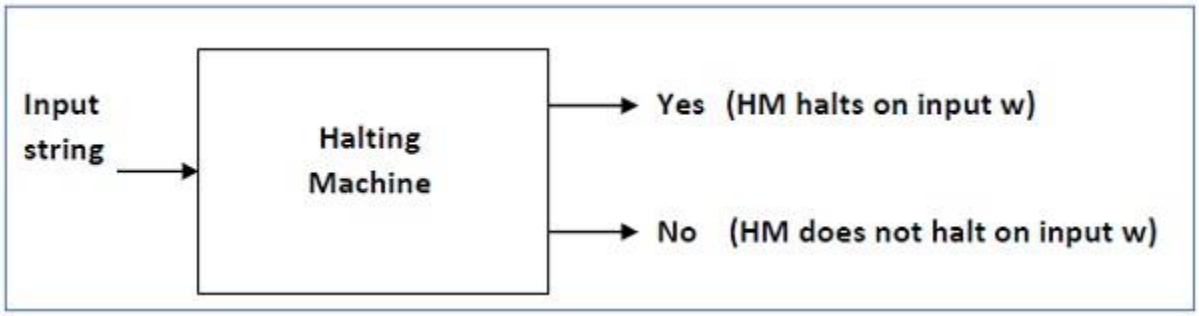
Halting Problem

Input – A Turing machine and an input string **w**.

Problem – Does the Turing machine finish computing of the string **w** in a finite number of steps? The answer must be either yes or no.

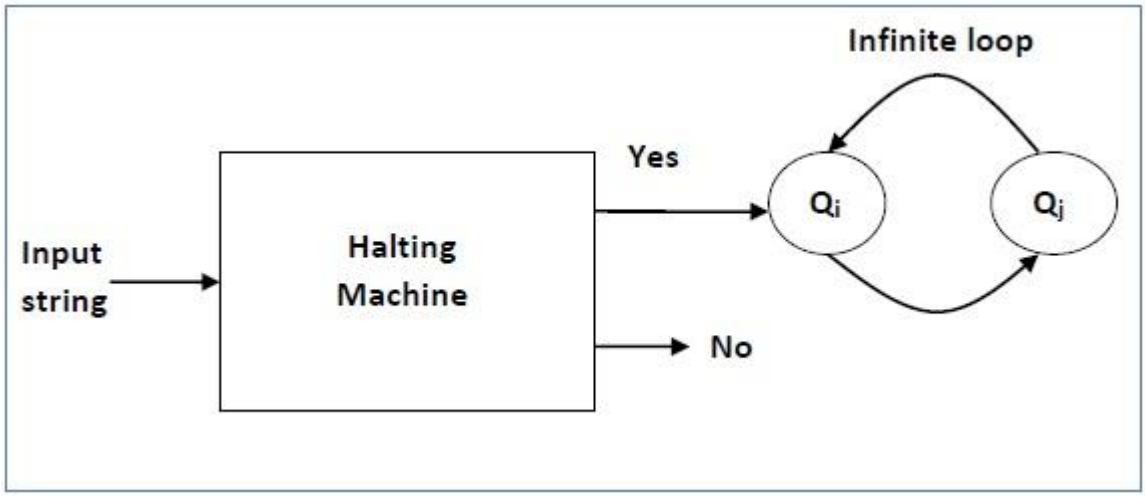
Proof – At first, we will assume that such a Turing machine exists to solve this problem and then we will show it is contradicting itself. We will call this Turing machine as a **Halting machine** that produces a 'yes' or 'no' in a finite amount of time. If the halting machine finishes in a finite amount of time, the output comes as 'yes', otherwise as 'no'. The following is the block diagram of a Halting machine –



Now we will design an **inverted halting machine (HM)'** as –

- If **H** returns YES, then loop forever.
- If **H** returns NO, then halt.

The following is the block diagram of an 'Inverted halting machine' –



Further, a machine $\mathbf{(HM)_2}$ which input itself is constructed as follows –

- If $(HM)_2$ halts on input, loop forever.
- Else, halt.

Here, we have got a contradiction. Hence, the halting problem is **undecidable**.