Sampling

Weight Distribution of Types

MechCivil

Type	Name	Nov 2017	May 2018	Nov 2018	May 2019	Nov 2019	May 2022	Nov 2022	May 2023	Dec 2023	May 2024	Dec 2024
I	Z test	06	05	05	05	05	05	---	---	---	05	---
II	t Test	08	14	06	08	08	07	06	12	11	06	05
III	Chi square Test	08	08	08	08	08	05	08	08	08	08	08
Total Marks		**22**	**27**	**19**	**21**	**21**	**17**	**14**	**20**	**19**	**19**	**13**

Comp/IT/AI

Type	Name	Nov 2017	May 2018	Nov 2018	May 2019	Nov 2019	May 2022	Nov 2022	May 2023	Dec 2023	May 2024	Dec 2024
I	t Test	06	11	06	06	11	05	05	05	06	06	06
II	Chi square Test	06	06	06	08	06	05	06	06	06	06	06
Total Marks		**12**	**17**	**12**	**14**	**17**	**10**	**11**	**11**	**12**	**12**	**12**

Type I: Z Test or Large Sampling $(n \geq 30)$ (Only for MechCivil)

A-Testing of Hypothesis that the population mean = μ

1. A random sample of 50 items gives the mean 6.2 & s.d 10.24. Can it be regarded as drawn from a normal population with mean 5.4 at 5% level of significance?
[M15/CompIT/4M][M19/MTRX/5M][M19/Chem/6M]
Solution:
$n = 50$
$\overline{x} = 6.2$
$\sigma = 10.24$
(i) Null Hypothesis: $\mu = 5.4$
Alternative Hypothesis: $\mu \neq 5.4$
(ii) Test statistic:
$$z = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| = \left|\frac{6.2-5.4}{\frac{10.24}{\sqrt{50}}}\right| = 0.55$$
(iii) L.O.S.: $\alpha = 0.05$
(iv) Critical value: $z_{\alpha} = 1.96$
(v) Decision: Since, the calculated value of z is less than the critical value, null hypothesis is accepted.
Thus, the sample is regarded as drawn from a normal population whose mean is 5.4

2. A sample of 50 pieces of certain type of string was tested. The mean breaking strength turned out to be 14.5 pounds. Test whether the sample is form a batch of a string having a mean breaking strength of 15.6 pounds and sd of 2.2 pounds.

[M14/MechCivil/5M]

Solution:

$n = 50$

$\bar{x} = 14.5$

$\sigma = 2.2$

(i) Null Hypothesis: $\mu = 15.6$

Alternative Hypothesis: $\mu \neq 15.6$

(ii) Test statistic:

$$z = \left| \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \right| = \left| \frac{14.5 - 15.6}{\frac{2.2}{\sqrt{50}}} \right| = 3.53$$

(iii) L.O.S.: $\alpha = 0.05$ (assumed)

(iv) Critical value: $z_{\alpha} = 1.96$

(v) Decision: Since, the calculated value of z is more than the critical value, null hypothesis is rejected.

Thus, the sample is not from a batch of string having a mean breaking strength of 15.6 pounds.

3. The mean breaking strength of cables supplied by a manufacturer is 1800 with standard deviation 100. By a new technique in the manufacturing process it is claimed that the breaking strength of the cables has increase. In order to test the claim a sample of 50 cables is tested. It is found that the mean breaking strength is 1850. Can we support the claim at 1% LOS?

[N18/Comp/6M]

Solution:

$n = 50$

$\overline{x} = 1850$

$\sigma = 100$

(i) Null Hypothesis: $\mu = 1800$

Alternative Hypothesis: $\mu \neq 1800$

(ii) Test statistic:

$$z = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| = \left|\frac{1850-1800}{\frac{100}{\sqrt{50}}}\right| = 3.5355$$

(iii) L.O.S.: $\alpha = 0.01$

(iv) Critical value: $z_{\alpha} = 2.58$

(v) Decision: Since, the calculated value of z is more than the critical value, null hypothesis is rejected.

Thus, the claim of the manufacturer is supported

4. A machine is set to produce metal plates of thickness 1.5 cm with s.d. 0.2 cm. A sample of 100 plates produced by the machine gave an average thickness of 1.52 cm. Is the machine fulfilling the purpose? Test at 1 % LOS
[M18/MechCivil/5M][N18/MTRX/6M][M19/Comp/5M]
Solution:
$n = 100$
$\overline{x} = 1.52$
$\sigma = 0.2$
(i) Null Hypothesis: $\mu = 1.5$
Alternative Hypothesis: $\mu \neq 1.5$
(ii) Test statistic:
$$z = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| = \left|\frac{1.52-1.5}{\frac{0.2}{\sqrt{100}}}\right| = 1$$
(iii) L.O.S.: $\alpha = 0.05$ (assumed)
(iv) Critical value: $z_\alpha = 1.96$
(v) Decision: Since, the calculated value of z is less than the critical value, null hypothesis is accepted.
Thus, the sample can be regarded as drawn from a population whose mean is 1.5 cm.
Therefore, the machine is fulfilling the purpose

5. A sample of 900 items is found to have mean 65.3 cms. Can it be regarded as drawn from a large population whose mean is 66.2 cm and sd 5 cm at 5% LOS.
[N14/MechCivil/5M][N17/MechCivil/6M]
Solution:
$n = 900, \overline{x} = 65.3$
$\sigma = 5$
(i) Null Hypothesis: $\mu = 66.2$
Alternative Hypothesis: $\mu \neq 66.2$
(ii) Test statistic:
$$z = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| = \left|\frac{65.3-66.2}{\frac{5}{\sqrt{900}}}\right| = 5.4$$
(iii) L.O.S.: $\alpha = 0.05$
(iv) Critical value: $z_\alpha = 1.96$
(v) Decision: Since, the calculated value of z is more than the critical value, null hypothesis is rejected.
Thus, the sample cannot be regarded as drawn from a large population whose mean is 66.2

6. Can it be concluded that average life span of an Indian is 70 years, if a random sample of 100 Indians has an average life span of 71.8 years with standard deviation 7.8 years? **[M15/MechCivil/6M]**

Solution:

$n = 100$

$\overline{x} = 71.8$

$\sigma = 7.8$

(i) Null Hypothesis: $\mu = 70$

Alternative Hypothesis: $\mu \neq 70$

(ii) Test statistic:

$$z = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| = \left|\frac{71.8-70}{\frac{7.8}{\sqrt{100}}}\right| = 2.308$$

(iii) L.O.S.: $\alpha = 0.05$ (assumed)

(iv) Critical value: $z_\alpha = 1.96$

(v) Decision: Since, the calculated value of z is more than the critical value, null hypothesis is rejected.

Thus, we cannot conclude that average life span of an Indian is 70 years at 5% LOS.

7. Can it be concluded that average life span of an Indian is 70 years, if a random sample of 100 Indians has an average life span of 71.8 years with standard deviation 8.9 years? **[N18/MechCivil/5M][N18/IT/4M]**

Solution:

$n = 100$

$\overline{x} = 71.8$

$\sigma = 8.9$

(i) Null Hypothesis: $\mu = 70$

Alternative Hypothesis: $\mu \neq 70$

(ii) Test statistic:

$$z = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| = \left|\frac{71.8-70}{\frac{8.9}{\sqrt{100}}}\right| = 2.0225$$

(iii) L.O.S.: $\alpha = 0.05$ (assumed)

(iv) Critical value: $z_\alpha = 1.96$

(v) Decision: Since, the calculated value of z is more than the critical value, null hypothesis is rejected.

Thus, we cannot conclude that average life span of an Indian is 70 years at 5% LOS.

8. Can it be concluded that average life span of an Indian is more than 70 years, if a random sample of 100 Indians has an average life span of 71.8 years with standard deviation 8.9 years?
[M24/MechCivil/5M]
Solution:
$n = 100,\ \bar{x} = 71.8,\ \sigma = 8.9$
(i) Null Hypothesis: $\mu > 70$
Alternative Hypothesis: $\mu < 70$
(ii) Test statistic:
$$z = \left|\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| = \left|\frac{71.8-70}{\frac{8.9}{\sqrt{100}}}\right| = 2.0225$$
(iii) L.O.S.: $\alpha = 0.05$ (assumed)
(iv) Critical value: $z_\alpha = 1.96$
(v) Decision: Since, the calculated value of z is more than the critical value, null hypothesis is rejected.
Thus, we cannot conclude that average life span of an Indian is not more than 70 years at 5% LOS.

9. Can it be concluded that average life span of an Indian is more than 71 years, if a random sample of 900 Indians has an average life span of 72.8 years with standard deviation 7.2 years?
[N19/MechCivil/5M]
Solution:
$n = 900,\ \bar{x} = 72.8,\ \sigma = 7.2$
(i) Null Hypothesis: $\mu > 71$
Alternative Hypothesis: $\mu \leq 71$
(ii) Test statistic:
$$z = \left|\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| = \left|\frac{72.8-71}{\frac{7.2}{\sqrt{900}}}\right| = 7.5$$
(iii) L.O.S.: $\alpha = 0.05$ (assumed)
(iv) Critical value: $z_\alpha = 1.645$
(v) Decision: Since, the calculated value of z is more than the critical value, null hypothesis is rejected.
Thus, we cannot conclude that average life span of an Indian is more than 71 years at 5% LOS.

10. A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160 cm. Can it be reasonably regarded that, in the population, the mean height is 165 cm and the standard deviation is 10 cm?
[N18/Comp/5M]
Solution:
$n = 100$
$\overline{x} = 160$
$\sigma = 10$
(i) Null Hypothesis: $\mu = 165$
Alternative Hypothesis: $\mu \neq 165$
(ii) Test statistic:
$$z = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| = \left|\frac{165-160}{\frac{10}{\sqrt{100}}}\right| = 5$$
(iii) L.O.S.: $\alpha = 0.05$ (assumed)
(iv) Critical value: $z_{\alpha} = 1.96$
(v) Decision: Since, the calculated value of z is more than the critical value, null hypothesis is rejected.
Thus, we cannot regard that the mean height of population is 165 at 5% LOS.

11. A random sample of 400 members is found to have a mean of 4.45 cms. Can it be reasonably regarded as a sample from large population whose mean is 5 cms and whose variance is 4 cms?
[M16/MechCivil/5M]
Solution:
$n = 400$
$\overline{x} = 4.45$
$\sigma = \sqrt{4} = 2$
(i) Null Hypothesis: $\mu = 5$
Alternative Hypothesis: $\mu \neq 5$
(ii) Test statistic:
$$z = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| = \left|\frac{4.45-5}{\frac{2}{\sqrt{400}}}\right| = 5.5$$
(iii) L.O.S.: $\alpha = 0.05$ (assumed)
(iv) Critical value: $z_{\alpha} = 1.96$
(v) Decision: Since, the calculated value of z is more than the critical value, null hypothesis is rejected.
Thus, the sample cannot be regarded as drawn from large population whose mean is 5 cms

12. A machine is claimed to produce nails of mean length 5 cm and standard deviation of 0.45 cm. A random sample of 100 nails gave 5.1 cm as average length. Does the performance of the machine justify the claim? Mention the level of significance you apply.
[N14/CompIT/5M][N19/Comp/6M]
Solution:
$n = 100$
$\bar{x} = 5.1$
$\sigma = 0.45$
(i) Null Hypothesis: $\mu = 5$
Alternative Hypothesis: $\mu \neq 5$
(ii) Test statistic:
$$z = \left|\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| = \left|\frac{5.1-5}{\frac{0.45}{\sqrt{100}}}\right| = 2.22$$
(iii) L.O.S.: $\alpha = 0.05$ (assumed)
(iv) Critical value: $z_\alpha = 1.96$
(v) Decision: Since, the calculated value of z is more than the critical value, null hypothesis is rejected.
Thus, the sample cannot be regarded as drawn from a population whose mean is 5 cm.
Therefore, the performance of machine do not justify the claim.

13. A tyre company claims that the lives of tyres have mean 42,000 km and standard deviation 4,000 km. A change in a production process is believed to result in a better product. A test sample of 81 new tyres has mean life of 42,500 km. Test at 5% LOS that the new product is significantly better than the old one.
[M17/CompIT/6M][M19/MechCivil/5M]
Solution:
$n = 81, \overline{x} = 42500, \sigma = 4000$
(i) Null Hypothesis: $\mu = 42000$
Alternative Hypothesis: $\mu \neq 42000$
(ii) Test statistic:
$$z = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| = \left|\frac{42500-42000}{\frac{4000}{\sqrt{81}}}\right| = 1.125$$
(iii) L.O.S.: $\alpha = 0.05$
(iv) Critical value: $z_\alpha = 1.96$
(v) Decision: Since, the calculated value of z is less than the critical value, null hypothesis is accepted.
Thus, the new product is not significantly better than the old one.

14. An ambulance service claims that it takes on an average 8.9 min to reach the destination in emergency calls. To check this the Licensing agency has then timed on 50 emergency calls, getting a mean of 9.3 min with an sd 1.6 min. Is the claim acceptable at 5% LOS?
[M18/Comp/6M]
Solution:
$n = 50$
$\overline{x} = 9.3$
$\sigma = 1.6$
(i) Null Hypothesis: $\mu = 8.9$
Alternative Hypothesis: $\mu \neq 8.9$
(ii) Test statistic:
$$z = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| = \left|\frac{9.3-8.9}{\frac{1.6}{\sqrt{50}}}\right| = 1.77$$
(iii) L.O.S.: $\alpha = 0.05$
(iv) Critical value: $z_\alpha = 1.96$
(v) Decision: Since, the calculated value of z is less than the critical value, null hypothesis is accepted.
Thus, the claim is acceptable

15. If the mean age at death of 64 men engaged in an occupation is 52.4 years with standard deviation of 10.2 years, what are the 98% confidence limits for the mean age of all men in that population? Also, determine can it be safely assumed at 5% level of significance that that mean age of death of population is 56?

[M22/MechCivil/5M]

Solution:

$n = 64$

$\overline{x} = 52.4$

$\sigma = 10.2$

98% confidence limits $= \overline{x} \pm z_\alpha \left[\frac{\sigma}{\sqrt{n}}\right] = 52.5 \pm z_{0.02} \left[\frac{10.2}{\sqrt{64}}\right]$

98% confidence limits $= 52.5 \pm (2.33)(1.28)$

98% confidence limits $= 52.5 \pm 2.98$

98% confidence limits $= 49.52$ to 55.48

(i) Null Hypothesis: $\mu = 56$

Alternative Hypothesis: $\mu \neq 56$

(ii) Test statistic:

$$z = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| = \left|\frac{52.4-56}{\frac{10.2}{\sqrt{64}}}\right| = 2.82$$

(iii) L.O.S.: $\alpha = 0.05$

(iv) Critical value: $z_\alpha = 1.96$

(v) Decision: Since, the calculated value of z is more than the critical value, null hypothesis is rejected. Thus, we cannot assume that mean age of death of population to be 56

B-Testing the difference between means

1. The means of 2 samples of sizes 1000 & 2000 respectively are 67.5 & 68 inches. Can the samples be regarded as drawn from the same population of standard deviation 2.5 inches? **[N15/MechCivil/6M]**

Solution:

$n_1 = 1000, n_2 = 2000$

$\overline{x}_1 = 67.5, \overline{x}_2 = 68$

$\sigma_1 = 2.5, \sigma_2 = 2.5$

(i) Null Hypothesis: $\mu_1 = \mu_2$

Alternative Hypothesis: $\mu_1 \neq \mu_2$

(ii) Test statistic:

$$S.E. = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{2.5^2}{1000} + \frac{2.5^2}{2000}} = 0.097$$

$$z = \left|\frac{\overline{x}_1 - \overline{x}_2}{S.E.}\right| = \left|\frac{67.5 - 68}{0.097}\right| = 5.15$$

(iii) L.O.S.: $\alpha = 0.01$ (assumed)

(iv) Critical value: $z_\alpha = 2.56$

(v) Decision: Since, the calculated value of z is more than the critical value, null hypothesis is rejected.

Thus, the samples cannot be regarded as drawn from the same population.

2. The average of marks scored by 32 boys is 72 with s.d. 8, while that of 36 girls is 70 with s.d. 6. Test at 1% level of significance whether the boys perform better than the girls. **[M14/CompIT/4M][N14/N15/N16/CompIT/6M][N16/M17/MechCivil/5M] [N17/M18/N18/Chem/6M][M18/IT/4M]**

Solution:

$n_1 = 32, n_2 = 36$

$\overline{x}_1 = 72, \overline{x}_2 = 70$

$\sigma_1 = 8, \sigma_2 = 6$

(i) Null Hypothesis: $\mu_1 = \mu_2$

Alternative Hypothesis: $\mu_1 \neq \mu_2$

(ii) Test statistic:

$$S.E. = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{8^2}{32} + \frac{6^2}{36}} = \sqrt{3}$$

$$z = \left|\frac{\overline{x}_1 - \overline{x}_2}{S.E.}\right| = \left|\frac{72-70}{\sqrt{3}}\right| = 1.15$$

(iii) L.O.S.: $\alpha = 0.01$

(iv) Critical value: $z_\alpha = 2.56$

(v) Decision: Since, the calculated value of z is less than the critical value, null hypothesis is accepted.

Thus, boys do not perform better than the girls

3. A man buys 100 electric bulbs of each of two well known makes taken at random from stock for testing purpose. He finds that make A has a mean life of 1300 hrs with sd of 82 hrs and make B has a mean life of 1248 hrs and sd of 93 hrs. discuss the significance of the result

[M19/Comp/6M]

Solution:

$n_1 = 100, n_2 = 100$

$\overline{x}_1 = 1300, \overline{x}_2 = 1248$

$\sigma_1 = 82, \sigma_2 = 93$

(i) Null Hypothesis: $\mu_1 = \mu_2$

Alternative Hypothesis: $\mu_1 \neq \mu_2$

(ii) Test statistic:

$$S.E. = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{82^2}{100} + \frac{93^2}{100}} = 12.3988$$

$$z = \left|\frac{\overline{x}_1 - \overline{x}_2}{S.E.}\right| = \left|\frac{1300 - 1248}{\sqrt{3}}\right| = 4.1939$$

(iii) L.O.S.: $\alpha = 0.01$

(iv) Critical value: $z_\alpha = 2.56$

(v) Decision: Since, the calculated value of z is more than the critical value, null hypothesis is rejected.

Thus, there is a significant difference

Type II: t Test or Small Sampling $(n < 30)$

A-Testing of Hypothesis that the population mean = μ

1. A test on breaking strengths of 6 ropes manufactured by a company showed a mean breaking strength of 7750 lb. and a standard deviation of 145 lb. whereas the manufacturer claimed a mean breaking strength of 8000 lb. Can we support the manufacturer's claim at a level of significance of 0.05.

Solution:

Given: $n = 6, \overline{x} = 7750, \sigma = 145$

(i) Null Hypothesis: $\mu = 8000$

Alternate Hypothesis: $\mu \neq 8000$

(ii) Test calculation:

$$t = \left|\frac{\overline{x}-\mu}{\sigma/\sqrt{n-1}}\right| = \left|\frac{7750-8000}{145/\sqrt{6-1}}\right| = 3.855$$

(iii) LOS: $\alpha = 0.05\ (5\%)$

(iv) Degree of Freedom (DOF): $\phi = n - 1 = 6 - 1 = 5$

(v) Critical Value: $t_\alpha = 2.571$

(vi) Decision: since the calculated value of t is greater than the critical value, null hypothesis is rejected. We do not support manufacturers claim.

2. A random sample size 16 from a normal population showed a mean of 103.75 cm and sum of squares of deviation from the mean is 843.75 cm. Can we say that population has a mean of 108.75 cm?

[M18/Comp/5M][N19/IT/6M]

Solution:

$n = 16,\ \overline{x} = 103.75$

$\sum(x - \overline{x})^2 = 843.75$

$$\sigma = \sqrt{\frac{\sum(x-\overline{x})^2}{n}} = \sqrt{\frac{843.75}{16}} = 7.2618$$

(i) Null Hypothesis: $\mu = 108.75$

Alternative Hypothesis: $\mu \neq 108.75$

(ii) Test statistic:

$$t = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n-1}}}\right| = \left|\frac{103.75-108.75}{\frac{7.2618}{\sqrt{16-1}}}\right| = 2.6667$$

(iii) L.O.S.: $\alpha = 0.05$ (assumed)

(iv) Degree of freedom: $\emptyset = n - 1 = 16 - 1 = 15$

(v) Critical value: $t_\alpha = 2.131$

(vi) Decision: Since, the calculated value of t is more than the critical value, null hypothesis is rejected. Thus, we cannot say that the population has a mean of 108.75 cm

3. A random sample of 9 items had the following values: 45, 47, 50, 52, 48, 47, 49, 53, and 51. Does the mean of 9 items differ significantly from the assumed population mean 47.5?
[M19/IT/6M][M22/MTRX/5M][M22/Chem/5M][N22/Chem/6M]
Solution:
$\overline{x} = \frac{45+47+50+52+48+47+49+53+51}{9} = \frac{442}{9} = 49.11$
$\sum x^2 = 45^2 + 47^2 + 50^2 + 52^2 + 48^2 + 47^2 + 49^2 + 53^2 + 51^2 = 21762$
$\sigma = \sqrt{\frac{\sum x^2}{N} - (\overline{x})^2} = 2.4696$
(i) Null Hypothesis: $\mu = 47.5$
Alternative Hypothesis: $\mu \neq 47.5$
(ii) Test statistic:
$$t = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n-1}}}\right| = \left|\frac{49.11-47.5}{\frac{2.4696}{\sqrt{9-1}}}\right| = 1.8439$$
(iii) L.O.S.: $\alpha = 0.05$ (assumed)
(iv) Degree of freedom: $\emptyset = n - 1 = 9 - 1 = 8$
(v) Critical value: $t_\alpha = 2.306$
(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted. There is no significant different between the means

4. A machine is designed to produce insulating washers for electrical devices of average thickness of 0.025 cm. A random sample of 10 washers was found to have average thickness of 0.024 cm; with standard deviation of 0.002 cm. Test the significance of the deviation.
Solution:
Given:
$n = 10, \overline{x} = 0.024, \sigma = 0.002$
(i) Null hypo: $\mu = 0.025$ (no significant difference)
Alt hypo: $\mu \neq 0.025$
(ii) Test calculation:
$$t = \left|\frac{\overline{x}-\mu}{\sigma/\sqrt{n-1}}\right| = 1.5$$
(iii) LOS: 0.05
(iv) Degree of freedom: $n - 1 = 10 - 1 = 9$
(v) Critical value: $t_\alpha = 2.262$
(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted. There is no significant difference.

5. Tests made on breaking strength of 10 pieces of a metal wire gave the following results: 578, 572, 570, 568, 572, 570, 570, 572, 596, and 584 in kg. Test if the breaking strength of a metal wire can be assumed to be 577 kg, at 5% L.O.S.
[N15/MechCivil/6M][M23/CompIT/5M]
Solution:

x	$d=(x-A)$	d^2
578	3	9
572	−3	9
570	−5	25
568	−7	49
572	−3	9
570	−5	25
570	−5	25
572	−3	9
596	21	441
584	9	81
Total = 5752	Total = 2	Total = 682

$n=10$

$\overline{x}=\frac{\sum x}{n}=\frac{5752}{10}=575.2\ (A=575)$

$\overline{d}=\frac{\sum d}{n}=\frac{2}{10}=0.2$

$\sigma=\sqrt{\frac{\sum d^2}{n}-\left(\overline{d}\right)^2}=\sqrt{\frac{682}{10}-(0.2)^2}=8.256$

(i) Null Hypothesis: $\mu=577$
Alternative Hypothesis: $\mu\neq 577$

(ii) Test statistic:

$$t=\left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n-1}}}\right|=\left|\frac{575.2-577}{\frac{8.256}{\sqrt{10-1}}}\right|=0.654$$

(iii) L.O.S.: $\alpha=0.05$

(iv) Degree of freedom: $\emptyset=n-1=10-1=9$

(v) Critical value: $t_\alpha=2.262$

(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted. Thus, the mean breaking strength of the metal wire can be assumed as 577 kg.

6. A random sample of 15 items gives the mean 6.2 and variance 10.24. Can it be regarded as drawn from a normal population with mean 5.4 at 5% LOS?
[M18/IT/4M]
Solution:
$n = 15,\ \bar{x} = 6.2,\ variance = 10.24$
$\sigma = \sqrt{variance} = \sqrt{10.24} = 3.2$
(i) Null Hypothesis: $\mu = 5.4$
Alternative Hypothesis: $\mu \neq 5.4$
(ii) Test statistic:
$$t = \left|\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n-1}}}\right| = \left|\frac{6.2-5.4}{\frac{3.2}{\sqrt{15-1}}}\right| = 0.9354$$
(iii) L.O.S.: $\alpha = 0.05$
(iv) Degree of freedom: $\emptyset = n - 1 = 15 - 1 = 14$
(v) Critical value: $t_\alpha = 2.145$
(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted. Thus, the sample can be regarded as drawn from a normal population with mean 5.4

7. 10 individuals are chosen at random from a population and their heights are found to be 63, 63, 64, 65, 66, 69, 69, 70, 70, 71 inches. Discuss the suggestion that the mean height of the universe is 65 inches?
[M15/CompIT/6M][N16/M17/MechCivil/6M][M17/CompIT/4M][N18/IT/4M][N18/MTRX/5M][N19/Comp/5M][N22/MTRX/8M]
Solution:

x	$(x-\overline{x})$	$(x-\overline{x})^2$
63	-4	16
63	-4	16
64	-3	9
65	-2	4
66	-1	1
69	2	4
69	2	4
70	3	9
70	3	9
71	4	16
Total = 670		Total = 88

$n=10$

$\overline{x}=\frac{\sum x}{n}=\frac{670}{10}=67$

$\sigma=\sqrt{\frac{\sum(x-\overline{x})^2}{n}}=\sqrt{\frac{88}{10}}=2.966$

(i) Null Hypothesis: $\mu=65$
Alternative Hypothesis: $\mu\neq 65$

(ii) Test statistic:

$$t=\left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n-1}}}\right|=\left|\frac{67-65}{\frac{2.966}{\sqrt{10-1}}}\right|=2.022$$

(iii) L.O.S.: $\alpha=0.05$ (assumed)

(iv) Degree of freedom: $\emptyset=n-1=10-1=9$

(v) Critical value: $t_\alpha=2.262$

(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted. Thus, the mean height of universe is 65 inches.

8. Ten individuals are chosen at random from a population and their heights are found to be 63, 63, 65, 66, 67, 68, 69, 70, 71 and 71 inches. Discuss the suggestion that the mean height of the universe is 66 inches?
[M18/MechCivil/6M]
Solution:

x	$d = (x - A)$	d^2
63	-4	16
63	-4	16
65	-2	4
66	-1	1
67	0	0
68	1	1
69	2	4
70	3	9
71	4	16
71	4	16
Total = 673	Total = 3	Total = 83

$n = 10$

$\overline{x} = \frac{\sum x}{n} = \frac{673}{10} = 67.3 \ (A = 67)$

$\sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2} = \sqrt{\frac{83}{10} - \left(\frac{3}{10}\right)^2} = 2.8653$

(i) Null Hypothesis: $\mu = 66$
Alternative Hypothesis: $\mu \neq 66$

(ii) Test statistic:

$$t = \left|\frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n-1}}}\right| = \left|\frac{67.3 - 66}{\frac{2.8653}{\sqrt{10-1}}}\right| = 1.3611$$

(iii) L.O.S.: $\alpha = 0.05$ (assumed)

(iv) Degree of freedom: $\emptyset = n - 1 = 10 - 1 = 9$

(v) Critical value: $t_\alpha = 2.262$

(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted. Thus, the mean height of universe is 66 inches.

9. Which of the following tests would be used to test the mean of a continuous random variable to a population mean?
[M22/MechCivil/2M] Ans. One sample t Test

10. If the parent population can be considered at least approximately as normal distribution and if the sample are ($n < 30$), then the test statistic is calculated by
[M22/MTRX/2M] Ans. t distribution
11. The sample size $n \geq 30$ (for any population) Population standard deviation is known then which test should be used
[M22/Chem/2M] Ans. z Test
12. The sample size $n \leq 30$ (for any population) Population standard deviation is unknown then which test should be used
[M22/Chem/2M] Ans. t Test

13. A machinist is expected to make engine parts with axle diameter of 1.75 cm. A random sample of 10 parts shows a mean diameter of 1.85 cm, with a S.D of 0.1 cm. Based on this sample, would you say that the work of the machinist is inferior?
[N22/MechCivil/6M]
Solution:
$n = 10$
$\overline{x} = 1.85$
$\sigma = 0.1$
(i) Null Hypothesis: $\mu = 1.75$
Alternative Hypothesis: $\mu \neq 1.75$
(ii) Test statistic:
$$t = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n-1}}}\right| = \left|\frac{1.85-1.75}{\frac{0.1}{\sqrt{10-1}}}\right| = 3$$
(iii) L.O.S.: $\alpha = 0.05$
(iv) Degree of freedom: $\emptyset = n - 1 = 10 - 1 = 9$
(v) Critical value: $t_{\alpha} = 2.262$
(vi) Decision: Since, the calculated value of t is more than the critical value, null hypothesis is rejected. Thus, work of the machinist is indeed inferior

14. A brochure inviting subscriptions for a new diet program states that the participants are expected to lose on an average 22 pounds in five weeks. Suppose that, from the data of the five-week weight losses of 26 participants, the sample mean and sample standard deviation are found to be 23.5 and 10.2, respectively. Could the statement in the brochure be substantiated based on these findings? Test at the $\alpha = 0.05$ level of significance.
[M22/MechCivil/5M]
Solution:
$n = 26, \overline{x} = 23.5, \sigma = 10.2$
(i) Null Hypothesis: $\mu = 22$
Alternative Hypothesis: $\mu \neq 22$
(ii) Test statistic:
$$t = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n-1}}}\right| = \left|\frac{23.5-22}{\frac{10.2}{\sqrt{26-1}}}\right| = 0.7353$$
(iii) L.O.S.: $\alpha = 0.05$
(iv) Degree of freedom: $\emptyset = n - 1 = 26 - 1 = 25$
(v) Critical value: $t_\alpha = 2.060$
(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted. Thus, the statement in the brochure be substantiated.

15. The mean lifetime of a sample of 25 bulbs is found as 1550 hours with standard deviation of 120 hours. The company of manufacturing bulbs claims that the average life of their bulbs is 1600 hours. Is the claim acceptable at 5% LOS?
[D23/D24/MechCivil/5M]
Solution:
$n = 25$
$\overline{x} = 1550$
$\sigma = 120$
(i) Null Hypothesis: $\mu = 1600$
Alternative Hypothesis: $\mu \neq 1600$
(ii) Test statistic:
$$t = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n-1}}}\right| = \left|\frac{1550-1600}{\frac{120}{\sqrt{25-1}}}\right| = 2.0412$$
(iii) L.O.S.: $\alpha = 0.05$
(iv) Degree of freedom: $\emptyset = n - 1 = 25 - 1 = 24$
(v) Critical value: $t_\alpha = 2.064$
(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted. Thus, the claim is acceptable

B-Testing the difference between means

1. The means of 2 random samples of size 9 & 7 is given as 196.42 & 198.82 respectively. The sums of the squares of the deviation from the means are 26.94 & 18.73 respectively. Can the samples be considered as drawn from the same population?
[N14/CompIT/6M][M15/N18/MechCivil/6M][M19/Comp/6M][M22/MTRX/5M] [N22/MTRX/6M][M24/CompITAI/6M]
Solution:
$n_1 = 9, n_2 = 7$
$\overline{x}_1 = 196.42, \overline{x}_2 = 198.82$
$\sum(x_1 - \overline{x}_1)^2 = 26.94, \sum(x_2 - \overline{x}_2)^2 = 18.73$
$\sigma_1 = \sqrt{\frac{\sum(x_1-\overline{x}_1)^2}{n_1}} = \sqrt{\frac{26.94}{9}} = 1.7301, \sigma_2 = \sqrt{\frac{\sum(x_2-\overline{x}_2)^2}{n_2}} = \sqrt{\frac{18.73}{7}} = 1.6358$
(i) Null Hypothesis: $\mu_1 = \mu_2$
Alternative Hypothesis: $\mu_1 \neq \mu_2$
(ii) Test statistic:
$$s_p = \sqrt{\frac{n_1\sigma_1^2+n_2\sigma_2^2}{n_1+n_2-2}} = \sqrt{\frac{9(1.7301)^2+7(1.6358)^2}{9+7-2}} = 1.806$$
$$S.E. = s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 1.806\sqrt{\frac{1}{9}+\frac{1}{7}} = 0.9102$$
$$t = \left|\frac{\overline{x}_1-\overline{x}_2}{S.E.}\right| = \left|\frac{196.42-198.82}{0.9102}\right| = 2.637$$
(iii) L.O.S.: $\alpha = 0.05$
(iv) Degree of freedom: $\emptyset = (n_1 - 1) + (n_2 - 1) = 8 + 6 = 14$
(v) Critical value: $t_\alpha = 2.145$
(vi) Decision: Since, the calculated value of t is more than the critical value, null hypothesis is rejected.
Thus, the samples cannot be regarded as drawn from the same populations

2. The means of 2 random samples of size 9 & 7 is given as 196 & 199 respectively. The sums of the squares of the deviation from the means are 27 & 19 respectively. Can the samples be considered as drawn from the same population?

[N18/Comp/6M]

Solution:

$n_1 = 9, n_2 = 7$

$\overline{x}_1 = 196, \overline{x}_2 = 199$

$\sum(x_1 - \overline{x}_1)^2 = 27, \sum(x_2 - \overline{x}_2)^2 = 19$

$\sigma_1 = \sqrt{\frac{\sum(x_1-\overline{x}_1)^2}{n_1}} = \sqrt{\frac{27}{9}} = 1.7321, \sigma_2 = \sqrt{\frac{\sum(x_2-\overline{x}_2)^2}{n_2}} = \sqrt{\frac{19}{7}} = 1.6475$

(i) Null Hypothesis: $\mu_1 = \mu_2$

Alternative Hypothesis: $\mu_1 \neq \mu_2$

(ii) Test statistic:

$s_p = \sqrt{\frac{n_1\sigma_1^2+n_2\sigma_2^2}{n_1+n_2-2}} = \sqrt{\frac{9(1.7321)^2+7(1.6475)^2}{9+7-2}} = 1.8127$

$S.E. = s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 1.8127\sqrt{\frac{1}{9} + \frac{1}{7}} = 0.9135$

$t = \left|\frac{\overline{x}_1-\overline{x}_2}{S.E.}\right| = \left|\frac{196-199}{0.9135}\right| = 3.2841$

(iii) L.O.S.: $\alpha = 0.05$

(iv) Degree of freedom: $\emptyset = (n_1 - 1) + (n_2 - 1) = 8 + 6 = 14$

(v) Critical value: $t_\alpha = 2.145$

(vi) Decision: Since, the calculated value of t is more than the critical value, null hypothesis is rejected.

Thus, the samples cannot be regarded as drawn from the same populations

3. If two independent random samples of sizes 15 & 8 have respectively the means and population standard deviations as $\overline{x_1} = 980, \overline{x_2} = 1012, \sigma_1 = 75, \sigma_2 = 80$. Test the hypothesis that $\mu_1 = \mu_2$ at 5% LOS

[N15/CompIT/4M][N17/CompIT/6M]

Solution:

$n_1 = 15, n_2 = 8$

$\overline{x}_1 = 980, \overline{x}_2 = 1012$

$\sigma_1 = 75, \sigma_2 = 80$

(i) Null Hypothesis: $\mu_1 = \mu_2$

Alternative Hypothesis: $\mu_1 \neq \mu_2$

(ii) Test statistic:

$$s_p = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{15(75)^2 + 8(80)^2}{15+8-2}} = 80.3489$$

$$S.E. = s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 80.3489\sqrt{\frac{1}{15} + \frac{1}{8}} = 35.1766$$

$$t = \left|\frac{\overline{x}_1 - \overline{x}_2}{S.E.}\right| = \left|\frac{980 - 1012}{35.1766}\right| = 0.91$$

(iii) L.O.S.: $\alpha = 0.05$

(iv) Degree of freedom: $\emptyset = (n_1 - 1) + (n_2 - 1) = 14 + 7 = 21$

(v) Critical value: $t_\alpha = 2.08$

(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted.

4. A sample of 8 students of 16 years each shown up a mean systolic blood pressure of 118.4 mm of Hg with sd of 12.17 mm. while a sample of 10 students of 17 years each showed the mean systolic blood pressure of 121.0 mm with sd of 12.88 mm during investigation. The investigator feels that the systolic BP is related to age. Do you think that the data provides enough reasons to support investigators feeling at 5% LOS? Assume the distribution of systolic BP to be normal.
[M14/CompIT/6M]
Solution:
$n_1 = 8, n_2 = 10$
$\overline{x}_1 = 118.4, \overline{x}_2 = 121$
$\sigma_1 = 12.17, \sigma_2 = 12.88$
(i) Null Hypothesis: $\mu_1 = \mu_2$
Alternative Hypothesis: $\mu_1 \neq \mu_2$
(ii) Test statistic:

$$s_p = \sqrt{\frac{n_1\sigma_1^2+n_2\sigma_2^2}{n_1+n_2-2}} = \sqrt{\frac{8(12.17)^2+10(12.88)^2}{8+10-2}} = 13.33$$

$$S.E. = s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 13.33\sqrt{\frac{1}{8}+\frac{1}{10}} = 6.3238$$

$$t = \left|\frac{\overline{x}_1-\overline{x}_2}{S.E.}\right| = \left|\frac{118.4-121}{6.3238}\right| = 0.411$$

(iii) L.O.S.: $\alpha = 0.05$
(iv) Degree of freedom: $\emptyset = (n_1 - 1) + (n_2 - 1) = 7 + 9 = 16$
(v) Critical value: $t_\alpha = 2.12$
(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted.
Thus, the two age group students have same systolic BP i.e. systolic BP is not related to age

5. Samples of two types of electric bulbs were tested for lengths of life and the following data were obtained:

	Type I	Type II
No. of samples	8	7
Mean	1134	1024
S.D.	35	40

Test at 5% L.O.S whether the difference in the sample means is significant.
[M23/MechCivil/6M]

Solution:

$n_1 = 8, n_2 = 7$

$\overline{x}_1 = 1134, \overline{x}_2 = 1024$

$\sigma_1 = 35, \sigma_2 = 40$

(i) Null Hypothesis: $\mu_1 = \mu_2$ (The difference is not significant)

Alternative Hypothesis: $\mu_1 \neq \mu_2$ (The difference is significant)

(ii) Test statistic:

$$s_p = \sqrt{\frac{n_1\sigma_1^2+n_2\sigma_2^2}{n_1+n_2-2}} = \sqrt{\frac{8(35)^2+7(40)^2}{8+7-2}} = 40.1918$$

$$S.E. = s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 40.1918\sqrt{\frac{1}{8}+\frac{1}{7}} = 20.8012$$

$$t = \left|\frac{\overline{x}_1-\overline{x}_2}{S.E.}\right| = \left|\frac{1134-1024}{20.8012}\right| = 5.2881$$

(iii) L.O.S.: $\alpha = 0.05$

(iv) Degree of freedom: $\emptyset = (n_1 - 1) + (n_2 - 1) = 7 + 6 = 13$

(v) Critical value: $t_\alpha = 2.160$

(vi) Decision: Since, the calculated value of t is more than the critical value, null hypothesis is rejected. The difference is significant

6. Samples of two types of electric bulbs were tested for lengths of life and the following data were obtained:

	Type I	Type II
No. of samples	10	9
Mean	1136	1034
S.D.	36	39

Test at 5% L.O.S whether the difference in the sample means is significant.
[N19/MechCivil/8M]

Solution:

$n_1 = 10, n_2 = 9$
$\overline{x}_1 = 1136, \overline{x}_2 = 1034$
$\sigma_1 = 36, \sigma_2 = 39$

(i) Null Hypothesis: $\mu_1 = \mu_2$ (The difference is not significant)
Alternative Hypothesis: $\mu_1 \neq \mu_2$ (The difference is significant)

(ii) Test statistic:

$$s_p = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2-2}} = \sqrt{\frac{10(36)^2+9(39)^2}{10+9-2}} = 39.5928$$

$$S.E. = s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 39.5928\sqrt{\frac{1}{10}+\frac{1}{9}} = 18.1916$$

$$t = \left|\frac{\overline{x}_1 - \overline{x}_2}{S.E.}\right| = \left|\frac{1136-1034}{18.1916}\right| = 5.607$$

(iii) L.O.S.: $\alpha = 0.05$

(iv) Degree of freedom: $\emptyset = (n_1 - 1) + (n_2 - 1) = 9 + 8 = 17$

(v) Critical value: $t_\alpha = 2.110$

(vi) Decision: Since, the calculated value of t is more than the critical value, null hypothesis is rejected. The difference is significant

7. The heights of 6 randomly chosen sailors are 63, 65, 68, 69, 71, 72 inches. The heights of 10 randomly chosen soldiers 61, 62, 65, 66, 69, 69, 70, 71, 72 & 73 inches. Discuss in the light that these data throw on the suggestion that the soldiers are on the average taller than the sailors. Test at 5% L.O.S.

[N17/M18/MechCivil/8M][M19/N19/MTRX/6M][M22/CompITAI/5M]

Solution:

x_1	x_1^2	x_2	x_2^2
63	3969	61	3721
65	4225	62	3844
68	4624	65	4225
69	4761	66	4356
71	5041	69	4761
72	5184	69	4761
		70	4900
		71	5041
		72	5184
		73	5329
Total = 408	Total = 27804	Total = 678	Total = 46122

$n_1 = 6, n_2 = 10$

$\overline{x}_1 = \frac{\sum x_1}{n_1} = 68, \overline{x}_2 = \frac{\sum x_2}{n_2} = 67.8$

$\sigma_1 = \sqrt{\frac{\sum x_1^2}{n_1} - (\overline{x}_1)^2} = 3.1623, \sigma_2 = \sqrt{\frac{\sum x_2^2}{n_2} - (\overline{x}_2)^2} = 3.9192$

(i) Null Hypothesis: $\mu_1 = \mu_2$

Alternative Hypothesis: $\mu_1 \neq \mu_2$

(ii) Test statistic:

$$s_p = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{6(3.16)^2 + 10(3.92)^2}{6+10-2}} = 3.91$$

$$S.E. = s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 3.91\sqrt{\frac{1}{6} + \frac{1}{10}} = 2.02$$

$$t = \left|\frac{\overline{x}_1 - \overline{x}_2}{S.E.}\right| = \left|\frac{68 - 67.8}{2.02}\right| = 0.099$$

(iii) L.O.S.: $\alpha = 0.05$

(iv) Degree of freedom: $\emptyset = (n_1 - 1) + (n_2 - 1) = 5 + 9 = 14$

(v) Critical value: $t_\alpha = 2.145$

(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted.

Thus, Soldiers are not taller than the sailors

8. Two independent samples of 8 and 7 items gave the following values.

Sample 1: 19 17 15 21 16 18 16 14

Sample 2: 15 14 15 19 15 18 16

Is the difference between the sample means significant?

[N14/MechCivil/6M]

Solution:

x_1	x_1^2	x_2	x_2^2
19	361	15	225
17	289	14	196
15	225	15	225
21	441	19	361
16	256	15	225
18	324	18	324
16	256	16	256
14	196		
Total = 136	Total = 2348	Total = 112	Total = 1812

$n_1 = 8, n_2 = 7$

$\overline{x}_1 = \frac{\sum x_1}{n_1} = 17, \overline{x}_2 = \frac{\sum x_2}{n_2} = 16$

$\sigma_1 = \sqrt{\frac{\sum x_1^2}{n_1} - (\overline{x}_1)^2} = 2.12, \sigma_2 = \sqrt{\frac{\sum x_2^2}{n_2} - (\overline{x}_2)^2} = 1.69$

(i) Null Hypothesis: $\mu_1 = \mu_2$

Alternative Hypothesis: $\mu_1 \neq \mu_2$

(ii) Test statistic:

$$s_p = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2-2}} = \sqrt{\frac{8(2.12)^2+7(1.69)^2}{8+7-2}} = 2.074$$

$$S.E. = s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 2.074\sqrt{\frac{1}{8} + \frac{1}{7}} = 1.0736$$

$$t = \left|\frac{\overline{x}_1 - \overline{x}_2}{S.E.}\right| = \left|\frac{17-16}{1.0736}\right| = 0.93$$

(iii) L.O.S.: $\alpha = 0.05$

(iv) Degree of freedom: $\emptyset = (n_1 - 1) + (n_2 - 1) = 7 + 6 = 13$

(v) Critical value: $t_\alpha = 2.16$

(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted.

Thus, there is no significant difference between the two samples.

9. The following data represent the marks obtained by 12 students in two tests, one held before coaching and the other after coaching

Test I	55	60	65	75	49	25	18	30	35	51	61	72
Test II	63	70	70	81	54	29	21	38	32	50	70	80

Do the data indicate that the coaching was effective in improving performance of the students

[D23/CompIT/6M]

Solution:

x_1	x_2	$x = (x_2 - x_1)$	x^2
55	63	8	64
60	70	10	100
65	70	5	25
75	81	6	36
49	54	5	25
25	29	4	16
18	21	3	9
30	38	8	64
35	32	-3	9
51	50	-1	1
61	70	9	81
72	80	8	64
		Total $= 62$	Total $= 494$

$n = 12$

$\overline{x} = \frac{\sum x}{n} = \frac{62}{12} = 5.1667$

$\sigma = \sqrt{\frac{\sum x^2}{n} - (\overline{x})^2} = \sqrt{\frac{494}{12} - (5.1667)^2} = 3.8042$

(i) Null Hypothesis: $\mu = 0$ (no change)

Alternative Hypothesis: $\mu \neq 0$

(ii) Test statistic:

$$t = \left|\frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n-1}}}\right| = \left|\frac{5.1667 - 0}{\frac{3.8042}{\sqrt{12-1}}}\right| = 4.5045$$

(iii) L.O.S.: $\alpha = 0.05$ (assumed)

(iv) Degree of freedom: $\emptyset = n - 1 = 12 - 1 = 11$

(v) Critical value: $t_\alpha = 2.201$

(vi) Decision: Since, the calculated value of t is more than the critical value, null hypothesis is rejected. Thus, the coaching was effective in improving the performance of students

10. Ten school boys were given a test in statistics and their scores were recorded. They were given a month special coaching and a second test was given to them in the same subject at the end of the coaching period. Test if the marks given below give evidence to the fact that the students are benefited by coaching.

Test I : 70 68 56 75 80 90 68 75 56 58

Test II : 68 70 52 73 75 78 80 92 54 55

[N18/MTRX/6M][M19/MechCivil/8M]

Solution:

x_1	x_2	$x = (x_2 - x_1)$	x^2
70	68	-2	4
68	70	2	4
56	52	-4	16
75	73	-2	4
80	75	-5	25
90	78	-12	144
68	80	12	144
75	92	17	289
56	54	-2	4
58	55	-3	9
		Total $= 1$	Total $= 643$

$n = 10$

$\overline{x} = \frac{\sum x}{n} = \frac{1}{10} = 0.1$

$\sigma = \sqrt{\frac{\sum x^2}{n} - (\overline{x})^2} = \sqrt{\frac{643}{10} - (0.1)^2} = 8.0181$

(i) Null Hypothesis: $\mu = 0$ (no change)

Alternative Hypothesis: $\mu \neq 0$

(ii) Test statistic:

$$t = \left|\frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n-1}}}\right| = \left|\frac{0.1 - 0}{\frac{8.0181}{\sqrt{10-1}}}\right| = 0.037$$

(iii) L.O.S.: $\alpha = 0.05$ (assumed)

(iv) Degree of freedom: $\emptyset = n - 1 = 10 - 1 = 9$

(v) Critical value: $t_\alpha = 2.262$

(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted. Thus, the students have not benefitted from coaching

11. The following data relates to the marks obtained by 11 students in two tests, one held at the beginning of the year and the other at the end of the year after giving intensive coaching.

Test I	19	23	16	24	17	18	20	18	21	19	20
Test II	17	24	20	24	20	22	20	20	18	22	18

Do the data indicate that the students have benefitted from coaching?

[M23/MechCivil/6M]

Solution:

x_1	x_2	$x = (x_2 - x_1)$	x^2
19	17	-2	4
23	24	1	1
16	20	4	16
24	24	0	0
17	20	3	9
18	22	4	16
20	20	0	0
18	20	2	4
21	18	-3	9
19	22	3	9
20	18	-2	4
		Total $= 10$	Total $= 72$

$n = 11$

$\overline{x} = \frac{\sum x}{n} = \frac{10}{11} = 0.9091$

$\sigma = \sqrt{\frac{\sum x^2}{n} - (\overline{x})^2} = \sqrt{\frac{72}{11} - (0.9091)^2} = 2.3914$

(i) Null Hypothesis: $\mu = 0$ (no change)

Alternative Hypothesis: $\mu \neq 0$

(ii) Test statistic:

$$t = \left|\frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n-1}}}\right| = \left|\frac{0.9091 - 0}{\frac{2.3914}{\sqrt{11-1}}}\right| = 1.217$$

(iii) L.O.S.: $\alpha = 0.05$ (assumed)

(iv) Degree of freedom: $\emptyset = n - 1 = 11 - 1 = 10$

(v) Critical value: $t_\alpha = 2.228$

(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted. Thus, the students have not benefitted from coaching

12. An I.Q. test was administered to 5 persons and after they were trained. The results are given below

Candidates	I	II	III	IV	V
I.Q. before training	110	120	123	132	125
I.Q. after training	120	118	125	136	121

Test whether there is any change in I.Q. after the training programme, use 1 % LOS
[N19/Comp/6M][D23/MechCivil/6M]

Solution:

x_1	x_2	$x = (x_2 - x_1)$	x^2
110	120	10	100
120	118	-2	4
123	125	2	4
132	136	4	16
125	121	-4	16
		Total $= 10$	Total $= 140$

$n = 5$

$\overline{x} = \frac{\sum x}{n} = \frac{10}{5} = 2$

$\sigma = \sqrt{\frac{\sum x^2}{n} - (\overline{x})^2} = \sqrt{\frac{140}{5} - (2)^2} = 4.8990$

(i) Null Hypothesis: $\mu = 0$ (no change)
Alternative Hypothesis: $\mu \neq 0$

(ii) Test statistic:

$$t = \left|\frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n-1}}}\right| = \left|\frac{2-0}{\frac{4.8990}{\sqrt{5-1}}}\right| = 0.8165$$

(iii) L.O.S.: $\alpha = 0.05$ (assumed)

(iv) Degree of freedom: $\emptyset = n - 1 = 5 - 1 = 4$

(v) Critical value: $t_\alpha = 2.776$

(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted. Thus, there is no change after the training programme.

13. Memory capacity of 9 students was tested before and after a course of meditation for a month. State whether the course was effective or not from the data below

Before	10	15	9	3	7	12	16	17	4
After	12	17	8	5	6	11	18	20	3

[M18/Comp/6M]

Solution:

x_1	x_2	$x = (x_2 - x_1)$	x^2
10	12	2	4
15	17	2	4
9	8	-1	1
3	5	2	4
7	6	-1	1
12	11	-1	1
16	18	2	4
17	20	3	9
4	3	-1	1
		Total = 7	Total = 29

$n = 9$

$\overline{x} = \frac{\sum x}{n} = \frac{7}{9} = 0.7758$

$\sigma = \sqrt{\frac{\sum x^2}{n} - (\overline{x})^2} = \sqrt{\frac{29}{9} - (0.7758)^2} = 1.6188$

(i) Null Hypothesis: $\mu = 0$ (no change)

Alternative Hypothesis: $\mu \neq 0$

(ii) Test statistic:

$$t = \left|\frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n-1}}}\right| = \left|\frac{0.7758 - 0}{\frac{1.6188}{\sqrt{9-1}}}\right| = 1.3555$$

(iii) L.O.S.: $\alpha = 0.05$ (assumed)

(iv) Degree of freedom: $\emptyset = n - 1 = 9 - 1 = 8$

(v) Critical value: $t_\alpha = 2.306$

(vi) Decision: Since, the calculated value of t is less than the critical value, null hypothesis is accepted. Thus, the course was not effective

14. The sales data of an item in six shops before and after a special promotional campaign is as follows

Shops	A	B	C	D	E	F
Before Campaign	53	28	31	48	50	42
After Campaign	58	29	30	55	56	45

Can the campaign be judged to be a success at 5% LOS?

[M24/MechCivil/6M]

Solution:

x_1	x_2	$x=(x_2-x_1)$	x^2
53	58	5	25
28	29	1	1
31	30	-1	1
48	55	7	49
50	56	6	36
42	45	3	9
		Total $=21$	Total $=121$

$n=6$

$\overline{x}=\frac{\sum x}{n}=\frac{21}{6}=3.5$

$\sigma=\sqrt{\frac{\sum x^2}{n}-(\overline{x})^2}=\sqrt{\frac{121}{6}-(3.5)^2}=2.8137$

(i) Null Hypothesis: $\mu=0$ (no change)

Alternative Hypothesis: $\mu\neq 0$

(ii) Test statistic:

$$t=\left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n-1}}}\right|=\left|\frac{3.5-0}{\frac{2.8137}{\sqrt{6-1}}}\right|=2.7815$$

(iii) L.O.S.: $\alpha=0.05$

(iv) Degree of freedom: $\emptyset=n-1=6-1=5$

(v) Critical value: $t_\alpha=2.571$

(vi) Decision: Since, the calculated value of t is more than the critical value, null hypothesis is rejected. Thus, the campaign was a success

15. A certain injection administered to 12 patients resulted in the following changes of blood pressure:
5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4
Can it be concluded that the injection will be in general accompanied by an increase in blood pressure?
[M14/M16/MechCivil/6M][M16/D24/CompIT/6M][N19/IT/6M][N22/CompITAI/5M]
Solution:

x	x^2
5	25
2	4
8	64
-1	1
3	9
0	0
6	36
-2	4
1	1
5	25
0	0
4	16
Total = 31	Total = 185

$n = 12$

$\overline{x} = \frac{\sum x}{n} = \frac{31}{12} = 2.5833$

$\sigma = \sqrt{\frac{\sum x^2}{n} - (\overline{x})^2} = \sqrt{\frac{185}{12} - (2.5833)^2} = 2.9569$

(i) Null Hypothesis: $\mu = 0$ (no change)
Alternative Hypothesis: $\mu \neq 0$

(ii) Test statistic:

$$t = \left|\frac{\overline{x}-\mu}{\frac{\sigma}{\sqrt{n-1}}}\right| = \left|\frac{2.5833-0}{\frac{2.9569}{\sqrt{12-1}}}\right| = 2.897$$

(iii) L.O.S.: $\alpha = 0.05$ (assumed)

(iv) Degree of freedom: $\emptyset = n - 1 = 12 - 1 = 11$

(v) Critical value: $t_\alpha = 2.201$

(vi) Decision: Since, the calculated value of t is more than the critical value, null hypothesis is rejected. Thus, there is rise in BP.

Type III: χ^2 Test

- If the observed frequencies and expected frequencies coincide, then the value of χ^2 will be
  [M22/MTRX/2M] Ans. 0
- If Contingency table has r rows and c columns, the degree of freedom is given by
  [M22/Chem/2M] Ans. $(r-1)(c-1)$
- If calculated value of χ^2 test is greater than the table value the hypothesis is
  [M22/Chem/2M] Ans. Rejected

1. The following table gives the number of accidents in a city during a week. Find whether the accidents are uniformly distributed over the week using χ^2 test

Days	:	Sun	Mon	Tue	Wed	Thurs	Fri	Sat
No. of accidents	:	13	15	9	11	12	10	14

[M15/MechCivil/8M][N17/CompIT/6M][M18/IT/6M][M22/CompITAI/5M]

Solution:

(i) Null Hypothesis: The accidents are uniformly distributed over a week
Alternative Hypothesis: The accidents are not uniformly distributed over a week

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
13	12	1	1	$1/12$
15	12	3	9	$9/12$
9	12	-3	9	$9/12$
11	12	-1	1	$1/12$
12	12	0	0	$0/12$
10	12	-2	4	$4/12$
14	12	2	4	$4/12$
Total				2.33

(iii) Degree of freedom: $\emptyset = n-1 = 6$

(iv) L.O.S: $\alpha = 0.05$

(v) Critical value: $\chi^2_\alpha = 12.592$

(vi) Decision: since, the calculated value is less than the critical value, null hypothesis is accepted. Thus, accidents are uniformly distributed over a week

2. The following table gives the number of accidents that occur during the various days of the week. Find whether the accidents are uniformly distributed over the week

Days	:	Sun	Mon	Tue	Wed	Thurs	Fri	Sat
No. of accidents	:	13	12	11	9	15	10	14

[M23/D23/MechCivil/8M]

Solution:

(i) Null Hypothesis: The accidents are uniformly distributed over a week

Alternative Hypothesis: The accidents are not uniformly distributed over a week

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
13	12	1	1	$1/12$
12	12	0	0	0
11	12	-1	1	$1/12$
9	12	-3	9	$9/12$
15	12	3	9	$9/12$
10	12	-2	4	$4/12$
14	12	2	4	$4/12$
Total				2.33

(iii) Degree of freedom: $\emptyset = n - 1 = 6$

(iv) L.O.S: $\alpha = 0.05$

(v) Critical value: $\chi^2_\alpha = 12.592$

(vi) Decision: since, the calculated value is less than the critical value, null hypothesis is accepted. Thus, accidents are uniformly distributed over a week

3. The number of car accidents in a metropolitan city was found to be 20, 17, 12, 6, 7, 15, 8, 5, 16 and 14 per month respectively. Use χ^2 Test to check whether these frequencies are in agreement with the belief that occurrence of accidents was the same during 10 months period. Test at 5% level of significance.
[N14/CompIT/8M][M22/Chem/5M][N22/Chem/8M][M24/CompITAI/6M]
Solution:
(i) Null Hypothesis: The accidents was same during 10 months period
Alternative Hypothesis: The accidents was not same during 10 months period
(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
20	12	8	64	64/12
17	12	5	25	25/12
12	12	0	0	0/12
6	12	-6	36	36/12
7	12	-5	25	25/12
15	12	3	9	9/12
8	12	-4	16	16/12
5	12	-7	49	49/12
16	12	4	16	16/12
14	12	2	4	4/12
		Total		20.33

(iii) Degree of freedom: $\emptyset = n - 1 = 9$
(iv) L.O.S: $\alpha = 0.05$
(v) Critical value: $\chi^2_\alpha = 16.919$
(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, accidents was not same during 10 months period

4. A die was thrown 132 times and the following frequencies were observed.

No. obtained :	1	2	3	4	5	6	Total
Frequency :	15	20	25	15	29	28	132

Test the hypothesis that the die is unbiased.

[N15/MechCivil/6M]M15/CompIT/6M][N16/CompIT/8M][N17/N22/MechCivil/8M]

Solution:

(i) Null Hypothesis: The die is unbiased

Alternative Hypothesis: The die is biased

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
15	22	-7	49	$49/22$
20	22	-2	4	$4/22$
25	22	3	9	$9/22$
15	22	-7	49	$49/22$
29	22	7	49	$49/22$
28	22	6	36	$36/22$
Total				8.91

(iii) Degree of freedom: $\emptyset = n-1 = 5$

(iv) L.O.S: $\alpha = 0.05$

(v) Critical value: $\chi^2_\alpha = 11.07$

(vi) Decision: since, the calculated value is less than the critical value, null hypothesis is accepted. Thus, the die is unbiased

5. 300 digits were chosen at random from a table of random numbers. The frequency of digits was as follows.

Digit	: 0	1	2	3	4	5	6	7	8	9
Frequency	: 28	29	33	31	26	35	32	30	31	25

Using χ^2Test examine the hypothesis that the digits were distributed in equal numbers in the table.

[M17/CompIT/6M][N19/N22/Chem/8M][M22/Chem/5M]

Solution:

(i) Null Hypothesis: The digits are uniformly distributed

Alternative Hypothesis: The digits are not uniformly distributed

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
28	30	-2	4	4/30
29	30	-1	1	1/30
33	30	3	9	9/30
31	30	1	1	1/30
26	30	-4	16	16/30
35	30	5	25	25/30
32	30	2	4	4/30
30	30	0	0	0
31	30	1	1	1/30
25	30	-5	25	25/30
Total				2.867

(iii) Degree of freedom: $\emptyset = n - 1 = 9$

(iv) L.O.S: $\alpha = 0.05$

(v) Critical value: $\chi^2_\alpha = 16.919$

(vi) Decision: since, the calculated value is less than the critical value, null hypothesis is accepted. Thus, digits are uniformly distributed

6. The following figures show the distribution of the digits in numbers chosen at random chosen from a telephone directory. Test at 5% level whether the digits may be taken to occur equally frequently in the directory

Digits	0	1	2	3	4	5	6	7	8	9
Frequency	1026	1107	997	966	1075	933	1107	972	964	853

[M22/MechCivil/5M]

Solution:

(i) Null Hypothesis: The digits are uniformly distributed

Alternative Hypothesis: The digits are not uniformly distributed

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
1026	1000	26	676	0.676
1107	1000	107	11449	11.449
997	1000	-3	9	0.009
966	1000	-34	1156	1.156
1075	1000	75	5625	5.625
933	1000	-67	4489	4.489
1107	1000	107	11449	11.449
972	1000	-28	784	0.784
964	1000	-36	1296	1.296
853	1000	-147	21609	21.609
Total				58.542

(iii) Degree of freedom: $\emptyset = n - 1 = 9$

(iv) L.O.S: $\alpha = 0.05$

(v) Critical value: $\chi^2_\alpha = 16.919$

(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, digits are not uniformly distributed

7. Investigate the association between the darkness of eye colour in father and son from the following data.

Father's / Son's	Dark	Not Dark	Total
Dark	48	90	138
Not Dark	80	782	862
Total	128	872	1000

[M19/N19/IT/8M]

Solution:

(i) Null Hypothesis: There is no association between the darkness of eye colour in father and son

Alternative Hypothesis: There is an association between the darkness of eye colour in father and son

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
48	$\frac{128\times138}{1000}=18$	30	900	900/18
90	$\frac{872\times138}{1000}=120$	-30	900	900/120
80	$\frac{128\times862}{1000}=110$	-30	900	900/110
782	$\frac{872\times862}{1000}=752$	30	900	900/752
Total				66.88

(iii) Degree of freedom: $\emptyset=(r-1)(c-1)=(2-1)(2-1)=1$

(iv) L.O.S: $\alpha=0.05$

(v) Critical value: $\chi^2_\alpha=3.841$

(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, there is an association between the darkness of eye colour in father and son

8. In an industry 200 workers employed for a specific job were classified according to their performance and training received. To test independence of training received and performance, the data is summarized as follows:

Performance	Good	Not Good	Total
Trained	100	50	150
Untrained	20	30	50
Total	120	80	200

Use χ^2 test for independence at 5% level of significance and write your conclusion.
[N19/Comp/6M]

Solution

(i) Null Hypothesis: There is no association between the training received and performance
Alternative Hypothesis: There is an association between the training received and performance

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
100	$\frac{150\times120}{200}=90$	10	100	$\frac{100}{90}$
50	$\frac{150\times80}{200}=60$	-10	100	$\frac{100}{60}$
20	$\frac{50\times120}{200}=30$	-10	100	$\frac{100}{30}$
30	$\frac{50\times80}{200}=20$	10	100	$\frac{100}{20}$
Total				11.11

(iii) Degree of freedom: $\emptyset=(r-1)(c-1)=(2-1)(2-1)=1$

(iv) L.O.S: $\alpha=0.05$

(v) Critical value: $\chi^2_\alpha=3.841$

(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, there is an association between training received and performance

9. Based on the following determine if there is a relation between literacy and smoking:

	Smokers	Non-smokers
Literates	83	57
Illiterates	45	68

[N14/MechCivil/8M][N18/Comp/6M][N22/D24/CompITAI/6M]

Solution:

The observed frequency table as given:

	Smokers	Non-smokers	Total
Literates	83	57	140
Illiterates	45	68	113
Total	128	125	253

(i) Null Hypothesis: There is no relation between literacy and smoking
Alternative Hypothesis: There is a relation between literacy and smoking

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
83	$\frac{140\times128}{253}=71$	12	144	144/71
57	$\frac{125\times140}{253}=69$	-12	144	144/69
45	$\frac{128\times113}{253}=57$	-12	144	144/57
68	$\frac{125\times113}{253}=56$	12	144	144/56
Total				9.213

(iii) Degree of freedom: $\emptyset=(r-1)(c-1)=(2-1)(2-1)=1$

(iv) L.O.S: $\alpha=0.05$

(v) Critical value: $\chi^2_\alpha=3.841$

(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, there is a relation between literacy and smoking

10. The following data is collected on two characters. Based on this, can you say that there is no relation between smoking and literacy? Use Chi square test at 5 % level of significance **[M19/Comp/8M]**

	Smokers	Non-smokers
Literates	40	35
Illiterates	35	85

Solution:

The observed frequency table as given:

	Smokers	Non-smokers	Total
Literates	40	35	75
Illiterates	35	85	120
Total	75	120	195

(i) Null Hypothesis: There is no relation between literacy and smoking

Alternative Hypothesis: There is a relation between literacy and smoking

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
40	$\frac{75\times 75}{195}=29$	11	121	121/29
35	$\frac{120\times 75}{195}=46$	-11	121	121/46
35	$\frac{75\times 120}{195}=46$	-11	121	121/46
85	$\frac{120\times 120}{195}=74$	11	121	121/74
Total				11.07

(iii) Degree of freedom: $\emptyset=(r-1)(c-1)=(2-1)(2-1)=1$

(iv) L.O.S: $\alpha=0.05$

(v) Critical value: $\chi_\alpha^2=3.841$

(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, there is a relation between literacy and smoking

11. Justify, if there is any relationship between sex and colour for the data:

Color	Male	Female
Red	10	40
White	70	30
Green	30	20

[M18/Comp/6M

Solution:

The observed frequency table as given:

Color	Male	Female	Total
Red	10	40	50
White	70	30	100
Green	30	20	50
Total	110	90	200

(i) Null Hypothesis: There is no association between sex and colour

Alternative Hypothesis: There is an association between sex and colour

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
10	$\frac{50\times110}{200}=27.5$	-17.5	306.25	306.25/27.5
40	$\frac{50\times90}{200}=22.5$	17.5	306.25	306.25/22.5
70	$\frac{100\times110}{200}=55$	15	225	225/55
30	$\frac{100\times90}{200}=45$	-15	225	225/45
30	$\frac{50\times110}{200}=27.5$	2.5	6.25	6.25/27.5
20	$\frac{50\times90}{200}=22.5$	-2.5	6.25	6.25/22.5
Total				34.3434

(iii) Degree of freedom: $\emptyset=(r-1)(c-1)=(2-1)(3-1)=2$

(iv) L.O.S: $\alpha=0.05$

(v) Critical value: $\chi^2_\alpha=5.991$

(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, there is an association between sex and colour

12. Table shows the performances of students in Mathematics and Physics. Test hypothesis that the performance in Mathematics is independent of performance in Physics.

		Grades in Maths		
		High	Medium	Low
Grades in Physics	High	56	71	12
	Medium	47	163	38
	Low	14	42	81

[M14/MechCivil/8M]

Solution:

The observed frequency table as given:

		Grades in Maths			Total
		High	Medium	Low	
Grades in Physics	High	56	71	12	139
	Medium	47	163	38	248
	Low	14	42	81	137
Total		117	276	131	524

(i) Null Hypothesis: There is no relation between the students' performance in Maths and Physics

Alternative Hypothesis: There is a relation between the students' performance in Maths and Physics

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
56	$\frac{139\times117}{524}=31$	25	625	625/31
71	$\frac{139\times276}{524}=73$	-2	4	4/73
12	$\frac{131\times139}{524}=35$	-23	529	529/35
47	$\frac{117\times248}{524}=55$	-8	64	64/55
163	$\frac{276\times248}{524}=131$	32	1024	1024/131
38	$\frac{131\times248}{524}=62$	-24	576	576/62
14	$\frac{117\times137}{524}=31$	-17	289	289/31
42	$\frac{276\times137}{524}=72$	-30	900	900/72
81	$\frac{131\times137}{524}=34$	47	2209	2209/34
Total				140.39

(iii) Degree of freedom: $\emptyset=(r-1)(c-1)=(3-1)(3-1)=4$

(iv) L.O.S: $\alpha=0.05$ (assumed)

(v) Critical value: $\chi^2_\alpha = 9.488$

(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, there is a relation between the students' performance in the two subjects.

13. The following table gives the result of opinion poll for three vehicles A, B, C. Test whether the age and the choice of the vehicle are independent at 5 % level of significance using χ^2 test

Age	Vehicle			Total
	A	B	C	
20 – 35	25	40	35	100
35 – 50	35	24	41	100
Above 50	40	36	24	100
Total	100	100	100	300

[N19/MechCivil/8M]

Solution:

(i) Null Hypothesis: There is no relation between age and choice of vehicles

Alternative Hypothesis: There is a relation between age and choice of vehicles

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
25	$\frac{100\times100}{300}=33$	-8	64	$64/33$
40	$\frac{100\times100}{300}=33$	7	49	$49/33$
35	$\frac{100\times100}{300}=34$	1	1	$1/34$
35	$\frac{100\times100}{300}=33$	2	4	$4/33$
24	$\frac{100\times100}{300}=33$	-9	81	$81/33$
41	$\frac{100\times100}{300}=34$	7	49	$49/34$
40	$\frac{100\times100}{300}=34$	6	36	$36/34$
36	$\frac{100\times100}{300}=34$	2	4	$4/34$
24	$\frac{100\times100}{300}=32$	-8	64	$64/32$
Total				10.65

(iii) Degree of freedom: $\emptyset=(r-1)(c-1)=(3-1)(3-1)=4$

(iv) L.O.S: $\alpha=0.05$ (assumed)

(v) Critical value: $\chi^2_\alpha=9.488$

(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, there is a relation between age and choice of vehicles

14. In an experiment of immunization of cattle from tuberculosis the following results were obtained

	Affected	Not Affected	Total
Inoculated	267	27	294
Not Inoculated	757	155	912
Total	1024	182	1206

Use χ^2 test to determine the effect of vaccine in preventing tuberculosis
[N15/CompIT/6M][M19/N19/MTRX/6M]
Solution:
(i) Null Hypothesis: There is no relation between vaccine and tuberculosis
Alternative Hypothesis: There is a relation between them
(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
267	$\frac{1024\times294}{1206}=250$	17	289	289/250
27	$\frac{182\times294}{1206}=44$	-17	289	289/44
757	$\frac{1024\times912}{1206}=774$	-17	289	289/774
155	$\frac{182\times912}{1206}=138$	17	289	289/138
Total				10.19

(iii) Degree of freedom: $\emptyset=(r-1)(c-1)=(2-1)(2-1)=1$
(iv) L.O.S: $\alpha=0.05$
(v) Critical value: $\chi^2_\alpha=3.841$
(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, there is a relation between vaccine and tuberculosis

15. Two batches of 12 animals each are given test of inoculation. One batch was inoculated and the other was not. The numbers of dead and surviving animals are given in the following table for both cases. Can the inoculation be regarded as effective against the disease at 5% level of significance.(Make Yates correction).

	Dead	Surviving	Total
Inoculated	2	10	12
Not-inoculated	8	4	12
Total	10	14	24

[M22/Chem/5M][N22/Chem/8M]

Solution:

(Note: Yates correction:- $\sum \frac{[|O-E|-0.5]^2}{E}$)

Observation table:

	Dead	Surviving	Total
Inoculated	2	10	12
Not-inoculated	8	4	12
Total	10	14	24

Expectation table:

	Dead	Surviving	Total
Inoculated	$\frac{10 \times 12}{24} = 5$	7	12
Not-inoculated	5	7	12
Total	10	14	24

(i) Null hypothesis: there is no association between inoculation and disease

Alt hypothesis: there is an association between inoculation and disease

(ii) Test calculation:

O	E	$O-E$	$\lvert O-E\rvert-0.5$	$(\lvert O-E\rvert-0.5)^2$	$\frac{(\lvert O-E\rvert-0.5)^2}{E}$
2	5	-3	2.5	6.25	6.25/5
10	7	3	2.5	6.25	6.25/7
8	5	3	2.5	6.25	6.25/5
4	7	-3	2.5	6.25	6.25/7
					$\chi^2 = 4.285$

(iii) LOS: 0.05

(iv) Degree of freedom: $\phi = (r-1)(c-1) = (2-1)(2-1) = 1$

(v) Critical value: $\chi^2_\alpha = 3.841$

(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, there is a relation between inoculation and disease

16. In an experiment of immunization of cattle from tuberculosis the following results were obtained

	Affected	Not Affected	Total
Inoculated	290	110	400
Not Inoculated	310	90	400
Total	600	200	800

Use χ^2 test to determine the effect of vaccine in preventing tuberculosis
[N18/MechCivil/8M]
Solution:
(i) Null Hypothesis: There is no relation between vaccine and tuberculosis
Alternative Hypothesis: There is a relation between vaccine and tuberculosis
(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
290	$\frac{600\times400}{800}=300$	-10	100	$\frac{100}{300}$
110	$\frac{200\times400}{800}=100$	10	100	$\frac{100}{300}$
310	$\frac{600\times400}{800}=300$	10	100	$\frac{100}{300}$
90	$\frac{200\times400}{800}=100$	-10	100	$\frac{100}{300}$
Total				1.3333

(iii) Degree of freedom: $\emptyset=(r-1)(c-1)=(2-1)(2-1)=1$
(iv) L.O.S: $\alpha=0.05$
(v) Critical value: $\chi^2_\alpha=3.841$
(vi) Decision: since, the calculated value is less than the critical value, null hypothesis is accepted. Thus, there is no relation between vaccine and tuberculosis

17. The following table gives the number of accounting clerks not committing errors among trained and untrained clerks working in an organization

	No. of clerks Committing errors	No. of clerks Not Committing errors	Total
Trained	70	530	600
Untrained	155	745	900
Total	225	1275	1500

Test the effectiveness of training in preventing errors (Table value of χ^2 for 1 d.f., 2 d.f, 3 d.f., 4 d.f., are 3.84, 5.99, 7.81, 4.99 respectively)

[M22/MTRX/5M]

Solution:

(i) Null Hypothesis: There is no relation between training and committing errors

Alternative Hypothesis: There is a relation between training and committing errors

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
70	$\frac{600\times225}{1500}=90$	-20	400	$\frac{400}{90}$
530	$\frac{600\times1275}{1500}=510$	20	400	$\frac{400}{510}$
155	$\frac{900\times225}{1500}=135$	20	400	$\frac{400}{135}$
745	$\frac{900\times1275}{1500}=765$	-20	400	$\frac{400}{765}$
Total				8.7146

(iii) Degree of freedom: $\emptyset=(r-1)(c-1)=(2-1)(2-1)=1$

(iv) L.O.S: $\alpha=0.05$

(v) Critical value: $\chi_\alpha^2=3.841$

(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, there is a relation between training and committing errors

18. A certain drug is claimed to be effective in curing cold in an experiment on 500 persons with cold. 300 of them were given drug and 200 of them were given sugar pills. The patients reaction to the treatment are recorded in the following table using χ^2 test (use 5% LOS)

	Helped	Harmed	No Effect	Total
Drug	200	40	60	300
Sugar pills	120	30	50	200
Total	320	70	110	500

Test the hypothesis that the drug is effective in curing cold

[D24/MechCivil/8M]

Solution:

The observed frequency table as given:

	Helped	Harmed	No Effect	Total
Drug	200	40	60	300
Sugar pills	120	30	50	200
Total	320	70	110	500

(i) Null Hypothesis: There is no association between drug and cold

Alternative Hypothesis: There is an association between drug and cold

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
200	$\frac{300\times320}{500}=192$	8	64	$64/192$
40	$\frac{300\times70}{500}=42$	-2	4	$4/42$
60	$\frac{300\times110}{500}=66$	-6	36	$36/66$
120	$\frac{200\times320}{500}=128$	-8	64	$64/128$
30	$\frac{200\times70}{500}=28$	2	4	$4/28$
50	$\frac{200\times110}{500}=44$	-6	36	$36/44$
Total				2.4351

(iii) Degree of freedom: $\emptyset=(r-1)(c-1)=(2-1)(3-1)=2$

(iv) L.O.S: $\alpha=0.05$

(v) Critical value: $\chi^2_\alpha=5.991$

(vi) Decision: since, the calculated value is less than the critical value, null hypothesis is accepted. Thus, there is no association between drug and cold

19. Out of a sample 120 persons in a village, 76 were administered a new drug for preventing influenza and out of them 24 persons were attacked by influenza. Out of these were not administered the new drugs, 12 persons were not attacked by influenza. Use chi square test method to find out whether the new drug is effective or not?
[M18/MechCivil/8M]
Solution:
The observed frequency table as given:

New drugs/Influenza	Affected	Unaffected	Total
Administered	24	52	76
Were not administered	32	12	44
Total	56	64	120

(i) Null Hypothesis: There is no association between new drug and influenza
Alternative Hypothesis: There is an association between new drug and influenza
(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
24	$\frac{76\times56}{120}=35$	11	121	121/35
52	$\frac{76\times64}{120}=41$	11	121	121/41
32	$\frac{44\times56}{120}=21$	11	121	121/21
12	$\frac{44\times64}{120}=23$	11	121	121/12
Total				22.2536

(iii) Degree of freedom: $\emptyset=(r-1)(c-1)=(2-1)(2-1)=1$
(iv) L.O.S: $\alpha=0.05$
(v) Critical value: $\chi_{\alpha}^{2}=3.841$
(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, there is an association between new drug and influenza

20. A sample of 400 students of undergraduates and 400 students of post graduate classes was taken to know their opinion about autonomous colleges. 290 of the undergraduate and 310 of the post graduate students favored the autonomous status. Present these facts in the form of a table and test at 5% level, that the opinion regarding autonomous status of colleges is independent of the level of classes of students.
[N16/M17/M19/MechCivil/8M]
Solution:
The observed frequency table as given:

Students/ Autonomous college	In favor	Opposed	Total
Undergraduates	290	110	400
Post graduates	310	90	400
Total	600	200	800

(i) Null Hypothesis: There is no association between autonomous status of college and the level of class of students

Alternative Hypothesis: There is an association between autonomous status of college and the level of class of students

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
290	$\frac{600\times 400}{800}=300$	-10	100	100/300
110	$\frac{200\times 400}{800}=100$	10	100	100/100
310	$\frac{600\times 400}{800}=300$	10	100	100/300
90	$\frac{200\times 400}{800}=100$	-10	100	100/100
Total				2.67

(iii) Degree of freedom: $\emptyset=(r-1)(c-1)=(2-1)(2-1)=1$
(iv) L.O.S: $\alpha=0.05$
(v) Critical value: $\chi^2_\alpha=3.841$
(vi) Decision: since, the calculated value is less than the critical value, null hypothesis is accepted. Thus, there is no association between autonomous status of college and the level of class of students

21. A total number of 3759 individuals were interviewed in a public opinion survey on a political proposal. Of them 1872 were men and the rest were women. A total of 2257 individuals were in favor of the proposal and 917 were opposed to it. A total of 243 men were undecided and 442 women were opposed to the proposal. Do you justify or contradict the hypothesis that there is no association between sex and attitude at 5% level of significance. **[M14/CompIT/6M]**

Solution:

The observed frequency table as given:

Sex/ Attitude	In favor	Opposed	Undecided	Total
Men	1154	475	243	1872
Women	1103	442	342	1887
Total	2257	917	585	3759

(i) Null Hypothesis: There is no association between sex and attitude

Alternative Hypothesis: There is an association between sex and attitude

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
1154	$\frac{2257\times1872}{3759}=1124$	30	900	900/1124
475	$\frac{917\times1872}{3759}=457$	18	324	324/457
243	$\frac{585\times1872}{3759}=291$	-48	2304	2304/291
1103	$\frac{2257\times1887}{3759}=1133$	-30	900	900/1133
442	$\frac{917\times1887}{3759}=460$	-18	324	324/460
342	$\frac{585\times1887}{3759}=294$	48	2304	2304/294
Total				18.76

(iii) Degree of freedom: $\emptyset=(r-1)(c-1)=(2-1)(3-1)=2$

(iv) L.O.S: $\alpha=0.05$

(v) Critical value: $\chi^2_\alpha=5.991$

(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, there is an association between sex and attitude

22. Out of 800 people 25 % were literate and 300 had travelled beyond the limits of the district, 40 % of the literates were among those who had not travelled. Prepare a 2 x 2 table and test at 5 % level of significance whether there is any relation between travelling and literacy **[N18/MTRX/6M]**

Solution:

The observed frequency table as given:

Literacy/ Travelling	Travelled	Not travelled	Total
Literate	120	80	200
Illiterate	180	420	600
Total	300	500	800

(i) Null Hypothesis: There is no association between travelling and literacy

Alternative Hypothesis: There is an association between travelling and literacy

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
120	$\frac{200\times300}{800}=75$	45	2025	2025/75
80	$\frac{200\times500}{800}=125$	-45	2025	2025/125
180	$\frac{600\times300}{800}=225$	-45	2025	2025/225
420	$\frac{600\times500}{800}=375$	45	2025	2025/375
Total				57.6

(iii) Degree of freedom: $\emptyset=(r-1)(c-1)=(2-1)(2-1)=1$

(iv) L.O.S: $\alpha=0.05$

(v) Critical value: $\chi^2_\alpha=3.841$

(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, there is an association between travelling and literacy

23. According to Genetic theory children having one parent of blood type M and other blood type N will always be one of three types M, MN and N and the average proportions of these types will be 1:2:1. Out of 300 children, having one M parent and one N parent, 30 percent were found to be of type M, 45 percent of MN and remaining of type N. Test the genetic theory by χ^2 test.

Solution:

(i) Null hypothesis: the theory is good

Alternate Hypothesis: the theory is not good

(ii) Test calculation:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
$\frac{30}{100}\times 300 = 90$	$\frac{1}{4}\times 300 = 75$	15	225	225/75
$\frac{45}{100}\times 300 = 135$	$\frac{2}{4}\times 300 = 150$	-15	225	225/150
$\frac{25}{100}\times 300 = 75$	$\frac{1}{4}\times 300 = 75$	0	0	0
Total = 300	300			$\chi^2 = 4.5$

(iii) LOS:0.05

(iv) Degree of freedom: $\phi = n - 1 = 3 - 1 = 2$

(v) Critical value: $\chi^2_\alpha = 5.991$

(vi) Decision: since, the calculated value is less than the critical value, null hypothesis is accepted. The theory is good

24. Theory predicts that the proportion of beans in the four groups A, B, C, D should be 9: 3: 3: 1. In an experiment among 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory?
[N18/IT/6M][M19/Chem/6M][M23/CompIT/6M][M24/MechCivil/8M]
Solution:
(i) Null Hypothesis: The experimental result support the theory
Alternative Hypothesis: The experimental result does not support the theory
(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
882	$\frac{9}{16}\times 1600 = 900$	-18	324	$324/900$
313	$\frac{3}{16}\times 1600 = 300$	13	169	$169/300$
287	$\frac{3}{16}\times 1600 = 300$	-13	169	$169/300$
118	$\frac{1}{16}\times 1600 = 100$	18	324	$324/100$
Total				4.7267

(iii) Degree of freedom: $\emptyset = n - 1 = 3$
(iv) L.O.S: $\alpha = 0.05$
(v) Critical value: $\chi^2_\alpha = 7.815$
(vi) Decision: since, the calculated value is less than the critical value, null hypothesis is accepted. Thus, The experimental result support the theory

25. According to theory of proportion of commodity in the four classes A, B, C, D should be 9:4:2:1. In a survey of 1600 items of this commodity the numbers in four classes were 882, 432, 168 and 118. Does the survey support the theory?
[M22/MTRX/5M][N22/MTRX/6M]
Solution:
(i) Null Hypothesis: The survey support the theory
Alternative Hypothesis: The survey does not support the theory
(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
882	$\frac{9}{16}\times 1600 = 900$	-18	324	324/900
432	$\frac{4}{16}\times 1600 = 400$	32	1024	1024/400
168	$\frac{2}{16}\times 1600 = 200$	-32	1024	1024/200
118	$\frac{1}{16}\times 1600 = 100$	18	324	324/100
Total				11.28

(iii) Degree of freedom: $\emptyset = n - 1 = 3$
(iv) L.O.S: $\alpha = 0.05$
(v) Critical value: $\chi^2_\alpha = 7.815$
(vi) Decision: since, the calculated value is more than the critical value, null hypothesis is rejected. Thus, the survey does not support the theory

26. In an experiment of pea breeding, the following frequencies of seeds were obtained

Round and Yellow	Wrinkled and Yellow	Round and Green	Wrinkled and Green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportion 9:3:3:1

Examine the correspondence between theory and experiment using chi square test

[M16/CompIT/8M]

Solution:

(i) Null Hypothesis: The experimental result support the theory

Alternative Hypothesis: The experimental result does not support the theory

(ii) Test Statistic:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
315	$\frac{9}{16}\times 556 = 313$	2	4	$4/313$
101	$\frac{3}{16}\times 556 = 104$	-3	9	$9/104$
108	$\frac{3}{16}\times 556 = 104$	4	16	$16/104$
32	$\frac{1}{16}\times 556 = 35$	3	9	$9/35$
Total				0.51

(iii) Degree of freedom: $\emptyset = n-1 = 3$

(iv) L.O.S: $\alpha = 0.05$

(v) Critical value: $\chi^2_\alpha = 7.815$

(vi) Decision: since, the calculated value is less than the critical value, null hypothesis is accepted. Thus, The experimental result support the theory

27. Of the 64 off-springs of a certain cross between guinea pigs, 34 were red, 10 were black and 20 were white. According to the generic model these numbers should be in the ratio 9:3:4. Use chi-square test to check whether the data are consistent with the model. **[D23/CompIT/6M]**

Solution:

(i) Null hypothesis: the data is consistent with the generic model

Alternate Hypothesis: the data is inconsistent with the generic model

(ii) Test calculation:

O	E	$O-E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
34	$\frac{9}{16}\times 64 = 36$	-2	4	4/36
10	$\frac{3}{16}\times 64 = 12$	-2	4	4/12
20	$\frac{4}{16}\times 64 = 16$	4	16	16/16
Total = 64	64			$\chi^2 = 1.444$

(iii) LOS:0.05

(iv) Degree of freedom: $\phi = n - 1 = 3 - 1 = 2$

(v) Critical value: $\chi^2_\alpha = 5.991$

(vi) Decision: since, the calculated value is less than the critical value, null hypothesis is accepted. The data is consistent with the generic model