Question Bank in Complex Variables

5 marks Questions

1. If $f(z) = qx^2y + 2x^2 + ry^3 - 2y^2 - i(px^3 - 4xy - 3xy^2)$ is analytic, find p, q, r

2. Find the constants a, b, c, d, e such that the following function is analytic:
   $f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$

3. Determine the constants a, b, c, d if
   $f(z) = (x^2 + 2axy + by^2) + i(cx^2 + 2dxy + y^2)$ is analytic.

4. Find the constants a, b, c, d, e such that the following function is analytic:
   $f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y + exy^3 - 4xy)$

5. If $f(z) = \sinh x \cos y + i \cosh x \sin y$ is analytic, find $f'(z)$ in terms of z

6. Show that $u = \sin x + 3x^2 - y^2 + 5y + 4$ cannot be the real part of an analytic function
   (or Check if there exists an analytic function whose real part is $u = \sin x + 3x^2 - y^2 + 5y + 4$.
   Justify your answer.)

7. Determine a, b such that the function $ax^3y - bxy^3$ is harmonic:

8. Prove that the following function is harmonic:
   $e^{2x}(x \cos 2y - y \sin 2y)$

9. Is $u = e^{-x} \cos y + e^{-y} \sin x$ harmonic?

10. Check whether the function $f(x, y) = 3x^2y - y^3$ is harmonic.

6 marks Questions

1. Construct an analytic function whose real part is
   $e^{-x} \cos y$

2. Construct an analytic function whose real part is
   $e^{-x}(x \sin y - y \cos y)$

3. Find the analytic function $f(z)$ whose imaginary part is $e^x \cos y + x^3 - 3xy^2$.

4. Find the analytic function $f(z) = u + iv$ whose imaginary part is $v = x^2 - y^2 + \dfrac{x}{x^2 + y^2}$.

5. Find an analytic function $f(z) = u + iv$ where $u + v = e^x(\cos y + \sin y)$

6. Find an analytic function $f(z) = u + iv$ such that
   $u + v = \dfrac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$

7. Find the analytic function $f(z) = u + iv$ where $u + v = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$, using Milne-Thompson's Method.

8. Find an analytic function $f(z) = u + iv$ where $u + v = e^x(\cos y + \sin y)$

9. Construct an analytic function $f(z) = u + iv$ if
$u - v = (x - y)(x^2 + 4xy + y^2)$

10. Construct an analytic function $f(z) = u + iv$ such that $u - v = e^{2x}(x \cos 2y - y \sin 2y)$

11. Find the orthogonal trajectories of the family of curves $3x^2y + 2x^2 - y^3 - 2y^2 = c$.

12. Determine the Harmonic Conjugate of u if $u + iv$ is analytic,
and $u = 3x^2y - y^3$.

13. If $v = e^x \sin y$, prove that v is a harmonic function.
Also find the corresponding harmonic conjugate.

14. Show that the function $u = \frac{1}{2}\log(x^2+y^2)$ is harmonic and find its corresponding analytic function and its harmonic conjugate.

15. Obtain the orthogonal trajectories of $3x^2y - y^3 = c$

16. Show that $u = (x - y)(x^2 + 4xy + y^2)$ is harmonic. Find its harmonic conjugate

17. Obtain the orthogonal trajectories of $e^x(x \cos y - y \sin y) = k$

8 marks Questions

1. Find the orthogonal trajectories of the family of curves given by $e^x \cos y - xy = c$

2. Show that the function $u = \sin x \cosh y + 2 \cos x \sinh y + x^2 - y^2 + 4xy$
satisfies Laplace's equation, also find the corresponding analytic function and the harmonic conjugate.

3. Find the orthogonal trajectories of the family of curves $e^{-x}(x \sin y - y \cos y) = c$

4. Find the orthogonal trajectories of the family of curves $3x^2y + 2x^2 - y^3 - 2y^2 = c$.

5. If $v = 3x^2y + 6xy - y^3$, show that v is harmonic and find the corresponding analytic function and the harmonic conjugate.

6. Show that the function $v = tan^{-1}\frac{y}{x}$ is harmonic. Find the corresponding analytic function and the harmonic conjugate.