[NLPP] - REVISION

(11) MAX, $Z = 6x_1 + 8x_2 - x_1^2 - x_2^2$

SUB, $4x_1 + 3x_2 = 16$

$3x_1 + 5x_2 = 15$

WHERE, $x_1, x_2 \geq 0$

LET, $f = 6x_1 + 8x_2 - x_1^2 - x_2^2$

LET, $h_1 = 4x_1 + 3x_2 - 16$

$h_2 = 3x_1 + 5x_2 - 15$

BY LANGRAGE'S MULTIPLIERS,

$L = f - \lambda_1 . h_1 - \lambda_2 h_2$

$L = 6x_1 + 8x_2 - x_1^2 - x_2^2 -$
$\lambda_1 [4x_1 + 3x_2 - 16] -$
$\lambda_2 [3x_1 + 5x_2 - 15]$

$L_{x_1} \Rightarrow 0$

$6 - 2x_1 - 4\lambda_1 - 3\lambda_2 = 0$

$2x_1 + 4\lambda_1 + 3\lambda_2 = 6$ — (1)

$L_{x_2} \Rightarrow 0$

$8 - 2x_2 - 3\lambda_1 - 5\lambda_2 = 0$

$2x_2 + 3\lambda_1 + 5\lambda_2 = 8$ — (2)

$L_{\lambda_1} \Rightarrow 0$

$-4x_1 - 3x_2 + 16 = 0$

$4x_1 + 3x_2 = 16$ — (3)

$L_{\lambda_2} \Rightarrow 0$

$-3x_1 - 5x_2 + 15 = 0$

$3x_1 + 5x_2 = 15$ — (4)

$x_1 = \frac{35}{11}, \quad x_2 = \frac{12}{11}$

$Z_{max} = 6\left(\frac{35}{11}\right) + 8\left(\frac{12}{11}\right) - \left(\frac{35}{11}\right)^2 - \left(\frac{12}{11}\right)^2$

$Z_{max} = \frac{210}{11} + \frac{96}{11} + -\frac{1225}{121} - \frac{144}{121}$

$\rightarrow \boxed{Z_{max} = 16.504}$

(14) OPTIMIZE, $Z = x_1^2 + x_2^2 + x_3^2$

SUB, $x_1 + x_2 + 3x_3 = 2$

$5x_1 + 2x_2 + x_3 = 5$

WHERE $x_1, x_2, x_3 \geq 0$

LET, $f = x_1^2 + x_2^2 + x_3^2$

LET, $h_1 = x_1 + x_2 + 3x_3 - 2$

$h_2 = 5x_1 + 2x_2 + x_3 - 5$

BY LAGRANGE'S MULTIPLIERS,

$L = f - \lambda_1 . h_1 - \lambda_2 . h_2$

$L = x_1^2 + x_2^2 + x_3^2 -$
$\lambda_1 [x_1 + x_2 + 3x_3 - 2] -$
$\lambda_2 [5x_1 + 2x_2 + x_3 - 5]$

$L_{x_1} \Rightarrow 0$

$2x_1 - \lambda_1 - 5\lambda_2 = 0$ — (1)

$L_{x_2} \Rightarrow 0$

$2x_2 - \lambda_1 - 2\lambda_2 = 0$ — (2)

$L_{x_3} \Rightarrow 0$

$2x_3 - 3\lambda_1 - \lambda_2 = 0$ — (3)

$L_{\lambda_1} \Rightarrow 0$

$-x_1 - x_2 - 3x_3 + 2 = 0$

$x_1 + x_2 + 3x_3 = 2$ — (4)

$L_{\lambda_2} \Rightarrow 0$

$-5x_1 - 2x_2 - x_3 + 5 = 0$

$5x_1 + 2x_2 + x_3 = 5$ — (5)

$x_1 = \frac{\lambda_1 + 5\lambda_2}{2}$

$x_2 = \frac{\lambda_1 + 2\lambda_2}{2}$

$x_3 = \frac{3\lambda_1 + \lambda_2}{2}$

PUT x_1, x_2, x_3 IN (4) & (5)

$\frac{\lambda_1 + 5\lambda_2}{2} + \frac{\lambda_1 + 2\lambda_2}{2} + \frac{9\lambda_1 + 3\lambda_2}{2} = 2$

$\frac{11\lambda_1 + 10\lambda_2}{2} = 2$

$11\lambda_1 + 10\lambda_2 = 4$ — (6)

$\frac{5\lambda_1 + 25\lambda_2}{2} + \frac{2\lambda_1 + 4\lambda_2}{2} + \frac{3\lambda_1 + \lambda_2}{2} = 5$

$\frac{10\lambda_1 + 30\lambda_2}{2} = 5$

$10\lambda_1 + 30\lambda_2 = 10$ — (7)

$\lambda_1 = \frac{2}{23}, \quad \lambda_2 = \frac{7}{23}$

$x_1 = \frac{\frac{2}{23} + 5\left(\frac{7}{23}\right)}{2} = \frac{37}{46}$

$x_2 = \frac{\frac{2}{23} + 2\left(\frac{7}{23}\right)}{2} = \frac{8}{23}$

$x_3 = \frac{3\left(\frac{2}{23}\right) + \frac{7}{23}}{2} = \frac{13}{46}$

$H^B = \begin{bmatrix} O & P \\ P' & Q \end{bmatrix}$

$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$P = \begin{bmatrix} -1 & -1 & -3 \\ -5 & -2 & -1 \end{bmatrix}$

$$P' = \begin{bmatrix} -1 & -5 \\ -1 & -2 \\ -3 & -1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$H^B = \begin{bmatrix} 0 & 0 & -1 & -1 & -3 \\ 0 & 0 & -5 & -2 & -1 \\ -1 & -5 & 2 & 0 & 0 \\ -1 & -2 & 0 & 2 & 0 \\ -3 & -1 & 0 & 0 & 2 \end{bmatrix}$$

— COFACTOR EXPANSION

$1 \cdot C_4 - 1 \cdot C_3$

$1 \cdot C_5 - 3 \cdot C_3$

$$H^B = \begin{bmatrix} 0 & 0 & \cancel{0} & -1 & 0 & 0 \\ 0 & 0 & \cancel{5} & -5 & 3 & 14 \\ -1 & -5 & \cancel{-2} & 2 & -2 & -6 \\ -1 & -2 & \cancel{2} & 0 & 2 & 0 \\ -3 & -1 & \cancel{0} & 0 & 0 & 2 \end{bmatrix}$$

$$H^B = \begin{bmatrix} 0 & 0 & 3 & 14 \\ -1 & -5 & -2 & -6 \\ -1 & -2 & 2 & 0 \\ -3 & -1 & 0 & 2 \end{bmatrix}$$

$$\Delta = 3 \begin{bmatrix} -1 & -5 & -6 \\ -1 & -2 & 0 \\ -3 & -1 & 2 \end{bmatrix} - 14 \begin{bmatrix} -1 & -5 & -2 \\ -1 & -2 & 2 \\ -3 & -1 & 0 \end{bmatrix}$$

$\Delta = 72 - -532$

$\Delta = 604$

$$Z_{min} = \left(\frac{37}{46}\right)^2 + \left(\frac{8}{23}\right)^2 + \left(\frac{13}{46}\right)^2$$

$\rightarrow$ $\boxed{Z_{min} = 0.8478}$

(8) OPTIMIZE, $Z = 12x_1 + 8x_2 + 6x_3 - x_1^2 - x_2^2 - x_3^2 - 23$

SUBJECT, $x_1 + x_2 + x_3 = 10$

WHERE, $x_1, x_2, x_3 \geq 0$

LET, $f = 12x_1 + 8x_2 + 6x_3 - x_1^2 - x_2^2 - x_3^2 - 23$

LET, $h = x_1 + x_2 + x_3 - 10$

BY LANGRANGE'S MULTIPLIER,

$L = f - \lambda . h$

$L = 12x_1 + 8x_2 + 6x_3 - x_1^2 - x_2^2 - x_3^2 - 23$
$\quad - \lambda \left[x_1 + x_2 + x_3 - 10\right]$

$(x_1 \Rightarrow 0$

$12 - 2x_1 - \lambda = 0$

$\therefore \quad x_1 = \dfrac{-\lambda + 12}{2}$

$L_{x_2} \Rightarrow 0$

$8 - 2x_2 - \lambda = 0$

$\therefore \quad x_2 = \dfrac{-\lambda + 8}{2}$

$L_{x_3} \Rightarrow 0$

$6 - 2x_3 - \lambda = 0$

$\therefore \quad x_3 = \dfrac{-\lambda + 6}{2}$

$L_{\lambda} \Rightarrow 0$

$-x_1 - x_2 - x_3 = -10$

$x_1 + x_2 + x_3 = 10$

$\therefore \quad \dfrac{-\lambda + 12}{2} + \dfrac{-\lambda + 8}{2} + \dfrac{-\lambda + 6}{2} = 10$

$\therefore \quad \dfrac{-3\lambda + 26}{2} = 10$

$\therefore \quad -3\lambda + 26 = 20$

$\therefore \quad -3\lambda = 20 - 26$

$\therefore \quad +3\lambda = +6$

$\therefore \quad \lambda = \dfrac{6}{3}$

$\therefore \quad \lambda = 2$

$x_1 = \dfrac{-\lambda + 12}{2} = \dfrac{-2 + 12}{2} = 5$

$x_2 = \dfrac{-\lambda + 8}{2} = \dfrac{-2 + 8}{2} = 3$

$x_3 = \dfrac{-\lambda + 6}{2} = \dfrac{-2 + 6}{2} = 2$

$$H' = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -2 & 0 \\ 1 & 0 & -2 \end{bmatrix}$$

$\Delta' = 4$

$$H'' = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & -2 & 0 \\ 1 & 0 & 0 & -2 \end{bmatrix}$$

$$\Delta'' = 0 \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} - 1 \begin{bmatrix} 1 & 0 & 0 \\ 1 & -2 & 0 \\ 1 & 0 & -2 \end{bmatrix} + 1 \begin{bmatrix} 1 & -2 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & -2 \end{bmatrix} - 1 \begin{bmatrix} 1 & -2 & 0 \\ 1 & 0 & -2 \\ 1 & 0 & 0 \end{bmatrix}$$

$\Delta'' = 0 - 4 - 4 - 4 = -12$

Δ' IS POSITIVE, Δ'' IS NEGATIVE

$\therefore \quad Z_{max} = 12(5) + 8(3) + 6(2) - (5)^2 - (3)^2 - (2)^2 - 23$

$\therefore \quad Z_{max} = 96 - 25 - 9 - 4 - 23$

$\rightarrow \therefore \quad \boxed{Z_{max} = 35}$

Q. MAX, $Z = 12x_1x_2 + 2x_1^2 - 7x_2^2$
SUB, $2x_1 + 5x_2 \leq 98$
WHERE, $x_1, x_2 \geq 0$

BY LANGRANGE'S MULTIPLIER,

$L = f - \lambda . h$

$L = 12x_1x_2 + 2x_1^2 - 7x_2^2 - \lambda \boxed{2x_1 + 5x_2 - 98}$

AS PER KUHN TUCKER CONDITIONS,

$L_{x_1} = 0$,
$\Rightarrow 12x_2 + 4x_1 - 2 = 0$
$\cancel{4x_1 + 12x_2 = 2} \quad — (1)$

$L_{x_2} = 0$
$\Rightarrow 12x_1 - 14x_2 - 5 = 0$
$\cancel{12x_1 - 14x_2 = 5} \quad — (2)$

$\lambda . h = 0$
$\Rightarrow \lambda \boxed{2x_1 + 5x_2 - 98} = 0 \quad — (3)$

$h \leq 0$
$\Rightarrow 2x_1 + 5x_2 - 98 \leq 0 \quad — (4)$

$x_1, x_2, \lambda \geq 0$

[CASE 1] $\lambda = 0$

$4x_1 + 12x_2 = 2$
$12x_1 - 14x_2 = 5$

$x_1 = \frac{11}{28}$, $x_2 = \frac{1}{28}$

$2\left(\frac{11}{28}\right) + 5\left(\frac{1}{28}\right) - 98 \leq 0$

$4x_1 + 12x_2 - 2\lambda = 0 \quad — (1)$
$12x_1 - 14x_2 - 5\lambda = 0 \quad — (2)$

[CASE 1] $\lambda = 0$

$x_1 = 0$, $x_2 = 0$

$2(0) + 5(1) - 98 \leq 0$

CONDITION SATISFIED

$Z_{Max} = 0$

[CASE 2] $\lambda \neq 0$

$2x_1 + 5x_2 + 0\lambda = 98 \quad — (4)$

$x_1 = 44$, $x_2 = 12$

$2(44) + 5(2) - 98 \leq 0$

CONDITION SATISFIED

$Z_{max} = 4900$

$\boxed{\text{OPTIMAL } Z_{max} = 4900}$

Q. MAX, $Z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$
SUB, $2x_1 + x_2 - 5 \leq 0$
WHERE, $x_1, x_2 \geq 0$

LET $f = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$
LET $h = 2x_1 + x_2 - 5$

BY LAGRANGE'S MULTIPLIERS,

$L = f - \lambda . h$

$L = 10x_1 + 4x_2 - 2x_1^2 - x_2^2 - \lambda[2x_1 + x_2 - 5]$

BY KUHN TUCKER CONDITIONS,

$L_{x_1} = 0$

$\Rightarrow 10 - 4x_1 - 2\lambda = 0$

$-4x_1 - 2\lambda = -10$

$4x_1 + 2\lambda = 10$ — (1)

$L_{x_2} = 0$

$\Rightarrow 4 - 2x_2 - \lambda = 0$

$-2x_2 - \lambda = -4$

$2x_2 + \lambda = 4$ — (2)

$\lambda . h = 0$

$\Rightarrow \lambda[2x_1 + x_2 - 5] = 0$ — (3)

$h \leq 0$

$\Rightarrow 2x_1 + x_2 - 5 \leq 0$ — (4)

$x_1, x_2, \lambda \geq 0$

$\boxed{\text{CASE 1}}$ — $\lambda = 0$

$4x_1 + 0.x_2 = 10$ — (5)

$0.x_1 + 2x_2 = 4$ — (6)

$x_1 = \frac{5}{2}, \quad x_2 = 2$

$2\left(\frac{5}{2}\right) + 2 - 5 \leq 0$

$2 \leq 0$

— CONDITION NOT SATISFIED

$\boxed{\text{CASE 2}}$ — $\lambda \neq 0$

$4x_1 + 0.x_2 + 2\lambda = 10$ — (7)

$0.x_1 + 2x_2 + \lambda = 4$ — (8)

$2x_1 + x_2 + 0\lambda = 5$ — (9)

$x_1 = \frac{11}{6}, \; x_2 = \frac{4}{3}, \; \lambda = \frac{4}{3}$

$2\left(\frac{11}{6}\right) + \frac{4}{3} - 5 \le 0$

$0 \le 0$

— CONDITION SATISFIED

$Z_{max} = 10\left(\frac{11}{6}\right) + 4\left(\frac{4}{3}\right) - 2\left(\frac{11}{6}\right)^2 - \left(\frac{4}{3}\right)^2$

$\rightarrow \boxed{Z_{max} = \frac{91}{6}}$

Q. MAX, $Z = 4x_1 + 6x_2 - x_1^2 - x_2^2 - x_3^2$

SUB, $x_1 + x_2 \le 2$

$2x_1 + 3x_2 \le 12$

WHERE, $x_1, x_2 \ge 0$

LET, $f = 4x_1 + 6x_2 - x_1^2 - x_2^2 - x_3^2$

LET, $h_1 = x_1 + x_2 - 2$

$h_2 = 2x_1 + 3x_2 - 12$

BY LAGRANGE'S MULTIPLIER,

$L = f - \lambda_1 . h_1 - \lambda_2 . h_2$

$L = 4x_1 + 6x_2 - x_1^2 - x_2^2 - x_3^2 - \lambda_1 . [x_1 + x_2 - 2] - \lambda_2 [2x_1 + 3x_2 - 12]$

BY KUHN TUCKER CONDITIONS,

$L_{x_1} = 0$

$\Rightarrow 4 - 2x_1 - \lambda_1 - 2\lambda_2 = 0$

$-2x_1 - \lambda_1 - 2\lambda_2 = -4$

$2x_1 + \lambda_1 + 2\lambda_2 = 4$

$L_{x_2} = 0$

$\Rightarrow 6 - 2x_2 - \lambda_1 - 3.\lambda_2 = 0$

$-2x_2 - \lambda_1 - 3\lambda_2 = -6$

$2x_2 + \lambda_1 + 3\lambda_2 = 6$

$L_{x_3} = 0$

$\Rightarrow 2x_3 = 0$

$\lambda_1 . h_1 = 0$

$\Rightarrow \lambda_1 [x_1 + x_2 - 2] = 0$

$\lambda_2 . h_2 = 0$

$\Rightarrow \lambda_2 [2x_1 + 3x_2 - 12] = 0$

$h_1 \le 0$

$\Rightarrow x_1 + x_2 - 2 \le 0$ — $\boxed{\text{CONDITION 1}}$

$h_2 \le 0$

$\Rightarrow 2x_1 + 3x_2 - 12 \le 0$ — $\boxed{\text{CONDITION 2}}$

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$x_1, x_2, x_3, \lambda_1, \lambda_2 \geq 0$ — $\boxed{\text{CONDITION 3}}$

$\boxed{\text{CASE 1}}$ — $\lambda_1 = 0, \quad \lambda_2 = 0$

$$2x_1 + 0x_2 + 0x_3 = 4 \rightarrow (1)$$
$$0x_1 + 2x_2 + 0x_3 = 6 \quad — (2)$$
$$0x_1 + 0x_2 + 2x_3 = 0 \quad — (3)$$

ON SOLVING (1), (2), (3)

$x_1 = 2, \quad x_2 = 3, \quad x_3 = 0$

$$x_1 + x_2 - 2 \leq 0$$
$$2 + 3 - 2 \leq 0$$
$$3 \leq 0$$

— CONDITION 1 NOT SATISFIED

$\boxed{\text{CASE 2}}$ — $\lambda_1 \neq 0, \quad \lambda_2 = 0$

$$2x_1 + 0x_2 + \lambda_1 = 4 \quad — (4)$$
$$0x_1 + 2x_2 + \lambda_1 = 6 \quad — (5)$$
$$x_1 + x_2 + 0\lambda_1 = 2 \quad — (6)$$

ON SOLVING (4), (5), (6)

$x_1 = \frac{1}{2}, \quad x_2 = \frac{3}{2}, \quad \lambda_1 = 3$

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$$x_1 + x_2 - 2 \leq 0$$
$$\frac{1}{2} + \frac{3}{2} - 2 \leq 0$$
$$0 \leq 0$$

— CONDITION 1 SATISFIED

$$2x_1 + 3x_2 - 12 \leq 0$$
$$2\left(\frac{1}{2}\right) + 3\left(\frac{3}{2}\right) - 12 \leq 0$$
$$\frac{-13}{2} \leq 0$$

— CONDITION 2 SATISFIED

$$Z_{max} = 4\left(\frac{1}{2}\right) + 6\left(\frac{3}{2}\right) - \left(\frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 0$$

$$Z_{max} = \frac{17}{2} = 8.5$$

$\boxed{\text{CASE 3}}$ — $\lambda_1 = 0, \quad \lambda \neq 0$

$$2x_1 + 0x_2 + 2\lambda = 4 \quad — (7)$$
$$0x_1 + 2x_2 + 3\lambda = 6 \quad — (8)$$
$$2x_1 + 3x_2 + 0\lambda_2 = 12 \quad — (9)$$

ON SOLVING (7), (8), (9)

$x_1 = \frac{24}{13}, \quad x_2 = \frac{36}{13}, \quad \lambda_2 = \frac{2}{13}$

$x_1 + x_2 - 2 \le 0$

$\frac{24}{13} + \frac{36}{13} - 2 \le 0$

$\frac{34}{13} \le 0$

— CONDITION 1 NOT SATISFIED

$\boxed{\text{CASE 4}}$ — $\lambda_1 \neq 0, \lambda_2 \neq 0$

$x_1 + x_2 = 2$ — (10)

$2x_1 + 3x_2 = 12$ — (11)

ON SOLVING (10), (11)

$x_1 = -6, \; x_2 = 8$

x_1 IS LESS THAN 0

— CONDITION 3 NOT SATISFIED

→ OPTIMAL $\boxed{Z_{max} = \frac{17}{2} = 8.5}$

Q. MAX, $Z = 10x_1 + 10x_2 - x_1^2 - x_2^2$

SUB, $x_1 + x_2 \le 8$

$-x_1 + x_2 \le 5$

WHERE, $x_1, x_2 \ge 0$

LET, $f = 10x_1 + 10x_2 - x_1^2 - x_2^2$

LET, $h_1 = x_1 + x_2 - 8$

$h_2 = -x_1 + x_2 - 5$

BY LAGRANGE'S MULTIPLIER,

$L = f - \lambda_1 h_1 + \lambda_2 h_2$

$L = 10x_1 + 10x_2 - x_1^2 - x_2^2 - \lambda_1 (x_1 + x_2 - 8) - \lambda_2 (-x_1 + x_2 - 5)$

BY KUHN-TUCKER CONDITIONS,

$L_{x_1} = 0$

$\Rightarrow 10 - 2x_1 - \lambda_1 + \lambda_2 = 0$

$-2x_1 - \lambda_1 + \lambda_2 = -10$

$2x_1 + \lambda_1 - \lambda_2 = 10$

$L_{x_2} = 0$

$\Rightarrow 10 - 2x_2 - \lambda_1 - \lambda_2 = 0$

$-2x_2 - \lambda_1 - \lambda_2 = -10$

$2x_2 + \lambda_1 + \lambda_2 = 10$

$\lambda_1 \cdot h_1 = 0$

$\Rightarrow \lambda_1 \boxed{x_1 + x_2 - 8} = 0$

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$\lambda_2 . h_2 = 0$

$\Rightarrow \lambda_2 (-x_1 + x_2 - 5)$

$h_1 \le 0$

$\Rightarrow x_1 + x_2 - 8 \le 0$ — $\boxed{\text{CONDITION } 1}$

$h_2 \le 0$

$\Rightarrow -x_1 + x_2 - 5 \le 0$ — $\boxed{\text{CONDITION } 2}$

$x_1, x_2, \lambda_1, \lambda_2 \ge 0$ — $\boxed{\text{CONDITION } 3}$

$\boxed{\text{CASE } 1}$ — $\lambda_1 = 0, \; \lambda_2 = 0$

$2x_1 + 0x_2 = 10$ — 1

$0x_1 + 2x_2 = 10$ — 2

SOLVING 1, 2

$x_1 = 5, \quad x_2 = 5$

$x_1 + x_2 - 8 \le 0$

$5 + 5 - 8 \le 0$

$2 \le 0$

— CONDITION 1 NOT SATISFIED

$\boxed{\text{CASE } 2}$ — $\lambda_1 \ne 0, \; \lambda_2 = 0$

$2x_1 + 0x_2 + \lambda_1 = 10$ — 3

$0x_1 + 2x_2 + \lambda_1 = 10$ — 4

$x_1 + x_2 + 0\lambda = 8$ — 5

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ON SOLVING 3, 4, 5

$x_1 = 4, \quad x_2 = 4, \quad \lambda_1 = 2$

$x_1 + x_2 - 8 \le 0$

$4 + 4 - 8$

$0 \le 0$

— CONDITION 1 SATISFIED

$-x_1 + x_2 - 5 \le 0$

$-4 + 4 - 5 \le 0$

$-5 \le 0$

— CONDITION 2 SATISFIED

$Z_{max} = 10.(4) + 10.(4) - (4^2) - (4^2)$

$Z_{max} = 48$

$\boxed{\text{CASE } 3}$ — $\lambda_1 = 0, \; \lambda_2 \ne 0$

$2x_1 + 0x_2 - \lambda_2 = 10$ — 6

$0x_1 + 2x_2 + \lambda_2 = 10$ — 7

$-x_1 + x_2 + 0\lambda_2 = 5$ — 8

ON SOLVING 6, 7, 8

$x_1 = \frac{5}{2}, \quad x_2 = \frac{15}{2}, \quad \lambda_2 = -5$

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$x_1 + x_2 - 8 \le 0$

$\frac{5}{2} + \frac{15}{2} - 8 \le 0$

$2 \le 0$

— CONDITION 1 NOT SATISFIED

$\boxed{\text{CASE } 4}$ — $\lambda_1 \neq 0$, $\lambda_2 \neq 0$

$x_1 + x_2 = 8$ — (9)

$-x_1 + x_2 = 5$ — (10)

ON SOLVING (9), (10)

$x_1 = \frac{3}{2}$, $x_2 = \frac{13}{2}$

$x_1 + x_2 - 8 \le 0$

$\frac{3}{2} + \frac{13}{2} - 8 \le 0$

$0 \le 0$

— CONDITION 1 SATISFIED

$-x_1 + x_2 - 5 \le 0$

$-\frac{3}{2} + \frac{13}{2} - 5 \le 0$

$0 \le 0$

— CONDITION 2 SATISFIED

$\rightarrow$ $Z_{max} = 7/2$, $\boxed{\text{OPTIMAL } Z_{max} = 48}$