Question Bank in Fourier Series

5 marks Questions

1. Find a Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$

2. Obtain half range Sine Series for $f(x) = x^2$, in $0 < x < 3$.

3. Find a Fourier series of $f(x) = x^2$ in $(0, a)$

4. Find a Fourier series of $f(x) = x$ in $(-1, 1)$

5. Find the Fourier series of $f(x) = x \cdot \cos x$ in $(-\pi, \pi)$

6. Find the half range Fourier Sine series of $f(x) = x^3, \quad -\pi < x < \pi$
   (Ans: $b_n = 2(-1)^n(\frac{6}{n^3} - \frac{\pi^2}{n})$)

7. Find the half range cosine series of $f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ x, & 1 \leq x \leq 2 \end{cases}$

8. Find the half range sine series of $f(x) = e^{ax}$ in $(0, \pi)$

9. Find the half range Fourier sine series for $f(x) = x^2 + 1$ where $x \; \epsilon \; (-\pi, \pi)$.

6 marks Questions

1. Determine the Half range sine series for $f(x) = \dfrac{x(\pi^2 - x^2)}{12}$, where $0 < x < \pi$.

2. Find the Fourier series of
   $f(x) = \frac{1}{2}(\pi - x)$ in $(0, 2\pi)$
   Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...$, given $f(x + 2\pi) = f(x)$.

3. Find the Fourier series of $f(x) = \sqrt{1 - \cos x}$ in $(0, 2\pi)$.
   Hence deduce that $\frac{1}{2} = \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$

4. Find the Fourier series of $f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$.
   Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...$

5. Find the Fourier series of $f(x) = |x|$ in the interval $[-\pi, \pi]$ and hence deduce that
   $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + - - -$

6. Find the Fourier series of function $f(x) = |\cos x|$ in $(-\pi, \pi)$

7. Find the Fourier series of $f(x) = \begin{cases} 0 & -5 < x < 0 \\ 3 & 0 < x < 5 \end{cases}$

8. Obtain the Fourier series of $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$

   (Ans: $f(x)$ is even, therefore $b_n = 0$ Then, $a_0 = 0,\ a_n = \dfrac{4}{n^2\pi^2}(1 - (-1)^n)$)

9. Find Half range sine series of function $f(x) = lx - x^2$ in $(0, l)$
   Hence, deduce that, $\dfrac{\pi^3}{32} = \dfrac{1}{1^3} - \dfrac{1}{3^3} + \dfrac{1}{5^3} - \dfrac{1}{7^3} + ...$

10. Obtain the Fourier series to represent $f(x) = 9 - x^2$ in $(-3, 3)$

11. Find the Fourier series of $f(x)$ in $(0, 2\pi)$ where $f(x) = \begin{cases} x, & 0 < x \le \pi \\ 2\pi - x, & \pi < x \le 2\pi \end{cases}$

    Hence deduce that $\dfrac{\pi^4}{96} = \dfrac{1}{1^4} + \dfrac{1}{3^4} + \dfrac{1}{5^4} + ...$

8 marks Questions

1. Find the Fourier series of $f(x) = x^2$ in (0,4).
   Hence deduce that $\dfrac{\pi^2}{6} = \dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + ...$

2. Find a Fourier series in $(-\pi, \pi)$ of $f(x) = \begin{cases} x + \frac{\pi}{2} & -\pi \le x \le 0 \\ \frac{\pi}{2} - x & 0 \le x \le -\pi \end{cases}$

   and hence deduce that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$ *and* $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}$

3. Find the Fourier series of $f(x)$ in $(0, 2\pi)$ where $f(x) = \begin{cases} x, & 0 < x \le \pi \\ 2\pi - x, & \pi < x \le 2\pi \end{cases}$

   Hence deduce that $\dfrac{\pi^4}{96} = \dfrac{1}{1^4} + \dfrac{1}{3^4} + \dfrac{1}{5^4} + ...$

4. Find the Fourier series of $f(x) = x \cdot \sin x$ in the interval $(0, 2\pi)$

5. Find the Half Range Cosine Series for $f(x) = x,\ 0 < x < 2$.
   Using Parseval's identity deduce that
   (i) $\dfrac{\pi^4}{96} = \dfrac{1}{1^4} + \dfrac{1}{3^4} + \dfrac{1}{5^4} + \cdots$
   (ii) $\dfrac{\pi^4}{90} = \dfrac{1}{1^4} + \dfrac{1}{2^4} + \dfrac{1}{3^4} + \dfrac{1}{4^4} + \dfrac{1}{5^4} + \cdots$

6. Obtain the expansion of $f(x) = x(\pi - x),\ 0 < x < \pi$ as a Half Range Cosine Series. Hence show that
   (i) $\displaystyle\sum_{1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ (ii) $\displaystyle\sum_{1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$ (iii) $\displaystyle\sum_{1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{96}$

7. Determine the Fourier series of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ over $(0, 2\pi)$.

   Hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + ... = \frac{\pi^2}{6}$

8. Obtain the Fourier expansion for $f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2 - x), & 1 \le x \le 2 \end{cases}$

   and hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + - - -$

9. Find a Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$ and hence prove that

   (i) $\frac{\pi^2}{6} = \sum_{1}^{\infty} \frac{1}{n^2}$ (ii) $\frac{\pi^2}{12} = \sum_{1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

   (iii) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ (iv) $\frac{\pi^4}{90} = \sum_{n=1}^{\infty} \frac{1}{n^4}$

10. Find a Fourier series in $(-\pi, \pi)$ of $f(x) = \begin{cases} x + \pi/2, & -\pi < x < 0 \\ \pi/2 - x, & 0 < x < \pi \end{cases}$.

    and hence deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$ and $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}$

11. Find a Cosine Series of period 2π to represent $\sin x$ in $0 \le x \le \pi$. Hence deduce that

    (i) $\frac{\pi^2 - 8}{16} = \frac{1}{1^2 \cdot 3^2} + \frac{1}{3^2 \cdot 5^2} + \frac{1}{5^2 \cdot 7^2} + \cdots$

    (ii) $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

    (iii) $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots = \frac{1}{2}$