Taylor's & Laurent's Series

Weight Distribution of Types

MechCivil

Year	May 2017	Nov 2017	May 2018	Nov 2018	May 2019	Nov 2019	May 2022	Nov 2022	May 2023	Dec 2023	May 2024	Dec 2024
Total Marks	08	06	06	06	06	06	05	08	08	08	08	08

Comp/IT/AI

Year	May 2017	Nov 2017	May 2018	Nov 2018	May 2019	Nov 2019	May 2022	Nov 2022	May 2023	Dec 2023	May 2024	Dec 2024
Total Marks	08	08	08	08	08	08	00	06	06	08	06	08

Extc

Year	May 2017	Nov 2017	May 2018	Nov 2018	May 2019	Nov 2019	May 2022	Nov 2022	May 2023	Dec 2023	May 2024	Dec 2024
Total Marks	08	08	08	08	08	08	05	08	08	08	08	08

Elect

Year	May 2017	Nov 2017	May 2018	Nov 2018	May 2019	Nov 2019	May 2022	Nov 2022	May 2023	Dec 2023	May 2024	Dec 2024
Total Marks	08	08	08	08	08	08	07	08	06	06	06	08

1. Expand $f(z)=\frac{1}{z(z+1)(z-2)}$ in Laurent's series when (i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$

[M16/CompIT/8M][M19/Chem/6M]

Solution:

We have, $f(z)=\frac{1}{z(z-2)(z+1)}$

Let $\frac{1}{z(z-2)(z+1)}=\frac{A}{z}+\frac{B}{z-2}+\frac{C}{z+1}$

$1=A(z-2)(z+1)+Bz(z+1)+Cz(z-2)$

$1=A(z^2-z-2)+B(z^2+z)+C(z^2-2z)$

On comparing the coefficients, we get

$A+B+C=0$

$-A+B-2C=0$

$-2A=1$

On solving, we get

$A=-\frac{1}{2},B=\frac{1}{6},C=\frac{1}{3}$

$f(z)=\frac{-\frac{1}{2}}{z}+\frac{\frac{1}{6}}{z-2}+\frac{\frac{1}{3}}{z+1}$

(i) $|z|<1$

$f(z)=-\frac{1}{2z}+\frac{\frac{1}{6}}{-2+z}+\frac{\frac{1}{3}}{1+z}$

$f(z)=-\frac{1}{2z}+\frac{\frac{1}{6}}{-2\left[1-\frac{z}{2}\right]}+\frac{\frac{1}{3}}{[1+z]}$

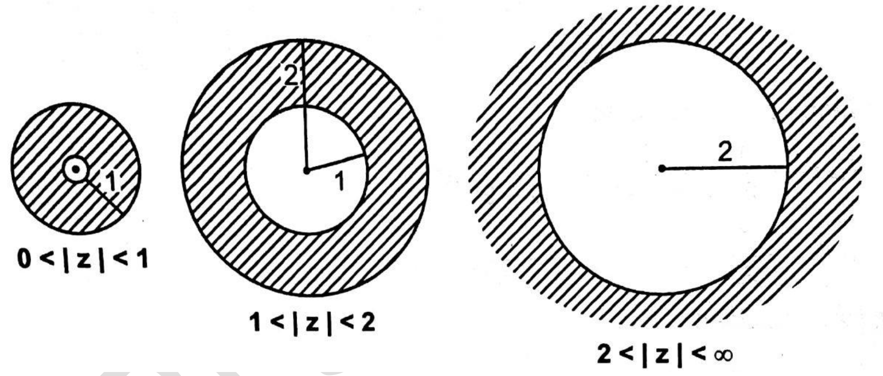
$$f(z) = -\frac{1}{2z} - \frac{1}{12}\left[1 - \frac{z}{2}\right]^{-1} + \frac{1}{3}[1 + z]^{-1}$$
$$f(z) = -\frac{1}{2z} - \frac{1}{12}\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots.\right] + \frac{1}{3}[1 - z + z^2 - z^3 + \cdots.]$$

(ii) $1 < |z| < 2$

$$f(z) = -\frac{1}{2z} + \frac{\frac{1}{6}}{-2+z} + \frac{\frac{1}{3}}{z+1}$$
$$f(z) = -\frac{1}{2z} + \frac{\frac{1}{6}}{-2\left[1-\frac{z}{2}\right]} + \frac{\frac{1}{3}}{z\left[1+\frac{1}{z}\right]}$$
$$f(z) = -\frac{1}{2z} - \frac{1}{12}\left[1 - \frac{z}{2}\right]^{-1} + \frac{1}{3z}\left[1 + \frac{1}{z}\right]^{-1}$$
$$f(z) = -\frac{1}{2z} - \frac{1}{12}\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots.\right] + \frac{1}{3z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots.\right]$$

(iii) $|z| > 2$

$$f(z) = -\frac{1}{2z} + \frac{\frac{1}{6}}{z-2} + \frac{\frac{1}{3}}{z+1}$$
$$f(z) = -\frac{1}{2z} + \frac{\frac{1}{6}}{z\left[1-\frac{2}{z}\right]} + \frac{\frac{1}{3}}{z\left[1+\frac{1}{z}\right]}$$
$$f(z) = -\frac{1}{2z} + \frac{1}{6z}\left[1 - \frac{2}{z}\right]^{-1} + \frac{1}{3z}\left[1 + \frac{1}{z}\right]^{-1}$$
$$f(z) = -\frac{1}{2z} + \frac{1}{6z}\left[1 + \frac{2}{z} + \frac{2^2}{z^2} + \frac{2^3}{z^3} + \cdots.\right] + \frac{1}{3z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots.\right]$$



2. Obtain all Taylor's and Laurent's expansions of $f(z)=\frac{z-1}{z^2-2z-3}$ indicating regions of convergence.
[N14/M16/ElexExtcElectBiomInst/8M][M18/N22/Extc/8M][M18/Elect/8M]
[N18/Elex/6M][M19/Comp/8M][N19/Inst/8M][M23/CompIT/6M]
[D24/MechCivil/8M]

Solution:

We have, $f(z)=\frac{z-1}{z^2-2z-3}=\frac{z-1}{(z-3)(z+1)}$

Let $\frac{z-1}{(z-3)(z+1)}=\frac{A}{z-3}+\frac{B}{z+1}$

$z-1=A(z+1)+B(z-3)$

Comparing the coefficients, we get

$A+B=1$

$A-3B=-1$

On solving, we get

$A=\frac{1}{2}, B=\frac{1}{2}$

$f(z)=\frac{\frac{1}{2}}{z-3}+\frac{\frac{1}{2}}{z+1}$

For ROC,

Put $z-3=0, z+1=0$

$z=3, z=-1$

$|z|=3, |z|=1$

The Region of Convergence are (i) $|z|<1$ (ii) $1<|z|<3$ (iii) $|z|>3$

The Taylors series is given by

(i) $|z|<1$

$$f(z)=\frac{\frac{1}{2}}{-3+z}+\frac{\frac{1}{2}}{1+z}$$

$$f(z)=\frac{\frac{1}{2}}{-3\left[1-\frac{z}{3}\right]}+\frac{\frac{1}{2}}{1+z}$$

$$f(z)=-\frac{1}{6}\left[1-\frac{z}{3}\right]^{-1}+\frac{1}{2}[1+z]^{-1}$$

$$f(z)=-\frac{1}{6}\left[1+\frac{z}{3}+\frac{z^2}{3^2}+\frac{z^3}{3^3}+\cdots.\right]+\frac{1}{2}[1-z+z^2-z^3+\cdots..]$$

The first Laurent's Series is given by

(ii) $1<|z|<3$

$$f(z)=\frac{\frac{1}{2}}{-3+z}+\frac{\frac{1}{2}}{z+1}$$

$$f(z)=\frac{\frac{1}{2}}{-3\left[1-\frac{z}{3}\right]}+\frac{\frac{1}{2}}{z\left[1+\frac{1}{z}\right]}$$

$$f(z)=-\frac{1}{6}\left[1-\frac{z}{3}\right]^{-1}+\frac{1}{2z}\left[1+\frac{1}{z}\right]^{-1}$$

$$f(z)=-\frac{1}{6}\left[1+\frac{z}{3}+\frac{z^2}{3^2}+\frac{z^3}{3^3}+\cdots.\right]+\frac{1}{2z}\left[1-\frac{1}{z}+\frac{1}{z^2}-\frac{1}{z^3}+\cdots..\right]$$

The second Laurent's Series is given by

(iii) $|z| > 3$

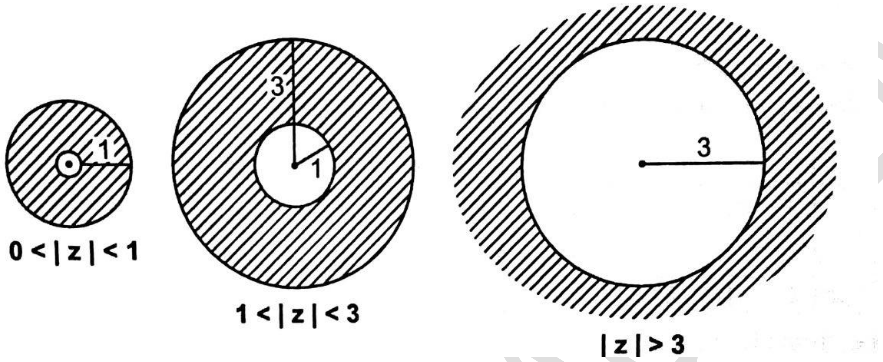
$$f(z) = \frac{\frac{1}{2}}{z-3} + \frac{\frac{1}{2}}{z+1}$$

$$f(z) = \frac{\frac{1}{2}}{z\left[1-\frac{3}{z}\right]} + \frac{\frac{1}{2}}{z\left[1+\frac{1}{z}\right]}$$

$$f(z) = \frac{1}{2z}\left[1-\frac{3}{z}\right]^{-1} + \frac{1}{2z}\left[1+\frac{1}{z}\right]^{-1}$$

$$f(z) = \frac{1}{2z}\left[1+\frac{3}{z}+\frac{3^2}{z^2}+\frac{3^3}{z^3}+\cdots.\right] + \frac{1}{2z}\left[1-\frac{1}{z}+\frac{1}{z^2}-\frac{1}{z^3}+\cdots..\right]$$

The regions of convergence are as below.



3. Find expansion of $f(z)=\frac{1}{(1+z^2)(z+2)}$ indicating region of convergence

[N15/ElexExtcElectBiomInst/8M]

Solution:

We have, $f(z)=\frac{1}{(1+z^2)(z+2)}$

Let $\frac{1}{(1+z^2)(z+2)}=\frac{Az+B}{1+z^2}+\frac{C}{z+2}$

$1=(Az+B)(z+2)+C(1+z^2)$

$1=A(z^2+2z)+B(z+2)+C(z^2+1)$

Comparing the coefficients, we get

$A+C=0$

$2A+B=0$

$2B+C=1$

On solving, we get

$A=-\frac{1}{5},B=\frac{2}{5},C=\frac{1}{5}$

$f(z)=\frac{-\frac{z}{5}+\frac{2}{5}}{1+z^2}+\frac{\frac{1}{5}}{z+2}=\frac{1}{5}\left[\frac{2-z}{1+z^2}+\frac{1}{z+2}\right]$

To find ROC, put $(1+z^2)=0, z+2=0$

$z^2=-1, z=-2$

$z=\pm i, z=-2$

$|z|=1,|z|=2$

The region of convergence are (i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$

(i) $|z|<1$

$f(z)=\frac{1}{5}\left[\frac{2-z}{1+z^2}+\frac{1}{2+z}\right]$

$f(z)=\frac{1}{5}\left[\frac{2-z}{1+z^2}+\frac{1}{2\left[1+\frac{z}{2}\right]}\right]$

$f(z)=\frac{2-z}{5}[1+z^2]^{-1}+\frac{1}{10}\left[1+\frac{z}{2}\right]^{-1}$

$f(z)=\frac{2-z}{5}[1-z^2+z^4-z^6+\cdots.]+\frac{1}{10}\left[1-\frac{z}{2}+\frac{z^2}{2^2}-\frac{z^3}{2^3}+\cdots\right]$

(ii) $1<|z|<2$

$f(z)=\frac{1}{5}\left[\frac{2-z}{z^2+1}+\frac{1}{2+z}\right]$

$f(z)=\frac{1}{5}\left[\frac{2-z}{z^2\left[1+\frac{1}{z^2}\right]}+\frac{1}{2\left[1+\frac{z}{2}\right]}\right]$

$f(z)=\frac{2-z}{5z^2}\left[1+\frac{1}{z^2}\right]^{-1}+\frac{1}{10}\left[1+\frac{z}{2}\right]^{-1}$

$f(z)=\frac{2-z}{5z^2}\left[1-\frac{1}{z^2}+\frac{1}{z^4}-\frac{1}{z^6}+\cdots.\right]+\frac{1}{10}\left[1-\frac{z}{2}+\frac{z^2}{2^2}-\frac{z^3}{2^3}+\cdots\right]$

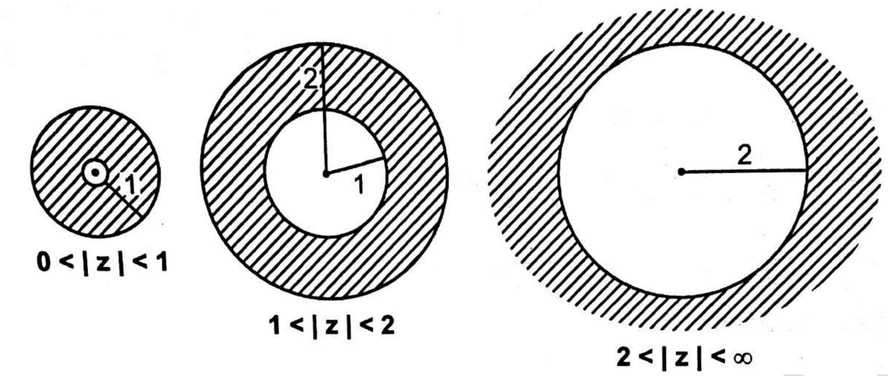
(iii) $|z|>2$

$f(z)=\frac{1}{5}\left[\frac{2-z}{z^2+1}+\frac{1}{z+2}\right]$

$$f(z)=\frac{1}{5}\left[\frac{2-z}{z^2\left[1+\frac{1}{z^2}\right]}+\frac{1}{z\left[1+\frac{2}{z}\right]}\right]$$

$$f(z)=\frac{2-z}{5z^2}\left[1+\frac{1}{z^2}\right]^{-1}+\frac{1}{5z}\left[1+\frac{2}{z}\right]^{-1}$$

$$f(z)=\frac{2-z}{5z^2}\left[1-\frac{1}{z^2}+\frac{1}{z^4}-\frac{1}{z^6}+\cdots.\right]+\frac{1}{5z}\left[1-\frac{2}{z}+\frac{2^2}{z^2}-\frac{2^3}{z^3}+\cdots\right]$$



4. Obtain two Laurent's series for $\frac{(z-2)(z+2)}{(z+1)(z+4)}$

Solution:

We have, $f(z) = \frac{(z-2)(z+2)}{(z+1)(z+4)} = \frac{z^2-4}{z^2+5z+4}$

$f(z) = \frac{z^2+5z+4-5z-4-4}{z^2+5z+4}$

$f(z) = 1 + \frac{-5z-8}{z^2+5z+4}$

Let $\frac{-5z-8}{(z+1)(z+4)} = \frac{A}{z+1} + \frac{B}{z+4}$

$-5z - 8 = A(z+4) + B(z+1)$

Comparing the coefficients, we get

$A + B = -5$

$4A + B = -8$

On solving, we get

$A = -1, B = -4$

$f(z) = 1 - \frac{1}{z+1} - \frac{4}{z+4}$

For ROC, put $z + 1 = 0, z + 4 = 0$

$z = -1, z = -4$

$|z| = 1, |z| = 4$

The ROCs are as follows (i) $|z| < 1$ (ii) $1 < |z| < 4$ (iii) $|z| > 4$

The Laurent's series is given by

(i) For $1 < |z| < 4$

$f(z) = 1 - \frac{1}{z+1} - \frac{4}{4+z}$

$f(z) = 1 - \frac{1}{z\left(1+\frac{1}{z}\right)} - \frac{4}{4\left(1+\frac{z}{4}\right)}$

$f(z) = 1 - \frac{1}{z}\left[1 + \frac{1}{z}\right]^{-1} - \left[1 + \frac{z}{4}\right]^{-1}$

$f(z) = 1 - \frac{1}{z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots..\right] - \left[1 - \frac{z}{4} + \frac{z^2}{16} - \frac{z^3}{64} + \cdots.\right]$

The second Laurent's series is given by

(ii) For $|z| > 4$

$f(z) = 1 - \frac{1}{z+1} - \frac{4}{z+4}$

$f(z) = 1 - \frac{1}{z\left(1+\frac{1}{z}\right)} - \frac{4}{z\left(1+\frac{4}{z}\right)}$

$f(z) = 1 - \frac{1}{z}\left[1 + \frac{1}{z}\right]^{-1} - \frac{4}{z}\left[1 + \frac{4}{z}\right]^{-1}$

$f(z) = 1 - \frac{1}{z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots..\right] - \frac{4}{z}\left[1 - \frac{4}{z} + \frac{16}{z^2} - \frac{64}{z^3} + \cdots.\right]$

5. Expand $f(z)=\frac{1}{z(1+z)(z-2)}$ (i) within the unit circle about the origin, (ii) within the annulus region between the concentric circles about the origin having radii 1 and 2 respectively, (iii) in the exterior of the circle with the centre at the origin and the radius 2.

[N17/CompIT/8M]

Solution:

We have, $f(z)=\frac{1}{z(z-2)(z+1)}$

Let $\frac{1}{z(z-2)(z+1)}=\frac{A}{z}+\frac{B}{z-2}+\frac{C}{z+1}$

$1=A(z-2)(z+1)+Bz(z+1)+Cz(z-2)$

$1=A(z^2-z-2)+B(z^2+z)+C(z^2-2z)$

On comparing the coefficients, we get

$A+B+C=0$

$-A+B-2C=0$

$-2A=1$

On solving, we get

$A=-\frac{1}{2},B=\frac{1}{6},C=\frac{1}{3}$

$f(z)=\frac{-\frac{1}{2}}{z}+\frac{\frac{1}{6}}{z-2}+\frac{\frac{1}{3}}{z+1}$

(i) $|z|<1$

$f(z)=-\frac{1}{2z}+\frac{\frac{1}{6}}{-2+z}+\frac{\frac{1}{3}}{1+z}$

$f(z)=-\frac{1}{2z}+\frac{\frac{1}{6}}{-2\left[1-\frac{z}{2}\right]}+\frac{\frac{1}{3}}{[1+z]}$

$f(z)=-\frac{1}{2z}-\frac{1}{12}\left[1-\frac{z}{2}\right]^{-1}+\frac{1}{3}[1+z]^{-1}$

$f(z)=-\frac{1}{2z}-\frac{1}{12}\left[1+\frac{z}{2}+\frac{z^2}{2^2}+\frac{z^3}{2^3}+\cdots.\right]+\frac{1}{3}[1-z+z^2-z^3+\cdots.]$

(ii) $1<|z|<2$

$f(z)=-\frac{1}{2z}+\frac{\frac{1}{6}}{-2+z}+\frac{\frac{1}{3}}{z+1}$

$f(z)=-\frac{1}{2z}+\frac{\frac{1}{6}}{-2\left[1-\frac{z}{2}\right]}+\frac{\frac{1}{3}}{z\left[1+\frac{1}{z}\right]}$

$f(z)=-\frac{1}{2z}-\frac{1}{12}\left[1-\frac{z}{2}\right]^{-1}+\frac{1}{3z}\left[1+\frac{1}{z}\right]^{-1}$

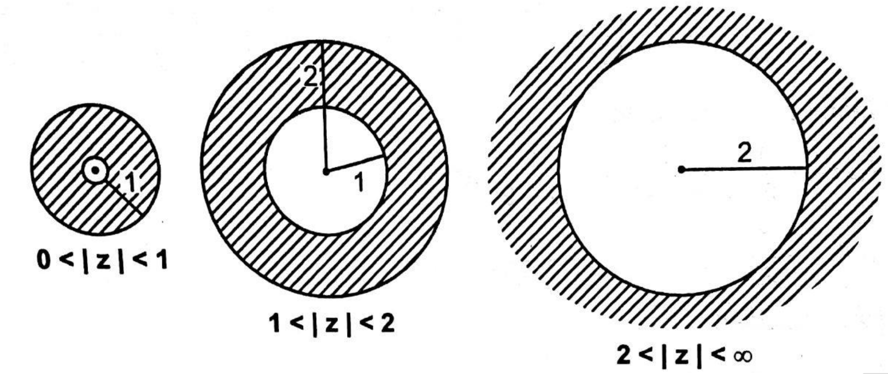
$f(z)=-\frac{1}{2z}-\frac{1}{12}\left[1+\frac{z}{2}+\frac{z^2}{2^2}+\frac{z^3}{2^3}+\cdots.\right]+\frac{1}{3z}\left[1-\frac{1}{z}+\frac{1}{z^2}-\frac{1}{z^3}+\cdots.\right]$

(iii) $|z|>2$

$f(z)=-\frac{1}{2z}+\frac{\frac{1}{6}}{z-2}+\frac{\frac{1}{3}}{z+1}$

$f(z)=-\frac{1}{2z}+\frac{\frac{1}{6}}{z\left[1-\frac{2}{z}\right]}+\frac{\frac{1}{3}}{z\left[1+\frac{1}{z}\right]}$

$$f(z) = -\frac{1}{2z} + \frac{1}{6z}\left[1 - \frac{2}{z}\right]^{-1} + \frac{1}{3z}\left[1 + \frac{1}{z}\right]^{-1}$$
$$f(z) = -\frac{1}{2z} + \frac{1}{6z}\left[1 + \frac{2}{z} + \frac{2^2}{z^2} + \frac{2^3}{z^3} + \cdots.\right] + \frac{1}{3z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots.\right]$$



6. Find all possible Laurent's expansions of the function $f(z) = \frac{1}{z(z-2)(z+1)}$

[D24/Extc/8M]

Solution:

We have, $f(z) = \frac{1}{z(z-2)(z+1)}$

Let $\frac{1}{z(z-2)(z+1)} = \frac{A}{z} + \frac{B}{z-2} + \frac{C}{z+1}$

$1 = A(z-2)(z+1) + Bz(z+1) + Cz(z-2)$

$1 = A(z^2 - z - 2) + B(z^2 + z) + C(z^2 - 2z)$

On comparing the coefficients, we get

$A + B + C = 0$

$-A + B - 2C = 0$

$-2A = 1$

On solving, we get

$A = -\frac{1}{2}, B = \frac{1}{6}, C = \frac{1}{3}$

$f(z) = \frac{-\frac{1}{2}}{z} + \frac{\frac{1}{6}}{z-2} + \frac{\frac{1}{3}}{z+1}$

For ROC,

Put $z - 2 = 0, z + 1 = 0$

$z = 2, z = -1$

$|z| = 2, |z| = 1$

The Region of Convergence are (i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$

(i) $|z| < 1$

$f(z) = -\frac{1}{2z} + \frac{\frac{1}{6}}{-2+z} + \frac{\frac{1}{3}}{1+z}$

$f(z) = -\frac{1}{2z} + \frac{\frac{1}{6}}{-2\left[1-\frac{z}{2}\right]} + \frac{\frac{1}{3}}{[1+z]}$

$f(z) = -\frac{1}{2z} - \frac{1}{12}\left[1 - \frac{z}{2}\right]^{-1} + \frac{1}{3}[1+z]^{-1}$

$f(z) = -\frac{1}{2z} - \frac{1}{12}\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots.\right] + \frac{1}{3}[1 - z + z^2 - z^3 + \cdots.]$

(ii) $1 < |z| < 2$

$f(z) = -\frac{1}{2z} + \frac{\frac{1}{6}}{-2+z} + \frac{\frac{1}{3}}{z+1}$

$f(z) = -\frac{1}{2z} + \frac{\frac{1}{6}}{-2\left[1-\frac{z}{2}\right]} + \frac{\frac{1}{3}}{z\left[1+\frac{1}{z}\right]}$

$f(z) = -\frac{1}{2z} - \frac{1}{12}\left[1 - \frac{z}{2}\right]^{-1} + \frac{1}{3z}\left[1 + \frac{1}{z}\right]^{-1}$

$f(z) = -\frac{1}{2z} - \frac{1}{12}\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots.\right] + \frac{1}{3z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots.\right]$

(iii) $|z| > 2$

$f(z) = -\frac{1}{2z} + \frac{\frac{1}{6}}{z-2} + \frac{\frac{1}{3}}{z+1}$

$$f(z) = -\frac{1}{2z} + \frac{\frac{1}{6}}{z\left[1-\frac{2}{z}\right]} + \frac{\frac{1}{3}}{z\left[1+\frac{1}{z}\right]}$$

$$f(z) = -\frac{1}{2z} + \frac{1}{6z}\left[1 - \frac{2}{z}\right]^{-1} + \frac{1}{3z}\left[1 + \frac{1}{z}\right]^{-1}$$

$$f(z) = -\frac{1}{2z} + \frac{1}{6z}\left[1 + \frac{2}{z} + \frac{2^2}{z^2} + \frac{2^3}{z^3} + \cdots.\right] + \frac{1}{3z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots.\right]$$

7. Expand $f(z)=\frac{2}{(z-1)(z-2)}$ in Laurent's series when (i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$
[N14/N15/N16/CompIT/8M][M18/MTRX/6M][N18/Chem/8M][M22/Extc/5M][M22/MTRX/5M][M23/Extc/8M]
Solution:
We have, $f(z)=\frac{2}{(z-1)(z-2)}$
Let $\frac{2}{(z-1)(z-2)}=\frac{A}{z-1}+\frac{B}{z-2}$
$2=A(z-2)+B(z-1)$
Comparing the coefficients, we get
$A+B=0$
$-2A-B=2$
On solving, we get
$A=-2, B=2$
$f(z)=-\frac{2}{z-1}+\frac{2}{z-2}$
(i) $|z|<1$
$f(z)=-\frac{2}{-1+z}+\frac{2}{-2+z}$
$f(z)=-\frac{2}{-[1-z]}+\frac{2}{-2\left[1-\frac{z}{2}\right]}$
$f(z)=2[1-z]^{-1}-1\left[1-\frac{z}{2}\right]^{-1}$
$f(z)=2[1+z+z^2+z^3+\cdots.]-\left[1+\frac{z}{2}+\frac{z^2}{2^2}+\frac{z^3}{2^3}+\cdots\right]$
(ii) $1<|z|<2$
$f(z)=-\frac{2}{z-1}+\frac{2}{-2+z}$
$f(z)=-\frac{2}{z\left[1-\frac{1}{z}\right]}+\frac{2}{-2\left[1-\frac{z}{2}\right]}$
$f(z)=-\frac{2}{z}\left[1-\frac{1}{z}\right]^{-1}-1\left[1-\frac{z}{2}\right]^{-1}$
$f(z)=-\frac{2}{z}\left[1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\cdots.\right]-\left[1+\frac{z}{2}+\frac{z^2}{2^2}+\frac{z^3}{2^3}+\cdots\right]$
(iii) $|z|>2$
$f(z)=-\frac{2}{z-1}+\frac{2}{z-2}$
$f(z)=-\frac{2}{z\left[1-\frac{1}{z}\right]}+\frac{2}{z\left[1-\frac{2}{z}\right]}$
$f(z)=-\frac{2}{z}\left[1-\frac{1}{z}\right]^{-1}+\frac{2}{z}\left[1-\frac{2}{z}\right]^{-1}$
$f(z)=-\frac{2}{z}\left[1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\cdots.\right]+\frac{2}{z}\left[1+\frac{2}{z}+\frac{2^2}{z^2}+\frac{2^3}{z^3}+\cdots\right]$

8. Find all possible Laurent's expansions of the function $f(z)=\frac{2}{(z-1)(z-2)}$ indicating the region of convergence

[M24/Extc/8M]

Solution:

We have, $f(z)=\frac{2}{(z-1)(z-2)}$

Let $\frac{2}{(z-1)(z-2)}=\frac{A}{z-1}+\frac{B}{z-2}$

$2=A(z-2)+B(z-1)$

Comparing the coefficients, we get

$A+B=0$

$-2A-B=2$

On solving, we get

$A=-2, B=2$

$f(z)=-\frac{2}{z-1}+\frac{2}{z-2}$

(i) $|z|<1$

$f(z)=-\frac{2}{-1+z}+\frac{2}{-2+z}$

$f(z)=-\frac{2}{-[1-z]}+\frac{2}{-2\left[1-\frac{z}{2}\right]}$

$f(z)=2[1-z]^{-1}-1\left[1-\frac{z}{2}\right]^{-1}$

$f(z)=2[1+z+z^2+z^3+\cdots.]-\left[1+\frac{z}{2}+\frac{z^2}{2^2}+\frac{z^3}{2^3}+\cdots\right]$

(ii) $1<|z|<2$

$f(z)=-\frac{2}{z-1}+\frac{2}{-2+z}$

$f(z)=-\frac{2}{z\left[1-\frac{1}{z}\right]}+\frac{2}{-2\left[1-\frac{z}{2}\right]}$

$f(z)=-\frac{2}{z}\left[1-\frac{1}{z}\right]^{-1}-1\left[1-\frac{z}{2}\right]^{-1}$

$f(z)=-\frac{2}{z}\left[1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\cdots.\right]-\left[1+\frac{z}{2}+\frac{z^2}{2^2}+\frac{z^3}{2^3}+\cdots\right]$

(iii) $|z|>2$

$f(z)=-\frac{2}{z-1}+\frac{2}{z-2}$

$f(z)=-\frac{2}{z\left[1-\frac{1}{z}\right]}+\frac{2}{z\left[1-\frac{2}{z}\right]}$

$f(z)=-\frac{2}{z}\left[1-\frac{1}{z}\right]^{-1}+\frac{2}{z}\left[1-\frac{2}{z}\right]^{-1}$

$f(z)=-\frac{2}{z}\left[1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\cdots.\right]+\frac{2}{z}\left[1+\frac{2}{z}+\frac{2^2}{z^2}+\frac{2^3}{z^3}+\cdots\right]$

9. Obtain Taylor's and Laurent's series for $f(z)=\frac{1}{(1+z^2)(z+2)}$ for (i) $1<|z|<2$ (ii) $|z|>2$

[N14/MechCivil/8M][N19/Chem/8M]

Solution:

We have, $f(z)=\frac{1}{(1+z^2)(z+2)}$

Let $\frac{1}{(1+z^2)(z+2)}=\frac{Az+B}{1+z^2}+\frac{C}{z+2}$

$1=(Az+B)(z+2)+C(1+z^2)$

$1=A(z^2+2z)+B(z+2)+C(z^2+1)$

Comparing the coefficients, we get

$A+C=0$

$2A+B=0$

$2B+C=1$

On solving, we get

$A=-\frac{1}{5},B=\frac{2}{5},C=\frac{1}{5}$

$f(z)=\frac{-\frac{z}{5}+\frac{2}{5}}{1+z^2}+\frac{\frac{1}{5}}{z+2}=\frac{1}{5}\left[\frac{2-z}{1+z^2}+\frac{1}{z+2}\right]$

(i) $1<|z|<2$

$f(z)=\frac{1}{5}\left[\frac{2-z}{z^2+1}+\frac{1}{2+z}\right]$

$f(z)=\frac{1}{5}\left[\frac{2-z}{z^2\left[1+\frac{1}{z^2}\right]}+\frac{1}{2\left[1+\frac{z}{2}\right]}\right]$

$f(z)=\frac{2-z}{5z^2}\left[1+\frac{1}{z^2}\right]^{-1}+\frac{1}{10}\left[1+\frac{z}{2}\right]^{-1}$

$f(z)=\frac{2-z}{5z^2}\left[1-\frac{1}{z^2}+\frac{1}{z^4}-\frac{1}{z^6}+\cdots.\right]+\frac{1}{10}\left[1-\frac{z}{2}+\frac{z^2}{2^2}-\frac{z^3}{2^3}+\cdots\right]$

(ii) $|z|>2$

$f(z)=\frac{1}{5}\left[\frac{2-z}{z^2+1}+\frac{1}{z+2}\right]$

$f(z)=\frac{1}{5}\left[\frac{2-z}{z^2\left[1+\frac{1}{z^2}\right]}+\frac{1}{z\left[1+\frac{2}{z}\right]}\right]$

$f(z)=\frac{2-z}{5z^2}\left[1+\frac{1}{z^2}\right]^{-1}+\frac{1}{5z}\left[1+\frac{2}{z}\right]^{-1}$

$f(z)=\frac{2-z}{5z^2}\left[1-\frac{1}{z^2}+\frac{1}{z^4}-\frac{1}{z^6}+\cdots.\right]+\frac{1}{5z}\left[1-\frac{2}{z}+\frac{2^2}{z^2}-\frac{2^3}{z^3}+\cdots\right]$

10. Obtain Taylor's or Laurent's series expansion of the function $f(z)=\frac{1}{z^2-3z+2}$ when (i) $|z|<1$ (ii) $1<|z|<2$

[N19/Extc/8M]

Solution:

We have, $f(z)=\frac{1}{(z-1)(z-2)}$

Let $\frac{1}{(z-1)(z-2)}=\frac{A}{z-1}+\frac{B}{z-2}$

$1=A(z-2)+B(z-1)$

Comparing the coefficients, we get

$A+B=0$

$-2A-B=1$

On solving, we get

$A=-1, B=1$

$f(z)=-\frac{1}{z-1}+\frac{1}{z-2}$

(i) $|z|<1$

$f(z)=-\frac{1}{-1+z}+\frac{1}{-2+z}$

$f(z)=-\frac{1}{-[1-z]}+\frac{1}{-2\left[1-\frac{z}{2}\right]}$

$f(z)=[1-z]^{-1}-\frac{1}{2}\left[1-\frac{z}{2}\right]^{-1}$

$f(z)=[1+z+z^2+z^3+\cdots.]-\frac{1}{2}\left[1+\frac{z}{2}+\frac{z^2}{2^2}+\frac{z^3}{2^3}+\cdots\right]$

(ii) $1<|z|<2$

$f(z)=-\frac{1}{z-1}+\frac{1}{-2+z}$

$f(z)=-\frac{1}{z\left[1-\frac{1}{z}\right]}+\frac{1}{-2\left[1-\frac{z}{2}\right]}$

$f(z)=-\frac{1}{z}\left[1-\frac{1}{z}\right]^{-1}-\frac{1}{2}\left[1-\frac{z}{2}\right]^{-1}$

$f(z)=-\frac{1}{z}\left[1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\cdots.\right]-\frac{1}{2}\left[1+\frac{z}{2}+\frac{z^2}{2^2}+\frac{z^3}{2^3}+\cdots\right]$

11. Obtain the Taylor's and Laurent's series which represent the function $f(z) = \frac{z}{(z+1)(z-2)}$ in the regions (i) $|z| < 1$ (ii) $1 < |z| < 2$

[N18/MTRX/6M]

Solution:

We have, $f(z) = \frac{z}{(z+1)(z-2)}$

Let $\frac{z}{(z+1)(z-2)} = \frac{A}{z+1} + \frac{B}{z-2}$

$z = A(z-2) + B(z+1)$

Comparing the coefficients, we get

$A + B = 1$

$-2A + B = 0$

On solving, we get

$A = \frac{1}{3}, B = \frac{2}{3}$

$f(z) = \frac{\frac{1}{3}}{z+1} + \frac{\frac{2}{3}}{z-2}$

(i) $|z| < 1$

$f(z) = \frac{\frac{1}{3}}{1+z} + \frac{\frac{2}{3}}{-2+z}$

$f(z) = \frac{\frac{1}{3}}{[1+z]} + \frac{\frac{2}{3}}{-2\left[1-\frac{z}{2}\right]}$

$f(z) = \frac{1}{3}[1+z]^{-1} - \frac{1}{3}\left[1-\frac{z}{2}\right]^{-1}$

$f(z) = \frac{1}{3}[1 - z + z^2 - z^3 + \cdots.] - \frac{1}{3}\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots\right]$

(ii) $1 < |z| < 2$

$f(z) = \frac{\frac{1}{3}}{z+1} + \frac{\frac{2}{3}}{-2+z}$

$f(z) = \frac{\frac{1}{3}}{z\left[1+\frac{1}{z}\right]} + \frac{\frac{2}{3}}{-2\left[1-\frac{z}{2}\right]}$

$f(z) = \frac{1}{3z}\left[1+\frac{1}{z}\right]^{-1} - \frac{1}{3}\left[1-\frac{z}{2}\right]^{-1}$

$f(z) = \frac{1}{3z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots.\right] - \frac{1}{3}\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots\right]$

12. Obtain the Taylor's and Laurent's series which represent the function $f(z) = \frac{z}{(z-1)(z-2)}$ in the regions (i) $|z| < 1$ (ii) $1 < |z| < 2$
[N17/M18/MechCivil/6M]
Solution:
We have, $f(z) = \frac{z}{(z-1)(z-2)}$
Let $\frac{z}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$
$z = A(z-2) + B(z-1)$
Comparing the coefficients, we get
$A + B = 1$
$-2A - B = 0$
On solving, we get
$A = -1, B = 2$
$f(z) = -\frac{1}{z-1} + \frac{2}{z-2}$
(i) $|z| < 1$
$f(z) = -\frac{1}{-1+z} + \frac{2}{-2+z}$
$f(z) = -\frac{1}{-[1-z]} + \frac{2}{-2\left[1-\frac{z}{2}\right]}$
$f(z) = [1-z]^{-1} - \left[1-\frac{z}{2}\right]^{-1}$
$f(z) = [1 + z + z^2 + z^3 + \cdots.] - \left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots\right]$
(ii) $1 < |z| < 2$
$f(z) = -\frac{1}{z-1} + \frac{2}{-2+z}$
$f(z) = -\frac{1}{z\left[1-\frac{1}{z}\right]} + \frac{2}{-2\left[1-\frac{z}{2}\right]}$
$f(z) = -\frac{1}{z}\left[1-\frac{1}{z}\right]^{-1} - \left[1-\frac{z}{2}\right]^{-1}$
$f(z) = -\frac{1}{z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots.\right] - \left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots\right]$

13. Obtain all possible Laurent's series expansion of $f(z)=\frac{z}{(z-1)(z-2)}$ about $z=0$

[M23/MechCivil/8M]

Solution:

We have, $f(z)=\frac{z}{(z-1)(z-2)}$

Let $\frac{z}{(z-1)(z-2)}=\frac{A}{z-1}+\frac{B}{z-2}$

$z=A(z-2)+B(z-1)$

Comparing the coefficients, we get

$A+B=1$

$-2A-B=0$

On solving, we get

$A=-1, B=2$

$f(z)=-\frac{1}{z-1}+\frac{2}{z-2}$

For ROC, put $z-1=0, z-2=0$

$z=1, z=2$

$|z|=1, |z|=2$

Thus, the Region of convergences are (i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$

(i) $|z|<1$

$f(z)=-\frac{1}{-1+z}+\frac{2}{-2+z}$

$f(z)=-\frac{1}{-[1-z]}+\frac{2}{-2\left[1-\frac{z}{2}\right]}$

$f(z)=[1-z]^{-1}-\left[1-\frac{z}{2}\right]^{-1}$

$f(z)=[1+z+z^2+z^3+\cdots.]-\left[1+\frac{z}{2}+\frac{z^2}{2^2}+\frac{z^3}{2^3}+\cdots\right]$

(ii) $1<|z|<2$

$f(z)=-\frac{1}{z-1}+\frac{2}{-2+z}$

$f(z)=-\frac{1}{z\left[1-\frac{1}{z}\right]}+\frac{2}{-2\left[1-\frac{z}{2}\right]}$

$f(z)=-\frac{1}{z}\left[1-\frac{1}{z}\right]^{-1}-\left[1-\frac{z}{2}\right]^{-1}$

$f(z)=-\frac{1}{z}\left[1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\cdots.\right]-\left[1+\frac{z}{2}+\frac{z^2}{2^2}+\frac{z^3}{2^3}+\cdots\right]$

(iii) $|z|>2$

$f(z)=-\frac{1}{z-1}+\frac{2}{z-2}$

$f(z)=-\frac{1}{z\left[1-\frac{1}{z}\right]}+\frac{2}{z\left[1-\frac{2}{z}\right]}$

$f(z)=-\frac{1}{z}\left[1-\frac{1}{z}\right]^{-1}+\frac{2}{z}\left[1-\frac{2}{z}\right]^{-1}$

$f(z)=-\frac{1}{z}\left[1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\cdots.\right]+\frac{2}{z}\left[1+\frac{2}{z}+\frac{2^2}{z^2}+\frac{2^3}{z^3}+\cdots\right]$

14. Obtain the Taylor's and Laurent's series which represent the function $f(z) = \frac{z-1}{z^2-2z-3}$ in the regions (i) $|z| < 1$ (ii) $1 < |z| < 3$

[N18/MechCivil/6M][M22/MechCivil/5M]

Solution:

We have, $f(z) = \frac{z-1}{(z+1)(z-3)}$

Let $\frac{z-1}{(z+1)(z-3)} = \frac{A}{z+1} + \frac{B}{z-3}$

$z - 1 = A(z-3) + B(z+1)$

Comparing the coefficients, we get

$A + B = 1$

$-3A + B = -1$

On solving, we get

$A = \frac{1}{2}, B = \frac{1}{2}$

$f(z) = \frac{\frac{1}{2}}{z+1} + \frac{\frac{1}{2}}{z-3}$

(i) $|z| < 1$

$f(z) = \frac{\frac{1}{2}}{1+z} + \frac{\frac{1}{2}}{-3+z}$

$f(z) = \frac{1}{2}[1+z]^{-1} + \frac{\frac{1}{2}}{-3\left[1-\frac{z}{3}\right]}$

$f(z) = \frac{1}{2}[1+z]^{-1} - \frac{1}{6}\left[1-\frac{z}{3}\right]^{-1}$

$f(z) = \frac{1}{2}[1 - z + z^2 - z^3 + \cdots.] - \frac{1}{6}\left[1 + \frac{z}{3} + \frac{z^2}{3^2} + \frac{z^3}{3^3} + \cdots\right]$

(ii) $1 < |z| < 3$

$f(z) = \frac{\frac{1}{2}}{z+1} + \frac{\frac{1}{2}}{-3+z}$

$f(z) = \frac{\frac{1}{2}}{z\left[1+\frac{1}{z}\right]} + \frac{\frac{1}{2}}{-3\left[1-\frac{z}{3}\right]}$

$f(z) = \frac{1}{2z}\left[1+\frac{1}{z}\right]^{-1} - \frac{1}{6}\left[1-\frac{z}{3}\right]^{-1}$

$f(z) = \frac{1}{2z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots.\right] - \frac{1}{6}\left[1 + \frac{z}{3} + \frac{z^2}{3^2} + \frac{z^3}{3^3} + \cdots\right]$

15. Obtain Laurent's series expansions of $f(z)=\frac{z-1}{z^2-2z-3}$; $|z|>3$

[N22/MechCivil/8M]

Solution:

We have, $f(z)=\frac{z-1}{(z+1)(z-3)}$

Let $\frac{z-1}{(z+1)(z-3)}=\frac{A}{z+1}+\frac{B}{z-3}$

$z-1=A(z-3)+B(z+1)$

Comparing the coefficients, we get

$A+B=1$

$-3A+B=-1$

On solving, we get

$A=\frac{1}{2},B=\frac{1}{2}$

$f(z)=\frac{\frac{1}{2}}{z+1}+\frac{\frac{1}{2}}{z-3}$

For $|z|>1$

$f(z)=\frac{\frac{1}{2}}{z+1}+\frac{\frac{1}{2}}{z-3}$

$f(z)=\frac{\frac{1}{2}}{z\left[1+\frac{1}{z}\right]}+\frac{\frac{1}{2}}{z\left[1-\frac{3}{z}\right]}$

$f(z)=\frac{1}{2z}\left[1+\frac{1}{z}\right]^{-1}+\frac{1}{2z}\left[1-\frac{3}{z}\right]^{-1}$

$f(z)=\frac{1}{2z}\left[1-\frac{1}{z}+\frac{1}{z^2}-\frac{1}{z^3}+\cdots.\right]+\frac{1}{2z}\left[1+\frac{3}{z}+\frac{3^2}{z^2}+\frac{3^3}{z^3}+\cdots\right]$

16. Obtain the Laurent's series which represent the function $f(z) = \frac{4z+3}{z(z-3)(z+2)}$ in the regions (i) $2 < |z| < 3$ (ii) $|z| > 3$

[N19/MechCivil/6M]

Solution:

We have, $f(z) = \frac{4z+3}{z(z-3)(z+2)}$

Let $\frac{4z+3}{z(z-3)(z+2)} = \frac{A}{z} + \frac{B}{z-3} + \frac{C}{z+2}$

$4z + 3 = A(z-3)(z+2) + Bz(z+2) + Cz(z-3)$

$4z + 3 = A(z^2 - z - 6) + B(z^2 + 2z) + C(z^2 - 3z)$

Comparing the coefficients, we get

$A + B + C = 0$

$-A + 2B - 3C = 4$

$-6A + 0B + 0C = 3$

On solving, we get

$A = -\frac{1}{2}, B = 1, C = -\frac{1}{2}$

$f(z) = \frac{-\frac{1}{2}}{z} + \frac{1}{z-3} - \frac{\frac{1}{2}}{z+2}$

(i) $2 < |\mathrm{z}| < 3$

$f(z) = -\frac{1}{2z} + \frac{1}{-3+z} - \frac{\frac{1}{2}}{z+2}$

$f(z) = -\frac{1}{2z} + \frac{1}{-3\left(1-\frac{z}{3}\right)} - \frac{\frac{1}{2}}{z\left(1+\frac{2}{z}\right)}$

$f(z) = -\frac{1}{2z} - \frac{1}{3}\left[1 - \frac{z}{3}\right]^{-1} - \frac{1}{2z}\left[1 + \frac{2}{z}\right]^{-1}$

$f(z) = -\frac{1}{2z} - \frac{1}{3}\left[1 + \frac{z}{3} + \frac{z^2}{3^2} + \frac{z^3}{3^3} + \cdots.\right] - \frac{1}{2z}\left[1 - \frac{2}{z} + \frac{2^2}{z^2} - \frac{2^3}{z^3} + \cdots\right]$

(ii) $|\mathrm{z}| > 3$

$f(z) = -\frac{1}{2z} + \frac{1}{z-3} - \frac{\frac{1}{2}}{z+2}$

$f(z) = -\frac{1}{2z} + \frac{1}{z\left(1-\frac{3}{z}\right)} - \frac{\frac{1}{2}}{z\left(1+\frac{2}{z}\right)}$

$f(z) = -\frac{1}{2z} + \frac{1}{z}\left[1 - \frac{3}{z}\right]^{-1} - \frac{1}{2z}\left[1 + \frac{2}{z}\right]^{-1}$

$f(z) = -\frac{1}{2z} + \frac{1}{z}\left[1 + \frac{3}{z} + \frac{3^2}{z^2} + \frac{3^3}{z^3} + \cdots.\right] - \frac{1}{2z}\left[1 - \frac{2}{z} + \frac{2^2}{z^2} - \frac{2^3}{z^3} + \cdots\right]$

17. Find the Laurent's series for $f(z)=\frac{4z+3}{z(z-3)(z+2)}$ valid for $2<|z|<3$

[N22/MTRX/6M][M24/CompITAI/6M]

Solution:

We have, $f(z)=\frac{4z+3}{z(z-3)(z+2)}$

Let $\frac{4z+3}{z(z-3)(z+2)}=\frac{A}{z}+\frac{B}{z-3}+\frac{C}{z+2}$

$4z+3=A(z-3)(z+2)+Bz(z+2)+Cz(z-3)$

$4z+3=A(z^2-z-6)+B(z^2+2z)+C(z^2-3z)$

Comparing the coefficients, we get

$A+B+C=0$

$-A+2B-3C=4$

$-6A+0B+0C=3$

On solving, we get

$A=-\frac{1}{2},B=1,C=-\frac{1}{2}$

$f(z)=\frac{-\frac{1}{2}}{z}+\frac{1}{z-3}-\frac{\frac{1}{2}}{z+2}$

For $2<|z|<3$

$f(z)=-\frac{1}{2z}+\frac{1}{-3+z}-\frac{\frac{1}{2}}{z+2}$

$f(z)=-\frac{1}{2z}+\frac{1}{-3\left(1-\frac{z}{3}\right)}-\frac{\frac{1}{2}}{z\left(1+\frac{2}{z}\right)}$

$f(z)=-\frac{1}{2z}-\frac{1}{3}\left[1-\frac{z}{3}\right]^{-1}-\frac{1}{2z}\left[1+\frac{2}{z}\right]^{-1}$

$f(z)=-\frac{1}{2z}-\frac{1}{3}\left[1+\frac{z}{3}+\frac{z^2}{3^2}+\frac{z^3}{3^3}+\cdots.\right]-\frac{1}{2z}\left[1-\frac{2}{z}+\frac{2^2}{z^2}-\frac{2^3}{z^3}+\cdots\right]$

18. Obtain the Taylor's and Laurent series which represent the function $\frac{1}{(z+1)(z+3)}$ in the regions (i) $|z| < 1$ (ii) $1 < |z| < 3$ (iii) $|z| > 3$

[N17/MTRX/6M]

Solution:

We have, $f(z) = \frac{1}{(z+1)(z+3)}$

Let $\frac{1}{(z+1)(z+3)} = \frac{A}{z+1} + \frac{B}{z+3}$

$1 = A(z+3) + B(z+1)$

Comparing the coefficients, we get

$A + B = 0$

$3A + B = 1$

On solving, we get

$A = \frac{1}{2}, B = -\frac{1}{2}$

$f(z) = \frac{\frac{1}{2}}{z+1} - \frac{\frac{1}{2}}{z+3}$

(i) $|z| < 1$

$f(z) = \frac{\frac{1}{2}}{1+z} - \frac{\frac{1}{2}}{3+z}$

$f(z) = \frac{1}{2}[1+z]^{-1} - \frac{\frac{1}{2}}{3\left[1+\frac{z}{3}\right]}$

$f(z) = \frac{1}{2}[1+z]^{-1} - \frac{1}{6}\left[1+\frac{z}{3}\right]^{-1}$

$f(z) = \frac{1}{2}[1 - z + z^2 - z^3 + \cdots.] - \frac{1}{6}\left[1 - \frac{z}{3} + \frac{z^2}{3^2} - \frac{z^3}{3^3} + \cdots\right]$

(ii) $1 < |z| < 3$

$f(z) = \frac{\frac{1}{2}}{z+1} - \frac{\frac{1}{2}}{3+z}$

$f(z) = \frac{\frac{1}{2}}{z\left[1+\frac{1}{z}\right]} - \frac{\frac{1}{2}}{3\left[1+\frac{z}{3}\right]}$

$f(z) = \frac{1}{2z}\left[1+\frac{1}{z}\right]^{-1} - \frac{1}{6}\left[1+\frac{z}{3}\right]^{-1}$

$f(z) = \frac{1}{2z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots.\right] - \frac{1}{6}\left[1 - \frac{z}{3} + \frac{z^2}{3^2} - \frac{z^3}{3^3} + \cdots\right]$

(iii) $|z| > 3$

$f(z) = \frac{\frac{1}{2}}{z+1} - \frac{\frac{1}{2}}{z+3}$

$f(z) = \frac{\frac{1}{2}}{z\left[1+\frac{1}{z}\right]} - \frac{\frac{1}{2}}{z\left[1+\frac{3}{z}\right]}$

$f(z) = \frac{1}{2z}\left[1+\frac{1}{z}\right]^{-1} - \frac{1}{2z}\left[1+\frac{3}{z}\right]^{-1}$

$f(z) = \frac{1}{2z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots.\right] - \frac{1}{2z}\left[1 - \frac{3}{z} + \frac{3^2}{z^2} - \frac{3^3}{z^3} + \cdots\right]$

19. Obtain the Taylor's and Laurent series which represent the function $\frac{z^2-1}{(z+3)(z+4)}$ in the regions (i) $|z| < 3$ (ii) $3 < |z| < 4$ (iii) $|z| > 4$

[M17/MechCivil/8M]

Solution:

We have, $f(z) = \frac{z^2-1}{(z+3)(z+4)}$

$f(z) = \frac{z^2-1}{z^2+7z+12}$

$f(z) = \frac{z^2+7z+12-7z-12-1}{z^2+7z+12}$

$f(z) = 1 + \frac{-7z-13}{z^2+7z+12}$

$f(z) = 1 + \frac{-7z-13}{(z+3)(z+4)}$

Let $\frac{-7z-13}{(z+3)(z+4)} = \frac{A}{z+3} + \frac{B}{z+4}$

$-7z - 13 = A(z+4) + B(z+3)$

On comparing the coefficients, we get

$A + B = -7$

$4A + 3B = -13$

On solving, we get

$A = 8, B = -15$

$f(z) = 1 + \frac{8}{z+3} - \frac{15}{z+4}$

(i) $|z| < 3$

$f(z) = 1 + \frac{8}{3+z} - \frac{15}{4+z} = 1 + \frac{8}{3\left(1+\frac{z}{3}\right)} - \frac{15}{4\left(1+\frac{z}{4}\right)}$

$f(z) = 1 + \frac{8}{3}\left[1 + \frac{z}{3}\right]^{-1} - \frac{15}{4}\left[1 + \frac{z}{4}\right]^{-1}$

$f(z) = 1 + \frac{8}{3}\left[1 - \frac{z}{3} + \frac{z^2}{3^2} - \frac{z^3}{3^3} + \cdots..\right] - \frac{15}{4}\left[1 - \frac{z}{4} + \frac{z^2}{4^2} - \frac{z^3}{4^3} + \cdots\right]$

(ii) $3 < |z| < 4$

$f(z) = 1 + \frac{8}{z+3} - \frac{15}{4+z} = 1 + \frac{8}{z\left(1+\frac{3}{z}\right)} - \frac{15}{4\left(1+\frac{z}{4}\right)}$

$f(z) = 1 + \frac{8}{z}\left[1 + \frac{3}{z}\right]^{-1} - \frac{15}{4}\left[1 + \frac{z}{4}\right]^{-1}$

$f(z) = 1 + \frac{8}{z}\left[1 - \frac{3}{z} + \frac{3^2}{z^2} - \frac{3^3}{z^3} + \cdots..\right] - \frac{15}{4}\left[1 - \frac{z}{4} + \frac{z^2}{4^2} - \frac{z^3}{4^3} + \cdots\right]$

(iii) $|z| > 4$

$f(z) = 1 + \frac{8}{z+3} - \frac{15}{z+4}$

$f(z) = 1 + \frac{8}{z\left(1+\frac{3}{z}\right)} - \frac{15}{z\left(1+\frac{4}{z}\right)}$

$f(z) = 1 + \frac{8}{z}\left[1 + \frac{3}{z}\right]^{-1} - \frac{15}{z}\left[1 + \frac{4}{z}\right]^{-1}$

$f(z) = 1 + \frac{8}{z}\left[1 - \frac{3}{z} + \frac{3^2}{z^2} - \frac{3^3}{z^3} + \cdots..\right] - \frac{15}{z}\left[1 - \frac{4}{z} + \frac{4^2}{z^2} - \frac{4^3}{z^3} + \cdots\right]$

20. Obtain Laurent's series expansion of $f(z)=\frac{1}{z^2+4z+3}$ where (i) $|z|<1$ (ii) $1<|z|<3$ (iii) $|z|>3$

[N22/CompITAI/6M]

Solution:

We have, $f(z)=\frac{1}{z^2+4z+3}=\frac{1}{(z+1)(z+3)}$

Let $\frac{1}{(z+1)(z+3)}=\frac{A}{z+1}+\frac{B}{z+3}$

$1=A(z+3)+B(z+1)$

Comparing the coefficients, we get

$A+B=0$

$3A+B=1$

On solving, we get

$A=\frac{1}{2},B=-\frac{1}{2}$

$f(z)=\frac{\frac{1}{2}}{z+1}-\frac{\frac{1}{2}}{z+3}$

(i) $|z|<1$

$f(z)=\frac{\frac{1}{2}}{1+z}-\frac{\frac{1}{2}}{3+z}$

$f(z)=\frac{1}{2}[1+z]^{-1}-\frac{\frac{1}{2}}{3\left[1+\frac{z}{3}\right]}$

$f(z)=\frac{1}{2}[1+z]^{-1}-\frac{1}{6}\left[1+\frac{z}{3}\right]^{-1}$

$f(z)=\frac{1}{2}[1-z+z^2-z^3+\cdots.]-\frac{1}{6}\left[1-\frac{z}{3}+\frac{z^2}{3^2}-\frac{z^3}{3^3}+\cdots\right]$

(ii) $1<|z|<3$

$f(z)=\frac{\frac{1}{2}}{z+1}-\frac{\frac{1}{2}}{3+z}$

$f(z)=\frac{\frac{1}{2}}{z\left[1+\frac{1}{z}\right]}-\frac{\frac{1}{2}}{3\left[1+\frac{z}{3}\right]}$

$f(z)=\frac{1}{2z}\left[1+\frac{1}{z}\right]^{-1}-\frac{1}{6}\left[1+\frac{z}{3}\right]^{-1}$

$f(z)=\frac{1}{2z}\left[1-\frac{1}{z}+\frac{1}{z^2}-\frac{1}{z^3}+\cdots.\right]-\frac{1}{6}\left[1-\frac{z}{3}+\frac{z^2}{3^2}-\frac{z^3}{3^3}+\cdots\right]$

(iii) $|z|>3$

$f(z)=\frac{\frac{1}{2}}{z+1}-\frac{\frac{1}{2}}{z+3}$

$f(z)=\frac{\frac{1}{2}}{z\left[1+\frac{1}{z}\right]}-\frac{\frac{1}{2}}{z\left[1+\frac{3}{z}\right]}$

$f(z)=\frac{1}{2z}\left[1+\frac{1}{z}\right]^{-1}-\frac{1}{2z}\left[1+\frac{3}{z}\right]^{-1}$

$f(z)=\frac{1}{2z}\left[1-\frac{1}{z}+\frac{1}{z^2}-\frac{1}{z^3}+\cdots.\right]-\frac{1}{2z}\left[1-\frac{3}{z}+\frac{3^2}{z^2}-\frac{3^3}{z^3}+\cdots\right]$

21. Expand $f(z)=\frac{1}{z^2(z-1)(z+2)}$ about $z=0$ for
(i) $|z|<1$ (ii) $1<|z|<2$
[N15/MechCivil/8M]
(iii) $|z|>2$ indicating the region of convergence in each case.
[M18/Chem/8M][N18/Extc/8M]
Solution:
We have, $f(z)=\frac{1}{z^2(z-1)(z+2)}$
Let $\frac{1}{z^2(z-1)(z+2)}=\frac{A}{z}+\frac{B}{z^2}+\frac{C}{z-1}+\frac{D}{z+2}$
$1=Az(z-1)(z+2)+B(z-1)(z+2)+Cz^2(z+2)+Dz^2(z-1)$
$1=A(z^3+z^2-2z)+B(z^2+z-2)+C(z^3+2z^2)+D(z^3-z^2)$
Comparing the coefficients, we get
$A+C+D=0$
$A+B+2C-D=0$
$-2A+B=0$
$-2B=1$
On solving, we get
$A=-\frac{1}{4}, B=-\frac{1}{2}, C=\frac{1}{3}, D=-\frac{1}{12}$
$f(z)=\frac{-\frac{1}{4}}{z}-\frac{\frac{1}{2}}{z^2}+\frac{\frac{1}{3}}{z-1}-\frac{\frac{1}{12}}{z+2}$
(i) $|z|<1$
$f(z)=-\frac{1}{4z}-\frac{1}{2z^2}+\frac{\frac{1}{3}}{-1+z}-\frac{\frac{1}{12}}{2+z}$
$f(z)=-\frac{1}{4z}-\frac{1}{2z^2}+\frac{\frac{1}{3}}{-(1-z)}-\frac{\frac{1}{12}}{2\left(1+\frac{z}{2}\right)}$
$f(z)=-\frac{1}{4z}-\frac{1}{2z^2}-\frac{1}{3}[1-z]^{-1}-\frac{1}{24}\left[1+\frac{z}{2}\right]^{-1}$
$f(z)=-\frac{1}{4z}-\frac{1}{2z^2}-\frac{1}{3}[1+z+z^2+z^3+\cdots.]-\frac{1}{24}\left[1-\frac{z}{2}+\frac{z^2}{2^2}-\frac{z^3}{2^3}+\cdots\right]$
(ii) $1<|z|<2$
$f(z)=-\frac{1}{4z}-\frac{1}{2z^2}+\frac{\frac{1}{3}}{z-1}-\frac{\frac{1}{12}}{2+z}$
$f(z)=-\frac{1}{4z}-\frac{1}{2z^2}+\frac{\frac{1}{3}}{z\left(1-\frac{1}{z}\right)}-\frac{\frac{1}{12}}{2\left(1+\frac{z}{2}\right)}$
$f(z)=-\frac{1}{4z}-\frac{1}{2z^2}+\frac{1}{3z}\left[1-\frac{1}{z}\right]^{-1}-\frac{1}{24}\left[1+\frac{z}{2}\right]^{-1}$
$f(z)=-\frac{1}{4z}-\frac{1}{2z^2}+\frac{1}{3z}\left[1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\cdots.\right]-\frac{1}{24}\left[1-\frac{z}{2}+\frac{z^2}{2^2}-\frac{z^3}{2^3}+\cdots\right]$
(ii) $|z|>2$
$f(z)=-\frac{1}{4z}-\frac{1}{2z^2}+\frac{\frac{1}{3}}{z-1}-\frac{\frac{1}{12}}{z+2}$

$$f(z) = -\frac{1}{4z} - \frac{1}{2z^2} + \frac{\frac{1}{3}}{z\left(1-\frac{1}{z}\right)} - \frac{\frac{1}{12}}{z\left(1+\frac{2}{z}\right)}$$

$$f(z) = -\frac{1}{4z} - \frac{1}{2z^2} + \frac{1}{3z}\left[1 - \frac{1}{z}\right]^{-1} - \frac{1}{12z}\left[1 + \frac{2}{z}\right]^{-1}$$

$$f(z) = -\frac{1}{4z} - \frac{1}{2z^2} + \frac{1}{3z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots.\right] - \frac{1}{12z}\left[1 - \frac{2}{z} + \frac{2^2}{z^2} - \frac{2^3}{z^3} + \cdots\right]$$

22. Obtain the Laurent's series for $f(z) = \frac{z}{(z-1)(z+3)}$ in $1 < |z| < 3$

[M22/Elect/5M]

Solution:

We have, $f(z) = \frac{z}{(z-1)(z+3)}$

Let $\frac{z}{(z-1)(z+3)} = \frac{A}{z-1} + \frac{B}{z+3}$

$z = A(z+3) + B(z-1)$

Comparing the coefficients, we get

$A + B = 1$

$3A - B = 0$

On solving, we get

$A = \frac{1}{4}, B = \frac{3}{4}$

$$f(z) = \frac{\frac{1}{4}}{z-1} + \frac{\frac{3}{4}}{z+3}$$

For $1 < |z| < 3$

$$f(z) = \frac{\frac{1}{4}}{z-1} + \frac{\frac{1}{4}}{3+z}$$

$$f(z) = \frac{\frac{1}{4}}{z\left[1-\frac{1}{z}\right]} + \frac{\frac{1}{4}}{3\left[1+\frac{z}{3}\right]}$$

$$f(z) = \frac{1}{4z}\left[1 - \frac{1}{z}\right]^{-1} + \frac{1}{12}\left[1 + \frac{z}{3}\right]^{-1}$$

$$f(z) = \frac{1}{4z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots.\right] + \frac{1}{12}\left[1 - \frac{z}{3} + \frac{z^2}{3^2} - \frac{z^3}{3^3} + \cdots\right]$$

23. Obtain the Taylor's and Laurent's series which represent the function $f(z) = \frac{z+2}{(z-1)(z-2)}$ in the regions (i) $|z| < 1$ (ii) $1 < |z| < 2$

[N19/MTRX/6M]

Solution:

We have, $f(z) = \frac{z+2}{(z-1)(z-2)}$

Let $\frac{z+2}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$

$z + 2 = A(z-2) + B(z-1)$

Comparing the coefficients, we get

$A + B = 1$

$-2A - B = 2$

On solving, we get

$A = -3, B = 4$

$f(z) = -\frac{3}{z-1} + \frac{4}{z-2}$

(i) $|z| < 1$

$f(z) = -\frac{3}{-1+z} + \frac{4}{-2+z}$

$f(z) = -\frac{3}{-[1-z]} + \frac{4}{-2\left[1-\frac{z}{2}\right]}$

$f(z) = 3[1-z]^{-1} - 2\left[1-\frac{z}{2}\right]^{-1}$

$f(z) = 3[1 + z + z^2 + z^3 + \cdots.] - 2\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots\right]$

(ii) $1 < |z| < 2$

$f(z) = -\frac{3}{z-1} + \frac{4}{-2+z}$

$f(z) = -\frac{3}{z\left[1-\frac{1}{z}\right]} + \frac{4}{-2\left[1-\frac{z}{2}\right]}$

$f(z) = -\frac{3}{z}\left[1-\frac{1}{z}\right]^{-1} - 2\left[1-\frac{z}{2}\right]^{-1}$

$f(z) = -\frac{3}{z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots.\right] - 2\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots\right]$

24. Find the Laurent series expansion of $\frac{z+2}{z^2-1}$ convergent in the domain $|z| > 1$

[N22/Elex/6M][M23/ElectECS/6M]

Solution:

We have, $f(z) = \frac{z+2}{z^2-1} = \frac{z+2}{(z+1)(z-1)}$

Let $\frac{z+2}{(z+1)(z-1)} = \frac{A}{z+1} + \frac{B}{z-1}$

$z + 2 = A(z-1) + B(z+1)$

Comparing the coefficients, we get

$A + B = 1$

$-A + B = 2$

On solving, we get

$A = -\frac{1}{2}, B = \frac{3}{2}$

$f(z) = -\frac{\frac{1}{2}}{z+1} + \frac{\frac{3}{2}}{z-1}$

For $|z| > 1$

$f(z) = -\frac{\frac{1}{2}}{z+1} + \frac{\frac{3}{2}}{z-1}$

$f(z) = -\frac{\frac{1}{2}}{z\left[1+\frac{1}{z}\right]} + \frac{\frac{3}{2}}{z\left[1-\frac{1}{z}\right]}$

$f(z) = -\frac{1}{2z}\left[1+\frac{1}{z}\right]^{-1} + \frac{3}{2z}\left[1-\frac{1}{z}\right]^{-1}$

$f(z) = -\frac{1}{2z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots.\right] + \frac{3}{2z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots\right]$

25. Find all possible expansions of $f(z) = \frac{1}{(z-1)(z-2)}$

[M19/Elex/8M][M19/Extc/8M]

Solution:

We have, $f(z) = \frac{1}{(z-1)(z-2)}$

Let $\frac{1}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$

$1 = A(z-2) + B(z-1)$

Comparing the coefficients, we get

$A + B = 0$

$-2A - B = 1$

On solving, we get

$A = -1, B = 1$

$f(z) = -\frac{1}{z-1} + \frac{1}{z-2}$

For ROC, put $z - 1 = 0, z - 2 = 0$

$z = 1, z = 2$

$|z| = 1, |z| = 2$

Thus, the ROC are (i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$

(i) $|z| < 1$

$f(z) = -\frac{1}{-1+z} + \frac{1}{-2+z}$

$f(z) = -\frac{1}{-[1-z]} + \frac{1}{-2\left[1-\frac{z}{2}\right]}$

$f(z) = [1-z]^{-1} - \frac{1}{2}\left[1-\frac{z}{2}\right]^{-1}$

$f(z) = [1 + z + z^2 + z^3 + \cdots.] - \frac{1}{2}\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots\right]$

(ii) $1 < |z| < 2$

$f(z) = -\frac{1}{z-1} + \frac{1}{-2+z}$

$f(z) = -\frac{1}{z\left[1-\frac{1}{z}\right]} + \frac{1}{-2\left[1-\frac{z}{2}\right]}$

$f(z) = -\frac{1}{z}\left[1-\frac{1}{z}\right]^{-1} - \frac{1}{2}\left[1-\frac{z}{2}\right]^{-1}$

$f(z) = -\frac{1}{z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots.\right] - \frac{1}{2}\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots\right]$

(iii) $|z| > 2$

$f(z) = -\frac{1}{z-1} + \frac{1}{z-2}$

$f(z) = -\frac{1}{z\left[1-\frac{1}{z}\right]} + \frac{1}{z\left[1-\frac{2}{z}\right]}$

$f(z) = -\frac{1}{z}\left[1-\frac{1}{z}\right]^{-1} + \frac{1}{z}\left[1-\frac{2}{z}\right]^{-1}$

$f(z) = -\frac{1}{z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots.\right] + \frac{1}{z}\left[1 + \frac{2}{z} + \frac{2^2}{z^2} + \frac{2^3}{z^3} + \cdots\right]$

26. Find all possible Laurent's series expansions of $f(z)=\frac{1}{(z+1)(z-2)}$ about $z=0$ indicating the region of convergence in each case

[D24/CompIT/8M]

Solution:

We have, $f(z)=\frac{1}{(z+1)(z-2)}$

Let $\frac{1}{(z+1)(z-2)}=\frac{A}{z+1}+\frac{B}{z-2}$

$1=A(z-2)+B(z+1)$

Comparing the coefficients, we get

$A+B=0$

$-2A+B=1$

On solving, we get

$A=-\frac{1}{3}, B=\frac{1}{3}$

$f(z)=\frac{-\frac{1}{3}}{z+1}+\frac{\frac{1}{3}}{z-2}$

(i) $|z|<1$

$f(z)=\frac{-\frac{1}{3}}{1+z}+\frac{\frac{1}{3}}{-2+z}$

$f(z)=\frac{-\frac{1}{3}}{[1+z]}+\frac{\frac{1}{3}}{-2\left[1-\frac{z}{2}\right]}$

$f(z)=-\frac{1}{3}[1+z]^{-1}-\frac{1}{6}\left[1-\frac{z}{2}\right]^{-1}$

$f(z)=-\frac{1}{3}[1-z+z^2-z^3+\cdots.]-\frac{1}{6}\left[1+\frac{z}{2}+\frac{z^2}{2^2}+\frac{z^3}{2^3}+\cdots\right]$

(ii) $1<|z|<2$

$f(z)=\frac{-\frac{1}{3}}{z+1}+\frac{\frac{1}{3}}{-2+z}$

$f(z)=\frac{-\frac{1}{3}}{z\left[1+\frac{1}{z}\right]}+\frac{\frac{1}{3}}{-2\left[1-\frac{z}{2}\right]}$

$f(z)=-\frac{1}{3z}\left[1+\frac{1}{z}\right]^{-1}-\frac{1}{6}\left[1-\frac{z}{2}\right]^{-1}$

$f(z)=-\frac{1}{3z}\left[1-\frac{1}{z}+\frac{1}{z^2}-\frac{1}{z^3}+\cdots.\right]-\frac{1}{6}\left[1+\frac{z}{2}+\frac{z^2}{2^2}+\frac{z^3}{2^3}+\cdots\right]$

(iii) $|z|>2$

$f(z)=\frac{-\frac{1}{3}}{z+1}+\frac{\frac{1}{3}}{z-2}$

$f(z)=\frac{-\frac{1}{3}}{z\left[1+\frac{1}{z}\right]}+\frac{\frac{1}{3}}{z\left[1-\frac{2}{z}\right]}$

$f(z)=-\frac{1}{3z}\left[1+\frac{1}{z}\right]^{-1}+\frac{1}{3z}\left[1-\frac{2}{z}\right]^{-1}$

$f(z)=-\frac{1}{3z}\left[1-\frac{1}{z}+\frac{1}{z^2}-\frac{1}{z^3}+\cdots.\right]+\frac{1}{3z}\left[1+\frac{2}{z}+\frac{2^2}{z^2}+\frac{2^3}{z^3}+\cdots\right]$

27. Obtain all possible Taylor's and Laurent's Series which represent the function $f(z) = \frac{z}{z^2-5z+6}$ indicating region of convergence

[M19/MechCivil/6M]

Solution:

We have, $f(z) = \frac{z}{z^2-5z+6}$

Let $\frac{z}{(z-2)(z-3)} = \frac{A}{z-2} + \frac{B}{z-3}$

$z = A(z-3) + B(z-2)$

Comparing the coefficients, we get

$A + B = 1$

$-3A - 2B = 0$

On solving, we get

$A = -2, B = 3$

$f(z) = \frac{-2}{z-2} + \frac{3}{z-3}$

For ROC, put $z - 2 = 0, z - 3 = 0$

$z = 2, z = 3$

$|z| = 2, |z| = 3$

The ROCs are as follows (i) $|z| < 2$ (ii) $2 < |z| < 3$ (iii) $|z| > 3$

(i) For $|z| < 2$

$f(z) = \frac{-2}{-2+z} + \frac{3}{-3+z}$

$f(z) = \frac{-2}{-2\left(1-\frac{z}{2}\right)} + \frac{3}{-3\left(1-\frac{z}{3}\right)}$

$f(z) = \left[1 - \frac{z}{2}\right]^{-1} - \left[1 - \frac{z}{3}\right]^{-1}$

$f(z) = \left[1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + \cdots..\right] - \left[1 + \frac{z}{3} + \frac{z^2}{9} + \frac{z^3}{27} + \cdots.\right]$

(ii) For $2 < |z| < 3$

$f(z) = \frac{-2}{z-2} + \frac{3}{-3+z}$

$f(z) = \frac{-2}{z\left(1-\frac{2}{z}\right)} + \frac{3}{-3\left(1-\frac{z}{3}\right)}$

$f(z) = -\frac{2}{z}\left[1 - \frac{2}{z}\right]^{-1} - \left[1 - \frac{z}{3}\right]^{-1}$

$f(z) = -\frac{2}{z}\left[1 + \frac{2}{z} + \frac{4}{z^2} + \frac{8}{z^3} + \cdots..\right] - \left[1 + \frac{z}{3} + \frac{z^2}{9} + \frac{z^3}{27} + \cdots.\right]$

(ii) For $|z| > 3$

$f(z) = \frac{-2}{z-2} + \frac{3}{z-3}$

$f(z) = \frac{-2}{z\left(1-\frac{2}{z}\right)} + \frac{3}{z\left(1-\frac{3}{z}\right)}$

$f(z) = -\frac{2}{z}\left[1 - \frac{2}{z}\right]^{-1} + \frac{3}{z}\left[1 - \frac{3}{z}\right]^{-1}$

$f(z) = -\frac{2}{z}\left[1 + \frac{2}{z} + \frac{4}{z^2} + \frac{8}{z^3} + \cdots..\right] + \frac{3}{z}\left[1 + \frac{3}{z} + \frac{9}{z^2} + \frac{27}{z^3} + \cdots.\right]$

28. Obtain all possible Laurent's series expansion of $f(z) = \frac{1}{z^2+3z+2}$ about $z = 0$

[D24/ElectECS/8M]

Solution:

We have, $f(z) = \frac{1}{z^2+3z+2} = \frac{1}{(z+1)(z+2)}$

Let $\frac{1}{(z+1)(z+2)} = \frac{A}{z+1} + \frac{B}{z+2}$

$1 = A(z+2) + B(z+1)$

Comparing the coefficients, we get

$A + B = 0$

$2A + B = 1$

On solving, we get

$A = 1, B = -1$

$f(z) = \frac{1}{z+1} - \frac{1}{z+2}$

(i) $|z| < 1$

$f(z) = \frac{1}{1+z} - \frac{1}{2+z}$

$f(z) = \frac{1}{[1+z]} - \frac{1}{2\left[1+\frac{z}{2}\right]}$

$f(z) = [1+z]^{-1} - \frac{1}{2}\left[1+\frac{z}{2}\right]^{-1}$

$f(z) = [1 - z + z^2 - z^3 + \cdots.] - \frac{1}{2}\left[1 - \frac{z}{2} + \frac{z^2}{2^2} - \frac{z^3}{2^3} + \cdots\right]$

(ii) $1 < |z| < 2$

$f(z) = \frac{1}{z+1} - \frac{1}{2+z}$

$f(z) = \frac{1}{z\left[1+\frac{1}{z}\right]} - \frac{1}{2\left[1+\frac{z}{2}\right]}$

$f(z) = \frac{1}{z}\left[1+\frac{1}{z}\right]^{-1} - \frac{1}{2}\left[1+\frac{z}{2}\right]^{-1}$

$f(z) = \frac{1}{z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots.\right] - \frac{1}{2}\left[1 - \frac{z}{2} + \frac{z^2}{2^2} - \frac{z^3}{2^3} + \cdots\right]$

(iii) $|z| > 2$

$f(z) = \frac{1}{z+1} - \frac{1}{z+2}$

$f(z) = \frac{1}{z\left[1+\frac{1}{z}\right]} - \frac{1}{z\left[1+\frac{2}{z}\right]}$

$f(z) = \frac{1}{z}\left[1+\frac{1}{z}\right]^{-1} - \frac{1}{z}\left[1+\frac{2}{z}\right]^{-1}$

$f(z) = \frac{1}{z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots.\right] - \frac{1}{z}\left[1 - \frac{2}{z} + \frac{2^2}{z^2} - \frac{2^3}{z^3} + \cdots\right]$

29. Obtain two distinct Laurent's series for $f(z) = \frac{2z-3}{z^2-4z+3}$ indicating the region of convergence

[M19/MTRX/6M][D23/MechCivil/8M]

Solution:

We have, $f(z) = \frac{2z-3}{(z-1)(z-3)}$

Let $\frac{2z-3}{(z-1)(z-3)} = \frac{A}{z-1} + \frac{B}{z-3}$

$2z - 3 = A(z-3) + B(z-1)$

Comparing the coefficients, we get

$A + B = 2$

$-3A - B = -3$

On solving, we get

$A = \frac{1}{2}, B = \frac{3}{2}$

$f(z) = \frac{\frac{1}{2}}{z-1} + \frac{\frac{3}{2}}{z-3}$

For ROC, put $z - 1 = 0, z - 3 = 0$

$z = 1, z = 3$

$|z| = 1, |z| = 3$

Thus, the ROC are (i) $|z| < 1$ (ii) $1 < |z| < 3$ (iii) $|z| > 3$

(i) $1 < |z| < 3$

$f(z) = \frac{\frac{1}{2}}{z-1} + \frac{\frac{3}{2}}{-3+z}$

$f(z) = \frac{\frac{1}{2}}{z\left[1-\frac{1}{z}\right]} + \frac{\frac{3}{2}}{-3\left[1-\frac{z}{3}\right]}$

$f(z) = \frac{1}{2z}\left[1 - \frac{1}{z}\right]^{-1} - \frac{1}{2}\left[1 - \frac{z}{3}\right]^{-1}$

$f(z) = \frac{1}{2z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots.\right] - \frac{1}{2}\left[1 + \frac{z}{3} + \frac{z^2}{3^2} + \frac{z^3}{3^3} + \cdots\right]$

(ii) $|z| > 3$

$f(z) = \frac{\frac{1}{2}}{z-1} + \frac{\frac{3}{2}}{z-3}$

$f(z) = \frac{\frac{1}{2}}{z\left[1-\frac{1}{z}\right]} + \frac{\frac{3}{2}}{z\left[1-\frac{3}{z}\right]}$

$f(z) = \frac{1}{2z}\left[1 - \frac{1}{z}\right]^{-1} + \frac{3}{2z}\left[1 - \frac{3}{z}\right]^{-1}$

$f(z) = \frac{1}{2z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots.\right] + \frac{3}{2z}\left[1 + \frac{3}{z} + \frac{3^2}{z^2} + \frac{3^3}{z^3} + \cdots\right]$

30. Find all possible expansions of $f(z) = \frac{2-z^2}{z(1-z)(2-z)}$

[N19/Elex/8M]

Solution:

We have, $f(z) = \frac{2-z^2}{z(2-z)(1-z)}$

Let $\frac{2-z^2}{z(2-z)(1-z)} = \frac{A}{z} + \frac{B}{2-z} + \frac{C}{1-z}$

$2 - z^2 = A(2-z)(1-z) + Bz(1-z) + Cz(2-z)$

$2 - z^2 = A(z^2 - 3z + 2) + B(z - z^2) + C(2z - z^2)$

On comparing the coefficients, we get

$A - B - C = -1$

$-3A + B + 2C = 0$

$2A + 0B + 0C = 2$

On solving, we get

$A = 1, B = 1, C = 1$

$f(z) = \frac{1}{z} + \frac{1}{2-z} + \frac{1}{1-z}$

For ROC, put $z = 0, 2 - z = 0, 1 - z = 0$

$z = 0, z = 1, z = 2$

$|z| = 1, |z| = 2$

Thus, the ROC are (i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$

(i) $|z| < 1$

$f(z) = \frac{1}{z} + \frac{1}{-2+z} + \frac{1}{1-z} = \frac{1}{z} + \frac{1}{-2\left[1-\frac{z}{2}\right]} + \frac{1}{[1-z]}$

$f(z) = \frac{1}{z} - \frac{1}{2}\left[1 - \frac{z}{2}\right]^{-1} + [1-z]^{-1}$

$f(z) = \frac{1}{z} - \frac{1}{2}\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots.\right] + [1 + z + z^2 + z^3 + \cdots.]$

(ii) $1 < |z| < 2$

$f(z) = \frac{1}{z} + \frac{1}{-2+z} + \frac{1}{z-1} = \frac{1}{z} + \frac{1}{-2\left[1-\frac{z}{2}\right]} + \frac{1}{z\left[1-\frac{1}{z}\right]}$

$f(z) = \frac{1}{z} - \frac{1}{2}\left[1 - \frac{z}{2}\right]^{-1} + \frac{1}{z}\left[1 - \frac{1}{z}\right]^{-1}$

$f(z) = \frac{1}{z} - \frac{1}{2}\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots.\right] + \frac{1}{z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots.\right]$

(iii) $|z| > 2$

$f(z) = \frac{1}{z} + \frac{1}{z-2} + \frac{1}{z-1}$

$f(z) = \frac{1}{z} + \frac{1}{z\left[1-\frac{2}{z}\right]} + \frac{1}{z\left[1-\frac{1}{z}\right]}$

$f(z) = \frac{1}{z} + \frac{1}{z}\left[1 - \frac{2}{z}\right]^{-1} + \frac{1}{z}\left[1 - \frac{1}{z}\right]^{-1}$

$f(z) = \frac{1}{z} + \frac{1}{z}\left[1 + \frac{2}{z} + \frac{2^2}{z^2} + \frac{2^3}{z^3} + \cdots.\right] + \frac{1}{z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots.\right]$

31. Find all possible Laurent's series expansions of the function

$f(z) = \frac{2-z^2}{z(1-z)(2-z)}$ about $z = 0$

[M24/MechCivil/8M]

Solution:

We have, $f(z) = \frac{2-z^2}{z(2-z)(1-z)}$

Let $\frac{2-z^2}{z(2-z)(1-z)} = \frac{A}{z} + \frac{B}{2-z} + \frac{C}{1-z}$

$2 - z^2 = A(2-z)(1-z) + Bz(1-z) + Cz(2-z)$

$2 - z^2 = A(z^2 - 3z + 2) + B(z - z^2) + C(2z - z^2)$

On comparing the coefficients, we get

$A - B - C = -1$

$-3A + B + 2C = 0$

$2A + 0B + 0C = 2$

On solving, we get

$A = 1, B = 1, C = 1$

$f(z) = \frac{1}{z} + \frac{1}{2-z} + \frac{1}{1-z}$

For ROC, put $z = 0, 2 - z = 0, 1 - z = 0$

$z = 0, z = 1, z = 2$

$|z| = 1, |z| = 2$

Thus, the ROC are (i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$

(i) $|z| < 1$

$f(z) = \frac{1}{z} + \frac{1}{-2+z} + \frac{1}{1-z} = \frac{1}{z} + \frac{1}{-2\left[1-\frac{z}{2}\right]} + \frac{1}{[1-z]}$

$f(z) = \frac{1}{z} - \frac{1}{2}\left[1 - \frac{z}{2}\right]^{-1} + [1-z]^{-1}$

$f(z) = \frac{1}{z} - \frac{1}{2}\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots.\right] + [1 + z + z^2 + z^3 + \cdots.]$

(ii) $1 < |z| < 2$

$f(z) = \frac{1}{z} + \frac{1}{-2+z} + \frac{1}{z-1} = \frac{1}{z} + \frac{1}{-2\left[1-\frac{z}{2}\right]} + \frac{1}{z\left[1-\frac{1}{z}\right]}$

$f(z) = \frac{1}{z} - \frac{1}{2}\left[1 - \frac{z}{2}\right]^{-1} + \frac{1}{z}\left[1 - \frac{1}{z}\right]^{-1}$

$f(z) = \frac{1}{z} - \frac{1}{2}\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots.\right] + \frac{1}{z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots.\right]$

(iii) $|z| > 2$

$f(z) = \frac{1}{z} + \frac{1}{z-2} + \frac{1}{z-1}$

$f(z) = \frac{1}{z} + \frac{1}{z\left[1-\frac{2}{z}\right]} + \frac{1}{z\left[1-\frac{1}{z}\right]}$

$f(z) = \frac{1}{z} + \frac{1}{z}\left[1 - \frac{2}{z}\right]^{-1} + \frac{1}{z}\left[1 - \frac{1}{z}\right]^{-1}$

$f(z) = \frac{1}{z} + \frac{1}{z}\left[1 + \frac{2}{z} + \frac{2^2}{z^2} + \frac{2^3}{z^3} + \cdots.\right] + \frac{1}{z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots.\right]$

32. Obtain all Taylors & Laurent's expansions of function $\frac{(z-2)(z+2)}{(z+1)(z+4)}$ about $z = 0$

[M14/CompIT/8M]

Solution:

We have, $f(z) = \frac{(z-2)(z+2)}{(z+1)(z+4)} = \frac{z^2-4}{z^2+5z+4}$

$f(z) = \frac{z^2+5z+4-5z-4-4}{z^2+5z+4}$

$f(z) = 1 + \frac{-5z-8}{z^2+5z+4}$

Let $\frac{-5z-8}{(z+1)(z+4)} = \frac{A}{z+1} + \frac{B}{z+4}$

$-5z - 8 = A(z+4) + B(z+1)$

Comparing the coefficients, we get

$A + B = -5$

$4A + B = -8$

On solving, we get

$A = -1, B = -4$

$f(z) = 1 - \frac{1}{z+1} - \frac{4}{z+4}$

For ROC, put $z + 1 = 0, z + 4 = 0$

$z = -1, z = -4$

$|z| = 1, |z| = 4$

The ROCs are as follows (i) $|z| < 1$ (ii) $1 < |z| < 4$ (iii) $|z| > 4$

The Taylors series is given by

(i) For $|z| < 1$

$f(z) = 1 - \frac{1}{1+z} - \frac{4}{4+z}$

$f(z) = 1 - \frac{1}{1+z} - \frac{4}{4\left(1+\frac{z}{4}\right)}$

$f(z) = 1 - [1+z]^{-1} - \left[1 + \frac{z}{4}\right]^{-1}$

$f(z) = 1 - [1 - z + z^2 - z^3 + \cdots..] - \left[1 - \frac{z}{4} + \frac{z^2}{16} - \frac{z^3}{64} + \cdots.\right]$

The Laurent's series is given by

(ii) For $1 < |z| < 4$

$f(z) = 1 - \frac{1}{z+1} - \frac{4}{4+z}$

$f(z) = 1 - \frac{1}{z\left(1+\frac{1}{z}\right)} - \frac{4}{4\left(1+\frac{z}{4}\right)}$

$f(z) = 1 - \frac{1}{z}\left[1 + \frac{1}{z}\right]^{-1} - \left[1 + \frac{z}{4}\right]^{-1}$

$f(z) = 1 - \frac{1}{z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots..\right] - \left[1 - \frac{z}{4} + \frac{z^2}{16} - \frac{z^3}{64} + \cdots.\right]$

The second Laurent's series is given by

(iii) For $|z| > 4$

$f(z) = 1 - \frac{1}{z+1} - \frac{4}{z+4}$

$$f(z) = 1 - \frac{1}{z\left(1+\frac{1}{z}\right)} - \frac{4}{z\left(1+\frac{4}{z}\right)}$$

$$f(z) = 1 - \frac{1}{z}\left[1 + \frac{1}{z}\right]^{-1} - \frac{4}{z}\left[1 + \frac{4}{z}\right]^{-1}$$

$$f(z) = 1 - \frac{1}{z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots ..\right] - \frac{4}{z}\left[1 - \frac{4}{z} + \frac{16}{z^2} - \frac{64}{z^3} + \cdots .\right]$$

33. Expand $f(z) = \frac{z^2-1}{z^2+5z+6}$ about $z = 0$
[M14/ElexExtcElectBiomInst/8M][M17/CompIT/8M][M18/Elex/8M]
[N18/N19/Comp/8M][N18/Inst/8M]
Solution:
We have, $f(z) = \frac{z^2-1}{z^2+5z+6}$
$f(z) = \frac{z^2+5z+6-5z-6-1}{z^2+5z+6}$
$f(z) = 1 + \frac{-5z-7}{z^2+5z+6}$
Let $\frac{-5z-7}{(z+2)(z+3)} = \frac{A}{z+2} + \frac{B}{z+3}$
$-5z - 7 = A(z+3) + B(z+2)$
Comparing the coefficients, we get
$A + B = -5$
$3A + 2B = -7$
On solving, we get
$A = 3, B = -8$
$f(z) = 1 + \frac{3}{z+2} - \frac{8}{z+3}$
For ROC, put $z + 2 = 0, z + 3 = 0$
$z = -2, z = -3$
$|z| = 2, |z| = 3$
The ROCs are as follows (i) $|z| < 2$ (ii) $2 < |z| < 3$ (iii) $|z| > 3$
(i) For $|z| < 2$
$f(z) = 1 + \frac{3}{2+z} - \frac{8}{3+z}$
$f(z) = 1 + \frac{3}{2\left(1+\frac{z}{2}\right)} - \frac{8}{3\left(1+\frac{z}{3}\right)}$
$f(z) = 1 + \frac{3}{2}\left[1 + \frac{z}{2}\right]^{-1} - \frac{8}{3}\left[1 + \frac{z}{3}\right]^{-1}$
$f(z) = 1 + \frac{3}{2}\left[1 - \frac{z}{2} + \frac{z^2}{4} - \frac{z^3}{8} + \cdots..\right] - \frac{8}{3}\left[1 - \frac{z}{3} + \frac{z^2}{9} - \frac{z^3}{27} + \cdots.\right]$
(ii) For $2 < |z| < 3$
$f(z) = 1 + \frac{3}{z+2} - \frac{8}{3+z}$
$f(z) = 1 + \frac{3}{z\left(1+\frac{2}{z}\right)} - \frac{8}{3\left(1+\frac{z}{3}\right)}$
$f(z) = 1 + \frac{3}{z}\left[1 + \frac{2}{z}\right]^{-1} - \frac{8}{3}\left[1 + \frac{z}{3}\right]^{-1}$
$f(z) = 1 + \frac{3}{z}\left[1 - \frac{2}{z} + \frac{4}{z^2} - \frac{8}{z^3} + \cdots..\right] - \frac{8}{3}\left[1 - \frac{z}{3} + \frac{z^2}{9} - \frac{z^3}{27} + \cdots.\right]$
(ii) For $|z| > 3$
$f(z) = 1 + \frac{3}{z+2} - \frac{8}{z+3}$
$f(z) = 1 + \frac{3}{z\left(1+\frac{2}{z}\right)} - \frac{8}{z\left(1+\frac{3}{z}\right)}$

$$f(z) = 1 + \frac{3}{z}\left[1 + \frac{2}{z}\right]^{-1} - \frac{8}{z}\left[1 + \frac{3}{z}\right]^{-1}$$
$$f(z) = 1 + \frac{3}{z}\left[1 - \frac{2}{z} + \frac{4}{z^2} - \frac{8}{z^3} + \cdots..\right] - \frac{8}{z}\left[1 - \frac{3}{z} + \frac{9}{z^2} - \frac{27}{z^3} + \cdots.\right]$$

34. Find all possible Laurent's series expansion of $\frac{2z+1}{z^2+5z+6}$ about the origin

[M24/ElectECS/6M]

Solution:

We have, $f(z) = \frac{2z+1}{z^2+5z+6}$

Let $\frac{2z+1}{(z+2)(z+3)} = \frac{A}{z+2} + \frac{B}{z+3}$

$2z + 1 = A(z + 3) + B(z + 2)$

Comparing the coefficients, we get

$A + B = 2$

$3A + 2B = 1$

On solving, we get

$A = -3, B = 5$

$f(z) = -\frac{3}{z+2} + \frac{5}{z+3}$

For ROC, put $z + 2 = 0, z + 3 = 0$

$z = -2, z = -3$

$|z| = 2, |z| = 3$

The ROCs are as follows (i) $|z| < 2$ (ii) $2 < |z| < 3$ (iii) $|z| > 3$

(i) For $|z| < 2$

$f(z) = -\frac{3}{2+z} + \frac{5}{3+z}$

$f(z) = -\frac{3}{2\left(1+\frac{z}{2}\right)} + \frac{5}{3\left(1+\frac{z}{3}\right)}$

$f(z) = -\frac{3}{2}\left[1 + \frac{z}{2}\right]^{-1} + \frac{5}{3}\left[1 + \frac{z}{3}\right]^{-1}$

$f(z) = -\frac{3}{2}\left[1 - \frac{z}{2} + \frac{z^2}{4} - \frac{z^3}{8} + \cdots..\right] + \frac{5}{3}\left[1 - \frac{z}{3} + \frac{z^2}{9} - \frac{z^3}{27} + \cdots.\right]$

(ii) For $2 < |z| < 3$

$f(z) = -\frac{3}{z+2} + \frac{5}{3+z}$

$f(z) = -\frac{3}{z\left(1+\frac{2}{z}\right)} + \frac{5}{3\left(1+\frac{z}{3}\right)}$

$f(z) = -\frac{3}{z}\left[1 + \frac{2}{z}\right]^{-1} + \frac{5}{3}\left[1 + \frac{z}{3}\right]^{-1}$

$f(z) = -\frac{3}{z}\left[1 - \frac{2}{z} + \frac{4}{z^2} - \frac{8}{z^3} + \cdots..\right] + \frac{5}{3}\left[1 - \frac{z}{3} + \frac{z^2}{9} - \frac{z^3}{27} + \cdots.\right]$

(ii) For $|z| > 3$

$f(z) = -\frac{3}{z+2} + \frac{5}{z+3}$

$f(z) = -\frac{3}{z\left(1+\frac{2}{z}\right)} + \frac{5}{z\left(1+\frac{3}{z}\right)}$

$f(z) = -\frac{3}{z}\left[1 + \frac{2}{z}\right]^{-1} + \frac{5}{z}\left[1 + \frac{3}{z}\right]^{-1}$

$f(z) = -\frac{3}{z}\left[1 - \frac{2}{z} + \frac{4}{z^2} - \frac{8}{z^3} + \cdots..\right] + \frac{5}{z}\left[1 - \frac{3}{z} + \frac{9}{z^2} - \frac{27}{z^3} + \cdots.\right]$

35. Given $f(z)=\frac{1}{2(z+1)}-\frac{1}{2(z+3)}$ for $1<|z|<3$, the expansion of $f(z)$ is

[M22/Elect/2M]

Solution:

$f(z)=\frac{1}{2(z+1)}-\frac{1}{2(z+3)}$

For $1<|z|<3$

$f(z)=\frac{1}{2(z+1)}-\frac{1}{2(3+z)}$

$f(z)=\frac{1}{2z\left(1+\frac{1}{z}\right)}-\frac{1}{6\left(1+\frac{z}{3}\right)}$

$f(z)=\frac{1}{2z}\left[1+\frac{1}{z}\right]^{-1}-\frac{1}{6}\left[1+\frac{z}{3}\right]^{-1}$

$f(z)=\frac{1}{2z}\left[1-\frac{1}{z}+\frac{1}{z^2}-\frac{1}{z^3}+\cdots.\right]-\frac{1}{6}\left[1-\frac{z}{3}+\frac{z^2}{3^2}-\frac{z^3}{3^3}+\cdots\right]$

36. Find Laurent's series for the function $f(z) = \frac{1}{(z-1)(z-2)}$ in the regions

(i) $1 < |z-1| < 2$ (ii) $1 < |z-3| < 2$

Solution:

We have, $f(z) = \frac{1}{(z-1)(z-2)}$

Let $\frac{1}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$

$1 = A(z-2) + B(z-1)$

Put $z = 1$

$1 = A(1-2)$

$A = -1$

Put $z = 2$

$1 = B(2-1)$

$B = 1$

$f(z) = \frac{-1}{z-1} + \frac{1}{z-2}$

(i) $1 < |z-1| < 2$

$f(z) = \frac{-1}{z-1} + \frac{1}{z-2}$

Let $z - 1 = u$ i.e. $z = u + 1$

Thus, ROC becomes $1 < |u| < 2$

$f(z) = \frac{-1}{u} + \frac{1}{u+1-2}$

$f(z) = -\frac{1}{u} + \frac{1}{u-1}$

$f(z) = -\frac{1}{u} + \frac{1}{u-1}$

$f(z) = -\frac{1}{u} + \frac{1}{u\left(1-\frac{1}{u}\right)}$

$f(z) = -\frac{1}{u} + \frac{1}{u}\left[1 - \frac{1}{u}\right]^{-1}$

$f(z) = -\frac{1}{u} + \frac{1}{u}\left[1 + \frac{1}{u} + \frac{1}{u^2} + \frac{1}{u^3} + \cdots.\right]$

$f(z) = -\frac{1}{z-1} + \frac{1}{z-1}\left[1 + \frac{1}{z-1} + \frac{1}{(z-1)^2} + \frac{1}{(z-1)^3} + \cdots\right]$

The above is a Laurent's series in powers of $(z-1)$ or series about $z = 1$

(ii) $1 < |z-3| < 2$

$f(z) = -\frac{1}{z-1} + \frac{1}{z-2}$

Let $z - 3 = u$ i.e. $z = u + 3$

Thus, ROC becomes $1 < |u| < 2$

$f(z) = -\frac{1}{u+3-1} + \frac{1}{u+3-2}$

$f(z) = -\frac{1}{u+2} + \frac{1}{u+1}$

$f(z) = -\frac{1}{2+u} + \frac{1}{u+1}$

$f(z) = -\frac{1}{2\left(1+\frac{u}{2}\right)} + \frac{1}{u\left(1+\frac{1}{u}\right)}$

$f(z) = -\frac{1}{2}\left[1 + \frac{u}{2}\right]^{-1} + \frac{1}{u}\left[1 + \frac{1}{u}\right]^{-1}$

$$f(z) = -\frac{1}{2}\left[1 - \frac{u}{2} + \frac{u^2}{2^2} - \frac{u^3}{2^3} + \cdots\right] + \frac{1}{u}\left[1 - \frac{1}{u} + \frac{1}{u^2} - \frac{1}{u^3} + \cdots..\right]$$

$$f(z) = -\frac{1}{2}\left[1 - \frac{z-3}{2} + \frac{(z-3)^2}{2^2} - \frac{(z-3)^3}{2^3} + ..\right] + \frac{1}{z-3}\left[1 - \frac{1}{z-3} + \frac{1}{(z-3)^2} + \cdots.\right]$$

Laurent's series about $z = 3$ or in powers of $(z - 3)$

37. Find all possible Laurent's series expansion of $\frac{4z^2+2z-4}{z^3-4z}$ about $z=2$ and specify their domain of convergence.

Solution:

Let $\frac{4z^2+2z-4}{z(z^2-4)} = \frac{4z^2+2z-4}{z(z-2)(z+2)} = \frac{A}{z} + \frac{B}{z-2} + \frac{C}{z+2}$

$4z^2+2z-4 = A(z-2)(z+2) + Bz(z+2) + Cz(z-2)$

Putting $z=0$, we get

$0-4 = A(0-2)(0+2)$

$-4 = A(-4)$

$A = 1$

Putting $z=2$, we get

$4(4)+2(2)-4 = B2(2+2)$

$16 = B(8)$

$B = 2$

Putting $z=-2$, we get

$4(4)+2(-2)-4 = C(-2)(-2-2)$

$8 = C(8)$

$C = 1$

$f(z) = \frac{1}{z} + \frac{2}{z-2} + \frac{1}{z+2}$

Let $z-2=u$ i.e. $z=2+u$

$f(z) = \frac{1}{u+2} + \frac{2}{u} + \frac{1}{u+2+2}$

$f(z) = \frac{1}{u+2} + \frac{2}{u} + \frac{1}{u+4}$

For ROC, put $(u+2)u(u+4) = 0$

$u=-2, u=0, u=-4$

$|u|=2, |u|=0, |u|=4$

Thus, ROC is $0<|u|<2, 2<|u|<4, |u|>4$

(i) $0<|u|<2$

$f(z) = \frac{1}{2+u} + \frac{2}{u} + \frac{1}{4+u}$

$f(z) = \frac{1}{2\left(1+\frac{u}{2}\right)} + \frac{2}{u} + \frac{1}{4\left(1+\frac{u}{4}\right)}$

$f(z) = \frac{1}{2}\left[1+\frac{u}{2}\right]^{-1} + \frac{2}{u} + \frac{1}{4}\left[1+\frac{u}{4}\right]^{-1}$

$f(z) = \frac{1}{2}\left[1-\frac{u}{2}+\frac{u^2}{2^2}-\frac{u^3}{2^3}+\cdots\right] + \frac{2}{u} + \frac{1}{4}\left[1-\frac{u}{4}+\frac{u^2}{4^2}-\frac{u^3}{4^3}+\cdots\right]$

$f(z) = \frac{1}{2}\left[1-\frac{z-2}{2}+\frac{(z-2)^2}{2^2}-\frac{(z-2)^3}{2^3}+\cdots\right] + \frac{2}{(z-2)} + \frac{1}{4}\left[1-\frac{(z-2)}{4}+\frac{(z-2)^2}{4^2}-\frac{(z-2)^3}{4^3}+\cdots\right]$

(ii) $2<|u|<4$

$f(z) = \frac{1}{u+2} + \frac{2}{u} + \frac{1}{4+u}$

$f(z) = \frac{1}{u\left(1+\frac{2}{u}\right)} + \frac{2}{u} + \frac{1}{4\left(1+\frac{u}{4}\right)}$

$f(z) = \frac{1}{u}\left[1+\frac{2}{u}\right]^{-1} + \frac{2}{u} + \frac{1}{4}\left[1+\frac{u}{4}\right]^{-1}$

$f(z) = \frac{1}{u}\left[1-\frac{2}{u}+\frac{2^2}{u^2}-\frac{2^3}{u^3}+\cdots\right] + \frac{2}{u} + \frac{1}{4}\left[1-\frac{u}{4}+\frac{u^2}{4^2}-\frac{u^3}{4^3}+\cdots\right]$

$$f(z)=\frac{1}{(z-2)}\left[1-\frac{2}{z-2}+\frac{2^2}{(z-2)^2}-\cdots\right]+\frac{2}{(z-2)}+\frac{1}{4}\left[1-\frac{(z-2)}{4}+\frac{(z-2)^2}{4^2}-\frac{(z-2)^3}{4^3}+\cdots\right]$$

(iii) $|u|>4$

$$f(z)=\frac{1}{u+2}+\frac{2}{u}+\frac{1}{u+4}$$

$$f(z)=\frac{1}{u\left(1+\frac{2}{u}\right)}+\frac{2}{u}+\frac{1}{u\left(1+\frac{4}{u}\right)}$$

$$f(z)=\frac{1}{u}\left[1+\frac{2}{u}\right]^{-1}+\frac{2}{u}+\frac{1}{u}\left[1+\frac{4}{u}\right]^{-1}$$

$$f(z)=\frac{1}{u}\left[1-\frac{2}{u}+\frac{2^2}{u^2}-\frac{2^3}{u^3}+\cdots\right]+\frac{2}{u}+\frac{1}{u}\left[1-\frac{4}{u}+\frac{4^2}{u^2}-\frac{4^3}{u^3}+\cdots\right]$$

$$f(z)=\frac{1}{(z-2)}\left[1-\frac{2}{z-2}+\frac{2^2}{(z-2)^2}-\cdots\right]+\frac{2}{(z-2)}+\frac{1}{(z-2)}\left[1-\frac{4}{z-2}+\frac{4^2}{(z-2)^2}-\frac{4^3}{(z-2)^3}+\cdots\right]$$

38. Find Taylor's series expansion of $f(z) = \frac{1}{z(z-1)}$ about $z = 2$.

Solution:

Let $\frac{1}{z(z-1)} = \frac{A}{z} + \frac{B}{z-1}$

$1 = A(z-1) + Bz$

Putting $z = 0$, we get

$1 = A(0-1) + B(0)$

$1 = -A$

$A = -1$

Putting $z = 1$, we get

$1 = A(1-1) + B(1)$

$1 = B$

$B = 1$

Thus,

$f(z) = -\frac{1}{z} + \frac{1}{z-1}$

Put $z - 2 = u$ i.e. $z = u + 2$

$f(z) = -\frac{1}{u+2} + \frac{1}{u+2-1}$

$f(z) = -\frac{1}{u+2} + \frac{1}{u+1}$

$f(z) = -\frac{1}{2+u} + \frac{1}{1+u}$

$f(z) = -\frac{1}{2\left(1+\frac{u}{2}\right)} + \frac{1}{1+u}$

$f(z) = -\frac{1}{2}\left[1 + \frac{u}{2}\right]^{-1} + [1+u]^{-1}$

$f(z) = -\frac{1}{2}\left[1 - \frac{u}{2} + \frac{u^2}{2^2} - \frac{u^3}{2^3} + \ldots\ldots\right] + [1 - u + u^2 - u^3 + \ldots\ldots]$

$f(z) = -\frac{1}{2} + \frac{u}{4} - \frac{u^2}{8} + \frac{u^3}{16} + \cdots\ldots\ldots + 1 - u + u^2 - u^3 + \cdots\ldots$

$f(z) = \frac{1}{2} - \frac{3u}{4} + \frac{7u^2}{8} - \frac{15u^3}{16} + \ldots\ldots\ldots\ldots$

$f(z) = \frac{1}{2} - \frac{3(z-2)}{4} + \frac{7(z-2)^2}{8} - \frac{15(z-2)^3}{16} + \cdots..$

39. Obtain Taylor's expansion of $f(z) = \frac{1-z}{z^2}$ in powers of $(z-1)$

Solution:

$f(z) = \frac{1-z}{z^2}$

$f(z) = \frac{1}{z^2} - \frac{z}{z^2}$

$f(z) = \frac{1}{z^2} - \frac{1}{z}$

Let $z - 1 = u$ i.e $z = u + 1$

$f(z) = \frac{1}{(u+1)^2} - \frac{1}{u+1}$

$f(z) = \frac{1}{(1+u)^2} - \frac{1}{1+u}$

$f(z) = [1+u]^{-2} - [1+u]^{-1}$

$f(z) = [1 - 2u + 3u^2 - 4u^3 + \cdots.] - [1 - u + u^2 - u^3 + \cdots]$

$f(z) = 1 - 1 - 2u + u + 3u^2 - u^2 - 4u^3 + u^3 + \cdots$ ...

$f(z) = -u + 2u^2 - 3u^3 + \cdots$ ....

$f(z) = -(z-1) + 2(z-1)^2 - 3(z-1)^3 + \cdots$ ....

40. Obtain two distinct Laurent's series for $\frac{2z-3}{z^2-4z+3}$ in powers of $(z-4)$ indicating the regions of convergence.

[N13/M15/MechCivil/8M][M15/M17/ElexExtcElectBiomInst/8M]

Solution:

We have, $f(z)=\frac{2z-3}{z^2-4z+3}=\frac{2z-3}{(z-3)(z-1)}$

Let $\frac{2z-3}{(z-3)(z-1)}=\frac{A}{z-3}+\frac{B}{z-1}$

$2z-3=A(z-1)+B(z-3)$

On comparing the coefficients, we get

$A+B=2$

$-A-3B=-3$

On solving, we get

$A=\frac{3}{2}, B=\frac{1}{2}$

$f(z)=\frac{\frac{3}{2}}{z-3}+\frac{\frac{1}{2}}{z-1}$

Now, put $z-4=u$ i.e $z=u+4$

$\therefore f(z)=\frac{\frac{3}{2}}{u+4-3}+\frac{\frac{1}{2}}{u+4-1}$

$\therefore f(z)=\frac{\frac{3}{2}}{u+1}+\frac{\frac{1}{2}}{u+3}$

The ROCs are as follows (i) $|u|<1$ (ii) $1<|u|<3$ (iii) $|u|>3$

Now, Laurent's series are as follows

(a) For $1<|u|<3$ i.e. $1<|z-4|<3$

$$f(z)=\frac{\frac{3}{2}}{u+1}+\frac{\frac{1}{2}}{3+u}$$

$$f(z)=\frac{\frac{3}{2}}{u\left[1+\frac{1}{u}\right]}+\frac{\frac{1}{2}}{3\left[1+\frac{u}{3}\right]}$$

$$f(z)=\frac{3}{2u}\left[1+\frac{1}{u}\right]^{-1}+\frac{1}{6}\left[1+\frac{u}{3}\right]^{-1}$$

$$f(z)=\frac{3}{2u}\left[1-\frac{1}{u}+\frac{1}{u^2}-\frac{1}{u^3}+\cdots\ldots\right]+\frac{1}{6}\left[1-\frac{u}{3}+\frac{u^2}{9}-\frac{u^3}{27}+\cdots\ldots\right]$$

$$f(z)=\frac{3}{2}\left[\frac{1}{u}-\frac{1}{u^2}+\frac{1}{u^3}-\cdots\ldots..\right]+\frac{1}{6}\left[1-\frac{u}{3}+\frac{u^2}{9}-\cdots\ldots\right]$$

$$f(z)=\frac{3}{2}\left[\frac{1}{z-4}-\frac{1}{(z-4)^2}+\frac{1}{(z-4)^3}-\cdots..\right]+\frac{1}{6}\left[1-\frac{z-4}{3}+\frac{(z-4)^2}{9}-\cdots..\right]$$

(b) For $|u|>3$ i.e. $|z-4|>3$

$$f(z)=\frac{\frac{3}{2}}{u+1}+\frac{\frac{1}{2}}{u+3}$$

$$f(z)=\frac{\frac{3}{2}}{u\left[1+\frac{1}{u}\right]}+\frac{\frac{1}{2}}{u\left[1+\frac{3}{u}\right]}$$

$$f(z)=\frac{3}{2u}\left[1+\frac{1}{u}\right]^{-1}+\frac{1}{2u}\left[1+\frac{3}{u}\right]^{-1}$$

$$f(z)=\frac{3}{2u}\left[1-\frac{1}{u}+\frac{1}{u^2}-\frac{1}{u^3}+\cdots\ldots\right]+\frac{1}{2u}\left[1-\frac{3}{u}+\frac{9}{u^2}-\frac{27}{u^3}+\cdots\ldots\right]$$
$$f(z)=\frac{3}{2}\left[\frac{1}{u}-\frac{1}{u^2}+\frac{1}{u^3}-\cdots\ldots..\right]+\frac{1}{2}\left[\frac{1}{u}-\frac{3}{u^2}+\frac{9}{u^3}-\cdots\ldots\right]$$
$$f(z)=\frac{3}{2}\left[\frac{1}{z-4}-\frac{1}{(z-4)^2}+\frac{1}{(z-4)^3}-\cdots..\right]+\frac{1}{2}\left[\frac{1}{z-4}-\frac{3}{(z-4)^2}+\frac{9}{(z-4)^3}-\cdots..\right]$$

41. Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the regions (i) $|z| < 1$ (ii) $1 < |z-1| < 2$

[M22/Elex/5M]

Solution:

We have, $f(z) = \frac{1}{(z-1)(z-2)}$

Let $\frac{1}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$

$1 = A(z-2) + B(z-1)$

Comparing the coefficients, we get

$A + B = 0$

$-2A - B = 1$

On solving, we get

$A = -1, B = 1$

$f(z) = -\frac{1}{z-1} + \frac{1}{z-2}$

(i) $|z| < 1$

$f(z) = -\frac{1}{-1+z} + \frac{1}{-2+z}$

$f(z) = -\frac{1}{-[1-z]} + \frac{1}{-2\left[1-\frac{z}{2}\right]}$

$f(z) = [1-z]^{-1} - \frac{1}{2}\left[1-\frac{z}{2}\right]^{-1}$

$f(z) = [1 + z + z^2 + z^3 + \cdots.] - \frac{1}{2}\left[1 + \frac{z}{2} + \frac{z^2}{2^2} + \frac{z^3}{2^3} + \cdots\right]$

(ii) $1 < |z-1| < 2$

Let $z - 1 = u$ i.e. $z = u + 1$

Thus, ROC becomes $1 < |u| < 2$

$f(z) = -\frac{1}{u} + \frac{1}{u-1}$

$f(z) = -\frac{1}{u} + \frac{1}{u\left(1-\frac{1}{u}\right)}$

$f(z) = -\frac{1}{u} - \frac{1}{u}\left[1-\frac{1}{u}\right]^{-1}$

$f(z) = -\frac{1}{u} - \frac{1}{u}\left[1 + \frac{1}{u} + \frac{1}{u^2} + \frac{1}{u^3} + \cdots\right]$

$f(z) = -\frac{1}{z-1} - \frac{1}{z-1}\left[1 + \frac{1}{(z-1)} + \frac{1}{(z-1)^2} + \frac{1}{(z-1)^3} + \cdots..\right]$

42. Find all possible Laurent's expansions of $\frac{7z-2}{z(z-2)(z+1)}$ about $z=-1$ indicating the region of convergence.
[M14/MechCivil/8M][M15/CompIT/8M][N22/Elect/8M]
Solution:
We have, $f(z)=\frac{7z-2}{z(z-2)(z+1)}$
Let $\frac{7z-2}{z(z-2)(z+1)}=\frac{A}{z}+\frac{B}{z-2}+\frac{C}{z+1}$
$7z-2=A(z-2)(z+1)+Bz(z+1)+Cz(z-2)$
$7z-2=A(z^2-z-2)+B(z^2+z)+C(z^2-2z)$
On comparing the coefficients, we get
$A+B+C=0$
$-A+B-2C=7$
$-2A=-2$
On solving, we get
$A=1, B=2, C=-3$
$f(z)=\frac{1}{z}+\frac{2}{z-2}-\frac{3}{z+1}$
Now, put $z+1=u$ i.e $z=u-1$
$\therefore f(z)=\frac{1}{u-1}+\frac{2}{u-1-2}-\frac{3}{u-1+1}$
$\therefore f(z)=\frac{1}{u-1}+\frac{2}{u-3}-\frac{3}{u}$
The ROCs are as follows (i) $|u|<1$ (ii) $1<|u|<3$ (iii) $|u|>3$
(i) For $|u|<1$ i.e. $|z+1|<1$
$f(z)=\frac{1}{-1+u}+\frac{2}{-3+u}-\frac{3}{u}$
$f(z)=\frac{1}{-1(1-u)}+\frac{2}{-3\left(1-\frac{u}{3}\right)}-\frac{3}{u}$
$f(z)=-1[1-u]^{-1}-\frac{2}{3}\left[1-\frac{u}{3}\right]^{-1}-\frac{3}{u}$
$f(z)=-[1+u+u^2+u^3+\cdots.]-\frac{2}{3}\left[1+\frac{u}{3}+\frac{u^2}{9}+\frac{u^3}{27}+\cdots.\right]-\frac{3}{u}$
$f(z)=-[1+(z+1)+(z+1)^2+\cdots.]-\frac{2}{3}\left[1+\frac{z+1}{3}+\frac{(z+1)^2}{9}+\cdots\right]-\frac{3}{z+1}$
(ii) For $1<|u|<3$ i.e. $1<|z+1|<3$
$f(z)=\frac{1}{u-1}+\frac{2}{-3+u}-\frac{3}{u}$
$f(z)=\frac{1}{u\left(1-\frac{1}{u}\right)}+\frac{2}{-3\left(1-\frac{u}{3}\right)}-\frac{3}{u}$
$f(z)=\frac{1}{u}\left[1-\frac{1}{u}\right]^{-1}-\frac{2}{3}\left[1-\frac{u}{3}\right]^{-1}-\frac{3}{u}$
$f(z)=\frac{1}{u}\left[1+\frac{1}{u}+\frac{1}{u^2}+\frac{1}{u^3}+\cdots.\right]-\frac{2}{3}\left[1+\frac{u}{3}+\frac{u^2}{9}+\frac{u^3}{27}+\cdots.\right]-\frac{3}{u}$
$f(z)=\frac{1}{z+1}\left[1+\frac{1}{(z+1)}+\frac{1}{(z+1)^2}+\cdots.\right]-\frac{2}{3}\left[1+\frac{z+1}{3}+\frac{(z+1)^2}{9}+\cdots\right]-\frac{3}{z+1}$

(iii) For $|u| > 3$ i.e. $|z+1| > 3$

$$f(z) = \frac{1}{u-1} + \frac{2}{u-3} - \frac{3}{u}$$

$$f(z) = \frac{1}{u\left(1-\frac{1}{u}\right)} + \frac{2}{u\left(1-\frac{3}{u}\right)} - \frac{3}{u}$$

$$f(z) = \frac{1}{u}\left[1-\frac{1}{u}\right]^{-1} + \frac{2}{u}\left[1-\frac{3}{u}\right]^{-1} - \frac{3}{u}$$

$$f(z) = \frac{1}{u}\left[1+\frac{1}{u}+\frac{1}{u^2}+\frac{1}{u^3}+\cdots.\right] + \frac{2}{u}\left[1+\frac{3}{u}+\frac{9}{u^2}+\frac{27}{u^3}+\cdots.\right] - \frac{3}{u}$$

$$f(z) = \frac{1}{z+1}\left[1+\frac{1}{(z+1)}+\frac{1}{(z+1)^2}+\cdots.\right] + \frac{2}{(z+1)}\left[1+\frac{3}{z+1}+\frac{9}{(z+1)^2}+\cdots\right] - \frac{3}{z+1}$$

43. Obtain Laurent's series for the function $f(z)=\frac{-7z-2}{z(z-2)(z+1)}$ about $z=-1$

[M16/MechCivil/8M]

Solution:

We have, $f(z)=\frac{-7z-2}{z(z-2)(z+1)}$

Let $\frac{-7z-2}{z(z-2)(z+1)}=\frac{A}{z}+\frac{B}{z-2}+\frac{C}{z+1}$

$-7z-2=A(z-2)(z+1)+Bz(z+1)+Cz(z-2)$

$-7z-2=A(z^2-z-2)+B(z^2+z)+C(z^2-2z)$

On comparing the coefficients, we get

$A+B+C=0$

$-A+B-2C=-7$

$-2A=-2$

On solving, we get

$A=1, B=-\frac{8}{3}, C=\frac{5}{3}$

$f(z)=\frac{1}{z}+\frac{-\frac{8}{3}}{z-2}+\frac{\frac{5}{3}}{z+1}$

Now, put $z+1=u$ i.e $z=u-1$

$\therefore f(z)=\frac{1}{u-1}-\frac{\frac{8}{3}}{u-1-2}+\frac{\frac{5}{3}}{u-1+1}$

$\therefore f(z)=\frac{1}{u-1}-\frac{\frac{8}{3}}{u-3}+\frac{\frac{5}{3}}{u}$

The ROCs are as follows (i) $|u|<1$ (ii) $1<|u|<3$ (iii) $|u|>3$

(i) For $|u|<1$ i.e. $|z+1|<1$

$f(z)=\frac{1}{-1+u}-\frac{\frac{8}{3}}{-3+u}+\frac{\frac{5}{3}}{u}$

$f(z)=\frac{1}{-1(1-u)}-\frac{\frac{8}{3}}{-3\left(1-\frac{u}{3}\right)}+\frac{5}{3u}$

$f(z)=-1[1-u]^{-1}+\frac{8}{9}\left[1-\frac{u}{3}\right]^{-1}+\frac{5}{3u}$

$f(z)=-[1+u+u^2+u^3+\cdots.]+\frac{8}{9}\left[1+\frac{u}{3}+\frac{u^2}{9}+\frac{u^3}{27}+\cdots.\right]+\frac{5}{3u}$

$f(z)=-[1+(z+1)+(z+1)^2+\cdots]+\frac{8}{9}\left[1+\frac{z+1}{3}+\frac{(z+1)^2}{9}+\cdots\right]+\frac{5}{3(z+1)}$

(ii) For $1<|u|<3$ i.e. $1<|z+1|<3$

$f(z)=\frac{1}{u-1}-\frac{\frac{8}{3}}{-3+u}+\frac{\frac{5}{3}}{u}$

$f(z)=\frac{1}{u\left(1-\frac{1}{u}\right)}-\frac{\frac{8}{3}}{-3\left(1-\frac{u}{3}\right)}+\frac{5}{3u}$

$f(z)=\frac{1}{u}\left[1-\frac{1}{u}\right]^{-1}+\frac{8}{9}\left[1-\frac{u}{3}\right]^{-1}+\frac{5}{3u}$

$f(z)=\frac{1}{u}\left[1+\frac{1}{u}+\frac{1}{u^2}+\frac{1}{u^3}+\cdots.\right]+\frac{8}{9}\left[1+\frac{u}{3}+\frac{u^2}{9}+\frac{u^3}{27}+\cdots.\right]+\frac{5}{3u}$

$$f(z)=\frac{1}{z+1}\left[1+\frac{1}{(z+1)}+\frac{1}{(z+1)^2}+\cdots.\right]+\frac{8}{9}\left[1+\frac{z+1}{3}+\frac{(z+1)^2}{9}+\cdots\right]+\frac{5}{3(z+1)}$$

(iii) For $|u|>3$ i.e. $|z+1|>3$

$$f(z)=\frac{1}{u-1}-\frac{\frac{8}{3}}{u-3}+\frac{\frac{5}{3}}{u}$$

$$f(z)=\frac{1}{u\left(1-\frac{1}{u}\right)}-\frac{\frac{8}{3}}{u\left(1-\frac{3}{u}\right)}+\frac{5}{3u}$$

$$f(z)=\frac{1}{u}\left[1-\frac{1}{u}\right]^{-1}-\frac{8}{3u}\left[1-\frac{3}{u}\right]^{-1}+\frac{5}{3u}$$

$$f(z)=\frac{1}{u}\left[1+\frac{1}{u}+\frac{1}{u^2}+\frac{1}{u^3}+\cdots.\right]-\frac{8}{3u}\left[1+\frac{3}{u}+\frac{9}{u^2}+\frac{27}{u^3}+\cdots.\right]+\frac{5}{3u}$$

$$f(z)=\frac{1}{z+1}\left[1+\frac{1}{(z+1)}+\frac{1}{(z+1)^2}+\cdots\right]-\frac{8}{3(z+1)}\left[1+\frac{3}{z+1}+\frac{9}{(z+1)^2}+\cdots\right]+\frac{5}{3(z+1)}$$

44. Find all possible Laurent's expansion of $\frac{z}{(z-1)(z-2)}$ about $z = -2$ indicating region of convergence

[N16/ElexExtcElectBiomInst/8M][M19/Elect/8M]

Solution:

We have, $f(z) = \frac{z}{(z-1)(z-2)}$

Let $\frac{z}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$

$z = A(z-2) + B(z-1)$

On comparing the coefficients, we get

$A + B = 1$

$-2A - B = 0$

On solving, we get

$A = -1, B = 2$

$f(z) = \frac{2}{z-2} - \frac{1}{z-1}$

Now, put $z + 2 = u$ i.e $z = u - 2$

$\therefore f(z) = \frac{2}{u-2-2} - \frac{1}{u-2-1}$

$\therefore f(z) = \frac{2}{u-4} - \frac{1}{u-3}$

The ROCs are as follows (i) $|u| < 3$ (ii) $3 < |u| < 4$ (iii) $|u| > 4$

The Taylors Series is given by

(i) For $|u| < 3$ i.e. $|z+2| < 3$

$f(z) = \frac{2}{-4+u} - \frac{1}{-3+u}$

$f(z) = \frac{2}{-4\left(1-\frac{u}{4}\right)} - \frac{1}{-3\left(1-\frac{u}{3}\right)}$

$f(z) = -\frac{1}{2}\left[1 - \frac{u}{4}\right]^{-1} + \frac{1}{3}\left[1 - \frac{u}{3}\right]^{-1}$

$f(z) = -\frac{1}{2}\left[1 + \frac{u}{4} + \frac{u^2}{16} + \frac{u^3}{64} + \cdots.\right] + \frac{1}{3}\left[1 + \frac{u}{3} + \frac{u^2}{9} + \frac{u^3}{27} + \cdots.\right]$

$f(z) = -\frac{1}{2}\left[1 + \frac{(z+2)}{4} + \frac{(z+2)^2}{4} + \cdots\right] + \frac{1}{3}\left[1 + \frac{z+2}{3} + \frac{(z+2)^2}{9} + \cdots\right]$

The first Laurent's Series is given by

(ii) For $3 < |u| < 4$ i.e. $3 < |z+2| < 4$

$f(z) = \frac{2}{-4+u} - \frac{1}{u-3}$

$f(z) = \frac{2}{-4\left(1-\frac{u}{4}\right)} - \frac{1}{u\left(1-\frac{3}{u}\right)}$

$f(z) = -\frac{1}{2}\left[1 - \frac{u}{4}\right]^{-1} - \frac{1}{u}\left[1 - \frac{3}{u}\right]^{-1}$

$f(z) = -\frac{1}{2}\left[1 + \frac{u}{4} + \frac{u^2}{16} + \frac{u^3}{64} + \cdots.\right] - \frac{1}{u}\left[1 + \frac{3}{u} + \frac{9}{u^2} + \frac{27}{u^3} + \cdots.\right]$

$f(z) = -\frac{1}{2}\left[1 + \frac{(z+2)}{4} + \frac{(z+2)^2}{4} + \cdots\right] - \frac{1}{z+2}\left[1 + \frac{3}{z+2} + \frac{9}{(z+2)^2} + \cdots\right]$

The second Laurent's Series is given by

(iii) For $|u| > 4$ i.e. $|z+2| > 4$

$f(z) = \frac{2}{u-4} - \frac{1}{u-3}$

$f(z) = \frac{2}{u\left(1-\frac{4}{u}\right)} - \frac{1}{u\left(1-\frac{3}{u}\right)}$

$f(z) = \frac{2}{u}\left[1-\frac{4}{u}\right]^{-1} - \frac{1}{u}\left[1-\frac{3}{u}\right]^{-1}$

$f(z) = \frac{2}{u}\left[1+\frac{4}{u}+\frac{16}{u^2}+\frac{64}{u^3}+\cdots.\right] - \frac{1}{u}\left[1+\frac{3}{u}+\frac{9}{u^2}+\frac{27}{u^3}+\cdots.\right]$

$f(z) = \frac{2}{z+2}\left[1+\frac{4}{z+2}+\frac{16}{(z+2)^2}+\cdots\right] - \frac{1}{z+2}\left[1+\frac{3}{z+2}+\frac{9}{(z+2)^2}+\cdots\right]$

45. Find all Taylor's and Laurent's expansion for $f(z)=\frac{z}{(z-3)(z-4)}$ about $z=1$ indicating region of convergence

[M18/Inst/8M][N18/Biom/8M][N18/Elect/8M]

Solution:

We have, $f(z)=\frac{z}{(z-3)(z-4)}$

Let $\frac{z}{(z-3)(z-4)}=\frac{A}{z-4}+\frac{B}{z-3}$

$z=A(z-3)+B(z-4)$

Comparing the coefficients, we get

$A+B=1$

$-3A-4B=0$

On solving, we get

$A=4, B=-3$

$f(z)=\frac{4}{z-4}-\frac{3}{z-3}$

Now, put $z-1=u$ i.e $z=u+1$

$\therefore f(z)=\frac{4}{u+1-4}-\frac{3}{u+1-3}$

$\therefore f(z)=\frac{4}{u-3}-\frac{3}{u-2}$

The ROCs are as follows (i) $|u|<2$ (ii) $2<|u|<3$ (iii) $|u|>3$

The Taylors series is given by

(i) For $|u|<2$ i.e. $|z-1|<2$

$f(z)=\frac{4}{-3+u}-\frac{3}{-2+u}$

$f(z)=\frac{4}{-3\left(1-\frac{u}{3}\right)}-\frac{3}{-2\left(1-\frac{u}{2}\right)}$

$f(z)=-\frac{4}{3}\left[1-\frac{u}{3}\right]^{-1}+\frac{3}{2}\left[1-\frac{u}{2}\right]^{-1}$

$f(z)=-\frac{4}{3}\left[1+\frac{u}{3}+\frac{u^2}{9}+\frac{u^3}{27}+\cdots..\right]+\frac{3}{2}\left[1+\frac{u}{2}+\frac{u^2}{4}+\frac{u^3}{8}+\cdots.\right]$

$f(z)=-\frac{4}{3}\left[1+\frac{z-1}{3}+\frac{(z-1)^2}{9}+\cdots.\right]+\frac{3}{2}\left[1+\frac{z-1}{2}+\frac{(z-1)^2}{4}+\cdots..\right]$

The Laurent's series is given by

(ii) For $2<|u|<3$ i.e. $2<|z-1|<3$

$f(z)=\frac{4}{-3+u}-\frac{3}{u-2}$

$f(z)=\frac{4}{-3\left(1-\frac{u}{3}\right)}-\frac{3}{u\left(1-\frac{2}{u}\right)}$

$f(z)=-\frac{4}{3}\left[1-\frac{u}{3}\right]^{-1}-\frac{3}{u}\left[1-\frac{2}{u}\right]^{-1}$

$f(z)=-\frac{4}{3}\left[1+\frac{u}{3}+\frac{u^2}{9}+\frac{u^3}{27}+\cdots..\right]-\frac{3}{u}\left[1+\frac{2}{u}+\frac{2^2}{u^2}+\frac{2^3}{u^3}+\cdots.\right]$

$f(z)=-\frac{4}{3}\left[1+\frac{z-1}{3}+\frac{(z-1)^2}{9}+\cdots.\right]-\frac{3}{z-1}\left[1+\frac{2}{z-1}+\frac{2^2}{(z-1)^2}+\cdots..\right]$

The Laurent's series is given by

(ii) For $|u| > 3$ i.e. $|z-1| > 3$

$$f(z) = \frac{4}{u-3} - \frac{3}{u-2}$$

$$f(z) = \frac{4}{u\left(1-\frac{3}{u}\right)} - \frac{3}{u\left(1-\frac{2}{u}\right)}$$

$$f(z) = \frac{4}{u}\left[1-\frac{3}{u}\right]^{-1} - \frac{3}{u}\left[1-\frac{2}{u}\right]^{-1}$$

$$f(z) = \frac{4}{u}\left[1+\frac{3}{u}+\frac{3^2}{u^2}+\frac{3^3}{u^3}+\cdots..\right] - \frac{3}{u}\left[1+\frac{2}{u}+\frac{2^2}{u^2}+\frac{2^3}{u^3}+\cdots.\right]$$

$$f(z) = \frac{4}{z-1}\left[1+\frac{3}{z-1}+\frac{3^2}{(z-1)^2}+\cdots.\right] - \frac{3}{z-1}\left[1+\frac{2}{z-1}+\frac{2^2}{(z-1)^2}+\cdots..\right]$$

46. Find all Taylor's and Laurent's expansion for $f(z) = \frac{z}{(z-2)(z-3)}$ about $z = 1$ indicating region of convergence

[M18/M19/Biom/8M][M19/Inst/8M]

Solution:

We have, $f(z) = \frac{z}{(z-2)(z-3)}$

Let $\frac{z}{(z-2)(z-3)} = \frac{A}{z-2} + \frac{B}{z-3}$

$z = A(z-3) + B(z-2)$

Comparing the coefficients, we get

$A + B = 1$

$-3A - 2B = 0$

On solving, we get

$A = -2, B = 3$

$f(z) = \frac{-2}{z-2} + \frac{3}{z-3}$

Now, put $z - 1 = u$ i.e $z = u + 1$

$\therefore f(z) = \frac{-2}{u+1-2} + \frac{3}{u+1-3}$

$\therefore f(z) = \frac{-2}{u-1} + \frac{3}{u-2}$

The ROCs are as follows (i) $|u| < 1$ (ii) $1 < |u| < 2$ (iii) $|u| > 2$

The Taylors series is given by

(i) For $|u| < 1$ i.e. $|z-1| < 1$

$f(z) = \frac{-2}{-1+u} + \frac{3}{-2+u}$

$f(z) = \frac{-2}{-1(1-u)} + \frac{3}{-2\left(1-\frac{u}{2}\right)}$

$f(z) = 2[1-u]^{-1} - \frac{3}{2}\left[1-\frac{u}{2}\right]^{-1}$

$f(z) = 2[1 + u + u^2 + u^3 + \cdots..] - \frac{3}{2}\left[1 + \frac{u}{2} + \frac{u^2}{4} + \frac{u^3}{8} + \cdots.\right]$

$f(z) = 2[1 + (z-1) + (z-1)^2 + \cdots.] - \frac{3}{2}\left[1 + \frac{z-1}{2} + \frac{(z-1)^2}{4} + \cdots..\right]$

The Laurent's series is given by

(ii) For $1 < |u| < 2$ i.e. $1 < |z-1| < 2$

$f(z) = \frac{-2}{u-1} + \frac{3}{-2+u}$

$f(z) = \frac{-2}{u\left(1-\frac{1}{u}\right)} + \frac{3}{-2\left(1-\frac{u}{2}\right)}$

$f(z) = -\frac{2}{u}\left[1-\frac{1}{u}\right]^{-1} - \frac{3}{2}\left[1-\frac{u}{2}\right]^{-1}$

$f(z) = -\frac{2}{u}\left[1 + \frac{1}{u} + \frac{1}{u^2} + \frac{1}{u^3} + \cdots..\right] - \frac{3}{2}\left[1 + \frac{u}{2} + \frac{u^2}{4} + \frac{u^3}{8} + \cdots.\right]$

$f(z) = -\frac{2}{z-1}\left[1 + \frac{1}{(z-1)} + \frac{1}{(z-1)^2} + \cdots.\right] - \frac{3}{2}\left[1 + \frac{z-1}{2} + \frac{(z-1)^2}{4} + \cdots..\right]$

The Laurent's series is given by

(ii) For $|u| > 2$ i.e. $|z-1| > 2$

$$f(z) = \frac{-2}{u-1} + \frac{3}{u-2}$$

$$f(z) = \frac{-2}{u\left(1-\frac{1}{u}\right)} + \frac{3}{u\left(1-\frac{2}{u}\right)}$$

$$f(z) = -\frac{2}{u}\left[1-\frac{1}{u}\right]^{-1} + \frac{3}{u}\left[1-\frac{2}{u}\right]^{-1}$$

$$f(z) = -\frac{2}{u}\left[1+\frac{1}{u}+\frac{1}{u^2}+\frac{1}{u^3}+\cdots..\right] + \frac{3}{u}\left[1+\frac{2}{u}+\frac{2^2}{u^2}+\frac{2^3}{u^3}+\cdots.\right]$$

$$f(z) = -\frac{2}{z-1}\left[1+\frac{1}{(z-1)}+\frac{1}{(z-1)^2}+\cdots.\right] + \frac{3}{z-1}\left[1+\frac{2}{z-1}+\frac{2^2}{(z-1)^2}+\cdots..\right]$$

47. Obtain Laurent's series for $f(z) = \frac{1}{z(z+2)(z+1)}$ about $z = -2$

[M18/Comp/8M]

Solution:

We have, $f(z) = \frac{1}{z(z+2)(z+1)}$

Let $\frac{1}{z(z+2)(z+1)} = \frac{A}{z} + \frac{B}{z+2} + \frac{C}{z+1}$

$1 = A(z+2)(z+1) + Bz(z+1) + Cz(z+2)$

$1 = A(z^2+3z-2) + B(z^2+z) + C(z^2+2z)$

On comparing the coefficients, we get

$A + B + C = 0$

$3A + B + 2C = 0$

$-2A = 1$

On solving, we get

$A = -\frac{1}{2}, B = -\frac{1}{2}, C = 1$

$f(z) = \frac{-\frac{1}{2}}{z} + \frac{-\frac{1}{2}}{z+2} + \frac{1}{z+1}$

Now, put $z + 2 = u$ i.e $z = u - 2$

$\therefore f(z) = \frac{-\frac{1}{2}}{u-2} - \frac{\frac{1}{2}}{u} + \frac{1}{u-2+1}$

$\therefore f(z) = -\frac{1}{2u} - \frac{1}{2(u-2)} + \frac{1}{u-1}$

The ROCs are as follows (i) $|u| < 1$ (ii) $1 < |u| < 2$ (iii) $|u| > 2$

(i) For $|u| < 1$ i.e. $|z+2| < 1$

$f(z) = -\frac{1}{2u} - \frac{1}{2(-2+u)} + \frac{1}{-1+u}$

$f(z) = -\frac{1}{2u} + \frac{1}{4\left(1-\frac{u}{2}\right)} - \frac{1}{1-u}$

$f(z) = -\frac{1}{2u} + \frac{1}{4}\left[1 - \frac{u}{2}\right]^{-1} - [1-u]^{-1}$

$f(z) = -\frac{1}{2u} + \frac{1}{4}\left[1 + \frac{u}{2} + \frac{u^2}{2^2} + \frac{u^3}{2^3} + \cdots.\right] - [1 + u + u^2 + u^3 + \cdots]$

$f(z) = -\frac{1}{2(z+2)} + \frac{1}{4}\left[1 + \frac{z+2}{2} + \frac{(z+2)^2}{2^2} + \cdots\right] - [1 + (z+2) + (z+2)^2 + ..]$

(ii) For $1 < |u| < 2$ i.e. $1 < |z+2| < 2$

$f(z) = -\frac{1}{2u} - \frac{1}{2(-2+u)} + \frac{1}{u-1}$

$f(z) = -\frac{1}{2u} + \frac{1}{4\left(1-\frac{u}{2}\right)} + \frac{1}{u\left(1-\frac{1}{u}\right)}$

$f(z) = -\frac{1}{2u} + \frac{1}{4}\left[1 - \frac{u}{2}\right]^{-1} + \frac{1}{u}\left[1 - \frac{1}{u}\right]^{-1}$

$f(z) = -\frac{1}{2u} + \frac{1}{4}\left[1 + \frac{u}{2} + \frac{u^2}{2^2} + \frac{u^3}{2^3} + \cdots.\right] + \frac{1}{u}\left[1 + \frac{1}{u} + \frac{1}{u^2} + \frac{1}{u^3} + \cdots\right]$

$f(z) = -\frac{1}{2(z+2)} + \frac{1}{4}\left[1 + \frac{z+2}{2} + \frac{(z+2)^2}{2^2} + \cdots\right] + \frac{1}{z+2}\left[1 + \frac{1}{(z+2)} + \frac{1}{(z+2)^2} + ..\right]$

(iii) For $|u| > 2$ i.e. $|z+2| > 2$

$$f(z) = -\frac{1}{2u} - \frac{1}{2(u-2)} + \frac{1}{u-1}$$

$$f(z) = -\frac{1}{2u} + \frac{1}{2u\left(1-\frac{2}{u}\right)} + \frac{1}{u\left(1-\frac{1}{u}\right)}$$

$$f(z) = -\frac{1}{2u} + \frac{1}{2u}\left[1-\frac{2}{u}\right]^{-1} + \frac{1}{u}\left[1-\frac{1}{u}\right]^{-1}$$

$$f(z) = -\frac{1}{2u} + \frac{1}{2u}\left[1+\frac{2}{u}+\frac{2^2}{u^2}+\frac{2^3}{u^3}+\cdots.\right] + \frac{1}{u}\left[1+\frac{1}{u}+\frac{1}{u^2}+\frac{1}{u^3}+\cdots\right]$$

$$f(z) = -\frac{1}{2(z+2)} + \frac{1}{2(z+2)}\left[1+\frac{2}{z+2}+\frac{2^2}{(z+2)^2}+\cdots\right] + \frac{1}{z+2}\left[1+\frac{1}{(z+2)}+\frac{1}{(z+2)^2}+..\right]$$

48. Obtain Laurent's series expansion of $f(z)=\frac{4z+3}{z^2-z-6}$ at $z=1$

[N17/ElexExtcElectBiomInst/8M]

Solution:

We have, $f(z)=\frac{4z+3}{z^2-z-6}=\frac{4z+3}{(z-3)(z+2)}$

Let $\frac{4z+3}{(z-3)(z+2)}=\frac{A}{z+2}+\frac{B}{z-3}$

$4z+3=A(z-3)+B(z+2)$

Comparing the coefficients, we get

$A+B=4$

$-3A+2B=3$

On solving, we get

$A=1, B=3$

$f(z)=\frac{1}{z+2}+\frac{3}{z-3}$

Now, put $z-1=u$ i.e $z=u+1$

$\therefore f(z)=\frac{1}{u+1+2}+\frac{3}{u+1-3}$

$\therefore f(z)=\frac{1}{u+3}+\frac{3}{u-2}$

The ROCs are as follows (i) $|u|<2$ (ii) $2<|u|<3$ (iii) $|u|>3$

The Taylors series is given by

(i) For $|u|<2$ i.e. $|z-1|<2$

$f(z)=\frac{1}{3+u}+\frac{3}{-2+u}$

$f(z)=\frac{1}{3\left(1+\frac{u}{3}\right)}+\frac{3}{-2\left(1-\frac{u}{2}\right)}$

$f(z)=\frac{1}{3}\left[1+\frac{u}{3}\right]^{-1}-\frac{3}{2}\left[1-\frac{u}{2}\right]^{-1}$

$f(z)=\frac{1}{3}\left[1-\frac{u}{3}+\frac{u^2}{9}-\frac{u^3}{27}+\cdots..\right]-\frac{3}{2}\left[1-\frac{u}{2}+\frac{u^2}{4}-\frac{u^3}{8}+\cdots.\right]$

$f(z)=\frac{1}{3}\left[1-\frac{z-1}{3}+\frac{(z-1)^2}{9}-\cdots.\right]-\frac{3}{2}\left[1-\frac{z-1}{2}+\frac{(z-1)^2}{4}-\cdots..\right]$

The Laurent's series is given by

(ii) For $2<|u|<3$ i.e. $2<|z-1|<3$

$f(z)=\frac{1}{3+u}+\frac{3}{u-2}$

$f(z)=\frac{1}{3\left(1+\frac{u}{3}\right)}+\frac{3}{u\left(1-\frac{2}{u}\right)}$

$f(z)=\frac{1}{3}\left[1+\frac{u}{3}\right]^{-1}+\frac{3}{u}\left[1-\frac{2}{u}\right]^{-1}$

$f(z)=\frac{1}{3}\left[1-\frac{u}{3}+\frac{u^2}{9}-\frac{u^3}{27}+\cdots..\right]+\frac{3}{u}\left[1-\frac{2}{u}+\frac{2^2}{u^2}-\frac{2^3}{u^3}+\cdots.\right]$

$f(z)=\frac{1}{3}\left[1-\frac{z-1}{3}+\frac{(z-1)^2}{9}-\cdots.\right]+\frac{3}{(z-1)}\left[1-\frac{2}{z-1}+\frac{2^2}{(z-1)^2}-\cdots..\right]$

The Laurent's series is given by

(ii) For $|u| > 3$ i.e. $|z-1| > 3$

$$f(z) = \frac{1}{u+3} + \frac{3}{u-2}$$

$$f(z) = \frac{1}{u\left(1+\frac{3}{u}\right)} + \frac{3}{u\left(1-\frac{2}{u}\right)}$$

$$f(z) = \frac{1}{u}\left[1+\frac{3}{u}\right]^{-1} + \frac{3}{u}\left[1-\frac{2}{u}\right]^{-1}$$

$$f(z) = \frac{1}{u}\left[1-\frac{3}{u}+\frac{3^2}{u^2}-\frac{3^3}{u^3}+\cdots..\right] + \frac{3}{u}\left[1-\frac{2}{u}+\frac{2^2}{u^2}-\frac{2^3}{u^3}+\cdots.\right]$$

$$f(z) = \frac{1}{(z-1)}\left[1-\frac{3}{z-1}+\frac{3^2}{(z-1)^2}-\cdots.\right] + \frac{3}{(z-1)}\left[1-\frac{2}{z-1}+\frac{2^2}{(z-1)^2}-\cdots..\right]$$

49. Expand $\frac{z^2-1}{z^2+5z+6}$ around $z=1$

[N16/MechCivil/8M]

Solution:

We have, $f(z)=\frac{z^2-1}{z^2+5z+6}$

$f(z)=\frac{z^2+5z+6-5z-6-1}{z^2+5z+6}$

$f(z)=1+\frac{-5z-7}{z^2+5z+6}$

Let $\frac{-5z-7}{(z+2)(z+3)}=\frac{A}{z+2}+\frac{B}{z+3}$

$-5z-7=A(z+3)+B(z+2)$

Comparing the coefficients, we get

$A+B=-5$

$3A+2B=-7$

On solving, we get

$A=3, B=-8$

$f(z)=1+\frac{3}{z+2}-\frac{8}{z+3}$

Now, put $z-1=u$ i.e $z=u+1$

$\therefore f(z)=1+\frac{3}{u+1+2}-\frac{8}{u+1+3}$

$\therefore f(z)=1+\frac{3}{u+3}-\frac{8}{u+4}$

The ROCs are as follows (i) $|u|<3$ (ii) $3<|u|<4$ (iii) $|u|>4$

The Taylors series is given by

(i) For $|u|<3$ i.e. $|z-1|<3$

$f(z)=1+\frac{3}{3+u}-\frac{8}{4+u}$

$f(z)=1+\frac{3}{3\left(1+\frac{u}{3}\right)}-\frac{8}{4\left(1+\frac{u}{4}\right)}$

$f(z)=1+\left[1+\frac{u}{3}\right]^{-1}-2\left[1+\frac{u}{4}\right]^{-1}$

$f(z)=1+\left[1-\frac{u}{3}+\frac{u^2}{9}-\frac{u^3}{27}+\cdots..\right]-2\left[1-\frac{u}{4}+\frac{u^2}{16}-\frac{u^3}{64}+\cdots.\right]$

$f(z)=1+\left[1-\frac{z-1}{3}+\frac{(z-1)^2}{9}-\cdots.\right]-2\left[1-\frac{z-1}{4}+\frac{(z-1)^2}{16}-\cdots..\right]$

The Laurent's series is given by

(ii) For $3<|u|<4$ i.e. $3<|z-1|<4$

$f(z)=1+\frac{3}{u+3}-\frac{8}{4+u}$

$f(z)=1+\frac{3}{u\left(1+\frac{3}{u}\right)}-\frac{8}{4\left(1+\frac{u}{4}\right)}$

$f(z)=1+\frac{3}{u}\left[1+\frac{3}{u}\right]^{-1}-2\left[1+\frac{u}{4}\right]^{-1}$

$f(z)=1+\frac{3}{u}\left[1-\frac{3}{u}+\frac{9}{u^2}-\frac{27}{u^3}+\cdots..\right]-2\left[1-\frac{u}{4}+\frac{u^2}{16}-\frac{u^3}{64}+\cdots.\right]$

$f(z) = 1 + \frac{3}{z-1}\left[1 - \frac{3}{z-1} + \frac{9}{(z-1)^2} - \cdots.\right] - 2\left[1 - \frac{z-1}{4} + \frac{(z-1)^2}{16} - \cdots..\right]$

The Laurent's series is given by

(ii) For $|u| > 4$ i.e. $|z-1| > 4$

$f(z) = 1 + \frac{3}{u+3} - \frac{8}{u+4}$

$f(z) = 1 + \frac{3}{u\left(1+\frac{3}{u}\right)} - \frac{8}{u\left(1+\frac{4}{u}\right)}$

$f(z) = 1 + \frac{3}{u}\left[1 + \frac{3}{u}\right]^{-1} - \frac{8}{u}\left[1 + \frac{4}{u}\right]^{-1}$

$f(z) = 1 + \frac{3}{u}\left[1 - \frac{3}{u} + \frac{9}{u^2} - \frac{27}{u^3} + \cdots..\right] - \frac{8}{u}\left[1 - \frac{4}{u} + \frac{16}{u^2} - \frac{64}{u^3} + \cdots.\right]$

$f(z) = 1 + \frac{3}{z-1}\left[1 - \frac{3}{z-1} + \frac{9}{(z-1)^2} - \cdots.\right] - \frac{8}{z-1}\left[1 - \frac{4}{z-1} + \frac{16}{(z-1)^2} - \cdots..\right]$

50. Find all Taylor's and Laurent's expansion for $f(z) = \frac{1}{(z-1)(z-2)}$ about $z = 3$ indicating region of convergence

[N19/Elect/8M]

Solution:

We have, $f(z) = \frac{1}{(z-1)(z-2)}$

Let $\frac{1}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$

$1 = A(z-2) + B(z-1)$

On comparing the coefficients, we get

$A + B = 0$

$-2A - B = 1$

On solving, we get

$A = -1, B = 1$

$f(z) = \frac{-1}{z-1} + \frac{1}{z-2}$

Now, put $z - 3 = u$ i.e $z = u + 3$

$\therefore f(z) = \frac{-1}{u+3-1} + \frac{1}{u+3-2}$

$\therefore f(z) = \frac{-1}{u+2} + \frac{1}{u+1}$

The ROCs are as follows (i) $|u| < 1$ (ii) $1 < |u| < 2$ (iii) $|u| > 2$

The Taylors Series is given by

(i) For $|u| < 1$ i.e. $|z-3| < 1$

$f(z) = \frac{-1}{2+u} + \frac{1}{1+u}$

$f(z) = \frac{-1}{2\left(1+\frac{u}{2}\right)} + \frac{1}{1+u}$

$f(z) = -\frac{1}{2}\left[1 + \frac{u}{2}\right]^{-1} + [1+u]^{-1}$

$f(z) = -\frac{1}{2}\left[1 - \frac{u}{2} + \frac{u^2}{4} - \frac{u^3}{8} + \cdots.\right] + [1 - u + u^2 - u^3 + \cdots.]$

$f(z) = -\frac{1}{2}\left[1 - \frac{z-3}{2} + \frac{(z-3)^2}{4} - \cdots\right] + [1 - (z-3) + (z-3)^2 - \cdots]$

The first Laurent's Series is given by

(ii) For $1 < |u| < 2$ i.e. $1 < |z-3| < 2$

$f(z) = \frac{-1}{2+u} + \frac{1}{u+1}$

$f(z) = \frac{-1}{2\left(1+\frac{u}{2}\right)} + \frac{1}{u\left(1+\frac{1}{u}\right)}$

$f(z) = -\frac{1}{2}\left[1 + \frac{u}{2}\right]^{-1} + \frac{1}{u}\left[1 + \frac{1}{u}\right]^{-1}$

$f(z) = -\frac{1}{2}\left[1 - \frac{u}{2} + \frac{u^2}{4} - \frac{u^3}{8} + \cdots.\right] + \frac{1}{u}\left[1 - \frac{1}{u} + \frac{1}{u^2} - \frac{1}{u^3} + \cdots.\right]$

$f(z) = -\frac{1}{2}\left[1 - \frac{z-3}{2} + \frac{(z-3)^2}{4} - \cdots\right] + \frac{1}{z-3}\left[1 - \frac{1}{(z-3)} + \frac{1}{(z-3)^2} - \cdots\right]$

The second Laurent's Series is given by

(iii) For $|u| > 2$ i.e. $|z-3| > 2$

$$f(z) = \frac{-1}{u+2} + \frac{1}{u+1}$$

$$f(z) = \frac{-1}{u\left(1+\frac{2}{u}\right)} + \frac{1}{u\left(1+\frac{1}{u}\right)}$$

$$f(z) = -\frac{1}{u}\left[1+\frac{2}{u}\right]^{-1} + \frac{1}{u}\left[1+\frac{1}{u}\right]^{-1}$$

$$f(z) = -\frac{1}{u}\left[1-\frac{2}{u}+\frac{4}{u^2}-\frac{8}{u^3}+\cdots.\right] + \frac{1}{u}\left[1-\frac{1}{u}+\frac{1}{u^2}-\frac{1}{u^3}+\cdots.\right]$$

$$f(z) = -\frac{1}{(z-3)}\left[1-\frac{1}{z-3}+\frac{4}{(z-3)^2}-\cdots\right] + \frac{1}{z-3}\left[1-\frac{1}{(z-3)}+\frac{1}{(z-3)^2}-\cdots\right]$$

51. Obtain all possible Taylor's and Laurent's Series expansions about $z = 0$ for the function $f(z) = \frac{z}{z^2+3z+2}$ indicating the region of convergence

[D23/Extc/8M]

Solution:

We have, $f(z) = \frac{z}{z^2+3z+2} = \frac{z}{(z+1)(z+2)}$

Let $\frac{z}{(z+1)(z+2)} = \frac{A}{z+1} + \frac{B}{z+2}$

$z = A(z+2) + B(z+1)$

Comparing the coefficients, we get

$A + B = 1$

$2A + B = 0$

On solving, we get

$A = -1, B = 2$

$f(z) = -\frac{1}{z+1} + \frac{2}{z+2}$

For ROC, put $z + 1 = 0, z + 2 = 0$

$z = -1, z = -2$

$|z| = 1, |z| = 2$

Thus, the ROC are (i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$

(i) $|z| < 1$

$f(z) = -\frac{1}{1+z} + \frac{2}{2+z}$

$f(z) = -\frac{1}{[1+z]} + \frac{2}{2\left[1+\frac{z}{2}\right]}$

$f(z) = -[1+z]^{-1} + \left[1+\frac{z}{2}\right]^{-1}$

$f(z) = [1 - z + z^2 - z^3 + \cdots.] + \left[1 - \frac{z}{2} + \frac{z^2}{2^2} - \frac{z^3}{2^3} + \cdots\right]$

(ii) $1 < |z| < 2$

$f(z) = -\frac{1}{z+1} + \frac{2}{2+z}$

$f(z) = -\frac{1}{z\left[1+\frac{1}{z}\right]} + \frac{2}{2\left[1+\frac{z}{2}\right]}$

$f(z) = -\frac{1}{z}\left[1+\frac{1}{z}\right]^{-1} + \left[1+\frac{z}{2}\right]^{-1}$

$f(z) = -\frac{1}{z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots.\right] + \left[1 - \frac{z}{2} + \frac{z^2}{2^2} - \frac{z^3}{2^3} + \cdots\right]$

(iii) $|z| > 2$

$f(z) = -\frac{1}{z+1} + \frac{2}{z+2}$

$f(z) = -\frac{1}{z\left[1+\frac{1}{z}\right]} + \frac{2}{z\left[1+\frac{2}{z}\right]}$

$f(z) = -\frac{1}{z}\left[1+\frac{1}{z}\right]^{-1} + \frac{2}{z}\left[1+\frac{2}{z}\right]^{-1}$

$f(z) = -\frac{1}{z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \cdots.\right] + \frac{2}{z}\left[1 - \frac{2}{z} + \frac{2^2}{z^2} - \frac{2^3}{z^3} + \cdots\right]$

52. Find all possible Laurent's Series expansions of $f(z) = \frac{1}{(z-1)(z+2)}$ about $z = 0$ indicating the region of convergence in each case

[D23/CompITAI/8M]

Solution:

We have, $f(z) = \frac{1}{(z-1)(z+2)}$

Let $\frac{1}{(z-1)(z+2)} = \frac{A}{z-1} + \frac{B}{z+2}$

$1 = A(z+2) + B(z-1)$

Comparing the coefficients, we get

$A + B = 0$

$2A - B = 1$

On solving, we get

$A = \frac{1}{3}, B = -\frac{1}{3}$

$f(z) = \frac{\frac{1}{3}}{z-1} - \frac{\frac{1}{3}}{z+2}$

For ROC, put $z - 1 = 0, z + 2 = 0$

$z = 1, z = -2$

$|z| = 1, |z| = 2$

Thus, the ROC are (i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$

(i) $|z| < 1$

$f(z) = \frac{\frac{1}{3}}{-1+z} - \frac{\frac{1}{3}}{2+z}$

$f(z) = \frac{\frac{1}{3}}{-[1-z]} - \frac{\frac{1}{3}}{2\left[1+\frac{z}{2}\right]}$

$f(z) = -\frac{1}{3}[1-z]^{-1} - \frac{1}{6}\left[1+\frac{z}{2}\right]^{-1}$

$f(z) = -\frac{1}{3}[1 + z + z^2 + z^3 + \cdots.] - \frac{1}{6}\left[1 - \frac{z}{2} + \frac{z^2}{2^2} - \frac{z^3}{2^3} + \cdots\right]$

(ii) $1 < |z| < 2$

$f(z) = \frac{\frac{1}{3}}{z-1} - \frac{\frac{1}{3}}{2+z}$

$f(z) = \frac{\frac{1}{3}}{z\left[1-\frac{1}{z}\right]} - \frac{\frac{1}{3}}{2\left[1+\frac{z}{2}\right]}$

$f(z) = \frac{1}{3z}\left[1-\frac{1}{z}\right]^{-1} - \frac{1}{6}\left[1+\frac{z}{2}\right]^{-1}$

$f(z) = \frac{1}{3z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots.\right] - \frac{1}{6}\left[1 - \frac{z}{2} + \frac{z^2}{2^2} - \frac{z^3}{2^3} + \cdots\right]$

(iii) $|z| > 2$

$f(z) = \frac{\frac{1}{3}}{z-1} - \frac{\frac{1}{3}}{z+2}$

$f(z) = \frac{\frac{1}{3}}{z\left[1-\frac{1}{z}\right]} - \frac{\frac{1}{3}}{z\left[1+\frac{2}{z}\right]}$

$$f(z) = \frac{1}{3z}\left[1 - \frac{1}{z}\right]^{-1} - \frac{1}{3z}\left[1 + \frac{2}{z}\right]^{-1}$$
$$f(z) = \frac{1}{3z}\left[1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots.\right] - \frac{1}{3z}\left[1 - \frac{2}{z} + \frac{2^2}{z^2} - \frac{2^3}{z^3} + \cdots\right]$$

53. Find the Laurent series expansion of $\frac{2}{(z+1)(z+3)}$ convergent in the region
i) $|z| < 1$ ii) $|z+1| > 2$
[D23/ElectECS/6M]
Solution:
We have, $f(z) = \frac{2}{(z+1)(z+3)}$
Let $\frac{2}{(z+1)(z+3)} = \frac{A}{z+1} + \frac{B}{z+3}$
$2 = A(z+3) + B(z+1)$
On comparing the coefficients, we get
$A + B = 0$
$3A + B = 2$
On solving, we get
$A = 1, B = -1$
$f(z) = \frac{1}{z+1} - \frac{1}{z+3}$
(i) For $|z| < 1$
$f(z) = \frac{1}{1+z} - \frac{1}{3+z}$
$f(z) = \frac{1}{1+z} - \frac{1}{3\left(1+\frac{z}{3}\right)}$
$f(z) = [1+z]^{-1} - \frac{1}{3}\left[1 + \frac{z}{3}\right]^{-1}$
$f(z) = [1 - z + z^2 - z^3 + \cdots .] - \frac{1}{3}\left[1 - \frac{z}{3} + \frac{z^2}{9} - \frac{z^3}{27} + \cdots .\right]$
(iii) For $|z+1| > 2$
$f(z) = \frac{1}{z+1} - \frac{1}{z+3}$
Let $z + 1 = u$ i.e. $z = u - 1$
Thus, for $|u| > 2$
$f(z) = \frac{1}{u} - \frac{1}{u+2} = \frac{1}{u} - \frac{1}{u\left(1+\frac{2}{u}\right)}$
$f(z) = \frac{1}{u} - \frac{1}{u}\left[1 + \frac{2}{u}\right]^{-1}$
$f(z) = \frac{1}{u} - \frac{1}{u}\left[1 - \frac{2}{u} + \frac{4}{u^2} - \frac{8}{u^3} + \cdots\right]$
$f(z) = \frac{1}{z+1} - \frac{1}{z+1}\left[1 - \frac{2}{(z+1)} + \frac{4}{(z+1)^2} - \frac{8}{(z+1)^3} + \cdots\right]$