Grammars

✓ Grammar is used for specifying the syntax of a language and is defined as follows

Defination $G = (V, T, P, S)$

where,

V = finite set of variables OR Non terminals

(Rd by → capital letters)

T = finite set of Terminals

(Rd by → small letters / operators)

P = finite set of Production OR Rewriting Rules

S = Start variable

✓ Context free Grammar (CFG)

Defination: [Grammar is CFG if all the productions are of the form $A \rightarrow \alpha$

where A – variable

α – some sentential form

(i.e any combination of V & T)]

eg $G = (\{S, A\}, \{a, b\}, P, S)$

P

$S \rightarrow aSb \mid aA \mid b$

$A \rightarrow Ab \mid aA \mid ba \mid b$

Derivation

It is a process to check whether the given G can deliver/generate the given sentence, using any combination of productions Rules, starting from start variables production

Left most derivation (LMD) -

Derivation is LMD, if at every step, we select & replace the leftmost variable by its production Rule

Right most Derivation (RMD)

Derivation is RMD, if at every step, we select & replace the leftmost variable by its production Rule.

$S \rightarrow aAS \mid a$

$A \rightarrow SbA \mid SS \mid ba$

Derive "aabbaa"

using LMD & RMD

Solution

LMD

$S \Rightarrow aAS$	using $S \rightarrow aAS$
$\Rightarrow aSbAS$	$A \rightarrow SbA$
$\Rightarrow aabAS$	$S \rightarrow a$
$\Rightarrow aabbaS$	$A \rightarrow ba$
$\Rightarrow aabbaa$	$S \rightarrow a$

RMD

$S \Rightarrow aAS$ using $S \to aAS$

$\xRightarrow{rm} aAa$ $S \to a$

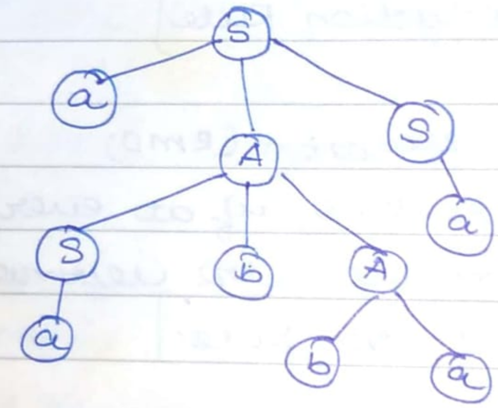
$\xRightarrow{rm} aSbAa$ $A \to SbA$

$\xRightarrow{rm} aSbbaa$ $A \to ba$

$\xRightarrow{rm} aabbaa$ $S \to a$

Derivation / Rule / Parse tree

It is a graphical representation of derivation process



$S \to aB \mid bA$

$A \to a \mid aS \mid bAA$

$B \to b \mid bS \mid aBB$

Derive using LMD & RMD

① aaabbb.

② bbaaba

③ abaa

① LMD $S \underset{lm}{\Rightarrow} aB$ using $S \to aB$

$\Rightarrow aaBB$ $\quad B \to aBB$

$\Rightarrow aaaBBB$ $\quad B \to aBB$

$\Rightarrow aaa\cancel{B}BB$ $\quad B \to b$

$\Rightarrow aaa\cancel{B}bB$ $\quad B \to b$

$\Rightarrow aaabbb$ $\quad B \to b$

RMD

$S \Rightarrow aB$ $\quad$ using $S \to aB$

$\Rightarrow aaBB$ $\quad B \to aBB$

$\Rightarrow aaBb$ $\quad B \to b$

$\Rightarrow aaaBBb$ $\quad B \to aBB$

$\Rightarrow aaaBbb$ $\quad B \to b$

$\Rightarrow aaabbb$ $\quad B \to b$

② LMD $S \Rightarrow bA$ $\quad S \to bA$

$\Rightarrow \cancel{b}bAA$ $\quad A \to bAA$

~~$\Rightarrow bbbAAA$~~

$\Rightarrow bbaA$ $\quad A \to a$

$\Rightarrow bbaaS$ $\quad A \to aS$

$\Rightarrow bbaabA$ $\quad S \to bA$

$\Rightarrow bbaaba$ $\quad A \to a$

RMD

$S \Rightarrow bA$	using $S \to bA$
$\Rightarrow bbAA$	$A \to bAA$
$\Rightarrow bbAa$	$A \to a$
$\Rightarrow bbaSa$	$A \to aS$
$\Rightarrow bbaaBa$	$S \to aB$
$\Rightarrow bbaaba$	$B \to b$

(3) LMD abaa

$S \Rightarrow aB$	$S \to aB$
$\Rightarrow abS$	$B \to bS$
$\Rightarrow abaB$	$S \to aB$
$\Rightarrow$	

RMD

The derivation of the sentence is not possible

Note If the sentence is derivable then it can be derived using LMD & RMD both ambiguity? Removal

ambiguous & unambiguous grammar

ambiguous - [Grammar is ambiguous if it can derive atleast one sentence using more than one LMD/RMD]

unambiguous - [Grammar is unambiguous if it can derive all sentences using exactly one LMD/RMD]

$E \rightarrow E+E \mid E*E \mid id$

Let us derive "id+id*id"

LMD $E \Rightarrow E+E$

$\Rightarrow id+E$

$\Rightarrow id+E*E$

$\Rightarrow id+id*id$

LMD $\Rightarrow E*E$

$\Rightarrow E+E*E$

$\Rightarrow id+id*E$

$\Rightarrow id+id*id$

∵ G can derive sentence using more than one LMD, G is ambiguous.

To eliminate ambiguity we rewrite as -

$E \rightarrow E+T \mid T \quad T \rightarrow T*F \mid F \quad F \rightarrow id$

Solution LMD 1 $\Rightarrow E+T$

$\Rightarrow T+T$

$\Rightarrow F+T$

$\Rightarrow id+T$

$\Rightarrow id+T$

$\Rightarrow id+T*F$

$\Rightarrow id+F*F$

$\Rightarrow id+id*F$

$\Rightarrow id+id*id$

Chomsky's Hierarchy/Types of Grammers/ classification of Grammers

Type Acc to chomsky 4 diff types of 0 grammers which have to classified based on the restrictions placed on the production Rules.

✓ Chomsky's Hierarchy / Types of Grammar / classification of Grammar

$\alpha \in (V+T)^+$ $\beta \in (V+T)^*$

Type	Grammar	Language	Restriction	eg	machine
0	Unrestricted grammar	Recursively ennumerable language	$\alpha \to \beta$	$S \to AB$ $AB \to AC$ $A \to a$ $B \to c$ $C \to b$	T.m
1	Context Sensitive grammar	CSL	$\lvert\alpha\rvert \le \lvert\beta\rvert$ $\alpha, \beta \in (V \cup T)^+$	$S \to aAb$ $aA \to bA$ $aA \to aaA$ $A \to c$	LBA linear Bounded automata.
2	Context free grammar	CFL	$A \to \alpha$	$S \to aSb \mid Aa$ $A \to Ab \mid b$	P.D.A
3	Regular Grammar	Regular language	$A \to$ any number of 'T's & atmost 1 V	LLG $S \to Ba$ $B \to Bb \mid a \mid bc$ RLG $S \to aA$ $A \to a \mid bA \mid b$	F.A

LLG, RLG } Right & Left linear grammar

examples of context free grammars

CFG

G	L(G)
$S \to a$	$\{a\}$
$S \to b$	$\{b\}$
$S \to a \mid b$	$\{a, b\}$
$S \to a \cdot b$	$\{a, b\}$
$S \to aS \mid \epsilon$	$\{\epsilon, a, aa, aaa \ldots\}$
$S \to aS \mid a$	$\{a, aa, aaa, \ldots\}$
$S \to abS \mid \epsilon$	$\{\epsilon, ab, abab, \ldots\}$
$S \to aS \mid bS \mid \epsilon$	$\{\epsilon, a, b, aa, ab, ba \ldots\}$

(1) write CFG to generate the foll

1) Set of all strings that start with 'a' over $\Sigma = \{a, b\}$

solution $S \to aA$

$A \to aA \mid bA \mid \epsilon$ $\qquad a \cdot (a+b)^*$

$G = (\{S, A\}, \{a, b\}, P, S)$

$L(G) = \{a, aa, ab, aaa, \ldots\}$

2) set of all strings that end in '0' over $\Sigma = \{0, 1\}$

$S \to B0$

$B \to 0B \mid 1B \mid \epsilon$

③ Set of all strings that start & end with diff symbol over $\Sigma = \{0,1\}$

$S \rightarrow 0X1 \mid 1X0$

$X \rightarrow 0X \mid 1X \mid \epsilon$

④ Set of all strings that start & end with same symbol over $\Sigma = \{0,1,2\}$

$S \rightarrow 0X0 \mid 1X1 \mid 2X2 \mid 0 \mid 1 \mid 2$

$X \rightarrow 0X \mid 1X \mid 2X \mid \epsilon$

⑤ Set of all strings over $\Sigma = \{a,b\}$ that contain –

① atleast two a's –

$S \rightarrow XaXaX$

$X \rightarrow aX \mid bX \mid \epsilon$

② exactly two a's

$S \rightarrow XaXaX$

$X \rightarrow bX \mid \epsilon$

③ utmost two a's

$S \rightarrow XaXaX \mid XaX \mid X$

$X \rightarrow bX \mid \epsilon$

① $L = \{a^n b^n \mid n \geq 1\}$

$n=1$ ab
$n=2$ aabb
$n=3$ aaabbb

$S \to aSb \mid ab$

$S \to aXb$

$X \to aXb \mid \epsilon$

Note
$n=0$ ϵ

$S \to aSb \mid \epsilon$ $n \geq 0$

② $L = \{a^m b^{2m} \mid m \geq 0\}$

$m=0$ ϵ
$m=1$ abb
$m=2$ aabbbb

$S \to aSbb \mid \epsilon$

$S \to aXbb \mid \epsilon$

$X \to aXbb \mid \epsilon$

③ $L = \{a^n b^{n+1} \mid n \geq 1\}$

$n=1$ ~~a~~ abb
$n=2$ aabbb
$n=3$ aaabbbb

$S \to aSb \mid abb$

$S \to aXbb$

$X \to aXb \mid \epsilon$

$S \to aXb$

$X \to aXb \mid b$

④ $L = \{a^m b^{m-3} \mid m \geq 3\}$

$m=3$ aaa
$m=4$ aaaab
$m=5$ aaaaabb

$S \to aaaX$

$X \to aXb \mid \epsilon$

$S \to aSb \mid aaa$

① $L = \{a^i b^j \mid i, j \geq 1\}$

$S \to XY$

$X \to aX / a$

$Y \to bY / b$

② $L = \{a^i b^j c^k \mid i, j, k \geq 1\}$

$S \to ABC$

$A \to aA / a$

$B \to bB / b$

$C \to cC / c$

③ $L = \{a^i b^i c^k \mid i, k \geq 1\}$

$S \to XY$

$X \to aXb / ab$

$Y \to cY / c$

④ $L = \{a^i b^k c^k\} \mid i, k = 1\}$

$S \to XY$

$X \to aX / a$

$Y \to bYc / bc$

⑤ $L = \{a^i b^j c^k \mid j = k + i,\ i, k \geq i\}$

$a^i b^j c^k$

$= a^i b^{k+i} c^k$

$= a^i b^{i+k} c^k$

$= a^i b^i b^k c^k$

$S \to XY$

$X \to aXb / ab$

$Y \to bYc / bc$

⑦ $L = \{a^i b^j c^k \mid i=j \text{ or } j=k ;\ i, j, k \geq 1\}$

$L = a^i b^j c^k$

$a^i b^j c^k$

$i = j$ | $j = k$

$\underbrace{a^i b^i}_{X} \underbrace{c^k}_{Y}$ | $\underbrace{a^i}_{A} \underbrace{b^j c^j}_{B}$

$\downarrow$ | $\downarrow$

S1 | S2

S2

$S \rightarrow S1 \mid S2$

$S1 \rightarrow XY$

$X \rightarrow aXb \mid ab$

$Y \rightarrow cY \mid c$

$S2 \rightarrow AB$

$A \rightarrow aA \mid a$

$B \rightarrow bBc \mid bc$

⑧ set of all strings over $\Sigma = \{a, b\}$ that contain

① even palindrome

$S \rightarrow aSa \mid bSb \mid e$

② odd palindrome

$S \rightarrow aSa \mid bSb \mid a \mid b$

③ palindrome
$S \to aSa \mid bSb \mid a \mid b \mid \epsilon$

Simplification of cfg

Elimination of useless productions
Defination of useless variable
A variable 'x' is useful if
$S \xRightarrow{*} \alpha x \beta \xRightarrow{*} w$. else x is useless
for some α, β where w is a sentence

Defination of useless Production
A production where useless variables are present is called useless production.

Elimination procedure
Given CFG = $G = (V, T, P, S)$
Define $G' = (V', T', P', S)$ be the CFG that does not contain any useless productions such that $L(G') = L(G)$

Step 1 Initialize P' to P

Step 2 Find useless productions
* variable is useless if it is not reachable from S
* variable is useless if it cannot derive any sentence (ie any combination of T's)

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Step3 Delete all useless productions

(1) Eliminate useless productions

$S \rightarrow aSb \mid a \mid bAB$

$A \rightarrow bA \mid c$

$B \rightarrow bB \mid dB$

given $G = (\{S, A, B\}, \{a, b, c, d\}, P, S\})$

$S \rightarrow aSb \mid a \mid bAB$

$A \rightarrow bA \mid c$

$B \rightarrow bB \mid dB$

Define $G' = (V', T', P', S)$ be the of G that does not contain any useless productions such that $L(G') = L(G)$

$P' = S \rightarrow aSb \mid a \mid bAB$

$A \rightarrow bA \mid c$

$B \rightarrow bB \mid dB$

$\because$ B cannot derive any sentence

B is useless

$S \rightarrow aSb \mid a$

$A \rightarrow bA \mid c$

A is not reachable from S

A is useless

$P' = S \rightarrow aSb \mid a$

$G' = (\{S\}, \{a, b\}, P', S)$

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Simplification of CFG

Elimination of unit productions

* Defination of unit production
A production of the form 'X→Y'
where X & Y are variables is called unit
production

* Defination of Non-Unit production
A production not of the form 'X→Y'
where X & Y are variables is called non-
unit production

* Elimination procedure
Given CFG $G = \{V, T, P, S\}$
Define $G' = (V', T', P', S)$ be the CFG that
does not contain any unit productions
such that $L(G') = L(G)$

Step1 Initialize P' to P

Step2 Find unit productions
* say 'X→Y' is a unit production of X
then add to X, the non unit production
of Y that are not present in X

Step3 Delete all unit productions

eliminate unit productions

$S \to aSb / a / A$

$A \to ab / b / a$

production — new productions

$S \to aSb$ —

$S \to a$ —

$S \to A$ — $S \to ab, S \to b$

$A \to ab$ —

$A \to b$ —

$A \to a$ —

P + n

$S \to aSb / a / A / ab / b$

$A \to ab / a / b$

After deleting unit productions

$S \to aSb / a / ab / b$

$A \to ab / a / b$

① simplify / Reduce the following

$S \to aS / A / D$

$A \to a$

$B \to aa$

$D \to aDb$

S1 elimination of unit productions

production	new production
$S \to aS$	–
$S \to A$	$S \to a$
$S \to D$	$D \to aDb$
$A \to a$	–
$B \to aa$	–
$D \to aDb$	–

p+np

$S \to aS / a / A / D / aDb$

$A \to a$

$B \to aa$

$D \to aDb$

after deleting unit productions

$S \to aS / a / aDb$

$A \to a$

$B \to aa$

$D \to aDb$

S2 elimination of useless production

$\because$ A & B are not reachable from S

A & B are useless

$\because$ D cannot derive any sentence D is useless

after deleting useless productions

$S \to aS / a$

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Simplification of CFG:-
Elimination of null productions

Defination of null production
A production of the form-
'$X \to \epsilon$' OR '$X \to \lambda$' OR '$X \to$ '
where X is a variable is called Null production

Defination of nullable variable-
A variable 'X' is nullable if '$X \to \epsilon$'
OR '$X \overset{*}{\Rightarrow} \epsilon$'

Elimination procedure
Given CFG. $G = (V, T, P, S)$
Define $G' = (V', T', P', S)$ be the CFG that does not contain any null productions such that $L(G') = L(G) - \{\epsilon\}$

S1 Initialize P' to P

S2 Find Nullable variables -
* say '$X \to \alpha$' is a production of X where α contains some nullable variables, then add to X, the produces obtained by deleting all possible subsets of nullable variables from α (not perfect in X)

Step3 Delete all null productions

① Eliminate null productions

$S \rightarrow aSb / a / bAB$
$A \rightarrow aA / \epsilon$
$B \rightarrow bB / \epsilon$

	production	new-production
$\{B\} = \{\{B\}, \{\}\}$	$B \rightarrow bB$	$B \rightarrow b$
	$B \rightarrow \epsilon$	–
$\{A\} = \{\{A\}, \{\}\}$	$A \rightarrow aA$	$A \rightarrow a$
	$A \rightarrow \epsilon$	–
	$S \rightarrow aSb$	–
	$S \rightarrow a$	–
$\{A,B\} = \{\{A\}, \{B\}, \{AB\}, \{\}\}$	$S \rightarrow bAB$	$S \rightarrow bB, S \rightarrow bA, S \rightarrow b$

$p + np$

$S \rightarrow aSb / a / bAB / bA / bB / b$
$A \rightarrow aA / a / \epsilon$
$B \rightarrow bB / b / \epsilon$

after deleting null productions

$S \rightarrow aSb / a / bAB / bA / bB / b$

$A \rightarrow aA / a$

$B \rightarrow bB / b$

Normal forms imply the rewriting of the CFG as per the specified conditions

Chomsky normal form (CNF)

Defination - Any ϵ free CFL can be generated by CFG in which all the productions are of the form
'$A \to BC$' OR '$A \to a$'
where A, B & C are variables a is a terminal
such a CFG is said to be in CNF

Conversion procedure from CFG to CNF

Step1 perform elimination of Null ①, unit ② & useless ③ productions

Step2 Add to the solution the productions that are already in CNF

Step3 For the remaining non CNF productions
* Replace the terminals by some variables
* Limit the number of variables on RHS to 2

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CNF:- $A \to BC$

$A \to a$

express CFG in CNF

$S \to aB \mid bA$

$A \to a \mid aS \mid bAA$

$B \to b \mid bSA \mid aBAB$

S1 S2 & S3

production	solution
$A \to a$	$A \to a$
$B \to b$	$B \to b$
$S \to aB$	$C_1 \to a$
$S \to C_1B$	$S \to C_1B$
$S \to bA$	$C_2 \to b$
$S \to C_2A$	$S \to C_2A$
$A \to aS$	
$A \to C_1S$	$A \to C_1S$
$A \to bAA$	
$A \to C_2AA$	$A \to C_2C_3$
$C_3 \to AA$	$C_3 \to AA$
$B \to bSA$	
$B \to C_2SA$	$B \to C_2C_4$
$C_4 \to SA$	$C_4 \to SA$
$B \to aBAB$	
$B \to C_1BAB$	$B \to C_1C_5$
$C_5 \to BAB$	$C_5 \to BC_6$
$C_6 \to AB$	$C_6 \to AB$

CFG in CNF is -

$S \rightarrow C_1 A / C_2 A$

$A \rightarrow a / C_1 S / C_2 C_3$

$B \rightarrow b / C_2 C_4 / C_1 C_5$

$C_1 \rightarrow a$

$C_2 \rightarrow b$

$C_3 \rightarrow AA$

$C_4 \rightarrow SA$

$C_5 \rightarrow B/C$

$C_6 \rightarrow AB$

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$A \rightarrow aBa / bBb$

$B \rightarrow aB / bB / \epsilon$

Solution Step1

~~production~~

~~$A \rightarrow aBa$~~

~~solution~~

Step1 Elimination of null productions

$\{B\} = \{\{B\}, \{\ \}\}$

p	np
$A \rightarrow aBa$	$A \rightarrow aa$
$A \rightarrow bBb$	$A \rightarrow bb$
$B \rightarrow aB$	$B \rightarrow a$
$B \rightarrow \epsilon$	—

p + np

$A \rightarrow aBa / aa / bBb / bb$

$B \rightarrow aB / a / bB / b / \epsilon$

after deleting null productions

$A \to aBa \mid aa \mid bBb \mid bb$

$B \to aB \mid a \mid bB \mid b$

<u>S2 & S3</u>

production	solution
$B \to a$	$B \to a$
$B \to b$	$B \to b$
$A \to aBa$	$C_1 \to a$
$A \to C_1BC_1$	$\boxed{A \to C_1C_2}$
$C_2 \to BC_1$	$C_2 \to BC_1$
$A \to aa$	
$A \to C_1C_1$	$\boxed{A \to C_1C_1}$
$A \to bBb$	$C_3 \to b$
$A \to C_3BC_3$	$\boxed{A \to C_3C_4}$
$C_4 \to BC_3$	$C_4 \to BC_3$
$A\ \cancel{B} \to bb$	
$A\ B \to C_3C_3$	$\boxed{A\ \cancel{B} \to C_3C_3}$
$B \to aB$	
$B \to C_1B$	$B \to C_1B$
$B \to bB$	
$B \to C_3B$	$B \to C_3B$

$A \to C_1C_2 \mid C_1C_1 \mid C_3C_4 \mid C_3C_3$

$B \to C_1B \mid a \mid C_3B \mid b$

② $S \to \sim S$
$S \to [S \supset S]$
$S \to p$
$S \to q$

S1 (done)

S2 & S3

production	solution
$S \to p$	$S \to p$
$S \to q$	$S \to q$
$S \to \sim S$	$C_1 \to \sim$
$S \to C_1 S$	$S \to C_1 S$
$S \to [S \supset S]$	$C_2 \to [$, $C_3 \to \supset$, $C_4 \to]$
$S \to C_2 S C_3 S C_4$	$S \to C_2 \cancel{S C_3 S C_4} C_5$
$\cancel{S \to C_2 C_5}$	
$C_5 \to S C_3 S C_4$	$C_5 \to S C_6$
$C_6 \to C_3 S C_4$	$C_6 \to C_3 C_7$
$C_7 \to S C_4$	$C_7 \to S C_4$

Normal forms

* [Greiback normal form (GNF)

Definition any ε-free CFL can be generated by CFG in which all the productions are of the following

$A \to a \gamma$

where A - variable
a - terminal
γ = string of variables
(can be empty)

Such a CFG is said to be in GNF

* Conversion procedure from CFG to GNF

Step 1 Perform elimination of null, unit & useless production

Step 2 Use any combination of Rule 1 & Rule 2 to get CFG in GNF

Rule 1 Let $A \rightarrow B\alpha$ be some A-production & let $B \rightarrow \beta_1 | \beta_2 | \cdots | \beta_n$ be B product then we can write A-produce as.

$$A \rightarrow \beta_1\alpha | \beta_2\alpha | \cdots | \beta_n\alpha$$

Rule 2 Let $A \rightarrow A\alpha_1 / A\alpha_2 | \cdots | A\alpha_5$ be some A-productions (starting with A) & let $A \rightarrow \beta_1 / \beta_2 | \cdots | \beta_3$ be the remaining A-production (not starting with A)

then,

B-production $B \rightarrow \alpha_i / \alpha_i B$ $\quad 1 \leq i \leq r$

A-production $A \rightarrow \beta_i / \beta_i B$ $\quad 1 \leq i \leq s$

Express CFG in GNF

(1) $S \to AB \mid BA \mid AA$
$A \to aA \mid b$
$B \to bB \mid a$

$B \to bB \mid a$
$A \to aA \mid b$

$S \to AB$ — $S \to aAB \mid bA$ (R1) ✓
$S \to BA$ — $S \to bBA \mid aA$ (R1) ✓
$S \to AA$ — $S \to aAA \mid bA$ (R1) ✓

(2) $A1 \to A2A2 \mid a$
$A2 \to A1A1 \mid b$

$A2 \to A1A1 \mid b$
$A2 \to A2A2A1 \mid aA1 \mid b$ (R1)
$A \to A\alpha_1 \quad \beta_1 \; \beta_2$

assumption

$B \to A2A1 \mid A2A1B$ ← $\alpha_1 \mid \alpha_1 B$
$\alpha 2 \quad \alpha 1$

$A2 \to aA1 \mid aA1B \mid b \mid bB$ $\quad \beta_1 \mid \beta_1 B$

$A1 \to aA1A2 \mid aA1BA2 \mid bA2 \mid bBA2 \mid a$ ✓ (R1)
$B \to aA1A1 \mid aA1BA1 \mid bA1 \mid bBA1$
$aA1A1B \mid aA1BA1B \mid bBA1B$

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③ a) $S \to AA / 0$
$A \to SS / 1$

$A \to SS / 1$

$A \to AAS / 0S / 1$ (R1)

$A \to A\alpha_1 \ \beta_1 \ \beta_2$

$B \to \alpha_1 / \alpha_i B$
$A \to \beta_i / \beta_i B$

$B \to AS / ASB$
$A \to 0S / 0SB / 1 / 1B$ (R2)

$S \to 0SA / 0SBA / 1A / 1BA / 0$ ✓ (R1)

$B \to 0SS / 0SBS / 1S / 1BS /$
$0SSB / 0SBSB / 1SB / 1BSB$ ✓ (R1)

① express CFG in GNF $A \to a\gamma$

① $S \to asa / bsb / \epsilon$

$S \to aSC_1 / bSC_2 / c$

$C_1 \to a$

$C_2 \to b$

② $S \to SS / aSb / ab$

$S \to SS / aSb / ab$

$A \to A\alpha_1 \ \beta_1 \ \beta_2$

$B \to \alpha_i / \alpha_i B$
$A \to \beta_i / \beta_i B$

$B \to S / SB$
$S \to aSb / aSbB / ab / abB$ (R2)

$S \to aSC_1 / aSC_1B / aC_1 / aC_1B$ ✓ (S·R)

$C_1 \to b$

$B \to aSC_1 / aSC_1B / aC_1 / aC_1B$
$aSC_1B / aSC_1BB / aC_1B / aC_1BB$ ✓ (R1)

⑧ Express the following language in CNF & GNF

$L = \{a^n b^n\}$ $n \geq 0$

CFG $S \rightarrow aSb / \epsilon$

Simplified CFG $S \rightarrow aSb / ab$

$A \rightarrow BC$

$A \rightarrow a$

$A \rightarrow a\gamma$

CNF

$C_1 \rightarrow a$

$C_2 \rightarrow b$

$S \rightarrow C_1 C_2$

$S \rightarrow C_1 C_3$

$C_3 \rightarrow S C_2$

GNF

$S \rightarrow aSC_1$

$S \rightarrow aC_1$

$C_1 \rightarrow b$