & 78 \\ -20 & 46 & 11 \\ 20 & 18 & 15 \end{bmatrix} - 9 \begin{bmatrix} 5 & 12 & 17 \\ -5 & 14 & 4 \\ 5 & 2 & 12 \end{bmatrix}$$

$$+ 25 \begin{bmatrix} \cancel{25}2 & 2 & 3 \\ -1 & 4 & 1 \\ 1 & 0 & 3 \end{bmatrix} - 20$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

RHS = 0

LHS = RHS

$\therefore$ CH THEOREM PROVED

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Date :

$\lambda^3 - 9\lambda^2 + 25\lambda - 20I = 0$

$\lambda^2 - 9\lambda + 25I - 20A^{-1} = 0$ — [MULTIPLY BY]

$\lambda^2 - 9\lambda + 25I - 20A^{-1} = 0$

$A^2 - 9A + 25I - 20A^{-1} = 0$

$20A^{-1} = A^2 - 9A + 25I$

$$20A^{-1} = \begin{bmatrix} 5 & 12 & 17 \\ -5 & 14 & 4 \\ 5 & 2 & 12 \end{bmatrix} - 9\begin{bmatrix} 2 & 2 & 3 \\ -1 & 4 & 1 \\ 1 & 0 & 3 \end{bmatrix} + 25\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = 20\begin{bmatrix} 3/5 & -3/10 & -1/2 \\ 1/5 & 3/20 & -1/4 \\ -1/5 & 1/10 & 1/2 \end{bmatrix}$$

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$A^9 - 6A^8 + 10A^7 - 3A^6 +$

IF $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \\ 1 & 0 \end{bmatrix}$

$|A| = 3$

CH. EQ = $|A -$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ -1 & 3-\lambda & 1 \\ 1 & 0 & 2-\lambda \end{bmatrix}$$

$\lambda^3 - 6\lambda^2 + [\,|$

$\lambda^3 - 6\lambda^2 + 10\lambda$

WE KNOW T

$= A^9 - 6A^8 + 10A^7$

$= (A^9 - 6A^8 + 10A$

$= A^6(A^3 - 6A^2$

FROM (1),

$\therefore = A + I$ —

$\therefore A = \begin{bmatrix} 2 & 2 \\ -1 & 4 \\ 1 & 0 \end{bmatrix}$