15/11/2012

Basic Concepts

* **Alphabet (Σ)**

It is defined as a finite set of I/P symbols

eg $\Sigma = \{0, 1, 2\}$

* **String / sentence / word**

It is defined as the finite sequence of symbols defined over the given Σ

eg - 0, 1, 2, 20, 10, 122 . . . .

* **String length**

It is defined as the number of symbols present in the given string

eg $x = 2120$ $|x| = 4$

<u>Note</u> String of length '0' will be denoted by ϵ (epsilon)

* **Language**

It is defined as the set of strings defined over the given Σ

Eg $L = \{x \mid x \text{ ends in "ba" over } \Sigma = \{a, b\}\}$

$L = \{ba, aba, bba, aaba\}$

eg

r	$L(r)$
a	$\{a\}$
b	$\{b\}$
$a+b$ or $a \mid b$	$\{a\} \cup \{b\} = \{a, b\}$
$a \cdot b$	$\{a\} \cdot \{b\} = \{ab\}$
a^*	$\{\epsilon, a, aa, aaa \ldots\}$
a^+	$\{a, aa, aaa \ldots\}$
$(ab)^*$	$\{\epsilon, ab, abab \ldots\}$
$(a+b)^*$	$\{\epsilon, ab, aa, ba, bb \ldots\}$

r	$L(r)$
$(00)^*$	$\{\epsilon, 00, 0000 \ldots\}$
$0 \cdot (00)^*$	$\{0, 000, 00000 \ldots\}$
$(00)^* \cdot 0$	$\{0, 000, 00000 \ldots\}$
$a \cdot a^*$	$\{a, aa, aaa \ldots\}$

Note

1. $0 \cdot \epsilon = \epsilon \cdot 0 = 0$
2. $1 \cdot \epsilon = \epsilon \cdot 1 = 1$
3. $a \cdot \epsilon = \epsilon \cdot a = a$
4. $a^+ = a \cdot a^*$
5. $L^+ = L \cdot L^*$

Write R·E for the following:-

① set of all strings that start with 'a' over $\Sigma = \{a, b\}$

Solution $r = a \cdot (a+b)^*$

$L(r) = \{a, aa, ab, aaa \ldots\}$

② Set of all strings that terminate either in '0' OR '11' over $\Sigma = \{0, 1\}$

$r = \{(0+1)^* 0 + (0+1)^* 11\}$

$L(r) = \{0, 00, 10, 000, 11, 011, 111 \ldots\}$

③ set of all strings that start with 'x' and end with 'y' over $\Sigma = \{x, y\}$

$r = x \cdot (x+y)^* \cdot y$

$L(r) = \{xy, xxy, xyy, xxxy, \ldots\}$

④ Set of all strings that start with 'ab' and end with 'ba' over $\Sigma = \{a, b\}$

$r = ab\,(a+b)^*\,ba + aba$

$L(r) = \{aba, abba, ababa, abba, \ldots\}$

* Operations on language

(1) Union of two languages

$L_1 \cup L_2 = \{x, y / \begin{matrix} x \in L_1 \\ \& \\ y \in L_2 \end{matrix}\}$ No combination

(2) Concatenation of two languages

$L_1 \cdot L_2 = L_1 L_2 \{x, y / \begin{matrix} x \in L_1 \\ \& \\ y \in L_2 \end{matrix}\}$ combination

eg = $L_1 = \{00, 10, 110\}$
$L_2 = \{aa, ba\}$

$L_1 \cup L_2 = \{00, 10, 110, aa, ba\}$

$L_1 \cdot L_2 = \{00aa, 00ba, 10aa, 10ba, 110aa, 110ba\}$

(3) Closure of a language zero/more

$L^* = \bigcup_{i=0}^{\infty} L^i$

(4) Positive closure of a language
One/more

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$L^+ = \bigcup_{i=1}^{\infty} L^i$

$L = \{10\} = L^1$

$L^2 = L \cdot L = \{1010\}$

$L^3 = L \cdot L \cdot L = \{101010\}$

$L^0 = \{\epsilon\}$

$L^* = L^0 \cup L^1 \cup L^2 \cup \cdots = \{\epsilon, 10, 1010, \cdots\}$

$L^+ = L^1 \cup L^2 \cup L^3 \cup \cdots = \{10, 1010, 101010, \cdots\}$

Defination Regular Expression.

Regular expression is used for specifying the strings of a regular language and is defined as following

1. ϕ is a RE for $\{\}$
2. ϵ is a R.E for $\{\epsilon\}$
3. 'a' is a R.E for $\{a\}$
4. Let R & S be two RE for specifying L_R & L_S respectively

   a. $(R)|(S)$ is a R.E for $L_R \cup L_S$
   b. $(R)\cdot(S)$ is a R.E for $L_R \cdot L_S$
   c. $(R)^*$ is a R.E for L_R^*

Teacher's Signature

write regular expression for the following.

1. Set of all strings that start with 'a' and end with 'ab' over $\Sigma = \{a,$

$$r = a \cdot (a+b)^* ab + ab .$$

$L(r) \Rightarrow \{ab, aab, aaab, abab, \ldots$

2. Set of all strings that start with xyyy and end with yyyx over $\Sigma = \{x, y\}$

Basic Concepts.

1. Prefix
A prefix of a string is the string formed by taking any no of symbols of the string
(For a string of length 'n' 'n+1' no of prefix

eg $w = 0111 \rightarrow \lambda, 0, 01, 011, 0111$

② Proper prefix
For a string, any prefix of the string other than the string itself is called as the proper prefix of the string.
eg $w = 0111$, proper prefix is λ, 0, 01, 011

③ Suffix
A suffix of a string is formed by taking any no of symbols from the end of the string
eg $w = 0110$ suffixes $\Rightarrow \lambda$, 0, 10, 110, 0110
(String of length n, there are n+1 no of suffixes)

④ Proper suffix
For a string, any suffix of the string other than the string itself is called as the proper suffix of the string.

⑤ Substring - A substring of a string is defined as a string formed by taking any number of symbols of the string.
$w = 012$, the substrings are
$\rightarrow \lambda$, 0, 1, 2, 01, 12 & 012

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* Basics of set Theory
A set is a well-defined collection of objects.
The objects used for constructing a set are called elements or the members of a set.

Subset
Let there be two sets S & S1, S1 is said to be a subset of S if every element S1 is an element of
Symbolically S1 ⊆ S

Finite & Infinite set
A set is said to be finite if it contains no element or a finite no elements otherwise it is an infinite set.

Equal
Two sets of S1 & S1 are said to be equal if S1 ⊆ S and S ⊆ S1.

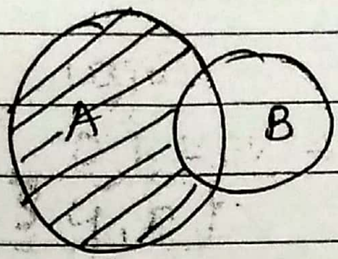
Teacher's Signature ....

Algebraic operations on sets

① Union ② Intersection ③ Difference.

$A = \{2, 3, 4, 5\}$ $B = \{3, 4\}$

$A - B = \{2, 5\}$



④ Cartesian product

$A \times B$

$A = \{2, 3, 4, 5\}$ $B = \{3, 4\}$

$A \times B = \{(2,3), (2,4), (3,3), (3,4), (4,3), (4,4), (5,3), (5,4)\}$

⑤ Power set

The power set of A is the set of all possible subsets of A. Let $A = \{a, b\}$.

Power set of $A = \{\{\phi\}, \{a\}, \{b\}, \{a, b\}\}$

for a set of elements 'n', the no of elements of power set of A is 2^n.

(6) Concatenation

If there are two sets A & B then concatenation is denoted by A·B

Let $A = \{a, b\}$ and $B = \{c, d\}$

$A \cdot B = \{ac, ad, bc, bd\}$

FSM

(Finite state machine)

FSM consists of finite set of states which alter on receiving the I/P set (I) to produce the output set

All the finite automata contain finite no of states, they are FSM

Applications :- sequence detector, incompletely specified machine, where some state and/or some output are not mentioned, definite machine, finite machine, information lossless machine, etc

$FSM = \{S, I, \delta, q_0, F\}$

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16/11/17

1. Design a FSM to check whether the given decimal no is divisible by 3.

Solution Step I Theory (Defination of FSM)

Step II Logic

$I = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$O = \{y, n\}$ $y \to f$ $n \to nf$

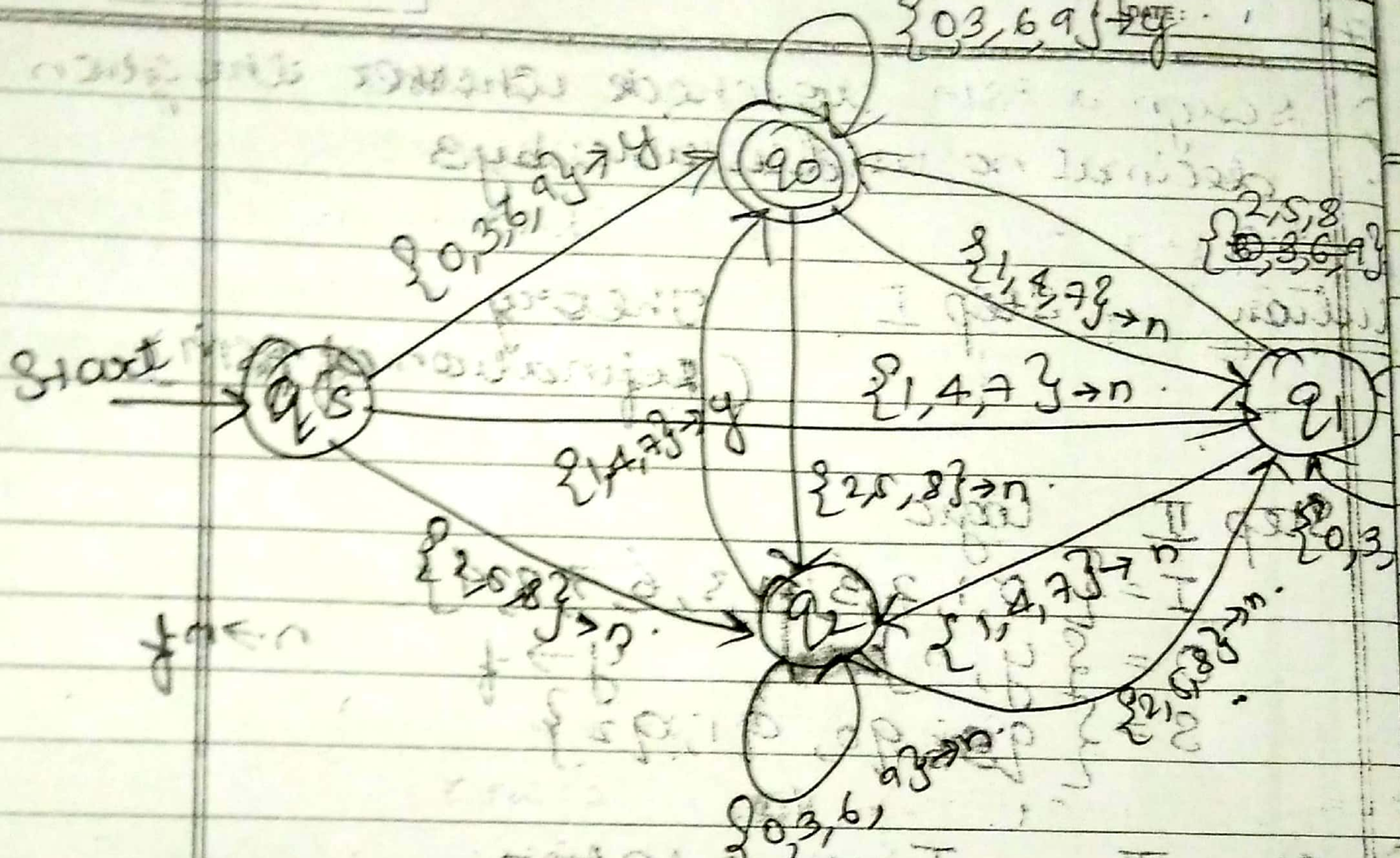
$S = \{q_s, q_0, q_1, q_2\}$

Step III Implementation

S \ I	{0,3,6,9}	{1,4,7}	{2,5,8}
$\to q_s$	q_0	q_1	q_2
0 q_0^*	q_0	q_1	q_2
1 q_1	q_1	q_2	q_0
2 q_2	q_2	q_0	q_1

S \ I	{0,3,6,9}	{1,4,7}	{2,5,8}
$\to q_s$	y	n	n
q_0^*	y	n	n
q_1	n	n	y
q_2	n	y	n

Teacher's Signature



⊢ → transition.

Step IV

(1) (qs, 5248}

⊢ {q2, 248}.
⊢ {q1, 48}
⊢ {q2, 8}.
⊢ q1 → n.

(2) (qs, 312}

⊢ (q0, 12).
⊢ (q1, 2)
⊢ (q0) → y

② Design a FSM to check whether the given decimal no is divisible by 4

Solution

$I = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$O = \{y, n\}$ y → f   n → nf

$S = \{q_s, q_0, q_1, q_2, q_3\}$

	S \ I	{0,4,8}	{1,5,9}	{2,6}	{3,7}
	→ q_s	q_0	q_1	q_2	q_3
0	q_0^*	q_0	q_1	q_2	q_3
1	q_1	q_2	q_3	q_0	q_1
2	q_2	q_0	q_1	q_2	q_3
3	q_3	q_2	q_3	q_0	q_1

S \ I	{0,4,8}	{1,5,9}	{2,6}	{3,7}
→ q_s	y	n	n	n
q_0^*	y	n	n	n
q_1	n	n	y	n
q_2	y	n	n	n
q_3	n	n	y	n

3R + I/P
NUM

(q3, 7328)
⊢ (q1, 328)
⊢ (q3, 28)
⊢ (q0 8)
⊢ (q0) → y

3. Design
I = {0, 1}
O = {y, n}
S = {qs, q0, q1, q2, q3}

	S \ I	0	1
	→ qs	q0	q1
100 (4) 0	q0*	q0	q1
101 (5) 1	q1	q2	q3
110 (6) 2	q2	q0	q1
111 (7) 3	q3	q2	q3

(qs, 101100)
⊢ (q1, 01100)
⊢ (q2, 1100)
⊢ (q1, 100)
⊢ (q3, 00)
⊢ (q2, 0)
⊢ (q0,)
⊢ (q0, 0)
⊢ (q0) → y

0 -	0	0	0	0
1 -	0	0	0	1
2 -	0	0	1	0
3 -	0	0	1	1
4 -	0	1	0	0
5 -	0	1	0	1
6 -	0	1	1	0
7 -	0	1	1	1
8 -	1	0	0	0
9 -	1	0	0	1
10 -	1	0	1	0
11 -	1	0	1	1
12 -	1	1	0	0
13 -	1	1	0	1
14 -	1	1	1	0
15 -	1	1	1	1

(4) Design FSM to check whether the given ternary no is divisible by 5.

$I = \{0, 1, 2\}$

$O = \{y, n\}$

$S = \{q_s, q_0, q_1, q_2, q_3, q_4\}$

	S \ I	0	1	2
	$\rightarrow q_s$	q_0	q_1	q_2
0	q_0^*	q_0	q_1	q_2
1	q_1	q_3	q_4	q_0
2	q_2	q_1	q_2	q_3
3	q_3	q_4	q_0	q_1
4	q_4	q_2	q_3	q_4

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(5) Design FSM in which the I/P is y if it ends in "100" over $\Sigma = \{0$

Solution :-

$I = \{0, 1\}$

$O = \{y, n\}$ $y \to f$ $n \to nf$

$S = \{q_s, q_0, q_1, q_2, q_3^*\}$

	S \ I	0	1
	$\to q_s$	q_0	q_1
0	q_0	q_0	q_1
1	q_1	q_2	q_1
10	q_2	q_3	q_1
100	q_3^*	q_0	q_1

$(q_s, \underline{1}0100)$

$\vdash (q_1, 0100)$

$\vdash (q_2, 100)$

$\vdash (q_1, 00)$

$\vdash (q_2, 0)$

$\vdash q_3 \to y.$

(6) Design FSM in which the I/P is valid if it ends in {bab b} over $\Sigma = \{a, b\}$

Solution $I = \{a, b\}$
$O = \{y, n\}$ $y \to f$ $n \to nf$
$S = \{q_s, q_0, q_1, q_2, q_3, q_4\}$

	S \ I	a	b
	$\to q_s$	q_0	q_1
a	q_0	q_0	q_1
b	q_1	q_2	q_1
ba	q_2	q_0	q_3
bab	q_3	q_2	q_4
babb*	q_4	q_2	q_1

(7) if it ends either in "101" or "110" over $\Sigma = \{0, 1\}$.

	S \ I	0	1
	$\to q_s$	q_0	q_1
0	q_0	q_0	q_1
1	q_1	q_2	q_4
10	q_2	q_0	q_3
101	q_3*	q_5	q_4
11	q_4	q_5	q_4
110	q_5*	q_0	q_3

Scanned by CamScanner

(8) if the second last symbol is 'a' over $\Sigma = \{a, b\}$

$I = \{a, b\}$

$O = \{n, y\}$

$S = \{$

"aa" "ab"

$\Sigma \backslash I$		a	b
	$\rightarrow q_s$	q_0	q_1
a	q_0	q_2	q_3
b	q_1	q_0	q_1
aa	q_2^*	q_2	q_3
ab	q_3^*	q_0	q_1

(9) if it contains "1011" over $\Sigma = \{0, 1\}$

	$S \backslash I$	0	1
	$\rightarrow q_s$	q_0	q_1
0	q_0	q_0	q_1
1	q_1	q_2	q_1
10	q_2	q_0	q_3
101	q_3	q_2	q_4
1011	q_4^*	q_4	q_4

$\rightarrow$ Trap state

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(10) if it contains atleast one occurrence of substring "bba" over Σ = {a,b}.

Σ \ I	a	b
→ qs	q0	q1
a q0	q0	q1
b q1	q0	q2
bb q2	q3	q2
bba * q3	q3	q3

state accept state

(11) if it does not contain any occurrence of three consecutive b's over Σ = {a,b}.

Σ \ I	a	b
→ * qs	q0	q1
a * q0	q0	q1
b * q1	q0	q2
bb * q2	q0	q3
bbb q3	q3	q3

= Dead state

(20)

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Points to remember

(1) There is always one start state

(2) There can be one / many states

(3) Start state can also be final
(in such cases blank i/p is va

(4) Dead state is a trap state con
is non-final

(5) Every dead state is a trap sta
a trap state may / may not be

(6) Start state $\longleftrightarrow$ initial sta
final state $\longleftrightarrow$ accepting st
non final state $\longleftrightarrow$ rejecting

(7) Dead state is always a rejecti
state

1/7/2017

(1) if it starts with three consecutive a's over $\Sigma = \{a, b\}$.

	$S \backslash I$	a	b
E	$\rightarrow q_s$	q_0	q_3
a	q_0	q_1	q_3
aa	q_1	q_2	q_3
aaa	q_2^*	q_2	q_2
dead state	q_3	q_3	q_3

(2) if it starts either with "011" or "100" over $\Sigma = \{0, 1\}$.

		$S \backslash I$	0	1
A	E	$\rightarrow q_s$	q_0	q_3
0	11	q_0	q_6	q_1
01	1	q_1	q_6	q_2
011	-	q_2^*	q_2	q_2
1	00	q_3	q_4	q_6
10	0	q_4	q_5	q_6
100	-	q_5^*	q_5	q_5
dead state		q_6	q_6	q_6

③ if it contains odd number of b over $\Sigma = \{a, b\}$

Solution.

	$S \diagdown I$	a	b
nb(x) →	q_s	q_0	q_1
even	q_0	q_0	q_1
odd *	q_1	q_1	q_0

④ if it contains even number of a and odd number of b's over Σ

"001" so "110"

na(x)	nb(x)	$S \diagdown I$	a	b
	→	q_s	q_2	q_1
E	E	q_0	q_2	q_1
E	O	q_1*	q_3	q_0
O	E	q_2	q_0	q_3
O	O	q_3	q_1	q_2

⑤ If it contains

① atleast 3a's
q_3 & q_4

② Exactly 3a's
q_3

③ atmost 3a's
q_s, q_0, q_1, q_2 & q_3

Over $\Sigma = \{a, b\}$

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	I	a	b
$n_a(x)$	$\rightarrow q_0$	q_1	q_0
0	q_0	q_1	q_0
1	q_1	q_2	q_1
2	q_2	q_3	q_2
3	q_3	q_4	q_3
more than 3	q_4	q_4	q_4

(6) If it contains 'b' at every even position over $\Sigma = \{a, b\}$

LD

(7) If it ends in a double letter
$\Sigma = \{a, b\}$

S \ I	a	b
→ qs	q0	q1
a q0	q2	q1
b q1	q0	q3
aa* q2	q2	q1
bb* q3	q0	q3

(one or more one)

(8) If it contains atleast one occ of a double letter over $\Sigma = \{$

Solution:

S \ I	x	y
→ qs	q0	q1
x q0	q2	q1
y q1	q0	q3
xx q2*	q2	q2
yy q3*	q3	q3

(9) If it does not contain any oc of a double letter over $\Sigma - \{ 0$

alternate way.

Design a FSM in which I/P is val if it contains alternating seq of 0's and 1's.

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01 010

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S \ I	0	1
→ q_s*	q_0	q_1
0 q_0*	q_2	q_1
1 q_1*	q_0	q_3
00 q_2	q_2	q_2
11 q_3	q_3	q_3

$(q_s, 010)$

q_0

$q_4 \Rightarrow$ dead

(10) If it starts with a double letter over $\Sigma = \{0,1\}$

ne)

$q_s \Rightarrow$ Start
$q_0 \Rightarrow$ Starting with 0
*$q_1 \Rightarrow$ Starting with 0
$q_2 \Rightarrow$ Starting with 1
*$q_3 \rightarrow$ Starting with 11

		S \ I	0	1
A	E	→ q_s	q_0	q_2
0	0	q_0	q_1	q_4
00*	–	*q_1	q_1	q_1
1		q_2	q_4	q_3
11*	–	*q_3	q_3	q_3
Dead state		q_4	q_4	q_4

	0	1
q_s	q_0	q_2
q_0	q_1	—
q_1	q_1	q_1
q_2	—	q_3
q_3	q_3	q_3

(11) Design FSM to O/P the remainder when binary no is divided by 3:

$I = \{0,1\}$

$O = \{0, 1, 2\}$

$S = \{q_s, q_0, q_1, q_2\}$

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Basic Concepts continued.

* Operations on language

① Union of two languages

$L_1 \cup L_2 = \{x, y \mid x \in L_1 \ \& \ y \in L_2\}$ NO combination

② Concatenation of two languages

$L_1 \cdot L_2 = L_1 \cdot L_2 = \{xy \mid x \in L_1 \ \& \ y \in L_2\}$ Combination.

eg: $L_1 = \{00, 10, 110\}$

$L_2 = \{aa, ba\}$

$L_1 \cup L_2 = \{00, 10, 110, aa, ba\}$

$L_1 \cdot L_2 = \{00aa, 00ba, 10aa, 10ba, 110aa, 110ba\}$

③ closure of a language zero/more

$$L^* = \bigcup_{i=0}^{\infty} L^i$$

④ Positive closure of a language one/more

$$L^+ = \bigcup_{i=1}^{\infty} L^i$$

$L = \{10\} = L^1$

$L^2 = L \cdot L = \{1010\}$

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$L^3 = \{101010\}$

$L^0 = \{\epsilon\}$

$L^* = L^0 \cup L^1 \cup L^2 \cup \dots = \{\epsilon, 10, 1010, \dots\}$

$L^+ = L^1 \cup L^2 \cup L^3 \cup \dots = \{10, 1010, 101010\}$

REGULAR EXPRESSIONS.

Defination :- R.E is used for specifying the strings of regular language and is defined as following

1. ϕ is a R.E for $\{\}$
2. ϵ is a R.E for $\{\epsilon\}$
3. 'a' is a R.E for $\{a\}$
4. Let R & S be two R.E for specifying LR & LS respectively

1. $(R)|(S)$ is a R.E for $LR \cup LS$
2. $(R).(S)$ is a R.E for $LR.LS$
3. $(R)^*$ is a R.E for LR^*

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 $(a+b)^+$

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Regular expression:

1. Set of all strings that start with 'a' over $\Sigma = \{a, b\}$
   Solution $r = a \cdot (a+b)^*$
   $L(r) = \{a, aa, ab, aaa \ldots\}$

2. Set of all strings that terminate either in '0' or '11' over $\Sigma = \{0, 1\}$
   $r = (0+1)^* 0 + (0+1)^* 11$
   $L(r) = \{0, 00, 10, 000, 11, 011, 111 \ldots\}$

3. Set of all strings that start with 'x' & end in 'y' over $\Sigma = \{x, y\}$
   $r = x \cdot (x+y)^* y$
   $L(r) = \{xy, xxy, xyy, xxxy, \ldots\}$.

4. Set of all strings that start with 'ab' and end with 'ba' over $\Sigma = \{a, b\}$
   $r = ab(a+b)^* ba + aba$
   $L(r) = \{aba, abba, ababa, abbba \ldots\}$.

(7)

(5) Set of all strings that start with 'a' and end with 'ab' over $\Sigma = \{a, b\}$

$a(a+b)^* ab + ab$

$L(r) = \{ ab, aab, aaab, abab \ldots \}$

(6) set of all strings that start with xyyy and end with yyy over $\Sigma = \{x, y\}$

$r = xyyy(x+y)^* yyyx + xyyyx$

$+ xyyyyx + xyyyyyx +$
$xyyyyyyx$

(7) Set of all strings that start & end with a different symbol over $\Sigma = \{0, 1\}$

$0(0+1)^* 1 + 1(0+1)^* 0$

$L(r) = \{ 01, 001, 011, 10, 100, 110 \ldots \}$

(8) Set of all strings that start and end with the same symbol $\Sigma = \{0, 1\}$

$0 \cdot (0+1+2)^* 0 + 1 \cdot (0+1+2)^* 1 + 2(0+1+2)^* 2$
$+ 0 + 1 + 2$.

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(51) 999

(9) Set of all strings that contain atleast one occurence of "aa" over $\Sigma = \{a, b\}$.

$r = (a+b)^* aa (a+b)^*$

(10) Set of all strings that contain atleast two b's over $\Sigma = \{a, b\}$

$r = b^* a b^* a b^*$

(11) Set of all strings that contain utmost two a's over $\Sigma = \{a, b\}$

$r = b^* a b^* a b^* + b^* a b^* + b^*$

(12) Set of all strings that contain atleast two a's over $\Sigma = \{a, b\}$

$r = (a+b)^* a (a+b)^* a (a+b)^*$

(13) Set of all strings that contain atleast one 'x' and atleast one 'y' over $\Sigma = \{x, y\}$

$b^* a b^* a b^* (a+b)^*$

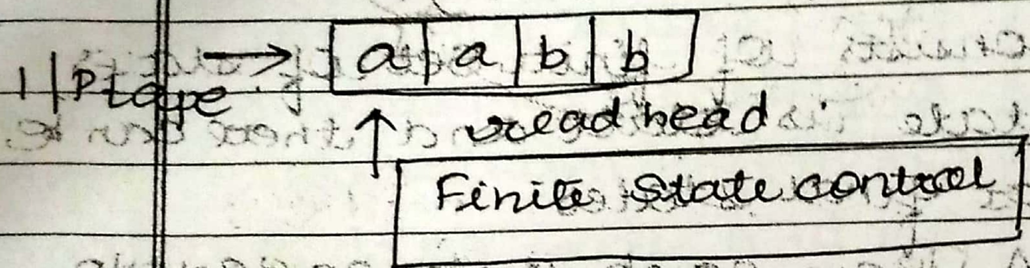
999

$r + b^* a b^* a b^*$

NOTE Regular expressions are said to be identical if they specify the same regular language.

FINITE AUTOMATA (FA)

Finite automata is considered to be a mathematical model of a machine or a system



Components of FA

Finite automada consists of fintle set of states, I/P tape & read head.

working of FA

Depending on the state and I/P symbol

* FA can change the state OR remain in same state.
* F.A moves the head to the right of the current cell

Variations of F.A

F.A with final state & no o/p

- DFA
- NFA

F.A with o/p and final o/p.

- moore
- mealey.

* Deterministic FA (DFA)

DFA consists of finite set of states. One state is start and there can be one/more final states.

In DFA from each state on each I/P symbol there is exactly one transition.

DFA can be represented mathematically as following

$m = (Q, \Sigma, \delta, q, F)$

Q = finite set of states

Σ = I/P alphabet

δ = transition f^n $\delta = Q \times \Sigma \to Q$.

q_0 = start state $q_0 \in Q$.

F = finite set of final states $F \subseteq Q$

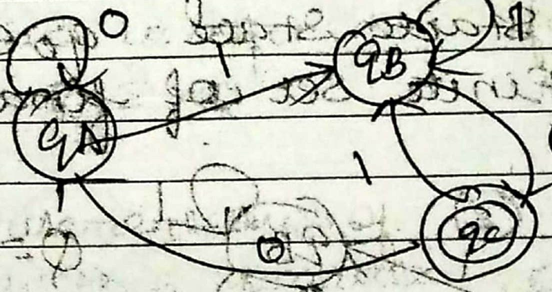
δ \ Σ	0	1
$\rightarrow q_A$	q_A	q_B
q_B	q_C	q_B
q_C^*	q_A	q_B

$Q = \{q_A, q_B, q_C\}$

$\Sigma = \{0,1\}$

$q_0 = q_A$

$F = \{q_C\}$



working

① $(q_A, 110)$
$\vdash (q_B, 10)$
$\vdash (q_B, 0)$
$\vdash (q_C, \epsilon)$
accept

② $(q_A, 1011)$
$\vdash (q_B, 011)$
$\vdash (q_C, 11)$
$\vdash (q_B, 1)$
$\vdash (q_B, \epsilon)$ Reject

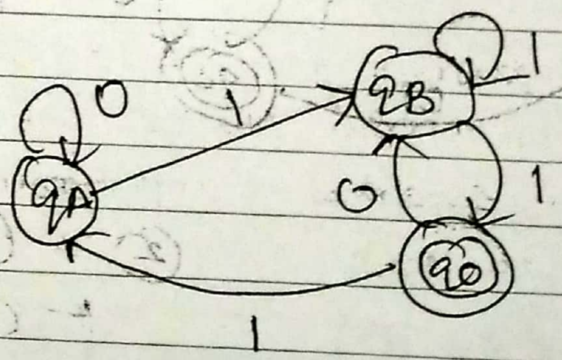
* Non-Deterministic FA (NFA) *

NFA consists of finite set of states one state is start state and there can be one/more final states. In NFA from each state on each I/P symbol there can be 0, or more transitions.

NFA can be represented as following
$m = (Q, \Sigma, \delta, q_0, F)$
where

Q = finite set of states
Σ = I/P alphabet
δ = transition fn $\quad \delta : Q \times \Sigma - 2^Q$
q_0 = Start State $\quad q_0 \in Q$
F_C = Finite set of final states $\subseteq Q$

eg



$Q = (q_A, q_B, q_C)$
$q_0 = q_A$
$\Sigma = \{0, 1\}$
$F = \{q_C\}$

Q \ Σ	0	1
$\rightarrow q_A$	q_A	q_B
q_B	$\{\}$	$\{q_B, q_C\}$
$* q_C$		

working

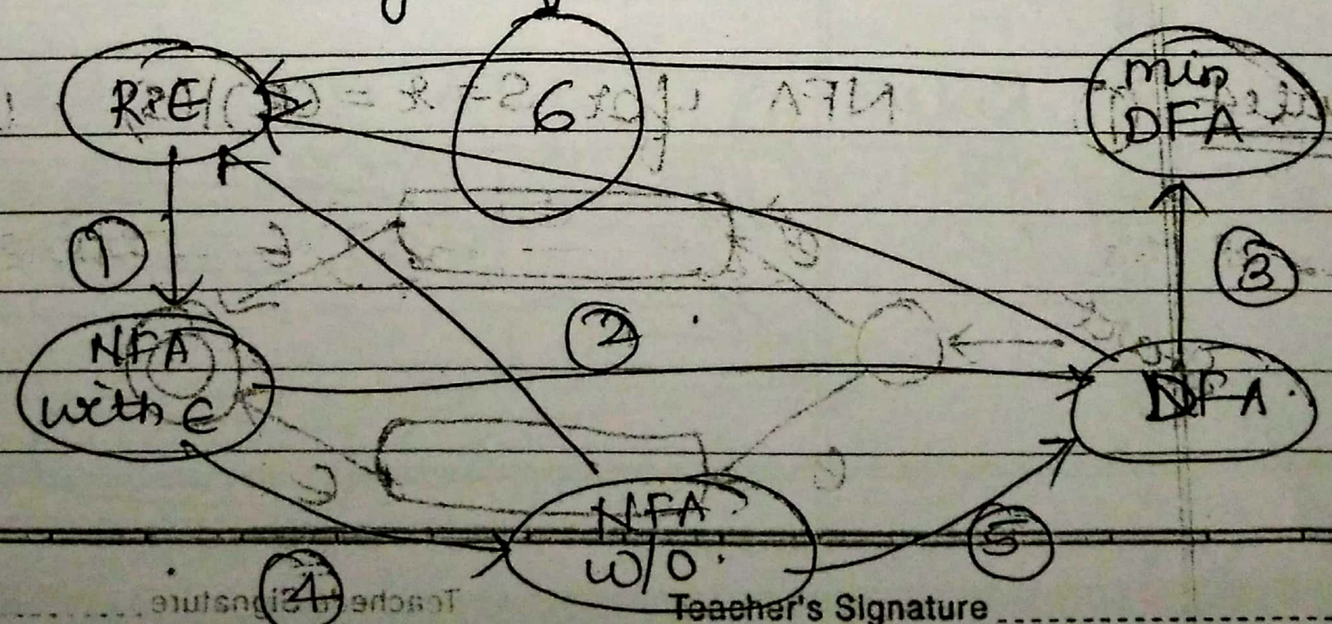
① $(q_A, 111)$
$\vdash (q_B, 11)$
$\vdash (q_B, 1) \quad \vdash (q_C, 1)$
$\vdash (q_B, \epsilon) \quad \vdash (q_C, \epsilon)$ accept

② $(q_A, 1000)$
$\vdash (q_B, 000)$
Reject

Teacher's Signature ....

DFA	NFA
(1) In DFA from each state each I/P symbol there is exactly one transition	(1) In NFA from each state on each I/P symbol there can be 0/1 or more transition
(2) In DFA the transition function is defined as $\delta: Q \times \Sigma \to Q$	(2) In NFA the transition function is defined as $\delta: Q \times \Sigma \to 2^Q$
(3) The implementation of DFA with a computer program is simple	(3) The implementation of NFA with the help of a computer program is difficult because of its non deterministic nature.
(4) In DFA there cannot be ϵ transition	(4) In NFA there can be ϵ transition.

Design of DFA & NFA :-



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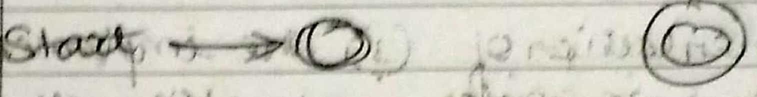
* Regular Expression to NFA

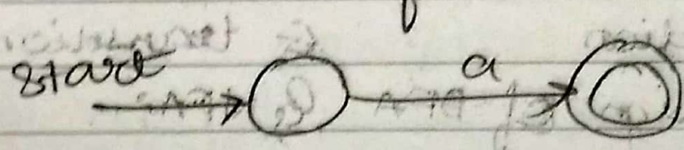
- Divide the given R.E into smaller subexpressions and create NFA for each using Rule 1,2&3
- Combine above NFA's using Rule 4

Rule 1: NFA for $r = \phi$ $\{\}$

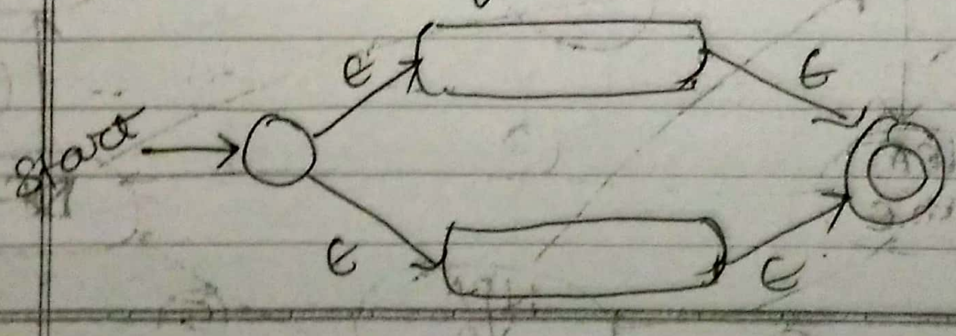


Rule 2: NFA for $r = \epsilon$ $\{\epsilon\}$

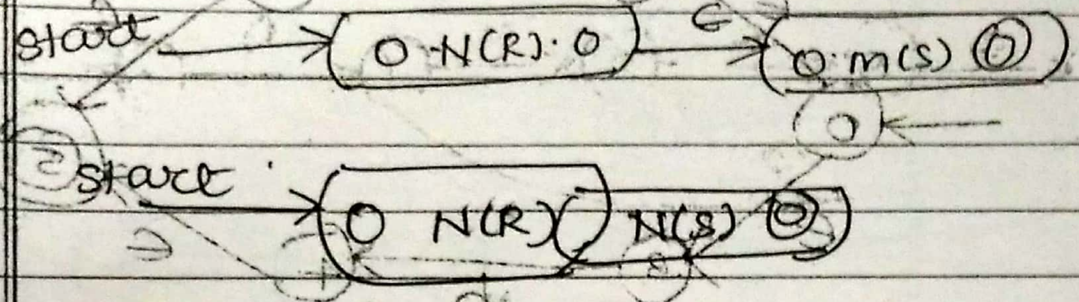
Rule 3: NFA for $r = a$ $\{a\}$



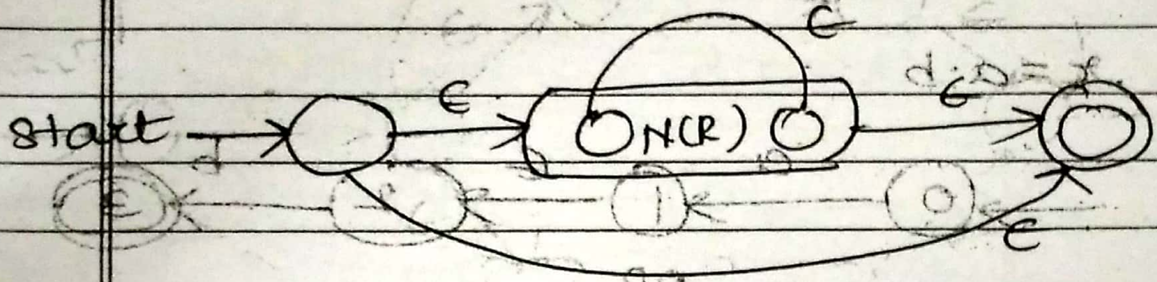
Rule 4: (1) NFA for $S = r = (R)|(S)$ $L_R \cup L_S$



(2) NFA for $r = (R)\cdot(S)$

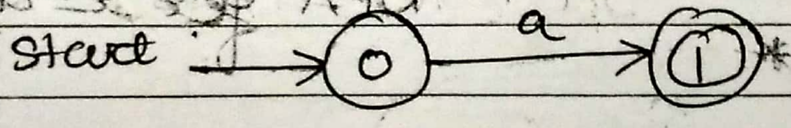


(3) NFA for $r = (R)^*$



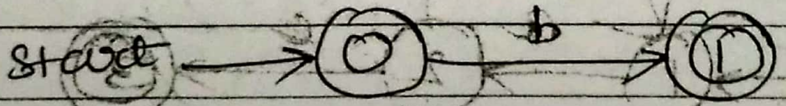
Convert NFA for the following

(1) $r = a$



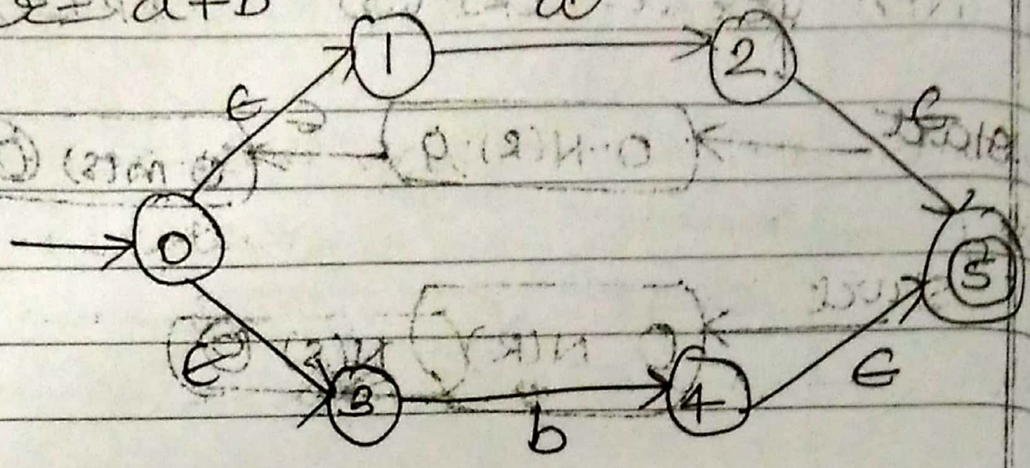
NFA for $r = a$

(2) $r = b$



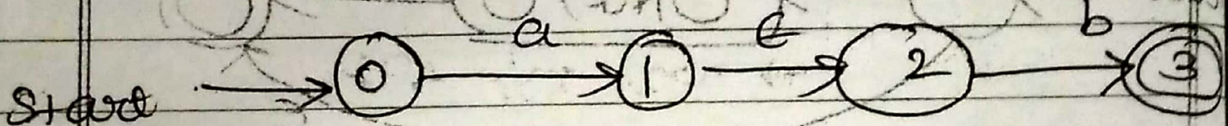
NFA for $r = b$.

(3) $r = a+b$

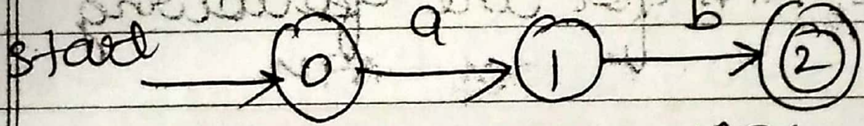


NFA for $r = a+b$.

(4) $r = a \cdot b$

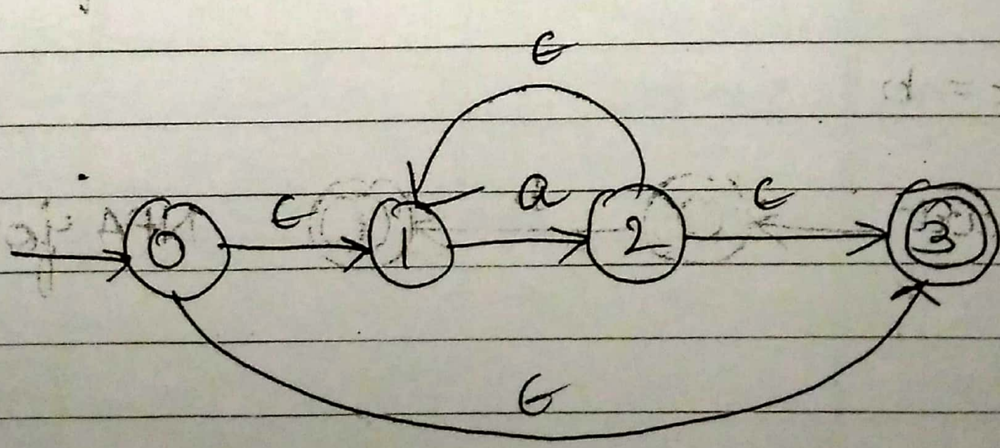


OR

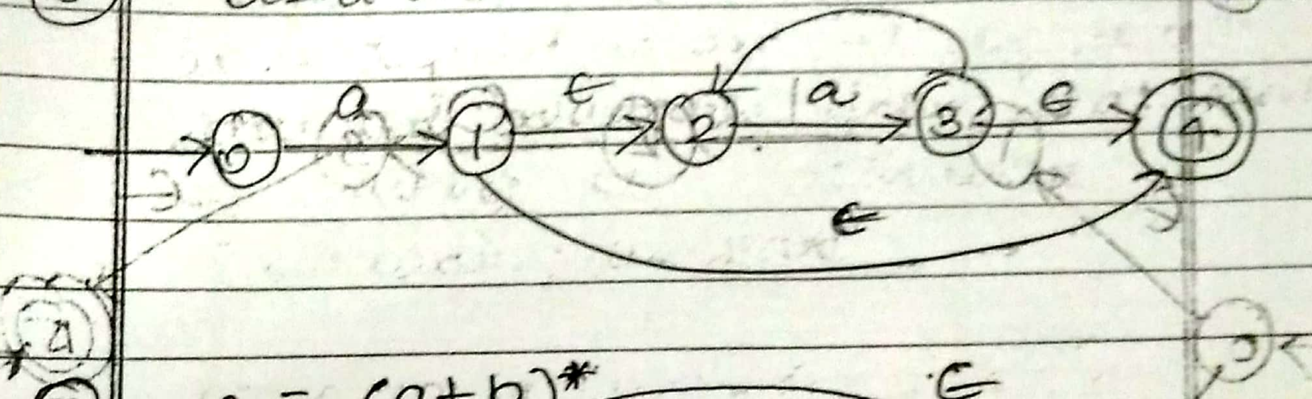


NFA for $r = a \cdot b$

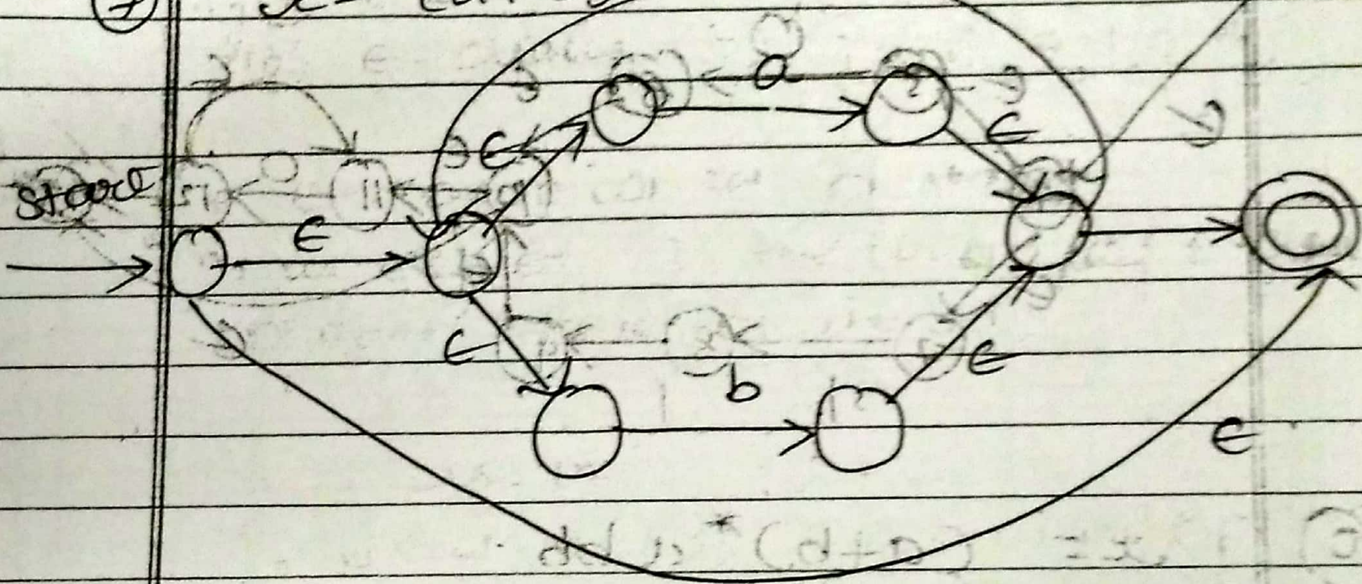
(5) $r = a^*$



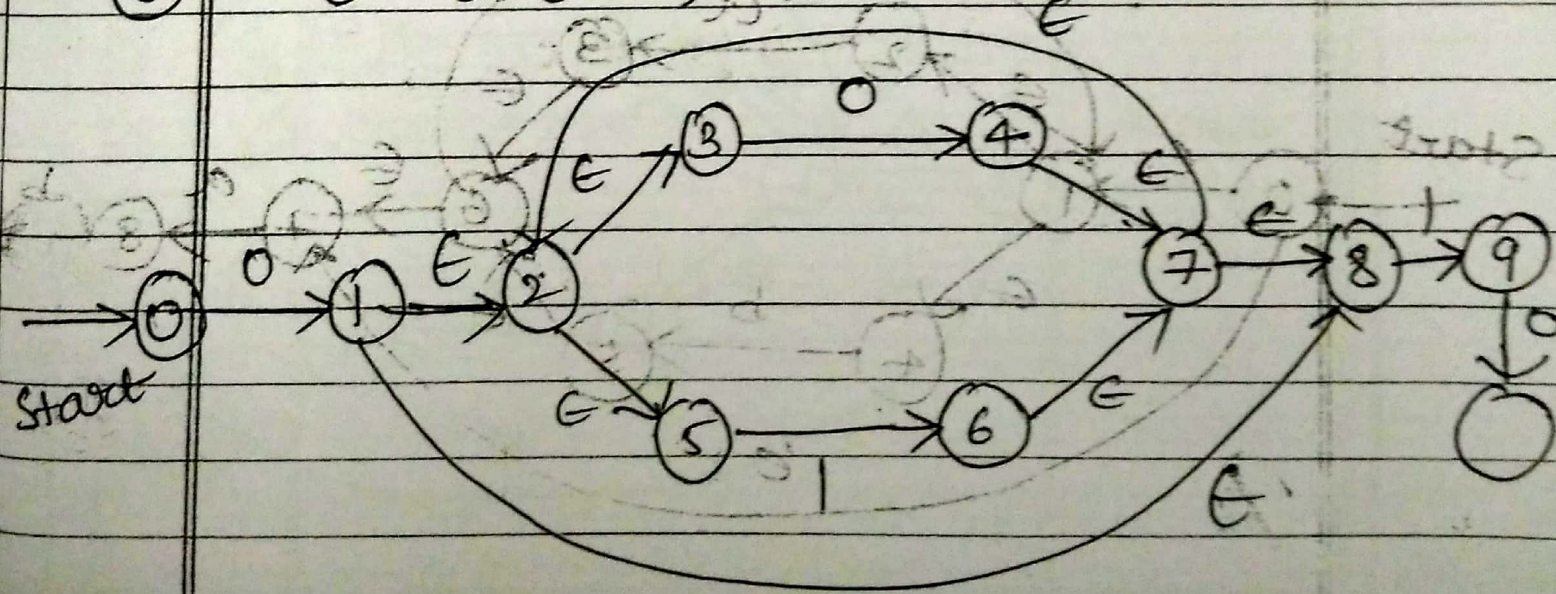
⑥ $r = a \cdot a^*$



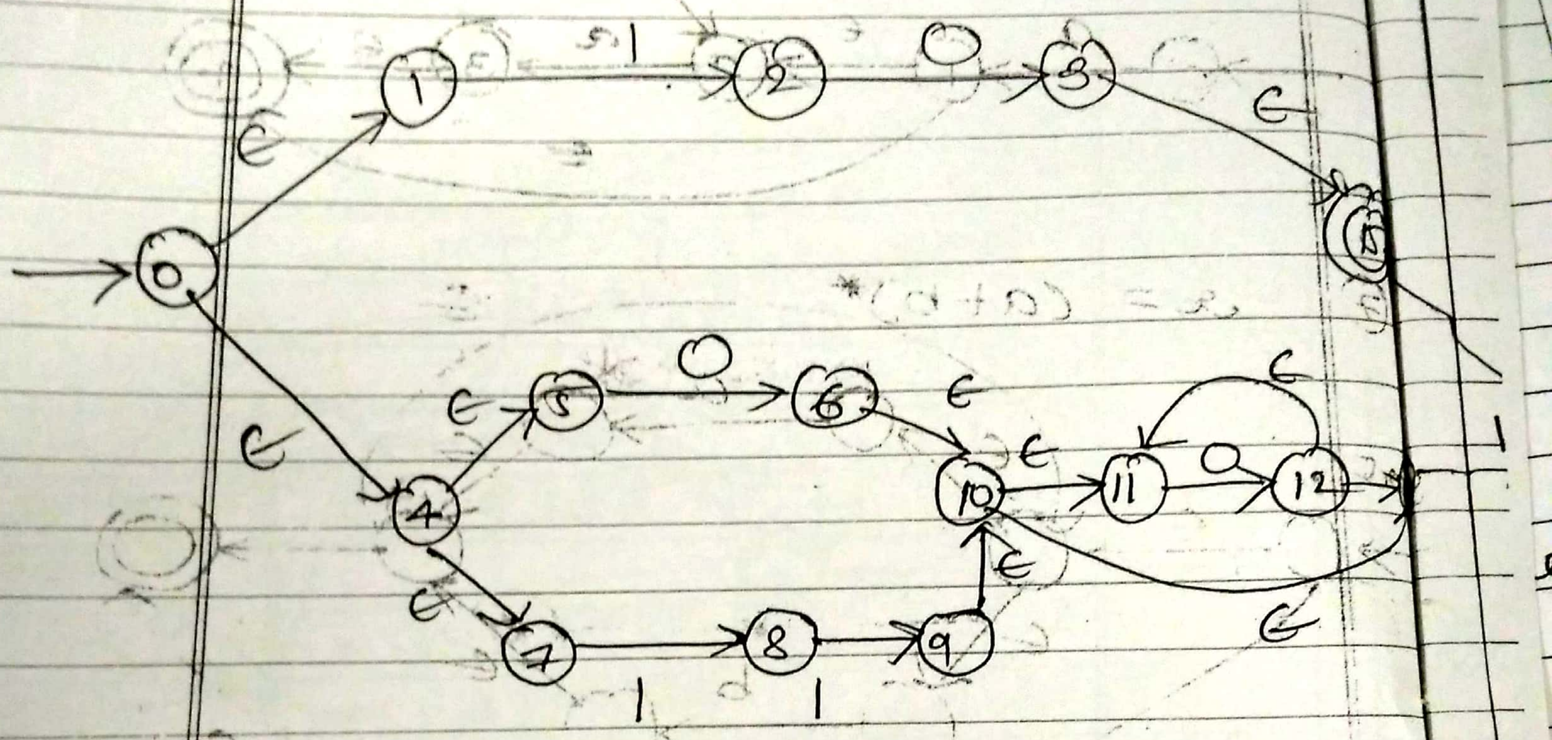
⑦ $r = (a+b)^*$



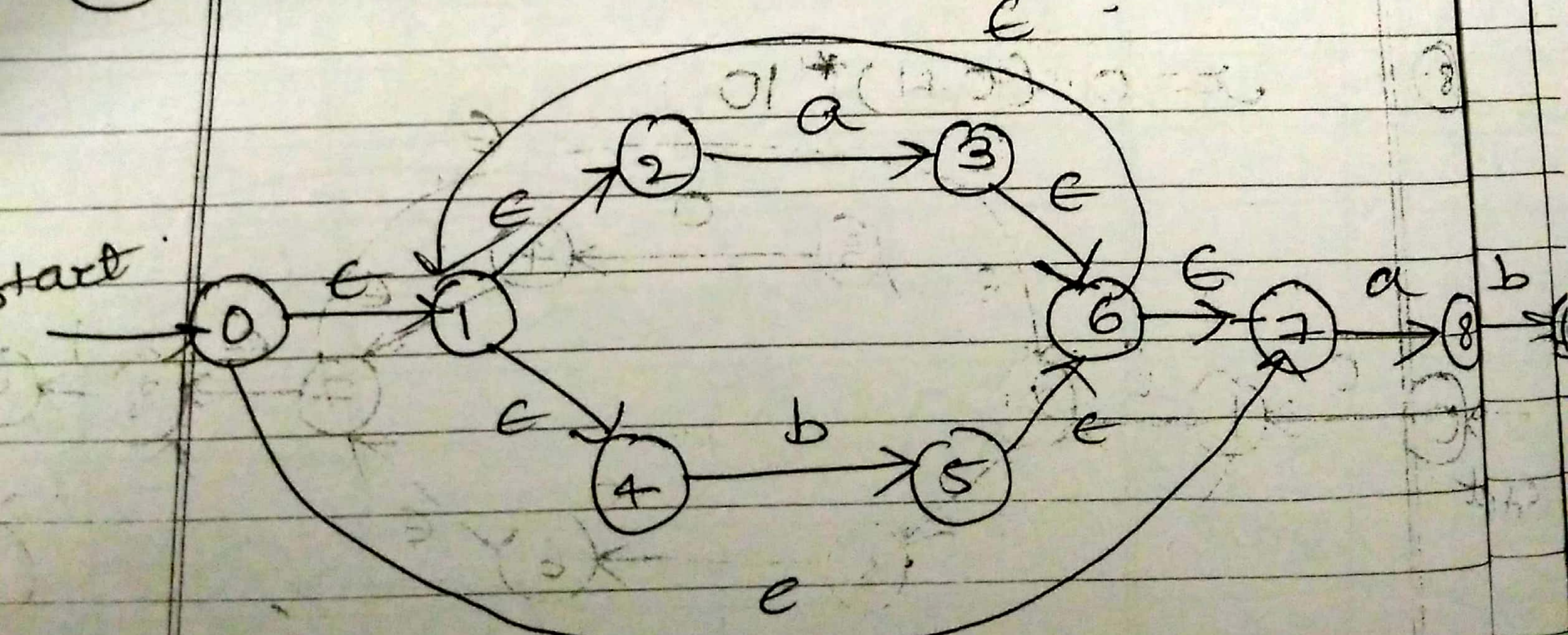
⑧ $r = 0 \cdot (0+1)^* 10$



(9) $r = 10 + (0+11)0^*1$



(10) $r = (a+b)^* abb$

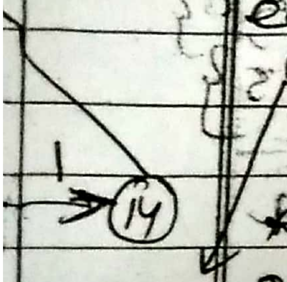


* ϵ-closure of a state

It is defined as the set of states that are reachable from that state by walking on ϵ transitions (including the state.

eg (1) ϵ closure $(0) = \{0, 1, 2, 4, 7\}$

(2) ϵ closure $(5) = \{5, 6, 7, 1, 2, 4\}$



* ϵ-closure of set of states

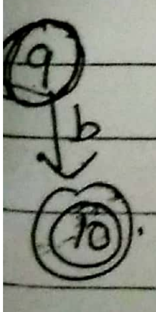
It is defined as the union of ϵ closure of each state of the set

eg. ϵ closure $(\{3, 8\})$

$= \epsilon$ closure $(3) \cup \epsilon$ closure of (8)

$= \{3, 6, 7, 1, 2, 4\} \cup \{8\}$

$= \{1, 2, 3, 4, 6, 7, 8\}$.



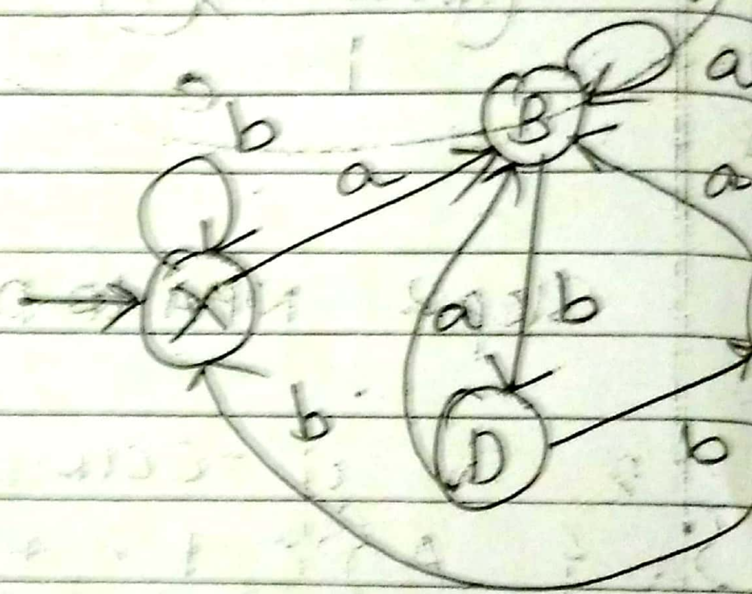
DFA to min DFA (Classical method)

⇒

States can be merged

if (all states have same transition) & (all are final or all are non final)

Q \ Σ	a	b
→ X	B	X
B	B	D
D	B	E
E*	B	X



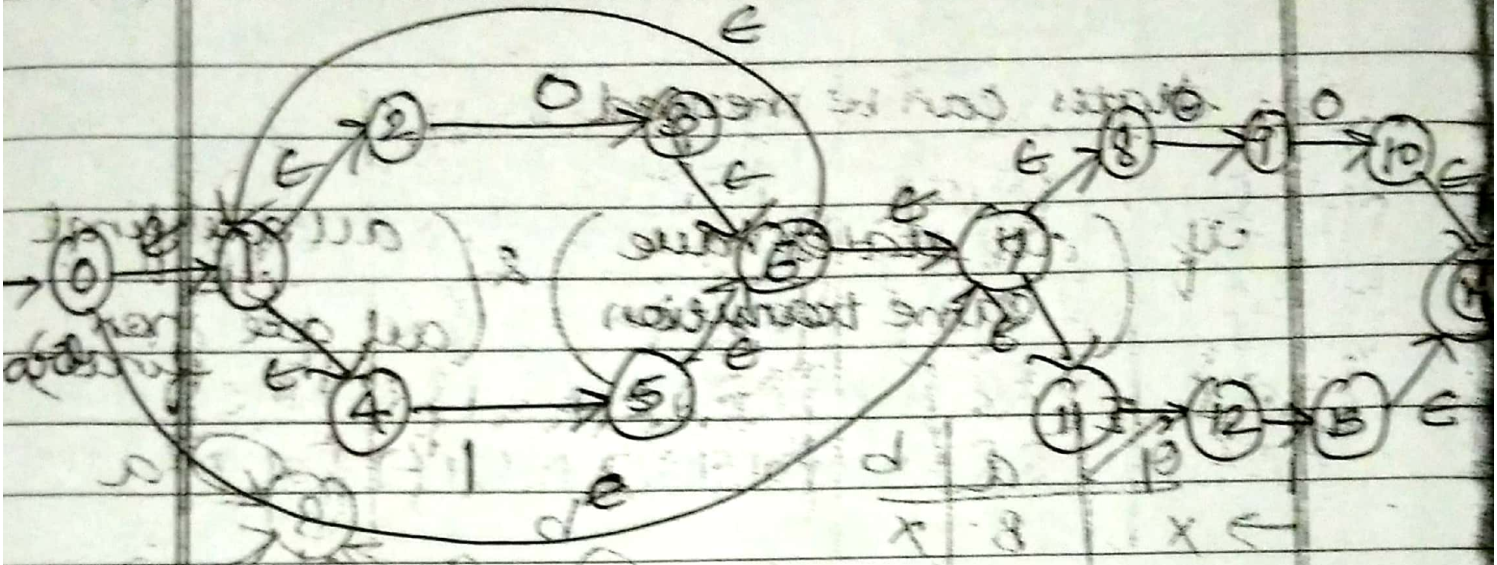
min DFA for $r = (a+b)^*$

Type 2

(1) Construct NFA from $r = (0+1)^*$ (and convert into min DFA.

→ Solution

Step I RE to NFA.



Step2 NFA to DFA

x	y = Eclosure(x)	$\delta(y,0)$	$\delta(y,1)$
{0}	A {0, 1, 2, 4, 7, 8, 11}	{3, 9}	{5, 12}
*{3, 9}	B {3, 6, 7, 8, 11, 1, 2, 4}	{9, 3}	{12, 5}
{5, 12}	C {5, 6, 7, 8, 11, 1, 2, 4, 12}	{3, 9, 3}	{5, 12, 13}
{3, 9, 10}	D {5, 6, 7, 8, 11, 1, 2, 4, 3, 10, 9}	{3, 9, 10}	{5, 12
{5, 12, 13}	E {1, 2, 4, 5, 6, 7, 8, 11, 12, 13, 14}	{3, 9}	{5, 12, 13

Q \ Σ	0	1
→ A	B	C
B	B	C
C	B	E
* D	D	C
* E	B	E

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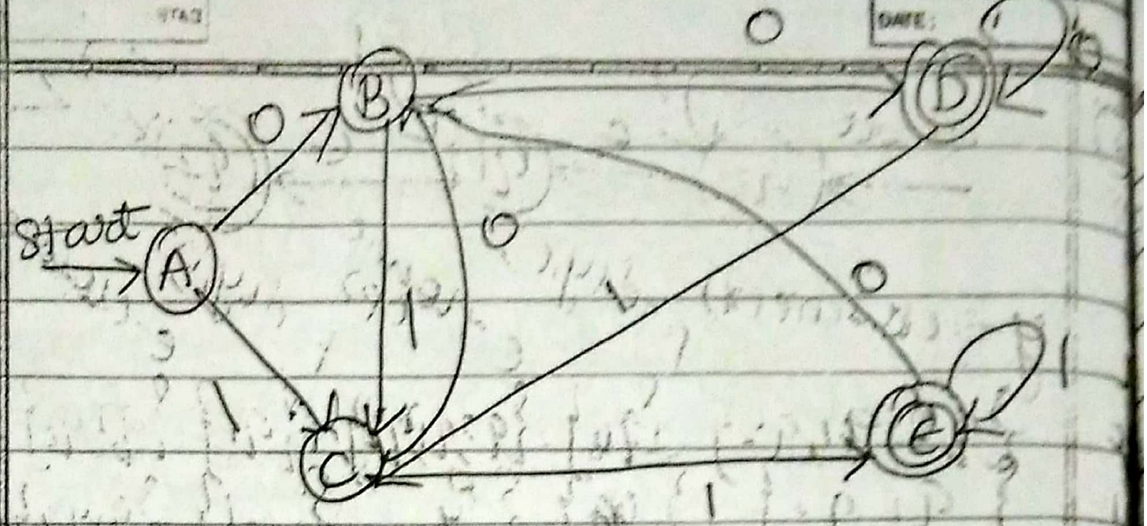
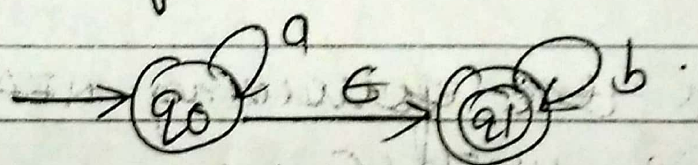


NFA with ∈ to NFA without ∈ guide...

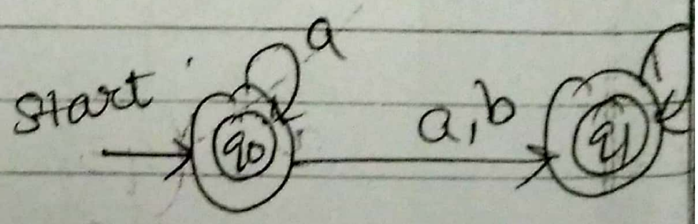
1. The number of states remain the s...
2. Start state remains the same
3. Final states remain the same
4. If the ∈ closure of a start state co... a final state then make the start s... also a final state

1. 

solution

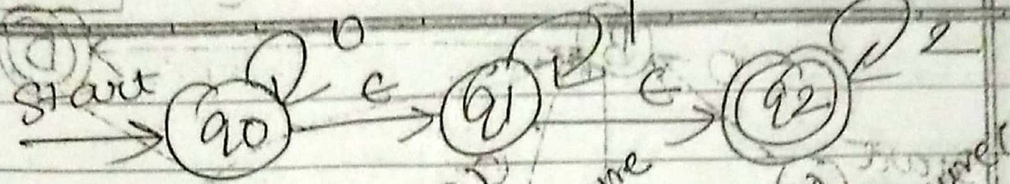
x	y-∈ closure of(x)	δ(y,a)	∈ closure of (a)	δ(y,b...
q_0	$\{q_0, q_1\}$	$\{q_0\}$	$\{q_0, q_1\}$	$\{q_1...$
q_1	$\{q_1\}$	$\{\}$	$\{\}$	$\{q_1\}$

Q \ Σ	a	b
$\rightarrow q_0$ *	$\{q_0, q_1\}$	$\{q_1\}$
q_1 *	$\{\}$	$\{q_1\}$



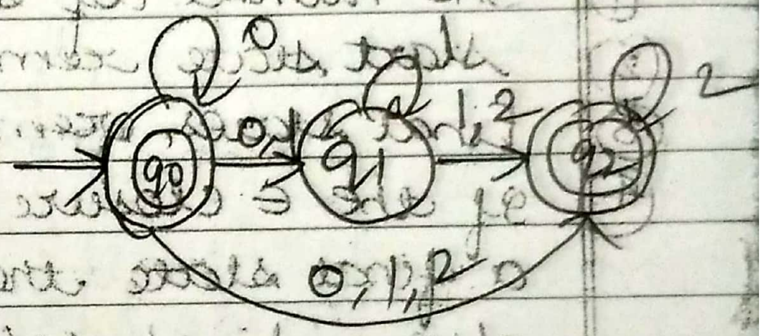
Teacher's Signature ....

(2)

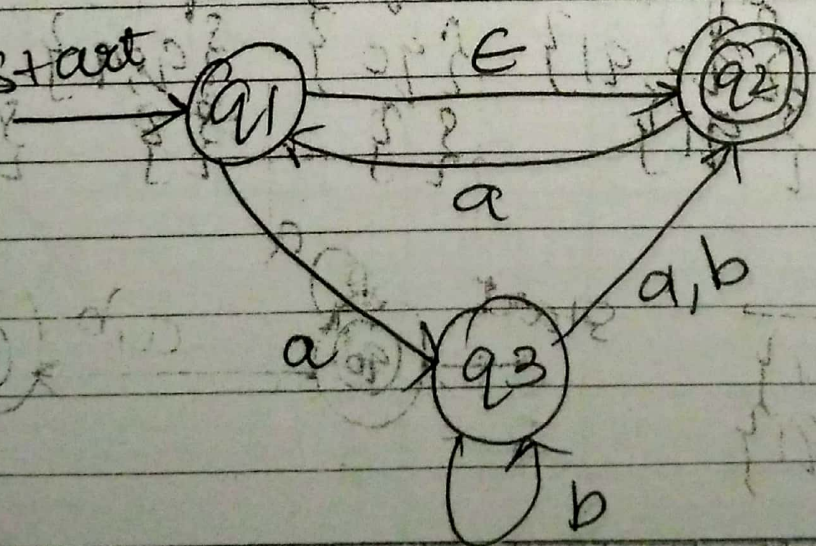


x	y ∈ closure(x)	p = δ(y,0)	ε-closure(p)	p = δ(y,1)	ε-closure(p)	p = δ(y,2)
q0	{q0, q1, q2}	{q0}	{q0, q1, q2}	{q1}	{q1, q2}	{q2}
q1	{q1, q2}	{ }	{ }	{q1}	{q1, q2}	{ }
q2	{q2}	{ }	{ }	{ }	{ }	{q2}

ε	0	1	2
→ q0*	{q0, q1, q2}	{q1, q2}	{q2}
q1	{ }	{q1, q2}	{q2}
q2*	{ }	{ }	{q2}



(3) Convert the following NFA with ε to NFA with ε



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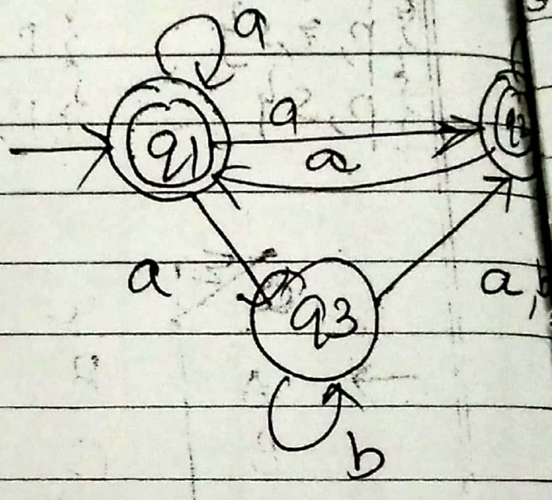
Solution

	x	y=εclosure(x)	p=δ(y,a)	εclosure(p)	q=δ(y,b)	ε-closure(q)
	q1	{q1, q2}	{q1 q3}	{q1, q2, q3}	{ }	{q2}
	q2	{q2}	{q1}	{q1, q2}	{ }	{
	q3	{q3}	{q2}	{q2}	{q3, q2}	{q2}

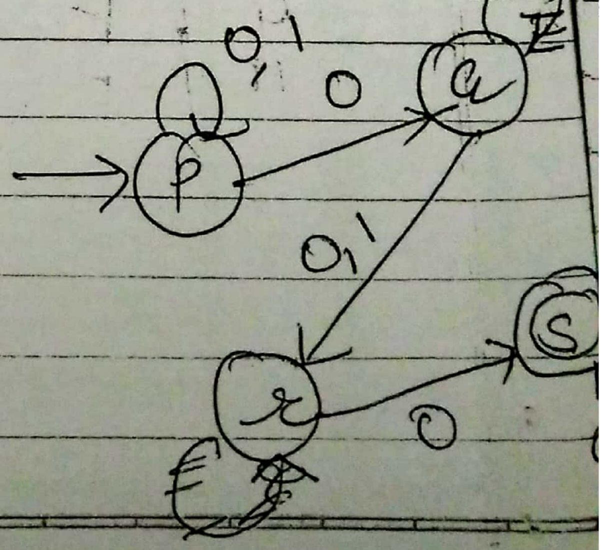
Q \ Σ	a	b
→ q1*	{q1 q2 q3}	{ }
q2*	{q1, q2}	{ }
q3	{q2}	{q2, q3}



Convert NFA without ε to DFA

({p, q, r, s} , {0, 1}, δ, p , {S})

Q \ Σ	0	1
p	p, q	p
q	r	r
r	s	-
s	s	s



Teacher's Signature ....

x	$y = \epsilon\text{-closure}(x)$		$\delta(y,0)$	$\delta(y,1)$
$\{p\}$	$\{p\}$	A	$\{p,q\}$	$\{p\}$
$\{p,q\}$	$\{p,q\}$	B	$\{p,q,r\}$	$\{p,r\}$
$\{p,q,r\}$	$\{p,q,r\}$	C	$\{p,q,r,s\}$	$\{p,r,\}$
$\{p,r\}$	$\{p,r\}$	D	$\{p,q,s\}$	$\{p,$ [illegible] $\}$
$\{p,q,r,s\}$	$\{p,q,r,s\}$	E	$\{p,q,r,s\}$	$\{p,r,s\}$
$\{p,q,s\}$	$\{p,q,s\}$	F	$\{p,q,r,s\}$	$\{p,r,s\}$
$\{p,r,s\}$	$\{p,r,s\}$	G	$\{p,q,s\}$	$\{p,s\}$
$\{p,s\}$	$\{p,s\}$	H	$\{p,q,s\}$	$\{p,S\}$

Q \ Σ	0	1
→ A	B	A
B	C	D
C	E	D
D	F	A
E	E	G
F	E	G
G	F	H
H	F	H

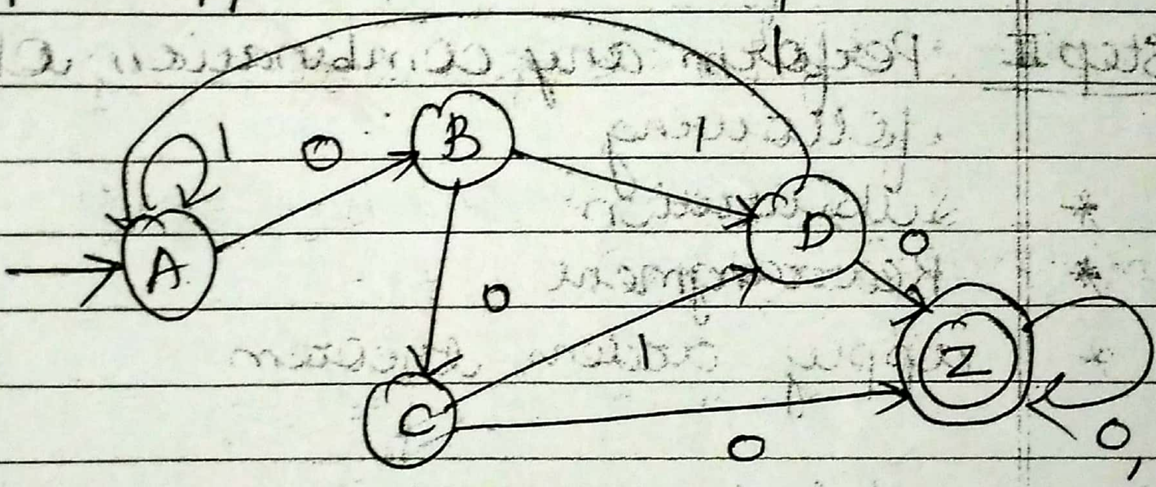
E, F → X

G, H → Y.

Σ / Q	0	1
→ A	B	A
B	C	D
C	X	D
D	X	A
X*	X	Y
Y*	X	Y

X, Y → Z

Σ / Q	0	1
A	B	A
B	C	D
C	Z	D
D	Z	A
Z*	Z	Z



Points to remember

1. NFA to DFA method remains the same irrespective of NFA is with ϵ or w/o ϵ
2. If you are asked to design a FA then always design DFA
3. always minimize the DFA.