231456

DATE:

Assignment → 04

Q1] show that $f(z) = z^2 + z$ is analytic also find $f'(z)$

$f(z) = z^2 + z$

$(x+iy)^2 + (x+iy)$

$x^2 + 2xyi - y^2 + x + iy$

$f(z) = x^2 - y^2 + x + (2xy + y)i$

$u = x^2 - y^2 + x$	$v = 2xy + y$
$u_x = 2x + 1$	$v_y = 2x + 1$
$u_y = -2y$	$v_x = 2y$

$u_x = v_y$ → condition 1 satisfied

$u_y = -v_x$ → condition 2 satisfied.

∴ It is analytic.

$f'(z) = u_x + i v_x$

$f'(z) = 2x + 1 - i(-2y)$

put $x = z$, $y = 0$ ∴ By M.T method.

$\boxed{f'(z) = 2x + 1}$

Q2] check whether $f(x) = (x^3 - 3xy^2 + 3x) + i(3x^2y - y^3 + 3y)$ is analytic also find $f'(z)$

$f(x) = (x^3 - 3xy^2 + 3x) + i(3x^2y - y^3 + 3y)$

$u = x^3 - 3xy^2 + 3x$	$v = 3x^2y - y^3 + 3y$
$u_x = 3x^2 - 3y^2 + 3$	$v_x = 6xy$
$u_y = -6xy$	$v_y = 3x^2 - 3y^2 + 3$

$u_y = -v_x$ --- condition 1 satisfied

$u_x = v_y$ --- condition 2 satisfied.

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f(z) is analytic

$f'(z) = u_x + iv_x = 3x^2 - 3y^2 + 3 - i(-6xy)$

put $x = z$, $y = 0$ ∴ By M.T

$3z^2 - 0 + 3 - i(0)$

$\boxed{f'(z) = 3z^2 + 3}$

Q3] find a, b, c, d, e if f(z) is analytic where f(z) = $(ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$

$f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$

$u = ax^3 + bxy^2 + 3x^2 + cy^2 + x$

$u_x = 3x^2a + by^2 + 6x + 1$

$u_y = 2ybx + 2cy$

$v = \cancel{dz} \; dx^2y - 2y^3 + exy + y$

$v_x = 2xyd + ey$

$v_y = dx^2 - 6y^2 + ex + 1$

since f(z) is analytic.

$u_x = v_y$ → .. condition 1

$3x^2a + by^2 + 6x + 1 = dx^2 - 6y^2 + ex + 1$

$\boxed{3a = d}$, $\boxed{b = -6}$, $\boxed{e = 6}$

Also,

$u_y = -v_x$

$2ybx + 2cy = -2xyd - ey$

$b = -d$; $2c = -e$

$\boxed{d = 6}$ $\boxed{c = -3}$

$3a = d$, $\boxed{a = 2}$

Q4] show that $(e^x \cos y + x^3 - 3xy^2)$ is Harmonic ..

$U = e^x \cos y + x^3 - 3xy^2$

$U_x = e^x \cos y + 3x^2 - 3y^2$

$U_{xx} = \cos y e^x + 6x$

$U_y = (-\sin y) e^x - 6xy$

$U_{yy} = -\cos y e^x - 6x$

$U_{xx} + U_{yy} = \cancel{\cos y e^x} + \cancel{6x} - \cancel{\cos y e^x} - \cancel{6x} = 0$

$= 0$

$e^x \cos y + x^3 - 3xy^2$ is Harmonic.

Hence proved.

Q5] find the analytic function $f(z)$ whose imaginary part is $v = e^{-x}[2xy \cos y + (y^2 - x^2) \sin y]$

$v = e^{-x}[2xy \cos y + e^{-x} y^2 \sin y - e^{-x} x^2 \sin y]$

$v_x = e^{-x}\, 2y \cos y + 2xy \cos y\, (-1) e^{-x} - e^{-x} y^2 \sin y - e^{-x} 2x \sin y + e^{-x} x^2 \sin y$

$v_y = e^{-x} 2xy(-\sin y) + \cos y\, e^{-x}\, 2x + e^{-x} y^2 \cos y - e^{-x} 2y \sin y - e^{-x} x^2 \cos y$

$f(z) = u + iv, \quad f'(z) = u_x + iv_x$

$= v_y + iv_x$

$-2x e^{-x}[y \sin y + \cos y] + e^{-x} y [y \cos y + 2\sin y] - e^{-x} x^2 \cos y + i[2e^{-x} y \cos y (1-x) + e^{-x} \sin y (-2x - y^2 + x^2)]$

By M.T.M put $x = z,\ y = 0$

$-2x e^{-x}(1) + 0 - e^{-x} x^2 (1) + i[0]$

$f'(z) = -2z e^{-z} - e^{-z} z^2$

Q5] find the analytic function whose real part is $u = x^2 - y^2 - 5x + y + 2$

$u = x^2 - y^2 - 5x + y + 2$

$\frac{\partial u}{\partial x} = 2x - 5$

$\frac{\partial u}{\partial y} = -2y + 1$

$f'(z) = u_x + i v_x$

$(2x-5) - i(-2y+1)$

By M.T.M put $x = z$ & $y = 0$

$f'(z) = (2z - 5) - i$

$\cancel{f(z) = f'(z)\,dz}$

$f'(z) = 2z - 5 - i(1)$

$\int f'(z) = \int 2z - 5 - i$

$\int (2z - 5)\,dz - \int (i)\,dz = \int (2z-5)\,dz - \int i\,dz$

$\cancel{2}\frac{z^2}{\cancel{2}} - 5z + iz$

$$\boxed{f(z) = z^2 - 5z + iz + c}$$

Req Solution.

Q7] find analytic function $f(z) = u+iv$ in terms of z if (i) $u-v = x^3 + x^2 - 3xy^2 - y^2 - 3x^2y + y^2 - 2xy$

(ii) $u+v = e^x(\cos y + \sin y) + \dfrac{x-y}{x^2+y^2}$

(i) Let $u = x^3 + x^2 - 3xy^2 - y^2 - 3x^2y + y^2 - 2xy$

$u_x = 3x^2 + 2x - 3y^2 - 6xy - 2y$

$u_y = -6xy - 2y - 3x^2 + 2y - 2x$

$f(z) = u + iv$

$f'(z) = u_x + iv_x$

$u_x + iv_y$

$= (3x^2 + 2x - 3y^2\ 6xy - 2y) - i(-6xy - 2y - 3x^2 + 2y - 2x)$

By M.T.M put $x = z$, $y = 0$

$f'(z) = (3z^3 + 2z) - i(-3z^2 - 2z)$

$(3z^2 + 2z) + i(3z^2 + 2z)$

$= (1+i)(3z^2 + 2z)$

$F(z) = \int f'(z)\,dz$

$(1+i)\int (3z^2 + 2z)\,dz$

$(1+i)\left(\cancel{3}\dfrac{z^3}{\cancel{3}} + \cancel{2}\dfrac{z^2}{\cancel{2}} + c\right.$

$f(z) = (1+i)(z^3 + z^2) + c$

$(1+i)f(z) = (1+i)(z^3 + z^2) + c$

$$\boxed{f(z) = (z^3 + z^2) + \frac{c}{(1+i)}}$$

Q8] Show that $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic find harmonic conjugate and then corresponding analytic $f(z)$.

→ $u_x = 6xy + 4x$

$u_{xx} = 6y + 4$

$u_y = 3x^2 - 3y^2 - 4y$

$u_{yy} = -6y - 4$

$u_{xx} + u_{yy} = \cancel{6y} + \cancel{4} - \cancel{6y} - \cancel{4}$

$= 0$

$u = 3x^2y + 2x^2 - y^3 - 2y^2$ is Harmonic

$f(z) = u + iv$

$f'(z) = u_x - iv_x$

$-(6xy + 4x) - i(3x^2 - 3y^2 - 4y)$

By MTM $x = z, y = 0$

~~$f'(z) = 4z$~~ -

$f(z) = \int f'(z)\, dz$

$\int (4z - i3z^2)\, dz$

$\frac{2\cancel{4}z^2}{\cancel{2}} - \cancel{3}i\left(\frac{z^3}{\cancel{3}}\right) + c$

$\boxed{2z^2 - iz^3}$

~~Required Solution~~:

$2(x+iy)^2 - i(x+iy)^3$

$2(x^2 + 2ixy - y^2) - i(x^3 - iy^3 + 3x^2iy - 3xy^2)$

$\cdot 2x^2 + 4ixy - 2y^2 - ix^3 - y^3 + 3x^2y + 3ixy^2$

$v = 4xy - x^3 + 3xy^2$

Q9] Find the orthogonal trajectory of the family of the curves given by $2x - x^3 + 3xy^2 = 9$

Let $u = 2x - x^3 + 3xy^2$

$u_x = 2 - 3x^2 + 3y^2$

$u_y = 6xy$

$f'(z) = u_x + iv_x$

$u_x - iu_y$

$(2 - 3x^2 + 3y^2) - i(6xy)$

By MTM put $x = z,\ y = 0$

$f'(z)(2 - 3z^2)$

$f(z) = \int f'(z)\,dz$

$\int 2z - \cancel{3}\frac{z^3}{\cancel{3}}$

$= \boxed{2z - z^3 + c}$

put $z = x + iy$

$f(z) = 2(x+iy) - (x+iy)^3 = (x+iy)(2-(x+iy)^2]$

$= (x+iy)(2 - x^2 - 2xiy + y^2) = 2x - x^3 - 2ix^2y + xy^2$

$= (2x - x^3 + 3xy^2) - i(2x^2y - 2yx^2y - y^3) + 2i - ix^2y$

$+ 2xy^2 + iy^3$

$\phi = 2x + 2iy - x^3 + i\,3x^2y + 3xy^2 + iy^3$

$\boxed{(3x^2y + 2x - x^3) + i(3x^2y - y^2 + 2y)}$

-x-x-x

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