231456
SE - IT Engg

Assignment : 06

Q1] Find the mean and variance of the following distribution.

x :	1	3	4	5
$P(x)$:	0.4	0.1	0.2	0.3

mean = $E(x) = \sum p_i x$

$= (0.4\times1) + (0.1\times3) + (0.2\times4) + (0.3\times5)$

$= 0.4 + 0.3 + 0.8 + 1.5$

$\boxed{E(x) = \text{mean} = 3}$

variance = $E(x^2) - [E(x)]^2$

$E(x^2) = (0.4\times1^2) + (0.1\times3^2) + (0.2\times4^2) + (0.3\times5^2)$

$= 0.4 + 0.9 + 3.2 + 7.5$

$E(x^2) = 12$

variance = $V(x) = E(x^2) - [E(x)]^2$

$12 - (3)^2$

$12 - 9$

$\boxed{V(x) = 3}$

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Q2] find k, mean and variance of the following distribution
$f(x) = k(1-x^2)$, $0 < x < 1$.

<u>For k:</u>

$$\int_0^1 k(1-x^2)\,dx = 1$$

$$k\int_0^1 (1-x^2)\,dx = 1$$

$$k\cdot\int_0^1 x - \frac{x^3}{3} = 1$$

$$k\cdot\left[x - \frac{x^3}{3}\right]_0^1$$

$$k\cdot\left[1 - \frac{1}{3}\right] = 1$$

$$k\cdot\frac{2}{3} = 1$$

$$\boxed{k = 1.5}$$

mean:
$$\int_0^1 x\cdot f(x)\,dx$$

$$k\int_0^1 x\cdot(1-x^2)\,dx$$

$$k\int_0^1 x - x^3\,dx$$

$$k\cdot\int_0^1 \frac{x^2}{2} - \frac{x^4}{4}$$

$$k\cdot\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$$

$$k\cdot 1/4$$

$$\boxed{E(x) = \frac{k}{4}} = \frac{3}{8}.$$

$$E(x^2) = k\cdot\int_0^1 x^2\cdot f(x)\,dx$$

$$E(x^2) = k\cdot\int_0^1 x^2(1-x^2)\,dx$$

$$= k\cdot\int_0^1 x^2 - x^4\,dx$$

$$k\cdot\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1$$

$$k\cdot\left[\frac{1}{3} - \frac{1}{5}\right]$$

$$k\cdot\frac{2}{15} \quad E(x^2) = \frac{1}{5}$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$\frac{1}{5} - \left(\frac{3}{8}\right)^2 = 0.0593$$

~~$f(x) = k($~~

Q3] A r.v. x has mgf $m(t) = \frac{3}{3-t}$ find mean & σ

Soln]: $E(x) = \mu_1' = \left[\frac{d}{dt}(M(t)\right)$

$3\left(-\frac{1}{(3-t)^2} \times (-1)\right)$

$= \frac{3}{(3-t)^2}$

for mean put $t=0$ above.

$E(x) \cdot \frac{3}{9-0} = \frac{\cancel{3}\,1}{\cancel{9}\,3} = \frac{1}{3} = \mu_1'$

$\sigma = \sqrt{v(x)}$

$v(x) = E(x^2) - [(Ex)]^2$

$v(x) = \mu_2 = (\mu_2' - (\mu_1')^2$

$\frac{d^2}{dt^2}(Mt) - \frac{1}{9}$

$3\left(-\frac{1}{(3-t)^4}\right. \; 2(3-t)(-1) - \frac{1}{9}$

$\frac{6\cancel{(3-t)}}{(3-\cancel{t})^4} - \frac{1}{9}$

$6 \times \frac{1}{(3-t)^3} - 1/9 = \frac{6}{27} - \frac{1}{9}$

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$v(x) = \frac{1}{9}$ $\therefore \sigma = \sqrt{1/9}$ $\boxed{= \sigma = 1/3}$

Q4] A r.v. x takes place 1, 2, 3, 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$ find $P(X=1)$ and mgf.

Let us assume $P(X=3) = K$

$2P(X=1) = K$

$P(X=1) = \frac{K}{2}$

$3P(X=2) = K$

$P(X=2) = \frac{K}{3}$

$5P(X=4) = K$

$P(X=4) = K/5$.

According to the probability defination sum of all probabilities = 1.

$\therefore \frac{K}{2} + \frac{K}{3} + K + \frac{K}{5} = 1$

$\frac{61K}{30} = 1$

$$\boxed{K = \frac{30}{61} = 0.4918}$$

$2P(X=1) = K =$

$P(X=1) = K/2 = \frac{30}{61 \times 2} = 0.2459$

mgf of x is $M(t) = E(e^{tx})$

$= \sum p_i e^{tx_i}$

$e^t \frac{15}{61} + e^{2t} \frac{10}{61} + e^{3t} \frac{30}{61} + e^{4t} \frac{6}{61}$

$$\boxed{M(t) = \frac{15e^t + 10e^{2t} + 30e^{3t} + 6e^{4t}}{61}}$$ Req Solution.

Q5] The first 4 moments of a distribution about origin of the random variable x are -1.5, 17, -30, 108, calculate mean, variance, μ_3 and μ_4.

$\mu_1' = -1.5$

$\mu_2' = 17$

$\mu_3' = -30$

$\mu_4' = 108$

The first moment of distribution is -1.5

$$\boxed{\text{mean}(x) = \mu_1' = -1.5}$$

$$V(x) = \mu_2 = \mu_2' - \mu_1'^2$$
$$= 17 - (-1.5)^2$$

$$\boxed{V(x) = 14.75}$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$
$$-30 - 3(17)(-1.5) + 2(-1.5)^3$$

$$\boxed{\mu_3 = 39.75}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_2' + 6\mu_2'\mu_1^2 - 3\mu_1^4$$

$$\mu_4 = 108 - 4(-30)(-1.5) + 6(17)(-1.5)^2 - 3(-1.5)^4$$

$$\boxed{\mu_4 = 142.31}$$

-x-x-x

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